Chapter 1

Review of Elementary Principles

1.1 INTRODUCTION

It is assumed that before entering the world of gas dynamics you have had a reasonable background in mathematics (through calculus) together with a course in elementary thermodynamics. An exposure to basic fluid mechanics would be helpful but is not absolutely essential. The concepts used in fluid mechanics are relatively straightforward and can be developed as we need them. On the other hand, some of the concepts of thermodynamics are more abstract and we must assume that you already understand the fundamental laws of thermodynamics as they apply to stationary systems. The extension of these laws to flow systems is so vital that we cover these systems in depth in Chapters 2 and 3.

This chapter is not intended to be a formal review of the courses noted above; rather, it should be viewed as a collection of the basic concepts and facts that will be used later. It should be understood that a great deal of background is omitted in this review and no attempt is made to prove each statement. Thus, if you have been away from this material for any length of time, you may find it necessary occasionally to refer to your notes or other textbooks to supplement this review. At the very least, the remainder of this chapter may be considered an assumed common ground of knowledge from which we shall venture forth.

At the end of this chapter a number of questions are presented for you to answer. No attempt should be made to continue further until you feel that you can answer all of these questions satisfactorily.

1.2 UNITS AND NOTATION

Dimension: a qualitative definition of a physical entity
(such as time, length, force)

Unit: an exact magnitude of a dimension
(such as seconds, feet, newtons)

In the United States most work in the area of thermo-gas dynamics (particularly in propulsion) is currently done in the English Engineering (EE) system of units. However, most of the world is operating in the metric or International System (SI) of units. Thus, we shall review both systems, beginning with Table 1.1.

Force and Mass

In either system of units, force and mass are related through *Newton's second law of motion*, which states that

$$\sum \mathbf{F} \propto \frac{d(\overrightarrow{\text{momentum}})}{dt} \tag{1.1}$$

The proportionality factor is expressed as $K = 1/g_c$, and thus

$$\sum \mathbf{F} = \frac{1}{g_c}\frac{d(\overrightarrow{\text{momentum}})}{dt} \tag{1.2}$$

For a mass that does not change with time, this becomes

$$\sum \mathbf{F} = \frac{m\mathbf{a}}{g_c} \tag{1.3}$$

where $\sum \mathbf{F}$ is the vector force summation acting on the mass m and $\mathbf{a}$ is the vector acceleration of the mass.

In the English Engineering system, we use the following definition:

A 1-pound force will give a 1-pound mass an acceleration of 32.174 ft/sec^2.

Table 1.1 Systems of Units[a]

	Basic Unit Used	
Dimension	English Engineering	International System
Time	second (sec)	second (s)
Length	foot (ft)	meter (m)
Force	pound force (lbf)	newton (N)
Mass	pound mass (lbm)	kilogram (kg)
Temperature	Fahrenheit (°F)	Celsius (°C)
Absolute Temperature	Rankine (°R)	kelvin (K)

[a] *Caution*: Never say *pound*, as this is ambiguous. It is either a *pound force* or a *pound mass*. Only for mass at the Earth's surface is it unambiguous, because here a pound mass weighs a pound force.

With this definition, we have

$$1\ \text{lbf} = \frac{1\ \text{lbm} \cdot 32.174\ \text{ft/sec}^2}{g_c}$$

and thus

$$g_c = 32.174 \frac{\text{lbm-ft}}{\text{lbf-sec}^2} \tag{1.4a}$$

Note that g_c is *not* the standard gravity (check the units). It is a proportionality factor whose value depends on the units being used. In further discussions we shall take the numerical value of g_c to be 32.2 when using the English Engineering system.

In other engineering fields of endeavor, such as statics and dynamics, the British Gravitational system (also known as the U.S. customary system) is used. This is very similar to the English Engineering system except that the unit of mass is the slug.

In this system of units we follow the definition:

> A 1-pound force will give a 1-slug mass an acceleration of 1 ft/sec^2.

Using this definition, we have

$$1\ \text{lbf} = \frac{1\ \text{slug} \cdot 1\ \text{ft/sec}^2}{g_c} \tag{1.4b}$$

and thus

$$g_c = 1 \frac{\text{slug-ft}}{\text{lbf-sec}^2}$$

Since g_c has the numerical value of unity, most authors drop this factor from the equations in the British Gravitational system. Consistent with the thermodynamics approach, we shall not use this system here. Comparison of the Engineering and Gravitational systems shows that 1 slug $\equiv$ 32.174 lbm.

In the SI system we use the following definition:

> A 1-N force will give a 1-kg mass an acceleration of 1 m/sec^2.

Now equation (1.3) becomes

$$1\,\text{N} = \frac{1\ \text{kg} \cdot 1\ \text{m/s}^2}{g_c}$$

and thus

$$g_c = 1\frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2} \tag{1.4c}$$

Since g_c has the numerical value of unity (and uses the dynamical unit of mass, i.e., the kilogram) most authors omit this factor from equations in the SI system. However, we shall leave the symbol g_c in the equations so that you may use any system of units with less likelihood of making errors.

