# ANSWERS TO ODD NUMBERED PROBLEMS

## Section 1.1

5 Slope: $-12/7$
Vertical intercept: $2/7$

7 Slope: 2
Vertical intercept: $-2/3$

9 $y = (1/2)x + 2$

11 $y = -\frac{1}{5}x + \frac{7}{5}$

13 Parallel: $y = m(x - a) + b$
Perpendicular:
$y = (-1/m)(x - a) + b$

15 (a) (V)
(b) (VI)
(c) (I)
(d) (IV)
(e) (III)
(f) (II)

17 Domain: $-2 \le x \le 2$
Range: $-2 \le y \le 2$

19 Domain: all $x$
Range: $y \ge 2$

21 $t \ge 4$ or $t \le -4$
$t = \pm 5$

23 $S = kh^2$

25 $N = k/l^2$

31 (a) \$0.025/cubic foot
(b) $c = 65 + 0.025w$
$c$ = cost of water
$w$ = cubic feet of water
(c) 2600 cubic feet

33 (a) $\Delta w/\Delta h$ constant
(b) $w = 5h - 174$; 5 lbs/in
(c) $h = 0.2w + 34.8$; 0.2 in/lb

35 (a) (i) $q = 320 - (2/5)p$
(ii) $p = 800 - (5/2)q$

37 (a) $R = k(350 - H)$
$(k \ge 0)$

## Section 1.2

1 Concave up

3 Neither

5 5; 7%

7 3.2; 3% (continuous)

9 (a) $P = 1000 + 50t$
(b) $P = 1000(1.05)^t$

11 (a) $x$-interval: $D$ to $E$, $H$ to $I$
(b) $x$-interval: $A$ to $B$, $E$ to $F$
(c) $x$-interval: $C$ to $D$, $G$ to $H$
(d) $x$-interval: $B$ to $C$, $F$ to $G$

15 (a) $P = 6(1.013)^t$
(b) 7.87 billion
(c) 53.7 years

17 (a) \$3486.78
(b) Approx 11 years (or 21 years from initial investment)

19 $y = 4(2^{-x})$

21 $y = 4(1 - 2^{-x})$

23 (a) 1.05
(b) 5%

25 $P = 2(0.61)^t$; decay

27 $P = 7(0.0432)^t$; decay

29 $d = 670(1.096)^{h/1000}$

31 25.5%

33 (a) $k(t)$
(b) $h(t)$
(c) $g(t)$

35 (a) $Q = Q_0\left(\frac{1}{2}\right)^{(t/1620)}$
(b) 80.7%

37 2.3 years

39 (a) $P = 2.5t + 50$
(b) $P = 50(1.035)^t$
(d) Exponential

## Section 1.3

1 (a) $h^2 + 6h + 11$
(b) 11
(c) $h^2 + 6h$

3 (a) $f(n) + g(n) = 3n^2 + n - 1$
(b) $f(n)g(n) = 3n^3 + 3n^2 - 2n - 2$
(c) $n \ne -1$
(d) $f(g(n)) = 3n^2 + 6n + 1$
(e) $g(f(n)) = 3n^2 - 1$

5 $2z + 1$

7 $2zh - h^2$

11 Length of column of mercury when temperature is 75°F

13 Not invertible

15 (a) $-1$

17 $y = (x - 2)^3 - 1$.

19 Neither

21 Not invertible

23 Invertible

25 $f(g(1)) \approx 0.4$

27 $f(f(1)) \approx -0.9$

31 $f(x) = x^3$
$g(x) = x + 1$

33 $n = 2y^2 - 5y + 3$

35 (a) $k = f(p) = (1/2.2)p$
(b) $p = 2.2k$; Weight in pounds given mass in kilograms

