Numbers and Operations

A **number** is a symbol used to represent a **quantity** (an amount). **Operations** are processes that are performed on numbers. Operations and **operational symbols** (figures representing math operations) include addition (+), subtraction (–), multiplication (×), and division (÷). An understanding of operations and the order in which they are to be performed gives kids the tools they will need later to discover the value of unknown variables in algebraic equations. Since some mathematical problems require more calculations than others, the use of a technological tool—the calculator—can speed up the process of finding the answer. The common operations of addition, subtraction, multiplication, and division are needed to solve problems containing fractions as well as to find unknown variables in algebraic equations.

Addition and Subtraction

TEACHING TIPS

Benchmarks

By the end of grade 5, students should be able to

- Demonstrate an understanding of operation patterns and properties.
- Use addition and subtraction to solve problems connected to everyday experiences.

By the end of grade 8, students should be able to

- Represent operations with models, words, and numbers.
- Compare and order integers.

In this chapter, students are expected to

- Use number lines to evaluate addition and subtraction expressions.
- Analyze word problems to choose an operation and write and evaluate an expression for the problem.

Preparing the Materials

Activity 1: Addition and Subtraction

- Make a copy of the Addition and Subtraction activity sheet for each student.

Activity 2: Problem Solving

- Make a copy of the Problem Solving activity sheet for each student.

Presenting the Math Concepts

1. Introduce the new terms:

addends Numbers that are added together.

addition The operation of adding together two or more numbers called addends, which are combined into a resulting number called the sum.

analyze To separate information into individual parts, examine those parts, and organize them to solve a problem.

commutative property for addition When numbers are added, the order of the addends may be changed without changing the sum.

equal (=) Symbol used to compare equal numbers or expressions.

equation A mathematical sentence that uses an equal symbol to show that two expressions are equal.

expression Numbers or letters or numbers and letters combined with one or more operational symbols.

inverse operations Operations that undo each other. Addition and subtraction are inverse operations.

number line A line divided into equal parts with one point chosen as the 0 point, or origin.

numerical expression Numbers combined with one or more operational symbols.

operations Processes such as addition and subtraction that are performed on numbers.

subtraction The operation that involves finding the difference between two numbers.

sum The number that is the result of adding two or more addends.

whole numbers Counting numbers and 0.

word problem A math problem using only words; a problem written in sentence form that needs to be solved using math.

2. Explore the new terms:

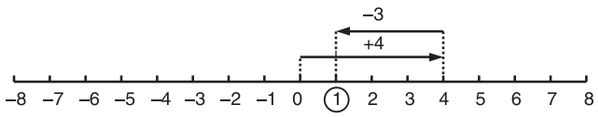
- Whole numbers are counting numbers and 0, which include 0, 1, 2, 3, 4, . . .
- Sometimes commas are used to write whole numbers with more than three digits to make the number easier to read. To place a comma in a whole number, count digits from the right-hand end and place a comma after every three digits. For example, the whole number 2307456 can be written as 2,307,456.
- Commas are also used when words are used to name a number. Thus, the name of 2,307,456 would be two million, three hundred seven thousand, four hundred fifty-six.
- The symbol for the operation of addition is the plus sign (+).
- The symbol for the operation of subtraction is the minus sign (–).
- Addition and subtraction are inverse operations, which means that if you start with any number and then add and subtract the same number to it, the result is the original number. For example, if you begin with the number 10, add 4, and then subtract 4, the result is 10. $10 + 4 - 4 = 10$.
- The commutative property for addition for real numbers a and b would be expressed as $a + b = b + a$.

TEACHING TIPS (continued)

- Examples of numeral expressions are 3 + 4 and 5 + 4 – 2.
- An equation uses the symbol = to compare equal numbers or expressions. For example: 2 + 3 = 5 or 2 + 3 = 4 + 1.
- To solve a word problem, you must first analyze it to determine what you know (the facts), what you want to know, and what operations are needed. You then write an expression in sentence form to find the answer.

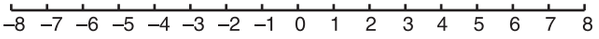
EXTENSION

1. Introduce the terms *negative numbers*, *positive numbers*, and *signed numbers*. Negative numbers are numbers with a value less than 0 and are found to the left of 0 on a horizontal number line. Positive numbers have a value geater than 0 and are found to the right of 0 on a horizontal number line. Signed numbers are numbers with a positive or negative sign. Negative numbers must have a negative sign, such as –5. But positive numbers can be written with or without a positive sign, for example, +4 or 4.
2. When using arrows to show addition of signed numbers, such as in the figure below, the length of the arrow represents the value of the number. On a horizontal number line, an arrow for a positive number points to the right and an arrow for a negative number points to the left. For example, using arrows and a number line to find the sum of 4 + (–3) would be:



Prepare an activity sheet for the addition of signed numbers, providing a number line for each problem as shown. For each problem, students can use a pencil to draw directed arrows on a number line to find the sum of each problem. Example problems:

Number Line

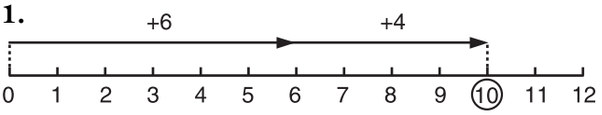


1. (–2) + (–1) **Answer:** (–3)
2. 6 + (–4) **Answer:** (2)
3. 7 + (–2) + (–4) **Answer:** (1)
4. 4 + (–2) + 1 + (–5) **Answer:** (–2)
5. (+3) + (–2) + (–6) **Answer:** (–5)

ANSWERS

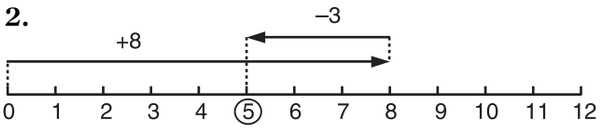
Activity 1: Addition and Subtraction

1.



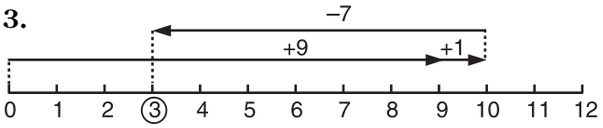
Answer: 6 + 4 = 10

2.



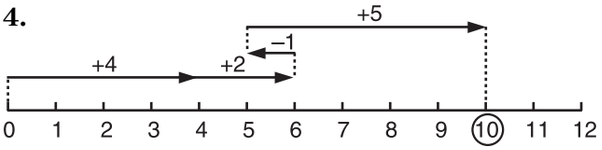
Answer: 8 – 3 = 5

3.



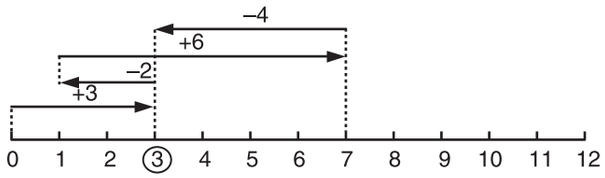
Answer: 9 + 1 – 7 = 3

4.



Answer: 4 + 2 – 1 + 5 = 10

5.



Answer: 3 – 2 + 6 – 4 = 3

Activity 2: Problem Solving

1. a. Total customers = 15
Customers given away = 7
b. How many customers did Kimberly keep?
c. Subtraction
d. 15 – 7
e. 15 – 7 = 8

2. a. Time Lacey has already spent baby-sitting = 2 hours
Time until parents return = 3 hours
b. Total time Lacey will baby-sit
c. Addition
d. 2 hours + 3 hours
e. 2 hours + 3 hours = 5 hours

3. a. Total miles to run = 4 miles
Miles left to run = 1 mile
b. How many miles has Ginger left to run?
c. Subtraction
d. 4 miles – 1 mile
e. 4 miles – 1 mile = 3 miles

4. a. Money paid to cut lawn = $15.00
Money paid to trim hedges = $10.00
Money paid to rake leaves = $5.00
b. How much did Travis earn in all?
c. Addition
d. $15.00 + $10.00 + $5.00
e. $15.00 + $10.00 + $5.00 = $30.00

Name

ACTIVITY 1

Addition and Subtraction

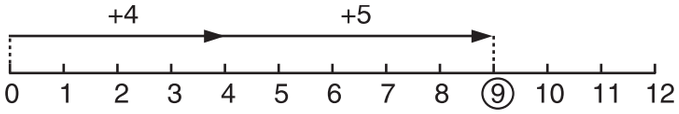
Operations are processes that are performed on numbers, including addition and subtraction. **Addition** is the operation of adding together two or more numbers called **addends**, which are combined into a resulting number called the **sum**. You can change the order of the addends without changing the sum. This property of addition is called the **commutative property for addition**. **Subtraction** is an operation that involves finding the difference between two numbers. Since addition is an operation that adds and subtraction is an operation that takes away, they are **inverse operations**, and they undo each other. A **number line**, which is a line divided into equal parts in which all points correspond to a number, can be used to show that addition and subtraction are inverse operations. An **expression** consists of numbers or letters or numbers and letters combined with one or more operational symbols. A **numerical expression** is an expression of numbers combined with one or more symbols. The **equal (=)** symbol is used to show that numbers or expressions are equal. An **equation** is a mathematical sentence that uses an equal symbol to show that two expressions are equal. In this activity **whole numbers**, which are counting numbers and 0, will be used.

Practice Problems

1. Use a number line to find the sum of 4 + 5.

Think!