Density and Specific Volume

Density is the mass per unit volume and is given the symbol ρ. It has units of lbm/ft^3, kg/m^3, or slug/ft^3.

Specific volume is the volume per unit mass and is given the symbol v. It has units of ft^3/lbm, m^3/kg, or ft^3/slug. Thus

$$\rho = \frac{1}{v} \tag{1.5}$$

Specific weight is the weight (due to the gravity force) per unit volume and is given the symbol γ. If we take a unit volume under the influence of gravity, its weight will be γ. Thus, from equation (1.3) we have

$$\gamma = \rho\frac{g}{g_c} \qquad \text{lbf/ft}^3 \text{ or N/m}^3 \tag{1.6}$$

Note that mass, density, and specific volume *do not* depend on the value of the local gravity. Weight and specific weight *do* depend on gravity. We shall not refer to specific weight in this book; it is mentioned here only to distinguish it from density. Thus the symbol γ may be used for another purpose [see equation (1.49)].

Pressure

Pressure is the normal force per unit area and is given the symbol p. It has units of lbf/ft^2 or N/m^2. Several other units exist, such as the pound per square inch (psi; lbf/in^2), the megapascal (MPa; 1×10^6 N/m^2), the bar (1×10^5 N/m^2), and the atmosphere (14.69 psi or 0.1013 MPa).

Absolute pressure is measured with respect to a perfect vacuum.

Gage pressure is measured with respect to the surrounding (ambient) pressure:

$$p_{\text{abs}} = p_{\text{amb}} + p_{\text{gage}} \tag{1.7}$$

When the gage pressure is negative (i.e., the absolute pressure is below ambient) it is usually called a (positive) vacuum reading:

$$p_{\text{abs}} = p_{\text{amb}} - p_{\text{vac}} \tag{1.8}$$

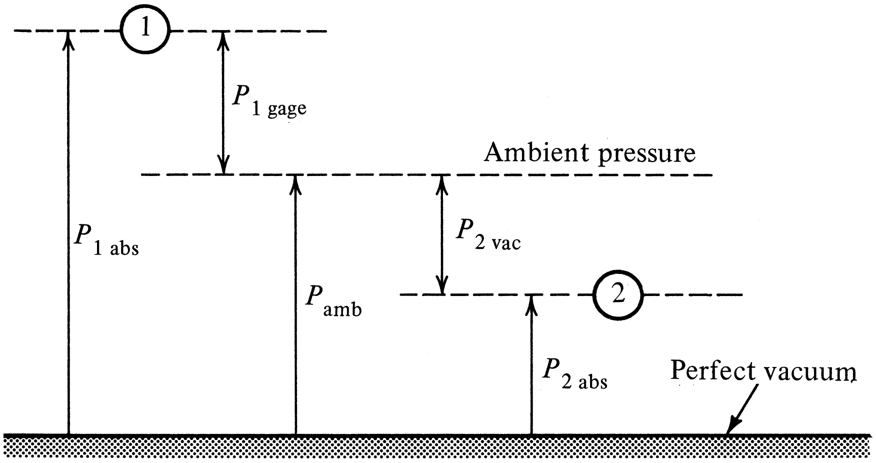

Figure 1.1 Absolute and gage pressures.

Two pressure readings are shown in Figure 1.1. Case 1 shows the use of equation (1.7) and case 2 illustrates equation (1.8). It should be noted that the surrounding (ambient) pressure does not necessarily have to correspond to standard atmospheric pressure. However, when *no* other information is available, one has to assume that the surroundings are at 14.69 psi or 0.1013 MPa. Most often, equations require the use of absolute pressure, and we shall use a numerical value of 14.7 when using the English Engineering system and 0.1 MPa (1 bar) when using the SI system.

Temperature

Degrees Fahrenheit (or Celsius) can safely be used only when *differences* in temperature are involved. However, most equations require the use of absolute temperature in Rankine (or kelvins).

$$^\circ\mathrm{R} = {}^\circ\mathrm{F} + 459.67 \tag{1.9a}$$

$$\mathrm{K} = {}^\circ\mathrm{C} + 273.15 \tag{1.9b}$$

The values 460 and 273 will be used in our calculations.

Viscosity

We shall be dealing with *fluids*, which are defined as

> Any substance that will continuously deform when subjected to a shear stress.

Thus the amount of deformation is of no significance (as it is with a solid), but rather, the *rate of deformation* is characteristic of each individual fluid and is indicated by the *viscosity*:

$$\text{viscosity} \equiv \frac{\text{shear stress}}{\text{rate of angular deformation}} \tag{1.10}$$

Viscosity, sometimes called *absolute viscosity*, is given the symbol μ and has the units lbf-sec/ft^2 or N · s/m^2.

For most common fluids, because viscosity is a function of the fluid, it varies with the fluid's state. Temperature has by far the greatest effect on viscosity, so most charts and tables display only this variable. Pressure has a slight effect on the viscosity of gases but a negligible effect on liquids.

