1

Introduction

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The laser as a directed, high brightness, coherent source of light was a dream come true at its first demonstration in 1960. A jump of several orders of magnitude improvement towards the ideal of a single frequency, 'zero' linewidth, spatially coherent, plane wave source of light had been made. A major new research field in physics and engineering – the development of different types of lasers, aiming to cover that part of the electromagnetic radiation spectrum that can be called 'light', grew rapidly. The 'solution without a problem', as the laser was unsupportively described, soon became the light source of choice in so many applications, that as we write, the research field of laser applications is a far larger one than lasers. Indeed, much of laser development has been motivated by the significant markets for their end use.

As laser physics and engineering grew, so did the knowledge that real lasers have outputs that are dynamically and spectrally diverse. The dynamics refer to output power variations in time and the spectrum means the time-averaged, optical-frequency-spectrum. The single-frequency, frequency-stabilised dye laser or titanium sapphire laser, generating output with a sub-MHz linewidth [1] is quite different from a Kerr-lens-mode-locked titanium sapphire laser propagated through an optical fibre, generating femtosecond pulses with an optical frequency spectrum made up of a comb of mode-locked modes covering hundreds of nanometres, spaced by the pulse repetition frequency [2]. Almost any laser can be made to generate an output that is unstable in time by optical feedback of part of the output light back into the laser cavity. In some cases the resulting output can be shown to follow a well-defined route to deterministically chaotic output. Thus, the diversity of dynamic and spectral outputs available from lasers is high. Semiconductor lasers, as a subset of all lasers, represent a category in which a very broad range of the possible dynamic and spectral outputs obtainable from lasers can be achieved – from the chaotic to the narrow-linewidth, single-frequency, for example.

The semiconductor laser has a history essentially as long as that of the laser itself, being first demonstrated in 1962. However, to a large extent, the development of the semiconductor laser has been quite separate from laser development more generally. This is primarily due to the knowledge, skills and infrastructure in semiconductor device fabrication being more

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closely aligned to semiconductor physics and electronics than to the atomic and solid state physics, and optics that underpin most other laser systems. A large part of the semiconductor laser research community knows little about other types of lasers and laser systems and vice versa. However, the areas of overlap are growing. Incoherent diode laser arrays have become the pump source of choice for solid state lasers [3] and tunable diode laser systems based on the same design principles as other tunable, single frequency lasers have become common in spectroscopic, interferometric, sensing and telecommunications applications. Also, the range of wavelengths achievable with semiconductor lasers continues to grow with efficient and reliable blue and violet gallium nitride lasers now being commercially available [4] and 4–12 micron quantum cascade lasers have been used for high resolution spectroscopy. The future for semiconductor lasers and their application is bright, and it is strengthened by the diversity of possible modes of operation of the devices and systems, in addition to the broad range of device structures and material systems.

The diversity of possible dynamic and spectral outputs from a semiconductor laser is well illustrated by the semiconductor laser subject to optical feedback. The feedback of part of the output light can be achieved using a mirror, a phase conjugate reflector or a diffraction grating. The latter will also frequency filter the optical feedback field. The coupled rate equations, or delay differential equations (DDEs), which describe these systems are among the classic examples of nonlinear science. Within the practically achievable parameter space (varying the strength of the optical feedback field, for example), sequences of bifurcations and transitions to chaos are seen. The nonlinear characteristics of these semiconductor laser systems are of interest both in their own right and in contrast to other biological, mechanical, hydrodynamical and electronic nonlinear systems. They also contrast with the nonlinear dynamics of other laser systems because of the high frequencies and small timescales (picoseconds) involved. The potential applicability, of chaotic semiconductor lasers, when synchronised in pairs and cascades of similar systems and devices, in secure optical communications has given a strong applications-based motivation for fully exploring the nature of chaotic outputs from semiconductor lasers with optical feedback. Optoelectronic feedback and optical injection (injection of light from a separate source) are also of interest.

In the chapters which follow, the semiconductor laser, including all the device structures in a highly developed stage, are introduced and discussed in the context of their behaviour when subject to optical feedback. The key theory and theoretical results are presented so that the nonlinear science of these systems can be fully understood and appreciated. The full range of dynamic and spectral outputs from the systems that have been demonstrated experimentally are covered, as are the key applications in commercial systems, and systems with commercial potential. The subject of the book, semiconductor lasers with optical feedback, has been synthesized for audiences in laser physics, semiconductor lasers, and nonlinear science. As such, the commonality and complementarity of the usual language and perspectives of these three sub-disciplines have been presented for the reader.

1.1 SEMICONDUCTOR LASER BASICS

1.1.1 Semiconductor Laser Materials and Output Wavelengths

Semiconductor lasers or laser diodes are the most widely used laser ever devised. They are normally pumped directly with an injection current. They are small, easily used devices which can be produced at low cost. Laser diodes are used in such everyday items as CD players and laser printers and are finding a host of new applications ranging from medical imaging to environmental sensing. The semiconductor laser is also the source which drives optical fibre communications. Indeed, it was this latter application which provided the motivation for progressing the development of semiconductor lasers. This started with simple structure prototype devices, made of relatively poor quality material, first operated pulsed, at liquid nitrogen temperatures [7–10]. It has progressed to the present sophisticated, versatile devices, made from high quality semiconductors, which are capable of reliable, long-lived, continuous wave (cw) operation in a range of environments.

A specific aspect of the progress made in semiconductor lasers is the wide wavelength range covered by the sources using different semiconductor materials. Laser diode sources from the ultra-violet through to the mid and far infrared are available. The III–V semiconductor materials used for the devices are summarized in Table 1.1, along with some of their key physical properties. The materials are grouped as nitrides, arsenides, phosphides and antimonides which sequentially lead to longer wavelength devices when combined as ternaries (see Figure 1.1). Quaternaries combining the arsenides and phosphides are used

Table 1.1 Key III–V Semiconductor materials used in semiconductor lasers which operate at room temperature and their key physical properties

III–V Compounds	Lattice Constant (Angstroms)	Electron (Conduction band) Effective Mass$^+$	Heavy Hole (Valence band) Effective Mass$^+$	Relative Dielectric Constant	Refractive Index (Near E_G)	Band Gap (E_G) at ~300 K(eV)
AlN	3.112$^\#$ 4.982	0.40	3.53	8.5	2.15	6.28
GaN	3.190 5.185	0.20	0.80	8.9	2.5	3.425
InN	3.545 5.703	0.11	1.63	15.3	2.9	1.20–1.9$^\$$
AlAs*	5.6611	0.146	0.76	10.1	3.2	2.168
GaAs	5.6533	0.067	0.45	13.1	3.4	1.42
InAs	6.0584	0.022	0.40	15.1	3.5	0.354
AlP*	5.4635	0.83	0.70	9.8	3.0	2.45
GaP*	5.4512	0.82	0.60	11.1	3.37	2.26
InP	5.8686	0.08	0.56	12.4	3.4	1.35
AlSb*	6.1355	0.33	0.47	12.0	3.5	1.63
GaSb	6.0959	0.041	0.27	15.7	3.9	0.70
InSb	6.4794	0.014	0.34	16.8	3.5	0.175

