## Chapter 1

# Sudoku Strategies for Advanced Gamers

### *In This Chapter*

- Choosing your options
- Watching for naked pairs and triplets
- Finding hidden pairs and triplets

The key to solving almost any sudoku, but especially the more difficult ones, is discovering each and every possible number for each square. These options can be found simply by checking each square and asking the question "Will such-and-such a number go here?" while checking whether the number is also in the row, column, or box in which the square resides. Once you have reached a point where all the obvious numbers have been solved, you have to search for options in a systematic way.

When you cannot prove which number should be in a square, you can often more easily prove what *cannot* be there. By a process of elimination, you can reduce the options for a square until you're left with only one.

Another basic discipline of sudoku solving is recognizing option groupings. As you will see, spotting the relationship of a group of options in one square to another group in another square is fundamental to solving difficult sudoku.

This chapter takes a look at some of the more useful strategies that help you to eliminate options and solve the puzzle. Sudoku is a puzzle that is solved with logic alone, so it follows that the schemes for solving should be examined in a logical way. We begin with the easiest and most fundamental schemes that will crack all but the most difficult puzzles. Learning about pairs and triplets gives you a good grounding in sudoku logic and enables you to understand the reasoning behind the more advanced strategies in Chapter 2.

Before we take a look at our first real strategy, a word about human strategy — we call it "eyeballing". In the first few minutes of confronting a new sudoku, you should familiarize yourself with the whole grid. During this eyeballing process, some numbers will almost solve themselves. It's a rare puzzle that has no squares that can't be solved simply by examining the grid. However, there will come a point where all that remain are squares that contain two or more options. At this point the logic strategies come into play, and we begin by looking for naked pairs.

# Naked Pairs

A *naked pair,* also known as a *twin,* is a set of two option numbers situated in two squares that belong to at least one element in common. That is, they reside in the same row, box, or column. Figure 1-1 shows you a naked pair, marked in gray.

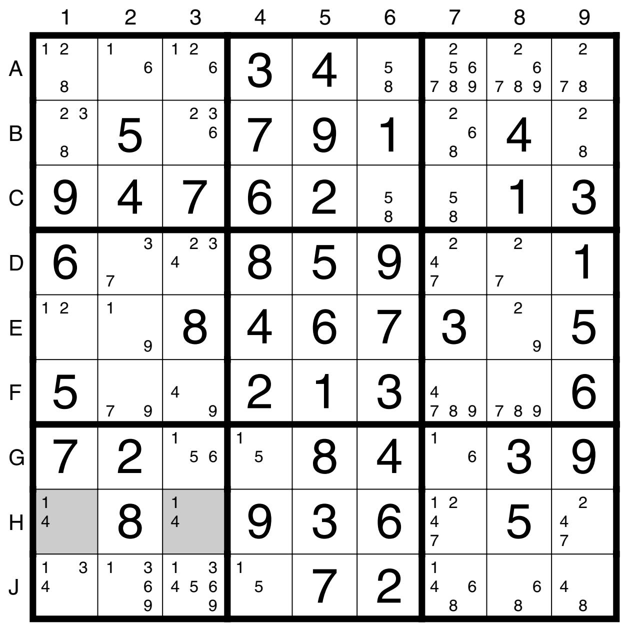

**Figure 1-1:** A naked pair.

It is clear that 1 and 4 must be the solutions to these two squares. Although you don't know in which order they will appear, you do know that neither 1 nor 4 can appear anywhere else in the box or the row of which they're both a part. Therefore you can eliminate any other 1 or 4 that appears in box 7 or row H. As a result, the square at J1 has been solved and you've revealed another naked pair at H7 and H9.

# Naked Triplets

You can extend the logic of naked pairs to three squares that contain the same three options. In this case, we refer to *naked triplets*. But a naked triplet is far more flexible than you may think. In fact, you don't need to have all three options in all three squares. The three numbers may be options for two squares, but just two of the numbers for the third square. This leaves you with the following possibilities for naked triplet arrangements, where x, y, and z are arbitrary, but different, numbers:

(xyz) (xyz) (xyz)

(xyz) (xyz) (xy)

(xyz) (xyz) (xz)

(xyz) (xyz) (yz)

(xyz) (xy) (yz) . . . (xyz) (xz) (yz)

(xy) (yz) (xz)

The last case is particularly interesting because it's often applied in more advanced strategies.