A number of engineering computations use a combination of (absolute) viscosity and density. This *kinematic viscosity* is defined as

$$\nu \equiv \frac{\mu g_c}{\rho} \tag{1.11}$$

Kinematic viscosity has the units ft^2/sec or m^2/s. We shall see more regarding viscosity in Chapter 9 when we deal with flow losses caused by duct friction.

Equation of State

In most of this book we consider all liquids as having constant density and all gases as following the perfect gas equation of state. Thus, for liquids we have the relation

$$\rho = \text{ constant} \tag{1.12}$$

The perfect gas equation of state is derived from kinetic theory and neglects molecular volume and intermolecular forces. Thus it is accurate under conditions of relatively low density which correspond to relatively low pressures and/or high temperatures. The form of the *perfect gas equation* normally used in gas dynamics is

$$p = \rho RT \tag{1.13}$$

where

$p \equiv$ absolute pressure	lbf/ft^2	or	N/m^2
$\rho \equiv$ density	lbm/ft^3	or	kg/m^3
$T \equiv$ absolute temperature	°R	or	K
$R \equiv$ *individual* gas constant	ft-lbf/lbm-°R	or	N · m/kg · K

The *individual* gas constant is found in the English Engineering system by dividing 1545 by the molecular mass of the gas chemical constituents. In the SI system, R

is found by dividing 8314 by the molecular mass. More exact numbers are given in Appendixes A and B.

Example 1.1 The (equivalent) molecular mass of air is 28.97.

$$R = \frac{1545}{28.97} = 53.3 \text{ ft-lbf/lbm-°R} \quad \text{or} \quad R = \frac{8314}{28.97} = 287 \text{ N} \cdot \text{m/kg} \cdot \text{K}$$

Example 1.2 Compute the density of air at 50 psia and 100°F.

$$\rho = \frac{p}{RT} = \frac{(50)(144)}{(53.3)(460 + 100)} = 0.241 \text{ lbm/ft}^3$$

Properties of selected gases are given in Appendixes A and B. In most of this book we use English Engineering units. However, there are many examples and problems in SI units. Some helpful conversion factors are also given in Appendixes A and B. You should become familiar with solving problems in both systems of units.

In Chapter 11 we discuss real gases and show how these may be handled. The simplifications that the perfect gas equation of state brings about are not only extremely useful but also accurate for ordinary gases because in most gas dynamics applications low temperatures exist with low pressures and high temperatures with high pressures. In Chapter 11 we shall see that deviations from ideality become particularly important at high temperatures and low pressures.

1.3 SOME MATHEMATICAL CONCEPTS

Variables

The equation

$$y = f(x) \tag{1.14}$$

indicates that a functional relation exists between the variables x and y. Further, it denotes that

x is the *independent variable*, whose value can be given anyplace within an appropriate range.

y is the *dependent variable*, whose value is fixed once x has been selected.

In most cases it is possible to interchange the dependent and independent variables and write

$$x = f(y) \tag{1.15}$$

Frequently, a variable will depend on more than one other variable. One might write

$$P = f(x, y, z) \tag{1.16}$$

indicating that the value of the dependent variable P is fixed once the values of the independent variables x, y, and z are selected.

Infinitesimal

A quantity that is eventually allowed to approach zero in the limit is called an *infinitesimal*. It should be noted that a quantity, say Δx, can initially be chosen to have a rather large finite value. If at some later stage in the analysis we let Δx approach zero, which is indicated by

$$\Delta x \to 0$$

Δx is called an infinitesimal.

Derivative

If $y = f(x)$, we define the *derivative* dy/dx as the limit of $\Delta y/\Delta x$ as Δx is allowed to approach zero. This is indicated by

$$\frac{dy}{dx} \equiv \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \tag{1.17}$$

For a unique derivative to exist, it is immaterial how Δx is allowed to approach zero.

If more than one independent variable is involved, *partial derivatives* must be used. Say that $P = f(x,y,z)$. We can determine the partial derivative $\partial P/\partial x$ by taking the limit of $\Delta P/\Delta x$ as Δx approaches zero, but in so doing we *must* hold the values of all other independent variables constant. This is indicated by

$$\frac{\partial P}{\partial x} \equiv \lim_{\Delta x \to 0} \left(\frac{\Delta P}{\Delta x}\right)_{y,z} \tag{1.18}$$

where the subscripts y and z denote that these variables remain fixed in the limiting process. We could formulate other partial derivatives as

$$\frac{\partial P}{\partial y} \equiv \lim_{\Delta y \to 0} \left(\frac{\Delta P}{\Delta y}\right)_{x,z} \qquad \text{and so on} \tag{1.19}$$