- 4 + 5 is an expression involving addition.
- Addition is represented on a number line by arrows that move to the right.
- Begin the first arrow at a point above the 0 on the number line. Use a pencil and ruler to draw the 4 arrow going toward the right. The length of the arrow is 4 divisions to the right on the number line, from 0 to 4. The head of the arrow is above the 4 on the number line.
- Starting at the 4 on the number line, count to the right 5 more divisions, from 4 to 9 on the number line. Use the pencil and ruler to draw the arrow from the tip of the first arrow 5 divisions to the right, so the head of the arrow is now above the 9 on the number line.



Answer: 4 + 5 = 9

2. Use a number line to find the solution of 3 + 6 – 4.

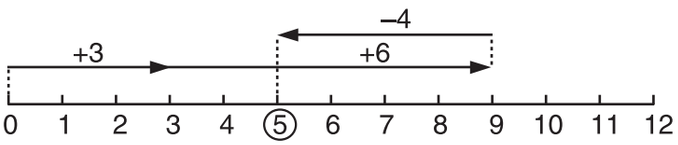
Think!

- 3 + 6 – 4 is a number expression involving addition and subtraction.
- Begin the first arrow at a point above the 0 on the number line. Use a pencil and ruler to draw the arrow going 3 spaces to the right, so the head of the arrow is above the 3 on the number line.

Name

ACTIVITY 1 (continued)

- Starting at the tip of the arrow that ends at the number 3, use the pencil and ruler to draw an arrow pointing 6 more spaces to the right. The head of the arrow is now above the 9 on the number line.
- Starting directly above the tip of the arrow that ends at the number 9, draw an arrow pointing 4 spaces to the left, from 9 to 5 on the number line.



- The tip of the last arrow is above the 5 on the number line.

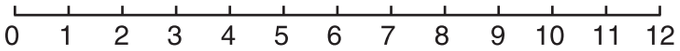
Answer: 3 + 6 – 4 = 5

On Your Own

Use a ruler and the number line to find the sum of each problem.

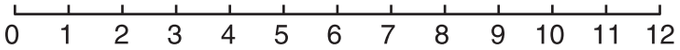
1. 6 + 4

Answer: _____



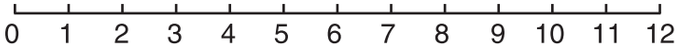
2. 8 – 3

Answer: _____



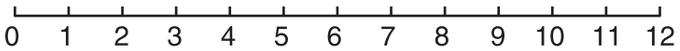
3. 9 + 1 – 7

Answer: _____



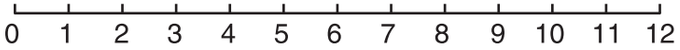
4. 4 + 2 – 1 + 5

Answer: _____



5. 3 – 2 + 6 – 4

Answer: _____



Name

ACTIVITY 2

Problem Solving

To **analyze** means to separate information into its individual parts, examining those parts and organizing them to solve a problem. A **word problem** is a math problem using only words; it is a problem written in sentence form that needs to be solved using math. To solve a word problem, you analyze it to determine what you know (the facts), what you want to know, and what operations are needed. Using this information, you write an expression. You solve the word problem by evaluating (or solving) the expression.

Practice Problems

Analyze each word problem, write an expression for the problem, then evaluate the expression.

1. Jennifer had 120 stuffed animals. She had so many animals that she gave away 50. How many animals did Jennifer keep?

Think!

- What do you know?
  Jennifer started with 120 stuffed animals and gave away 50 of them.
- What do you want to know?
  How many animals Jennifer kept.
- What operation is needed?
  Since Jennifer is *giving away* stuffed animals, the operation is subtraction.
- What expression represents the problem?
  120 animals – 50 animals
- Evaluate the expression.
  120 animals – 50 animals = ?

Answer: Jennifer kept 70 animals.

2. David watched television for 1 hour. His parents then left the house and said they'd be back in 2 hours. If David watches television until his parents return, how long will he have watched television?

Think!

- What do you know?
  David watched television for 1 hour, and will watch for another 2 hours.
- What do you want to know?
  Total time watching television
- What operation is needed?
  Since the question is about a *total* amount of time, the operation is addition.
- What expression represents the problem?
  1 hour + 2 hours

Name

- Evaluate the expression.
  1 hour + 2 hours = ?

Answer: David will have watched television for 3 hours.

On Your Own

Analyze each word problem, write an expression for the problem, then evaluate the expression.

1. Kimberly watched dogs for 15 customers. This was more dogs than she could handle, so she gave 7 customers to Lauren. How many customers did Kimberly keep?

a. What do you know? _____

b. What do you want to know? _____

c. What operation is needed? _____

d. What expression represents the problem? _____

e. Evaluate the expression. _____

2. Lacey has been baby-sitting Jacob for 2 hours. Jacob's parents will be home in 3 hours. How long will Lacey have baby-sat Jacob by the time his parents return?

a. What do you know? _____

b. What do you want to know? _____

c. What operation is needed? _____

d. What expression represents the problem? _____

e. Evaluate the expression. _____

3. Ginger has 1 mile left to run. If she wants to run 4 miles, how many miles has she already run?

a. What do you know? _____

b. What do you want to know? _____

c. What operation is needed? _____

d. What expression represents the problem? _____

e. Evaluate the expression. _____

4. Travis was paid $15.00 to cut the lawn, $10.00 to trim the hedges, and $5.00 to rake leaves. How much did he earn in all?

a. What do you know? _____

b. What do you want to know? _____

c. What operation is needed? _____

d. What expression represents the problem? _____

e. Evaluate the expression. _____

Multiplication

TEACHING TIPS

Benchmarks

By the end of grade 5, students should be able to

- Demonstrate an understanding of operation patterns and properties.
- Make generalizations from patterns and justify why an answer is reasonable.

By the end of grade 8, students should be able to

- Represent operations with models, words, and numbers.

In this chapter, students are expected to

- Use models to demonstrate multiplication as repeated addition.
- Write multiplication equations.
- Make and learn multiplication tables for numbers 1 through 10.

Preparing the Materials

Activity: Multiplication Grid

- Make a copy of the Multiplication Grid activity sheet for each student.
- Make a copy of the Multiplication Answers sheet for each student.
- Students can use crayons or colored markers to color in the squares.

Investigation: Multiplication Strips

- Make a copy of the Multiplication Strips investigation sheet for each student.
- On white card stock, make one copy of Multiplication Tables I and II for each student. Note: If card stock is not available, make copies of the pages on white copy paper and have students glue the strips to heavy paper, such as file folders.

Presenting the Math Concepts

1. Introduce the new terms:

factors Numbers multiplied together to obtain a product.

multiplication An operation involving repeated addition.

product The number obtained after multiplying.

2. Explore the new terms:

- Multiplication is used to find the total amount for a problem when a certain number of equal amounts are given.
- Multiplication is a shortcut for the addition of equal addends. In other words, multiplication is the process by which the same number is added to itself an indicated number of times. For example, if you multiply 5 three times (5×3), one way to calculate the answer is to add five 3s ($3 + 3 + 3 + 3 + 3 = 15$).
- Multiplication is an example of an equation, which shows that two expressions are equal. For example, $2 \times 3 = 6$.

EXTENSIONS

1. Show students how they can check the product when multiplying by 9 because the sum of the digits of each product always add up to 9. For example:

$9 \times 2 = 18$	$1 + 8 = 9$
$9 \times 3 = 27$	$2 + 7 = 9$
$9 \times 6 = 54$	$5 + 4 = 9$
$9 \times 10 = 90$	$9 + 0 = 9$

2. Define *integers*, which are positive or negative whole numbers; for example, –2, –1, 0, 1, 2. Then show students the rules for multiplying integers.
   - When two integers have like signs, the product will be positive.

