4

Multiuser environment: code division multiple access

4.1 Multiuser systems and the multiple access problem

Many modern wireless systems are of multiuser type. In a *multiuser* system multiple communication links are arranged within the total time–frequency resource so that every individual user is allowed to transmit or receive his specific data in parallel with the others and independently of them. An instructive example of a multiuser system where a single transmitter transmits data to multiple users is the downlink of a satellite system or of a cellular terrestrial system. Each user receiver in such a system should be able to filter out the data addressed to it individually from the observed signal containing data sent to many users. Another case is the uplink of a satellite or a terrestrial cellular system, where there are a number of parallel transmitters and the only receiver should separate and detect the data of each individual user in the resulting observed signal.

In designing any multiuser system the principal issue is how to provide *multiple access*, i.e. the ability for many subscribers to use the communication channel simultaneously with minimal mutual interference. To describe this problem mathematically, suppose that the kth user's data form a sequence $\mathbf{b}_k = (b_{k,0}, b_{k,1}, \ldots)$, where $b_{k,i}$ stands for the ith symbol in the data stream of the kth user. This sequence in one way or another modulates the specific kth user's signal $s_k(t)$, producing the modulated signal $s_k(t; \mathbf{b}_k)$. Passing through the channel every such signal may acquire amplitude A_k and time delay τ_k and is summed with the signals of other users so that the overall or *group* signal reaching a receiver is:

$$s(t; \mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_K) = \sum_{k=1}^{K} A_k s_k(t - \tau_k; \mathbf{b}_k)$$

where K is the number of active, i.e. actively transmitting, users, and arguments after the semicolon in the group signal stress its dependence on data of all active users.

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Certainly, the group signal is accompanied by channel noise $n(t)$ and the resultant observation is:

$$y(t) = s(t; \mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_K) + n(t) = \sum_{k=1}^{K} A_k s_k(t - \tau_k; \mathbf{b}_k) + n(t) \tag{4.1}$$

The receiver should retrieve the user's data from the observation $y(t)$. According to the general ideas presented in Section 2.1, the key role in decisions on the received data $\mathbf{b}_k,\ k = 1, 2, \ldots, K$ for the AWGN channel belongs to the (squared) Euclidean distance between observation $y(t)$ and various copies of the group signal $s(t; \mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_K)$ corresponding to all possible combinations of data of K users:

$$d^2(\mathbf{s}, \mathbf{y}) = \int_0^T [y(t) - s(t; \mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_K)]^2 \, \mathrm{d}t \tag{4.2}$$

Substituting (4.1) into (4.2) and opening the brackets leads to the equation:

$$d^2(\mathbf{s}, \mathbf{y}) = \|\mathbf{y}\|^2 - 2\sum_{k=1}^{K} A_k z_k(\mathbf{b}_k) + \sum_{k=1}^{K}\sum_{l=1}^{K} A_k A_l \int_0^T s_k(t - \tau_k; \mathbf{b}_k) s_l(t - \tau_l; \mathbf{b}_l) \, \mathrm{d}t \tag{4.3}$$

where $z_k(\mathbf{b}_k)$ is the correlation (inner product) of the observation $y(t)$ and the kth user's signal modulated by the data sequence $\mathbf{b}_k$ and delayed by τ_k:

$$z_k(\mathbf{b}_k) = \int_0^T y(t) s_k(t - \tau_k; \mathbf{b}_k) \, \mathrm{d}t \tag{4.4}$$

Typically, estimating intensities and delays of all user signals precedes the decision on data sequences so that parameters $A_k,\ \tau_k,\ k = 1, 2, \ldots, K$ in (4.3) and (4.4) may be assumed known precisely. Then the optimal (ML or minimum distance) strategy of recovering user data consists in substitution of all possible realizations of the sequences $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_K$ in (4.3) and selecting those of them which jointly minimize the squared distance (4.3).[1] Such a decision rule, called *multiuser detection*, may appear quite impractical in a typical situation where the number of users K is measured in the tens or more. As an example take the simplest case of a synchronous system with zero mutual delays $\tau_k = 0, k = 1, 2, \ldots, K$ and binary data transmission. With the observation interval spanning only one bit, retrieving the individual bits of $K = 40$ users would require testing $2^{40} > 10^{12}$ bit patterns of all users, which looks absolutely infeasible from an implementation standpoint. We will revisit the issue of multiuser detection in Chapters 7 and 10.

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[1] This rule remains adequate even if the receiver is intended to recover only the individual (kth) user information sequence, as e.g. takes place in the downlink of a cellular mobile system. After estimating data of all users the receiver just discards the unnecessary data of all users but the kth one.

The so-called *conventional* or *single-user* receiver realizes an alternative decision rule estimating each of the data sequences $\mathbf{b}_k$ separately by maximizing the correlation (4.4). It is evident that this strategy coincides with the optimal (multiuser) one if and only if the third term of (4.3) does not depend on data sequences $\mathbf{b}_k,\ k = 1, 2, \ldots, K$ at all. To meet the latter condition, one can use the modulation scheme possessing the following properties: (a) the user's signal energy does not depend on transmitted data (PSK, FSK); (b) all user signals are orthogonal regardless of transmitted data. Both these requirements are expressed by the equation:

$$\int_0^T s_k(t-\tau_k;\mathbf{b}_k)s_l(t-\tau_l;\mathbf{b}_l)\,\mathrm{d}t = E\delta_{kl} \tag{4.5}$$

Calling this multiple access mode orthogonal and returning to the material of Sections 2.3 and 2.4, we recollect that the maximal number of orthogonal signals is limited by the total signal space dimension, and within the total bandwidth W_t and time resource T_t no more than $2W_tT_t$ bandpass orthogonal signals may exist. To derive from this the maximal available number of users in the orthogonal multiple access scheme, let us limit ourselves to M-ary PSK digital data transmission with fixed rate R bps. Assuming that all user signals should be orthogonal on the time interval equal to the M-ary symbol duration, we arrive at the equation $T_t = (\log_2 M)/R$. Therefore, the maximal signal space dimension is $2W_tT_t = (2W_t \log_2 M)/R$. When $M = 2$ (BPSK), each user occupies only a one-dimensional subspace of the signal space, since only two antipodal pulses (i.e. two collinear vectors) are necessary to transmit one bit (see Figure 2.5a). In this case the maximal number of users coincides with the total signal space dimension. With $M > 2$ each user needs a two-dimensional subspace (i.e. a plane; see Figure 2.6c), and all those subspaces should be orthogonal according to (4.5), so that the maximal number of users becomes two times smaller than the total signal space dimension. Combining these results gives the upper bound of the maximal number of users in the orthogonal multiple access scheme:

$$K = \begin{cases} \dfrac{2W_t}{R}, M = 2 \\ \dfrac{W_t \log_2 M}{R}, M > 2 \end{cases} \tag{4.6}$$

In the following three sections we discuss briefly traditional ways of carrying orthogonal multiple access into effect.

