CHAPTER 1

BASIC ELECTRIC AND MAGNETIC CIRCUITS

1.1 INTRODUCTION TO ELECTRIC CIRCUITS

In elementary physics classes you undoubtedly have been introduced to the fundamental concepts of electricity and how real components can be put together to form an electrical circuit. A very simple circuit, for example, might consist of a battery, some wire, a switch, and an incandescent lightbulb as shown in Fig. 1.1. The battery supplies the energy required to force electrons around the loop, heating the filament of the bulb and causing the bulb to radiate a lot of heat and some light. Energy is transferred from a *source*, the battery, to a *load*, the bulb. You probably already know that the *voltage* of the battery and the electrical *resistance* of the bulb have something to do with the amount of *current* that will flow in the circuit. From your own practical experience you also know that no current will flow until the switch is closed. That is, for a circuit to do anything, the loop has to be completed so that electrons can flow from the battery to the bulb and then back again to the battery. And finally, you probably realize that it doesn't much matter whether there is one foot or two feet of wire connecting the battery to the bulb, but that it probably would matter if there is a mile of wire between it and the bulb.

Also shown in Fig. 1.1 is a model made up of *idealized* components. The battery is modeled as an ideal source that puts out a constant voltage, V_B, no matter what amount of current, i, is drawn. The wires are considered to be perfect

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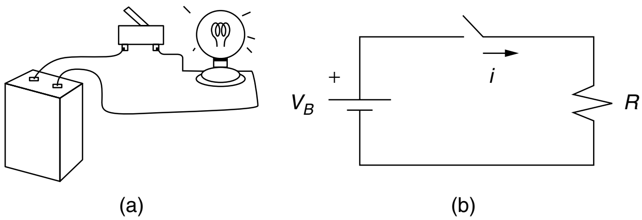


Figure 1.1 (a) A simple circuit. (b) An idealized representation of the circuit.

conductors that offer no resistance to current flow. The switch is assumed to be open or closed. There is no arcing of current across the gap when the switch is opened, nor is there any bounce to the switch as it makes contact on closure. The lightbulb is modeled as a simple resistor, R, that never changes its value, no matter how hot it becomes or how much current is flowing through it.

For most purposes, the idealized model shown in Fig. 1.1b is an adequate representation of the circuit; that is, our prediction of the current that will flow through the bulb whenever the switch is closed will be sufficiently accurate that we can consider the problem solved. There may be times, however, when the model is inadequate. The battery voltage, for example, may drop as more and more current is drawn, or as the battery ages. The lightbulb's resistance may change as it heats up, and the filament may have a bit of inductance and capacitance associated with it as well as resistance so that when the switch is closed, the current may not jump instantaneously from zero to some final, steady-state value. The wires may be undersized, and some of the power delivered by the battery may be lost in the wires before it reaches the load. These subtle effects may or may not be important, depending on what we are trying to find out and how accurately we must be able to predict the performance of the circuit. If we decide they are important, we can always change the model as necessary and then proceed with the analysis.

The point here is simple. The combinations of resistors, capacitors, inductors, voltage sources, current sources, and so forth, that you see in a circuit diagram are merely models of real components that comprise a real circuit, and a certain amount of judgment is required to decide how complicated the model must be before sufficiently accurate results can be obtained. For our purposes, we will be using very simple models in general, leaving many of the complications to more advanced textbooks.

1.2 DEFINITIONS OF KEY ELECTRICAL QUANTITIES

We shall begin by introducing the fundamental electrical quantities that form the basis for the study of electric circuits.

1.2.1 Charge

An atom consists of a positively charged nucleus surrounded by a swarm of negatively charged electrons. The charge associated with one electron has been found to be 1.602×10^{-19} coulombs; or, stated the other way around, one coulomb can be defined as the charge on 6.242×10^{18} electrons. While most of the electrons associated with an atom are tightly bound to the nucleus, good conductors, like copper, have *free electrons* that are sufficiently distant from their nuclei that their attraction to any particular nucleus is easily overcome. These conduction electrons are free to wander from atom to atom, and their movement constitutes an electric current.

1.2.2 Current

In a wire, when one coulomb's worth of charge passes a given spot in one second, the current is defined to be one *ampere* (abbreviated A), named after the nineteenth-century physicist André Marie Ampère. That is, current i is the net rate of flow of charge q past a point, or through an area:

$$i = \frac{dq}{dt} \tag{1.1}$$

In general, charges can be negative or positive. For example, in a neon light, positive ions move in one direction and negative electrons move in the other. Each contributes to current, and the total current is their sum. By convention, the direction of current flow is taken to be the direction that positive charges would move, whether or not positive charges happen to be in the picture. Thus, in a wire, electrons moving to the right constitute a current that flows to the left, as shown in Fig. 1.2.

When charge flows at a steady rate in one direction only, the current is said to be *direct current*, or *dc*. A battery, for example, supplies direct current. When charge flows back and forth sinusoidally, it is said to be *alternating current*, or *ac*. In the United States the ac electricity delivered by the power company has a frequency of 60 cycles per second, or 60 hertz (abbreviated Hz). Examples of ac and dc are shown in Fig. 1.3.

1.2.3 Kirchhoff's Current Law

Two of the most fundamental properties of circuits were established experimentally a century and a half ago by a German professor, Gustav Robert Kirchhoff (1824–1887). The first property, known as Kirchhoff's current law (abbreviated

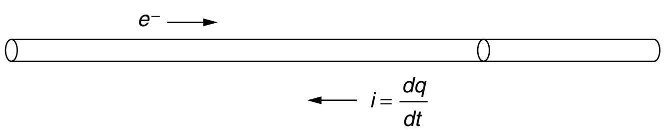


Figure 1.2 By convention, negative charges moving in one direction constitute a positive current flow in the opposite direction.

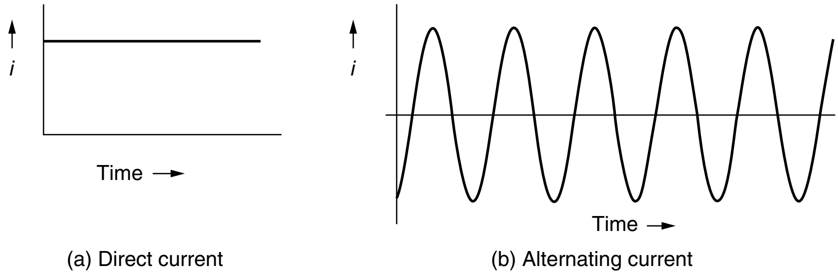


Figure 1.3 (a) Steady-state direct current (dc). (b) Alternating current (ac).

KCL), states that *at every instant of time the sum of the currents flowing into any node of a circuit must equal the sum of the currents leaving the node*, where a node is any spot where two or more wires are joined. This is a very simple, but powerful concept. It is intuitively obvious once you assert that current is the flow of charge, and that charge is conservative—neither being created nor destroyed as it enters a node. Unless charge somehow builds up at a node, which it does not, then the rate at which charge enters a node must equal the rate at which charge leaves the node.

There are several alternative ways to state Kirchhoff's current law. The most commonly used statement says that the sum of the currents into a node is zero as shown in Fig. 1.4a, in which case some of those currents must have negative values while some have positive values. Equally valid would be the statement that the sum of the currents leaving a node must be zero as shown in Fig. 1.4b (again some of these currents need to have positive values and some negative). Finally, we could say that the sum of the currents entering a node equals the sum of the currents leaving a node (Fig. 1.4c). These are all equivalent as long as we understand what is meant about the direction of current flow when we indicate it with an arrow on a circuit diagram. Current that actually flows in the direction shown by the arrow is given a positive sign. Currents that actually flow in the opposite direction have negative values.

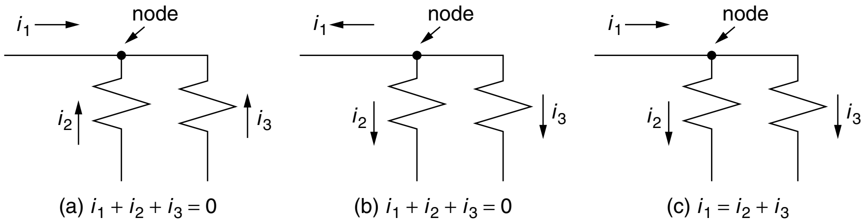
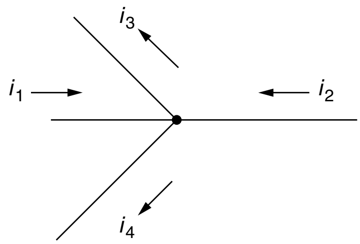


Figure 1.4 Illustrating various ways that Kirchhoff's current law can be stated. (a) The sum of the currents into a node equals zero. (b) The sum of the currents leaving the node is zero. (c) The sum of the currents entering a node equals the sum of the currents leaving the node.

Note that you can draw current arrows in any direction that you want—that much is arbitrary—but once having drawn the arrows, you must then write Kirchhoff's current law in a manner that is consistent with your arrows, as has been done in Fig. 1.4. The algebraic solution to the circuit problem will automatically determine whether or not your arbitrarily determined directions for currents were correct.

Example 1.1 Using Kirchhoff's Current Law. A node of a circuit is shown with current direction arrows chosen arbitrarily. Having picked those directions, $i_1 = -5$ A, $i_2 = 3$ A, and $i_3 = -1$ A. Write an expression for Kirchhoff's current law and solve for i_4.



Solution. By Kirchhoff's current law,

$$i_1 + i_2 = i_3 + i_4$$
$$-5 + 3 = -1 + i_4$$

so that

$$i_4 = -1 \text{ A}$$

That is, i_4 is actually 1 A flowing into the node. Note that i_2, i_3, and i_4 are all entering the node, and i_1 is the only current that is leaving the node.

1.2.4 Voltage

Electrons won't flow through a circuit unless they are given some energy to help send them on their way. That "push" is measured in volts, where voltage is defined to be the amount of energy (w, joules) given to a unit of charge,

$$v = \frac{dw}{dq} \tag{1.2}$$

A 12-V battery therefore gives 12 joules of energy to each coulomb of charge that it stores. Note that the charge does not actually have to move for voltage to have meaning. Voltage describes the potential for charge to do work.

While currents are measured *through* a circuit component, voltages are measured *across* components. Thus, for example, it is correct to say that current through a battery is 10 A, while the voltage across that battery is 12 V. Other ways to describe the voltage across a component include whether the voltage rises across the component or drops. Thus, for example, for the simple circuit in Fig. 1.1, there is a voltage rise across the battery and voltage drop across the lightbulb.

Voltages are always measured with respect to something. That is, the voltage of the positive terminal of the battery is "so many volts" with respect to the negative terminal; or, the voltage at a point in a circuit is some amount with respect to some other point. In Fig. 1.5, current through a resistor results in a voltage drop from point A to point B of V_{AB} volts. V_A and V_B are the voltages at each end of the resistor, measured with respect to some other point.

The reference point for voltages in a circuit is usually designated with a *ground* symbol. While many circuits are actually grounded—that is, there is a path for current to flow directly into the earth—some are not (such as the battery, wires, switch, and bulb in a flashlight). When a ground symbol is shown on a circuit diagram, you should consider it to be merely a reference point at which the voltage is defined to be zero. Figure 1.6 points out how changing the node labeled as ground changes the voltages at each node in the circuit, but does not change the voltage drop across each component.

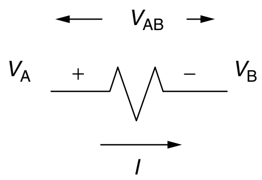


Figure 1.5 The voltage drop from point A to point B is V_{AB}, where $V_{AB} = V_A - V_B$.

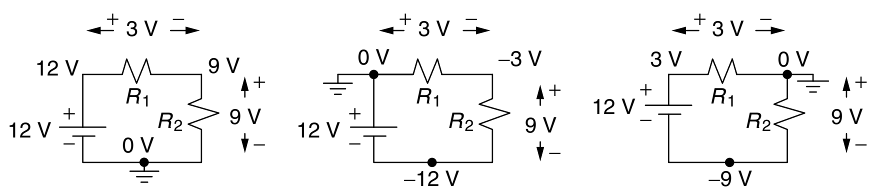


Figure 1.6 Moving the reference node around (ground) changes the voltages at each node, but doesn't change the voltage drop across each component.

1.2.5 Kirchhoff's Voltage Law

The second of Kirchhoff's fundamental laws states that *the sum of the voltages around any loop of a circuit at any instant is zero*. This is known as Kirchhoff's voltage law (KVL). Just as was the case for Kirchhoff's current law, there are alternative, but equivalent, ways of stating KVL. We can, for example, say that the sum of the voltage rises in any loop equals the sum of the voltage drops around the loop. Thus in Fig. 1.6, there is a voltage rise of 12 V across the battery and a voltage drop of 3 V across R_1 and a drop of 9 V across R_2. Notice that it doesn't matter which node was labeled ground for this to be true. Just as was the case with Kirchhoff's current law, we must be careful about labeling and interpreting the signs of voltages in a circuit diagram in order to write the proper version of KVL. A plus (+) sign on a circuit component indicates a reference direction under the assumption that the potential at that end of the component is higher than the voltage at the other end. Again, as long as we are consistent in writing Kirchhoff's voltage law, the algebraic solution for the circuit will automatically take care of signs.