Differential

For functions of a single variable such as $y = f(x)$, the *differential* of the dependent variable is defined as

$$dy \equiv \frac{dy}{dx}\Delta x \tag{1.20}$$

The differential of an independent variable is defined as its increment; thus

$$dx \equiv \Delta x \tag{1.21}$$

and one can write

$$dy = \frac{dy}{dx} dx \tag{1.22}$$

For functions of more than one variable, such as $P = f(x,y,z)$, the differential of the dependent variable is defined as

$$dP \equiv \left(\frac{\partial P}{\partial x}\right)_{y,z} \Delta x + \left(\frac{\partial P}{\partial y}\right)_{x,z} \Delta y + \left(\frac{\partial P}{\partial z}\right)_{x,y} \Delta z \tag{1.23a}$$

or

$$dP \equiv \left(\frac{\partial P}{\partial x}\right)_{y,z} dx + \left(\frac{\partial P}{\partial y}\right)_{x,z} dy + \left(\frac{\partial P}{\partial z}\right)_{x,y} dz \tag{1.23b}$$

It is important to note that quantities such as ∂P, ∂x, ∂y, and ∂z by themselves are *never* defined and *do not exist*. Under no circumstance can one "separate" a partial derivative. This is an error frequently made by students when integrating partial differential equations.

Maximum and Minimum

If a plot is made of the functional relation $y = f(x)$, *maximum* and/or *minimum* points may be exhibited. At these points $dy/dx = 0$. If the point is a maximum, d^2y/dx^2 will be negative; whereas if it is a minimum point, d^2y/dx^2 will be positive.

Natural Logarithms

From time to time you will be required to manipulate expressions containing *natural logarithms*. For this you need to recall that

$$\ln A = x \quad \text{means} \quad e^x = A \tag{1.24}$$

$$\ln CD = \ln C + \ln D \tag{1.24a}$$

$$\ln E^n = n \ln E \tag{1.24b}$$

Taylor Series

When the functional relation $y = f(x)$ is not known but the values of y together with those of its derivatives are known at a particular point (say, x_1), the value of y may be found at any other point (say, x_2) through the use of a *Taylor series expansion*:

$$\begin{aligned} f(x_2) = f(x_1) &+ \frac{df}{dx}(x_2 - x_1) + \frac{d^2 f}{dx^2}\frac{(x_2 - x_1)^2}{2!} \\ &+ \frac{d^3 f}{dx^3}\frac{(x_2 - x_1)^3}{3!} + \cdots \end{aligned} \tag{1.25}$$

To use this expansion the function must be continuous and possess continuous derivatives throughout the interval x_1 to x_2. It should be noted that all derivatives in the expression above must be evaluated about the point of expansion x_1.

If the increment $\Delta x = x_2 - x_1$ is small, only a few terms need be evaluated to obtain an accurate answer for $f(x_2)$. If Δx is allowed to approach zero, all higher-order terms may be dropped and

$$f(x_2) \approx f(x_1) + \left(\frac{df}{dx}\right)_{x=x_1} dx \qquad \text{for } dx \to 0 \tag{1.26}$$

1.4 THERMODYNAMIC CONCEPTS FOR CONTROL MASS ANALYSIS

We apologize for the length of this section, but a good understanding of thermodynamic principles is essential to a study of gas dynamics.

General Definitions

Microscopic approach: deals with individual molecules, and with their motion and behavior, on a statistical basis. It depends on our understanding of the structure and behavior of matter at the atomic level. Thus this view is being refined continually.

Macroscopic approach: deals directly with the average behavior of molecules through observable and measurable properties (temperature, pressure, etc.). This classical approach involves no assumptions regarding the molecular structure of matter; thus no modifications of the basic laws are necessary. The macroscopic approach is used in this book through the first 10 chapters.

Control mass: a fixed quantity of mass that is being analyzed. It is separated from its surroundings by a boundary. A control mass is also referred to as a *closed system*. Although no matter crosses the boundary, energy may enter or leave the system.

Control volume: a region of space that is being analyzed. The boundary separating it from its surroundings is called the *control surface*. Matter as well as energy may cross the control surface, and thus a control volume is also referred to as an *open system*. Analysis of a control volume is introduced in Chapters 2 and 3.

Properties: characteristics that describe the state of a system; any quantity that has a definite value for each definite state of a system (e.g., pressure, temperature, color, entropy).

Intensive property: depends only on the state of a system and is independent of its mass (e.g., temperature, pressure).

Extensive property: depends on the mass of a system (e.g., internal energy, volume).

Types of properties:

1. *Observable:* readily measured
   (pressure, temperature, velocity, mass, etc.)

2. *Mathematical:* defined from combinations of other properties (density, specific heats, enthalpy, etc.)
3. *Derived:* arrived at as the result of analysis
    a. *Internal energy* (from the first law of thermodynamics)
    b. *Entropy* (from the second law of thermodynamics)

State change: comes about as the result of a change in any property.

Path or process: represents a series of consecutive states that define a unique path from one state to another. Some special processes:

Adiabatic → no heat transfer
Isothermal → $T =$ constant
Isobaric → $p =$ constant
Isentropic → $s =$ constant

Cycle: a sequence of processes in which the system is returned to the original state.

Point functions: another way of saying *properties*, since they depend only on the state of the system and are independent of the history or process by which the state was obtained.

