1

Fundamentals of Array Signal Processing

1.1 INTRODUCTION

The robust design of an adaptive array system is a multi-disciplinary process, where component technologies include: signal processing, transceiver design, array design, antenna element design, and signal propagation characteristics. The 'glue' between the technologies is provided by the system engineer who specifies the requirements for each, so that the complete adaptive array system will operate according to the required performance criteria.

This chapter introduces the fundamentals that enable the design of these component technologies and sets the scene for much of the remainder of the book. It is an important prerequisite for chapters 2–5 where the fundamentals are expended to include an understanding of adaptive antenna arrays as well as practical issues related to adaptive antenna design.

The first section of this chapter is dedicated to the very fundamentals of antennas, where the reasons why antennas are capable of transmitting and receiving an electromagnetic signal are explained. Surprisingly, transmission and reception are facilitated by a small asymmetry in Maxwell's equations, which triggers an electromagnetic wave to decouple from (transmission) and couple into (reception) a medium carrying free electric charges (antenna). The application of Maxwell's equations to the most fundamental radiation element, the Hertzian dipole, is then explained. It allows some common antenna terminology to be defined, which is not confined to Hertzian dipoles only but is also applicable to practical antenna elements.

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This brings us to a brief description of typically occurring antenna elements, which themselves might be part of larger antenna arrays which are subsequently introduced. Here, commonly occurring antenna arrays, such as the linear and circular antenna arrays, are discussed. Antenna arrays are then applied to achieve spatial filtering which enhances the signal strength (beamforming) and weakens the interference power (nulling) in wireless communication systems.

A favourable candidate to accomplish spatial filtering is the adaptive antenna array, where channel parameters are fed into the algorithms controlling the adaptive beamforming so as to optimise the performance. The conditions under which beamforming can be achieved and which inter-element spacings are required are then explained.

The chapter is finalised with the conclusions and some problems related to the fundamentals of antenna arrays.

1.2 THE KEY TO TRANSMISSION

1.2.1 Maxwell's Equations

Starting from some assumptions and observations, Maxwell derived a set of mutually coupled equations, the so called *Maxwell Equations*, which paved the way to the field of electrodynamics, part of which allows a proper understanding and design of antenna elements and arrays. In differential form, the four equations are:

$$\operatorname{div}\mathbf{D}(\mathbf{r},t) = \rho(\mathbf{r},t) \tag{1.1}$$

$$\operatorname{div}\mathbf{B}(\mathbf{r},t) = 0 \tag{1.2}$$

$$\operatorname{curl}\mathbf{E}(\mathbf{r},t) = -\frac{\partial\mathbf{B}(\mathbf{r},t)}{\partial t} \tag{1.3}$$

$$\operatorname{curl}\mathbf{H}(\mathbf{r},t) = \frac{\partial\mathbf{D}(\mathbf{r},t)}{\partial t} + \mathbf{J}(\mathbf{r},t) \tag{1.4}$$

where $\mathbf{E}(\mathbf{r},t)$ in [V/m] is the vector representing the *electric field intensity*, $\mathbf{D}(\mathbf{r},t)$ in [C/m^2] is the *electric flux density*, $\mathbf{H}(\mathbf{r},t)$ in [A/m] is the *magnetic field intensity*, $\mathbf{B}(\mathbf{r},t)$ in [T] is the *magnetic flux density*, $\rho(\mathbf{r},t)$ in [C/m^3] is the *charge density* and $\mathbf{J}(\mathbf{r},t)$ in [A/m^2] is the *current density*. All of the above electromagnetic field variables depend on the spatial position with respect to some coordinate system, $\mathbf{r}$ in [m], and the elapsed time, t in [s].

The electric and magnetic field vectors can be related through the material equations, i.e.

$$\mathbf{D}(\mathbf{r},t) = \epsilon_0\epsilon_r\mathbf{E}(\mathbf{r},t) \tag{1.5}$$

$$\mathbf{B}(\mathbf{r},t) = \mu_0\mu_r\mathbf{H}(\mathbf{r},t) \tag{1.6}$$

where $\epsilon_0 \approx 8.85 \cdot 10^{-12}$ [F/m] is the free space permittivity, ϵ_r is the material dependent *relative permittivity* (also called the *dielectric constant*), $\mu_0 \approx 1.257 \cdot 10^{-6}$ [H/m] is the *free space permeability* and μ_r is the material dependent *relative permeability.*

Finally, the *div* operation characterises how much a vector field linearly diverges and the *curl* operation characterises the strength of the curl (rotation) in the field. Both relate to spatial operations, i.e. they do not involve any operations with respect to time.

1.2.2 Interpretation

To understand the meaning of the mathematical formulation of the above equations, let's scrutinise them one by one.

$\operatorname{div}\mathbf{D}(\mathbf{r},t) = \rho(\mathbf{r},t)$ can be rewritten as $\rho(\mathbf{r},t) = \operatorname{div}\mathbf{D}(\mathbf{r},t)$, i.e. static or dynamic charges in a given volume are responsible for a diverging electric field. This implies that there must be a distinct source and sink for the electric field since a field cannot possibly (linearly) diverge and start and end in the same location.

$\operatorname{div}\mathbf{B}(\mathbf{r},t) = 0$ can be rewritten as $0 = \operatorname{div}\mathbf{B}(\mathbf{r},t)$, i.e., there is nothing available in nature which makes a magnetic field diverge. This equation comes from the observation that there are no magnetic 'charges' known to physics. Note that magnetic charges are sometimes introduced in theoretical electrodynamics so as to simplify the derivation of certain theories.

$\operatorname{curl}\mathbf{E}(\mathbf{r},t) = -\partial\mathbf{B}(\mathbf{r},t)/\partial t$ means that a spatially varying (curling) electric field will cause a time-varying magnetic field. Alternatively, it can be rewritten as $-\partial\mathbf{B}(\mathbf{r},t)/\partial t = \operatorname{curl}\mathbf{E}(\mathbf{r},t)$, i.e. a time-varying electric field will cause a curl in the magnetic field.

Finally, and most importantly as shown shortly, $\operatorname{curl}\mathbf{H}(\mathbf{r},t) = \partial\mathbf{D}(\mathbf{r},t)/\partial t + \mathbf{J}(\mathbf{r},t)$ can be read as follows. A spatially varying (curling) magnetic field will cause a time-varying electric field and, if existent, also a current through a medium capable of carrying a flow of electric charges. The equation can also be read as either a current flow through a medium or a time-varying electric field producing a spatially curling magnetic field.

1.2.3 Key to Antennas

The first two equations yield separately an insight into the properties of the electric field and magnetic field, respectively. The remaining two equations, however, show that both fields are closely coupled through spatial (curl) and temporal ($\partial/\partial t$) operations. It can also be observed that the equations are entirely symmetric - apart from the current density $\mathbf{J}(\mathbf{r},t)$. It turns out that this minor asymmetry is responsible for any radiation process occurring in nature, including the transmission and reception of electromagnetic waves. For the ease of explanation, the last two of Maxwell equations are rewritten as

$$\operatorname{curl}\mathbf{E}(\mathbf{r},t) = -\mu_0\mu_r\frac{\partial\mathbf{H}(\mathbf{r},t)}{\partial t} \tag{1.7}$$

$$\operatorname{curl}\mathbf{H}(\mathbf{r},t) = \epsilon_0\epsilon_r\frac{\partial\mathbf{E}(\mathbf{r},t)}{\partial t} + \mathbf{J}(\mathbf{r},t) \tag{1.8}$$

Let us assume first that there is a static current density $\mathbf{J}(\mathbf{r})$ available which, according to (1.8), causes a spatially curling magnetic field $\mathbf{H}(\mathbf{r})$; however, it fails to generate a temporally varying magnetic field which means that $\partial\mathbf{H}(\mathbf{r})/\partial t = 0$. According to (1.7), this in turn fails to generate a spatially and temporally varying electric field $\mathbf{E}(\mathbf{r})$. Therefore, a magnetic field is only generated in the location where a current density $\mathbf{J}(\mathbf{r})$ is present. Since the focus is on making a wave propagating in a wire<u>less</u> environment where no charges (and hence current densities) can be supported, a static current density $\mathbf{J}(\mathbf{r})$ is of little use.