Kirchhoff's voltage law has a simple mechanical analog in which weight is analogous to charge and elevation is analogous to voltage. If a weight is raised from one elevation to another, it acquires potential energy equal to the change in elevation times the weight. Similarly, the potential energy given to charge is equal to the amount of charge times the voltage to which it is raised. If you decide to take a day hike, in which you start and finish the hike at the same spot, you know that no matter what path was taken, when you finish the hike the sum of the increases in elevation has to have been equal to the sum of the decreases in elevation. Similarly, in an electrical circuit, no matter what path is taken, as long as you return to the same node at which you started, KVL provides assurance that the sum of voltage rises in that loop will equal the sum of the voltage drops in the loop.

1.2.6 Power

Power and *energy* are two terms that are often misused. Energy can be thought of as the ability to do work, and it has units such as joules or Btu. Power, on the other hand, is the *rate* at which energy is generated or used, and therefore it has rate units such as joules/s or Btu/h. There is often confusion about the units for electrical power and energy. Electrical power is measured in watts, which is a rate (1 J/s = 1 watt), so electrical energy is watts multiplied by time—for example, watt-hours. Be careful not to say "watts per hour," which is incorrect (even though you will see this all too often in newspapers or magazines).

When a battery delivers current to a load, power is generated by the battery and is dissipated by the load. We can combine (1.1) and (1.2) to find an expression for instantaneous power supplied, or consumed, by a component of a circuit:

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi \tag{1.3}$$

Equation (1.3) tells us that the power supplied at any instant by a source, or consumed by a load, is given by the current through the component times the voltage across the component. When current is given in amperes, and voltage in volts, the units of power are *watts* (W). Thus, a 12-V battery delivering 10 A to a load is supplying 120 W of power.

1.2.7 Energy

Since power is the rate at which work is being done, and energy is the total amount of work done, energy is just the integral of power:

$$w = \int p\,dt \tag{1.4}$$

In an electrical circuit, energy can be expressed in terms of joules (J), where 1 watt-second = 1 joule. In the electric power industry the units of electrical energy are more often given in watt-hours, or for larger quantities kilowatt-hours (kWh) or megawatt-hours (MWh). Thus, for example, a 100-W computer that is operated for 10 hours will consume 1000 Wh, or 1 kWh of energy. A typical household in the United States uses approximately 750 kWh per month.

1.2.8 Summary of Principal Electrical Quantities

The key electrical quantities already introduced and the relevant relationships between these quantities are summarized in Table 1.1.

Since electrical quantities vary over such a large range of magnitudes, you will often find yourself working with very small quantities or very large quantities. For example, the voltage created by your TV antenna may be measured in millionths of a volt (microvolts, μV), while the power generated by a large power station may be measured in billions of watts, or gigawatts (GW). To describe quantities that may take on such extreme values, it is useful to have a system of prefixes that accompany the units. The most commonly used prefixes in electrical engineering are given in Table 1.2.

TABLE 1.1 Key Electrical Quantities and Relationships

Electrical Quantity	Symbol	Unit	Abbreviation	Relationship
Charge	q	coulomb	C	$q = \int i\,dt$
Current	i	ampere	A	$i = dq/dt$
Voltage	v	volt	V	$v = dw/dq$
Power	p	joule/second	J/s	$p = dw/dt$
		or watt	W	
Energy	w	joule	J	$w = \int p\,dt$
		or watt-hour	Wh	

TABLE 1.2 Common Prefixes

Small Quantities			Large Quantities		
Quantity	Prefix	Symbol	Quantity	Prefix	Symbol
10^{-3}	milli	m	10^3	kilo	k
10^{-6}	micro	μ	10^6	mega	M
10^{-9}	nano	n	10^9	giga	G
10^{-12}	pico	p	10^{12}	tera	T

1.3 IDEALIZED VOLTAGE AND CURRENT SOURCES

Electric circuits are made up of a relatively small number of different kinds of circuit *elements*, or *components*, which can be interconnected in an extraordinarily large number of ways. At this point in our discussion, we will concentrate on idealized characteristics of these circuit elements, realizing that real components resemble, but do not exactly duplicate, the characteristics that we describe here.

1.3.1 Ideal Voltage Source

An ideal voltage source is one that provides a given, known voltage v_s, no matter what sort of load it is connected to. That is, regardless of the current drawn from the ideal voltage source, it will always provide the same voltage. Note that an ideal voltage source does not have to deliver a constant voltage; for example, it may produce a sinusoidally varying voltage—the key is that that voltage is not a function of the amount of current drawn. A symbol for an ideal voltage source is shown in Fig. 1.7.

A special case of an ideal voltage source is an ideal battery that provides a constant dc output, as shown in Fig. 1.8. A real battery approximates the ideal source; but as current increases, the output drops somewhat. To account for that drop, quite often the model used for a real battery is an ideal voltage source in series with the internal resistance of the battery.

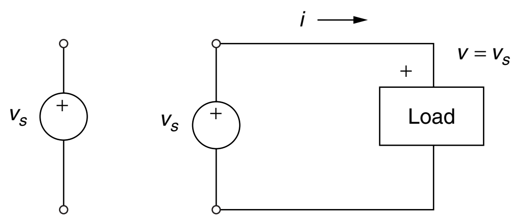


Figure 1.7 A constant voltage source delivers v_s no matter what current the load draws. The quantity v_s can vary with time and still be ideal.

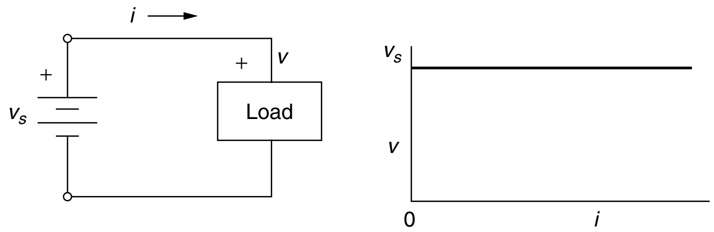


Figure 1.8 An ideal dc voltage.

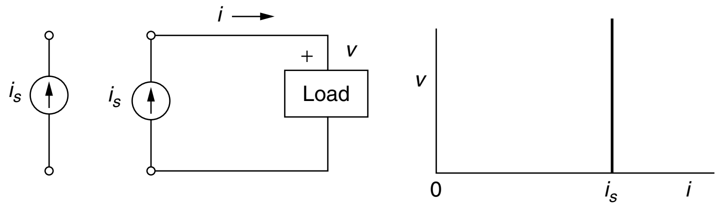


Figure 1.9 The current produced by an ideal current source does not depend on the voltage across the source.

1.3.2 Ideal Current Source

An ideal current source produces a given amount of current i_s no matter what load it sees. As shown in Fig. 1.9, a commonly used symbol for such a device is circle with an arrow indicating the direction of current flow. While a battery is a good approximation to an ideal voltage source, there is nothing quite so familiar that approximates an ideal current source. Some transistor circuits come close to this ideal and are often modeled with idealized current sources.

1.4 ELECTRICAL RESISTANCE

For an ideal *resistance* element the current through it is directly proportional to the voltage drop across it, as shown in Fig. 1.10.

1.4.1 Ohm's Law

The equation for an ideal resistor is given in (1.5) in which v is in volts, i is in amps, and the constant of proportionality is resistance R measured in ohms (Ω). This simple formula is known as *Ohm's law* in honor of the German physicist, Georg Ohm, whose original experiments led to this incredibly useful and important relationship.

$$v = Ri \tag{1.5}$$

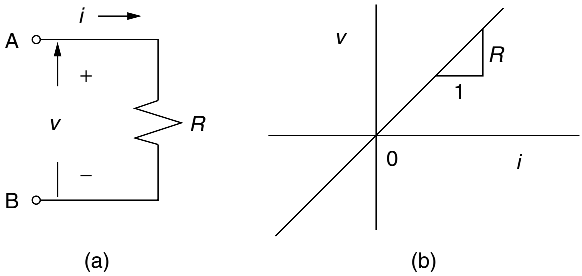


Figure 1.10 (a) An ideal resistor symbol. (b) voltage–current relationship.

Notice that voltage v is measured across the resistor. That is, it is the voltage at point A with respect to the voltage at point B. When current is in the direction shown, the voltage at A with respect to B is positive, so it is quite common to say that there is a *voltage drop* across the resistor.

An equivalent relationship for a resistor is given in (1.6), where current is given in terms of voltage and the proportionality constant is conductance G with units of siemens (S). In older literature, the unit of conductance was mhos.

$$i = Gv \tag{1.6}$$

By combining Eqs. (1.3) and (1.5), we can easily derive the following equivalent relationships for power dissipated by the resistor:

$$p = vi = i^2 R = \frac{v^2}{R} \tag{1.7}$$

Example 1.2 Power to an Incandescent Lamp. The current–voltage relationship for an incandescent lamp is nearly linear, so it can quite reasonably be modeled as a simple resistor. Suppose such a lamp has been designed to consume 60 W when it is connected to a 12-V power source. What is the resistance of the filament, and what amount of current will flow? If the actual voltage is only 11 V, how much energy would it consume over a 100-h period?

Solution. From Eq. (1.7),

$$R = \frac{v^2}{p} = \frac{12^2}{60} = 2.4\ \Omega$$

and from Ohm's law,

$$i = v/R = 12/2.4 = 5\ \text{A}$$

Connected to an 11-V source, the power consumed would be

$$p = \frac{v^2}{R} = \frac{11^2}{2.4} = 50.4\ \text{W}$$

Over a 100-h period, it would consume

$$w = pt = 50.4\ \text{W} \times 100\ \text{h} = 5040\ \text{Wh} = 5.04\ \text{kWh}$$

1.4.2 Resistors in Series

We can use Ohm's law and Kirchhoff's voltage law to determine the equivalent resistance of resistors wired in *series* (so the same current flows through each one) as shown in Fig. 1.11.

For R_s to be equivalent to the two series resistors, R_1 and R_2, the voltage–current relationships must be the same. That is, for the circuit in Fig. 1.11a,

$$v = v_1 + v_2 \tag{1.8}$$

and from Ohm's law,

$$v = iR_1 + iR_2 \tag{1.9}$$

For the circuit in Fig. 1.11b to be equivalent, the voltage and current must be the same:

$$v = iR_s \tag{1.10}$$

By equating Eqs. (1.9) and (1.10), we conclude that

$$R_s = R_1 + R_2 \tag{1.11}$$

And, in general, for n-resistances in series the equivalent resistance is

$$R_s = R_1 + R_2 + \cdots + R_n \tag{1.12}$$

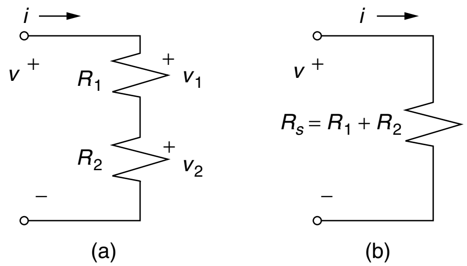


Figure 1.11 R_s is equivalent to resistors R_1 and R_2 in series.

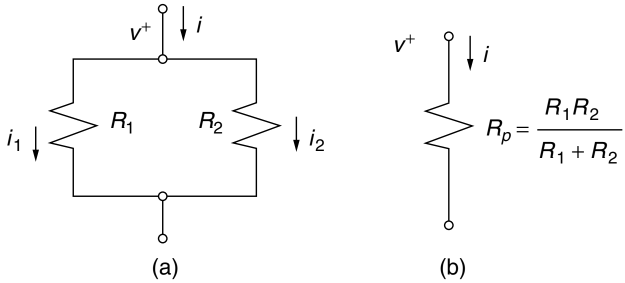


Figure 1.12 Equivalent resistance of resistors wired in parallel.

1.4.3 Resistors in Parallel

When circuit elements are wired together as in Fig. 1.12, so that the same voltage appears across each of them, they are said to be in *parallel*.

To find the equivalent resistance of two resistors in parallel, we can first incorporate Kirchhoff's current law followed by Ohm's law:

$$i = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2} = \frac{v}{R_p} \tag{1.13}$$

so that

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_p} \qquad \text{or} \qquad G_1 + G_2 = G_p \tag{1.14}$$

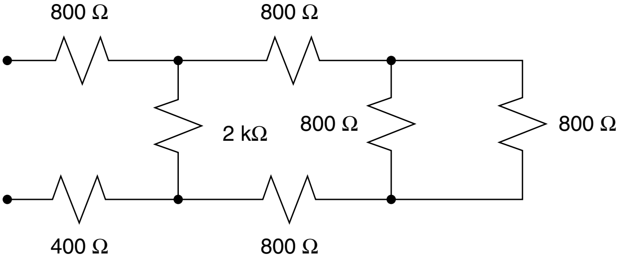
Notice that one reason for introducing the concept of conductance is that the conductance of a parallel combination of n resistors is just the sum of the individual conductances.

For two resistors in parallel, the equivalent resistance can be found from Eq. (1.14) to be

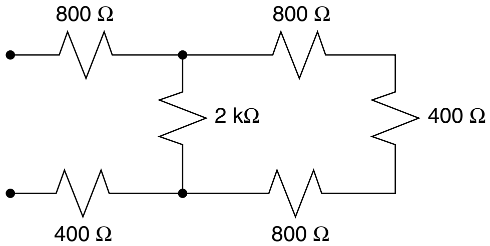
$$R_p = \frac{R_1 R_2}{R_1 + R_2} \tag{1.15}$$

Notice that when R_1 and R_2 are of equal value, the resistance of the parallel combination is just one-half that of either one. Also, you might notice that the parallel combination of two resistors always has a lower resistance than either one of those resistors.