4.2 Frequency division multiple access

One of the simplest ways to fulfil requirement (4.5) is employing user signals whose spectra do not overlap. The idea is fully allied to that of frequency-shift orthogonal coding discussed in Section 2.7.2. This multiple access mode called *frequency division multiple access* (FDMA) is illustrated by Figure 4.1a. If M-ary PSK is used for data transmission at the rate R, the data symbol duration is $T_t = (\log_2 M)/R$ so that each

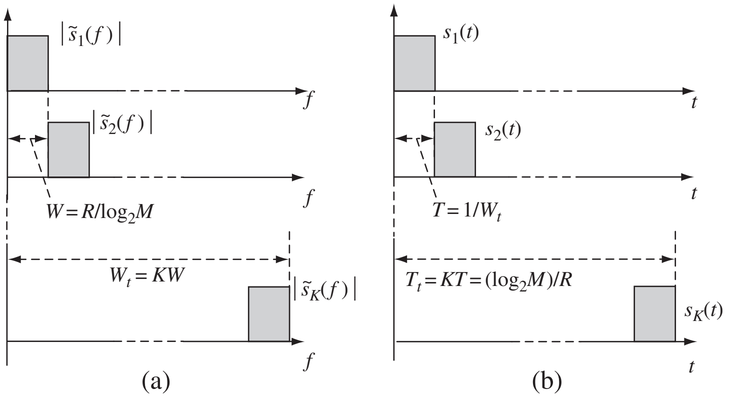

Figure 4.1 Frequency (a) and time (b) division multiple access

user's signal occupies bandwidth no smaller than $W = 1/T_t = R/\log_2 M$. Then the total allowed bandwidth W_t could accommodate no greater than $W_t/W = (W_t \log_2 M)/R$ non-overlapping spectra. This is exactly the maximal number of users if $M > 2$, which is the case shown in Figure 4.1a. When $M = 2$ and phase coherence is available each of those spectra may be utilized by two users whose carriers differ by only quadrature phase shift. As a result, the potential number of users in the FDMA scheme is subject to bound (4.6). In practice, non-ideal filtering, master clock generator drifts and Doppler frequency shifts may cause partial overlapping of the adjacent spectra, i.e. mutual interference between different user signals. To neutralize these effects and preserve separation of user signals a system designer is often forced to introduce guard frequency intervals between adjacent spectra, which decrease the achievable number of users as compared to bound (4.6).

FDMA is the oldest and classical multiple access mode commonly used in both analog and digital wireless systems (radio broadcasting, TV, mobile radio etc.). Non-overlapping spectra secure orthogonality, and hence separability of the user signals, regardless not only of data but also of time delays, thanks to which no synchronization between user signals is required. This fact is frequently referred to as a serious advantage of FDMA (see Section 4.5 for more detail).

4.3 Time division multiple access

Another popular orthogonal multiple access scheme is *time division multiple access* (TDMA), in which user signals do not overlap in the time domain (Figure 4.1b). The idea is again borrowed from time-shift orthogonal coding (see Section 2.7.1). Specifically to the case of M-ary PSK, it means that the whole available time resource $T_t = (\log_2 M)/R$ (in TDMA systems it is often called a *frame*) is divided into non-overlapping slots of duration T. If $M > 2$ (this case is shown in Figure 4.1b) every slot may be used by only one user and the duration of the data symbol transmitted by it cannot be smaller than the inverse total bandwidth $1/W_t$. Therefore, the total number of users is limited by the figure $T_t/T = (W_t \log_2 M)/R$. When $M = 2$ in the phase-coherent case two users may use the same slot having quadrature shifted carriers. As a result we again come to the bound (4.6) for the maximal value of K, demonstrating the theoretical equivalence of FDMA and TDMA in terms of the potential number of users accommodated.

TDMA mode has found application in various systems, e.g. in 2G mobile radio (GSM, IS-136 etc.). Although its simplicity is superficially attractive, limiting factors should be mentioned too. First, every user's signal occupies only a Kth (or perhaps a $K/2$th) part of the frame, which entails increasing peak-power K (or $K/2$) times as compared to the case of continuous emission in order to preserve the necessary signal energy, i.e. SNR. Implementation problems related to this have been repeatedly pointed out before. Second, strict synchronization is necessary between user signals at the receiver input, since otherwise they will overlap and create mutual interference. At the same time, in systems with migrating users, like the uplink of a mobile telephone, the lengths of the propagation paths between the user transmitters and the central station receiver are changing continually and over a wide range. Clearly, synchronization of the user signals at the receiver input in such situations, although possible in principle, might appear problematic technologically. The conventional way of getting around these obstructions consists in introducing guard time intervals between adjacent user signals, preventing them from being superimposed on each other within the whole range of variation of their delays. Rather commonly the guard intervals appear significant and may dramatically reduce the number of users compared to the upper bound (4.6). The severity of the problem is typically alleviated when an individual user's slot carries not a single data symbol (e.g. a bit), but, instead, a burst of n_b symbols. Then guard intervals are necessary for separating only the bursts of different users, which entails n_b times smaller guard-time overhead. On the other hand, pauses between the successive bursts of the same user become n_b times longer, too. In many systems (such as mobile telephone, where continuous speech exchange should be maintained) long pauses are not acceptable, and this frequently puts a tough limit on the length of bursts.

For these reasons, 'pure' TDMA is not often encountered in practice. For example, the 2G mobile telephone standards combine TDMA and FDMA.

