Chapter 1

Simplifying Sudoku

This chapter introduces the sudoku-solving ground rules, giving you all the tools you need to approach each puzzle with the best strategy. It also shows you how to take your sudoku-solving skills up a notch with more advanced solving strategies.

Understanding the Rules

A blank sudoku grid like the one in Figure 1-1 consists of a grid of nine rows and nine columns subdivided into nine 3×3 subgrids. Throughout this book I refer to a particular square by its coordinates — row first, then column: 1,3 is the top row, third square from the left and 9,8 the bottom row, 8 squares from the left. I refer to a 3×3 subgrid as a *box,* numbered as shown in Figure 1-1.

Sudoku has two simple rules:

- Each column, each row, and each box must contain each of the numbers 1 to 9.
- Therefore, no column, row, or box can contain two squares with the same number.

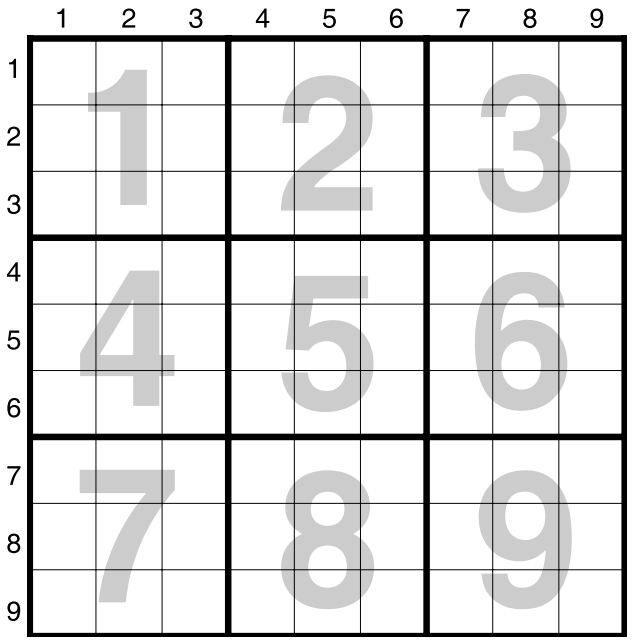

Figure 1-1: A blank sudoku showing grid coordinates and box numbers.

Getting Down to Basics

Each puzzle starts with a set of clue numbers placed on the grid as shown in Figure 1-2.

Logic is all you need to solve a sudoku. No addition, subtraction, division, or multiplication is required. However, you do need to ask yourself questions like "If so-and-so number is in this column, will such-and-such number go in this other box?" Your answer to these questions will always be either "yes", "no", or "maybe". When the going gets a little tougher — if you're working on a more difficult puzzle — you may find yourself asking more complex logic questions, but for now I'll just stick to the basics.

Use a pencil and eraser. Solving sudoku, especially the more difficult puzzles, requires you to make notes of possible number options. These notes change as the puzzle progresses,

so you need to erase them as you solve — or partially solve — any squares.

6		7	4					
					9		8	6
	9			6		5		
			1		6		4	
7		8				6		1
	3		9		7			
		9		1			6	
8	6		7					
					2	8		3

Figure 1-2: A moderately difficult sudoku. Tempted?

Taking the puzzle in pieces

Your first tip to getting started with a sudoku: Don't try to look at the whole grid at first. Take the puzzle in sections, as I've done in Figure 1-3. You could try using a sheet of paper to help block off the part of the grid you aren't looking at.

As you can see in the first three columns, you have a 9 in box 1 and a 9 in box 7, but no 9 in box 4. The 9 in column 2 precludes any 9 appearing in column 2 of box 4, and the 9 in column 3 stops a 9 being placed in column 3 of box 4. That means that the 9 of box 4 has to be in column 1, but could appear in either of two squares. I've shown these options as small numbers in the corner of the squares. Writing in options like this as you solve helps you keep things straight and saves you time.