Take a look at the example in Figure 1-2. Row B shows a naked triplet with the numbers 6, 7, and 8. The number 8 doesn't appear in B3, but various combinations of the numbers 6, 7, and 8 exist in all three highlighted squares and are the only options for these squares. Because this triplet is arranged in a row, you can safely eliminate 6, 7, and 8 as options for the remaining squares in this row. Eliminating 7 from B9 reveals a pair with the numbers 4 and 9 in A9 and B9.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | **3** | 4 8 | **2** | 4 6 8 9 | **7** | **1** | **5** | 4 6 8 | 4 9 |
| B | 6 7 8 | **1** | 6 7 | 3 4 6 8 9 | **5** | **2** | 6 7 8 | 3 4 6 7 8 | 4 7 9 |
| C | **9** | 4 5 7 8 | 5 6 7 | 3 4 6 8 | 3 4 6 8 | 3 6 8 | 2 6 7 8 | 2 3 4 6 7 8 | **1** |
| D | 2 5 7 | **6** | **8** | 2 4 5 7 | **1** | 5 7 | **3** | **9** | 2 4 7 |
| E | **1** | 5 7 | **9** | 2 3 4 5 7 | 3 4 | 3 5 7 | 2 7 8 | 2 4 7 8 | **6** |
| F | 2 7 | **3** | **4** | 6 8 | 6 8 | **9** | **1** | **5** | 2 7 |
| G | **4** | **2** | 5 6 7 | 3 5 6 7 | 3 6 | 3 5 6 7 | **9** | **1** | **8** |
| H | 6 7 8 | 7 8 | **3** | **1** | **9** | **4** | 2 6 7 | 2 6 7 | **5** |
| J | 5 6 7 8 | **9** | **1** | 5 6 7 8 | **2** | 5 6 7 8 | **4** | 6 7 | **3** |

**Figure 1-2:** A naked triplet.

With all this talk of naked pairs and triplets, you may be asking yourself whether there is such a thing as a naked *quadruplet*, and if so, is it of any use? Quads are indeed possible, but occur infrequently and are not so easy to spot. Practically, naked quads are more often than not accompanied by pairs and triplets that are much easier to spot and which eliminate some of the options of the quad. So, quads are not to be forgotten as a strategy, but they are of limited usefulness.

# Hidden Pairs

*Hidden pairs* are real work horses for eliminating options and form the basis of many advanced strategies. Remember the naked pair in Figure 1-1? You didn't know which number belonged in which of the squares, just that you had only two options for each of the two squares. But what happens if you discover another option for one of the two squares. Does this change anything? The answer is no. The same two options are still the only possible options for the squares in question. The new interloper option simply obstructs your view a little bit, "hiding" the pair. Because you know which options make up the pair, you can safely eliminate the other third option.

So here's the basic rule on hidden pairs: If any two numbers occur only twice in two squares within a row, column, or box, you can safely remove all other options from the two squares.

Take a look at Figure 1-3. You can see two sets of hidden pairs in this sudoku puzzle.

The numbers 1 and 5 only occur in two squares within box 2; however, these numbers are hidden by the 3 and the 8 in C4, and by the 3, 7, and 8 in C6. Because you know that the 1 and the 5 are the true pair, you can now eliminate the other numbers from these two squares. The picture is similar in the center box: Squares D5 and F5 contain the pair 3 and 9, so you can eliminate the 1 and the 5 in both squares.

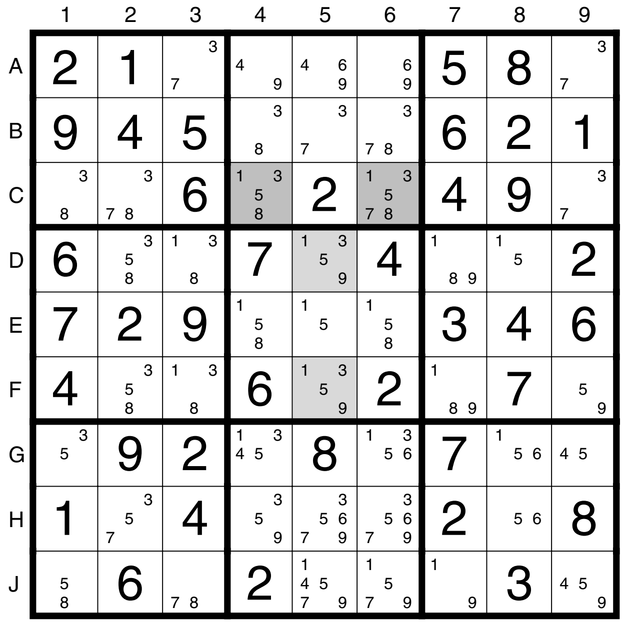

**Figure 1-3:** Two hidden pairs.

Once you reveal a hidden pair in this way, it becomes a naked pair. You can then apply the strategy for naked pairs to see whether you can eliminate other options at other positions on the grid.

# Hidden Triplets

The logic of hidden pairs can be extended to hidden triplets (or even hidden quads). Remember that a naked triplet consists of three options in three squares within the same row, column, or box, such as (4,8,9), (4,8,9) and — surprise, surprise — (4,8,9). Just like with hidden pairs, other options can cloud the view of the numbers in the triplet.