Path functions: quantities that are *not* functions of the state of the system but rather depend on the path taken to move from one state to another. *Heat* and *work* are path functions. They can be observed crossing the system's boundaries *during* a process.

Laws of Classical Thermodynamics

0^2	Relation among properties
0	Thermal equilibrium
1	Conservation of energy
2	Degradation of energy (irreversibilities)

The *0^2 law* (sometimes called the *00 law*) is seldom listed as a formal law of thermodynamics; however, one should realize that without such a statement our entire thermodynamic structure would collapse. This law states that we may assume the existence of a relation among the properties, that is, an *equation of state*. Such an equation might be extremely complicated or even undefined, but as long as we know that such a relation exists, we can continue our studies. The equation of state can also be given in the form of tabular or graphical information.

For a single component or pure substance only three *independent* properties are required to fix the state of the system. Care must be taken in the selection of these properties; for example, temperature and pressure are not independent if the substance exists in more than one phase (as in a liquid together with its vapor). When dealing with a unit mass, only two independent properties are required to fix the state. Thus one can express any property in terms of two other known independent properties with a relation such as

$$P = f(x,y)$$

If two systems are separated by a nonadiabatic wall (one that permits heat transfer), the state of each system will change until a new equilibrium state is reached for the combined system. The two systems are then said to be in *thermal equilibrium* with each other and will then have one property in common which we call the *temperature*.

The *zeroth law* states that two systems in thermal equilibrium with a third system are in thermal equilibrium with each other (and thus have the same temperature). Among other things, this allows the use of thermometers and their standardization.

First Law of Thermodynamics

The *first law* deals with conservation of energy, and it can be expressed in many equivalent ways. Heat and work are two extreme types of energy in transit. *Heat* is transferred from one system to another when an effect occurs solely as a result of a temperature difference between the two systems.

Heat is always transferred from the system at the higher temperature to the one at the lower temperature.

Work is transferred from a system if the total external effect can be reduced to the raising of a mass in a gravity field. For a closed system that executes a complete cycle,

$$\sum Q = \sum W \tag{1.27}$$

where

Q = heat transferred *into* the system

W = work transferred *from* the system

Other sign conventions are sometimes used but we shall adopt those above for this book.

For a closed system that executes a process,

$$Q = W + \Delta E \tag{1.28}$$

where E represents the total energy of the system. On a unit mass basis, equation (1.28) is written as

$$q = w + \Delta e \tag{1.29}$$

The total energy may be broken down into (at least) three types:

$$e \equiv u + \frac{V^2}{2g_c} + \frac{g}{g_c}z \tag{1.30}$$

where

u = the intrinsic internal energy manifested by the motion of the molecules within the system

$\frac{V^2}{2g_c}$ = the kinetic energy represented by the movement of the system as a whole

$\frac{g}{g_c}z$ = the potential energy caused by the position of the system in a field of gravity

It is sometimes necessary to include other types of energy (such as dissociation energy), but those mentioned above are the only ones that we are concerned with in this book.

For an infinitesimal process, one could write equation (1.29) as

$$\delta q = \delta w + de \tag{1.31}$$

Note that since heat and work are *path functions* (i.e., they are a function of how the system gets from one state point to another), infinitesimal amounts of these quantities are not exact differentials and thus are written as δq and δw. The infinitesimal change in internal energy is an exact differential since the internal energy is a point function or property. For a stationary system, equation (1.31) becomes

$$\delta q = \delta w + du \tag{1.32}$$

The reversible work done by pressure forces during a change of volume for a stationary system is

$$\delta w = p\,dv \tag{1.33}$$

Combination of the terms u and pv enters into many equations (particularly for open systems) and it is convenient to define the property *enthalpy:*

$$h \equiv u + pv \tag{1.34}$$

Enthalpy is a property since it is defined in terms of other properties. It is frequently used in differential form:

$$dh = du + d(pv) = du + p\,dv + v\,dp \tag{1.35}$$

Other examples of defined properties are the specific heats at constant pressure (c_p) and constant volume (c_v):

$$c_p \equiv \left(\frac{\partial h}{\partial T}\right)_p \tag{1.36}$$

$$c_v \equiv \left(\frac{\partial u}{\partial T}\right)_v \tag{1.37}$$

Second Law of Thermodynamics

The *second law* has been expressed in many equivalent forms. Perhaps the most classic is the statement by Kelvin and Planck stating that it is impossible for an engine operating in a *cycle* to produce *net* work output when exchanging heat with only one temperature source. Although by itself this may not appear to be a profound statement, it leads the way to several corollaries and eventually to the establishment of a most important property (entropy).

The *second law* also recognizes the degradation of energy quality by irreversible effects such as internal fluid friction, heat transfer through a finite temperature difference, lack of pressure equilibrium between a system and its surroundings, and so on. All real processes have some degree of irreversibility present. In some cases these effects are very small and we can envision an ideal limiting condition that has none of these effects and thus is reversible. A *reversible process* is one in which *both* the system and its surroundings can be restored to their original states.