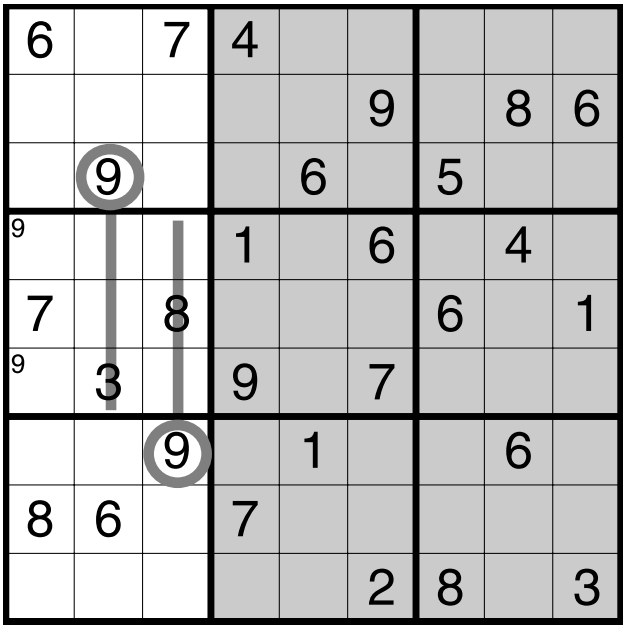

Figure 1-3: Look at a sudoku in sections rather than trying to work out the whole puzzle at first glance.

Looking at the (slightly) bigger picture

"Well," you might say, "looking at one column solves nothing." But wait . . . By revealing the next column, as in Figure 1-4, you expose a 9 in row 6. Obviously, with a 9 in this row, the option of 9 in box 4 at row 6 has been disproved, and the option can be erased. The 9 has to go in the only other available square in row 4, column 1 (or square 4,1). This is the first solved number. Whew! That wasn't so difficult, was it?

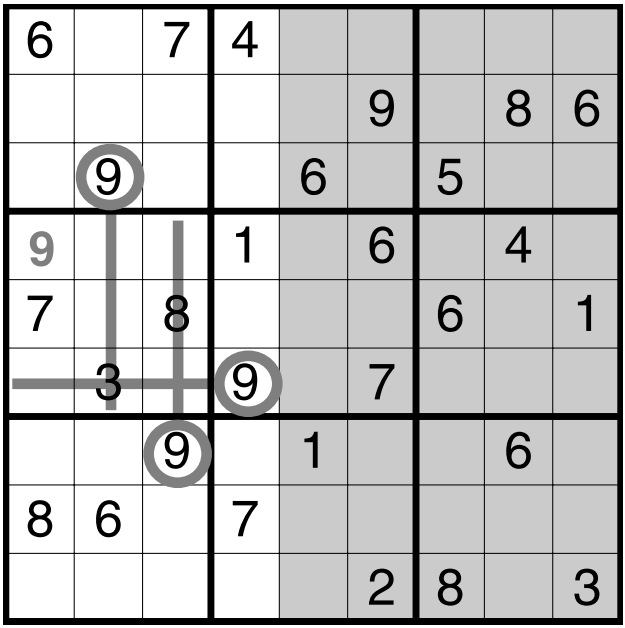

Figure 1-4: Gradually look at more and more of the puzzle for clues, keeping one number in mind.

Solving for the second number

The fewer empty squares you have in any box, row, or column, the better your chances of proving the empty squares, so look for the most populated rows, columns, and boxes. For example, concentrate on the middle three rows. You have a 6 in box 5 and a 6 in box 6, but no 6 in box 4, so that seems like a good number to focus on.

The 6s in rows 4 and 5 mean that the 6 of box 4 must be at either 6,1 or 6,3, so you can pencil in those options. Revealing the rest of column 1 in Figure 1-5, you find a 6 already in that column. So the 6 of box 4 can't be at 6,1 and must be in the only remaining square at 6,3. The second number of the puzzle has been solved.