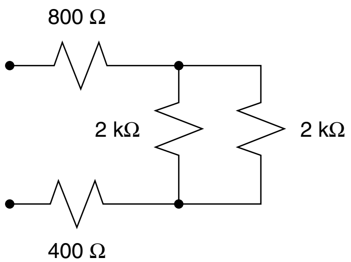
Example 1.3 Analyzing a Resistive Circuit. Find the equivalent resistance of the following network.



Solution. While this circuit may look complicated, you can actually work it out in your head. The parallel combination of the two 800-Ω resistors on the right end is 400 Ω, leaving the following equivalent:



The three resistors on the right end are in series so they are equivalent to a single resistor of 2 kΩ (=800 Ω + 400 Ω + 800 Ω). The network now looks like the following:



The two 2-kW resistors combine to 1 kΩ, which is in series with the 800-Ω and 400-Ω resistors. The total resistance of the network is thus 800 Ω + 1 kΩ + 400 Ω = 2.2 kΩ.

1.4.4 The Voltage Divider

A voltage divider is a deceptively simple, but surprisingly useful and important circuit. It is our first example of a two-port network. Two-port networks have a pair of input wires and a pair of output wires, as shown in Fig. 1.13.

The analysis of a voltage divider is a straightforward extension of Ohm's law and what we have learned about resistors in series.

As shown in Fig. 1.14, when a voltage source is connected to the voltage divider, an amount of current flows equal to

$$i = \frac{v_{\text{in}}}{R_1 + R_2} \tag{1.16}$$

Since $v_{\text{out}} = iR_2$, we can write the following voltage-divider equation:

$$v_{\text{out}} = v_{\text{in}} \left(\frac{R_2}{R_1 + R_2} \right) \tag{1.17}$$

Equation (1.17) is so useful that it is well worth committing to memory.

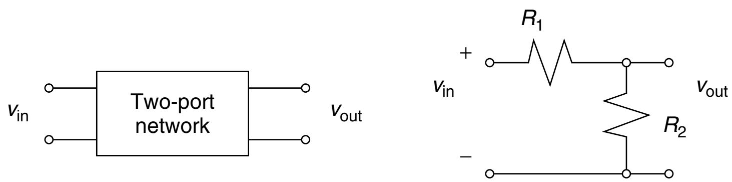


Figure 1.13 A voltage divider is an example of a two-port network.

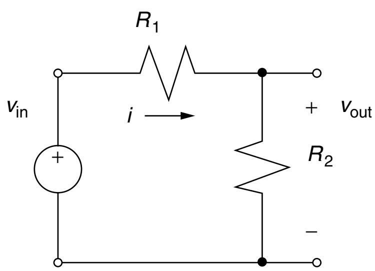
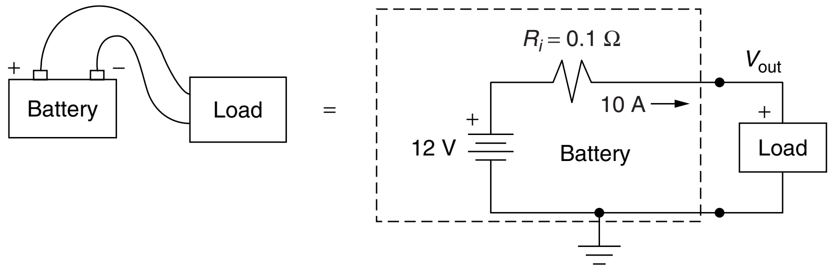


Figure 1.14 A voltage divider connected to an ideal voltage source.

Example 1.4 Analyzing a Battery as a Voltage Divider. Suppose an automobile battery is modeled as an ideal 12-V source in series with a 0.1-Ω internal resistance.

a. What would the battery output voltage drop to when 10 A is delivered?
b. What would be the output voltage when the battery is connected to a 1-Ω load?

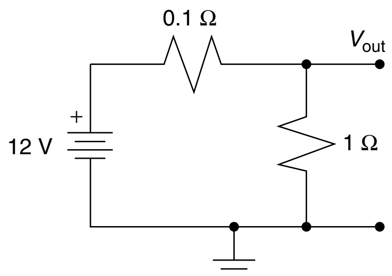


Solution

a. With the battery delivering 10 A, the output voltage drops to

$$V_{\text{out}} = \text{V}_B - IR_i = 12 - 10 \times 0.1 = 11 \text{ V}$$

b. Connected to a 1-Ω load, the circuit can be modeled as shown below:



We can find V_{out} from the voltage divider relationship, (1.17):

$$v_{\text{out}} = v_{\text{in}} \left(\frac{R_2}{R_1 + R_2} \right) = 12 \left(\frac{1.0}{0.1 + 1.0} \right) = 10.91 \text{ V}$$

1.4.5 Wire Resistance

In many circumstances connecting wire is treated as if it were perfect—that is, as if it had no resistance—so there is no voltage drop in those wires. In circuits delivering a fair amount of power, however, that assumption may lead to serious errors. Stated another way, an important part of the design of power circuits is choosing heavy enough wire to transmit that power without excessive losses. If connecting wire is too small, power is wasted and, in extreme cases, conductors can get hot enough to cause a fire hazard.

The resistance of wire depends primarily on its length, diameter, and the material of which it is made. Equation (1.18) describes the fundamental relationship for resistance (Ω):

$$R = \rho \frac{l}{A} \tag{1.18}$$

where ρ is the resistivity of the material, l is the wire length, and A is the wire cross-sectional area.

With l in meters (m) and A in m^2, units for resistivity ρ in the SI system are Ω-m (in these units copper has $\rho = 1.724 \times 10^{-8}$ Ω-m). The units often used in the United States, however, are tricky (as usual) and are based on areas expressed *in circular mils*. One circular mil is the area of a circle with diameter 0.001 in. (1 mil = 0.001 in.). So how can we determine the cross-sectional area of a wire (in circular mils) with diameter d (mils)? That is the same as asking how many 1-mil-diameter circles can fit into a circle of diameter d mils.

$$A = \frac{\frac{\pi}{4} d^2 \text{ sq mil}}{\frac{\pi}{4} \cdot 1^2 \text{ sq mil/cmil}} = d^2 \text{ cmil} \tag{1.19}$$

Example 1.5 From mils to Ohms. The resistivity of annealed copper at 20°C is 10.37 ohm-circular-mils/foot. What is the resistance of 100 ft of wire with diameter 80.8 mils (0.0808 in.)?

Solution

$$R = \rho \frac{l}{A} = 10.37\ \Omega - \text{cmil/ft} \cdot \frac{100 \text{ ft}}{(80.8)^2 \text{cmil}} = 0.1588\ \Omega$$

Electrical resistance of wire also depends somewhat on temperature (as temperature increases, greater molecular activity interferes with the smooth flow of electrons, thereby increasing resistance). There is also a phenomenon, called the *skin effect*, which causes wire resistance to increase with frequency. At higher frequencies, the inherent inductance at the core of the conductor causes current to flow less easily in the center of the wire than at the outer edge of conductor, thereby increasing the average resistance of the entire conductor. At 60 Hz, for modest loads (not utility power), the skin effect is insignificant. As to materials, copper is preferred, but aluminum, being cheaper, is sometimes used by professionals, but never in home wiring systems. Aluminum under pressure slowly

TABLE 1.3 Characteristics of Copper Wire

Wire Gage (AWG No.)	Diameter (inches)	Area cmils	Ohms per 100 ft[a]	Max Current (amps)
000	0.4096	168,000	0.0062	195
00	0.3648	133,000	0.0078	165
0	0.3249	106,000	0.0098	125
2	0.2576	66,400	0.0156	95
4	0.2043	41,700	0.0249	70
6	0.1620	26,300	0.0395	55
8	0.1285	16,500	0.0628	40
10	0.1019	10,400	0.0999	30
12	0.0808	6,530	0.1588	20
14	0.0641	4,110	0.2525	15

[a] dc, at 68°F.

deforms, which eventually loosens connections. That, coupled with the high-resistivity oxide that forms over exposed aluminum, can cause high enough I^2R losses to pose a fire hazard.

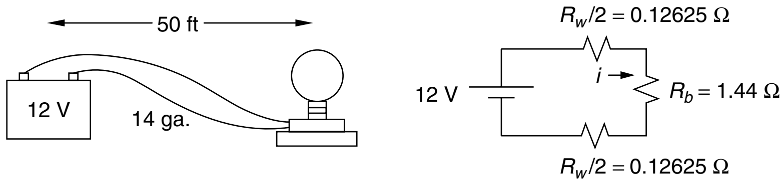
Wire size in the United States with diameter less than about 0.5 in. is specified by its American Wire Gage (AWG) number. The AWG numbers are based on wire resistance, which means that larger AWG numbers have higher resistance and hence smaller diameter. Conversely, smaller gage wire has larger diameter and, consequently, lower resistance. Ordinary house wiring is usually No. 12 AWG, which is roughly the diameter of the lead in an ordinary pencil. The largest wire designated with an AWG number is 0000, which is usually written 4/0, with a diameter of 0.460 in. For heavier wire, which is usually stranded (made up of many individual wires bundled together), the size is specified in the United States in thousands of circular mills (kcmil). For example, 1000-kcmil stranded copper wire for utility transmission lines is 1.15 in. in diameter and has a resistance of 0.076 ohms per mile. In countries using the metric system, wire size is simply specified by its diameter in millimeters. Table 1.3 gives some values of wire resistance, in ohms per 100 feet, for various gages of copper wire at 68°F. Also given is the maximum allowable current for copper wire clad in the most common insulation.

Example 1.6 Wire Losses. Suppose an ideal 12-V battery is delivering current to a 12-V, 100-W incandescent lightbulb. The battery is 50 ft from the bulb, and No. 14 copper wire is used. Find the power lost in the wires and the power delivered to the bulb.

Solution. The resistance, R_b, of a bulb designed to use 100 W when it is supplied with 12 V can be found from (1.7):

$$P = \frac{v^2}{R} \quad \text{so} \quad R_b = \frac{v^2}{P} = \frac{12^2}{100} = 1.44\ \Omega$$

From Table 1.3, 50 ft of 14 ga. wire has 0.2525 Ω/100 ft, so since we have 50 ft of wire to the bulb and 50 ft back again, the wire resistance is $R_w = 0.2525\ \Omega$. The circuit is as follows:



From Ohm's law, the current flowing in the circuit is

$$i = \frac{v}{R_{\text{tot}}} = \frac{12\ \text{V}}{(0.12625 + 0.12625 + 1.44)\ \Omega} = 7.09\ \text{A}$$

So, the power delivered to the lightbulb is

$$P_b = i^2 R_b = (7.09)^2 \cdot 1.44 = 72.4\ \text{W}$$

and the power lost in the wires is

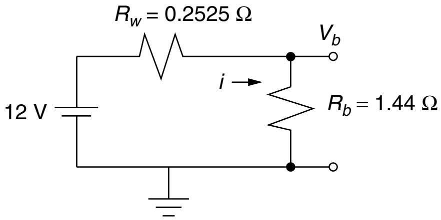
$$P_w = i^2 R_w = (7.09)^2 \cdot 0.2525 = 12.7\ \text{W}$$

Notice that our bulb is receiving only 72.4 W instead of 100 W, so it will not be nearly as bright. Also note that the battery is delivering

$$\text{P}_{\text{battery}} = 72.4 + 12.7 = 85.1\ \text{W}$$

of which, quite a bit, about 15%, is lost in the wires (12.7/85.1 = 0.15).

Alternate Solution: Let us apply the concept of a voltage divider to solve this problem. We can combine the wire resistance going to the load with the wire resistance coming back, resulting in the simplified circuit model shown below:



Using (1.17), the voltage delivered to the load (the lightbulb) is

$$v_{\text{out}} = v_{\text{in}}\left(\frac{R_2}{R_1 + R_2}\right) = 12\left(\frac{1.44}{0.2525 + 1.44}\right) = 10.21\ \text{V}$$

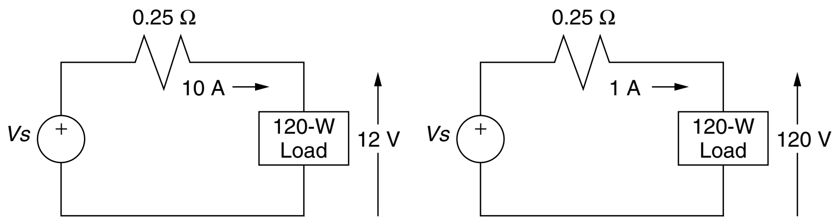
The 1.79-V difference between the 12 V supplied by the battery and the 10.21 V that actually appears across the load is referred to as the *voltage sag*.

Power lost in the wires is thus

$$P_w = \frac{V_w^2}{R_w} = \frac{(1.79)^2}{0.2525} = 12.7\ \text{W}$$

Example 1.6 illustrates the importance of the resistance of the connecting wires. We would probably consider 15% wire loss to be unacceptable, in which case we might want to increase the wire size (but larger wire is more expensive and harder to work with). If feasible, we could take the alternative approach to wire losses, which is to increase the supply voltage. Higher voltages require less current to deliver a given amount of power. Less current means less i^2R power losses in the wires as the following example demonstrates.