     both integers are positive $2 \times 4 = 8$

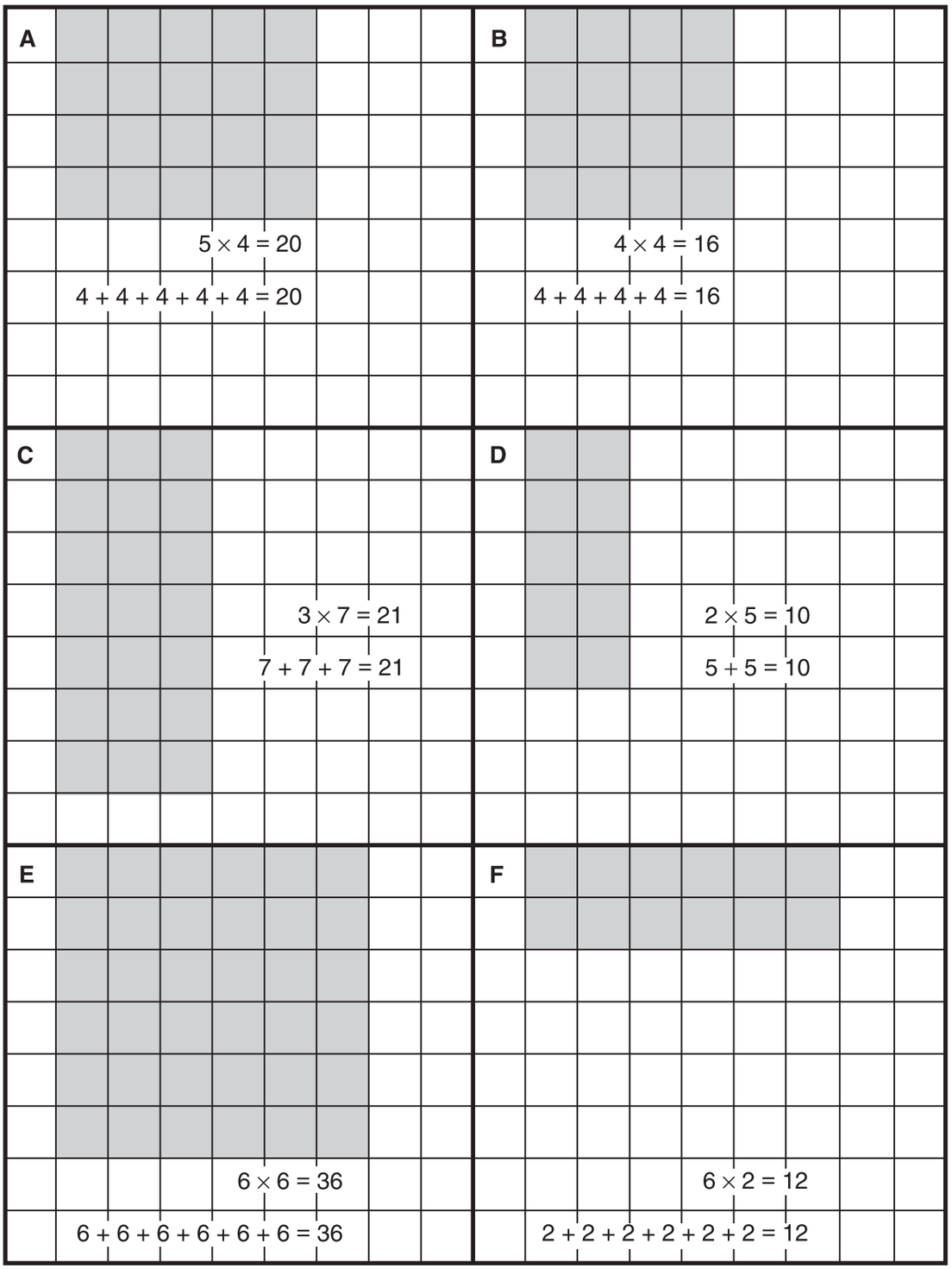
     both integers are negative $-2 \times -4 = 8$
   - When two integers have unlike signs, the product will be negative.

     one integer is negative and the other is positive: $-2 \times 4 = -8$

     one integer is positive and the other is negative: $2 \times -4 = -8$

ANSWERS

Activity: Multiplication Grid



2

Name

ACTIVITY

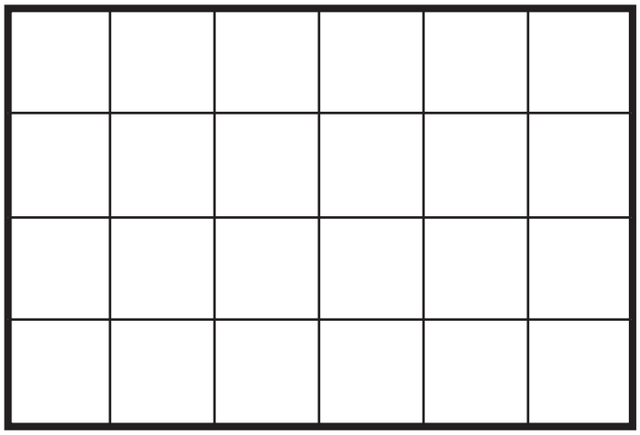
Multiplication Grid

Multiplication is an operation involving repeated addition. **Factors** are the numbers multiplied together to obtain a **product**, which is the number obtained after multiplying.

Practice Problems

Complete the following for the multiplication problem 4×2.

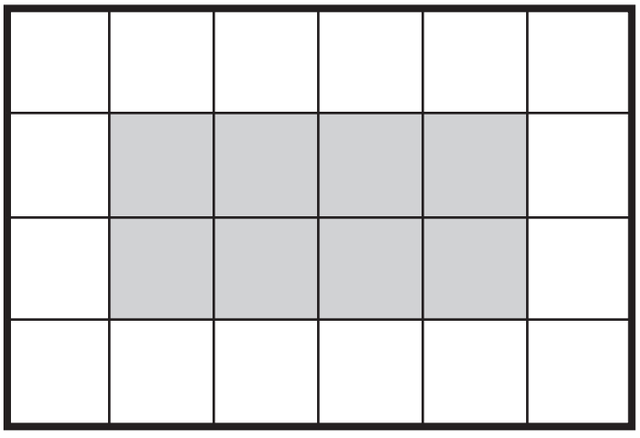
1. Use crayons or colored markers to color squares on the grid to represent the multiplication problem. Let the first factor equal the number of rows (horizontal grouping) and the second factor the number of squares per row.



Think!

- The factors of a multiplication problem are the numbers multiplied together, in this case 4 and 2, to obtain a product.
- The first factor is 4 and the second factor is 2, so there will be 4 colored rows with 2 squares in each row.

Answer:



Name

ACTIVITY (continued)

2. Write a multiplication equation for the problem.

Think!

- The product, which is 8, is the number obtained after multiplying.
- An equation shows that two expressions are equal.

Answer: $4 \times 2 = 8$

3. Write the multiplication problem as an addition problem.

Think!

- Multiplication is a shortcut for the addition of equal addends.
- The product of 4×2 is the same as the sum of four 2s.

Answer:

$2 + 2 + 2 + 2 = 8$

On Your Own

On the Multiplication Answers sheet, complete the following for these problems:

A. 5×4

B. 4×4

C. 3×7

D. 2×5

E. 6×6

F. 6×2

1. Color squares on a grid on the answer sheet to represent each multiplication problem. Let the first factor equal the number of rows and the second factor the number of squares per row.

2. Write a multiplication equation for the problem under the colored squares.

3. Write the multiplication problem as an addition problem under the multiplication equation.

Name

ACTIVITY (continued)

Multiplication Answers

A									B								
C									D								
E									F								

Name

INVESTIGATION 2

Multiplication Strips

PURPOSE

To prepare multiplication table strips for numbers 1 through 10.

Materials

copy of Multiplication Tables I and II
ten different-color crayons
(your choice of colors)
pen
scissors
pencil
paper hole punch
paper brad

Procedure

1. In each of the ten tables, use a different crayon to lightly color the shaded boxes, which contain numbers.
2. Fill in the blanks for each of the multiplication tables. For example, for the 2s table, multiply each number in the vertical column times 2.
3. Use the scissors to cut the tables apart by cutting along the dashed lines.
4. Use the paper hole punch to cut a hole in the bottom of each table where indicated.
5. Lay the multiplication tables one on top of the other in numerical order from 1s to 10s.
6. Insert the paper brad through all the holes and secure it.
7. Study one of the tables and try to memorize it. Then try to multiply the numbers without looking at the table. You can check yourself by looking. Repeat this until you can correctly multiply the numbers in each table without looking.

2s	
	2
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18
10	20
○ — hole	

Results

You have made a set of multiplication strips for the numbers 1 through 10. Use the strips periodically to refresh your memory of the multiplication of the numbers in the tables.

Name

INVESTIGATION (continued)

Multiplication Tables I

1s		2s		3s		4s		5s		
1		2		3		4		5		
	1		1		1		1		1	○
	2		2		2		2		2	
	3		3		3		3		3	
	4		4		4		4		4	
	5		5		5		5		5	
	6		6		6		6		6	
	7		7		7		7		7	
	8		8		8		8		8	
	9		9		9		9		9	
	10		10		10		10		10	○

Name

INVESTIGATION (continued)

2

Multiplication Tables II

6s		7s		8s		9s		10s		
6		7		8		9		10		
	1		1		1		1		1	○
	2		2		2		2		2	
	3		3		3		3		3	
	4		4		4		4		4	
	5		5		5		5		5	
	6		6		6		6		6	
	7		7		7		7		7	
	8		8		8		8		8	
	9		9		9		9		9	
	10		10		10		10		10	○
○		○		○		○		○		

Division

TEACHING TIPS

Benchmarks

By the end of grade 5, students should be able to

- Use inverse operations to check quotients for division problems or products for multiplication problems.
- Demonstrate an understanding of operation patterns and properties.
- Make generalizations from patterns and justify why an answer is reasonable.

By the end of grade 8, students should be able to

- Represent operations with models, words, and numbers.

In this chapter, students are expected to

- Use models to demonstrate division as sharing and repeated subtractions.
- Use multiplication to check quotients for division problems.

Preparing the Materials

Activity: Dividing Counters

- Make a copy of the Dividing Counters activity sheet for each student.
- Provide small objects such as coins, buttons, or paper clips to be used as counters.
- 3-ounce (90-mL) paper cups can be used as group holders.

Investigation: Equal Groups

- Make a copy of the Equal Groups investigation sheet for each student or group.
- Make sure each student or group has a set of crayons and a sheet of paper.

Presenting the Math Concepts

1. Introduce the new terms:

dividend A number that is divided in a division problem.

divisible A number that can be divided by another number without leaving a remainder.

division An operation that tells how many groups there are or how many are in each group.

divisor The number by which a dividend is divided.

quotient The number other than the whole number remainder that is the result of division; the answer to a division problem.

remainder The number less than the divisor that remains when division is finished.