A naked triplet doesn't require all three options to occur in all three squares, and neither does a hidden triplet. Figure 1-4 shows the options (1,4,6), (1,4), and (1,4,6) in the three highlighted squares, but these options are not alone.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | 1 7 8 | 1 7 8 | **3** | **4** | **2** | **5** | **6** | 7 8 9 | 7 8 9 |
| B | 5 7 8 | **9** | **2** | 7 8 | **1** | **6** | **4** | **3** | 5 7 8 |
| C | **6** | **4** | 5 7 | **9** | 7 8 | **3** | 2 5 7 | **1** | 2 5 7 8 |
| D | 1 2 5 7 8 | 1 7 8 | 4 5 7 | **3** | 4 5 6 7 | **9** | 2 5 7 | 2 5 7 8 | 1 2 4 5 6 7 8 |
| E | 1 2 3 5 7 8 | 1 3 7 8 | **6** | 1 7 | 4 5 7 | 1 4 | **9** | 2 5 7 8 | 1 2 3 4 5 7 8 |
| F | 1 3 5 7 9 | 1 3 7 | 4 5 7 9 | **2** | 4 5 6 7 | **8** | 3 5 7 | 5 7 | 1 3 4 5 6 7 |
| G | 3 7 9 | **2** | 7 9 | 1 8 | 4 8 | 1 4 | 3 5 7 | **6** | 3 5 7 |
| H | 3 7 | **6** | **8** | **5** | **9** | **2** | **1** | **4** | 3 7 |
| J | **4** | **5** | **1** | **6** | **3** | **7** | **8** | 2 9 | 2 9 |

**Figure 1-4:** Hidden triplets.

But because 1, 4, and 6 don't appear in any other squares of the common box and column, you know that they are a triplet. So you can remove all the other options from the three squares to reveal the naked triplet.

# Moving On with Intersection Removal

The previous sections have talked about pairs, triplets, and possibly quads, within a single element (a row, a column, or a box). But you can also use pairs and triplets that are members of one element to eliminate options from another element. This is called *intersection removal*.

For example, in the bottom third of Figure 1-5, you can see multiple occurrences of the number 3, particularly in row G. In contrast to this, the bottom center box only has two occurrences of the number 3. These two squares are the only ones in the box where a 3 is possible.

Fortunately, they are both in the same row. So if you apply a little logic to this scenario, you can assume that all the other 3s in row G are out of place because 3 has to occur in one of the two squares in the central box. Therefore, you can remove all the other 3s in the row, which are shown in small circles in Figure 1-5. You could also say that the pair of 3s in the central box *points* to the other redundant 3s in row G, which makes it a *pointing pair* in sudoku jargon.

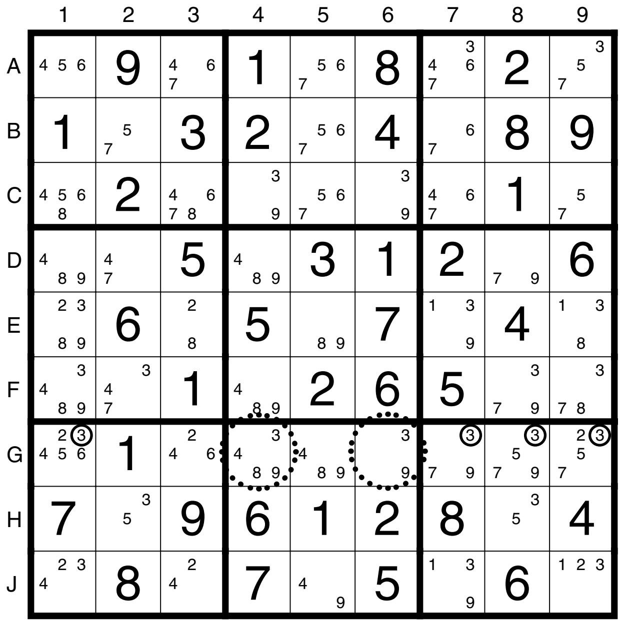

**Figure 1-5:** Intersection removal, or pointing pairs.

Pointing triplets are also possible. Figure 1-6 shows a triplet of 3s in the upper right box and in row A. This triplet lets you safely eliminate the 3s in the other squares in this box, which are circled.