## Section 1.4

1 $A$: $e^x$
$B$: $x^2$
$C$: $x^{1/2}$
$D$: $\ln x$

3 $(\log 2)/(\log 17) \approx 0.24$

5 $(\log(2/5))/(\log 1.04) \approx -23.4$

7 $(\log(4/7))/(\log(5/3)) \approx -1.1$

9 $-3.26$

11 6.212

13 0.26

15 1

17 $(\log a)/(\log b)$

19 $(\log Q - \log Q_0)/(n \log a)$

21 $\ln(a/b)$

23 $1/2$

25 $5A^2$

27 $-1 + \ln A + \ln B$

29 $P = 15e^{0.41t}$

31 $P = 174e^{-0.1054t}$

33 $p^{-1}(t) \approx 58.708 \log t$

35 $f^{-1}(t) = e^{t-1}$

37 (a) 10 mg
(b) 18%
(c) 3.04 mg
(d) 11.60 hours

39 About 14.21 years

41 16 kg

43 $B = 78e^{0.555t}$
55.5%

45 1990

47 (a) 81%
(b) 32.9 hours

49 96.34 years

## Section 1.5

1 Negative
0
Undefined

3 Positive
Positive
Positive

5 Positive
Positive
Positive

7 Positive
Negative
Negative

9 Negative
Positive
Negative

11 0.588

13 (a) 1
(b) $2\pi/3$

15 $f(x) = 2\sin(x/4)$

17 $f(x) = 2 - \sin x$

19 $f(x) = 2\sin(x/4) + 2$

21 $f(x) = (\sin x) + 2$

23 $f(x) = \sin(2(\pi/5)x)$

25 $f(x) = 2\cos(5x)$

27 $(\sin^{-1}(2/5))/3 \approx 0.1372$

29 20.94 to 52.36 rad/sec

31 If $f(x) = \sin x$ and $g(x) = x^2$ then
$\sin x^2 = f(g(x))$
$\sin^2 x = g(f(x))$
$\sin(\sin x) = f(f(x))$

33 (a) $(2.3, -4.4)$
(b) Once around circle and back

35 (a) Maximum displacement from equilibrium
(b) $\omega$

37 US: 156 volts max, 60 cycles/sec
Eur: 339 volts max, 50 cycles/sec

39 (b) $P = 800 - 100\cos(\pi t/6)$

41 $hw + h^2/\tan\theta$

## Section 1.6

1 $10 \cdot 2^x$

3 As $x \to \infty$, $y \to \infty$.
As $x \to -\infty$, $y \to -\infty$.

5 (I) Degree $\ge 3$, negative
(II) Degree $\ge 4$, positive
(III) Degree $\ge 4$, negative
(IV) Degree $\ge 5$, negative
(V) Degree $\ge 5$, positive

7 $y = \frac{1}{5}(x+2)(x-1)(x-5)$

9 (a) $-\infty, -\infty$
(b) $3/2, 3/2$
(c) $0, +\infty$

11 $f(x) = kx(x+3)(x-4)$
$(k < 0)$

13 $f(x) = k(x+2)(x-2)^2(x-5)$
$(k < 0)$

15 (a) $1.3\ \text{m}^2$
(b) $86.8$ kg
(c) $h = 112.6s^{4/3}$

17 (a) (i) $V = 3\pi r^2$
(ii) $V = \pi r^2 h$

19 $5\pi x^2$

21 44.25 ft and 708 ft

23 (a) 0
(b) $t = 2v_0/g$
(c) $t = v_0/g$
(d) $(v_0)^2/(2g)$

25 (a) (i) $1 = a + b + c$
(ii) $b = -2a$ and $c = 1 + a$
(iii) $c = 6$
(b) $y = 5x^2 - 10x + 6$

27 $g$ is an exponential;
$f$ is a cubic;
$k$ is a quadratic

29 1, 2, 3, 4, or 5 roots

31 $g(x) = 2x^2$
$h(x) = x^2 + k$
$(k > 0)$

33 $-10^5 \le x \le 10^5,\ -10^{15} \le y \le 10^{15}$

35 (a) $a(v) = \frac{1}{m}(F_E - kv^2)$
$(k > 0)$

## Section 1.7

1 Yes

3 Yes

5 Yes

7 No

9 No

11 $f(t) = \begin{cases} 6, & 0 < t \le 7 \\ 12, & 7 < t \end{cases}$
Not continuous on any interval around 7

13 $k = 6$

15 20

19 $Q = \begin{cases} 1.2t & 0 \le t \le 0.5 \\ 0.6e^{0.001}e^{-.002t} & 0.5 < t \end{cases}$