The observations, however, change when a time-varying current density $\mathbf{J}(\mathbf{r},t)$ is generated, which, according to (1.8), generates a spatially and temporally varying magnetic field $\mathbf{H}(\mathbf{r},t)$. Clearly, $\partial\mathbf{H}(\mathbf{r},t)/\partial t \neq 0$ which, according to (1.8), generates a spatially and temporally varying electric field $\mathbf{E}(\mathbf{r},t)$, i.e., $\partial\mathbf{E}(\mathbf{r},t)/\partial t \neq 0$. With reference to (1.8), this generates a spatially and temporally varying magnetic field $\mathbf{H}(\mathbf{r},t)$, even in the <u>absence</u> of a current density $\mathbf{J}(\mathbf{r},t)$, and so on.

A wave is hence born where the electric field stimulates the magnetic field and vice versa. From now on, this wave is referred to as an electromagnetic (EM) wave, since it contains both magnetic and electric fields. From the above it is clear that such a wave can now propagate without the need of a charge-bearing medium; however, such a medium can certainly enhance or weaken the strength of the electromagnetic wave by means of an actively or passively created current density $\mathbf{J}(\mathbf{r},t)$.

In summary, to make an electromagnetic wave decouple from a transmitting antenna, a medium capable of carrying a time-varying current density $\mathbf{J}(\mathbf{r},t)$ is required. A medium which achieves this with a high efficiency is called an *antenna*. As simple as that! An antenna can hence be anything: a rod, wire, metallic volumes and surfaces, etc.

Remember that $\mathbf{J}(\mathbf{r},t) = \partial Q/\partial t$, where Q in [C] is the electric charge; therefore, if a time-varying current density for which $\partial\mathbf{J}(\mathbf{r},t)/\partial t \neq 0$ is required, it must be ensured that $\partial^2 Q/\partial t^2 \neq 0$, i.e. that the charges are accelerated. Sometimes such acceleration happens unintentionally, e.g. in bent wires where electrons in the outer radius move faster than the ones in the inner radius of the bend. Therefore, any non-straight piece of wire will emit electromagnetic waves which, in the case of antennas, is desirable, but in the case e.g. of wiring between electric components in a computer, not at all.

1.3 HERTZIAN DIPOLE

A wire of infinitesimal small length δl is known as a Hertzian dipole. It plays a fundamental role in the understanding of finite length antenna elements, because any of these consists of an infinite number of Hertzian dipoles. Antenna arrays, on the other hand, can be represented by means of a plurality of finite length antenna elements. This, somehow, justifies the importance of properly understanding the radiation behaviour of a Hertzian dipole.

The Hertzian dipole can be fed by a current $I(t)$ of any temporal characteristics; however, since an arbitrary signal can be resolved into its spectral harmonics, the focus is on the analysis of sinusoidal feeding, i.e. a single frequency. Thus currents of the form $I(t) = I_{\max} \cdot \exp(j\omega t)$ are considered, where $I_{\max}$ is the maximum current and ω in [rad/s] is the angular frequency. Once the response to a particular spectral component has been determined, the total response to an arbitrary temporal excitation is obtained by linearly superimposing the individual spectral contributions.

With these assumptions, it is straightforward to show that a feeding current of $I(t) = I_{\max} \cdot \exp(j\omega t)$ will cause an EM wave of the form $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) \cdot \exp(j\omega t)$ and $\mathbf{H}(\mathbf{r}, t) = \mathbf{H}(\mathbf{r}) \cdot \exp(j\omega t)$. Focus is given to the spatial dependencies of the field components and henceforth we omit the harmonic temporal factor $\exp(j\omega t)$.

A Hertzian dipole with a feeding generator is depicted in figure 1.1. For obvious reasons, the coordinate system of choice is spherical where a point in space is characterised by the azimuth $\phi \in (1, 2\pi)$, elevation $\theta \in (1, \pi)$ and distance from the origin r. The field vectors can therefore be represented as $\mathbf{E}(\mathbf{r}) = (E_\phi, E_\theta, E_r)$ and $\mathbf{H}(\mathbf{r}) = (H_\phi, H_\theta, H_r)$.

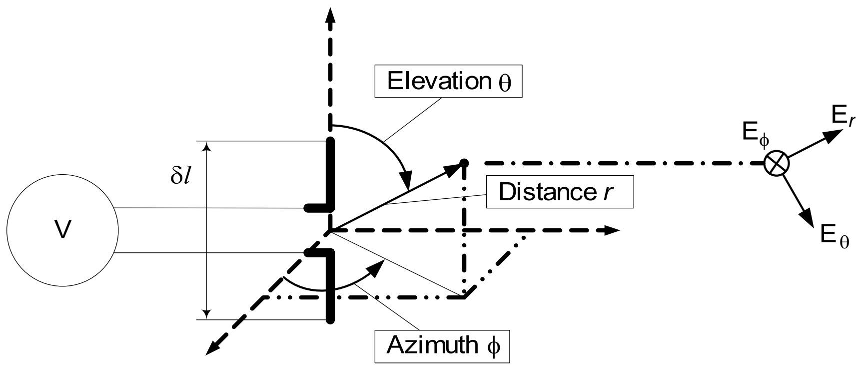


Fig. 1.1 Feeding arrangement and coordinate system for Hertzian dipole.

To obtain these electromagnetic field contributions radiated by a Hertzian dipole, one needs to solve Maxwell's equations (1.1)–(1.4). The procedure, although straightforward, is a bit lengthy and is hence omitted here. The interested

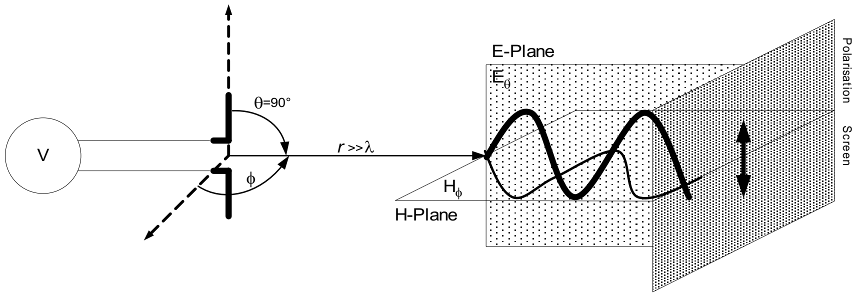


Fig. 1.2 Relationship between **E** and **H** in the far-field.

reader is referred to [9]. The final solution is of interest to us, where the non-zero EM field components are found to be

$$E_\theta = -\frac{\eta\, k^2}{4\pi}\, I\,\delta l\ \sin(\theta)\ e^{-jkr}\ \left[\frac{1}{jkr} + \left(\frac{1}{jkr}\right)^2 + \left(\frac{1}{jkr}\right)^3\right] \tag{1.9}$$

$$E_r = -\frac{\eta\, k^2}{2\pi}\, I\,\delta l\ \cos(\theta)\ e^{-jkr}\ \left[\left(\frac{1}{jkr}\right)^2 + \left(\frac{1}{jkr}\right)^3\right] \tag{1.10}$$

$$H_\phi = -\frac{k^2}{4\pi}\, I\,\delta l\ \sin(\theta)\ e^{-jkr}\ \left[\frac{1}{jkr} + \left(\frac{1}{jkr}\right)^2\right] \tag{1.11}$$

and all remaining components are zero, i.e. $E_\phi = 0$ and $H_\theta = H_r = 0$. Here, $k = 2\pi/\lambda$ is the wave number and λ in [m] is the wavelength. These equations can be simplified for distances near to and far from the dipole, where the latter is obviously of more importance to the aspects covered here, i.e., far-field operation.

The far-field, also referred to as the Fraunhofer region, is characterised by $k\,r \gg 1$, which allows simplification of (1.9)–(1.11) to

$$E_\theta = \eta\, H_\phi \tag{1.12}$$

$$H_\phi = j\frac{k^2}{4\pi}\, I\,\delta l\ \frac{\sin(\theta)\, e^{-jkr}}{kr} \tag{1.13}$$

and $E_\phi = E_r = 0$ and $H_\theta = H_r = 0$. The wave is visualised in figure 1.2 for $\theta = \pi/2$, i.e. perpendicular to the Hertzian dipole. In the far-field, **E** and **H** are clearly in in-phase, i.e. they have maxima and minima at the same locations and times. Also, the EM wave turns into a *plane wave* consisting of two mutually orthogonal electromagnetic field components.

1.4 ANTENNA PARAMETERS & TERMINOLOGY

The brief introduction to the radiation behaviour of an infinitesimal radiating element allows us to introduce some concepts which are vital for the characterisation of antenna elements and arrays. The following introduces important antenna terminology that are commonly used when characterising antennas and arrays.