Example 1.7 Raising Voltage to Reduce Wire Losses Suppose a load that requires 120 W of power is located 50 ft from a generator. The load can be designed to operate at 12 V or 120 V. Using No. 14 wire, find the voltage sag and power losses in the connecting wire for each voltage.



(a) 12-V system (b) 120-V system

Solution. There are 100 ft of No. 14 wire (to the load and back) with total resistance of 0.2525 Ω (Table 1.3).

At 12 V: To deliver 120 W at 12 V requires a current of 10 A, so the voltage sag in the 0.2525-Ω wire carrying 10 A is

$$V_{\text{sag}} = iR = 10 \text{ A} \times 0.2525\ \Omega = 2.525 \text{ V}$$

The power loss in the wire is

$$P = i^2 R = (10)^2 \times 0.2525 = 25.25 \text{ W}$$

That means the generator must provide $25.25 + 120 = 145.25$ W at a voltage of $12 + 2.525 = 14.525$ V. Wire losses are $25.25/145.25 = 0.174 = 17.4\%$ of the power generated. Such high losses are generally unacceptable.

At 120 V: The current required to deliver 120 W is only 1 A, which means the voltage drop in the connecting wire is only

$$\text{Voltage sag} = iR = 1 \text{ A} \times 0.2525\ \Omega = 0.2525 \text{ V}$$

The power loss in the wire is

$$P_w = i^2 R = (1)^2 \times 0.2525 = 0.2525 \text{ W (1/100th that of the 12-V system)}$$

The source must provide $120 \text{ W} + 0.2525 \text{ W} = 120.2525$ W, of which the wires will lose only 0.21%.

Notice that $i^2 R$ power losses in the wires are 100 times larger in the 12-V circuit, which carries 10 A, than they are in the 120-V circuit carrying only 1 A. That is, increasing the voltage by a factor of 10 causes line losses to decrease by a factor of 100, which is why electric power companies transmit their power at such high voltages.

1.5 CAPACITANCE

Capacitance is a parameter in electrical circuits that describes the ability of a circuit component to store energy in an electrical field. Capacitors are discrete components that can be purchased at the local electronics store, but the capacitance effect can occur whenever conductors are in the vicinity of each other. A capacitor can be as simple as two parallel conducting plates (Fig. 1.15), separated by a nonconducting dielectric such as air or even a thin sheet of paper.

If the surface area of the plates is large compared to their separation, the capacitance is given by

$$C = \varepsilon \frac{A}{d} \quad \text{farads} \tag{1.20}$$

where C is capacitance (farads, F), ε is permittivity (F/m), A is area of one plate (m^2), and d is separation distance (m).

Example 1.8 Capacitance of Two Parallel Plates. Find the capacitance of two 0.5-m^2 parallel conducting plates separated by 0.001 m of air with permittivity 8.8×10^{-12} F/m.

Solution

$$C = 8.8 \times 10^{-12}\ \text{F/m} \cdot \frac{0.5\ \text{m}^2}{0.001\ \text{m}} = 4.4 \times 10^{-9}\ \text{F} = 0.0044\ \mu\text{F} = 4400\ \text{pF}$$

Notice even with the quite large plate area in the example, the capacitance is a very small number. In practice, to achieve large surface area in a small volume, many capacitors are assembled using two flexible sheets of conductor, separated by a dielectric, rolled into a cylindrical shape with connecting leads attached to each plate.

Capacitance values in electronic circuits are typically in the microfarad (10^{-6} F = μF) to picofarad (10^{-12} = pF) range. Capacitors used in utility power systems are much larger, and are typically in the millifarad range. Later, we will see how a different unit of measure, the kVAR, will be used to characterize the size of large, power-system capacitors.

While Eq. (1.20) can be used to determine the capacitance from physical characteristics, of greater importance is the relationship between voltage, current, and capacitance. As suggested in Fig. 1.15, when charge q builds up on the

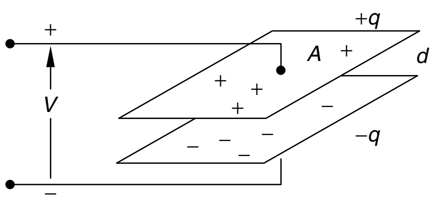


Figure 1.15 A capacitor can consist of two parallel, charged plates separated by a dielectric.

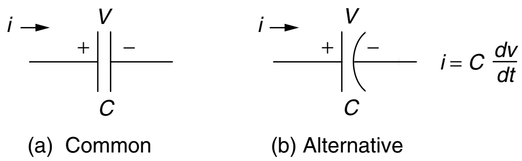


Figure 1.16 Two symbols for capacitors.

plates of a capacitor, a voltage v is created across the capacitor. This leads to the fundamental definition of capacitance, which is that capacitance is equal to the amount of charge required to create a 1-V potential difference between the plates:

$$C\,(\text{farads}) = \frac{q\,(\text{coulombs})}{v\,(\text{volts})} \tag{1.21}$$

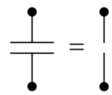
Since current is the rate at which charge is added to the plates, we can rearrange (1.21) and then take the derivative to get

$$i = \frac{dq}{dt} = C\frac{dv}{dt} \tag{1.22}$$

The circuit symbol for a capacitor is usually drawn as two parallel lines, as shown in Fig. 1.16a, but you may also encounter the symbol shown in Fig. 1.16b. Sometimes, the term *condenser* is used for capacitors, as is the case in automobile ignition systems.

From the defining relationship between current and voltage (1.22), it can be seen that if voltage is not changing, then current into the capacitor has to be zero. That is, under dc conditions, the capacitor appears to be an open circuit, through which no current flows.

$$\text{dc:} \qquad \frac{dv}{dt} = 0, \quad i = 0, \tag{1.23}$$



Kirchhoff's current and voltage laws can be used to determine that the capacitance of two capacitors in parallel is the sum of their capacitances and that the capacitance of two capacitors in series is equal to the product of the two over the sum, as shown in Fig. 1.17.

Another important characteristic of capacitors is their ability to store energy in the form of an electric field created between the plates. Since power is the rate of change of energy, we can write that energy is the integral of power:

$$W_c = \int P\,dt = \int vi\,dt = \int vC\frac{dv}{dt}\,dt = C\int v\,dv$$

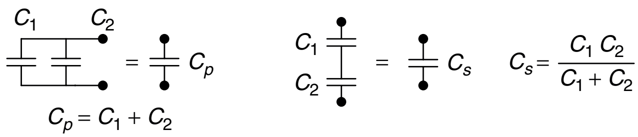


Figure 1.17 Capacitors in series and capacitors in parallel.

So, we can write that the energy stored in the electric field of a capacitor is

$$W_c = \tfrac{1}{2} C v^2 \tag{1.24}$$

One final property of capacitors is that the *voltage across a capacitor cannot be changed instantaneously*. To change voltage instantaneously, charge would have to move from one plate, around the circuit, and back to the other plate in zero time. To see this conclusion mathematically, write power as the rate of change of energy,

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2} C v^2\right) = C v \frac{dv}{dt} \tag{1.25}$$

and then note that if voltage could change instantaneously, dv/dt would be infinite, and it would therefore take infinite power to cause that change, which is impossible—hence, the conclusion that voltage cannot change instantaneously. An important practical application of this property will be seen when we look at rectifiers that convert ac to dc. Capacitors resist rapid changes in voltages and are used to smooth the dc voltage produced from such dc power supplies. In power systems, capacitors have a number of other uses that will be explored in the next chapter.

1.6 MAGNETIC CIRCUITS

Before we can introduce inductors and transformers, we need to understand the basic concept of electromagnetism. The simple notions introduced here will be expanded in later chapters when electric power quality (especially harmonic distortion), motors and generators, and fluorescent ballasts are covered.

1.6.1 Electromagnetism

Electromagnetic phenomena were first observed and quantified in the early nineteenth century—most notably, by three European scientists: Hans Christian Oersted, André Marie Ampère, and Michael Faraday. Oersted observed that a wire carrying current could cause a magnet suspended nearby to move. Ampère, in

1825, demonstrated that a wire carrying current could exert a force on another wire carrying current in the opposite direction. And Faraday, in 1831, discovered that current could be made to flow in a coil of wire by passing a magnet close to the circuit. These experiments provided the fundamental basis for the development of all electromechanical devices, including, most importantly, motors and generators.

What those early experiments established was that electrical current flowing along a wire creates a magnetic field around the wire, as shown in Fig. 1.18a. That magnetic field can be visualized by showing lines of magnetic flux, which are represented with the symbol ϕ. The direction of that field that can be determined using the "right hand rule" in which you imagine wrapping your right hand around a wire, with your thumb pointing in the direction of current flow. Your fingers then show the direction of the magnetic field. The field created by a coil of wire is suggested in Fig. 1.18b.

Consider an iron core wrapped with N turns of wire carrying current i as shown in Fig. 1.19. The magnetic field formed by the coil will take the path of least resistance—which is through the iron—in much the same way that electric current stays within a copper conductor. In essence, the iron is to a magnetic field what a wire is to current.

What Faraday discovered is that current flowing through the coil not only creates a magnetic field in the iron, it also creates a voltage across the coil that is proportional to the rate of change of magnetic flux ϕ in the iron. That voltage is called an electromotive force, or emf, and is designated by the symbol e.

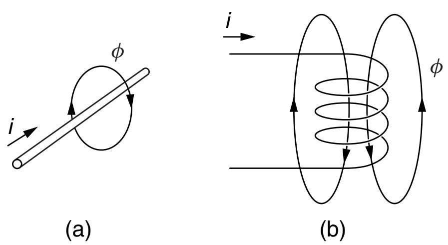


Figure 1.18 A magnetic field is formed around a conductor carrying current.

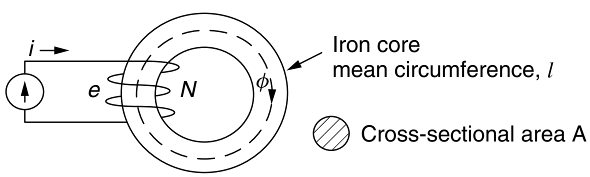


Figure 1.19 Current in the N-turn winding around an iron core creates a magnetic flux ϕ. An electromotive force (voltage) e is induced in the coil proportional to the rate of change of flux.

Assuming that all of the magnetic flux ϕ links all of the turns of the coil, we can write the following important relationship, which is known as *Faraday's law of electromagnetic induction*:

$$e = N\frac{d\phi}{dt} \tag{1.26}$$

The sign of the induced emf is always in a direction that opposes the current that created it, a phenomenon referred to as *Lenz's law*.

1.6.2 Magnetic Circuits

Magnetic phenomena are described using a fairly large number of terms that are often, at first, somewhat difficult to keep track of. One approach that may help is to describe analogies between electrical circuits, which are usually more familiar, and corresponding magnetic circuits. Consider the electrical circuit shown in Fig. 1.20a and the analogous magnetic circuit shown in Fig 1.20b. The electrical circuit consists of a voltage source, v, sending current i through an electrical load with resistance R. The electrical load consists of a long wire of length l, cross-sectional area A, and conductance ρ.

The resistance of the electrical load is given by (1.18):

$$R = \rho\frac{l}{A} \tag{1.18}$$

The current flowing in the electrical circuit is given by Ohm's law:

$$i = \frac{v}{R} \tag{1.5}$$

In the magnetic circuit of Fig. 1.20b, the driving force, analogous to voltage, is called the *magnetomotive force* (mmf), designated by $\mathcal{F}$. The magnetomotive force is created by wrapping N turns of wire, carrying current i, around a toroidal

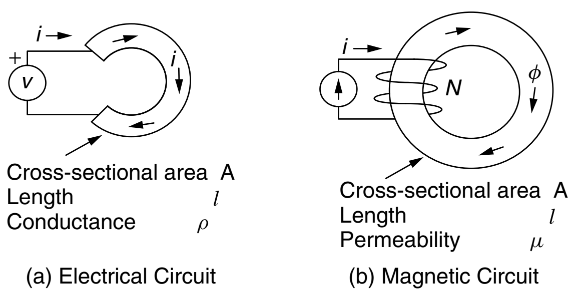


Figure 1.20 Analogous electrical and magnetic circuits.

core. By definition, the magnetomotive force is the product of *current* × *turns*, and has units of *ampere-turns*.

$$\textit{Magnetomotive force (mmf)}\,\mathcal{F} = Ni \quad (\textit{ampere} - \textit{turns}) \tag{1.27}$$

The response to that mmf (analogous to current in the electrical circuit) is creation of magnetic flux ϕ, which has SI units of *webers* (Wb). The magnetic flux is proportional to the mmf driving force and inversely proportional to a quantity called *reluctance* $\Re$, which is analogous to electrical resistance, resulting in the "Ohm's law" of magnetic circuits given by

$$\mathcal{F} = \Re\,\phi \tag{1.28}$$

From (1.28), we can ascribe units for reluctance $\Re$ as amp-turns per weber (A-t/Wb).

Reluctance depends on the dimensions of the core as well as its materials:

$$\textit{reluctance} = \Re = \frac{l}{\mu A} \qquad (A\text{-}t/Wb) \tag{1.29}$$

Notice the similarity between (1.29) and the equation for resistance given in (1.18).