4.4 Synchronous code division multiple access

Both FDMA and TDMA distribute the total available time–frequency resource between different users so that each user utilizes only his 'personal', user-specific fraction of it and no users share common fractions. In FDMA this fragmentation is done in the frequency domain (Figure 4.2a), and at the kth user's disposal are the whole time resource ($T = T_t$) but only part W of the total frequency resource W_t. When the maximal number of users is a top priority, $W = 1/T \sim W_t/K$. Splitting the time domain in TDMA (Figure 4.2b) makes it possible for the single user to occupy the whole available frequency range ($W = W_t$) but only part of the total time frame ($T = 1/W \sim T_t/K$). If the number of users needs to be maximized the resource fragmentation in both these orthogonal multiple access schemes makes each user signal plain since its time frequency product $WT = 1$.

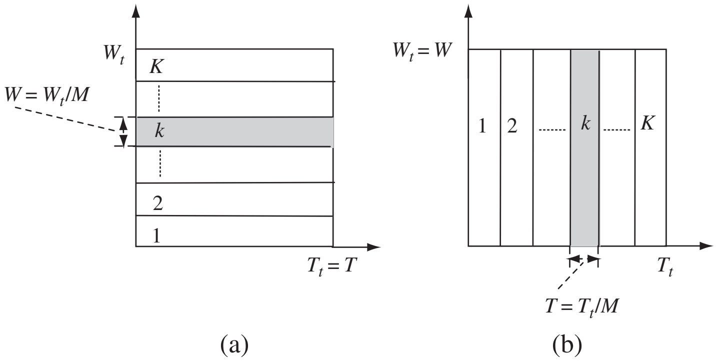

Figure 4.2 Resource distribution in FDMA (a) and TDMA (b)

On the other hand, with a large necessary number of users K the total time–frequency product has to be large too ($W_tT_t \gg 1$; see (4.6)), and if every user's signal occupied both the total available bandwidth ($W = W_t$) and time interval ($T = T_t$) we would have an orthogonal multiple access scheme in which all user signals are spread spectrum ones. Such a multiuser system would enjoy all the advantages of spread spectrum technology studied in the previous chapter.

Let us assume that the transmission may be arranged in a manner providing zero mutual delays between all user signals at the receiver input. Then without sacrificing generality all the absolute delays can be set equal to zero: $\tau_k = 0,\ k = 1, 2, \ldots, K$. Take an arbitrary family of W_tT_t orthogonal spread spectrum signals (see Section 2.7.3), e.g. Walsh functions, and employ each of them as a user signal for M-ary PSK data transmission. An individual spread spectrum signal assigned to the kth user is called the kth *signature*. Every signature occupies the total bandwidth W_t and the total time frame T_t (Figure 4.3), transmitting $\log_2 M$ bits of data over the interval T_t. If $M > 2$ this multiple access mode may serve up to $K = WT = W_tT_t = (W_t \log_2 M)/R$ users, while BPSK allows doubling of K by permitting two different users to exploit quadrature-phase-shifted copies of the same signature. Obviously, we again have the maximal possible number of users determined by (4.6), exactly as for both FDMA and TDMA.

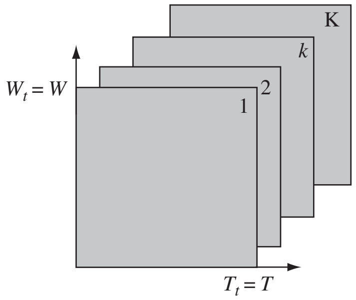

Figure 4.3 Resource utilization in spread-spectrum orthogonal multiple access

In the considered multiple access mode an appropriate signature encoding provides orthogonality of the user signals instead of fragmentation of the time or frequency domain. That is why it has the name *code division multiple access* (CDMA). The advantages of CDMA compared to classical FDMA and TDMA (jamming immunity, low detection probability, opportunity to involve the RAKE algorithm etc) follow automatically from the spread spectrum nature of CDMA signatures. At the same time, signature synchronism is critical for their orthogonality and user separation at the receiving side. In order to distinguish this version of CDMA from the one to be studied in the next section, the term *synchronous* CDMA (S-CDMA) is used. Synchronous mode is easily achievable in systems where only a single transmitter (like the base station of a cellular network) transmits simultaneously individual data streams, each being addressed to a specific user (e.g. mobile station). That is why S-CDMA constitutes the basis of the physical layer of downlinks in 2G (IS-95) and 3G (UMTS, cdma2000) CDMA cellular networks. In parallel, the core idea of S-CDMA is used in both downlink and uplink of 3G standards for arranging the so-called multi-code transmission (see Section 11.3).

4.5 Asynchronous CDMA

A situation typical of numerous applications is where delays τ_k may change over a wide range, making synchronization of signatures at the receiver input problematic or even impossible. An instructive example of this is an uplink of a mobile cellular system where users migrate over the cell, as a result of which the distances between them and the base station change constantly, as do the arrival times of the user signals at the base station receiver. In principle, each user knowing his instantaneous location relative to the base station, and therefore propagation delay τ_k, is capable of transmitting his signal with an advance τ_k. Thereby all path delays will be compensated for and all the user signals will be synchronized at the base station receiver. This operational mode, however, places excessive demands on the complexity of equipment, and in many cases can hardly be regarded as commercially viable.

Let us analyse the consequence of the asynchronous character of the received user signals. First of all, is it possible to preserve the orthogonality of signals in a wide range of mutual time shifts? Take two signals $u(t)$ and $v(t)$ and calculate their cross-correlation function (CCF) $R_{uv}(\tau)$, i.e. the inner product of $u(t)$ and a copy of $v(t)$ time-shifted by τ as a function of the argument τ:

$$R_{uv}(\tau) = \int_{-\infty}^{\infty} u(t)v(t-\tau)\,\mathrm{d}t$$

Applying the Parseval theorem gives:

$$R_{uv}(\tau) = \int_{-\infty}^{\infty} \tilde{u}(f)\tilde{v}^*(f)\exp(-j2\pi f\tau)\,\mathrm{d}f$$

If the goal is signal orthogonality independently of mutual delay τ, the equality $R_{uv}(\tau) = 0$ should hold under all values τ, which due to the Fourier transform linearity is possible if and only if $\tilde{u}(f)\tilde{v}(f) = 0$ everywhere in the frequency domain. This tells us that two signals are orthogonal under arbitrary time shifts if and only if their spectra do not overlap. But the multiple access scheme with non-overlapping spectra is FDMA! Hence, asynchronous orthogonal multiple access is realizable only with FDMA, which is often proclaimed as one of the main advantages of FDMA.