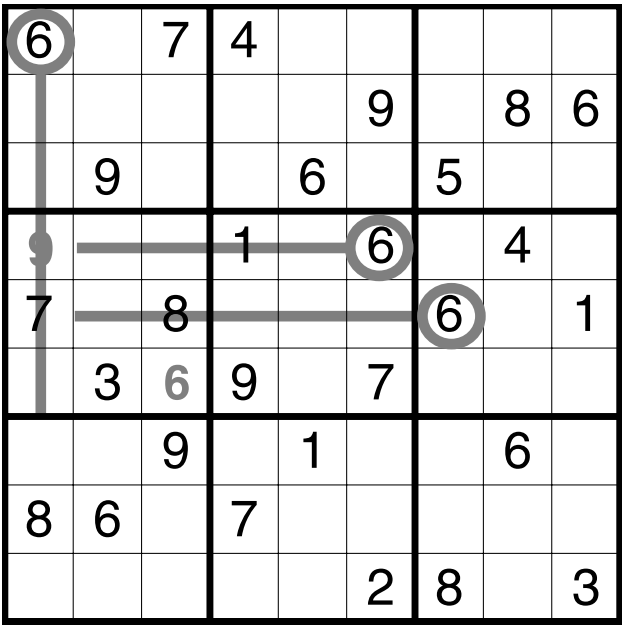

Figure 1-5: Searching for sixes to solve the sudoku.

Incidentally, when the rest of the grid was revealed, did you spot the 6 in column 2? If you hadn't already excluded a 6 from column 2 of box 4 with the 6s at row 4 and row 5, this 6 would have done the job nicely. Having such a surfeit of riches is rare, but worth pointing out.

Cracking open the first box

Moving on in this exploration of the given clues: look at the 1 in row 4 and the 1 in row 5 in Figure 1-6. Between them they stop any 1 appearing anywhere in box 4 other than in the only square available at 6,1. No penciling required here, it's the only place for a 1 to go.

Now look at the 4 in row 4. It very nicely stops another 4 from going in box 4 at row 4. Because you've already solved some numbers in this box, only one possible square is left for the 4, at 5,2. Box 4 is filling up quite nicely and you have only the 2 and the 5 left to solve. Either of these numbers could go into 4,2 or 4,3. You don't have an obvious way of proving the

correct square at this stage from the clues provided, so you're stuck for the moment.

6		7	4					
					9		8	6
	9			6		5		
9			1		6		4	
7	4	8				6		1
1	3	6	9		7			
		9		1			6	
8	6		7					
					2	8		3

Figure 1-6: The 4 at 4,8 means the only place for a 4 in box 4 is at 5,2.

Before you move on, pencil in the two options of 5 and 2 for both squares: At some stage you'll be able to prove one of the numbers and solve the box.

You can draw a very important implication from the two unsolved squares: Both contain either 5 or 2 as you have proved, but that must mean that these two squares can be the only place for a 5 or 2, not only in that box, but also for the remaining unsolved squares in that row. Only one 5 and one 2 can be in the row and you've just proved where they are.

What you just discovered are a matched pair of twins. A *twin* is a number that has been proved to appear in either of two squares that helps disprove its presence in another part of the grid. A *matched pair of twins* are two numbers that must go in either of only two squares, which helps to solve other problems as the puzzles get harder.

Using your clues

By now, you've probably familiarized yourself with the position of given numbers and can dispense with blocking off parts of the grid, although it's a useful tool when you're concentrating on a specific part of the grid. The "good" clues eventually start to jump out at you as in Figure 1-7. Here you have a combination of 6s stopping another 6 from appearing anywhere but in 9,4 of box 8. The 6 of this box doesn't really help you to solve any other numbers, so you move on.