The parameter in (1.29) that indicates how readily the core material accepts magnetic flux is the material's *permeability* μ. There are three categories of magnetic materials: *diamagnetic*, in which the material tends to exclude magnetic fields; *paramagnetic*, in which the material is slightly magnetized by a magnetic field; and *ferromagnetic*, which are materials that very easily become magnetized. The vast majority of materials do not respond to magnetic fields, and their permeability is very close to that of free space. The materials that readily accept magnetic flux—that is, ferromagnetic materials—are principally iron, cobalt, and nickel and various alloys that include these elements. The units of permeability are webers per amp-turn-meter (*Wb/A-t-m*).

The permeability of free space is given by

$$\text{Permeability of free space } \mu_0 = 4\pi \times 10^{-7} \text{ Wb/A-t-m} \tag{1.30}$$

Oftentimes, materials are characterized by their *relative permeability*, μ_r, which for ferromagnetic materials may be in the range of hundreds to hundreds of thousands. As will be noted later, however, the relative permeability is not a constant for a given material: It varies with the magnetic field intensity. In this regard, the magnetic analogy deviates from its electrical counterpart and so must be used with some caution.

$$\text{Relative permeability} = \mu_r = \frac{\mu}{\mu_0} \tag{1.31}$$

Another important quantity of interest in magnetic circuits is *the magnetic flux density*, B. As the name suggests, it is simply the "density" of flux given by the following:

$$\text{Magnetic flux density } B = \frac{\phi}{\text{A}} \text{ webers/m}^2 \text{ or teslas (T)} \tag{1.32}$$

When flux is given in webers (Wb) and area A is given in m^2, units for B are teslas (T). The analogous quantity in an electrical circuit would be the current density, given by

$$\text{Electric current density } J = \frac{i}{\text{A}} \tag{1.33}$$

The final magnetic quantity that we need to introduce is the *magnetic field intensity*, H. Referring back to the simple magnetic circuit shown in Fig. 1.20b, the magnetic field intensity is defined as the magnetomotive force (mmf) per unit of length around the magnetic loop. With N turns of wire carrying current i, the mmf created in the circuit is Ni ampere-turns. With l representing the mean path length for the magnetic flux, the magnetic field intensity is therefore

$$\text{Magnetic field intensity } H = \frac{Ni}{l} \text{ ampere-turns/meter} \tag{1.34}$$

An analogous concept in electric circuits is the electric field strength, which is voltage drop per unit of length. In a capacitor, for example, the intensity of the electric field formed between the plates is equal to the voltage across the plates divided by the spacing between the plates.

Finally, if we combine (1.27), (1.28), (1.29), (1.32), and (1.34), we arrive at the following relationship between magnetic flux density B and magnetic field intensity H:

$$B = \mu H \tag{1.35}$$

Returning to the analogies between the simple electrical circuit and magnetic circuit shown in Fig. 1.20, we can now identify equivalent circuits, as shown in Fig. 1.21, along with the analogs shown in Table 1.4.

TABLE 1.4 Analogous Electrical and Magnetic Circuit Quantities

Electrical	Magnetic	Magnetic Units
Voltage v	Magnetomotive force $\mathfrak{F} = Ni$	Amp-turns
Current i	Magnetic flux ϕ	Webers Wb
Resistance R	Reluctance $\mathfrak{R}$	Amp-turns/Wb
Conductivity $1/\rho$	Permeability μ	Wb/A-t-m
Current density J	Magnetic flux density B	Wb/m^2 = teslas T
Electric field E	Magnetic field intensity H	Amp-turn/m

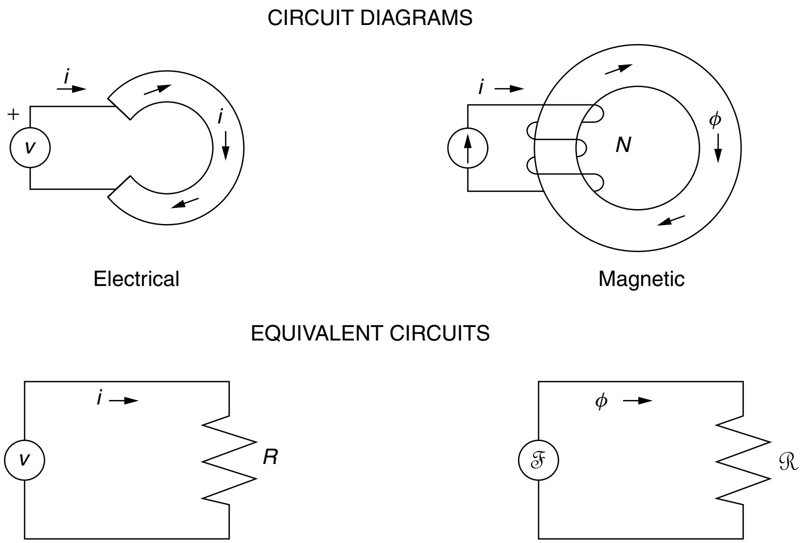


Figure 1.21 Equivalent circuits for the electrical and magnetic circuits shown.

1.7 INDUCTANCE

Having introduced the necessary electromagnetic background, we can now address inductance. Inductance is, in some sense, a mirror image of capacitance. While capacitors store energy in an electric field, inductors store energy in a magnetic field. While capacitors prevent voltage from changing instantaneously, inductors, as we shall see, prevent current from changing instantaneously.

1.7.1 Physics of Inductors

Consider a coil of wire carrying some current creating a magnetic field within the coil. As shown in Fig 1.22, if the coil has an air core, the flux can pretty much go where it wants to, which leads to the possibility that much of the flux will not link all of the turns of the coil. To help guide the flux through the coil, so that flux leakage is minimized, the coil might be wrapped around a ferromagnetic bar or ferromagnetic core as shown in Fig. 1.23. The lower reluctance path provided by the ferromagnetic material also greatly increases the flux ϕ.

We can easily analyze the magnetic circuit in which the coil is wrapped around the ferromagnetic core in Fig. 1.23a. Assume that all of the flux stays within the low-reluctance pathway provided by the core, and apply (1.28):

$$\phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{Ni}{\mathcal{R}} \tag{1.36}$$

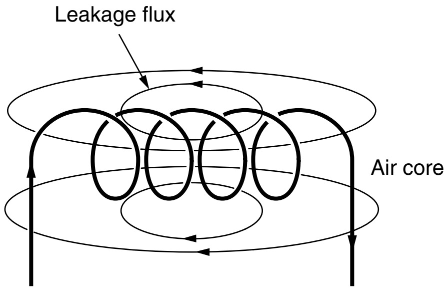


Figure 1.22 A coil with an air core will have considerable leakage flux.

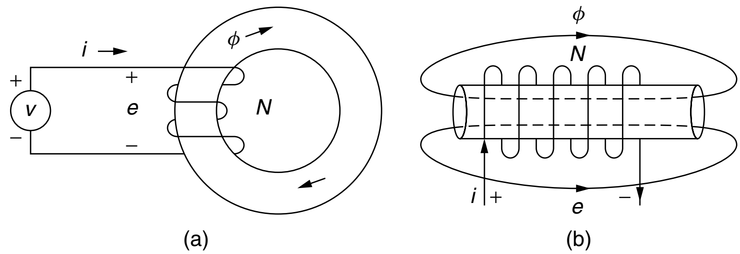


Figure 1.23 Flux can be increased and leakage reduced by wrapping the coils around a ferromagnetic material that provides a lower reluctance path. The flux will be much higher using the core (a) rather than the rod (b).

From Faraday's law (1.26), changes in magnetic flux create a voltage e, called the electromotive force (emf), across the coil equal to

$$e = N\frac{d\phi}{dt} \tag{1.26}$$

Substituting (1.36) into (1.26) gives

$$e = N\frac{d}{dt}\left(\frac{Ni}{\Re}\right) = \frac{N^2}{\Re}\frac{di}{dt} = L\frac{di}{dt} \tag{1.37}$$

where inductance L has been introduced and defined as

$$\text{Inductance } L = \frac{N^2}{\Re} \text{ henries} \tag{1.38}$$

Notice in Fig. 1.23a that a distinction has been made between e, the emf voltage induced across the coil, and v, a voltage that may have been applied to the circuit to cause the flux in the first place. If there are no losses in the connecting wires between the source voltage and the coil, then $e = v$ and we have the final defining relationship for an inductor:

$$v = L\frac{di}{dt} \tag{1.39}$$

As given in (1.38), inductance is inversely proportional to reluctance $\mathcal{R}$. Recall that the reluctance of a flux path through air is much greater than the reluctance if it passes through a ferromagnetic material. That tells us if we want a large inductance, the flux needs to pass through materials with high permeability (not air).

Example 1.9 Inductance of a Core-and-Coil. Find the inductance of a core with effective length $l = 0.1$ m, cross-sectional area $A = 0.001\ \text{m}^2$, and relative permeability μ_r somewhere between 15,000 and 25,000. It is wrapped with $N = 10$ turns of wire. What is the range of inductance for the core?

Solution. When the core's permeability is 15,000 times that of free space, it is

$$\mu_{\text{core}} = \mu_r\mu_0 = 15{,}000 \times 4\pi \times 10^{-7} = 0.01885\ \text{Wb/A-t-m}$$

so its reluctance is

$$\mathcal{R}_{\text{core}} = \frac{l}{\mu_{\text{core}}A} = \frac{0.1\ \text{m}}{0.01885\ (\text{Wb/A-t-m}) \times 0.001\ \text{m}^2} = 5305\ \text{A-t/Wb}$$

and its inductance is

$$L = \frac{N^2}{\mathcal{R}} = \frac{10^2}{5305} = 0.0188\ \text{henries} = 18.8\ \text{mH}$$

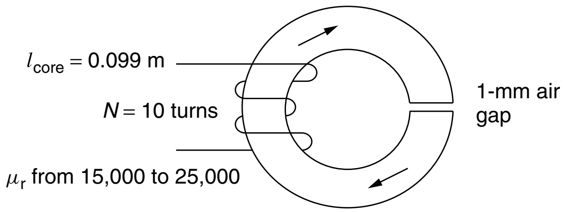
Similarly, when the relative permeability is 25,000 the inductance is

$$L = \frac{N^2}{\mathcal{R}} = \frac{N^2\mu_r\mu_0 A}{l} = \frac{10^2 \times 25{,}000 \times 4\pi \times 10^{-7} \times 0.001}{0.1}$$
$$= 0.0314\ \text{H} = 31.4\ \text{mH}$$

The point of Example 1.9 is that the inductance of a coil of wire wrapped around a solid core can be quite variable given the imprecise value of the core's permeability. Its permeability depends on how hard the coil is driven by mmf so you can't just pick up an off-the-shelf inductor like this and know what its inductance is likely to be. The trick to getting a more precise value of inductance given the uncertainty in permeability is to sacrifice some amount of inductance by building into the core a small air gap. Another approach is to get the equivalent of an air gap by using a powdered ferromagnetic material in which the spaces between particles of material act as the air gap. The air gap reluctance, which is determined strictly by geometry, is large compared to the core reluctance so the impact of core permeability changes is minimized.

The following example illustrates the advantage of using an air gap to minimize the uncertainty in inductance. It also demonstrates something called *Ampère's circuital law*, which is the magnetic analogy to Kirchhoff's voltage law. That is, the rise in magnetomotive force (mmf) provided by N turns of wire carrying current i is equal to the sum of the mmf drops $\mathcal{R}\,\phi$ around the magnetic loop.

Example 1.10 An Air Gap to Minimize Inductance Uncertainty. Suppose the core of Example 1.9 is built with a 0.001-m air gap. Find the range of inductances when the core's relative permeability varies between 15,000 and 25,000.



Solution. The reluctance of the ferromagnetic portion of the core when its relative permeability is 15,000 is

$$\mathcal{R}_{core} = \frac{l_{core}}{\mu_{core} A} = \frac{0.099}{15{,}000 \times 4\pi \times 10^{-7} \times 0.001} = 5252 \text{ A-t/Wb}$$

And the air gap reluctance is

$$\mathcal{R}_{air\ gap} = \frac{l_{air\ gap}}{\mu_0 A} = \frac{0.001}{4\pi \times 10^{-7} \times 0.001} = 795{,}775 \text{ A-t/Wb}$$

So the total reluctance of the series path consisting or core and air gap is

$$\mathcal{R}_{Total} = 5252 + 795{,}775 = 801{,}027 \text{ A-t/Wb}$$

And the inductance is

$$L = \frac{N^2}{\mathcal{R}} = \frac{10^2}{801{,}027} = 0.0001248 \text{ H} = 0.1248 \text{ mH}$$

When the core's relative permeability is 25,000, its reluctance is

$$\Re_{\text{core}} = \frac{l_{\text{core}}}{\mu_{\text{core}} A} = \frac{0.099}{25{,}000 \times 4\pi \times 10^{-7} \times 0.001} = 3151 \text{ A-t/Wb}$$

And the new total inductance is

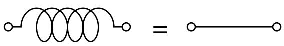
$$L = \frac{N^2}{\Re} = \frac{10^2}{3151 + 795{,}775} = 0.0001251 \text{ H} = 0.1251 \text{ mH}$$

This is an insignificant change in inductance. A very precise inductance has been achieved at the expense of a sizable decrease in inductance compared to the core without an air gap.