1.4.1 Polarisation

From (1.12) and figure 1.2, it is observed that in the far-field from the dipole the E-wave oscillates in a plane. It has also been seen that the H-wave oscillates perpendicular to the E-wave. Since both waves always occur together, it is, unless otherwise mentioned, from now on referred to as the E-wave.

If a plane orthogonal to the direction of propagation is cut, indicated as the polarisation screen in figure 1.2, the electric field vector E_θ is observed oscillating on a straight line. This polarisation state is referred to as *linear polarisation* and is further illustrated in figure 1.3.

If two orthogonal dipoles are considered instead of one, and fed with in-phase currents, then this will trigger two decoupled EM waves to be orthogonal in the far-field. A tilted but straight line on the orthogonally cut screen is then observed; this polarisation state is often referred to as *linear tilted polarisation.* If, on the other hand, both dipoles are fed with currents in quadrature phase, i.e. shifted by 90°, then the resulting E-field will be *circularly polarised.* Finally, if the amplitudes of the two decoupled fields are different due to different feeding current amplitudes, then the resulting polarisation will be *elliptical.* In dependency of whether the two feeding currents are $+90^\circ$ or -90° shifted, the polarisation will be left or right circular or elliptical polarised. Finally, if the two feeding currents deviate by a phase different from 90° and/or more Hertzian dipoles are used, then more complicated polarisation patterns can be obtained.

1.4.2 Power Density

The instantaneous power density, w, in [W/m^2] is defined as

$$w = \mathrm{Re}\left\{\mathbf{E} \times \mathbf{H}^*\right\} \tag{1.14}$$

where $\mathrm{Re}(x)$ denotes the real part of x, and $\mathbf{H}^*$ is the complex conjugate of $\mathbf{H}$. The average power density can be obtained from (1.14) by assuming that the EM-wave is harmonic, which yields

$$\overline{w} = \mathrm{Re}\left\{\mathbf{S}\right\} = \mathrm{Re}\left\{\frac{1}{2}\mathbf{E} \times \mathbf{H}^*\right\} \tag{1.15}$$

where $\mathbf{S}$ is referred to as the Pointing vector.

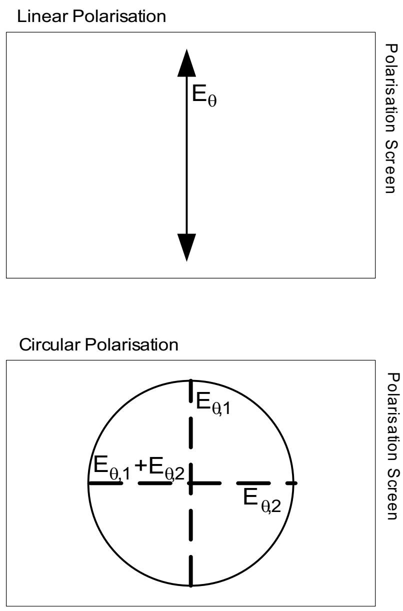


Fig. 1.3 Linear, linear tilted, circular and elliptical polarisation states.

As an example, let us calculate the average power density of an EM-wave in the far-field for a Hertzian dipole, where equations (1.12) and (1.13) are utilised to obtain

$$\operatorname{Re}\{\mathbf{S}\} = \operatorname{Re}\left\{ \frac{1}{2} \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_\theta & \mathbf{e}_\phi \\ 0 & E_\theta & 0 \\ 0 & 0 & H_\phi^* \end{vmatrix} \right\} \tag{1.16}$$

$$= \frac{\eta I^2 \sin^2(\theta)}{8r^2} \left(\frac{\delta l}{\lambda}\right)^2 \mathbf{e}_r \tag{1.17}$$

where $\mathbf{e}_r, \mathbf{e}_\theta$ and $\mathbf{e}_\phi$ are the unit vectors of the perpendicular spherical coordinates.

1.4.3 Radiated Power

Given the power density, the total average power passing through a sphere with surface area σ is calculated as

$$P = \int_\sigma \operatorname{Re}\{\mathbf{S}\}\, d\sigma \tag{1.18}$$

$$= \int_0^{2\pi}\int_0^{\pi} \operatorname{Re}\{\mathbf{S}\}\, r^2 \sin(\theta)\, d\theta\, d\phi \tag{1.19}$$

which yields for the Hertzian dipole

$$P = \frac{\pi \eta I^2}{3} \left(\frac{\delta l}{\lambda} \right)^2 . \tag{1.20}$$

Clearly, the average radiated power is independent of the distance but does depend on the geometrical (δl) and electrical (λ) properties of the dipole.

1.4.4 Radiation Resistance

The radiation resistance, R_r, is defined as *the value of a hypothetical resistor which dissipates a power equal to the power radiated by the antenna when fed by the same current I,* i.e.

$$\frac{1}{2} IU = \frac{1}{2} I^2 R_r = P \tag{1.21}$$

With reference to (1.20), this simply yields for the Hertzian dipole:

$$R_r = \frac{2\pi\eta}{3} \left(\frac{\delta l}{\lambda} \right)^2 = 80\pi^2 \left(\frac{\delta l}{\lambda} \right)^2 = 789 \left(\frac{\delta l}{\lambda} \right)^2 \tag{1.22}$$

the unit of which is [Ω].

1.4.5 Antenna Impedance

The antenna impedance, Z_a, is defined as *the ratio of the voltage at the feeding point* $V(0)$ *of the antenna to the resulting current flowing in the antenna* I, i.e.

$$Z_a = \frac{V(0)}{I_{\text{antenna}}} \tag{1.23}$$

where

- If $I_{\text{antenna}} = I_{\max}$, then the impedance Z_a is referred to as the *loop current*;
- If $I_{\text{antenna}} = I(0)$, then the impedance Z_a is referred to as the *base current*;

Referring the impedance to the base current, it is written as

$$Z_a = \frac{V(0)}{I(0)} = R_a + jX_a \tag{1.24}$$

where X_a is the antenna reactance and $R_a = R_r + R_l$ is the antenna resistance, where R_r is the radiation resistance and R_l is the ohmic loss occurring in the antenna.

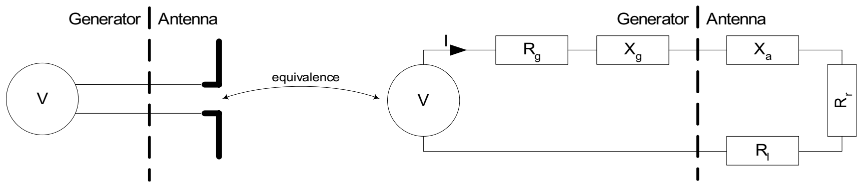


Fig. 1.4 Equivalence between physical radiating antenna and linear circuit.

1.4.6 Equivalent Circuit

The behaviour of a radiating antenna has therefore been reduced to that of an equivalent impedance. This is not a coincidence as Maxwell's equations treat both radiated EM-waves and guided EM-waves equally. The duality allows us to introduce an equivalent circuit as depicted in figure 1.4. It depicts a generator, here a voltage source, with internal impedance consisting of resistance R_g and reactance X_g being connected to the antenna with impedance consisting of resistance R_a and reactance X_a.

1.4.7 Antenna Matching

The equivalent circuit allows us to tune the impedance of the generator so as to maximise the radiated power delivered from the generator. This procedure is also often referred to as *impedance matching*. The average power delivered to the antenna is clearly given as

$$P = \frac{1}{2} I^2 R_a \tag{1.25}$$

where

$$I = \frac{V_g}{(R_a + R_g) + j \cdot (X_a + X_g)} \tag{1.26}$$

The power in (1.25) is maximised if conjugate matching is deployed, i.e. $R_g = R_a$ and $X_g = -X_a$, which yields a delivered power of

$$P = \frac{1}{8} \frac{V_g^2}{R_a} \tag{1.27}$$

1.4.8 Effective Length and Area

The effective length, l_e, *characterises the antenna's ability to transform the impinging electric field E into a voltage at the feeding point $V(0)$, and vice versa*, and it is defined as

$$l_e = \frac{V(0)}{E} \tag{1.28}$$

which can be interpreted as the more voltage that is induced with less electric field strength, the bigger the effective length of the antenna.

The effective area, A_e, *characterises the antenna's ability to absorb the incident power density* w *and to deliver it to the load*, and it is defined as

$$A_e = \frac{P_{\text{load}}}{w} \tag{1.29}$$

which can be interpreted as the higher the delivered power with respect to the incident power density, the higher the effective area.