But what sort of penalty will accompany an attempt to realize CDMA when signatures are not synchronous at the receiver input? Since the signatures of the different users within CDMA have overlapping spectra, they cannot remain orthogonal in a wide range of mutual delays, and equality (4.5) cannot be true for arbitrary values of τ_k, τ_l and data sequences $\mathbf{b}_k, \mathbf{b}_l$. As a result inter-user interference emerges, which is a non-zero response of the receiver intended for the kth user to signals of other users.

Consider the kth user conventional receiver. With no loss of generality we may put $\tau_k = 0$, rewriting (4.4) as:

$$z_k(\mathbf{b}_k) = \int_0^T y(t)s_k(t;\mathbf{b}_k)\,\mathrm{d}t \tag{4.7}$$

According to the single-user rule, the estimate $\hat{\mathbf{b}}_k$ of data $\mathbf{b}_k$ should maximize the decision statistic $z_k(\mathbf{b}_k)$ as a function of $\mathbf{b}_k$. Substituting (4.1) (at this step we should replace $\mathbf{b}_k$ of (4.1) by $\mathbf{b}'_k$ in order to mark differently a genuine transmitted data $\mathbf{b}'_k$ from that assumed in the course of decision making $\mathbf{b}_k$) into (4.7) presents $z_k(\mathbf{b}_k)$ in the form:

$$\begin{aligned} z_k(\mathbf{b}_k) = {} & A_k \int_0^T s_k(t;\mathbf{b}'_k)s_k(t;\mathbf{b}_k)\,\mathrm{d}t \\ & + \sum_{\substack{l=1 \\ l\neq k}}^{K} A_l \int_0^T s_l(t-\tau_l;\mathbf{b}'_l)s_k(t;\mathbf{b}_k)\,\mathrm{d}t + \int_0^T n(t)s_k(t;\mathbf{b}_k)\,\mathrm{d}t \end{aligned} \tag{4.8}$$

The first and last terms of (4.8) give, respectively, the contribution of the proper, i.e. kth, user signal, and thermal additive noise into the kth user receiver effect. If no side users were present ($K = 1$) the second addend would be zero and the whole problem would be no different to the one considered in Chapter 2. With $K > 1$ and arbitrary signature delays this term differs from zero, expressing the contribution of other user signals to the kth receiver output effect, i.e. mutual or *multiple-access interference* (MAI).

An easy way to assess the influence of MAI is to treat all the alien signals as random noise-like processes similarly to what was frequently done in Chapter 3. In any practical asynchronous CDMA system, measures should be taken to equalize the level of all user signals at the receiver input in order to mitigate the *near–far* problem. The latter implies that MAI created by alien users which are much closer to the receiver than the kth user may significantly overpower the useful signal of the latter due to the strong dependence of the received power on distance (see Section 3.5.2). Therefore, we may assume that, thanks to the efficient *power control*, all A_k, $k = 1, 2, \ldots, K$ are the same; in other words, the powers of all signals are identical and equal to P. Then an lth side-user noise-like signal whose energy is assumed to be uniformly spread over the bandwidth W creates extra noise power spectrum density $N_l = P/W$ added to the thermal noise spectrum. Since there are $K - 1$ independent alien users altogether, the total MAI spectrum density is $N_I = (K-1)N_l = (K-1)P/W$. Now we may determine the power SIR, i.e. signal-to-interference ratio q_I^2 embracing both MAI and thermal noise:

$$q_I^2 = \frac{2E}{N_0 + N_I} = \frac{2E}{N_0 + (K-1)(P/W)} \tag{4.9}$$

Typically the number of users K or/and processing gain of every signature are large enough to enforce the mechanism of the central limit theorem and treat the second sum in (4.8) as a Gaussian random value. This justifies the Gaussian approximation of MAI used universally and meaning that all the results obtained in Chapter 2 for the classical reception problems (error probabilities, estimation precision etc.) are applicable to the similar multiuser problems after replacing SNR q^2 by SIR q_I^2. For example, if data are transmitted by BPSK the bit error probability for any user is calculated via (2.19) where q_I^2 substitutes for $q^2 = 2E/N_0$.

Equation (4.9) makes it possible to estimate the maximal number of users which asynchronous CDMA can accommodate within the total time–frequency resource WT. It is readily seen that in the multiuser environment, absence of thermal noise does not lead to error-free decisions at the receiving side since MAI retains SIR finite and equal to so-called *floor* SIR:

$$q_{If}^2 = \frac{2E}{(K-1)(P/W)} = \frac{2PT}{(K-1)(P/W)} = \frac{2WT}{K-1} \tag{4.10}$$

The last result shows that the floor SIR and, hence, floor reception fidelity is exhaustively determined by the time–frequency product, i.e. spread spectrum processing gain WT and number of users. As long as inequality $q_I^2 \leq q_{If}^2$ is always true the maximal possible number of users may be limited by the relation:

$$K \leq \frac{2WT}{q_I^2} + 1 \tag{4.11}$$

where q_I^2 stands for the required SIR dictated by the necessary fidelity of reception in the analysed system. To be specific, consider a BPSK or QPSK data transmission system where the bit error probability should be provided no worse than $P_e = 10^{-2}$. From (2.19) or Figure 3.16 (dashed line) it may be seen that in the no-fading case SIR of about 7 dB ($q_I^2 = 5$) is necessary to meet this demand. This produces the following estimate of the potential number of users:

$$K \leq \frac{2WT}{5} + 1 \tag{4.12}$$

At the same time, FDMA is capable of accommodating WT users[2] within the same total time–frequency resource ($W_t = W, T_t = T$), which is about 2.5 times greater than the right-hand side of (4.12). This leaves a rather bleak impression about the prospects of asynchronous CDMA in comparison to FDMA. In the next section, however, we will demonstrate that in the systems where the frequency resource needs to be reused in spatially distant areas (e.g. cellular ones), asynchronous CDMA significantly outperforms FDMA in the maximal number of users.