1.7.2 Circuit Relationships for Inductors

From the defining relationship between voltage and current for an inductor (1.39), we can note that when current is not changing with time, the voltage across the inductor is zero. That is, for dc conditions an inductor looks the same as a short-circuit, zero-resistance wire:

$$\text{dc:} \quad v = L\frac{di}{dt} = L \cdot 0 = 0 \qquad (1.40)$$



When inductors are wired in series, the same current flows through each one so the voltage drop across the pair is simply:

$$v_{\text{series}} = L_1\frac{di}{dt} + L_2\frac{di}{dt} = (L_1 + L_2)\frac{di}{dt} = L_{\text{series}}\frac{di}{dt} \qquad (1.41)$$

where L_{series} is the equivalent inductance of the two series inductors. That is,

$$L_{\text{series}} = L_1 + L_2 \qquad (1.42)$$

Consider Fig. 1.24 for two inductors in parallel.

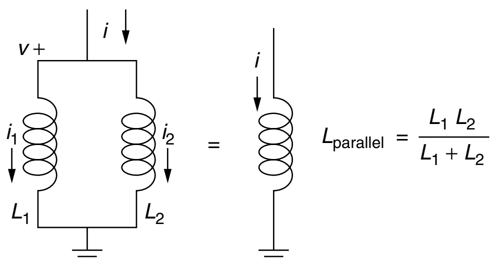


Figure 1.24 Two inductors in parallel.

The total current flowing is the sum of the currents:

$$i_{\text{parallel}} = i_1 + i_2 \tag{1.43}$$

The voltages are the same across each inductor, so we can use the integral form of (1.39) to get

$$\frac{1}{L_{\text{parallel}}} \int v\, dt = \frac{1}{L_1} \int v\, dt + \frac{1}{L_2} \int v\, dt \tag{1.44}$$

Dividing out the integral gives us the equation for inductors in parallel:

$$L_{\text{parallel}} = \frac{L_1\ L_2}{L_1 + L_2} \tag{1.45}$$

Just as capacitors store energy in their electric fields, inductors also store energy, but this time it is in their magnetic fields. Since energy W is the integral of power P, we can easily set up the equation for energy stored:

$$W_L = \int P\, dt = \int vi\, dt = \int \left(L\frac{di}{dt}\right) i\, dt = L \int i\, di \tag{1.46}$$

This leads to the following equation for energy stored in an inductor's magnetic field:

$$W_L = \tfrac{1}{2} L\ i^2 \tag{1.47}$$

If we use (1.47) to learn something about the power dissipated in an inductor, we get

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt} \tag{1.48}$$

From (1.48) we can deduce another important property of inductors: *The current through an inductor cannot be changed instantaneously.* For current to change instantaneously, di/dt would be infinite, which (1.48) tells us would require infinite power, which is impossible. It takes time for the magnetic field, which is storing energy, to collapse. *Inductors, in other words, make current act like it has inertia.*

Now wait a minute. If current is flowing in the simple circuit containing an inductor, resistor, and switch shown in Fig 1.25, why can't you just open the switch and cause the current to stop instantaneously? Surely, it doesn't take infinite power to open a switch. The answer is that the current has to keep going for at least a short interval just after opening the switch. To do so, current momentarily must jump the gap between the contact points as the switch is

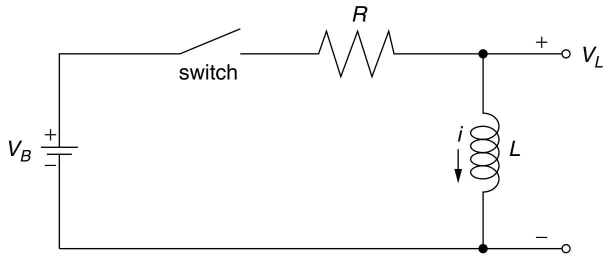


Figure 1.25 A simple R–L circuit with a switch.

opened. That is, the switch "arcs" and you get a little spark. Too much arc and the switch can be burned out.

We can develop an equation that describes what happens when an open switch in the R–L circuit of Fig. 1.25 is suddenly closed. Doing so gives us a little practice with Kirchhoff's voltage law. With the switch closed, the voltage rise due to the battery must equal the voltage drop across the resistance plus inductance:

$$V_B = iR + L\frac{di}{dt} \tag{1.49}$$

Without going through the details the solution to (1.49), subject to the initial condition that $i = 0$ at $t = 0$, is

$$i = \frac{V_B}{R}\left(1 - e^{-\frac{R}{L}t}\right) \tag{1.50}$$

Does this solution look right? At $t = 0$, $i = 0$, so that's OK. At $t = \infty$, $i = V_B/R$. That seems alright too since eventually the current reaches a steady-state, dc value, which means the voltage drop across the inductor is zero ($v_L = L\,di/dt = 0$). At that point, all of the voltage drop is across the resistor, so current is $i = V_B/R$. The quantity L/R in the exponent of (1.50) is called the *time constant*, τ.

We can sketch out the current flowing in the circuit of Fig. 1.25 along with the voltage across the inductor as we go about opening and closing the switch (Fig 1.26). If we start with the switch open at $t = 0^-$ (where the minus suggests just before $t = 0$), the current will be zero and the voltage across the inductor, V_L will be 0 (since $V_L = L\,di/dt$ and $di/dt = 0$).

At $t = 0$, the switch is closed. At $t = 0^+$ (just after the switch closes) the current is still zero since it cannot change instantaneously. With zero current, there is no voltage drop across the resistor ($v_R = i\ R$), which means the entire battery voltage appears across the inductor ($v_L = V_B$). Notice that there is no restriction on how rapidly voltage can change across an inductor, so an instantaneous jump is allowed. Current climbs after the switch is closed until dc conditions are reached, at which point $di/dt = 0$ so $v_L = 0$ and the entire battery voltage is dropped across the resistor. Current i asymptotically approaches V_B/R.

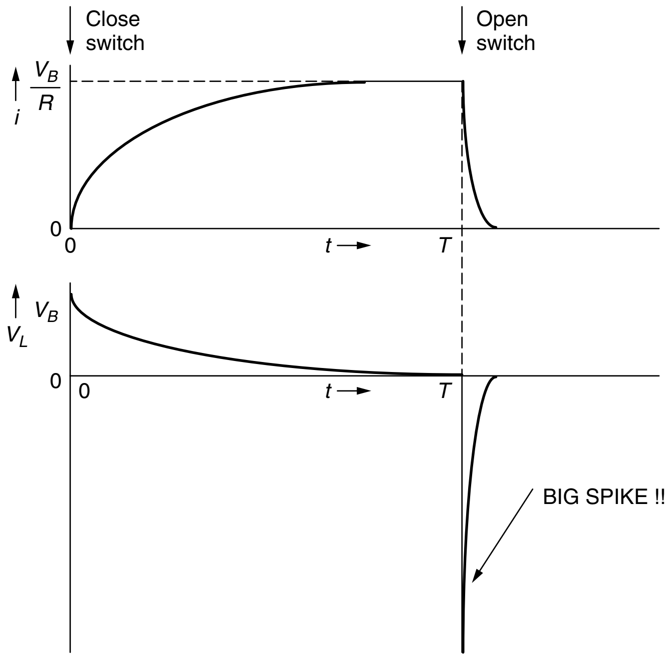


Figure 1.26 Opening a switch at $t = T$ produces a large spike of voltage across the inductor.

Now, at time $t = T$, open the switch. Current quickly, but not instantaneously, drops to zero (by arcing). Since the voltage across the inductor is $v_L = L\,di/dt$, and di/dt (the slope of current) is a very large negative quantity, v_L shows a precipitous, downward spike as shown in Fig. 1.26. This large spike of voltage can be much, much higher than the little voltage provided by the battery. In other words, with just an inductor, a battery, and a switch, we can create a very large voltage spike as we open the switch. This peculiar property of inductors is used to advantage in automobile ignition systems to cause spark plugs to ignite the gasoline in the cylinders of your engine. In your ignition system a switch opens (it used to be the points inside your distributor, now it is a transistorized switch), thereby creating a spike of voltage that is further amplified by a transformer coil to create a voltage of tens of thousands of volts—enough to cause an arc across the gap in your car's spark plugs. Another important application of this voltage spike is to use it to start the arc between electrodes of a fluorescent lamp.

1.8 TRANSFORMERS

When Thomas Edison created the first electric utility in 1882, he used dc to transmit power from generator to load. Unfortunately, at the time it was not possible to change dc voltages easily from one level to another, which meant transmission was at the relatively low voltages of the dc generators. As we have seen, transmitting significant amounts of power at low voltage means that high currents must flow, resulting in large i^2R power losses in the wires as well as high voltage drops between power plant and loads. The result was that power plants had to be located very close to loads. In those early days, it was not uncommon for power plants in cities to be located only a few blocks apart.

In a famous battle between two giants of the time, George Westinghouse solved the transmission problem by introducing ac generation using a transformer to boost the voltage entering transmission lines and other transformers to reduce the voltage back down to safe levels at the customer's site. Edison lost the battle but never abandoned dc—a decision that soon led to the collapse of his electric utility company.

It would be hard to overstate the importance of transformers in modern electric power systems. Transmission line power losses are proportional to the square of current and are inversely proportional to the square of voltage. Raising voltages by a factor of 10, for example, lowers line losses by a factor of 100. Modern systems generate voltages in the range of 12 to 25 kV. Transformers boost that voltage to hundreds of thousands of volts for long-distance transmission. At the receiving end, transformers drop the transmission line voltage to perhaps 4 to 25 kV at electrical substations for local distribution. Other transformers then drop the voltage to safe levels for home, office and factory use.

1.8.1 Ideal Transformers

A simple transformer configuration is shown in Fig. 1.27. Two coils of wire are wound around a magnetic core. As shown, the primary side of the transformer has N_1 turns of wire carrying current i_1, while the secondary side has N_2 turns carrying i_2.

If we assume an ideal core with no flux leakage, then the magnetic flux ϕ linking the primary windings is the same as the flux linking the secondary. From Faraday's law we can write

$$e_1 = N_1 \frac{d\phi}{dt} \tag{1.51}$$

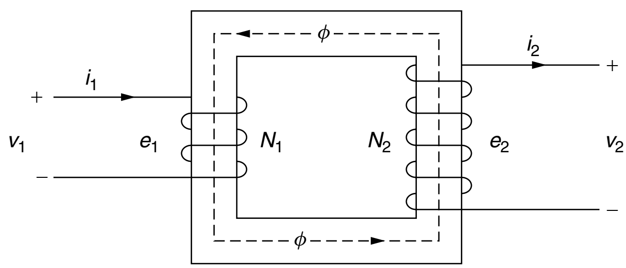


Figure 1.27 An idealized two-winding transformer.

and

$$e_2 = N_2 \frac{d\phi}{dt} \tag{1.52}$$

Continuing the idealization of the transformer, if there are no wire losses, then the voltage on the incoming wires, v_1, is equal to the emf e_1, and the voltage on the output wires, v_2, equals e_2. Dividing (1.52) by (1.51) gives

$$\frac{v_2}{v_1} = \frac{e_2}{e_1} = \frac{N_2(d\phi/dt)}{N_1(d\phi/dt)} \tag{1.53}$$

Before canceling out the $d\phi/dt$, note that we can only do so if $d\phi/dt$ is not equal to zero. That is, *the following fundamental relationship for transformers (1.53) is not valid for dc conditions*:

$$v_2 = \left(\frac{N_2}{N_1}\right) v_1 = (\textit{turns ratio}) \cdot v_1 \tag{1.54}$$

The quantity in the parentheses is called *the turns ratio*. If voltages are to be raised, then the turns ratio needs to be greater than 1; to lower voltages it needs to be less than 1.

Does (1.54), which says that we can easily increase the voltage from primary to secondary, suggest that we are getting something for nothing? The answer is, as might be expected, no. While (1.54) suggests an easy way to raise ac voltages, energy still must be conserved. If we assume that our transformer is perfect; that is, it has no energy losses of its own, then power going into the transformer on the primary side, must equal power delivered to the load on the secondary side. That is,

$$v_1\, i_1 = v_2\, i_2 \tag{1.55}$$

Substituting (1.54) into (1.55) gives

$$i_2 = \left(\frac{v_1}{v_2}\right) i_1 = \left(\frac{N_1}{N_2}\right) i_1 \tag{1.56}$$

What (1.56) shows is that if we increase the voltage on the secondary side of the transformer (to the load), we correspondingly reduce the current to the load. For example, bumping the voltage up by a factor of 10 reduces the current delivered by a factor of 10. On the other hand, decreasing the voltage by a factor of 10 increases the current 10-fold on the secondary side.

Another important consideration in transformer analysis is what a voltage source "sees" when it sends current into a transformer that is driving a load. For example, in Fig. 1.28 a voltage source, transformer, and resistive load are shown. The symbol for a transformer shows a couple of parallel bars between the windings, which is meant to signify that the coil is wound around a metal (steel) core (not an air core). The dots above the windings indicate the polarity

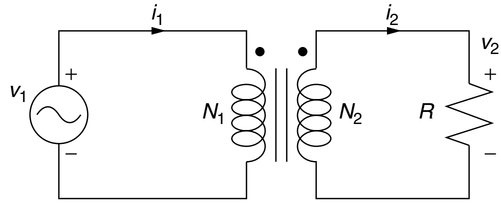


Figure 1.28 A resistance load being driven by a voltage source through a transformer.

of the windings. When both dots are on the same side (as in Fig. 1.28) a positive voltage on the primary produces a positive voltage on the secondary.