1.4.9 Radiation Intensity

The radiation intensity, U, is defined as *the power* P *per solid angle* Ω, and it is defined as

$$U = \frac{dP}{d\Omega} \tag{1.30}$$

This can be rewritten as

$$U = \frac{\text{Re}\{\mathbf{S}\}\, d\sigma}{d\Omega} = \frac{\text{Re}\{\mathbf{S}\}\, r^2\, d\Omega}{d\Omega} = \text{Re}\{\mathbf{S}\}\, r^2 \tag{1.31}$$

which, for the Hertzian dipole, yields

$$U = \frac{\eta I^2 \sin^2(\theta)}{8} \left(\frac{\delta l}{\lambda}\right)^2 \tag{1.32}$$

and is clearly independent of the distance of observation.

1.4.10 Radiation Pattern

Since the radiation intensity U is independent of the distance of observations but only depends upon the antenna's inherent parameters, it can be taken to describe the radiation pattern of an antenna, an example of which is depicted in figure 1.5(a). If this radiation pattern is rolled out and depicted on a cartesian coordinate system, it will show a graph as depicted in figure 1.5(b). With reference to the example figure, the following is defined:

- *Radiation null:* Angle at which the radiated power is zero.
- *Main lobe:* The angular region between two radiation nulls which contains the angle with the strongest radiation intensity U_{max}.
- *Side lobes:* The angular regions between two radiation nulls which do not contain the angle with the strongest radiation power.

- *Half-power beamwidth (HPBW):* The angle spanned by the intensity region for which $U_{\max}/2 \leq U \leq U_{\max}$. The HPBW is associated with the ability of an antenna to direct a beam. The HPBW is often referred to as the 3 dB beamwidth for obvious reasons.

- *First null beamwidth (FNBW):* The angle spanned by the main lobe. The FNBW is associated with the ability of an antenna to reject interference.

- *Sidelobe level (SLL):* The power of the highest sidelobe relative to the peak of the main beam.

1.4.11 Bandwidth

The bandwidth B is defined as the *frequency band ranging from* f_{lower} *to* f_{upper} *within which the performance of the antenna, with respect to some characteristics, conforms to a specified standard, e.g. a drop by 3 dB.* Such a definition is fairly broad and includes characteristics such as radiation pattern, beamwidth, antenna gain, input impedance, and radiation efficiency. An antenna is said to be

- *narrowband* if $B_{\text{narrow}} = (f_{\text{upper}} - f_{\text{lower}})/f_{\text{centre}} < 5\%$;

- *wideband* if $f_{\text{upper}} : f_{\text{lower}} > 10 : 1$; and

- *frequency independent* if $f_{\text{upper}} : f_{\text{lower}} > 40 : 1$.

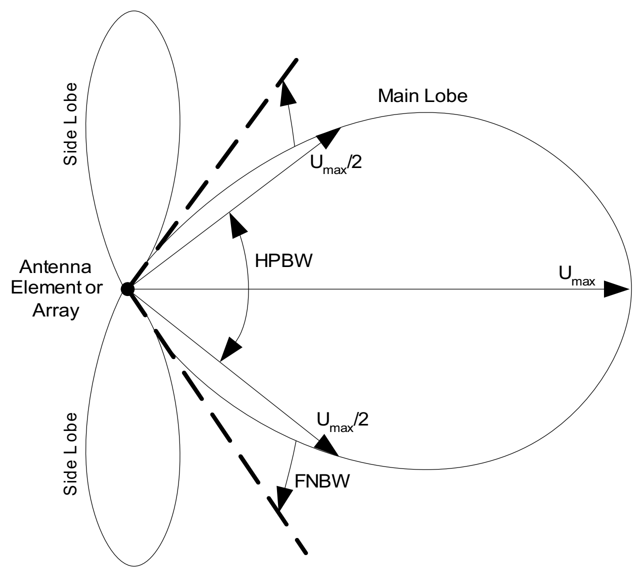
For example, an antenna is designed such that it radiates a total power of 0 dBm at a centre frequency of 1.8 GHz. It can be said that the specified standard is a power drop of 3 dB. Therefore, if the radiated power does not drop to -3 dBm within $f_{\text{lower}} = 1.755$ GHz and $f_{\text{lower}} = 1.845$ GHz but falls below -3 dBm out of that range, then the antenna is narrowband. If, however, the radiated power does not fall below -3 dBm well beyond 18 GHz, then the antenna is wideband.

1.4.12 Directive Gain, Directivity, Power Gain

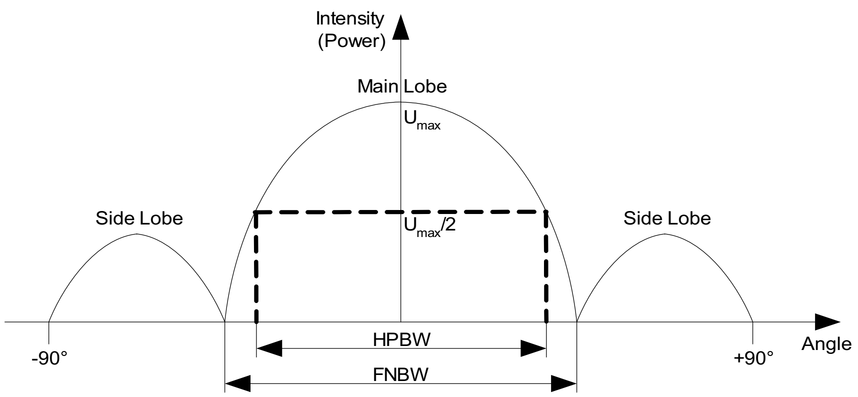
An isotropic radiator is defined as *a radiator which radiates the same amount of power in all directions.* It is a purely hypothetical radiator used to aid the analysis of realisable antenna elements. For an isotropic radiator, the radiation intensity, U_0, is defined to be $U_0 = P/(4\pi)$.

This allows us to define the directive gain, g, as *the ratio of the radiation intensity* U *of the antenna to that of an isotropic radiator* U_0 *radiating the same amount of power*, which can be formulated as

$$g = \frac{U}{U_0} = 4\pi \frac{U}{P} \tag{1.33}$$



(a)



(b)

Fig. 1.5 Two different representations of a radiation pattern: (a) polar plot, (b) Cartesian plot.

For the Hertzian dipole,

$$g(\theta) = 1.5 \sin^2(\theta) \tag{1.34}$$

which is clearly a function of direction, but not distance from the radiator.

From the above, the directivity D is defined as *the ratio of the maximum radiation intensity* $U_{\max}$ *of the antenna to that of an isotropic radiator* U_0 *radiating the same amount of power*, giving

$$D = \frac{U_{max}}{U_0} = 4\pi \frac{U_{\max}}{P} \tag{1.35}$$

and For the Hertzian dipole

$$D = 1.5 \tag{1.36}$$

which is simply a value and not a function of direction anymore.

This allows us finally to define the power gain G as *the ratio of the radiation intensity* U *of the antenna to that of an isotropic radiator* U_0 *radiating an amount of power equal to the power accepted by the antenna*, i.e.

$$G = \frac{U}{U_{0,input}} = 4\pi \frac{U}{P_{input}} \tag{1.37}$$

1.4.13 Radiation Efficiency

The radiation efficiency, e, is defined as *the ratio of the radiated power* P *to the total power* P_{input} *accepted by the antenna, where* $P_{input} = P + P_{loss}$, i.e.

$$e = \frac{P}{P_{input}} = \frac{P}{P + P_{loss}} \tag{1.38}$$

which can also be related to previous antenna parameters as $e = G/g$.

1.5 BASIC ANTENNA ELEMENTS

Any wire antenna can be viewed as the superimposition of an infinite amount of Hertzian dipoles which, theoretically speaking, allows one to calculate the resulting EM-field at any point in space by adding the field contributions of each Hertzian dipole. From (1.12) and (1.13) it has been shown that to calculate the field contributions, the current (and its direction) through the Hertzian dipole should be known.

For many antenna configurations, the current distribution can be calculated or estimated and the aforementioned theory applied. In this case, the antennas are referred to as *wire antennas*. In many other configurations, however, it is difficult to obtain an exact picture of the current distribution. In this case, it is easier to utilise Huygen's principle and deduce the radiated EM-field at any point in space from an estimate of the EM-field at a well-defined surface; such antennas are also referred to as *aperture antennas*.