Notes:

* Indirect Band Gap Compound, all others Direct Gap

Nitrides have a hexagonal (Wurtzite) crystal structure, hence two lattice constants, a_o and c_o, c/a~1.633 from theory for closest packed arrangement

+ Relative to the rest mass of an electron

$ Earlier value of 0.70 is now regarded as incorrect, actual value measured varies in different materials

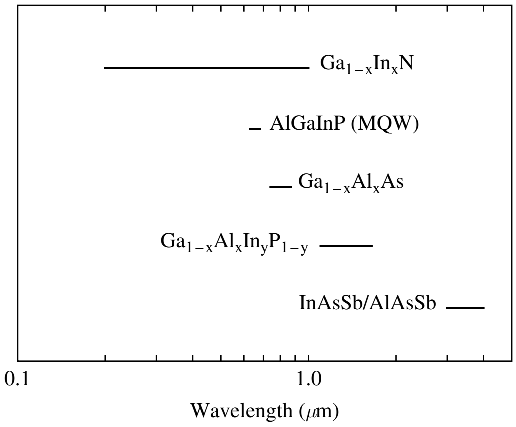

Figure 1.1 Range of emission wavelengths possible with different III–V semiconductor material compositions.

for devices at the key telecommunications wavelengths of 1.3 and 1.55 μm. In all the materials, lasers of shorter wavelength than that associated with the energy band gap of the bulk semiconductor material, a binary, ternary, or quaternary, can be obtained using quantum wells. The visible semiconductor lasers based on AlGaInP are quantum well (QW) devices, usually multiple QWs. Longer wavelength lasers (4–12 μm) have been produced using intersub-band transitions in quantum cascade lasers [5]. Strain can also be used to vary the energy band gap of the active layer material in a semiconductor laser [11].

The II–VI semiconductor materials, such as ZnSe, ZnS and $Zn_xCd_{1-x}S$, have been used to develop short wavelength devices, and the lead salts such as PbS_xSe_{1-x}, $Pb_xCd_{1-x}Se$, $Pb_xGe_{1-x}Te$, $Pb_xGe_{1-x}Te$, $Pb_xSn_{1-x}Se$, and $Pb_xSn_{1-x}Te$ have been developed as tunable systems emitting in the 2–30 μm range. These lasers operate at cryogenic temperatures. The study of optical feedback effects in semiconductor lasers has been primarily confined to room temperature systems and thus, these devices that operate at cryogenic temperatures will not be described further.

1.1.2 Semiconductor Laser Structures

In the 40 years since their first demonstration the design of semiconductor lasers has undergone an almost continuous evolution. Laser action arises in laser diodes due to a recombination of charge carriers injected into semiconductor material using an electrical contact. In the first homojunction (p-n) lasers, optical gain was achieved in a volume essentially defined by the area of the electrical contact and a thickness determined by the charge diffusion and recombination processes. The major advance, which achieved laser diode operation at room temperature, was to utilise layers of dissimilar semiconductor materials – so-called heterostructures – to effect control over the thickness of the active volume. The double heterostructure semiconductor laser [12] is now regarded as the basic, standard semiconductor laser structure. This is shown in Figure 1.2. It achieves confinement of the injected carriers in the active layer region via potential barriers as indicated in the band structure for a forward biased ppn double heterostructure laser diode shown in Figure 1.3 [12, 13].

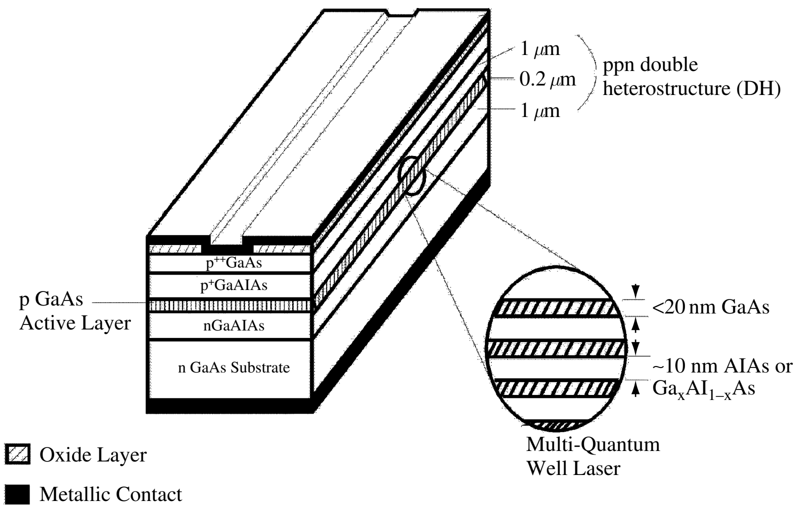

Figure 1.2 Standard GaAs/$Ga_{1-x}Al_xAs$ stripe double heterostructure laser diode (DH LD). In the case of a QW stripe DH LD the active layer is made up of one or more quantum wells as indicated by the inset expansion.

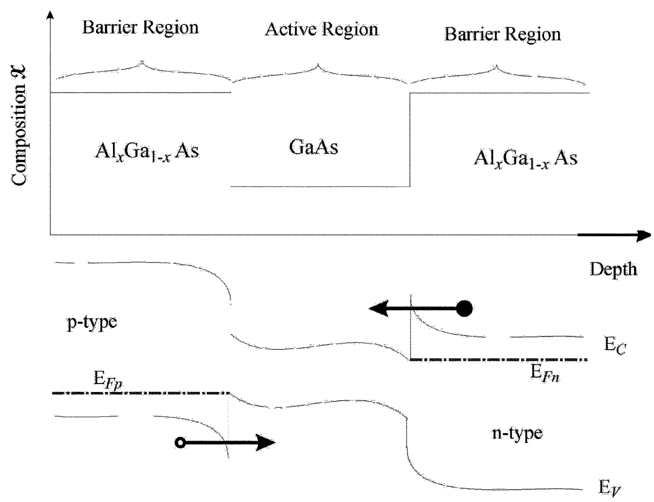

Figure 1.3 Energy band diagram of a ppn double heterostructure showing the potential confinement for the conduction band electrons injected from the n-type $Ga_{1-x}Al_xAs$ into the GaAs (active) layer and for the holes in the valence band injected from the p-type $Ga_{1-x}Al_xAs$ into the GaAs layer.