4.6 Asynchronous CDMA in the cellular networks

4.6.1 The resource reuse problem and cellular systems

When creating a new commercial multiuser wireless system the system designer naturally intends to serve as many subscribers as possible, at the same time being tightly bound by some fundamental limitations. The first of these is the power constraint limiting the spatial zone covered by a single transmitter. The curvature of the Earth and fast attenuation of signal intensity with distance, which is characteristic of the UHF band utilized by systems akin to mobile radio (see Section 3.3), rule out a practical opportunity of covering zones whose radius exceeds tens of kilometres. Another tough restriction is imposed by the time–frequency resource, i.e. allocated spectrum band and data rate required. Take, for example, the physical layer bandwidth of cdmaOne (IS-95) $W_t = 1.25\,\text{MHz}$. With rate of encoded voice data $R = 19.2\,\text{bps}$ and BPSK data modulation used in the downlink the potential number of active users according to (4.6) is $K = 130$. This number is obviously too small for coverage of a densely populated urban area and this is all the more true if the service should also include high-speed (e.g. multimedia) data transmission along with a telephone connection.

An effective way of getting around these obstructions is offered by a cellular network topology involving multiple base stations, each servicing its specific zone (cell) and covering collectively the total necessary area. The base station (BS) transmitter of relatively low power sends signals to the users (or mobile stations, MS), which are located within the served cell, and MS receivers form the downlink. The uplink includes the MS transmitters and BS receiver. All BS operate in strong coordination and the whole network has connections with fixed telephone and data transmission networks. When an MS moving across the system coverage zone leaves a current cell, the BS of the adjacent cell automatically takes over servicing this MS: this procedure is called *handover*. Within the framework of the cellular philosophy the wave attenuation manifests its favourable feature, allowing reuse of the same physical sub-channels (e.g. frequency sub-bands in FDMA or time slots in TDMA) by different transmitters, provided they are distant enough to secure a low level of their signals over the foreign coverage zones. As a result, just increasing the number of cells may flexibly solve the problem of raising the number of users and extension of a coverage area. In sparsely populated regions macrocells (measured by kilometres to tens of kilometres) may meet

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[2] We ignore here the potential doubling of the number of users by reuse of the same subcarrier frequency with a quadrature phase shift, since this opportunity is not feasible if different users are not time-synchronized.

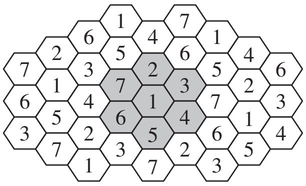

Figure 4.4 Cellular network configuration

coverage demands while in congested zones microcells (hundreds of metres) or even picocells (tens of metres) are likely to be necessary. It is universally accepted that an individual cell of a cellular network be approximated by a hexagon, owing to which the network pattern resembles a honeycomb (Figure 4.4).

Let us estimate the efficiency of utilization of the time–frequency resource in a cellular system employing classical FDMA or TDMA multiple access schemes. To avoid unnecessary repetition and allowing for equivalence of FDMA and TDMA in number of users (Section 4.3), we will use only FDMA terminology. It is obvious that the cell radius cannot be bigger than the radius of total wave attenuation but the latter, as was pointed out, should be at least two times smaller than the distance between the centres of cells utilizing user signals with identical frequencies. If the first condition fails, MS near the edge of a cell will receive too weak a BS signal and contact with the BS will appear unreliable. Violation of the second condition will entail inter-cell interference, since again the MS travelling near the edge of the cell may receive along with the proper signal of its own BS the signal of an alien BS communicating with another MS (serviced by this alien BS) at the same frequency. In other words, the frequency sets of all cells around any specific cell should differ from the set used by the central cell. Thus, a configuration arises called a *cluster* within which no frequency set may be reused. A regular honeycomb structure where frequency alternation between cells meets the condition above may exist for only some specific sizes of cluster. The most typical is the seven-cell cluster highlighted in Figure 4.4. Hence, only one seventh of the total number of physical channels (frequencies) granted by the total time–frequency resource $W_t T_t$ of the system may be utilized by a single cell. This gives the following estimation of the maximal number of users per cell in an FDMA or TDMA system:

$$K_c = \frac{W_t T_t}{7} \tag{4.13}$$

where asynchronous operation typical of uplink is assumed. In the light of this result the pessimistic conclusion about the prospects of asynchronous CDMA made in the previous section needs a serious revision.

4.6.2 Number of users per cell in asynchronous CDMA

Let us recall that asynchronous CDMA is spread spectrum based and every signature occupies the whole available time–frequency resource. Consider the uplink of a CDMA cellular system in which all cells share the same frequency band with no distribution of the spectral resource between them. In other words, signatures of all cells, including adjacent ones, occupy the same spectral band, and the cluster consists of only a single cell. Clearly, the BS receiver of a specific cell will receive MAI not only from users of the cell but also from MS served by alien base stations. The natural question arises: how great is the contribution to the total MAI of the component caused by the MS transmitters of the outer cells? To estimate the intensity of this *inter-cell* MAI, look at Figure 4.5, where two contiguous cells C_1, C_2 are approximated by circles of radius D_c. There are two BS marked as BS1 and BS2 and the MS within the zone of coverage of BS2. Despite the fact that the MS is served by BS2 its signal will also fall at the input of receiver BS1, contributing to the inter-cell MAI. Denote the distances from the MS to BS1 and BS2 as D_1 and D_2, respectively, and recall that precise power control is essential for any asynchronous CDMA system to get around the near–far problem. Thanks to the control loop, the power of the signal received by BS2 from the MS is permanently maintained constant and equal to P. If the power transmitted by MS is P_t then according to the propagation model introduced in Section 3.5 $P = kP_t/D_2^e$. On the other hand, the signal propagating from MS to BS1 will suffer attenuation defined by the distance D_1, so that the power received by BS1 $P_{r1} = kP_t/D_1^e$. We can use the previous equation to express P_{r1} in terms of power P of the useful signal at its 'own' BS receiver input:

$$P_{r1}(D_2, \theta) = \left(\frac{D_2}{D_1}\right)^e P \tag{4.14}$$

where MS coordinates D_2, θ (see Figure 4.5) emphasize the dependence of P_{r1} on the MS position inside C_2.