Basic wire antennas include dipoles of finite length, loop and helix antennas. Basic aperture antennas include the horn and slot antenna, as well as parabolic dishes. Examples of common antenna elements are shown in figure 1.6. To obtain a feeling for the properties of these basic antenna elements, some of the most important are briefly considered in the following sections.

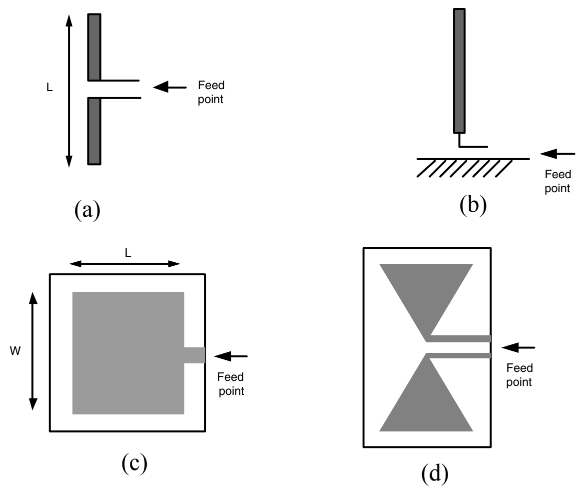


Fig. 1.6 Common types of antenna elements: (a) dipole, (b) mono-pole, (c) square patch, (d) bow tie.

1.5.1 Finite-Length Dipole

Dipoles of finite length L are of practical interest, an example of which is depicted in figure 1.6(a). If it is assumed that the dipole is fed with a sinusoidal voltage generator, then the resulting current in the finite-length dipole will also be (approximately) harmonic with a maximum amplitude of $I_{\max}$. Such a feeding procedure is referred to as *balanced*, i.e. one feeding wire carries a current that is in anti-phase to the current in the other. That allows us to integrate over the field

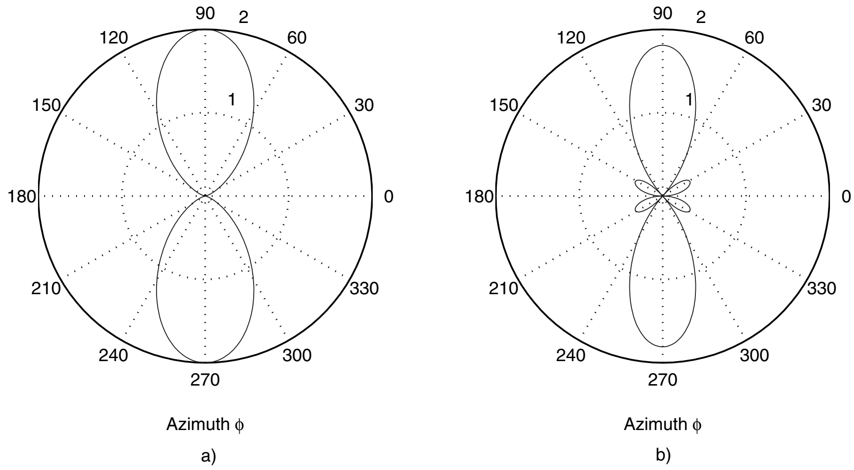


Fig. 1.7 Radiation pattern of a dipole of length: (a) $L = 1.0\lambda$ and (b) $L = 1.2\lambda$.

contributions (1.12) and (1.13) to arrive at [9]

$$E_\theta = \eta\, H_\phi \tag{1.39}$$

$$H_\phi = \frac{jI_{\max}e^{-jkr}}{2\pi r} \cdot P(\theta) \tag{1.40}$$

where $P(\theta)$ is the pattern factor given by

$$P(\theta) = \frac{\cos\left(\frac{1}{2}kL\cos(\theta)\right) - \cos\left(\frac{1}{2}kL\right)}{\sin(\theta)} \tag{1.41}$$

With (1.39) and (1.40), it is possible to calculate the antenna parameters introduced in section 1.4.

For example, the radiation resistance of a $\lambda/2-$dipole, i.e. $L = \lambda/2$, can be calculated to be $R_r = 73\Omega$ and the directivity is $D = 1.64$. Also, whilst radiating power uniformly over the azimuth plane, the finite-length dipole develops some interesting radiation patterns over the elevation plane. It can be shown that for $L < 1.1\lambda$ only one main lobe exists (figure 1.7(a), whereas for $L \geq 1.1\lambda$ the radiation pattern develops multilobes (figure 1.7(b). From this figure it is also observed that the power radiated along the main lobe decreases once multilobes develop, which is detrimental in most wireless applications. Note that the variation in radiation patterns observed between figures 1.7(a) and (b) also depict the change that occurs with frequency, since the radiation characteristics depend on the ratio between the physical length L and the wavelength λ.

1.5.2 Mono-pole

A mono-pole antenna is often considered as half of a dipole placed above a ground-plane, where the ground-plane acts as an electric mirror thus creating the other half of the dipole, as illustrated in figure 1.8. In contrast to the balanced feed for the dipole, a mono-pole is fed with a single-ended feeder where one wire carries the signal to the antenna and the other is connected to the ground-plane. This has the advantage of being directly connected to most receiver and transmitter modules that are often designed around a single-ended grounding system, and is fully compatible with co-axial cables and associated connectors.

Since half of the radiating plane is cut-off by the ground plane, the radiated power (and hence the radiation resistance) of a mono-pole is only half compared to the dipole with the same current; however, the directivity is doubled. Therefore, a $\lambda/4$ mono-pole has a radiation resistance of $R_r = 36.5\Omega$ and a directivity of $D = 3.28$. The latter, together with the compact spatial realisation and the simple feeding mechanism, is the main incentive to use mono-poles.

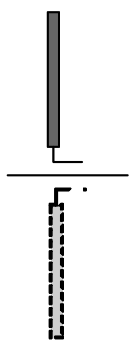


Fig. 1.8 Mono-pole above a ground-plane.

1.5.3 Printed Antennas

The antennas discussed so far have required a wire structure as a convenient way of realisation. Alternatively antennas can be produced using printed circuit techniques where one side of a copper clad dielectric is etched to the desired shape and the other is left as the ground-plane. Such antennas are often referred to as *printed antennas*. Dipoles can also be produced in this way, but this section focuses on patch antennas.

Patch antennas are practical and popular owing to their ease of manufacturing and the flexibility of design in terms of shape and topology. Many applications require antennas which are capable of conforming to the shape of the surface onto which they are mounted, and are required to have a planar profile for aesthetic or mechanical reasons. Patch antennas are a good solution for such applications. Figure 1.6(c) shows an example of a square patch antenna. Although the surface conductor can be practically any shape, rectangular and circular tend to be the most common.

The far-field radiation pattern is orientated orthogonal to the surface conductor, so in figure 1.6(c) it projects towards the user. As a rule of thumb, length L is approximately $\lambda_g/2$ and controls the operating frequency and width W is $0.9\lambda_g$ and controls the radiation resistance [10], where $\lambda_g = 1/\sqrt{\varepsilon_r}$ which is defined as the *normalised* wavelength in a media with relative permeability ε_r.

Note that contrary to common belief, the surface conductor does not form the radiating element as it does in a dipole. Instead, radiation occurs from along edges L and W, and which edge depends upon the electromagnetic mode of radiation the antenna is operating in. The radiation pattern of a square patch operating in the TE_{10} mode is $E_\phi \approx \cos(\theta)$.

It is evident from figure 1.6(c) that a single-ended feeder is required, as described in section 1.5.2 for the mono-pole antenna, and therefore has the characteristics associated with a single-ended feeder. Furthermore, although patch antennas have the advantages mentioned, they have a narrow operating bandwidth and low radiation efficiency compared to a dipole. Advanced structures can improve these parameters, an example of which is the stacked patch, which consists of several layers sandwiched together, where the size of each layer and the distance between the layers is carefully chosen.

1.5.4 Wideband Elements

With reference to the definition given in section 1.4.11, an antenna is said to be wideband if given characteristics, e.g. input impedance, do not change over a frequency band from f_{lower} to f_{upper} where $f_{upper} : f_{lower} > 10 : 1$. Simple antenna elements, such as the Hertzian dipole, finite-length dipole and mono-pole, are not capable of maintaining any characteristics over such wide bandwidths. More sophisticated antenna structures are hence required, an example of which are bow-tie antennas and horn antennas.

From previous analysis, it is shown that radiation characteristics depend on the ratio between the physical length L and the wavelength λ. From this fact, Rumsey observed that if an antenna design is only described by angles and is itself infinite in length, then it is inherently self-scaling and thus frequency-independent [9].