A typical semiconductor laser chip has a volume of 100 μm × 200 μm × 500 μm. This small size also brings with it a simple physical limitation on the output power which can be extracted from such lasers. Typically, single-stripe laser diode output powers are cited in milliwatts (mW). This limitation has been addressed with some considerable success so that laser diode arrays delivering many watts of output power are now commercially available. Broad area and tapered waveguide devices are also of interest for output power scaling, but in common with coherently coupled laser arrays, they introduce spatial beam quality issues which need to be addressed. One approach to obtaining high power and high brightness is to use high-power laser diode arrays as a pump laser for solid-state lasers [3].

The narrowness of the active laser stripe (0.2 μm as shown in Figure 1.2, and the limited width of the material excited laterally (~2–10 microns, determined by the width of the electroded stripe into which current is injected (Figure 1.2) or by the use of other carrier and light confinement techniques) means the beam emitted by the semiconductor laser is highly divergent, more so perpendicular to the stripe than parallel to it. Typical values of 20–25 degrees and 5–10 degrees, respectively, are seen in real devices (Figure 1.4(a)). This necessitates the use of short focal length lenses of high numerical aperture to collimate the output beam so that it may be utilized in applications. Direct butt coupling of the output into optical fibre is also common (Figure 1.4(b)).

Further innovations in device structures have been enabled by advances in material growth and material processing which have allowed practical implementation of advanced device concepts. The vigorous development of laser diodes has involved the development of a rather large range of structures which have been advanced in order to enhance one or more aspect of laser performance, e.g. output power, output beam quality, emission spectrum, and laser dynamical response. From this rich variety of designs it is possible to identify two general classes of structures whose distinction has some considerable significance in relation to their dynamical behaviour. We use the terms *horizontal cavity* and *vertical cavity* to denote these two classes of device. In the former, the laser cavity is defined by mirrors which are perpendicular to the plane of the heterostructure layers, as in Figure 1.2; in the latter, the cavity mirrors are parallel to the heterostructure layers as shown in Figure 1.5. It is generally the case that the orientation of the cavity determines the manner in which light is emitted from the lasers. With the configuration illustrated in Figure 1.2, the laser emission occurs through the cavity mirrors giving rise to the term *edge-emitting lasers*. In contrast,

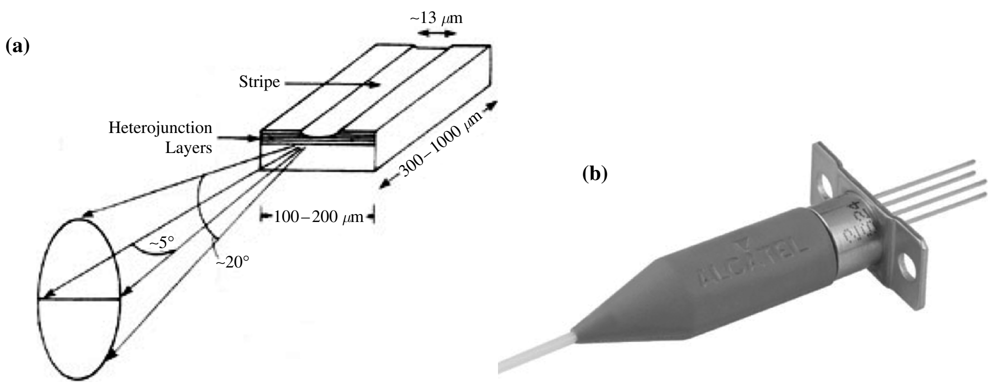

Figure 1.4 (a) shows the elliptical, divergent beam emitted by a standard semiconductor laser. This output gets collimated using a short focal length lens or the output is coupled directly to an optical fibre in a fibre pig-tailed device (b).

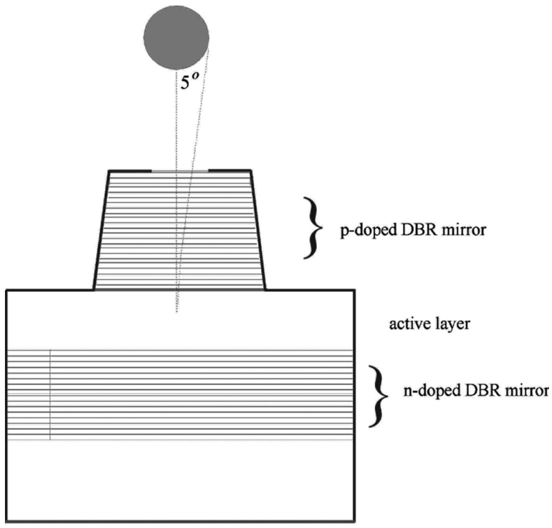

Figure 1.5 VCSEL semiconductor laser structure showing the two distributed Bragg reflector (DBR) high reflectance mirrors and the active layer of order of micron thickness.

the device in Figure 1.5 is said to be a vertical cavity *surface emitting* laser (VCSEL). It is noted that surface emission can be achieved in horizontal cavity laser diodes and conversely that edge-emission can be derived from vertical cavity structures.

The horizontal cavity laser has been the mainstay of most laser diode designs and consequently much effort has been productively employed to optimize device designs. Specific attention has been given to the use of waveguide structures which offer high beam quality by ensuring that the laser can operate stably in the lowest order spatial mode of defined polarisation. In its simplest form, with the cleaved surfaces of the semiconductor forming the mirrors, it is a moderate finesse Fabry–Perot cavity. Such Fabry–Perot cavity semiconductor lasers tend to support lasing action on a number of longitudinal cavity modes. This has profound implications for both the spectral and dynamical properties of the lasers. Robust single longitudinal mode output can be obtained using distributed feedback (DFB), distributed Bragg reflector and external or extended cavity laser structure and systems as shown in Figure 1.6.

The motivation for developing VCSELs was to utilise very short optical cavities – typically of the order of the emission wavelength – in order to ensure that only one longitudinal cavity mode would lie within the gain spectrum of the laser material. One consequence of the geometry of such devices is that they are generally found to support multiple transverse modes whose emission polarization cannot be determined *a priori*.

From the above it is clear that laser diodes are a rich source of dynamics in their own right. The opportunities for accessing interesting dynamics are immeasurably increased when the laser is subject to optical feedback. This is the subject of this book. However, we would indicate that further varieties of dynamical behaviour can be generated in laser diodes, e.g. by subjecting them to external optical injection and opto-electronic feedback. Semiconductor lasers are thus seen as an ideal test-bed for exploring concepts in nonlinear dynamics.

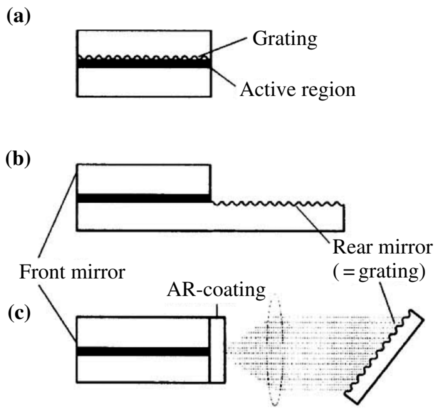

Figure 1.6 Laser structures ((a) DFB, (b) DBR, (c) external cavity semiconductor laser) for achieving robust single longitudinal mode output from semiconductor lasers, and which also allow some wavelength tunability.