Now average the result (4.14) over all the cell C_2 assuming that all positions of MS within the cell are equally likely, i.e. joint PDF of the polar coordinates $W(D_2, \theta) = D_2/\pi D_c^2$ inside C_2 and zero outside. Then the mean power $\overline{P_{r1}}$ of MAI created by a single alien MS from the neighbouring cell is:

$$\overline{P_{r1}} = \iint\limits_{C_2} P_{r1}(D_2, \theta) W(D_2, \theta)\, \mathrm{d}D_2\, \mathrm{d}\theta = \frac{P}{\pi D_c^2} \int\limits_0^{D_c} \int\limits_0^{2\pi} \frac{D_2^{e+1}}{D_1^e}\, \mathrm{d}\theta\, \mathrm{d}D_2.$$

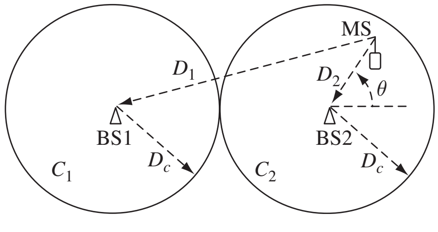

Figure 4.5 Computation of inter-cell MAI

With attenuation exponent $e = 4$ appropriate for many mobile communication scenarios, integration in the last equation may be completed analytically [20]. By the cosine theorem $D_1^2 = (2D_c)^2 + D_2^2 + 2(2D_c)D_2 \cos\theta$ and:

$$\overline{P_{r1}} = \frac{P}{\pi D_c^2} \int_0^{D_c} \int_0^{2\pi} \frac{D_2^5}{(4D_c^2 + D_2^2 + 4D_c D_2 \cos\theta)^2} \, \mathrm{d}\theta \, \mathrm{d}D_2 = \frac{2P}{\pi} \int_0^1 \int_0^{\pi} \frac{x^5}{(x^2 + 4x\cos\theta + 4)^2} \, \mathrm{d}\theta \, \mathrm{d}x$$

The internal integral here may be evaluated by trigonometric substitution or found in tables (e.g. [21]), after which the integrand in x becomes $(4x^5 + x^7)/(4 - x^2)^3$. Resorting again to the integral tables [21], we arrive at:

$$\overline{P_{r1}} = P\left(16 \ln\frac{4}{3} - \frac{41}{9}\right) < 0.05P$$

This figure should be multiplied by the number of users per cell K_c and also by the number of adjacent cells surrounding the specific one. With hexagonal cell representation the latter is 6 and the total power of the inter-cell MAI induced by all adjacent cells is no greater than $6 \times 0.05 \times K_c P = 0.3K_c P$. Strictly speaking, this estimation should be further incremented to cover inter-cell MAI from the more remote cells than just the neighbouring ones. However, it is quite predictable from the calculation above that this contribution will be negligible in comparison with the estimation just obtained [20]. Reserving some safety margin, we can therefore say that overall inter-cell MAI power $P_{I,ext} \leq 0.5K_c P$, while internal MAI created by $K_c - 1$ 'own' mobiles has, as earlier, power $P_{I,in} = (K_c - 1)P$. The floor SIR (4.10) may now be modified to allow for both internal and external MAI:

$$q_{If}^2 = \frac{2E}{(K_c - 1)(P/W) + 0.5K_c(P/W)} = \frac{2WT}{1.5K_c - 1} \tag{4.15}$$

This result admits further revision for 'pure' telephoning, since in the dialogue every party does not talk continuously and spends some time reflecting and listening. Certainly, during such pauses the transmitter of a silent speaker may be switched off or at least operate at much lower power. Actually, this opportunity has already been exploited in 2G non-CDMA mobile telephone systems to extend battery lifetime. However, only in CDMA standards does it allow MAI to be reduced simultaneously, thereby, potentially increasing the number of users served by one cell.

A typical figure for the *voice activity factor*, i.e. the fraction of total conversation time during which the phone speaker is talking, is 3/8. Correspondingly, weighting the average MAI power by this factor will transform the floor SIR above as follows:

$$q_{If}^2 = \frac{16WT}{4.5K_c - 3}$$

Solving this with respect to K_c gives a much more encouraging estimation of the number of users in CDMA as compared to the original one (4.12), obtained irrespective of a specific system topology:

$$K_c \leq \frac{32WT}{9q_{If}^2} + \frac{2}{3} \tag{4.16}$$

Substituting in (4.16) the former reference figure of 7 dB for the required SIR gives:

$$K_c \leq \frac{32WT}{45} + \frac{2}{3} \tag{4.17}$$

The estimation produced by this inequality is about five times greater in comparison with that of (4.13), the time–frequency resource being assumed the same. This shows that in cellular systems asynchronous CDMA is significantly more promising than the traditional orthogonal multiple access schemes FDMA and TDMA.

Example 4.6.1. Suppose FDMA is used to provide multiple access within the bandwidth of 5 MHz typical of 3G systems. With the transmission rate of encoded speech 19.2 kbps and BPSK, up to $K_c = W_t/7R = 37$ users per cell may be potentially accommodated. At the same time, the asynchronous CDMA alternative, as follows from (4.17), is significantly better, allowing upto $K_c = 32W/45R + 2/3 \approx 185$ users to be served by a single cell site.

Estimations of the sort of (4.16) and (4.17) may seem excessively optimistic, since they ignore the thermal noise component. A practical situation to which they are more applicable implies that the power P transmitted by every mobile is so great that overall MAI dominates the AWGN noise. On the other hand, a designer may be interested in employing as low transmitted power as possible, e.g. for reasons of battery lifetime or electromagnetic compatibility. Returning to (4.15), adding AWGN spectrum density to the denominator and allowing again for the voice activity factor, it is easy to show that when a 'pure' (i.e. not covering MAI) power SNR is q^2 and SIR q_I^2 covering MAI plus noise is required, (4.16) changes to:

$$K_c \leq \frac{32WT}{9q_I^2}\left(1 - \frac{q_I^2}{q^2}\right) + \frac{2}{3} \tag{4.18}$$

If, for example, an overall MAI power should be of the same level as noise power within the signal bandwidth then $q^2 = 2q_I^2$ and:

$$K_c \leq \frac{16WT}{9q_I^2} + \frac{2}{3} \tag{4.19}$$

that is, about half of that calculated from the floor SIR (see (4.16)). For the required SIR specified as before (7 dB):

$$K_c \leq \frac{16WT}{45} + \frac{2}{3} \tag{4.20}$$

Although with signal power reduction the number of users per cell dropped by two times, it nevertheless remained more than twofold greater than in FDMA or TDMA.