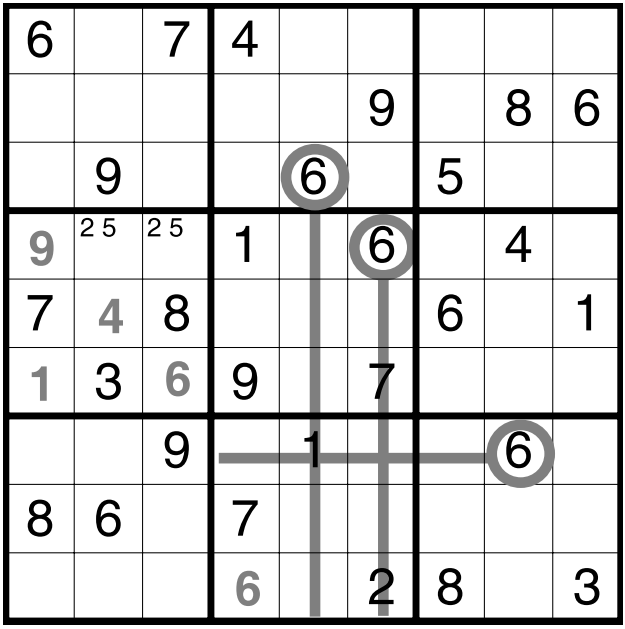

Figure 1-7: Separating out the "good" clues.

You're getting a good handle on the grid in Figure 1-8. The 8s aren't helping to solve the 8 of box 6 immediately, but they allow you to note that the 8 could be in 4,9 or 6,9. Although the 8s don't solve anything immediately, making these observations is always helpful for use at a later stage of solving.

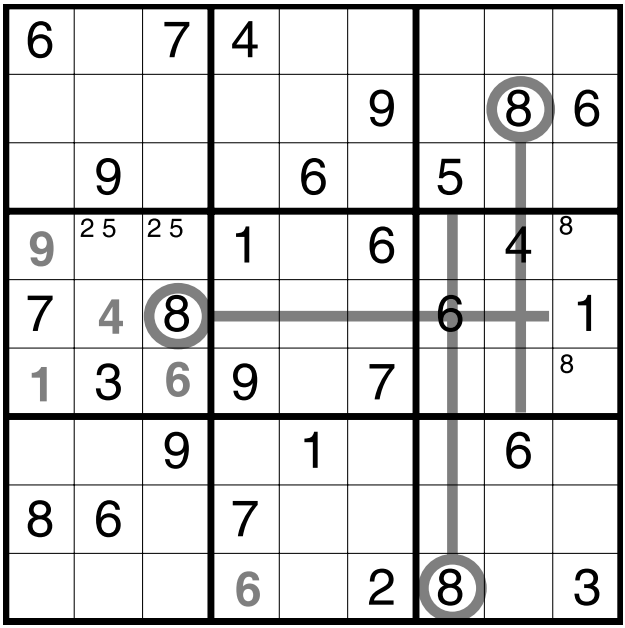

Figure 1-8: Getting a good grip on the grid.

You can still make plenty of observations and solutions from the given clues and those squares that you've already solved. For example, look at the 7 in column 4 and the 7 in column 6. Together with the 7 in row 1 they solve the 7 in box 2.

Taking Sudoku Up a Notch

It's time to start being methodical in solving and get serious about discovering the secrets of each individual square. Depending on the grade of difficulty of the puzzle, you can simply bite the bullet and write in all the options for every square right now, or take it gradually box-by-box (or row-by-row or column-by-column). As this puzzle is only of moderate difficulty, take this discovery process gradually.

All the options for the squares of box 6 have been noted in Figure 1-9. I found them by asking at each square "Will such-and-such number go here?" from 1 through to 9 while

checking to see whether that number is already in its box, row, or column. Try doing the exercise to check the numbers for yourself (I'm not perfect, you know!).

You've already proved that at 4,7 and 4,9 neither 2 nor 5 can appear because of the pair of twins in box 4 that have already fixed their position.

6		7	4					
					9		8	6
	9			6		5		
9	2 5	2 5	1		6	3 7	4	7 8
7	4	8				6	2 3 5 9	1
1	3	6	9		7	2	2 5	2 5 8
		9		1			6	
8	6		7					
			6		2	8		3

Figure 1-9: Taking a stab at box 6.

Singling Out Lone Numbers

Look at the bottom left hand square of box 6 and you see that the only number that can go into that square is a 2. You don't have any clues around in the rows or columns that might indicate that a 2 is the solution to that square, and only by eliminating all the other options could you work out that the 2 goes in 6,7. Discovering a number by a process of elimination is a *lone number*.