1.1.3 Semiconductor Laser Gain and Output Power versus Injection Current

The gain coefficient, g(ν) in a laser based on an atomic transition is commonly derived as given in Equation (1.1) [14, Eq. 8-4-4]. This is the coefficient of exponential growth of the irradiance of the resonant light field as it propagates through the gain medium (Eq. 1.2) if the pumping (usually optical) and atomic level population densities (N_1 and N_2, with degeneracies ρ_1 and ρ_2) are spatially uniform. The variation of the gain with frequency is given by the lineshape function $l(\nu)$, λ is the centre wavelength in vacuum, n is the refractive index and τ_{sp} is the spontaneous lifetime of the laser transition. The lineshape is typically a Doppler profile for a gaseous discharge.

$$g(\nu) = \frac{\left(N_2 - N_1 \dfrac{\rho_2}{\rho_1}\right)\lambda^2}{8\pi n^2 \tau_{sp}} l(\nu) \tag{1.1}$$

$$I_\nu(z) = I_\nu(0)e^{g(\nu)z} \tag{1.2}$$

Growth in the irradiance requires a population inversion, i.e. a positive value for $(N_2 - N_1/(\rho_2/\rho_1))$ This is simply interpreted as requiring population density for the upper laser level to be greater than that (weighted) for the lower laser level, for lasing on an atomic transition. This simple interpretation does not apply to semiconductor lasers as here the laser ‘levels’ are bands (the upper level is the conduction band and the lower level is the valence band), with energy widths of several eV, made up of a quasi-continuum of energy levels described statistically by a density of states. Nearly all the energy levels in the valence band are occupied by electrons and it is not probable that more electrons can be elevated to the conduction band than are left behind in the nearly filled valence band. However, statistically, there are many electrons in the conduction band and many ‘holes’ (vacancies) in the upper levels of the valence band to allow efficient direct radiative recombination of electrons and holes. The band theory of semiconductors for non-thermal equilibrium systems, utilizing Fermi Dirac statistics and quasi-Fermi levels, is required to describe the inverted semiconductor [11, 15]. The resulting gain per unit length can be written in the form [11, Eq. 4.37]:

$$g_{21} = g_{\max}(E_{21})(f_2 - f_1) \tag{1.3}$$

where

$$g_{\max}(E_{21}) = \frac{\pi q^2 \hbar^2}{n\varepsilon_o c m_o^2 h\nu_{21}} \left|M_T(E_{21})\right|^2 \rho_r(E_{21})$$

$$f_1 = \frac{1}{\exp(E_1 - E_{Fv}) + 1}$$

and

$$f_2 = \frac{1}{\exp(E_2 - E_{Fc}) + 1}\,.$$

E_{Fv} and E_{Fc} are the nonequilibrium quasi-Fermi levels for the valence band and conduction band, respectively. E_{21}(k) is the energy between the conduction band and valence band, $\rho_r(E_{21})$ is the reduced density of states which has a different functional form for bulk semiconductor and reduced dimension structures such as quantum wells. $|M_T(E_{21})|^2$ is the transition matrix element, ν_{21} is the transition frequency, q is the magnitude of the electron charge, $\hbar$ is Planck’s constant divided by 2π, n is the refractive index of the semiconductor material, m_o is the rest mass of the electron, c is the speed of light in vacuum and ε_o is the permittivity of free space. The full derivation of Equation (1.3) can be found in Chapter 4 and appendices of [11]. Another excellent discussion is found in Chapter 11 of [15]. Values of g_{21} of 10^3–10^4 cm^{-1} are achieved for different semiconductor laser structures using high quality materials.

Describing the semiconductor laser phenomenologically, when the laser is operating in a steady state the gain per unit length must equal the loss per unit length. The loss includes the light transmitted out of the cavity through the partially reflecting laser mirrors and a second component describing the distributed loss per unit length, α due to scattering, free carrier absorption etc., in the cavity. Also, only a factor $\Gamma(0 < \Gamma < 1)$ of the light is confined to the volume which is subject to gain via the injection of carriers. Γ is called the confinement factor. For one round trip in the cavity under steady state conditions this gives:

$$\text{Gain} = 1 + \exp[(\Gamma \text{g} - \alpha)L]R_1 \exp[(\Gamma \text{g} - \alpha)L]R_2 \tag{1.4}$$

where L is the length of the laser cavity and, R_1 and R_2 are the reflectances of the semiconductor laser facets. Solving this yields:

$$\Gamma \text{g}_{\text{th}} = \alpha - (1/2L)\ln(R_1 R_2) \tag{1.5}$$

Making the link between Equation (1.1) and Equation (1.5) [13], the threshold current density is

$$J_{th}(0) = \frac{qd\,8\pi\,\Delta\nu}{\eta_i \lambda^2}\left(\alpha - \frac{1}{2L\Gamma}\ln(R_1 R_2)\right) + J(T) \tag{1.6}$$

where $\Delta\nu$ is the full width at half maximum of the gain profile, d is the stripe thickness for a DH semiconductor laser and η_i is the internal quantum efficiency, (the number of photons produced per injected carrier) and $J(T)$ is a term to take account of the effective non-zero $N_1(\rho_2/\rho_1)$ value at temperatures above 0 K. The threshold current I_{th}, which is easily measured in practice, can be calculated from the threshold current density if the effective area over which the carriers are injected can be determined. The temperature dependence of I_{th}, or J_{th}, is of the form:

$$J_{th}(T) = J_{th}(0)\,e^{T/T_0} \tag{1.7}$$

where T_0 is called the characteristic temperature. It has values >120 K for near infrared GaAs/GaAlAs DH lasers and ~150–180 K is determined for QW GaAs/GaAlAs lasers. The stimulated optical power inside the laser cavity increases linearly with the injection current according to:

$$P_{in} = \frac{(I - I_{th})}{q}\eta_i h\nu \tag{1.8}$$

The external power, P_{ex} is:

$$P_{ex} = \frac{\ln\left(\dfrac{1}{R_1 R_2}\right)}{2\alpha L + \ln\left(\dfrac{1}{R_1 R_2}\right)}\frac{(I - I_{th})}{q}\eta_i h\nu \tag{1.9}$$

The external differential quantum efficiency, η_{ex} is:

$$\eta_{ex} = \frac{\ln\left(\dfrac{1}{R_1 R_2}\right)}{2\alpha L + \ln\left(\dfrac{1}{R_1 R_2}\right)}\eta_i \tag{1.10}$$