## Chapter 1 Review

1 (a) $[0, 7]$
(b) $[-2, 5]$
(c) 5
(d) $(1, 7)$
(e) Concave up
(f) 1
(g) No

5 Amplitude: 2
Period: $2\pi/5$

7 $y = e^{0.4621x}$

9 $y = -k(x^2 + 5x)$
$(k > 0)$

11 $z = 1 - \cos\theta$

13 $x = k(y^2 - 4y)$
$(k > 0)$

15 $y = -(x+5)(x+1)(x-3)^2$

17 Simplest is $y = 1 - e^{-x}$

19 Not continuous

21 Not continuous

23 $f(x) = x^3$
$g(x) = \ln x$

25 $Q(m) = T + L + Pm$
$T$ = fuel for take-off
$L$ = fuel for landing
$P$ = fuel per mile in the air
$m$ = length of the trip (miles)

27 13,500 bacteria

29 2010

31 21,153%

33 (a) $S = 2\pi r^2 + 2V/r$
(b) $S \to \infty$ as $r \to \infty$

35 Depth $= d = 7 + 1.5\sin(\pi t/3)$

37 (a) $f$ is invertible
(b) $f^{-1}(400) \approx 1979$ is the year in which 400 million motor vehicles were registered in the world.

39 (a) $2\pi$

41 (a) $r(p) = kp(A - p)$
$(k > 0)$
(b) $p = A/2$

43 (a) $f(x) = (x-a)(x+a)(x+b)(x-c)$

45 (a) $f(x) \to \infty$ as $x \to \infty$
$f(x) \to 16$ as $x \to -\infty$
(b) $(e^x+1)(e^{2x}-2)(e^x-2)(e^{2x}+2e^x+4)$
Two zeros
(c) $(\ln 2)/2, \ln 2$
One twice other

47 (a) $p(x) = x^2 + 3x + 9$
$r(x) = -3$, $q(x) = x - 3$
(b) $f(x) \approx -3/(x-3)$ for $x$ near 3
(c) $f(x) \approx x^2 + 3x + 9$ as $x \to \pm\infty$

49 (a) $1 - 8\cos^2 x + 8\cos^4 x$
(b) $1 - 8\sin^2 x + 8\sin^4 x$

## Ch. 1 Understanding

1 False

3 True

5 True

7 True

9 False

11 False

13 True

15 False

17 False

19 True

21 True

23 True

25 False

27 Possible answer
$f(x) = \begin{cases} x & x \le 3 \\ 2x & x > 3 \end{cases}$

29 Possible answer
$f(x) = 1/\left((x-1)(x-2)\cdots(x-17)\right)$

31 $f(x) = 1.5x$, $g(x) = 1.5x + 3$

33 Impossible

35 Impossible

37 False; $f(x) = \log x$ on $[1, 2]$

39 True; $f(x) = (0.5)^x$

41 False, $f(x) = \begin{cases} 1 & x \le 3 \\ 2 & x > 3 \end{cases}$

43 (a) Follows
(b) Does not follow (although true)
(c) Follows
(d) Does not follow

## Section 2.1

1 $265/3$ km/hr

5 27

7 $1.9\ldots$

9 2

11 $0.01745\ldots$

17 15.47, 57.65, 135.90, 146.35, 158.55 people/min

## Section 2.2

1 (a) 3
(b) 7
(c) Does not exist
(d) 8

3 (b) 1
(d) $-0.0033 < x < 0.0033$, $0.99 < y < 1.01$

5 (b) 0
(d) $-0.005 < x < 0.005$, $-0.01 < y < 0.01$

7 (b) 2
(d) $-0.0865 < x < 0.0865$, $1.99 < y < 2.01$

9 (b) 1
(d) $-0.0198 < x < 0.0198$, $0.99 < y < 1.01$

11 (b) 4
(d) $1.99 < x < 2.01$, $3.99 < y < 4.01$

13 (b) 0
(d) $1.55 < x < 1.59$, $-0.01 < y < 0.01$

15 4

17 $-2$

19 $-1/16$

21 $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = \lim_{x\to 2} f(x) = 0$

23 0.05

29 $2.71828\ldots$

31 $1/3$

33 $2/3$

35 $3/2$

37 4 or $-4$

39 4

41 Any $k$

43 0.46, 0.21, 0.09

45 (a) $x = 1/(n\pi)$, $n = 1, 2, 3, \ldots$
(b) $x = 2/(n\pi)$, $n = 1, 5, 9, \ldots$
(c) $x = 2/(n\pi)$, $n = 3, 7, 11, \ldots$