By prudent application of the second law it can be shown that the integral of $\delta Q/T$ for a reversible process is independent of the path. Thus this integral must represent the change of a *property*, which is called *entropy:*

$$\Delta S \equiv \int \frac{\delta Q_R}{T} \tag{1.38}$$

where the subscript R indicates that it must be applied to a reversible process. An alternative expression on a unit mass basis for a differential process is

$$ds \equiv \frac{\delta q_R}{T} \tag{1.39}$$

Although you have no doubt used entropy for many calculations, plots, and so on, you probably do not have a good feeling for this property. In Chapter 3 we divide entropy changes into two parts, and by using it in this fashion for the remainder of this book we hope that you will gain a better understanding of this elusive “creature.”

Property Relations

Some extremely important relations come from combinations of the first and second laws. Consider the first law for a stationary system that executes an infinitesimal process:

$$\delta q = \delta w + du \tag{1.32}$$

If it is a reversible process,

$$\delta w = p\,dv \quad (1.33) \qquad \text{and} \qquad \delta q = T\,ds \quad (\text{from } 1.39)$$

Substitution of these relations into the first law yields

$$T\,ds = du + p\,dv \tag{1.40}$$

Differentiating the enthalpy, we obtained

$$dh = du + p\,dv + v\,dp \tag{1.35}$$

Combining equations (1.35) and (1.40) produces

$$T\,ds = dh - v\,dp \tag{1.41}$$

Although the assumption of a reversible process was made to derive equations (1.40) and (1.41), the results are equations that *contain only properties and thus are valid relations to use between any end states*, whether reached reversibly or not. These are important equations that are used throughout the book.

$$T\,ds = du + p\,dv \tag{1.40}$$

$$T\,ds = dh - v\,dp \tag{1.41}$$

If you are uncomfortable with the foregoing technique (one of making special assumptions to derive a relation which is then generalized to be always valid since it involves only properties), perhaps the following comments might be helpful. First let's write the first law in an alternative form (as some authors do):

$$\delta q - \delta w = du \tag{1.32a}$$

Since the internal energy is a property, changes in *u* depend only on the end states of a process. Let's now substitute an irreversible process *between the same end points* as our reversible process. Then du must remain the same for both the reversible and irreversible cases, with the following result:

$$(\delta q - \delta w)_{\text{rev}} = du = (\delta q - \delta w)_{\text{irrev}}$$

For example, the extra work that would be involved in an ireversible compression process must be compensated by *exactly the same amount of heat released* (an equivalent argument applies to an expansion). In this fashion, irreversible effects will appear to be "washed out" in equations (1.40) and (1.41) and we cannot tell from them whether a particular process is reversible or irreversible.

Perfect Gases

Recall that for a unit mass of a single component substance, any one property can be expressed as a function of at most *two* other independent properties. However, for substances that follow the perfect gas equation of state,

$$p = \rho RT \tag{1.13}$$

it can be shown (see p. 173 of Ref. 4) that *the internal energy and the enthalpy are functions of temperature only.* These are extremely important results, as they permit us to make many useful simplifications for such gases.

Consider the specific heat at constant volume:

$$c_v \equiv \left(\frac{\partial u}{\partial T}\right)_v \tag{1.37}$$

If $u = f(T)$ only, it does not matter whether the volume is held constant when computing c_v; thus the partial derivative becomes an ordinary derivative. Thus

$$c_v = \frac{du}{dt} \tag{1.42}$$

or

$$du = c_v\, dT \tag{1.43}$$

Similarly, for the specific heat at constant pressure, we can write for a perfect gas:

$$dh = c_p\, dT \tag{1.44}$$

It is important to realize that equations (1.43) and (1.44) are applicable to *any and all* processes (as long as the gas behaves as a perfect gas). If the specific heats remain reasonably constant (normally good over limited temperature ranges), one can easily integrate equations (1.43) and (1.44):

$$\Delta u = c_v\ \Delta T \tag{1.45}$$

$$\Delta h = c_p\ \Delta T \tag{1.46}$$

In gas dynamics one simplifies calculations by introducing an arbitrary base for internal energy. We let $u = 0$ when $T = 0$ absolute. Then from the definition of enthalpy, h also equals zero when $T = 0$. Equations (1.45) and (1.46) can now be rewritten as

$$u = c_v T \tag{1.47}$$

$$h = c_p T \tag{1.48}$$

Typical values of the specific heats for air at normal temperature and pressure are $c_p = 0.240$ and $c_v = 0.171$ Btu/lbm-°R. Learn these numbers (or their SI equivalents)! You will use them often.

Other frequently used relations in connection with perfect gases are

$$\gamma \equiv \frac{c_p}{c_v} \tag{1.49}$$

$$c_p - c_v = \frac{R}{J} \tag{1.50}$$

Notice that the conversion factor

$$J = 778 \text{ ft-lbf/Btu} \tag{1.51}$$

has been introduced in (1.50) since the specific heats are normally given in units of Btu/lbm-°R. This factor will be omitted in future equations and it will be left for you to consider when it is required. It is hoped that by this procedure you will develop careful habits of checking units in all your work. What units are used for specific heat and R in the SI system? (See the table on gas properties in Appendix B.) Would this require a J factor in equation (1.50)?