One more merit of asynchronous CDMA is its favourable blocking character. In all real multiuser systems physical channels (frequency sub-bands in FDMA, time slots in TDMA, code signatures in CDMA) are not assigned to the customers once and for all. Instead, the network itself controls the bank of traffic channels and grants one of them to a user only when he requests access to the network. Of course, some system resource in this case should be reserved to arrange the request channel. In FDMA or TDMA systems the number of physical channels is fixed and from time to time blocking may happen, i.e. the situation where the network rejects a user's request since all the channels are busy. A figure of about 2% is often assumed as tolerable probability of blocking and the number of channels should meet this requirement. Sometimes the pattern of distribution of subscribers over the network coverage area may change so seriously that at some cells blocking becomes intolerably probable. Then the network operator may face the challenge of reconfiguring the network, which entails frequency replanning affecting all cells.

Scenarios typical of CDMA are critically different. First, if the number of users already active equals the nominal one calculated by (4.16) or (4.20), and one more request is received, it may be satisfied by assigning a signature which differs from those already employed. This will lead to some (as a rule slight) reduction of SIR and thereby quality of service for all active users. Therefore, instead of an outright denial a smooth blocking happens. Second, when in the course of time traffic in some area increases dramatically the operator may place an additional base station at the 'hot spot' without frequency replanning or any other radical affecting of the other cell sites.

Based on the analysis of this chapter, the conclusion is justified that spread spectrum appears quite a flexible and efficient aid in providing multiple access. In particular, cellular systems are among those where the advantages of CDMA manifest themselves most persuasively.

Problems

4.1. A digital FDMA data transmission system should serve at least 100 users. Estimate the minimal total bandwidth occupied by the system if the necessary data rate per user is 20 kbps and BPSK is used as the data modulation mode. How will the bandwidth change if QPSK replaces BPSK? Answer the same questions if TDMA is preferred to FDMA.

4.2. An FDMA QPSK system is intended for digital data exchange between aircrafts and operates at frequencies around 3 GHz. The maximal vehicle velocity is 1800 km/h, master clock drift is 2×10^{-7} and the guard interval due to non-rectangular filtering is 1 kHz. Find the maximal number of users accommodated within the bandwidth 2.32 MHz if the necessary data rate per one user is 20 kbps.

4.3. A digital TDMA multiuser system should serve at least 100 users. The modulation mode is 8-PSK. Estimate the minimum bandwidth occupied by the system if the necessary transmission rate per user is 20 kbps.

4.4. A single uplink frequency subchannel of a TDMA BPSK digital cellular system is allowed to occupy bandwidth of 200 kHz. The time interval between consecutive data bursts of every user should not be longer than 5 ms, the necessary data transmission rate per user is around 20 kbps and the maximal cell radius is 30 km. Find the maximal number of TDMA channels per frequency subchannel.

4.5. In a synchronous CDMA system 128 physical channels should be arranged. The data transmission mode is 8-PSK and the necessary rate per user is 20 kbps. Estimate the minimum bandwidth demanded.

4.6. A synchronous CDMA system has 50 physical channels operating in 16-PSK mode with a data rate of 20 kbps per user. The total bandwidth occupied is 500 kHz. What is the processing gain of the system? Can the system be free of MAI? What if the bandwidth were four times smaller?

4.7. There are two users within one cell of a cellular CDMA telephone system located at distances 500 m and 5 km from the base station. A more distant mobile emits power of 100 mW. Find the power emitted by the closer mobile under the assumption of perfect power control.

4.8. In the uplink of an IS-95 cellular telephone asynchronous CDMA is used. Data are transmitted at a rate of 28.8 kbps by means of orthogonal signals comprising 6-bit data blocks. The signal bandwidth can be assumed as 1.25 MHz. What is the number of users per cell if the minimum required SIR is 7 dB, voice activity factor is 3/8, external MAI adds 50% to internal interference and thermal noise is neglected. What will change if the SNR for thermal noise only is 9 dB?

4.9. Estimate the number of necessary cell sites under the conditions of Problem 4.8 (AWGN is not neglected) to service an area with 50 000 subscribers if the probability of the active state of a subscriber is 0.02. Compare the result with that for an FDMA system.

4.10. How will the potential number of users change if under the conditions of Problem 4.8 orthogonal 6-bit signalling is replaced by BPSK, the bandwidth and data rate remaining the same?

4.11. Based on the Ocumura–Hata model, prove that asynchronous CDMA is inoperative in a typical macrocell cellular system uplink without an effective power control. Neglect the thermal noise component.

4.12. There is synchronous downlink in a cellular CDMA system. Within one cell the maximal number of orthogonal signatures is used. If no time–frequency resource distribution between cells is involved, how can the effect of the surrounding base stations on the mobile receivers of the given cell be estimated? What would you recommend as a general approach to choosing signatures in such a system?

Matlab-based problems

4.13. Write a program illustrating the principle of FDMA. Example plots are shown in Figure 4.6.

(a) Form the $K \times 100$ matrix of K subcarriers ($K = 2-10$ is recommended). Take frequencies so that for each subcarrier the exact integer number of periods per 100 points is one greater than for the previous one. For the first one 4–6 periods are advised.

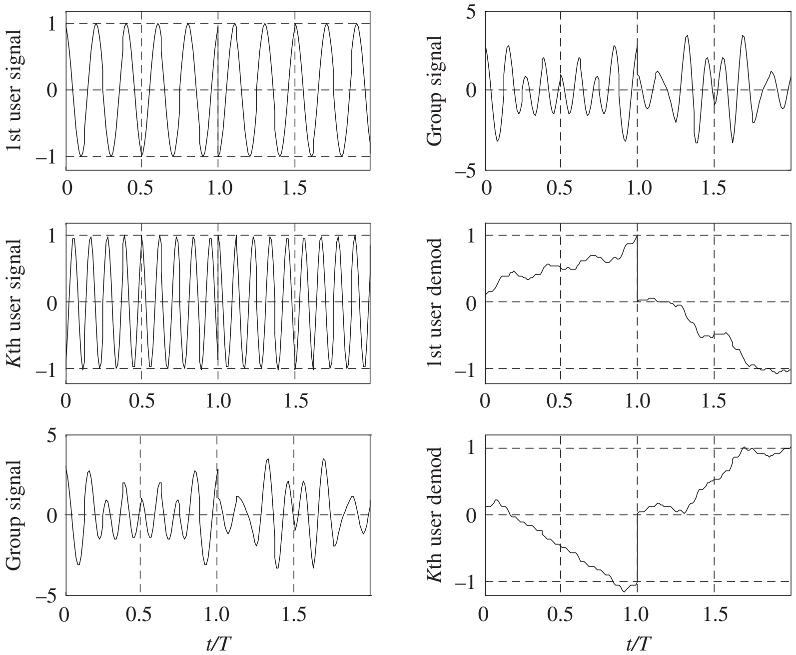

Figure 4.6 Simulation of the principle of FDMA

(b) Take 2–3 random information bits for every subcarrier and perform BPSK modulation of all subcarriers. Plot the modulated signals for two selected users.