An example of a radiating structure obeying Rumsey's principle is depicted in figure 1.9(a). Since it is impossible to build the depicted radiating structure, which is infinite in size, several techniques have been suggested to make them effectively infinite. One of them is simply to truncate the infinite sheet, leading to a radiating

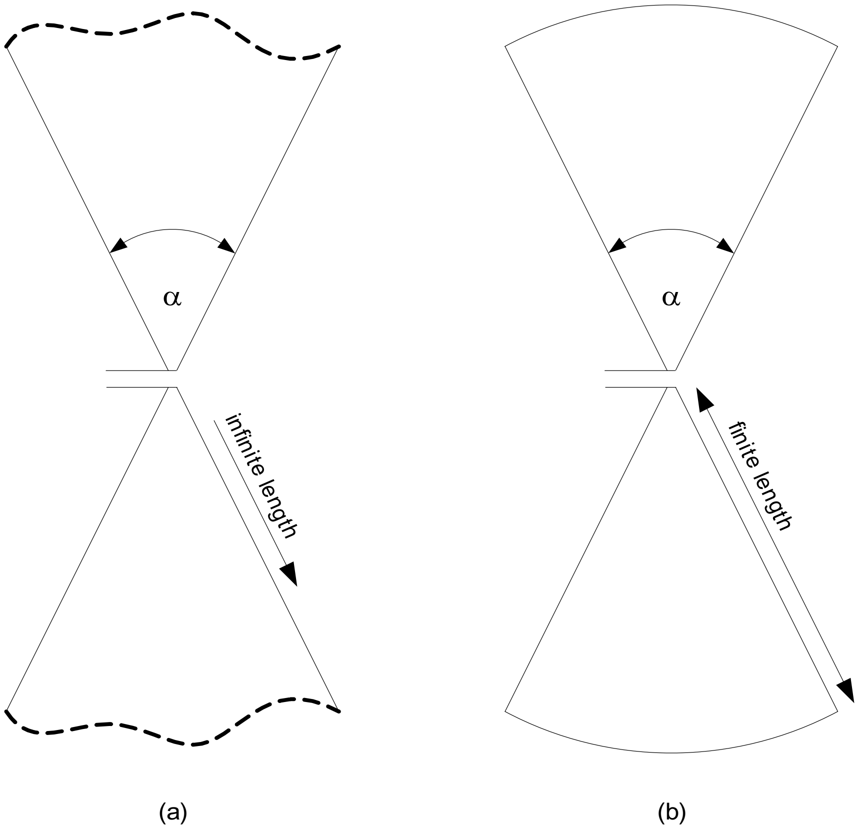


Fig. 1.9 A frequency-independent (wideband) antenna in (a) theoretical realisation and (b) practical realisation (bow-tie antenna).

structure as depicted in figure 1.9(b); such an antenna is also referred to as a *bow-tie antenna*. Since Rumsey's design properties are violated, the antenna will not be frequency-independent anymore; however, it exhibits parameter stability over a very wide frequency band, where exact numbers depend on the specific antenna realisation.

More sophisticated antenna elements are the *log-period toothed antenna, log-period trapezoidal wire antenna* and *log-period dipole array*. Due to their log-period self-scaling nature, they extend the effective length of the antenna structure. Finally, antennas which, by nature, are self-scaling, are fractal antennas.

1.5.5 Dual Polarised Elements

In section 1.3, the polarisation states of linear, circular and elliptical polarisation were introduced, which can be generated by means of two dipoles displaced by 90°. Such a radiation structure is referred to as a *dual polarised element* and is depicted in figure 1.10.

In addition to the polarisation states, it is possible to distinguish horizontal and vertical polarisation, which refers to the orientation of the polarisation with respect to some reference plane, such as ground or another surface. Therefore, if a linearly polarised wave has its E-field vector oscillating in a plane vertical to the ground, then such polarisation is referred to as *vertical polarisation*. If, on the other hand, the plane of the E-field vector oscillates in a plane parallel to the ground, then such polarisation is referred to as *horizontal polarisation*. Since the EM-field is linear, it is possible to decompose a field of any polarisation into vertically and horizontally polarised waves with respective phase differences.

Polarisation can be exploited in wireless systems to give operational advantages and therefore antenna elements are required that will efficiently transmit and receive electromagnetic waves with the required polarisation. Such antennas can be employed as sensors in adaptive antenna systems.

The dipole in figure 1.1 will generate or receive a vertically polarised electromagnetic wave with respect to the ground. By rotating the antenna through 90°, the antenna is orientated for horizontal polarisation. Therefore the simplest way to simultaneously operate in vertical and horizontally polarised modes is to employ two orthogonal dipoles as shown in figure 1.10. Notice that this configuration has two feedpoints, one for each polarisation.

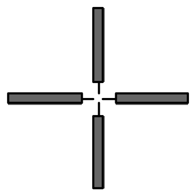


Fig. 1.10 Dual polarised dipole.

Patch antennas can conveniently be configured for dual polarised operation by simply employing an additional feed point for the second polarisation at the appropriate location.

Circular polarisation operation is obtained in the same manner as linear polarisation except that appropriate phasing of the two waves is required between the two feed points, i.e. 90°.

1.5.6 Sonar Sensors

Sonar systems are a water-borne equivalent of radar systems. They operate using transverse waves. Due to this difference, they require different sonar sensors to receive and transmit the signals. Note, also, that the operative wavelength in the acoustic spectrum becomes comparable to a radar operating in the microwave EM spectrum.

Sonar systems are often classified as either *active* or *passive*. Active systems employ a transmitter and receiver and send out a signal which is subsequently detected by the receiver and processed to give information about the environment, e.g. other vessels or fish. Passive sonar only consists of a receiver which monitors the surrounding signal sources.

The receiving microphone is termed a *hydrophone* and the transmitting loudspeaker is termed a *projector*. In the case of a sensor having the dual purpose of transmitting and receiving, the sensor is referred to as a *transducer*.

The output power from the transducer is termed *source power level* (SPL). Like antenna elements, transducers can be designed to have directional properties and a 3 dB beamwidth, θ. The beamwidth can only be made as small as the diffraction limit of the transducer, which is a function of the diameter, D, of the transducer's aperture. The beamwidth is given by:

$$\theta = \frac{2\lambda}{D} \tag{1.42}$$

For example, assuming a transducer operating in sea water with $D = 10$ cm and $\lambda = 15$ cm, then a 3 dB-beamwidth of $\theta = 3°$ is obtained.

1.6 ANTENNA ARRAYS

From section 1.4.9, it can be shown that the HPBW of a Hertzian dipole is 90°. In most wireless terrestrial and space applications, a narrower HPBW is desired because it is desirable to direct power in one direction and no power into other directions. For example, if a Hertzian dipole is used for space communication, only a fraction of the total power would reach the satellite and most power would simply be lost (space applications require HPBWs of well below 5°).

With a finite-length dipole, it would be possible to decrease the HPBW down to 50° by increasing the length to $L \approx 1.1\lambda$; a further increase in length causes the HPBW to decrease further; however, unfortunately multilobes are generated hence further decreasing the useful power radiated into the specified direction.

It turns out that with the aid of antenna arrays, it is possible to construct radiation patterns of arbitrary beamwidth and orientation, both of which can be controlled electronically. An *antenna array* is by definition a radiating configuration consisting of more than one antenna element. The definition does not specify which antenna elements are used to form the array, nor how the spatial arrangement ought to be. This allows us to build antenna arrays consisting of different elements, feeding arrangements and spatial placement, hence resulting in radiating structures of different properties.

Example antenna arrays consisting of several patch antennas are depicted in figure 1.11, and briefly dealt with below.

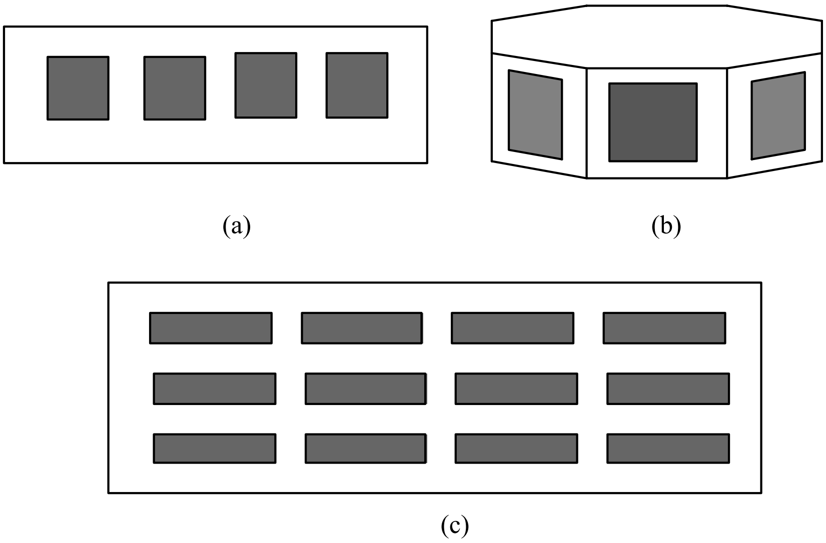


Fig. 1.11 Common types of antenna arrays: (a) linear array, (b) circular array, (c) planar array.