The slope efficiency listed in the specification sheets of commercial semiconductor lasers is given in W/A and can be calculated from η_{ex} as:

$$\eta = \eta_{ex}\left(\frac{h\nu}{e}\right) \tag{1.11}$$

Often it is the slope efficiency pertaining to the output from one of the output facets of the device that is of interest. For a laser with a large output coupling, the values associated with facets of R_1 and R_2 are given by [16]:

$$\begin{aligned} \eta_1 &= \eta\frac{(1 - R_1)\sqrt{R_2}}{\left(\sqrt{R_1} + \sqrt{R_2}\right)\left(1 - \sqrt{R_1 R_2}\right)} \\ \eta_2 &= \eta\frac{(1 - R_2)\sqrt{R_1}}{\left(\sqrt{R_1} + \sqrt{R_2}\right)\left(1 - \sqrt{R_1 R_2}\right)} \end{aligned} \tag{1.12}$$

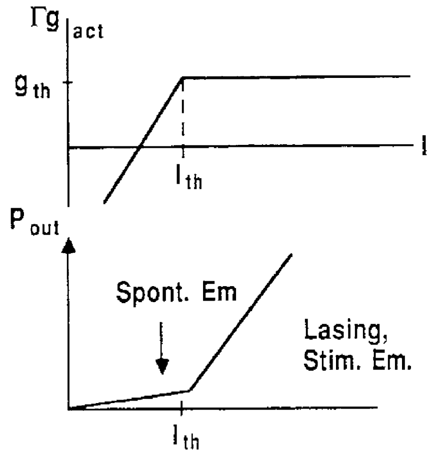

Figure 1.7 Gain per unit length and output power as a function of the injection current. *Source*: Reproduced with permission from [17].

A typical output power versus injection current graph for a semiconductor laser is shown in Figure 1.7. Semiconductor lasers operate as LEDs (spontaneous emission predominantly) below the injection current threshold for lasing. Figure 1.7 indicates that this contribution due to spontaneous emission is large. It also contributes to noise in the laser output making semiconductor laser noisy compared to other lasers.

1.1.4 Semiconductor Laser Relaxation Oscillations, Noise, Modulation and Linewidth Enhancement Factor

Other distinguishing features of semiconductor lasers, relative to most other lasers, that contribute to their dynamic timescales and diversity, and the extreme sensitivity to optical feedback include enhanced spontaneous emission; enhanced linewidth (due to spontaneous emission, and the strong coupling between variations in intracavity optical power and frequency, through the irradiance and carrier density dependent refractive index); high relaxation oscillation frequency, which scales as the square root of the injection current above threshold; a modulation bandwidth up to the relaxation oscillation frequency, and mixed FM and AM arising when the injection current is modulated, again, due to the coupling between power variation and frequency variation. All these topics are interrelated but they are dealt with separately, mostly, in standard texts on semiconductor lasers. Some key results and references are summarized here.

Relaxation oscillation of the output power has been observed in most lasers and occurs with a time characteristically long compared to the laser cavity decay time, or the cavity round trip time. Typical periods of the relaxation oscillations in, for example, solid state lasers such as Nd:YAG are ~ 0.1–$1\ \mu$s [14]. The basic mechanism is an interplay between the oscillation field in the resonator and the atomic/molecular/solid state inversion. An increase in the intracavity power leads to a reduction in the inversion due to the increased rate of stimulated emission. This in turn leads to a reduction in the power, and so on cyclically.

In semiconductor lasers the competing timecales are very much shorter. The inversion can change dynamically on nanosecond timescales τ_c and the photon lifetime τ_p in the short, relatively low finesse semiconductor laser cavities is 1–3 picoseconds, typically. The relaxation oscillations in this case occur at GHz frequencies, f_r. The relaxation oscillation frequency scales as the square root of the injection current for DH semiconductor lasers. The relaxation oscillations are damped with a rate f_d which scales as $f_r^{\,2}$.

$$f_r = \frac{1}{2}\left(\frac{1}{\tau_c \tau_p}\right)^{\frac{1}{2}}\left(\frac{I}{I_{th}} - 1\right)^{\frac{1}{2}} \tag{1.13}$$

Small signal modulation of a semiconductor laser is achieved by adding a small sinusoidal current of frequency f, to a dc injection current that operates the laser well above threshold. In this case the rate equations which describe the carrier density and photon density, with the modulation, can be linearised and solved. The general power modulation response, as the modulation frequency is varied, is given by the modulation transfer function (Eq. 1.14), the magnitude squared of which is shown in Figure 1.8. The power modulation follows the current modulation up to frequencies near the relaxation oscillation frequency, with a resonant response at a frequency close to, but slightly lower than the f_R (Eq. 1.13). The modulation transfer function also depends on the damping constant, $f_d(\omega_x = 2\pi f_x)$.

$$H(\omega_m) = \frac{1}{1 + \dfrac{i\omega_m \omega_d}{\omega_r^2} + \left(\dfrac{i\omega_m}{\omega_r}\right)^2} \tag{1.14}$$

The modulation of the carrier density modulates both the power and the refractive index. This gives rise to frequency modulation which in semiconductor lasers dominates the power modulation. The ratio of the frequency modulation index to the power or irradiance modulation index is proportional to the linewidth enhancement factor, α, which is given by

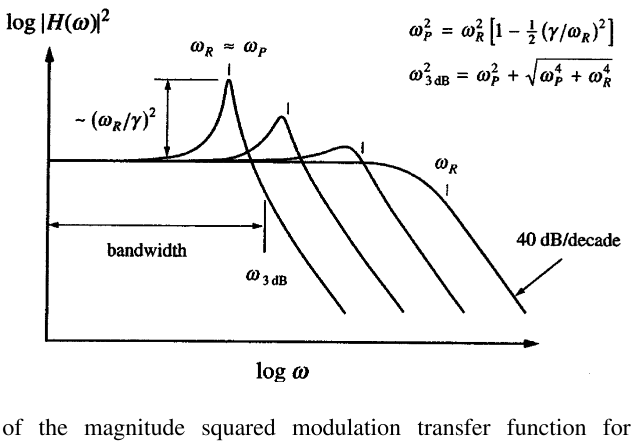

Figure 1.8 Log of the magnitude squared modulation transfer function for power/irradiance modulation as a function of (log circular) modulation frequency. The function broadens and flattens as f_r and f_d increase.
Source: After [11].

Equation (1.15). This quantity is the ratio of the change in the real part of the refractive index with carrier density to the change in the imaginary part of the refractive index with carrier density. It has typical values of 1–7 and the fact that it is usually larger than 1 has major implications for the dynamics as described here and the behaviour of semiconductor lasers with optical feedback, as discussed in the subsequent chapters of this book.

$$\alpha \equiv \frac{dn/dN}{dn_i/dN} = -\frac{4\pi}{\lambda}\frac{dn/dN}{dg/dN} \tag{1.15}$$

The qualitative comparison of the FM and IM (irradiance modulation) response of a semiconductor laser is shown in Figure 1.9.