Entropy Changes

The change in entropy between any two states can be obtained by integrating equation (1.39) along any reversible path or combination of reversible paths connecting the points, with the following results for perfect gases:

$$\Delta s_{1-2} = c_p \ln \frac{v_2}{v_1} + c_v \ln \frac{p_2}{p_1} \tag{1.52}$$

$$\Delta s_{1-2} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \tag{1.53}$$

$$\Delta s_{1-2} = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \tag{1.54}$$

Remember, absolute values of pressures and temperatures must be used in these equations; volumes may be either total or specific, but both volumes must be of the same type. Watch the units on c_p, c_v, and R.

Process Diagrams

Many processes in the gaseous region can be represented as a *polytropic process*, that is, one that follows the relation

$$pv^n = \text{const} = C_1 \tag{1.55}$$

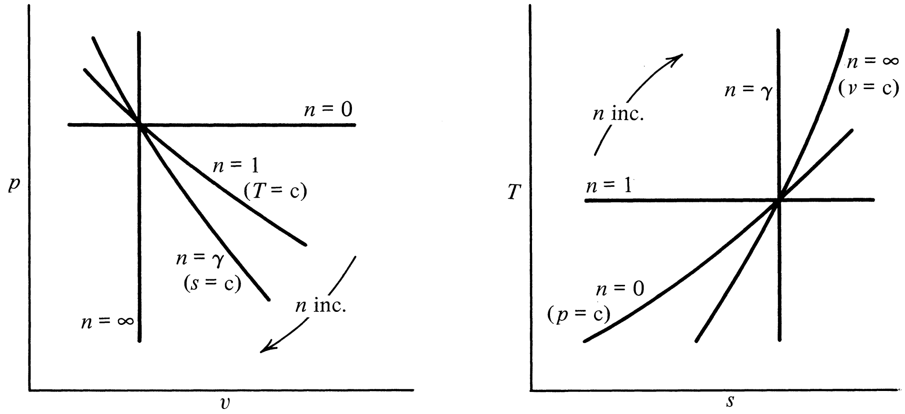

Figure 1.2 General polytropic process plots for perfect gases.

where n is the polytropic exponent, which can be any positive number. If the fluid is a perfect gas, the equation of state can be introduced into (1.55) to yield

$$Tv^{n-1} = \text{const} = C_2 \tag{1.56}$$

$$Tp^{(1-n)/n} = \text{const} = C_3 \tag{1.57}$$

Keep in mind that C_1, C_2, and C_3 in the equations above are different constants. It is interesting to note that certain values of n represent particular processes:

$$n = 0 \quad \rightarrow \quad p = \text{const}$$
$$n = 1 \quad \rightarrow \quad T = \text{const}$$
$$n = \gamma \quad \rightarrow \quad s = \text{const}$$
$$n = \infty \quad \rightarrow \quad v = \text{const}$$

These plot in the p–v and T–s diagrams as shown in Figure 1.2, *Learn these diagrams*! You should also be able to figure out how temperature and entropy vary in the p–v diagram and how pressure and volume vary in the T–s diagram (Try drawing several T = const lines in the p–v plane. Which one represents the highest temperature?).

REVIEW QUESTIONS

A number of questions follow that are based on concepts that you have covered in earlier calculus and thermodynamic courses. State your answers as clearly and concisely as possible using any source that you wish (although all the material has been covered in the preceding review). Do not proceed to Chapter 2 until you fully understand the correct answers to all questions and can write them down without reference to your notes.

1.1. How is an ordinary derivative such as dy/dx defined? How does this differ from a partial derivative?

1.2. What is the Taylor series expansion, and what are its applications and limitations?

1.3. State Newton's second law as you would apply it to a control mass.

1.4. Define a 1-pound force in terms of the acceleration it will give to a 1-pound mass. Give a similar definition for a newton in the SI system.

1.5. Explain the significance of g_c in Newton's second law. What are the magnitude *and units* of g_c in the English Engineering system? In the SI system?

1.6. What is the relation between degrees Fahrenheit and degrees Rankine? Degrees Celsius and Kelvin?

1.7. What is the relationship between density and specific volume?

1.8. Explain the difference between absolute and gage pressures.

1.9. What is the distinguishing characteristic of a fluid (as compared to a solid)? How is this related to viscosity?

1.10. Describe the difference between the microscopic and macroscopic approach in an analysis of fluid behavior.

1.11. Describe the control volume approach to problem analysis and contrast it to the control mass approach. What kinds of systems are these also called?

1.12. Describe a property and give at least three examples.

1.13. Properties may be categorized as either intensive or extensive. Define what is meant by each, and list examples of each type of property.

1.14. When dealing with a unit mass of a single component substance, how many independent properties are required to fix the state?