The second result of solving that 2 in 6,7 is that all other optional 2s in that box, row, and column may now be eliminated. The new situation is illustrated in Figure 1-10. Having removed all the optional 2s, to the right of the solved 2 you can see that the 5 is on its own — another lone number solved. Solving the 5 (so all the optional 5s are removed) leaves the 8 on its own in 6,9, which means the other optional 8s are gone, leaving a lone 7 at 4,9. Carry on this process as far as you can with this box. Think about the 3 and the 9 at 5,7. What can you conclude, although there appear to be two options in that square?

6		7	4					
					9		8	6
	9			6		5		
9	25	25	1		6	37	4	78
7	4	8				6	359	1
1	3	6	9		7	2	5	58
		9		1			6	
8	6		7					
			6		2	8		3

Figure 1-10: Solving for lone numbers.

When you've finished your number-fest in that box, check the consequences of solving those numbers in the associated rows, columns, and boxes. I hand the puzzle in Figure 1-11 over to you for solving as far as you can, using the techniques and strategies you've picked up so far.

6		7	4					
					9		8	6
	9			6		5		
9	25	25	1		6	37	4	78
7	4	8				6	39	1
1	3	6	9		7	2	5	8
		9		1			6	
8	6		7					
			6		2	8		3

Figure 1-11: Try your hand at this puzzle, keeping an eye out for lone numbers.

Serious Sudoku Solving

The new puzzle in Figure 1-12 challenges you to look beyond the obvious. Consider the number 4:

- The 4s in columns 1 and 3 and the 4 in square 9,5 preclude a 4 going anywhere in box 7 except 7,2 or 8,2.
- Because row 8 is full in box 9, the 4 in box 9 will have to go in row 7 — so the 4 in box 7 has to be in row 8, at 8,2.
- The 4 in square 5,8 precludes a 4 from going anywhere else in column 8. This means the 4 in box 9 cannot go at 7,8, and because the 4 at 9,5 precludes the 4 from going anywhere in row 9 in box 9, 7,9 is the only option left.

You should see another 4 to solve very easily in the middle three columns. And you can now solve the 4 in box 7.

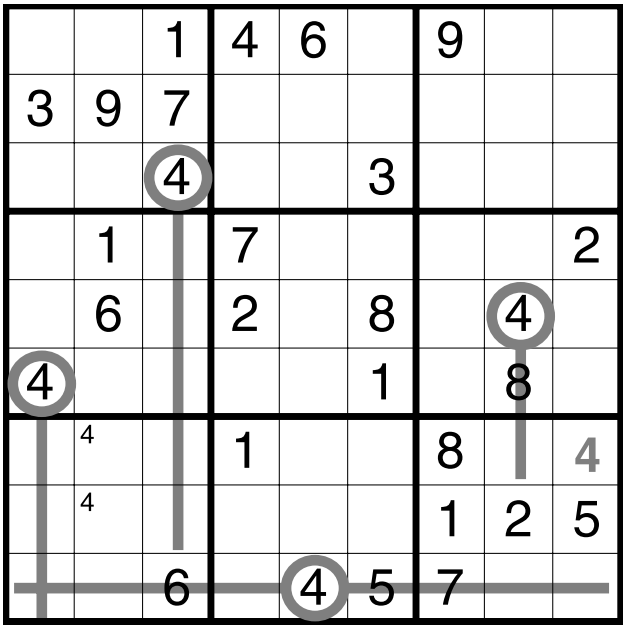

Figure 1-12: Look beyond the obvious.

Getting Rid of Extraneous Options

Now on to some solving strategies for the more difficult levels of sudoku. First, you have to meticulously discover all the options for every unsolved square in the grid. The method for this is the same as when you did it for box 6: Look at each square and ask the question "Can such-and-such number go in this square?" for each number 1 to 9.

As shown in Figure 1-13, writing down all the options automatically solves many of the squares, but some remain with their penciled options. So how do you move on? With logic, of course.

2 8	2 8	1	4	6	7	9	5	3
3	9	7	5 8	5 8	2	4	1	6
6	5	4	9	1	3	2	7	8
5 8	1	3 5 8	7	3 5 8	4	6	9	2
7	6	3 5 9	2	3 5 9	8	3 5	4	1
4	2 3	2 3 5 9	6	3 5 9	1	3 5	8	7
5 9	7	3 5 9	1	2	6 9	8	3 6	4
8 9	4	3 8 9	3 8	7	6 9	1	2	5
1	2 3 8	6	3 8	4	5	7	3	9

Figure 1-13: Looking at the options — narrowly.