2. Explore the new terms:

- In division, a dividend is divided by a divisor, producing the quotient. In the problem $a \div b = c$, a is the dividend, b is the divisor, and c is the quotient.
- In division, if the dividend cannot be divided into an equal number of groups, there will be a remainder. The letter R is used to identify the remainder. For example, in the following problem, 3 is the remainder: $43 \div 5 = 8$ R3.
- Division can be thought of as:
  - **a.** SHARING An amount is equally divided into parts. For example, 18 pieces of candy are given to 3 children, with an equal number of pieces given to each child. Each child has 6 pieces of candy, as determined by the calculation $18 \div 3 = 6$.
  - **b.** REPEATED SUBTRACTION A specific amount is repeatedly subtracted from the whole. For example, 18 pieces of candy have 6 pieces repeatedly subtracted from it and placed in separate piles. The subtraction would be $18 - 6 = 12$; $12 - 6 = 6$; $6 - 6 = 0$. There would be 3 piles with 6 candies in each. This is determined by the calculation $18 \div 6 = 3$.
  - **c.** OPPOSITE OF MULTIPLICATION The dividend divided by the divisor equals the quotient; thus the quotient multiplied by the divisor equals the dividend. For example, $18 \div 6 = 3$, so $3 \times 6 = 18$.
- Multiplication and division are inverse operations, which means that if you start with a particular number and then multiply and divide by the same number, the result is the original number. For example, begin with the number 12 and multiply by 4, then divide by 4; the result is 12. $12 \times 4 \div 4 = 12$.
- The quotient does not include the remainder if the remainder is written as a whole number. For example, $90 \div 40 = 2$ R10. To check the answer, the quotient of 2 is multiplied by the divisor: $2 \times 40 = 80$. Then the remainder is added to the product, and the sum will be equal to the dividend: $80 + 10 = 90$.

EXTENSIONS

1. Show students how to divide integers using these rules:
   a. When two integers (dividend and divisor) have like signs, the quotient will be positive.
      both integers are positive: 18 ÷ 6 = 3
      both integers are negative: –18 ÷ –6 = 3
   b. When two integers (dividend and divisor) have unlike signs, the quotient will be negative.
      the dividend is positive and the divisor is negative: 18 ÷ –6 = –3
      the dividend is negative and the divisor is positive: –18 ÷ 6 = –3
2. Explain to students that if the remainder is a fraction or a decimal, it is part of the quotient. For example, 9 ÷ 4 = 2¼ or 2.25. Remember that the quotient times the divisor equals the dividend. So 2¼ and 2.25 are correct quotients for the problem 9 ÷ 4, since $2\frac{1}{4} \times 4 = 9$ and $2.25 \times 4 = 9$.
3. Explain that a *set* is a collection of numbers and that *real numbers* can be defined as the members in the set {rational numbers plus irrational numbers}. Compare rational and irrational numbers. A number that has no decimal, has a finite decimal, or has a decimal with a repeating number or block of numbers is called a *rational number*. For example, 4 ÷ 2 = 2; thus the quotient 2 is a rational number. 10 ÷ 3 = 3.3333 . . . . . For a rational number, the dots indicate a repeat of the last number. Placing a line under or above the last number also indicates that the number is repeated; thus the quotient $3.3\underline{3}$ is a rational number.

   A number with a decimal that has no repeating number or pattern is called an *irrational number*. For example, pi (π) = $\frac{22}{7}$ = 3.1415926535 . . . . For an irrational number, the dots indicate no repeating number or pattern. Pi has been divided by computers to 100 million decimal places, although this computation has no practical purpose.

ANSWERS

Activity: Dividing Counters

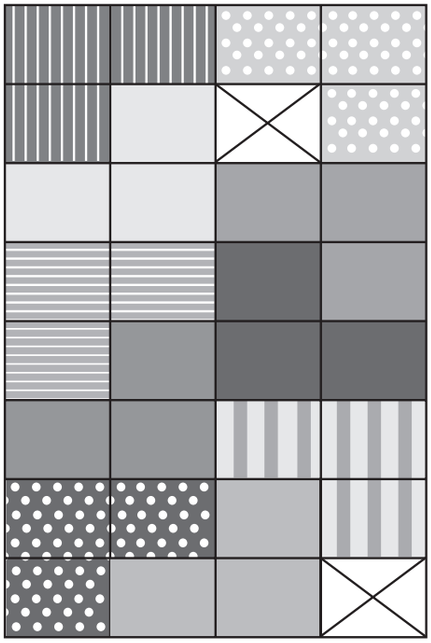
1. 14 ÷ 2 = 7
Check: 7 × 2 = 14

2. 18 ÷ 6 = 3
Check: 3 × 6 = 18

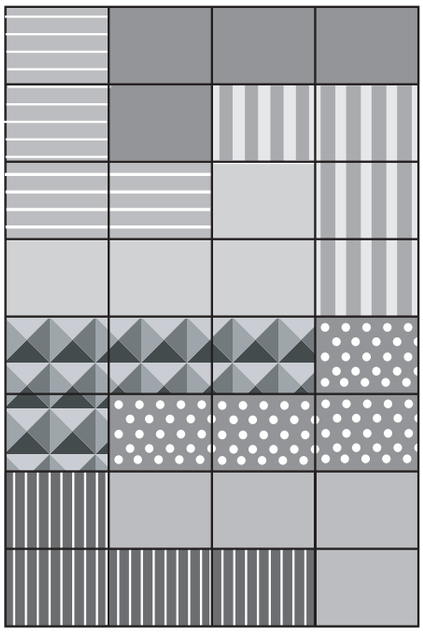
3. 10 – 3 = 7, 7 – 3 = 4, 4 – 3 = 1; 10 ÷ 3 = 3 R1
Check: 3 × 3 = 9, 9 + 1 = 10

4. 15 – 6 = 9, 9 – 6 = 3; 15 ÷ 6 = 2 R3
Check: 6 × 2 = 12, 12 + 3 = 15

Investigation: Equal Groups



10 shapes
2 remaining boxes



8 shapes
0 remaining boxes

3

Name

ACTIVITY

Dividing Counters

Division is an operation that tells how many groups there are or how many are in each group. In division, a **dividend** is divided by a **divisor**, producing the **quotient** (the answer). A number is **divisible** if it can be divided by another number an even number of times. If a number is not divisible, there is a **remainder**, which is the number less than the divisor that remains when division is finished.

Practice Problems

1. Use counters to think about solving the division problem 12 ÷ 3 as sharing. For example, if 12 coins are shared with 3 people, how many coins will each receive?

Think!

- In the problem 12 ÷ 3 = ?, 12 is the dividend, 3 is the divisor, and ? represents the quotient.
- The divisor number tells how many groups the dividend is divided into.
- In sharing, an amount is equally divided into parts. Imagine that 12 coins are divided into 3 groups, with an equal number of coins in each group. Place 3 coins on a table in 3 separate places. Continue to place 1 coin in each group until all the coins are placed. How many coins are in each group? 4.

Answer: 12 ÷ 3 = 4

2. Use counters to think about solving the division problem 12 ÷ 3 as repeated subtraction. For example, if 12 coins are divided so that each person receives 3 coins, how many people receive coins?

Think!

- In repeated subtraction, a group of objects is repeatedly subtracted from the whole. Imagine taking away 3 coins from the 12 coins until you have 0 left. Place each group of 3 coins removed in a separate pile on the table. The subtraction would be 12 – 3 = 9; 9 – 3 = 6; 6 – 3 = 3; 3 – 3 = 0.

- The number of piles of coins (4) equals the number of times each group of 3 coins is removed.

Answer: $12 \div 3 = 4$

3. Use multiplication to check the answer for the above problem, $12 \div 3$.

Think!

- The quotient of a division problem multiplied by the divisor equals the dividend.
- In the problem $12 \div 3 = 4$, the quotient is 4, the divisor is 3, and the dividend is 12.

Answer: $4 \times 3 = 12$

4. Use counters to show repeated subtractions for the problem $7 \div 2$.

Think!

- Take away 2 coins from the 7 coins until there is 0 or less than 2 coins left. Place each group of 2 coins removed in a separate pile on the table. The subtraction would be $7 - 2 = 5$; $5 - 2 = 3$; $3 - 2 = 1$.
- The number of piles of coins (3) equals the number of times each group of 2 coins is removed; the number of coins less than 2 that remain is called the remainder.
- R1 indicates a remainder of 1.

Answer: $7 \div 2 = 3$ R1

5. Use multiplication to check the answer for the problem $7 \div 2$.

Think!

- In the problem $7 \div 2 = 3$ R1, the quotient is 3 and the remainder is 1.
- The quotient multiplied by the divisor plus the remainder equal the dividend.

Answer: $2 \times 3 = 6$, $6 + 1 = 7$

On Your Own

Use counters to solve the following division problems by sharing. Use multiplication to check the answer for each.

1. $14 \div 2$ _____

2. $18 \div 6$ _____

Use counters to solve the following division problems by repeated subtraction. Use multiplication to check the answer for each.

3. $10 \div 3$ _____

4. $15 \div 6$ _____

3

Name

INVESTIGATION

Equal Groups

PURPOSE

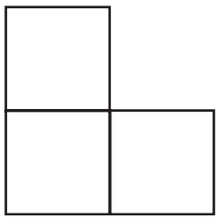
To show how something can be divided into equal groups.

Materials

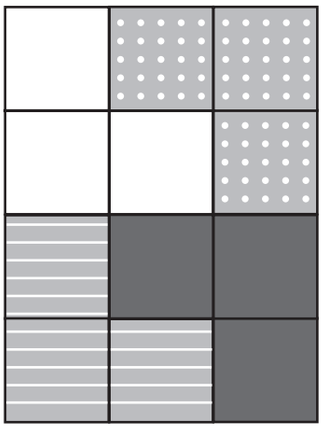
sheet of copy paper
crayons

Procedure

1. Fold the paper in half five times. First, fold the paper three times from top to bottom. Then fold it two times from side to side.
2. Unfold the paper. The fold lines divide the sheet into 32 boxes.
3. Look at the shape drawn to the right, made up of 3 boxes in an L shape.