1.15. Of what use is an equation of state? Write down one with which you are familiar.

1.16. Define point functions and path functions. Give examples of each.

1.17. What is a process? What is a cycle?

1.18. How does the zeroth law of thermodynamics relate to temperature?

1.19. State the first law of thermodynamics for a closed system that is executing a single process.

1.20. What are the sign conventions used in this book for heat and work?

1.21. State any form of the second law of thermodynamics.

1.22. Define a reversible process for a thermodynamic system. Is any real process ever completely reversible?

1.23. What are some effects that cause processes to be irreversible?

1.24. What is an adiabatic process? An isothermal process? An isentropic process?

1.25. Give equations that define enthalpy and entropy.

1.26. Give differential expressions that relate entropy to

(a) internal energy and

(b) enthalpy.

1.27. Define (in the form of partial derivatives) the specific heats c_v and c_p. Are these expressions valid for a material in any state?

1.28. State the perfect gas equation of state. Give a consistent set of units for each term in the equation.

1.29. For a perfect gas, specific internal energy is a function of which state variables? How about specific enthalpy?

1.30. Give expressions for Δu and Δh that are valid for perfect gases. Do these hold for any process?

1.31. For perfect gases, at what temperature do we arbitrarily assign $u = 0$ and $h = 0$?

1.32. State any expression for the entropy change between two arbitrary points which is valid for a perfect gas.

1.33. If a perfect gas undergoes an isentropic process, what equation relates the pressure to the volume? Temperature to the volume? Temperature to the pressure?

1.34. Consider the general polytropic process (pv^n = const) for a perfect gas. In the p–v and T–s diagrams shown in Figure RQ1.34, label each process line with the correct value of n and identify which fluid property is held constant.

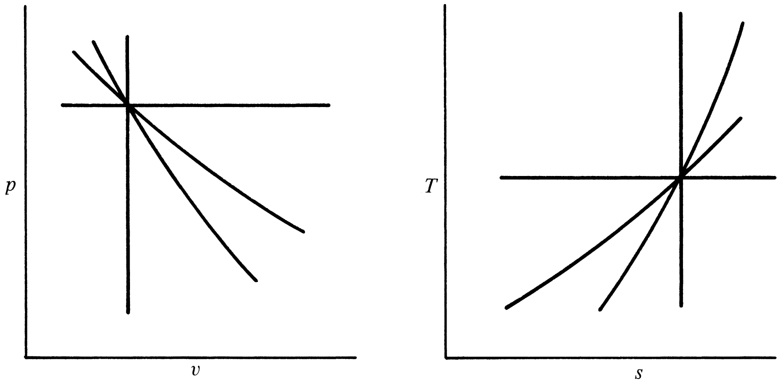

Figure RQ1.34

REVIEW PROBLEMS

If you have been away from thermodynamics for a long time, it might be useful to work the following problems.

1.1. How well is the relation $c_p = c_v + R$ represented in the table of gas properties in Appendix A? Use entries for hydrogen.

1.2. A perfect gas having specific heats $c_v = 0.403$ Btu/lbm-°R and $c_p = 0.532$ Btu/lbm-°R undergoes a reversible polytropic process in which the polytropic exponent $n = 1.4$. Giving clear reasons, answer the following:

(a) Will there be any heat transfer in the process?

(b) Which would this process be *nearest*, a horizontal or a vertical line on a p–v or a T–s diagram? (Alternatively, state between which constant property lines the process lies.)

1.3. Nitrogen gas is reversibly compressed from 70°F and 14.7 psia to one-fourth of its original volume by (1) a T = const process or (2) a p = const process followed by a v = const process to the same end point as (1).

(a) Which compression involves the least amount of work? Show clearly on a p–v diagram.

(b) Calculate the heat and work interaction for the isothermal compression.

1.4. For the reversible cycle shown in Figure RP1.4, compute the cyclic integrals $[\oint d(\cdot)]$ of dE, δQ, dH, δW, and dS.

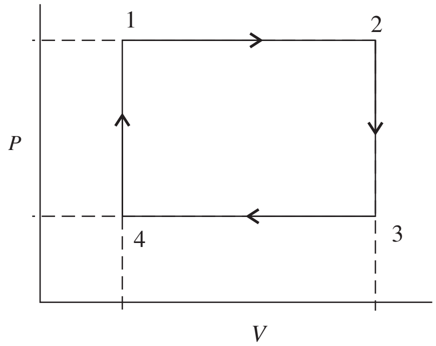

Figure RP1.4

$p_1 = p_2 = 1.0 \times 10^6$ Pa

$p_3 = p_4 = 0.4 \times 10^6$ Pa

$V_1 = V_4 = 0.6\,\mathrm{m}^3$

$V_2 = V_3 = 1.0\,\mathrm{m}^3$

1.5. A perfect gas (methane) undergoes a reversible, polytropic process in which the polytrotic exponent is 1.4.

(a) Using the first law, arrive at an expression for the heat transfer per unit mass solely as a function of the temperature difference ΔT. This should be some numerical value (use SI units).

(b) Would this heat transfer be equal to either the enthalpy change or the internal energy change for the same ΔT?