The electric field (and hence the output power) of a semiconductor laser has fluctuations in amplitude and phase which result in power (irradiance) noise and phase/frequency noise. There are fundamental limits associated with spontaneous emission and statistical fluctuations in the carrier density. It can be shown that [18] the power spectral density function, $S_A(f)$, ('intensity' noise) is given by:

$$S_A(f) = \frac{A_0^2(\delta f)_{ST}}{\pi}\frac{f^2 + (\omega_d/2\pi)^2}{(f^2 - f_r^2)^2 + (\omega_d/2\pi)^2 f^2} \tag{1.16}$$

and the power spectral density of the frequency fluctuations is:

$$S_F(f) = \frac{(\delta f)_{ST}}{\pi}\left[1 + \frac{\alpha^2 f_r^4}{(f_r^2 - f^2)^2 + (\omega_d/2\pi)^2 f^2}\right] \tag{1.17}$$

Both these noise spectra broaden as the output power (injection current) increases. The fundamental laser linewidth, the Schawlow Townes linewidth, $(\delta f)_{ST}$, from standard laser physics appears in these expressions. This also includes a spontaneous emission factor that is greater than 1 (typically 2.5–3) which takes account of the large spontaneous emission contribution. The laser line profile, and the associated linewidth, can be determined from the power spectral density of the electric field (an autocorrelation). The functional form is a

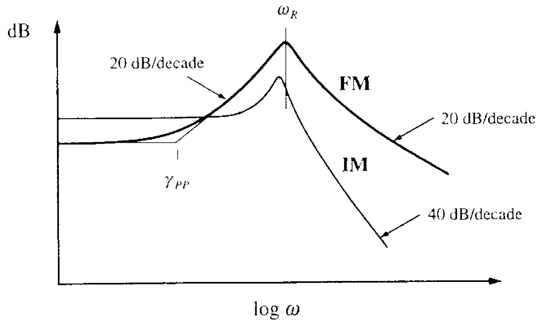

Figure 1.9 Qualitative comparison of the FM and PM/IM response of a semiconductor laser. *Source*: After [11].

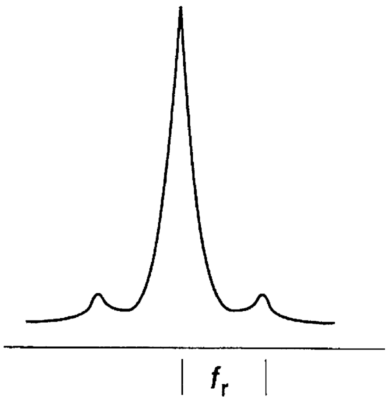

Figure 1.10 Line profile of a semiconductor laser – a Lorentzian with satellite peaks at the relaxation oscillation frequency.

Lorentzian with sidebands at the relaxation oscillation frequency as shown in Figure 1.10. These satellite peaks are suppressed in heavily damped semiconductor lasers. The linewidth is enhanced by the factor $(1 + \alpha^2)$ compared to the Schawlow Townes linewidth.

1.2 NONLINEAR DYNAMICAL SYSTEMS

The study of nonlinear systems has long antecedents, having its origins in studies pursued by Poincaré in the nineteenth century and has experienced steady development – particularly within mathematical domains. The stimulus which has brought-more general awareness of the relevant mathematical techniques is the realization that chaos is a universal phenomenon which can arise in relatively simple nonlinear systems. The popular imagination has been caught by the application of chaos theory to weather forecasting as exemplified in the work of Lorenz. For laser specialists, a key event was the 1975 demonstration by Haken that equations of laser dynamics are isomorphic to the Lorenz equations. This observation stimulated significant experimental and theoretical analysis of routes to chaos in a variety of laser systems. In this way methods of nonlinear dynamical systems were absorbed into the mathematical capabilities of laser physicists. The need to study a number of practical issues had also caused several workers to use nonlinear systems theory to study, in particular, dynamical instabilities in laser diodes.

Nonlinear dynamical systems theory is largely concerned with classifying the conditions under which systems undergo a qualitative change in dynamics. Such changes in dynamics are termed *bifurcations*. The simplest example is the process by which a system changes from a steady state to an oscillatory state – this is termed a Hopf bifurcation. A very familiar bifurcation sequence is one where a change in a nonlinear parameter causes a system to undergo successive doublings of the period of regular oscillations. The period-doubling bifurcation sequence is a classical 'route to chaos'.

Central to identifying the conditions for bifurcations is the need to define the stability of a given dynamical state. In essence, the general approach taken is to examine the effects of applying a small perturbation to the state of interest. That process allows linearization of the system dynamics and thus enables classical techniques such as Routh–Hurwitz analysis to be applied to test the system stability.

Techniques of nonlinear system theory have been applied to several aspects of laser diode dynamics including studies of their response to direct-current modulation and external optical injection.

1.3 SEMICONDUCTOR LASERS WITH OPTICAL FEEDBACK

The simplest experimental configuration in which a semiconductor laser is subject to optical feedback is illustrated in Figure 1.11. Here the optical feedback is derived from a plane mirror which reflects a part of the laser output back into the laser. It is the behaviour of this experimental configuration which is the attention of a substantial part of this book. It is perhaps somewhat surprising that this simple experimental arrangement should admit such detailed study as to require an entire book to describe the phenomena which thereby arise. It is then even more surprising to appreciate that the study of this arrangement is still far from complete. The present volume seeks to represent the current understanding of the behaviour of the laser in this configuration. However, this book also points to the need for further studies of important experimental and theoretical issues which remain unresolved. Moreover, this book also treats other scenarios where the behaviour of semiconductor lasers can be influenced by alternative forms of optical feedback, such as phase conjugate optical feedback.

The importance of studying optical feedback effects in semiconductor lasers is due to the confluence of a number of significant features arising from both practical and theoretical considerations. The coincidence of all these factors in an experimental configuration of vital commercial importance provides a powerful impetus for gaining a detailed understanding of the laser behaviour in this configuration.

It is the commercial application of semiconductor lasers in optical fibre communication systems which provided the primary practical motivation for studying the behaviour of semiconductor lasers subject to optical feedback. In optical fibre communication systems, information is impressed on light generated in laser diodes. The information-carrying light is transmitted, generally over quite long distances, by optical fibre waveguides. In order to operate successfully, such an optical fibre communication system must exhibit a number of basic features expected in all communications systems. In particular, the communication channel must be sufficiently free of extraneous signals – noise – to ensure accurate recovery of the transmitted information. It is precisely in this respect that the study of optical feedback effects on semiconductor lasers becomes of major importance.