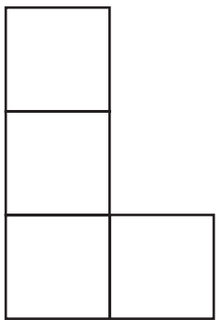


4. Determine how many Ls the 32 boxes can be divided into with the least number of boxes remaining that do not form an L shape. Do this by coloring each shape a different color on the paper. Follow this example:



4 shapes
0 remaining boxes

5. Look at the bigger L shape, using 4 boxes, below.



6. Turn the paper over and determine how many of the larger shape the paper can be divided into with the least number of boxes remaining that do not form an L shape made of 4 boxes. A clue is to turn the paper with one of the longer sides at the top.

Results

The paper can be divided into 10 smaller L shapes, with 2 remaining boxes. The paper can be divided into 8 large L shapes (4 boxes), with no remaining boxes.

Why?

The five folds divide the paper into 32 boxes. Dividing the paper into L shapes of 3 boxes is the same as the division problem 32 ÷ 3 = 10 R2. Dividing the paper into L shapes of 4 boxes is the same as the division problem 32 ÷ 4 = 8.

4 Decimals

TEACHING TIPS

Benchmarks

By the end of grade 5, students are expected to

- Read, write, compare, and order decimal numbers through the thousandths place.

By the end of grade 8, students are expected to

- Compare and order decimal numbers.
- Communicate mathematical ideas using models.

In this chapter, students are expected to

- Compare equivalent decimals.
- Compare and order decimals.
- Write the decimal value of money.

Preparing the Materials

Activity 1: Decimals

- Make a copy of the Decimals activity sheet for each student.

Activity 2: Comparing Decimals

- Make a copy of the Comparing Decimals activity sheet for each student.

Presenting the Math Concepts

1. Introduce the new terms:

decimal A number that uses a decimal point to show tenths, hundredths, thousandths, and so on.

decimal point A dot placed between the ones place and the tenths place in decimals.

equivalent decimals Decimals that name the same amount.

greater than (>) Symbol meaning "is greater than."

hundredth One out of 100 equal parts of a whole; the second place after a decimal point.

less than (<) Symbol meaning "is less than."

tenth One out of 10 equal parts of a whole; the first place after a decimal point.

thousandth One out of 1,000 equal parts of a whole; the third place after a decimal point.

2. Explore the new terms:

- The words for decimal parts end in *th* or *ths*. Read aloud these numbers: 2, 0.2, 32, 0.32, 4,000, 0.004 (two, two tenths, thirty-two, thirty-two hundredths, four thousand, four thousandths). Students can practice saying whole numbers and decimals.
- A decimal point always separates the ones and the tenths in a decimal.
- Money can be used to explain hundredths. It takes 100 pennies to make 1 dollar. One penny is written using a decimal: $0.01 (one hundredth of a whole dollar). The symbol for money is $.
- One penny is called 1 cent. There are 100 cents in 1 whole dollar. So 85 cents is less than 1 whole dollar and is written as $0.85 (eighty-five hundredths of 1 whole dollar).
- For amounts less than 1 using a decimal, a 0 is written in the ones place, such as in 0.234, which is read as "two hundred thirty-four thousandths."
- Decimal numbers are made up of two parts, the whole number part and the fractional part. The parts are separated by a decimal point.
- The following steps are used to name decimal numbers:

  (1) Name the whole number.

  (2) Say "and" at the decimal point.

  (3) Name the fractional part in the same manner that whole numbers are named.

  (4) Say the place value of the last digit of the fractional part.

  For example: the name for 300.034 is three hundred and thirty-four thousandths.
- Name different decimal amounts and show the value of each numeral making up the total number in a Place-Value Data table, as in the following examples:

  a. 23.45 (twenty-three and forty-five hundredths)

  b. 0.653 (six hundred fifty-three thousandths)

  c. 0.8 (eight tenths)

  d. 3,400.07 (three thousand, four hundred and seven hundredths)

TEACHING TIPS (continued)

PLACE-VALUE DATA

thousands	hundreds	tens	ones	decimal	tenths	hundredths	thousandths
		2	3	.	4	5	
			0	.	6	5	3
			0	.	8		
3	4	0	0	.	0	7	

- If there is a 0 in the hundredths place of a 2-digit decimal, the number can be changed to tenths, and the numbers will be equivalent. For example, fifty hundredths (0.50) equals five tenths (0.5). The symbol = is used to show that numbers or expressions have the same value. For example: 0.50 = 0.5.
- The symbol < is used to show that a number or expression is less than another. For example: 0.04 < 0.60.
- The symbol > is used to show that a number or expression is greater than another. For example: 0.60 > 0.04.

EXTENSIONS

1. *Scientific notations,* also called *power-of-10 notation,* is a way of writing any number, but is often used when writing extremely large or small numbers. A *power* is an *exponent* (a number on the right hand of and above a base number that tells how many times the base is multiplied by itself). A *base number* is a number multiplied by itself the number of times equal to the value of the exponent. In scientific notation the base is ten, and the power stands for the number of decimal places. If the power is positive, the decimal places are in front of the decimal point. So, 4×10^3 means "move the decimal point 3 places to the right and fill the empty places with zeros." Another way of saying this is to multiply by 10 three times. $4 \times 10^3 = 4 \times 10 \times 10 \times 10 = 4{,}000$.

   If the power is negative, the decimal places are behind the decimal point. So, 4×10^{-3} means "move the decimal point 3 places to the left and fill the empty places with zeros." Another way of saying this is to multiply by $\frac{1}{10}$ three times.
   $4 \times 10^{-3} = 4 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.0004$

   The usual way of writing scientific notation, called the *standard form,* is to move the decimal behind, or to the right of, the first digit greater than or equal to 1 and multiply by 10 raised to the power representing the decimal place.

EXAMPLES

A. Express 356 in scientific notation.

Think!

- Count how many places the decimal point must be moved so it is to the right of the first digit greater than or equal to 1. In the example, this is two decimal places.
- The number of places the decimal is moved is the exponent of 10. Since the decimal is moved to the left, then the exponent is positive.
- Multiply 3.56 by 10^2.

Answer: 3.56×10^2

B. Express 0.564 in scientific notation.

Think!

- Move the decimal so it is to the right of the first digit greater than or equal to 1.
- The number of places the decimal is moved is one decimal place. So 1 is the exponent of 10. Since the decimal is moved to the right, then the exponent is negative.
- Multiply 5.64 by 10^{-1}.

Answer: 5.64×10^{-1}

2. To make sure everyone gets the same answer for a problem, a set of rules known as the *order of operations* is used. The order of operations is (1) simplify inside parentheses; (2) simplify exponents; (3) multiply and divide from left to right; (4) add and subtract from left to right.

EXAMPLES

- $6 \times (3 + 2)$

  $6 \times (3 + 2) = 6 \times 5$ simplify inside parentheses first

  $= 30$ multiply

- $3^2 - 5$

  $3^2 - 5 = 9 - 5$ simplify exponents first

  $= 4$ subtract

- $8 + 6 \times 4$

  $8 + 6 \times 4 = 8 + 24$ multiply first

  $= 32$ add

- $20 \div 4 \times 6$

  $20 \div 4 \times 6 = 5 \times 6$ do left part first

  $= 30$ do right part

ANSWERS

Activity 1: Decimals

1. 0.43

2. 0.7

3. 2.55

4. five and eight hundredths

5. two and five tenths

6. one thousandth

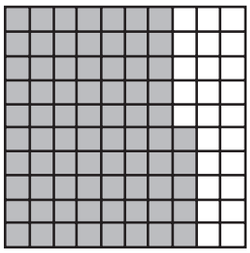
7. twenty-three hundredths

8. ten and forty-four hundredths

9. ninety-nine hundredths

10. eight tenths

11.

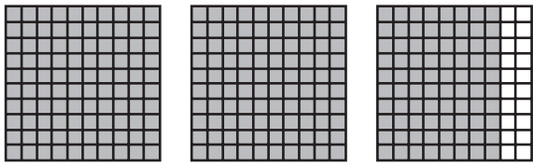


a. zero

b. seven

c. seventy-five hundredths

12.



a. two

b. eight

c. two and eight tenths

Activity 2: Comparing Decimals

1. a. 1.2 **d.** 25.60

b. 0.30 **e.** 0.040

c. 0.5 **f.** 0.770

2. a. < **d.** >

b. > **e.** =

c. < **f.** =

4

Name

ACTIVITY 1

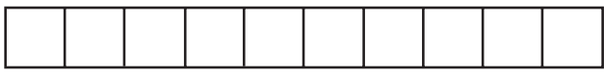
Decimals

A **decimal** is a number that uses a decimal point to show tenths, hundredths, thousandths, and so on. A **decimal point** is a dot placed between the ones place and the tenths place in decimals, such as in 2.5. A **tenth** is 1 out of 10 equal parts of a whole and is the first place after a decimal point, such as the 5 in 2.5. A **hundredth** is 1 out of 100 equal parts of a whole and is the second place after a decimal point, such as the 6 in 2.56. A **thousandth** is 1 out of 1,000 equal parts of a whole and is the third place after a decimal point, such as the 1 in 2.561.

Practice Problems

Write the name of each decimal. Then shade in the grids to show each decimal.