1.6.1 Linear Array

The most common and most analysed structure is the *linear antenna array*, which consists of antenna elements separated on a straight line by a given distance. Although each single antenna element can have a large HPBW, the amplitude and phase of the feeding current to each element can be adjusted in a controlled manner such that power is transmitted (received) to (from) a given spatial direction.

If adjacent elements are equally spaced then the array is referred to as a *uniform linear array* (ULA). If, in addition, the phase α_n of the feeding current to the nth antenna element is increased by $\alpha_n = n\alpha$, where α is a constant, then the array is a *progressive phaseshift array*. Finally, if the feeding amplitudes are constant, i.e. $I_n = I$, then this is referred to as a *uniform array*.

The uniform array, depicted in figure 1.11(a), with an inter-element spacing of $\lambda/2$ is the most commonly deployed array as it allows simple feeding, beamsteering and analysis. For example, the power radiated in azimuth at an elevation of $\theta = \pi/2$ for such an array consisting of $\lambda/2$-dipoles can be calculated as

$$P(\theta = \pi/2, \phi) \propto \left| \frac{1}{N} \frac{\sin(\frac{1}{2}N(\pi\cos(\phi) + \alpha))}{\sin(\frac{1}{2}(\pi\cos(\phi) + \alpha))} \right|^2 \tag{1.43}$$

where α is the progressive phase shift, N is the total number of antenna elements, and it has been assumed that $I_n = 1/N$.

Figure 1.12 depicts the dependency of the radiation pattern on the number of antenna elements, where $\alpha = 0$. It can be observed that increasing the number of elements N, also decreases the HPBW where the width is inversely proportional to the number of elements. It is also observed that, independent of the number of elements, the ratio between the powers of the main lobe and the first side lobe is approximately 13.5 dB. Therefore, if such a level does not suit the application, antenna arrays different from uniform arrays have to be deployed.

Figure 1.13 illustrates the dependency of the beam direction on the progressive feeding phase α, where $N = 10$. Increasing α increases the angle between the broadside direction and the main beam, where the dependency is generally not linear.

In summary, by means of a simply employable uniform antenna array, the HPBW can be reduced by increasing the number of antenna elements and steering the main beam into an arbitrary direction by adjusting the progressive phase shift.

1.6.2 Circular Array

If the elements are arranged in a circular manner as depicted in figure 1.11(b), then the array is referred to as an *uniform circular array* (UCA). With the same number of elements and the same spacing between them, the circular array produces beams of a wider width than the corresponding linear array. However, it outperforms the linear array in terms of diversity reception.

If maximal ratio combining is used, then it can be shown that the UCA outperforms the ULA on average for small and moderate angle spread for similar aperture sizes. However, the ULA outperforms the UCA for near-broadside angles-of-arrival with medium angular spreads. It can also be shown that the central angle-of-arrival has a significant impact on the performance of the ULA, whereas the UCA is less susceptible to it due to its symmetrical configuration.

1.6.3 Planar Array

Both linear and circular arrangements allow one to steer the beam into any direction in the azimuth plane; however, the elevation radiation pattern is dictated

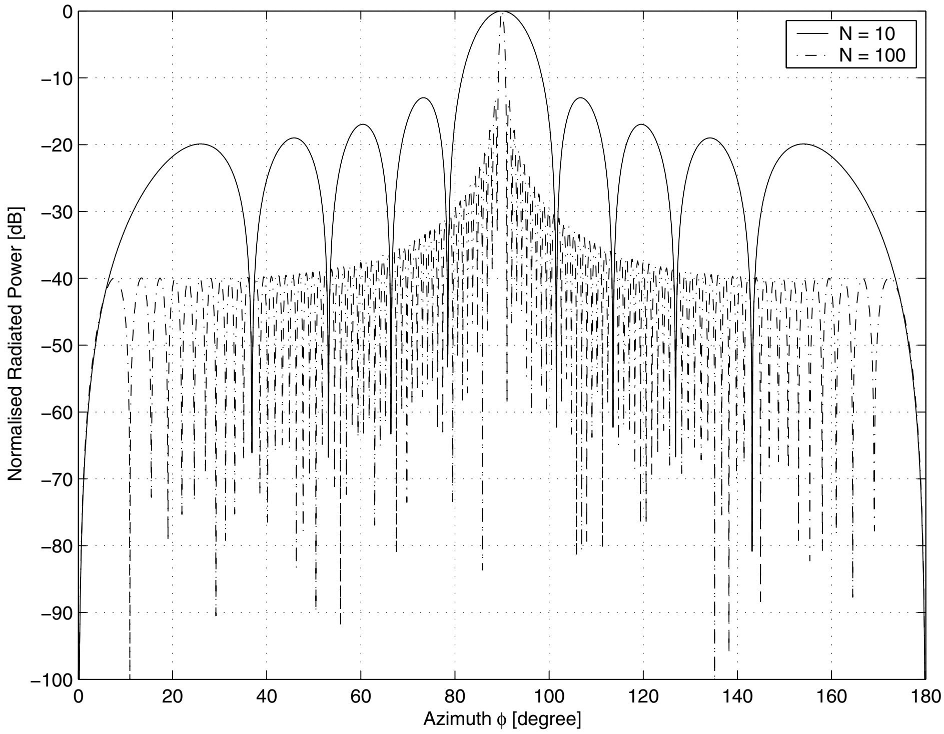


Fig. 1.12 Dependency of the radiation pattern on the number of elements N.

by the radiation pattern of the antenna elements. In contrast, the *planar antenna array* allows one also to steer the beam in elevation, thereby producing so-called *pencil beams*. An example realisation of a planar array is depicted in figure 1.11(c). For the planar antenna array, the same observations as for the linear array hold.

1.6.4 Conformal Arrays

The array types discussed so far have all been based upon a regular, symmetrical design. This is suitable for applications where such mounting is possible; however, this is not the case for many scenarios where the surface is irregular or the space is confined. For these situations a conformal array is required which, as the name suggests, conforms to the surrounding topology. The arrays could be elliptical or follow a more complex topology. The challenge for the design of these antennas is to make sure that the main lobe beamwidth and side lobe levels fall within the required specification. Specialised antenna elements are also required since

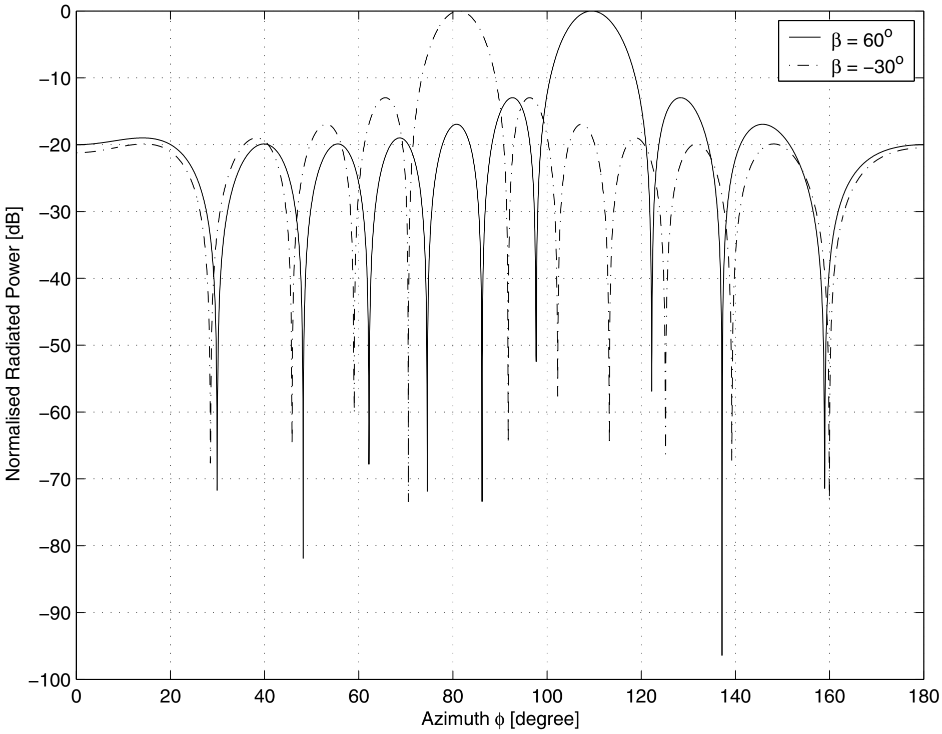


Fig. 1.13 Dependency of the beam direction on the progressive feeding phase α.

they will be mounted on an irregular surface and must therefore follow the surface contours. This particularly applies to patch antennas which usually require a flat surface.