Back to the question of the equivalent load seen by the input voltage source for the circuit of Fig. 1.28. If we call that load R_{in}, then we have

$$v_1 = R_{\text{in}} i_1 \tag{1.57}$$

Rearranging (1.57) and substituting in (1.55) and (1.56) gives

$$R_{\text{in}} = \left(\frac{v_1}{i_1}\right) = \frac{(N_1/N_2)v_2}{(N_2/N_1)i_2} = \left(\frac{N_1}{N_2}\right)^2 \cdot \frac{v_2}{i_2} = \left(\frac{N_1}{N_2}\right)^2 R \tag{1.58}$$

where $v_2/i_2 = R$ is the resistance of the transformer load.

As far as the input voltage source is concerned, the load it sees is the resistance on the secondary side of the transformer divided by the square of the turns ratio. This is referred to as a resistance transformation (or more generally an *impedance* transformation).

Example 1.11 Some Transformer Calculations. A 120- to 240-V step-up transformer is connected to a 100-Ω load.

a. What is the turns ratio?
b. What resistance does the 120-V source see?
c. What is the current on the primary side and on the secondary side?

Solution

a. The turns ratio is the ratio of the secondary voltage to the primary voltage,

$$\text{Turns ratio} = \frac{N_2}{N_1} = \frac{v_2}{v_1} = \frac{240 \text{ V}}{120 \text{ V}} = 2$$

b. The resistance seen by the 120 V source is given by (1.58):

$$R_{\text{in}} = \left(\frac{N_1}{N_2}\right)^2 R = \left(\frac{1}{2}\right)^2 100 = 25\ \Omega$$

c. The primary side current will be

$$i_{\text{primary}} = \frac{v_1}{R_{\text{in}}} = \frac{120\ \text{V}}{25\ \Omega} = 4.8\ \text{A}$$

On the secondary side, current will be

$$i_{\text{secondary}} = \frac{v_2}{R_{\text{load}}} = \frac{240\ \text{V}}{100\ \Omega} = 2.4\ \text{A}$$

Notice that power is conserved:

$$v_1 \cdot i_1 = 120\ \text{V} \cdot 4.8\ \text{A} = 576\ \text{W}$$

$$v_2 \cdot i_2 = 240\ \text{V} \cdot 2.4\ \text{A} = 576\ \text{W}$$

1.8.2 Magnetization Losses

Up to this point, we have considered a transformer to have no losses of any sort associated with its performance. We know, however, that real windings have inherent resistance so that when current flows there will be voltage and power losses there. There are also losses associated with the magnetization of the core, which will be explored now.

The orientation of atoms in ferromagnetic materials (principally iron, nickel, and cobalt as well as some rare earth elements) are affected by magnetic fields. This phenomenon is described in terms of unbalanced spins of electrons, which causes the atoms to experience a torque, called a *magnetic moment*, when exposed to a magnetic field.

Ferromagnetic metals exist in a crystalline structure with all of the atoms within a particular portion of the material arranged in a well-organized lattice. The regions in which the atoms are all perfectly arranged is called a subcrystalline *domain*. Within each magnetic domain, all of the atoms have their spin axes aligned with each other. Adjacent domains, however, may have their spin axes aligned differently. The net effect of the random orientation of domains in an unmagnetized ferromagnetic material is that all of the magnetic moments cancel each other and there is no net magnetization. This is illustrated in Fig. 1.29a.

When a strong magnetic field H is imposed on the domains, their spin axes begin to align with the imposed field, eventually reaching saturation as shown in Fig. 1.29b. After saturation is reached, increasing the magnetizing force causes no increase in flux density, B. This suggests that the relationship between magnetic

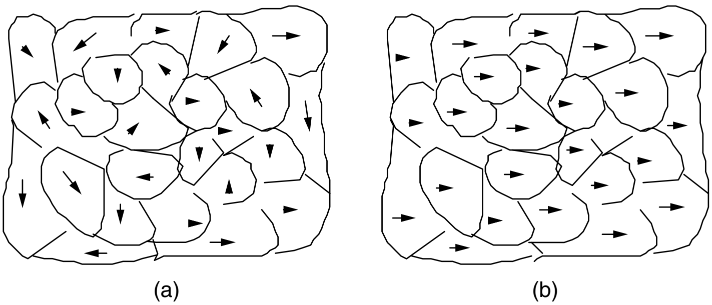


Figure 1.29 Representation of the domains in (a) an unmagnetized ferromagnetic material and (b) one that is fully magnetized.

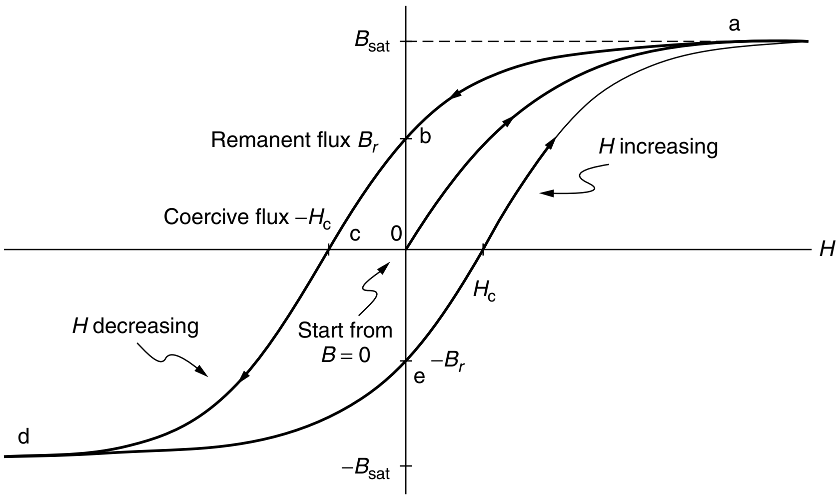
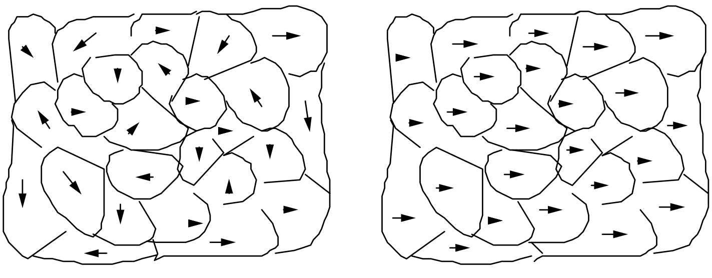


Figure 1.30 Cycling an imposed mmf on a ferromagnetic material produces a hysteresis loop.

field H and flux density B will not be linear, as was implied in (1.35), and in fact will exhibit some sort of s-shaped behavior. That is, permeability μ is not constant.

Figure 1.30 illustrates the impact that the imposition of a magnetic field H on a ferromagnetic material has on the resulting magnetic flux density B. The field causes the magnetic moments in each of the domains to begin to align. When the magnetizing force H is eliminated, the domains relax, but don't return to their original random orientation, leaving a remanent flux B_r; that is, the material becomes a "permanent magnet." One way to demagnetize the material is to heat it to a high enough temperature (called the *Curie temperature*) that the domains once again take on their random orientation. For iron, the Curie temperature is 770°C.



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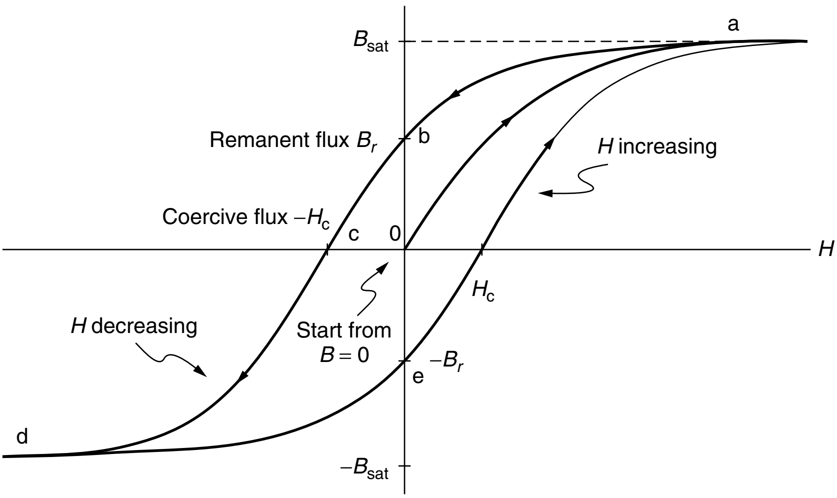

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Consider what happens to the $B-H$ curve as the magnetic domains are cycled back and forth by an imposed ac magnetomagnetic force. On the $B-H$ curve of Fig 1.30, the cycling is represented by the path o–a followed by the path a–b. If the field is driven somewhat negative, the flux density can be brought back to zero (point c) by imposing a *coercive force*, H_c; forcing the applied mmf even more negative brings us to point d. Driving the mmf back in the positive direction takes us along path d–e–a.

The phenomenon illustrated in the $B-H$ curve is called hysteresis. Cycling a magnetic material causes the material to heat up; in other words, energy is being wasted. It can be shown that the energy dissipated as heat in each cycle is proportional to the area contained within the hysteresis loop. Each cycle through the loop creates an energy loss; therefore the rate at which energy is lost, which is power, is proportional to the frequency of cycling and the area within the hysteresis loop. That is, we can write an equation of the sort

$$\text{Power loss due to hysteresis} = k_1 f \tag{1.59}$$

where k_1 is just a constant of proportionality and f is the frequency.

Another source of core losses is caused by small currents, called *eddy currents*, that are formed within the ferromagnetic material as it is cycled. Consider a cross section of core with magnetic flux ϕ aligned along its axis as shown in Fig. 1.31a. We know from Faraday's law that anytime a loop of electrical conductor has varying magnetic flux passing through it, there will be a voltage (emf) created in that loop proportional to the rate of change of ϕ. That emf can create its own current in the loop. In the case of our core, the ferromagnetic material is the conductor, which we can think of as forming loops of conductor wrapped around flux creating the eddy currents shown in the figure.

To analyze the losses associated with eddy currents, imagine the flux as a sinusoidal, time-varying function

$$\phi = \sin(\omega t) \tag{1.60}$$

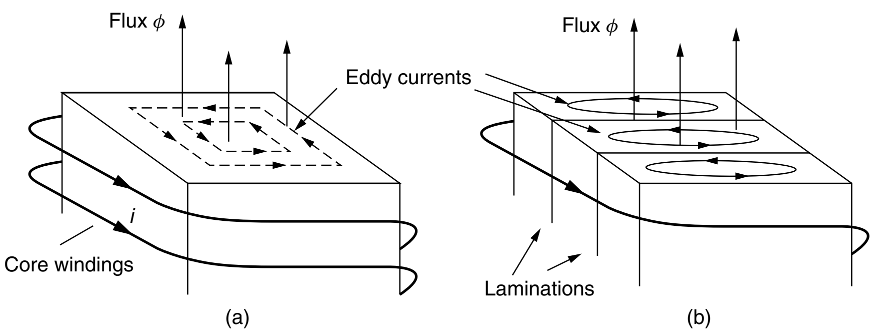


Figure 1.31 Eddy currents in a ferromagnetic core result from changes in flux linkages: (a) A solid core produces large eddy current losses. (b) Laminating the core yields smaller losses.

The emf created by changing flux is proportional to $d\phi/dt$

$$e = k_2 \frac{d\phi}{dt} = k_2 \omega \cos(\omega t) \tag{1.61}$$

where k_2 is just a constant of proportionality. The power loss in a conducting "loop" around this changing flux is proportional to voltage squared over loop resistance:

$$\text{Eddy current power loss} = \frac{e^2}{R} = \frac{1}{R}[k_2 \omega \cos(\omega t)]^2 \tag{1.62}$$

Equation (1.62) suggests that power loss due to eddy currents is inversely proportional to the resistance of the "loop" through which the current is flowing. To control power losses, therefore, there are two approaches: (1) Increase the electrical resistance of the core material, and (2) make the loops smaller and tighter. Tighter loops have more resistance (since resistance is inversely proportional to cross-sectional area through which current flows) and they contain less flux ϕ (emf is proportional to the rate of change of flux, not flux density).

Real transformer cores are designed to control both causes of eddy current losses. Steel cores, for example, are alloyed with silicon to increase resistance; otherwise, high-resistance magnetic ceramics, called ferrites, are used instead of conventional alloys. To make the loops smaller, cores are usually made up of many thin, insulated, lamination layers as shown in Fig. 1.31b.

The second, very important conclusion from Eq. (1.62) is that eddy current losses are proportional to frequency squared:

$$\text{Power loss due to eddy currents} = k_3 f^2 \tag{1.63}$$

Later, when we consider harmonics in power circuits, we will see that some loads cause currents consisting of multiples of the fundamental 60-Hz frequency. The higher-frequency harmonics can lead to transformer core burnouts due to the eddy current dependence on frequency squared.

Transformer hysteresis losses are controlled by using materials with minimal B–H hysteresis loop area. Eddy current losses are controlled by picking core materials that have high resistivity and then laminating the core with thin, insulated sheets of material. Leakage flux losses are minimized not only by picking materials with high permeability but also by winding the primary and secondary windings right on top of each other. A common core configuration designed for overlapping windings is shown in Fig. 1.32. The two windings are wrapped around the center section of core while the outer two sections carry the flux in closed loops. The top of a laminated slice of this core is a separate piece in order to facilitate wrapping the windings around core material. With the top off, a mechanical winder can easily wrap the core, after which the top bar is attached.

A real transformer can be modeled using a circuit consisting of an idealized transformer with added idealized resistances and inductors as shown in Fig. 1.33.