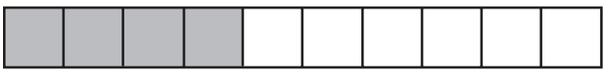
1. 0.4



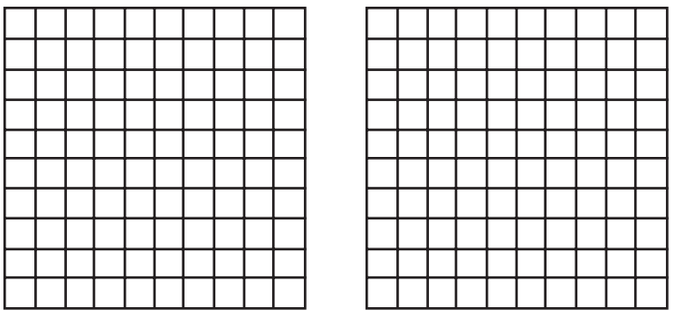
Think!

- The grid is divided into 10 equal boxes. Each box represents one tenth of the whole grid.
- The decimal 0.4 is read as "four tenths." Thus four one-tenth boxes represent the decimal.

Answer: Four tenths



2. 1.29



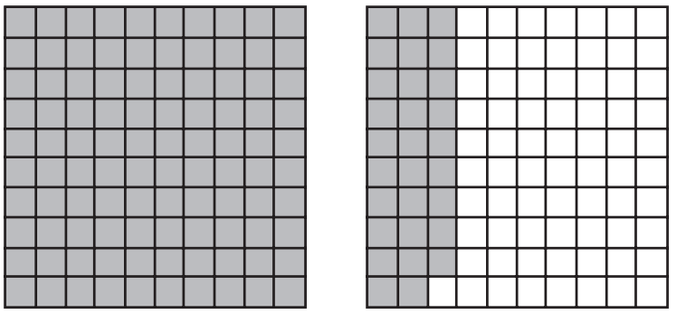
Name

ACTIVITY 1 (continued)

Think!

- Each grid is divided into 100 boxes. Each box represents one hundredth of the whole grid.
- The decimal 1.29 is read as "one and twenty-nine hundredths." Thus 1 whole grid plus 29 boxes on a second grid represent the decimal.

Answer: One and twenty-nine hundredths



On Your Own

Write each number in decimal form.

1. forty-three hundredths _____

2. seven tenths _____

3. two and fifty-five hundredths _____

Write the name for each decimal.

4. 5.08 _____

5. 2.5 _____

6. 0.001 _____

7. 0.23 _____

8. 10.44 _____

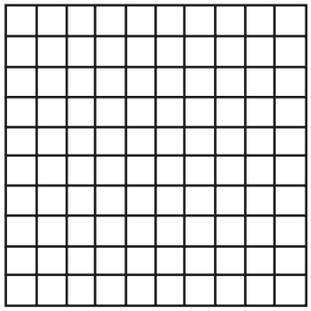
9. 0. 99 _____

10. 0.8 _____

Name

ACTIVITY 1 (continued)

11. Shade in the grids to show the decimal 0.75. Then answer the questions.

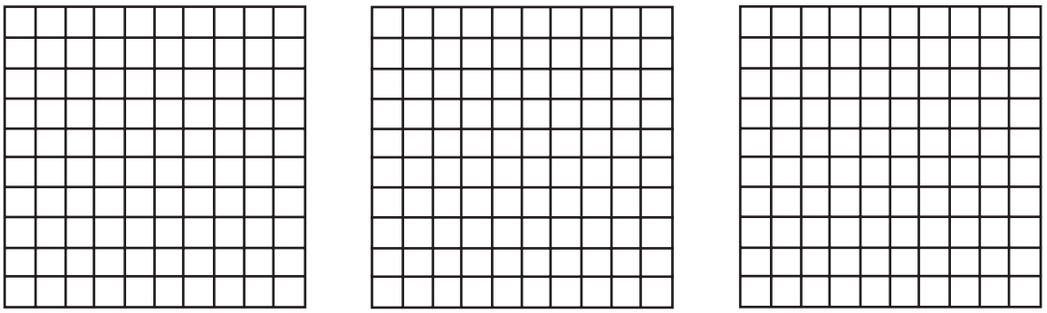


a. How many ones? _____

b. How many tenths? _____

c. What is the name of the decimal? _____

12. Shade in the grids to show the decimal 2.8. Then answer the questions.



a. How many ones? _____

b. How many tenths? _____

c. What is the name of the decimal?_____

Name

ACTIVITY 2

4

Comparing Decimals

Equivalent decimals are decimals that name the same amount. The symbols =, <, and > are used to compare decimals. The symbol = means "equal to." The symbol < means "is **less than**." The symbol > means "is **greater than**."

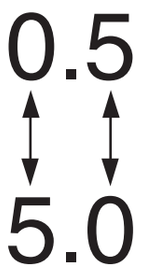
Practice Problems

1. Compare the decimals below. Use = , < , or > for each answer.

a. 0.5 _____ 5.0

Think!

- Write the numbers with their decimal points lined up.
- Compare the digits in each place, moving from left to right.
- In comparing the digits in the ones place, 0 < 5, so 0.5 < 5.0.



Answer: 0.5 < 5.0

b. 2.56 _____ 2.562

Think!

- Write the numbers with their decimal points lined up.
- So that each decimal has a number in the thousandths place, a 0 can be added to the right of 2.56; thus 2.560.
- Compare the digits in each place, moving from left to right.
- The digits in the ones, tenths, and hundredths place are equal. But in the thousandths place, 2 > 0. So 2.562 > 2.560.

Answer: 2.562 > 2.560

Name

ACTIVITY 2 (continued)

On Your Own

1. Write an equivalent decimal for each.

a. 1.20 _____

b. 0.3 _____

c. 0.50 _____

d. 25.6 _____

e. 0.04 _____

f. 0.77 _____

2. Write = , < , or > to compare each decimal.

a. 0.04 _____ 0.40

b. 22.0 _____ 0.22

c. 0.09 _____ 9.0

d. 6.5 _____ 0.065

e. 0.7 _____ 0.70

f. 3.5 _____ 3.500

5 Fractions

TEACHING TIPS

Benchmarks

By the end of grade 5, students should be able to

- Determine fractional parts and write fractions in lowest form.
- Use a model to represent fractional parts.
- Identify fractions in everyday situations.

By the end of grade grade 8, students should be able to

- Determine relationships between fractions.

In this chapter, students are expected to

- Count the number making up a whole and determine the fractional parts of the whole.

Preparing the Materials

Activity: Fractions

- Make a copy of the Fractions activity sheet for each student.

Investigation: Candy Parts

- Students should work in groups of three, but smaller or larger groups also will work.
- Make three copies of the Candy Parts investigation sheet for each student.
- For each group, prepare 3 resealable bags (sandwich size works well) of candy. Number the bags 1, 2, 3. Select 4 colors of candy and place varying numbers of each color in the bags. Each bag should have a different total number of candies: bag 1 should have 6 candies, bag 2 should have 12 candies, and bag 3 should have 18 candies. In each group, one Candy Parts investigation sheet will be completed for each bag. One recorder can be appointed for the group, or each student can record the results for a bag. Tell the students not to open the candy bags before or during the activity.

Presenting the Math Concepts

1. Introduce the new terms:

circle graph A graph in the form of a circle that is divided into sections showing how the whole is broken into parts.

data Observations and/or measured facts.

denominator The number below the line in a fraction; the total number of equal parts in the whole.

division bar The line separating the numerator and the denominator in a fraction.

fraction A number used to express part of a whole and made up of two numbers separated by a line.

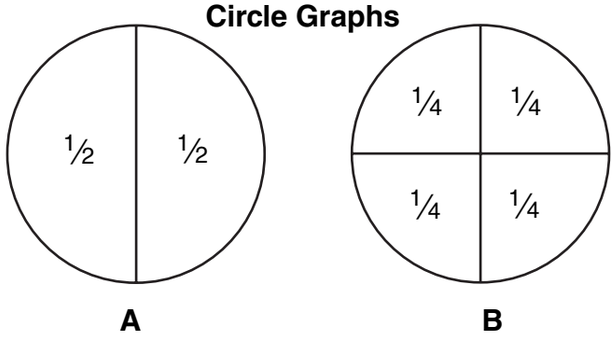
graph A drawing that shows data in an organized way.

mixed number A number made up of a whole number and a fraction.

numerator The number above the line in a fraction; the number of equal parts being considered.

2. Explore the new terms:

- A fraction tells how many parts a whole is broken into.
- A fraction is made up of two numbers separated by a line called the division bar. For example, $\frac{3}{4}$ is a fraction. The top number of all fractions is called the numerator, and the bottom number is called the denominator. In this example, 3 is the numerator and 4 is the denominator.
- A fraction is read by saying the numerator first and then the denominator. For example, $\frac{1}{8}$ is read as "one eighth."
- A fraction is a number that compares part of an object or set to the whole object or set. Fractions are expressed in terms of $\frac{a}{b}$ where b is not 0.
- A fraction can be expressed as a decimal by dividing the numerator by the denominator.
- Examples of mixed numbers are $1\frac{1}{2}$ and $23\frac{2}{3}$.
- A circle graph is a way to show fractional parts. Graph A is broken into two equal parts so each part represents the fraction $\frac{1}{2}$. Graph B is broken into four equal parts with each part representing the fraction $\frac{1}{4}$.