Covering certain columns helps to concentrate on column 4. If you look at box 8 there are two sets of options, but right now look at just the pair of 3 and 8 at 8,4 and 9,4. Although you don't know which order the 3 and 8 go in, you know that these two squares must be the only two squares to contain these numbers in all of column 4. So you now know that the 8 in that column cannot be at 2,4 with the 5. That leaves only the 5 in that square, so it is solved and as a consequence the 8 in box 2 must be at 2,5.

Matched pairs of twins like 3 and 8 can help solve the stickiest of problems. Sometimes you might see a pair such as 3 8 9 and 3 8 in a box, row, or column. If the 9 is somewhere else in the options of that element (it would have to be, otherwise it's the 9 for that element), then you can remove it from the 3 8 9 group. Why? Because you know that those two squares are the only squares for a 3 or an 8: If a 3 is in one square and an 8 in the other there's no room for the 9. In the lingo, that's called a *hidden matched pair*. In Figure 1-13 the matched pair eliminated just one 8, but sometimes such a matched pair can get rid of large numbers of extraneous options.

Probably the most difficult construct to get your head round is a step up from matched pairs. A *triplet* is where three numbers share three squares in a box, row, or column and the three squares contain these three numbers exclusively. For example, if the three numbers are 2, 5, and 9 they may appear in the options of an element as, say, 2 5, 5 9, 2 5 9 or 2 5, 2 5 9, 2 5 9, or simply 2 5 9, 2 5 9, and 2 5 9.

Look at column 1 in the left side of Figure 1-14. The fourth, seventh, and eighth squares must be filled by the numbers 5, 8, or 9. Following the rule of triplets, 5, 8, and 9 can be in those squares only and nowhere else in the column, so the first square in the column must be 2. And if the number in 1,1 must be 2, the number in square 1,2 must be 8, as the right side of Figure 1-14 shows. While this example eliminated only a single option, you can usually cull more with this method of using twins and triplets to solve numbers in other parts of the puzzle.

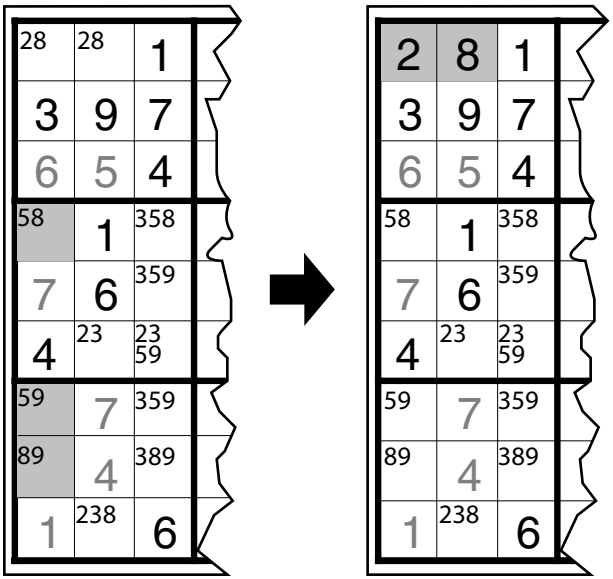

Figure 1-14: A triplet — three squares sharing three numbers exclusively — in column one (left) means 1,1 must be 2 and 1,2 must be 8 (right).

The strategies for solving sudoku thus far allow you to solve all but the most difficult and extreme sudoku.

The more difficult constructs and strategies are best learned by practice. The more sudoku you solve the easier it becomes. Twins, triplets, and pairs pop out at you all over the place. And when they don't, you'll be able to find help on one of the many Web sites and sudoku forums that have been set up just for sudoku solvers. Two of the best are `www.sudoku.com` and `www.sudoku.org.uk`.