## Section 2.3

1 12

3 (b) 8.69
(c) 7.7

5 $f'(1) \approx 0.43427$

7 $f'(d) = 0$, $f'(b) = 0.5$, $f'(c) = 2$, $f'(a) = -0.5$, $f'(e) = -2$

9 (a) $x = 1$ and $x = 3$
(b) $f(5)$
(c) $f'(1)$

11 12

13 3

15 $-1/4$

17 $y = 12x + 16$

19 $y = -2x + 3$

21 $(4, 25)$; $(4.2, 25.3)$; $(3.9, 24.85)$

25 From smallest to largest:
$0, f'(3), f(3) - f(2), f'(2)$

27 (a) $(f(b) - f(a))/(b - a)$
(b) Slopes same
(c) Yes

29 (a) $f'(0) \approx 0.01745$

31 $f'(0) \approx -1$
$f'(1) \approx 3.5$

33 16.0 million people/year
16.4 million people/year

## Section 2.4

11 (a) 3
(b) Positive: $0 < x < 4$
Negative: $4 < x < 12$

13 $-1/x^2$

15 $4x$

21 $f'(x)$ positive:
$4 \le x \le 8$
$f'(x)$ negative:
$0 \le x \le 3$
$f'(x)$ greatest:
at $x \approx 8$

35 (a) $x_3$
(b) $x_4$
(c) $x_5$
(d) $x_3$

37 (a) Graph II
(b) Graph I
(c) Graph III

39 (a) Periodic: period 1 year
(b) Max of 4500 on July $1^{\text{st}}$
Min of 3500 on Jan $1^{\text{st}}$
(c) Growing fastest:
around April $1^{\text{st}}$
Decreasing fastest:
around Oct $1^{\text{st}}$
(d) $\approx 0$

## Section 2.5

1 (a) Negative
(b) Degrees/min

3 (a) Costs \$350 for 200 gallons
(b) Costs \$1.40 for $201^{\text{st}}$ gallon

5 Dollars/year

7 Dollars/percent; positive

9 Feet/mile; negative

11 (b) Pounds/(Calories/day)

13 1.25 billion people; growing at 0.0174 billion people per year

17 mpg/mph

19 (a) $f'(a)$ is always positive
(c) $f'(100) = 2$: more
$f'(100) = 0.5$: less

21 (a) Liters per centimeter
(b) About 0.042 liters per centimeter
(c) Cannot expand much more

## Section 2.6

1 (a) Negative
(b) Negative
(c) Positive

3 $B$

9 $f'(x) = 0$
$f''(x) = 0$

11 $f'(x) < 0$
$f''(x) > 0$

13 $f'(x) < 0$
$f''(x) < 0$

15 $0 \le t \le 1$:
acceleration $= 30$ ft/sec$^2$
$1 \le t \le 2$:
acceleration $= 22$ ft/sec$^2$

17 (a) $dP/dt > 0$, $d^2P/dt^2 > 0$

(b) $dP/dt < 0$, $d^2P/dt^2 > 0$
(but $dP/dt$ is close to zero.)

19 A positive second derivative indicates a successful campaign.
A negative second derivative indicates an unsuccessful campaign.

23 (a) B and E
(b) A and D

## Section 2.7

1 (a) $x = 1$
(b) $x = 1, 2, 3$

3 (a) No
(b) Yes, $x = 0$

5 Yes

7 Yes

9 Yes

11 (a) Yes
(b) Not at $t = 0$

13 (a) Yes
(b) No

15 (a) Yes
(b) No

## Chapter 2 Review

7 $10x + 1$

9 (b) 0
(d) $-0.015 < x < 0.015$, $-0.01 < y < 0.01$

11 (b) 0
(d) $1.570 < x < 1.5715$, $-0.01 < y < 0.01$

13 $2a$

15 $-2/a^3$

17 $-1/(2(\sqrt{a})^3)$

19 $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0} f(x) = 1$

21 (a) $3, -1$
(b) $-3, 1$

23 357

25 (a) $f(7) = 3$
(b) $f'(7) = 4$

31 (a) $f'(0.6) \approx 0.5$
$f'(0.5) \approx 2$
(b) $f''(0.6) \approx -15$
(c) Maximum: near $x = 0.8$
minimum: near $x = 0.3$

33 0.45, 0.0447, 0.00447

35 (a) At $(0, \sqrt{19})$:
slope $= 0$
At $(\sqrt{19}, 0)$:
slope is undefined.
(b) slope $\approx \frac{1}{2}$
(c) At $(-2, \sqrt{15})$:
slope $\approx \frac{1}{2}$
At $(-2, -\sqrt{15})$:
slope $\approx -\frac{1}{2}$
At $(2, \sqrt{15})$:
slope $\approx -\frac{1}{2}$.