EXTENSION

1. Prepare an activity sheet with a Group Color Fraction Data table, such as the one shown here. Include questions such as the following that require students to use the data table. Note: The letters A, B, C, and D are used in the sample questions. Replace these with the colors of candy used.
   a. Compare the fractional parts of bags 1 and 2 to answer these questions:
      - List the fractional parts of the A-colored candy for the two bags in order from the greatest to the least.
      - Which bag has the greatest fractional part of C-colored candy?_____
   b. Compare the fractional parts of all three bags to answer these questions:
      - List the fractional parts of the D-colored candy for each bag in order from the greatest to the least. _____
        _____
      - List the fractional parts of the B-colored candy for each bag in order from the least to the greatest. _____
        _____
   c. Prepare a data table using decimals instead of fractions.

GROUP COLOR FRACTION DATA				
Bag	**Candy Colors**			
1				
2				
3				

ANSWERS

Activity: Fractions

1. a. 3/5

b. 2/5

2. a. 3/7

b. 2/7

c. 1/7

3. 1⅛ or 1½

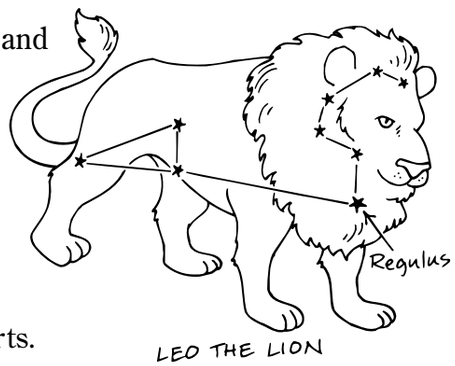
Name

ACTIVITY

5

Fractions

A **fraction** is a number used to express part of a whole and is made up of two numbers separated by a line. For example, $\frac{2}{3}$ and $\frac{6}{15}$ are both fractions. The **division bar** is the line separating the numerator and the denominator in a fraction. The number above the line in a fraction is called the **numerator** and is the number of equal parts being considered. The number below the line in a fraction is called the **denominator** and is the total number of equal parts in the whole. A **mixed number** is a number made up of a whole number and a fraction, such as $1\frac{1}{2}$. A **graph** is a drawing that shows **data** (observations and/or measured facts) in an organized way. A **circle graph** is a graph in the form of a circle that is divided into sections showing how the whole is broken into parts.



Practice Problems

1. What fractional part of the stars make up the lion's head?

Think!

- In this problem, the numerator is the number of stars in the lion's head, which is 6.
- The denominator is the total number of stars, which is 9.
- 6 of the 9 stars are in the lion's head.

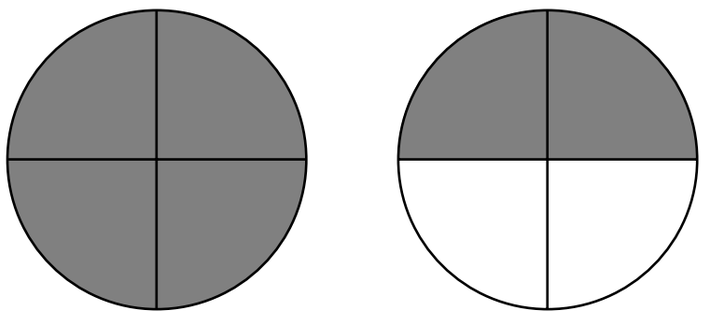
Answer: $\frac{6}{9}$

2. What mixed number do the shaded parts of the circle graphs represent?

Think!

- One circle graph is completely shaded, thus it equals 1 whole circle.
- The second circle graph has two of the four parts shaded, thus it equals $\frac{2}{4}$ or $\frac{1}{2}$.
- The mixed number representing the circle graphs is the sum of $1 + \frac{1}{2}$.

Answer: $1\frac{1}{2}$



5

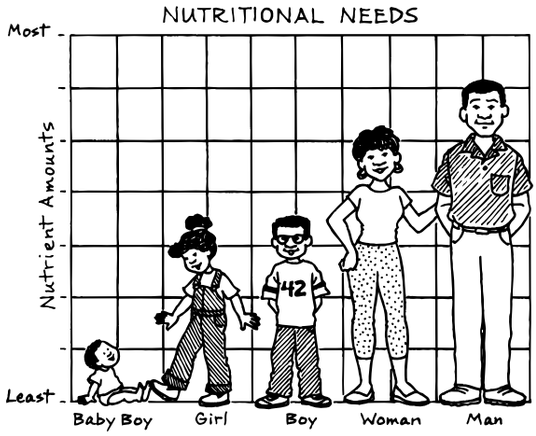
Name

ACTIVITY (continued)

On Your Own

1. a. What fractional part of the family in the drawing are males? _____

b. What fractional part of the family are adults? _____

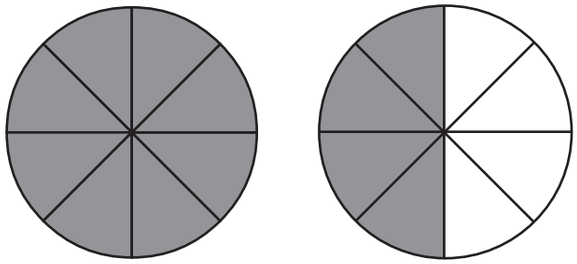


2. a. What fractional part of the animals can fly? _____

b. What fractional part of the animals are on lily pads? _____

c. What fractional part of the animals have no legs? _____

3. What mixed number do the shaded parts of the circle graphs represent? _____



Name

INVESTIGATION 5

Candy Parts

PURPOSE

To determine the fractional parts of a bag of candy.

Material

plastic bag of candy

Procedure

1. Without opening the bag, look at the candy through the bag.
2. Record the different colors of candy in the first column of the Candy Fraction Data table.
3. Count the number of each candy color and record it in the data table.
4. Determine the total number of candy pieces in the bag by adding the numbers of each candy color.
5. Use the total number of candy pieces and the number of each color of candy to write the fractional part for each color in the data table.

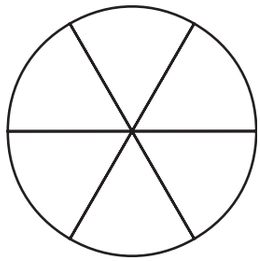
CANDY FRACTION DATA		
Candy Color	**Number of Pieces**	**Fractional Part**

6. Use the completed data table to answer these questions:
   - **a.** Which color makes up the largest fractional part?_____
   - **b.** Which color makes up the least fractional part? _____
   - **c.** How many colors have an equal fractional part? _____

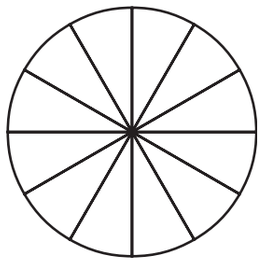
7. Select the appropriate circle graph A, B, or C and color in the sections to represent the fraction of each candy color in the candy bag.

Circle Graphs

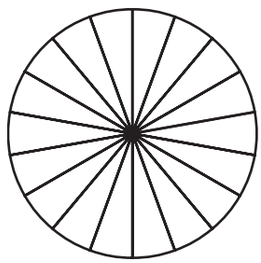
A



B



C



Results

Results will vary.

6 Percents

TEACHING TIPS

Benchmarks

By the end of grade 5, students should be able to

- Write percents as fractions and decimals, and fractions and decimals as percents.

By the end of grade 8, students should be able to

- Represent percents with fractions and decimals.

In this chapter, students are expected to

- Convert numbers among percents, decimals, and fractions.

Preparing the Materials

Activity 1: Connecting Percents, Fractions, and Decimals

- Make a copy of the Connecting Percents, Fractions, and Decimals activity sheet for each student.

Activity 2: Percent of a Number

- Make a copy of the Percent of a Number activity sheet for each student.

Presenting the Math Concepts

1. Introduce the new term:

   percent (%) Per hundred. A way to compare a number to 100.

2. Explore the new term:
   - Percent is a way to compare a number to 100.
   - The percent symbol, %, means hundredths. So 60% is read as "sixty percent" and means $^{60}/_{100}$ or sixty hundredths.
   - Percent numbers can be expressed as a decimal number by first expressing the percent as a fraction, then dividing the numerator by the denominator, which is 100. For example, 60% = $^{60}/_{100}$ = 0.6.
   - Percent used to be written as a fraction with the numerator over 100. Over time, the bar of the fraction and the 100 were combined to become the % symbol used today.
   - Information on a circle graph is usually shown as a percentage or fraction. The larger the area of the graph used, the greater the percentage represented.
   - The whole circle of a circle graph represents 100% or the total amount.
   - A circle graph is often called a pie chart because when divided into sections each section looks like a piece of pie.

EXTENSION

Bags of different-colored candy pieces can be used to study the percent of different colors of candy in each bag. Have students count the number of candies of each color in their bags and use that number and the total number of candies in the bag to come up with percentages of each color in the bag. You may wish to have extra candy that is not part of the investigation for eating.

ANSWERS

Activity 1: Connecting Percents, Fractions, and Decimals

1. **a.** 35%
   b. 0.2%
2. **a.** 0.68
   b. 0.72
3. **a.** 50%
   b. 50%
   c. 75%
   d. 25%

Activity 2: Percent of a Number

1. **a.** 24
   b. 4.5
   c. 2.5
2. **a.** 7.5
   b. 100
   c. 225
3. **a.** 3
   b. 12
   c. 9
   d. 6

Name

ACTIVITY 1

6

Connecting Percents, Fractions, and Decimals

Percent (%) means per hundred. Percent is a way to compare a number to 100. A fraction is a number used to express part of a whole. A decimal is a number that uses a decimal point to show tenths, hundredths, thousandths, and so on.