Sudoku on Steroids

If you thought some of the 9×9 sudoku were difficult, this volume contains a few puzzles with 16×16 grids to blow your mind. As you'd expect, the rules for the 16×16 puzzles are slightly different. As well as numbers, you use the letters A to G. Each of the numbers 1 to 9 and the letters A to G must go in each row, each column, and each 4×4 box. Figure 1-15 demonstrates a completed 16×16 grid.

3	8	F	6	2	B	7	G	5	4	C	A	D	9	E	1
9	5	B	A	4	1	E	3	G	F	7	D	6	C	8	2
4	G	C	1	D	6	5	8	E	9	3	2	B	F	A	7
2	7	D	E	F	A	C	9	8	6	1	B	G	5	4	3
G	4	A	5	E	C	F	2	6	D	B	8	7	1	3	9
D	B	8	7	6	G	3	A	1	C	F	9	2	E	5	4
6	E	3	F	7	9	D	1	2	5	A	4	C	B	G	8
C	9	1	2	5	4	8	B	7	3	G	E	A	6	F	D
F	1	4	B	8	7	A	5	9	G	E	C	3	2	D	6
8	A	6	D	C	2	B	E	3	7	4	1	F	G	9	5
5	2	7	9	G	3	1	D	B	8	6	F	4	A	C	E
E	C	G	3	9	F	4	6	A	2	D	5	8	7	1	B
A	3	9	4	1	D	6	7	F	E	2	G	5	8	B	C
1	6	2	C	A	8	9	F	D	B	5	3	E	4	7	G
7	D	5	8	B	E	G	4	C	A	9	6	1	3	2	F
B	F	E	G	3	5	2	C	4	1	8	7	9	D	6	A

Figure 1-15: Use the numbers 1–9 and the letters A–G for 16×16 puzzles.

Don't panic. All the strategies used in 9×9 sudoku work in 16×16 puzzles, and you'll be surprised how quickly you become familiar with the extra characters.

Taking On the Target: Sudoku in the Round

If you're getting square eyes from doing regular sudoku, I've got some relief for you in the form of circular sudoku, sometimes called *target sudoku*. The target sudoku in Figure 1-16 is a 4-ring circle. Think of the puzzle as a big pie cut into eight slices, each slice with four bites. Your goal is to place a number into each bite of pie (so four numbers to a slice) so that every two adjacent slices contain all of the numbers from 1 to 8. Every ring also must contain all the numbers 1 to 8.

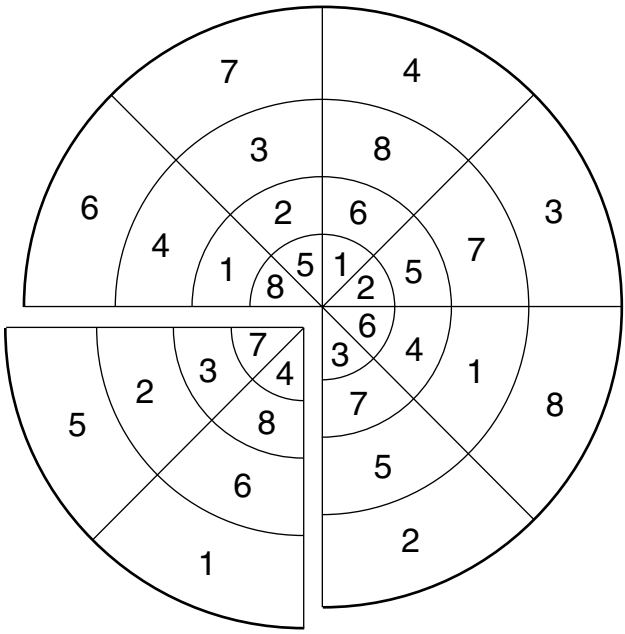

Figure 1-16: In a circular sudoku, every two adjacent slices contain the digits 1–8.

Every other slice contains the same numbers — but not in the same order because every number must be represented in every ring.

In the target sudoku section, you'll also find a 3-ring puzzle, for which you use the digits 1–6, and 5-ring puzzles, for which you use the digits 0–9, all of various difficulty levels to tempt you.