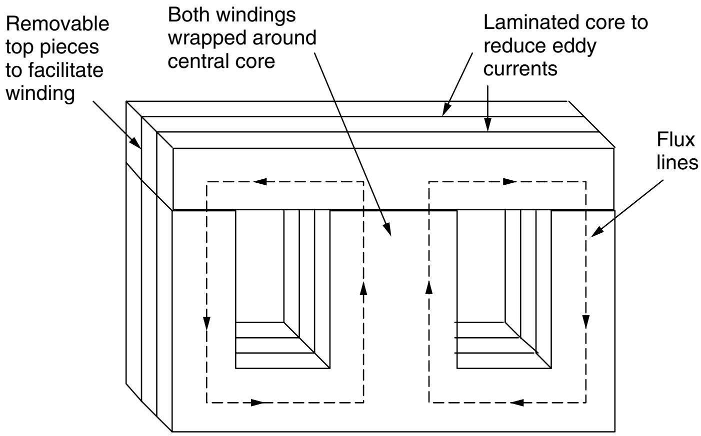


Figure 1.32 A type "E-1" laminated core for a transformer showing the laminations and the removable top pieces to enable machine winding. Windings are wound on top of each other on the central portion of the core.

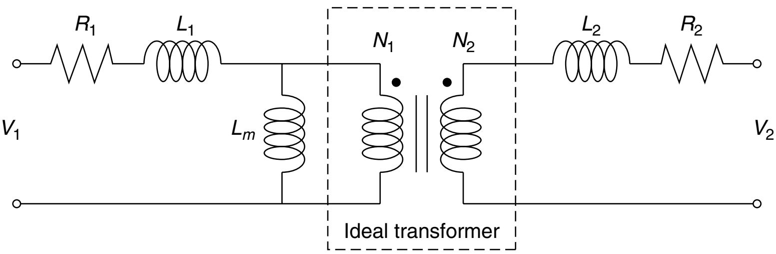


Figure 1.33 A model of a real transformer accounts for winding resistances, leakage fluxes, and magnetizing inductance.

Resistors R_1 and R_2 represent the resistances of the primary and secondary windings. L_1 and L_2 represent the inductances associated with primary and secondary leakage fluxes that pass through air instead of core material. Inductance L_m, *the magnetizing inductance*, allows the model to show current in the primary windings even if the secondary is an open circuit with no current flowing.

PROBLEMS

1.1. Either a resistor, capacitor or inductor is connected through a switch to a current source. At $t = 0$, the switch is closed and the following applied current results in the voltage shown. What is the circuit element and what is its magnitude?

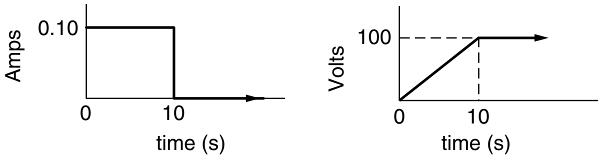


Figure P1.1

1.2. A voltage source produces the square wave shown below. The load, which is either an ideal resistor, capacitor or inductor, draws current current as shown below.

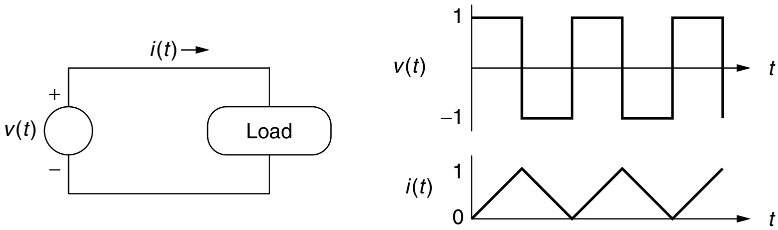


Figure P1.2

a. Is the "Load" a resistor, capacitor or inductor?

b. Sketch the power delivered to the load versus time.

c. What is the average power delivered to the load?

1.3. A single conductor in a transmission line dissipates 6,000 kWh of energy over a 24-hour period during which time the current in the conductor was 100 amps. What is the resistance of the conductor?

1.4. A core-and-coil inductor has a mean cross-sectional area of 0.004 m^2 and a mean circumference of 0.24 m. The iron core has a relative permeability of 20,000. It is wrapped with 100 turns carrying 1 amp of current.

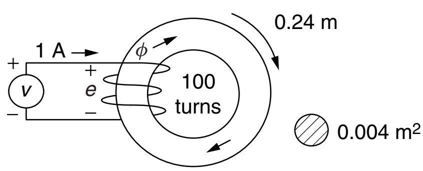


Figure P1.4

a. What is the reluctance of the core $\mathcal{R}$ (A-t/Wb)?

b. What is the inductance of the core and coil L (henries)?

c. What is the magnetic field intensity H (A-t/m)?

d. What is the magnetic flux density B (Wb/m^2)

1.5. The resistance of copper wire increases with temperature in an approximately linear manner that can be expressed as

$$R_{T2} = R_{T1}[1 + \alpha(T_2 - T_1)]$$

where $\alpha = 0.00393/°\mathrm{C}$. Assuming the temperature of a copper transmission line is the same as the ambient temperature, how hot does the weather have to get to cause the resistance of a transmission line to increase by 10% over its value at 20°C?

1.6. A 52-gallon electric water heater is designed to deliver 4800 W to an electric-resistance heating element in the tank when it is supplied with 240 V (it doesn't matter if this is ac or dc).

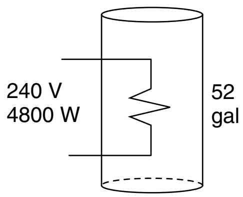


Figure P1.5

a. What is the resistance of the heating element?

b. How many watts would be delivered if the element is supplied with 208 V instead of 240 V?

c. Neglecting any losses from the tank, how long would it take for 4800 W to heat the 52 gallons of water from 60°F to 120°F? The conversion between kilowatts of electricity and Btu/hr of heat is given by 3412 Btu/hr = 1 kW. Also, one Btu heats 1 lb of water by 1°F and 1 gallon of water weighs 8.34 lbs.

1.7. Suppose an automobile battery is modeled as an ideal 12-V battery in series with an internal resistance of 0.01 Ω as shown in (a) below.

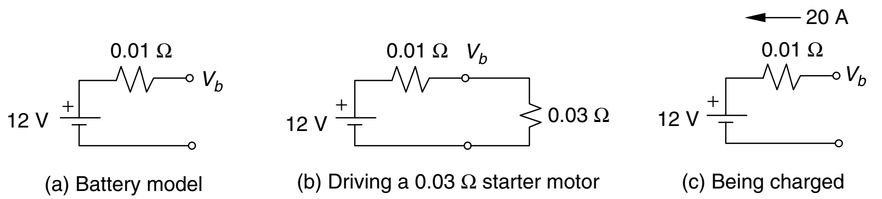


Figure P1.7

a. What current will be delivered when the battery powers a 0.03 Ω starter motor, as in (b)? What will the battery output voltage be?

b. What voltage must be applied to the battery in order to deliver a 20-A charging current as in (c)?

1.8. Consider the problem of using a low-voltage system to power a small cabin. Suppose a 12-V system powers a pair of 100-W lightbulbs (wired in parallel).

a. What would be the (filament) resistance of a bulb designed to use 100 W when it receives 12 V?

b. What would be the current drawn by two such bulbs if each receives a full 12 V?

c. What gage wire should be used if it is the minimum size that will carry the current.

d. Suppose a 12-V battery located 80-ft away supplies current to the pair of bulbs through the wire you picked in (c). Find:

1. The equivalent resistance of the two bulbs plus the wire resistance to and from the battery.
2. Current delivered by the battery
3. The actual voltage across the bulbs
4. The power lost in the wires
5. The power delivered to the bulbs
6. The fraction of the power delivered by the battery that is lost in the wires.

1.9. Repeat Problem 1.8 using a 60-V system using the same 12 gage wire.

1.10. Suppose the lighting system in a building draws 20 A and the lamps are, on the average, 100 ft from the electrical panel. Table 1.3 suggests that 12 ga wire meets code, but you want to consider the financial merits of wiring the circuit with bigger 10 ga wire. Suppose the lights are on 2500 hours per year and electricity costs $0.10 per kWh.

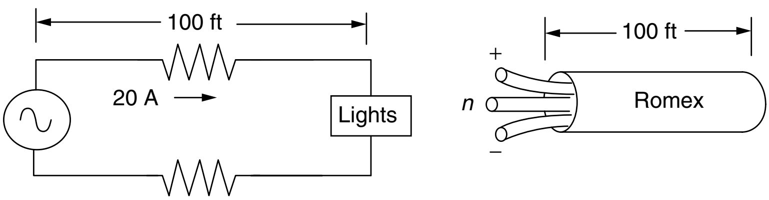


Figure P1.10

a. Find the energy savings per year (kWhr/yr) that would result from using 10 ga instead of 12 ga wire.

b. Suppose 12 ga wire costs $25 per 100 ft of "Romex" (2 conductors, each 100-ft long, plus a ground wire in a tough insulating sheath) and 10 ga costs $35 per 100 ft. What would be the "simple payback" period (simple payback = extra 1st cost/annual $ savings) when utility electricity costs $0.10/kWh?

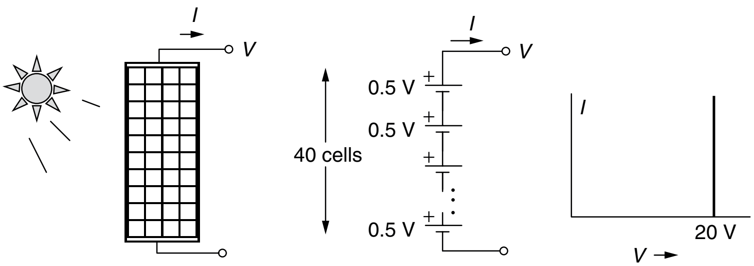
c. An effective way to evaluate energy efficiency projects is by calculating the annual cost associated with conservation and dividing it by the annual energy saved. This is the *cost of conserved energy* (CCE) and is described more carefully in Section 5.4. CCE is defined as follows

$$\text{CCE} = \frac{\text{annual cost of saved electricity(\$/yr)}}{\text{annual electricity saved (kWhr/yr)}} = \frac{\Delta P \cdot \text{CRF}(i, n)}{\text{kWhr/yr}}$$

where ΔP is the extra cost of the conservation feature (heavier duty wire in this case), and CRF is the capital recovery factor (which means your annual loan payment on \$1 borrowed for n years at interest rate i.

What would be the "cost of conserved energy" CCE (cents/kWhr) if the building (and wiring) is being paid for with a 7-%, 20-yr loan with CRF = 0.0944/yr. How does that compare with the cost of electricity that you don't have to purchase from the utility at 10¢ /kWhr?

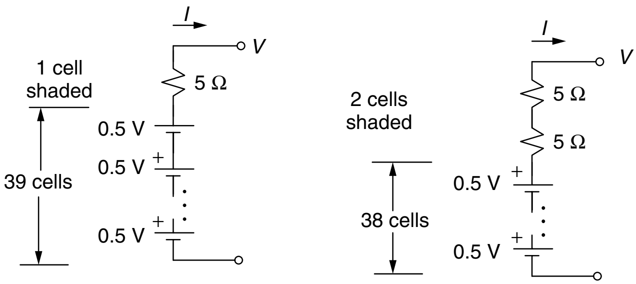
1.11. Suppose a photovoltaic (PV) module consists of 40 individual cells wired in series, (a). In some circumstances, when all cells are exposed to the sun it can be modeled as a series combination of forty 0.5-V ideal batteries, (b). The resulting graph of current versus voltage would be a straight, vertical 20-V line as shown in (c).



(a) 40-cell PV module (b) 40 cells in sun (c) full-sun I-V curve

Figure P1.11

a. When an individual cell is shaded, it looks like a 5-Ω resistor instead of a 0.5-V battery, as shown in (d). Draw the I-V curve for the PV module with one cell shaded.



(d) one cell shaded (e) two cells shaded

Figure P1.11

b. With two cells shaded, as in (e), draw the I-V curve for the PV module on the same axes as you have drawn the full-sun and 1-cell shaded I-V lines.

1.12. If the photovoltaic (PV) module in Problem 1.11 is connected to a 5-Ω load, find the current, voltage, and power that will be delivered to the load under the following conditions:

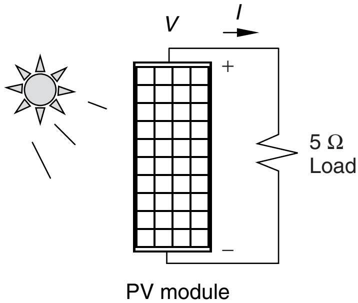


Figure P1.12

a. Every cell in the PV module is in the sun.

b. One cell is shaded.

c. Two cells are shaded.

Use the fact that the same current and voltage flows through both the PV module and the load so solve for I, V, and P.

1.13. When circuits involve a source and a load, the same current flows through each one and the same voltage appears across both. A graphical solution can therefore be obtained by simply plotting the current-voltage (I-V) relationship for the source onto the same axes that the I-V relationship for the load is plotted, and then finding the crossover point where both are satisfied simultaneously. This is an especially powerful technique when the relationships are nonlinear.

For the photovoltaic module supplying power to the 5-W resistive load in Problem 1.12, solve for the resulting current and voltage using the graphical approach:

a. For all cells in the sun.

b. For one cell shaded

c. For two cells shaded