To change a percent to a decimal, divide the number by 100. To change a decimal to a percent, multiply the number by 100. To change a fraction to a percent, first divide the numerator by the denominator, then multiply by 100.

Practice Problems

1. Change 45% to a decimal.

Think!

- To change a percent to a decimal, divide the number by 100.
  $45 \div 100 = ?$

Answer: 0.45

2. Change 0.25 to a percent.

Think!

- To change a decimal to a percent, multiply the number by 100.
  $0.25 \times 100 = ?$

Answer: 25%

3. Change $\frac{1}{5}$ to a percent.

Think!

- To change a fraction to a percent, first divide the numerator by the denominator.
  $1 \div 5 = 0.20$
- Then multiply by 100.
  $0.20 \times 100 = ?$

Answer: 20%

On Your Own

1. Change these decimals to percents:

a. 0.35 _____ **b.** 0.002 _____

2. Change these percents to decimals:

a. 68% _____ **b.** 72% _____

3. Change these fractions to percents:

a. $\frac{1}{2}$ _____ **c.** $\frac{3}{4}$ _____

b. $\frac{2}{4}$ _____ **d.** $\frac{25}{100}$ _____

Name

ACTIVITY 2

Percent of a Number

To find the percent of any number, you can use one of two methods:

Method 1: Use a Fraction

- Express the percent as a fraction per 100.
- Multiply the fraction times the number.
- Divide the numerator by the denominator.

Method 2: Use a Decimal

- Express the percent as a decimal.
- Multiply the decimal times the number.

Practice Problem

Find 12% of 20.

Think!

Method 1: Use a Fraction

- Express the percent as a fraction per 100: $12\% = {}^{12}/_{100}$.
- Multiply the fraction times the number: ${}^{12}/_{100} \times 20 = {}^{240}/_{100}$.
- Divide the numerator by the denominator: $240 \div 100 = ?$

Answer: 2.4

Method 2: Use a Decimal

- Express the percent as a decimal: $12\% = {}^{12}/_{100} = 0.12$.
- Multiply the decimal times the number: $0.12 \times 20 = ?$

Answer: 2.4

On Your Own

1. Find the indicated percent of each number using Method 1.

a. 60% of 40 _____

b. 30% of 15 _____

c. 5% of 50 _____

Name

ACTIVITY 2 (continued)

6

2. Find the indicated percent of each number using Method 2.

a. 15% of 50 _____

b. 50% of 200 _____

c. 75% of 300 _____

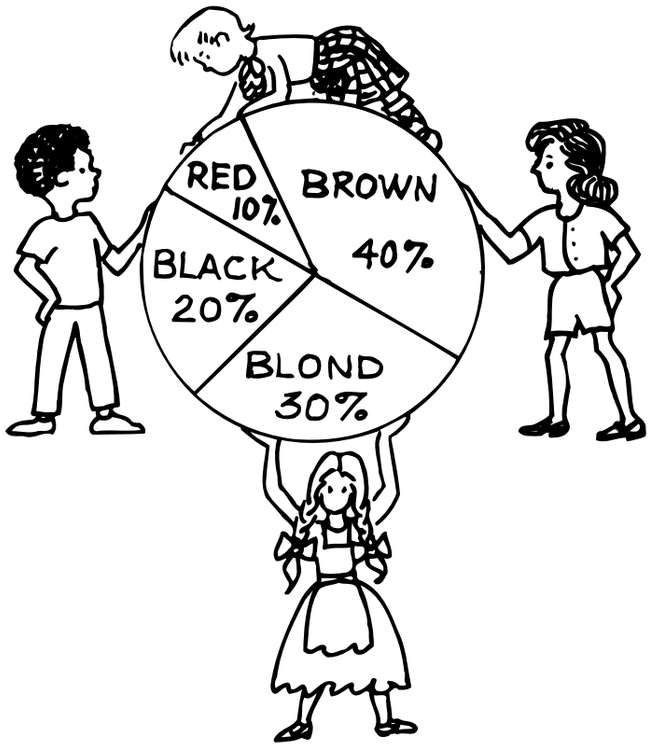
3. The circle graph below shows the percentage of different hair colors in a class of 30 students. Use the graph to determine how many students have each hair color.

a. Red _____

b. Brown _____

c. Blond _____

d. Black _____



7 Ratios

TEACHING TIPS

Benchmarks

By the end of grade 5, students should be able to
- Use ratios to describe relationships mathematically.

By the end of grade 8, students should be able to
- Represent ratios with models.

In this chapter, students are expected to
- Write ratios in three different ways.

Preparing the Materials

Investigation 1: Red to White
- Make a copy of the Red to White investigation sheet for each student.
- Prepare 1 bag of beans for each pair of students. Place 10 red beans and 10 white beans in each bag.

Investigation 2: Color Ratio
- Make a copy of the Color Ratio investigation sheet for each student.
- Prepare colored water by adding 10 drops of food coloring to 1 cup (250 mL) of tap water.
- Provide a bag of blue and a bag of yellow water for each pair of students. Place about ¼ cup (63 mL) of colored water in each quart-size (liter-size) resealable bag.

Presenting the Math Concepts

1. Introduce the new term:

ratio A pair of numbers used to compare quantities.

2. Explore the new term:
- A ratio can be expressed in three different ways: (1) in words, (2) with a colon, or (3) as a fraction. For example, in comparing the number of your eyes to the number of your toes, the ratio could look like this:

Comparison	In words, using "to"	With a colon	As a fraction
Eyes to toes	2 to 10	2:10	$\frac{2}{10}$

- A ratio is generally written as a simplified fraction; for the comparison of eyes to toes, the ratio would be 1 to 5, 1:5, or $\frac{1}{5}$.
- A ratio is not written as a mixed number (a number containing a whole number and a fraction).
- Order is important when writing a ratio. For the eyes-to-toes comparison, the ratio is 1 to 5, not 5 to 1.
- A ratio can compare a part to a part (P/P), a part to a whole (P/W), or a whole to a part (W/P).

EXTENSION

Redo the Color Ratio investigation, this time adding a bag of red water for each group. Allow students to create their own Ratio Data tables. Ratios of two colors as well as ratios of three colors can be used.

Name

INVESTIGATION 1

7

Red to White

PURPOSE

To determine the ratio of objects.

Materials

1 bag of 10 red and 10 white beans
pencil
red crayon

Procedure

1. Without looking, scoop out a small handful of beans from the bag.
2. Make a drawing of the beans on the Bean Ratio Data table. Use the crayon to color the red beans.

BEAN RATIO DATA			
Drawing			
Comparison of Beans	**Simplified Ratio**		
	In words, using "to"	**With a colon**	**As a fraction**
red to white			
white to red			
red to whole			
white to whole			
whole to red			
whole to white			

3. Write the ratio of the beans in the data table in the ways indicated. "Red" is for all the red beans, "white" is for all of the white beans, and "whole" is for total number of beans scooped out of the bag.

Results

Results will vary depending on how many beans you pick. For example, if 3 red beans and 4 white beans are scooped out of the bag, the table would look like this:

BEAN RATIO DATA			
Comparison of Beans	**Simplified Ratio**		
	In words, using "to"	**With a colon**	**As a fraction**
red to white	3 to 4	3:4	$\frac{3}{4}$
white to red	4 to 3	4:3	$\frac{4}{3}$
red to whole	3 to 7	3:7	$\frac{3}{7}$
white to whole	4 to 7	4:7	$\frac{4}{7}$
whole to red	7 to 3	7:3	$\frac{7}{3}$
whole to white	7 to 4	7:4	$\frac{7}{4}$

Why?

A **ratio** is a pair of numbers used to compare quantities. It is expressed in words using the word "to," with a colon, or as a fraction. A ratio compares a part to a part (P/P), a part to a whole (P/W), or a whole to a part (W/P). In this activity, the parts are red beans and white beans. The whole is the sum of red and white beans.

7

Name

INVESTIGATION 2

Color Ratio

PURPOSE

To determine the colors produced in mixtures made from different ratios of colors.

Materials

marker
3 sandwich-size resealable plastic bags
two 1-teaspoon (5-mL) measuring spoons
2 bags of colored water, yellow and blue
helper

Procedure

1. Use the marker to label the empty plastic bags A, B, and C.
2. Look at the Color Ratio Data table below. Write the ratio for each mixture on the label of the bag indicated.
3. Prepare each mixture by asking your helper to hold open the bags of colored water one at a time while you dip out a measured amount. For example, the mixture in bag A will be a combination of 1 teaspoon of blue water and 1 teaspoon of yellow water. (Use a different measuring spoon for each color of water.)
4. Seal bag A, then gently shake it.
5. Repeat steps 3 and 4 to prepare mixtures B and C.
6. Hold the three bags up to a light and compare their colors. Write a description of the colors in bags A, B, and C in the Resulting Color column of the data table.

Results

The combination of different ratios of blue and yellow water produced various shades of green.

COLOR RATIO DATA

Mixture	Ratio	Combination	Resulting Color
A blue to yellow	1:1	blue—1 spoon yellow—1 spoon	
B blue to yellow	2:1	blue—2 spoons yellow—1 spoon	
C yellow to blue	2:1	yellow—2 spoons blue—1 spoon	

Why?

Colored materials have *colorants* (chemical substances that give color to materials). Colorants that dissolve in liquids are called *dyes.* When two different colorants are mixed, a third color is produced. For example, when blue water is mixed with yellow water, the resulting mixture appears green. The ratio of the parts produces different shades of green. The more yellow in the mixture, the lighter the green shade.