37 (a) Period 12 months
(b) Max of 4500 on June $1^{\text{st}}$
(c) Min of 3500 on Feb $1^{\text{st}}$
(d) Growing fastest:
April $1^{\text{st}}$
Decreasing fastest:
July 15 and Dec 15
(e) About 400 deer/month

39 (a) Concave down
(b) $120^\circ < T < 140^\circ$
(c) $135^\circ < T < 140^\circ$
(d) $45 < t < 50$

41 (a) $f'(0) = 1.00000$
$f'(0.3) = 1.04534$
$f'(0.7) = 1.25521$
$f'(1) = 1.54314$
(b) They are about the same.

43 (a) $2\cos^2 x - 2\sin^2 x$
Answers may vary
(b) $(d/dx)\sin(2x) = 2\cos(2x)$

45 (a) $1/x$
(b) Graphs same shape, shifted vertically

47 (a) $\cos x, -\sin x, \cos^2 x - \sin^2 x$
Form of answers may vary
(b) No

## Ch. 2 Understanding

1 False

3 True

5 True

7 False

9 True

11 False

13 True

15 True

17 True

19 True

21 True; $f(x) = |x - 3|$

23 False; $f(x) = |x|$

25 False

27 True

29 True

31 False

33 True

35 False

37 False

39 (a) Not a counterexample
(b) Counterexample
(c) Not a counterexample
(d) Not a counterexample

## Section 3.1

3 $11x^{10}$

5 $11x^{-12}$

7 $-12x^{-13}$
9 $3x^{-1/4}/4$
11 $-4x^{-5}$
13 $ex^{e-1}$
15 $6t-4$
17 $2t-k/t^2$
19 $6/w^4+3/(2\sqrt{w})$
21 $15t^4-\frac{5}{2}t^{-1/2}-7t^{-2}$
23 $2z-\frac{1}{2}z^{-2}$
25 $(z^2-1)/3z^2$
27 $1/(2\sqrt{\theta})+1/(2\theta^{3/2})$
29 $(8\pi rb)/3$
31 $2ax+b$
33 $-(5x^4+2)/2$
35 $5z^4+20z^3-1$
37 Do not apply.
39 $6x$
(power rule and sum rule)
41 $-2/3z^3$
(power rule and sum rule)
43 $x\geq 1$ or $x\leq -2$
45 $x>1$
47 $r=3\sqrt{2}$
49 $y=2x-1$
51 $y=2x$ and $y=-6x$
53 $n=4$, $a=3/32$
55 Height $=625$ cm,
Changing (eroding) at $-30$ cm/year
57 (a) 15.2 m/sec
(b) 5.4 m/sec
(c) $-9.8$ m/sec$^2$
(d) 34.9 m
(e) 5.2 sec
59 (a) $dT/dl=\pi/\sqrt{gl}$
(b) Positive, so the period increases as the length increases
61 $V(r)=4\pi r^3/3$
$\frac{dV}{dr}=4\pi r^2=$ surface area of a sphere

## Section 3.2

1 $2e^x+2x$
3 $(\ln 5)5^x$
5 $10x+(\ln 2)2^x$
7 $4(\ln 10)10^x-3x^2$
9 $((\ln 3)3^x)/3-(33x^{-3/2})/2$
11 $e^{1+x}$
13 $e^{\theta-1}$
15 $(\ln 4)^2 4^x$
17 $3x^2+3^x\ln 3$
19 $\pi^x\ln\pi$
21 $(\ln\pi)\pi^x$
23 $a^x\ln a+ax^{a-1}$
25 $(2\ln 3)z+(\ln 4)e^z$
27 $2x+(\ln 2)2^x$
29 Our rules do not apply here.
31 $e^{x+5}$
33 The methods of the section do not apply here.
35 The methods of the section do not apply here.
37 $\approx 7.95$ ¢/year
39 $\approx 22.5(1.35)^t$
41 (a) $P'(t)=kP(t)$
43 $c=-1/\ln 2$
47 $e$