Rows and columns intersect on three squares with boxes, but between a row and a column the overlap is confined to one square. So the principle of intersection must always involve a box. This also restricts the size of the intersection to pairs and triplets, as only three squares in a box can be aligned on a row or column.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | 1 6 | **9** | 4 5 | **7** | 1 4 | **2** | 3 4 5 8 | 1 3 4 5 6 8 | 1 3 5 8 |
| B | **8** | 1 3 6 | 4 5 | 1 3 | **9** | 3 4 | **2** | 1 3 4 5 6 | **7** |
| C | 1 7 | 1 3 7 | **2** | **5** | **8** | **6** | 3 4 | 1 3 4 | **9** |
| D | **9** | **4** | **6** | **2** | 3 5 | **7** | **1** | 3 5 8 | 3 5 8 |
| E | **3** | **5** | **7** | **8** | **6** | **1** | **9** | **2** | **4** |
| F | **2** | **8** | **1** | **4** | 3 5 | **9** | **6** | **7** | 3 5 |
| G | 1 5 | 1 2 | **9** | **6** | **7** | **8** | 3 4 5 | 1 3 4 5 | 1 2 3 5 |
| H | **4** | 1 7 | **3** | **9** | **2** | **5** | 7 8 | 1 8 | **6** |
| J | 5 6 7 | 2 6 7 | **8** | 1 3 | 1 4 | 3 4 | 5 7 | **9** | 2 5 |

**Figure 1-6:** Pointing triplet.

You can take intersection removal a step further and eliminate options within a box based on their position within a row or column, a variation called *box-row interactions*. Take a closer look at the central area of the sudoku puzzle in Figure 1-7.

The pair of 1s in row E are the only possible 1s in the row. Both are also within the same box. Because row E has to contain a 1 and it must go in either E4 or E6, you can safely remove all the other 1s in box 5.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | 3 | 4 7 | 8 | 6 | 4 7 | 2 | 9 | 1 5 | 1 5 |
| B | 1 | 2 4 9 | 6 | 5 | 4 9 | 3 | 2 4 | 8 | 7 |
| C | 7 9 | 2 4 7 9 | 5 | 4 7 9 | 1 | 8 | 2 4 6 | 2 3 4 6 | 3 6 |
| D | 4 | 8 | 1 9 | 1 3 7 9 | 3 6 7 9 | 1 5 6 9 | 1 6 7 | 5 6 7 9 | 2 |
| E | 2 | 6 | 7 | 1 4 9 | 8 | 1 5 9 | 3 | 4 5 9 | 5 9 |
| F | 5 | 3 | 1 9 | 1 4 7 9 | 2 | 1 6 9 | 1 4 6 7 | 4 6 7 9 | 8 |
| G | 8 | 7 9 | 4 | 1 3 9 | 5 | 1 6 9 | 2 6 7 | 2 3 6 7 9 | 3 6 9 |
| H | 6 | 1 | 2 | 8 | 3 9 | 7 | 5 | 3 9 | 4 |
| J | 7 9 | 5 | 3 | 2 | 6 9 | 4 | 8 | 1 6 7 9 | 1 6 9 |

**Figure 1-7:** Box-row interactions with twins.

You can also apply this strategy to triplets (see Figure 1-8). You can see a triplet of 3s in row A. Because they are all in the same box, you can eliminate all the other 3s in box 3.

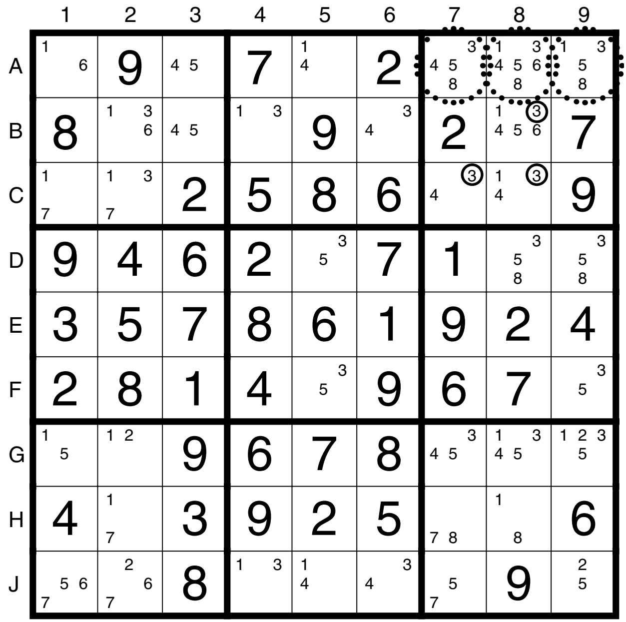


**Figure 1-8:** Box-row interactions with triplets.

REMEMBER

When a given number occurs two or three times in a single row, column, or box, you can eliminate this number from the intersection with another group. The following list gives you examples of the different types of intersection removal (*n* is the number which is paired or tripled):

- **A pair or triplet in a box:** If it occurs within a row, you can remove all other occurrences of *n* from the rest of the row.
- **A pair or triplet in a box:** If it occurs within a column, you can remove all other occurrences of *n* from the rest of the column.

- **A pair or triplet in a row:** If it occurs within a box, you can remove all other occurrences of *n* from the rest of the box.
- **A pair or triplet in a column:** If it occurs within a box, you can remove all other occurrences of *n* from the rest of the box.