1.7 SPATIAL FILTERING

From the previous section, it has been shown that appropriate feeding allows antenna arrays to steer their beam and nulls towards certain directions, which is often referred to as *spatial filtering*. Spatial filtering is of particular importance in mobile communication systems since their performance tends to be interference limited.

The very first mobile communication systems had base stations with an omnidirectional antenna element, i.e. the transmit power was equally spread over the entire cell and hence serving everybody equally. However, since many such communication cells are placed beside each other, they interfere with each other. This is because a mobile terminal at a cell fringe receives only a weak signal from its own base station, whereas the signals from the interfering base stations grow stronger.

Modern wireless communication systems deploy antenna arrays, as depicted in figure 1.14. Here, a base station communicates with several active users by directing the beam towards them and it nulls users which cause interference. This has two beneficial effects: first, the target users receive more power compared to the omnidirectional case (or, alternatively, transmit power can be saved); and, second, the interference to adjacent cells is decreased because only very selected directions are targeted.

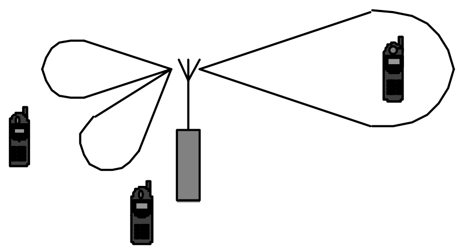


Fig. 1.14 Principle of spatial filtering applied to a mobile communication system.

The above-described spatial filtering can be realised by means of the following mechanisms:

- **Sectorisation:** The simplest way is to deploy antenna elements which inherently have a sectorised radiation pattern. For instance, in current second-generation mobile phone systems, three base station antennas with 120° sectors are deployed to cover the entire cell.

- **Switched beam:** A beam is generated by switching between separate directive antennas or predefined beams of an antenna array. Traditionally, algorithms are in place which guarantee that the beam with the strongest signal is chosen.

- **Phased antenna array:** By adjusting the feeding phase of the currents, a moveable beam can be generated (see section 1.6.1 and figure 1.13). The feeding phase is adjusted such that the signal level is maximised.

- **Adaptive antenna array:** As with the phased array, a main lobe is generated in the direction of the strongest signal component. Additionally, side lobes are generated in the direction of multipath components and also interferers are nulled. Here, not only the signal power is maximised but also the interference power is minimised, which clearly requires algorithms of higher complexity to be deployed when compared to the switched beam and phased arrays. Since adaptive antenna arrays are currently of great importance and also the main topic of this book, their functioning is briefly introduced below.

1.8 ADAPTIVE ANTENNA ARRAYS

Why adaptive? Clearly, adaptability is required to change its characteristics to satisfy new requirements, which may be due to

- changing propagation environment, such as moving vehicles;
- changing filtering requirements such as new targets or users requiring processing.

This is particularly the case for spatial filtering which can dynamically update the main beam width and direction, side-lobe levels and direction of nulls as the filtering requirement and/or operating environment changes. This is achieved by employing an antenna array where the feeding currents can be updated as required. Since the feeding current is characterised by amplitude and phase, it can be represented by a complex number. For this reason, from now on explicit reference to the feeding current is omitted and instead we refer to the *complex array weights*. Such a system with adaptive array weights is shown in figure 1.15. Clearly, the aim of any analysis related to adaptive antenna arrays is to find the optimum array weights so as to satisfy a given performance criterion.

1.9 MUTUAL COUPLING & CORRELATION

Any form of beamforming relies on the departure of at least two phase-shifted and coherent waves. If either the phase-shift or the coherence is violated, then no beamforming can take place; this may happen as follows.

If two antenna elements are placed very closely together, then the radiated EM field of one antenna couples into the other antenna and vice versa and therefore erasing any possible phase-shift between two waves departing from these elements; this effect is known as *mutual coupling*. The antenna array designer has hence to make sure that the mutual coupling between any of the elements of an antenna array is minimised. This is usually accomplished for inter-element spacing larger than $\lambda/2$. If, on the other hand, the spacing is too large, then the departing waves do not appear coherent anymore, hence, also preventing an EM wave to form a directed beam.

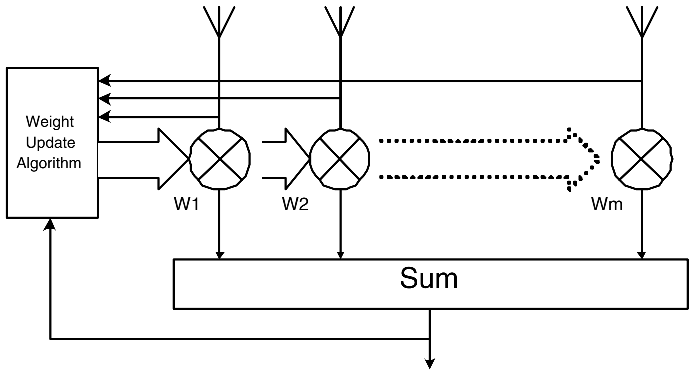


Fig. 1.15 Concept of an adaptive antenna array.

From a purely antenna array point of view, a suitable inter-element spacing for beamforming is thus in the order of $\lambda/2$. Sometimes, however, the antenna array is not utilised as a beamformer, but as a diversity array. This requires the arriving and departing signals to be as de-correlated as possible. It can be shown that if the antenna array is uniformly surrounded by clutter, then the spatial correlation function of the channel requires a minimal inter-antenna element spacing of $\lambda/2$ for the signals to appear de-correlated. Such ideal clutter arrangement, however, is rarely found in real-world applications where the correlation between antenna elements increases as the angular spread of the impinging waves decreases. For instance, a base station mounted on the rooftop with little clutter around requires spacings of up to 10λ to achieve de-correlation between the elements.

From this short overview, it is clear that the antenna array designer faces many possible realisations where the optimum choice will depend on the application at hand.

1.10 CHAPTER SUMMARY

This chapter has introduced the most basic concepts required to understand the remainder of this book. The functioning of an antenna element has been explained by means of Maxwell's equations. This facilitated the analytical description of the radiation properties of the smallest possible element, the infinitesimal small Hertzian dipole. Some antenna parameters have then been introduced and quantified for the Hertzian dipole, but which are also applicable to any other form of radiating element.

Typically occurring antenna elements have then been described, which can be viewed as a superposition of an infinite number of Hertzian dipoles. These elements include the finite length half-wavelength dipole, the mono-pole and other radiating structures.

Placed together, these antenna elements form antenna arrays, and distinction has been made between linear, circular and planar antenna arrays. Their advantages and disadvantages have also been discussed. Finally, the antenna arrays were applied to achieve spatial filtering by means of adapting the feeding currents.

1.11 PROBLEMS

Problem 1. Why is the Hertzian dipole the basis of any radiating structure?

Problem 2. Prove equations (1.12) and (1.13) from equations (1.9)–(1.11).

Problem 3. Show that equation (1.17) holds by utilising equations (1.12) and (1.13).

Problem 4. Demonstrate that the complex conjugate matching indeed maximises the radiated power, and show that equation (1.27) holds.

Problem 5. What is the difference between the bandwidth and beamwidth of an antenna?

Problem 6. With the help of equation (1.43), show that the ratio between the mainlobe and first sidelobe is approximately 13.5 dB and, for N being large, independent of the number of antenna elements N forming the array.

Problem 7. Explain the advantage of employing an adaptive antenna in a mobile communications system.

Problem 8. Explain why 'adaptivity' is fundamental to successful tracking of a missile using an antenna array.

Problem 9. Explain how an array may improve the performance of a sonar system.