## Section 3.3

1 $5x^4+10x$
3 $e^x(x+1)$
5 $2^x/(2\sqrt{x})+\sqrt{x}(\ln 2)2^x$
7 $4s^3-1$
9 $(t^3-4t^2-14t+1)e^t$
11 $(50x-25x^2)/(e^x)$
13 $6/(5r+2)^2$
15 $1/(5t+2)^2$
17 $(t^2+2t+2)/(t+1)^2$
19 $2y-6$, $y\neq 0$
21 $\sqrt{z}(3-z^{-2})/2$
23 $2r(r+1)/(2r+1)^2$
25 $17e^x(1-\ln 2)/2^x$
27 $\frac{(-4x^2-8x-1)}{(2+3x+4x^2)^2}$
29 $(3t^2+5)(t^2-7t+2)+(t^3+5t)(2t-7)$
31 $x<2$
33 $y=7x-5$
35 $f'(x)=2e^{2x}$
37 $\frac{d}{dx}e^{4x}=4e^{4x}$
41 (a) 3
(b) 14
(c) 13/8
43 $f(x)=x^{10}e^x$
45 $r_2^2/(r_1+r_2)^2$
47 (a) $g(v)=\frac{1}{f(v)}$
$g(80)=20\frac{\text{km}}{\text{liter}}$
$g'(80)=-\frac{1}{5}\frac{\text{km}}{\text{liter}}$ for each $1\frac{\text{km}}{\text{hr}}$ increase in speed.
(b) $h(v)=v\cdot f(v)$
$h(80)=4\frac{\text{liters}}{\text{hr}}$
$h'(80)=0.09\frac{\text{liters}}{\text{hr}}$ for each $1\frac{\text{km}}{\text{hr}}$ inc. in speed.
49 (a) $f'(x)=$
$(x-2)+(x-1)$
(b) $f'(x)=$
$(x-2)(x-3)+$
$(x-1)(x-3)+$
$(x-1)(x-2)$
(c) $f'(x)=$
$(x-2)(x-3)(x-4)+$
$(x-1)(x-3)(x-4)+$
$(x-1)(x-2)(x-4)+$
$(x-1)(x-2)(x-3)$
51 (a) $(FGH)'=$
$F'GH+FG'H+FGH'$
(c) $f_1'f_2f_3\cdots f_n+$
$f_1f_2'f_3\cdots f_n+\cdots+$
$f_1\cdots f_{n-1}f_n'$

## Section 3.4

1 $99(x+1)^{98}$
3 $200t(t^2+1)^{99}$
5 $50(\sqrt{t}+1)^{99}/\sqrt{t}$
7 $5(w^4-2w)^4(4w^3-2)$
9 $\pi e^{\pi x}$
11 $(\ln\pi)\pi^{(x+2)}$
13 $4(x^3+e^x)^3(3x^2+e^x)$
15 $(2t-ct^2)e^{-ct}$
17 $6(1+3t)e^{(1+3t)^2}$
19 $5\cdot\ln 2\cdot 2^{5t-3}$
21 $\frac{3}{2}e^{\frac{3}{2}w}$
23 $3s^2/2\sqrt{s^3+1}$
25 $e^{-t^2}(1-2t^2)$
27 $e^{-z}/2\sqrt{z}-\sqrt{z}e^{-z}$
29 $e^{5-2t}(1-2t)$
31 $\frac{\sqrt{x+3}(x^2+6x-9)}{[2\sqrt{x^2+9}(x+3)^2]}$
33 $-(3e^{3x}+2x)/(e^{3x}+x^2)^2$
35 $(\ln 2)(3e^{3x})2^{e^{3x}}$
37 $e^{-\theta}/(1+e^{-\theta})^2$
39 $2we^{w^2}(5w^2+8)$
41 $-(\ln 10)(10^{\frac{5}{2}-\frac{y}{2}})/2$
43 $2ye^{[e^{(y^2)}+y^2]}$
45 $abe^{bt}$
47 $ae^{-bx}-abxe^{-bx}$
49 $y=3x-5$
51 $-\frac{1}{\sqrt{2}}<x<\frac{1}{\sqrt{2}}$
53 (a) $H(4)=1$
(b) $H'(4)=30$
(c) $H(4)=4$
(d) $H'(4)=56$
(e) $H'(4)=-1$
55 (a) $\pi\sqrt{2}$
(b) $7e$
(c) $\pi e$
57 Yes
59 \$596.73/yr.
61 6 billion people,
0.078 billion people per year,
6.833 billion people,
0.089 billion people per year
63 (a) $dH/dt=-60e^{-2t}$
(b) $dH/dt<0$
(c) At $t=0$
65 $400\pi$ cm$^2$/sec
67 (a) For $t<0$, $I=0$
For $t>0$
$I=-\frac{Q_0}{RC}e^{-t/RC}$
(b) No
(c) No
69 56 cm/sec$^2$