The unfortunate fact is that semiconductor lasers are highly prone to generating noise which may impair the operation of optical fibre communication systems. In part, this

Figure 1.11 Semiconductor laser with facets with power reflectance R_1 and R_2 subject to optical feedback from an external mirror with reflectance R_3.

noise-generating propensity arises from the basic physics of semiconductor lasers but more pertinently the process of coupling laser light into an optical fibre provides a further potential source of significant noise. The origin of this noise is the effect on the laser of reflections of very small amount of light from the ends of the optical fibre to which the light is being coupled. It is emphasized that such reflections are normally extremely small but it is now widely appreciated that they may have a profound effect on the behaviour of the laser. In many cases those effects are deleterious and specifically they may lead to a dramatic increase in the laser output noise. In turn, this will affect the performance of the optical fibre communication system.

In the awareness of the dangers inherent in even very small back-reflections from the tips of optical fibres, the system designer can find very effective means for reducing these threats to successful optical communications. Specifically, laser diode transmitter modules may accommodate optical isolators to provide strong immunity of the lasers to optical feedback effects. A rather severe penalty is literally paid in adopting this approach: effective optical isolation is rather expensive and hence the commercial advantage of using low-cost laser diodes is offset. Nevertheless the approach is perfectly reasonable for applications – such as long-haul high-data-rate communication systems – where high returns on investment are expected. However, the approach becomes questionable when emphasis is given to deploying optical communication systems in a more general context where low-cost components are required. A constructive approach to this problem would be to design a semiconductor laser which is inherently immune to optical feedback effects. Such a challenge provides a fresh practical impetus for gaining a thorough understanding of optical feedback effects on laser diodes.

Although the demands of optical fibre communication systems are quite properly seen as the initial practical reason for studying optical feedback effects, it is not the case that this exhausts the practical motivation for investigating these effects. Semiconductor lasers are increasingly the laser of choice for many practical applications in a wide range of technical areas spanning aeronautics, biotechnology, dentistry, environmental sensing, medicine, metrology, nanotechnology, security and transportation. In many, if not all, of these areas the demands are for versatile, low-cost, high-performance devices which operate in a reliable manner. The utilization of laser diodes in these contexts will often bring dangers of the impact of optical reflections – indeed, in some cases, the utilization will rely on optical feedback effects – and again will demand an understanding of the behaviour of the laser in these contexts.

The provision of the underpinning knowledge required for the practical use of laser diodes is a compelling reason for sustaining research effort on optical feedback effects. Such studies also have legitimacy as an end in themselves. The basic physics of semiconductor lasers is a fascinating discipline whose vitality is sustained by the versatility and variety of laser diodes. There are rather profound reasons for enlarging the field of activity by also undertaking fundamental studies of the effects of optical feedback.

Optical feedback may be used as a convenient tool for probing the basic physics of semiconductor laser operation. In this context, controlled optical feedback would be deliberately introduced and the response of the laser determined. A proper interpretation of the observed response relies on the availability of fundamental knowledge of optical feedback effects in laser diodes. Largely unexplored avenues for investigation would include elucidating ultrafast nanoscale dynamical processes in a variety of semiconductor lasers.

Optical feedback effects on laser diodes have a particularly important place in the fundamental study of optical chaos in particular, and of chaos, in general. Here the ease of operation of laser diodes makes them particularly attractive for experimental investigations. Counter-posed to their ease of use is the complexity of the basic physics of laser diodes. These features provide a potent combination which is expected to present significant scientific challenges for many years to come.

Finally, it is important to draw attention to the opportunities which arise in the theoretical analysis of these effects. It is significant that a small but growing community of mathematicians has begun to appreciate the challenges inherent in studying laser diode optical feedback dynamics. It is foreseen that this will also remain a fertile area for activity for many years.

1.4 LANDMARK RESULTS: THEORY AND EXPERIMENT

The response of semiconductor lasers to the effects of optical feedback has attracted interest for almost as long as semiconductor lasers have existed. In 1970 Broom *et al.* in Berne, Switzerland (where half a century earlier Einstein had worked on the basis concepts of lasers) reported the observation of dynamical effects arising in semiconductor lasers coupled to an external resonator [19]. Later Morikawa *et al.* discussed the appearance of oscillations in the output of semiconductor lasers subject to optical reflections [20]. A particular early focus for such investigations was provided by the recognition of the importance of optical feedback effects in determining the noise properties of lasers. Such considerations were particularly driven by the requirements of optical communications systems whose reliability would be expected to be compromised by noise penalties. This was the subject of theoretical investigations by Hirota and Suematsu [21]. Interestingly this paper stressed the effects of noise in analogue modulation systems. This paper pointed out that the behaviour of the laser depends upon the distance between the laser and the relevant external reflector. Specifically, the behaviour was classified as being either that of a ‘double cavity state’ or an ‘external injection state’ depending whether the distance to the external reflector was smaller or greater than the coherence length of the laser. On the basis of their analysis, the authors offered practical strategies for the reduction of noise effects – by driving the laser well above threshold – and also their possible elimination by the use of optical isolation. Nevertheless it was recognized in this paper that optical feedback induced noise would be expected to play a central role in many perceived applications of semiconductor lasers.

Appreciation of the fundamental significance of optical feedback effects in understanding the behaviour of semiconductor lasers emerged following complementary studies which, paradoxically, indicated that the effects of optical feedback could actually be beneficial for semiconductor laser performance. The positive aspects of optical feedback were seen when consideration was given to the spectral properties of semiconductor lasers. The mechanisms for mode selection in semiconductor lasers represent a continuing area for fundamental investigation which has long antecedents in the pioneering work of Bogatov *et al.* [22]. External feedback was shown to enhance longitudinal mode selection and hence could be useful in narrowing the emission spectrum of semiconductor lasers [23]. The intimate connections between the dynamics and spectra of semiconductor lasers were identified in early work [24] and remain an ever-present theme in the development of semiconductor lasers.

The contrasting effects which arise with variation of the distance between the laser and an external reflector were reported by Chinone *et al.* [25]. This work showed that enhancements of relaxation oscillations or induced self-sustained oscillations occurred with relatively large distances to the external mirrors while reduction in the relaxation oscillations was achieved when the external mirror was less than a few centimetres from the laser.

It was in this context that an extremely important paper was prepared at NEC, Kawasaki, by Lang and Kobayashi [26]. In introducing their work, the authors of this paper allude to previous efforts to understand the variety of behaviour which had already been observed due to optical feedback effects on semiconductor lasers. In this discussion the authors observe that: 'The compound cavity effects in conventional lasers have been well known. However, previous experimental observations of external feedback effects in semiconductor lasers did not appear to permit simple interpretation.' With the benefits of hindsight, this may be regarded as a significant under-statement.