(c) Sum all modulated signals to come to a group signal and plot that.

(d) Demodulate each transmitted bit for every user, multiplying the group signal by a relevant subcarrier and then integrating over the bit duration. Plot selected demodulator outputs. Take decisions on the received bits for all users and compare them with the transmitted ones.

(e) Run the program, changing the number of users, and interpret the results.

4.14. Use the program from Problem 4.13 to demonstrate the influence of inter-channel interference accompanying frequency drifts in an FDMA scheme. Enter a frequency shift of $+0.25$ in the first subchannel and fix identical bit patterns in all subchannels. Reducing the amplitude of the first signal, note the value under which wrong decisions on the channel bits arise. Run the program, varying frequency drift and channel amplitudes, and comment on the results.

4.15. Write a program confirming experimentally equation (4.10) for the floor SIR in asynchronous CDMA.

(a) Set $N = 50-80$ and form the $K \times N$ binary matrix (taking on values ± 1) of independent random numbers.

(b) Use the rows of the matrix as the signatures of K users. Assume the first of them is a useful one, the rest being interfering.

(c) Sum all rows except the first to simulate MAI.
(d) Calculate output receive MAI as the inner product of the input MAI (item (c)) and the first signature.
(e) Repeat items (a)–(d) 5000 times, calculate the variance of MAI and SIR, and compare them with the one evaluated by (4.10). What value of W should be used in (4.10) to make the experimental results fit the theoretical prediction? Give your explanation as to why $WT = N$ appears inappropriate.
(f) Plot a histogram of MAI over all 5000 experiments and verify its closeness to the Gaussian PDF with an equal variance (Figure 4.7 presents example plots).
(g) Run the program for combinations of N, K and check the validity of (4.12).

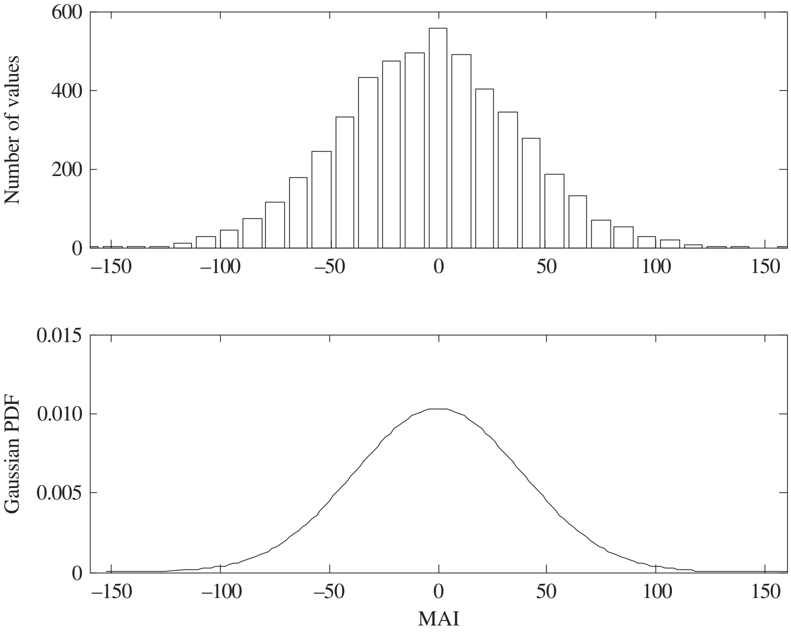

Figure 4.7 Histogram of MAI and its Gaussian approximation

4.16. Write a program illustrating the low contribution of outer mobiles to an overall MAI in an asynchronous cellular CDMA. Advised steps:

(a) Set $N = 80-100$ and number of users per cell $K = 20-25$.
(b) Using uniform random generator, form and plot internal MAI as the sum of $K - 1$ random binary (consisting of elements ±1) signatures of length N.
(c) Form the $6K \times 2$ matrix of random polar coordinates D_c, θ of outer mobiles (see Figure 4.4) to simulate uniform distribution of mobiles throughout a cell. Note that to realize this the angle should be uniformly distributed over $[-\pi, \pi]$, but the radius has to have linearly rising PDF. The latter is simulated by generation of uniformly distributed random numbers and afterwards taking their square roots.

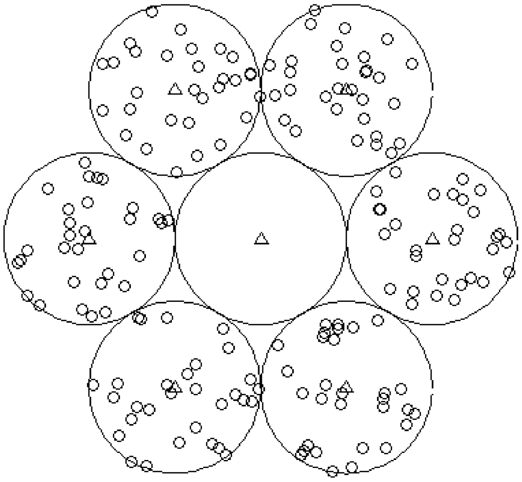

Figure 4.8 Random distribution of users over the surrounding cells

(d) Form the $6K$ dimensional vector of ratios of distances D_2 (from an outer mobile to the BS serving it) and D_1 (to the BS serving internal mobiles):

$$\frac{D_2}{D_1} = \frac{D_2}{\sqrt{4D_c^2 + D_2^2 + 4D_c D_2 \cos\theta}}$$

where cell radius D_c may be set equal to 1.

(e) Form and plot the external MAI as sum of all outer signatures, each weighted with an amplitude attenuation factor. The latter is calculated on the basis of previous item with power attenuation exponent $e = 3.8$ (see (4.14)).

(f) Form and plot total MAI as the sum of the internal and external ones, and estimate the increase of the total MAI versus the internal one.

(g) Produce a scatter plot demonstrating the random distribution of mobiles in outer cells (Figure 4.8 shows an example).

(h) Run the program repeatedly for a range of N, K, compare the results with the theoretical prediction and give your comments.