In making their observations Lang and Kobayashi had very clear physical reasons for expecting rather complex behaviour to emerge from semiconductor lasers. They pointed out that the semiconductor lasers available to early experimentalists were susceptible to transverse mode instabilities and generally supported multi-longitudinal mode oscillation and hence 'exhibited erratic behaviour even without external feedback'. Moreover, Lang and Kobayashi drew attention to aspects of the basic physics of semiconductor laser gain media which were likely to lead to complex behaviour under conditions of external feedback: broad gain spectrum; temperature dependence of material refractive index; carrier-density dependence of the refractive index. The paper includes experimental measurements of the behaviour of a semiconductor laser subject to optical reflections from a mirror a few centimetres from the laser. Among a number of interesting features of device behaviour the authors reported experimental observation of bistability and hysteresis in the light output versus drive current characteristics of the lasers.

The Lang–Kobayashi paper has, however, become synonymous with the theoretical model, it attempts to describe the behaviour of single-mode semiconductor lasers subject to external optical feedback. Use of this model or its generalizations is ubiquitous in the subsequent literature. This widespread application of the model is explained rather simply in terms of its remarkable ability to describe almost all salient experimental features of laser diodes subject to optical feedback. This ability is underlined by the frequent citations of this model through the present volume as well as in the research literature.

The appreciation of the subtleties inherent in the response of semiconductor lasers to external optical feedback led to increased and widespread efforts to record and understand the variety of dynamical and spectral features which could thereby be accessed in semiconductor lasers. The manner in which the single-mode linewidth of semiconductor lasers could be influenced by optical feedback was of particular interest in the context of developing coherent optical communications systems. Investigations showed that the laser linewidth could be both narrowed [27–30] and broadened [31, 32]. These seemingly contradictory responses were relatively easily understood as being a consequence of the sensitivity of the response to the phase of the reflected light.

The noise properties of semiconductor lasers subject to optical feedback have remained a theme of considerable theoretical and practical interest [33–36]. More generally, much effort has been directed at measuring and modelling the impact of optical feedback on the dynamical behaviour of semiconductor lasers (for early work see, for example, [37–40].

1.5 OVERVIEW OF FEEDBACK RESPONSE: REGIMES I–V

Hirota and Suematsu [21] had pointed to the significant role played by the distance between the laser facet and the external mirror reflector in determining the nature of the response of semiconductor lasers to optical feedback. An experimental study of the so-called 'regimes of feedback', of semiconductor lasers was undertaken by Tkach and Chaprylyvy [41]. Here attention was paid to both the distance between the laser and the external reflector and the strength of the optical feedback. From this study a widely utilized terminology has emerged of five identified operating regimes which are conventionally labelled Regimes I to V. The regimes may be characterized by reference to either the dynamical or spectral properties of the laser when subject to appropriate feedback strengths. In Regime I, with weak optical feedback, the laser linewidth can be either narrowed or broadened depending upon the phase of the optical feedback; Regime II can be characterized by the appearance of longitudinal mode hopping; in regime III the laser becomes stable and locks to the mode with minimum single-mode linewidth; with increased feedback the linewidth of the laser broadens dramatically – a phenomenon referred to as 'coherence collapse' – this is Regime IV; for further increase in feedback strength into Regime V the laser enters a stable external cavity mode of operation. The laser facet needs to be AR coated to allow feedback levels for Regime V to be achieved. Rather extensive studies have been made of the five regimes of semiconductor laser operation (see, for example, [34]). Considerable effort has, in particular, been given to determining the nature of the laser dynamics in the coherence collapse regime first reported by Lenstra *et al.* [42]. Of specific interest for proposed applications of nonlinear dynamical effects in laser diodes was the relation between coherence collapse and chaotic dynamics which had been identified in external cavity lasers by Cho and Umeda [43].

The details of the theoretical models introduced here and the resultant predicted behaviour are covered in Chapters 2 and 3. In Chapter 2 the Lang–Kobayashi model for optical feedback in a single mode laser, and its further development by subsequent researchers, are fully described. The effect of the distance from the laser output facet to the external reflector on the dynamics is reviewed. An iterative model valid for multi-mode semiconductor lasers and arbitrarily large values of the optical feedback strength is introduced and the results are contrasted with the Lang–Kobayashi model within the parameter space that the LK model is valid. In Chapter 3 a theory of generalized optical feedback is developed. This approach uses the recent developments in techniques for solving delay differential equations. It is applied to the special cases of frequency filtered feedback and phase conjugate feedback.

In Chapter 4 the enormous range of experimental studies of optical feedback, including phase conjugate and frequency filtered feedback, in standard, commercial, single stripe, semiconductor lasers is fully reviewed. Comparisons are made with the theoretical predictions in the cases covered.

In Chapter 5 a bifurcation analysis approach from a more mathematical perspective is introduced. This shows that with the newly available mathematical tools it is possible to explore the nonlinear dynamics of the system of a semiconductor laser with optical feedback in a much more complete way within the very large parameter space available, by varying things like the feedback strength, injection current to the laser, etc. Such analyses are expected to lead to new predictions of useful and interesting dynamic behaviours in the near future.

Chapter 6 introduces the synchronization of two similar chaotic semiconductor lasers with optical feedback. This is one of the synchronized chaos sources being researched for possible application in private optical communication systems. This application is described more fully in Chapter 9.

1.6 OUTLINE OF APPLICATIONS

It is a salutary observation that novel applications for laser diodes subject to optical feedback were actually identified very early in the study of this behaviour. It is perhaps unsurprising that one of the earliest proposals for such a novel application emanated from one of the key figures in this activity: Kobayashi – who suggested the use of optical feedback to reduce waveform distortion in laser modulation [44]. Similar early inventiveness was displayed in the proposal for using a laser diode as both a source and a detector [45].

Chapters 7 to 9 cover the key systems for applications, and the applications themselves, of semiconductor lasers with optical feedback. In Chapter 7 the development of self-mixing interferometry which can give sub-nanometre sensitivity in pathlength measurements, is reviewed. In Chapter 8 the development of tunable, single frequency semiconductor lasers and systems are reviewed, along with the enormous range of applications in sensing and optical communications. The appearance of chaotic dynamics in semiconductor laser subject to optical feedback has been identified as a means of developing private communications systems exploiting this behaviour. Chapter 9 deals with this topic.

With increased understanding of the behaviour of semiconductor lasers subject to optical feedback it is to be expected that further applications will be found for this configuration. Major areas where applications have and continue to be identified include sensors and spectroscopy. The former area can exploit the changes in the optical emission intensity which occur in lasers subject to optical feedback. In the latter area the more subtle changes in the spectra of semiconductor lasers can be put to good effect.

As the range of semiconductor lasers expands, so it can be envisaged that there will follow an expansion in the applications where the response of those lasers to optical feedback is utilised to good effect. One of the functions of this book is to provide the fundamental knowledge which will stimulate the broadening of the applications base of semiconductor laser subject to optical feedback.

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