## Section 3.5

3 $\cos^2\theta-\sin^2\theta=\cos 2\theta$
5 $3\cos(3x)$
7 $3\pi\sin(\pi x)$
9 $2x\cos x-x^2\sin x$
11 $-\sin xe^{\cos x}$
13 $e^{\cos\theta}-\theta(\sin\theta)e^{\cos\theta}$
15 $\cos(\tan\theta)/\cos^2\theta$
17 $\sin x/2\sqrt{1-\cos x}$
19 $\cos x/\cos^2(\sin x)$
21 $2\sin(3x)+6x\cos(3x)$
23 $e^{-2x}[\cos x-2\sin x]$

25 $5\sin^4\theta\cos\theta$

27 $-3e^{-3\theta}/\cos^2(e^{-3\theta})$

29 $\cos t - t\sin t + 1/\cos^2 t$

31 $5\sin^4\alpha\cos^4\alpha - 3\sin^6\alpha\cos^2\alpha$

33 $\dfrac{\sqrt{1-\cos x}(1-\cos x-\sin x)}{2\sqrt{1-\sin x}(1-\cos x)^2}$

35 $(\cos y + a + y\sin y)/\left((\cos y + a)^2\right)$

37 $(\ln 2)(2^{2\sin x+e^x})(2\cos x + e^x)$

39 $\theta^2\cos\theta$

41 $-2(\cos w\sin w + w\sin(w^2))$

43 $e^x/\sin x$

45 Decreasing, concave up

47 (a) $dy/dt = -\frac{4.9\pi}{6}\sin\left(\frac{\pi}{6}t\right)$ ft/hr
(b) Occurs at $t = 6$, 12, 18, and 24 hrs

49 (a) $t = (\pi/2)(m/k)^{\frac{1}{2}}$;
$t = 0$;
$t = (3\pi/2)(m/k)^{\frac{1}{2}}$
(b) $T = 2\pi(m/k)^{\frac{1}{2}}$
(c) $dT/dm = \pi/\sqrt{km}$;
Positive sign means an increase in mass causes the period to increase

51 $k = 7.46$, $(3\pi/4, 1/\sqrt{2})$

## Section 3.6

1 $2t/(t^2+1)$

3 2

5 $-1/z(\ln z)^2$

7 $\frac{e^{-x}}{1-e^{-x}}$

9 $(e^x)/(e^x+1)$

11 $ae^{ax}/(e^{ax}+b)$

13 7

15 $-\tan(w-1)$

17 $2y/\sqrt{1-y^4}$

19 1

21 $-\sin(\ln t)/t$

23 $\arcsin w + \frac{w}{\sqrt{1-w^2}}$

25 $2/\sqrt{1-4t^2}$

27 $-3\sin(\arctan 3x)/(1+9x^2)$

29 $\ln x/(1+\ln x)^2$

31 $(\cos x - \sin x)/(\sin x + \cos x)$

33 $1/(1+2u+2u^2)$

35 $-(x+1)/(\sqrt{1-(x+1)^2})$

37 $\frac{d}{dx}(\arcsin x) = 1/\sqrt{1-x^2}$
$(-1 < x < 1)$

39 (a) $(-0.99, -0.16)$
(b) $v_x = 0.32$
$v_y = -1.98$

41 $-43.4$

43 (a) $f'(x) = 0$
(b) $f$ is a constant function.

45 (a) $y = -x^2/2 + 2x - 3/2$
(b) From graph, notice that around $x = 1$, the values of $\ln x$ and its approximation are very close.
(c) At $x = 1.1$, $y \approx 0.095$
At $x = 2$, $y = 0.5$

47 (a) $k \approx 0.067$
(b) $t \approx 10.3$ hours
(c) Formula:
$T(24) \approx 74.1^\circ$F,
rule of thumb: $73.6^\circ$F

49 2513.3 cm$^3$/sec

51 2 cm/sec

53 (a) $z = \sqrt{0.25 + x^2}$
(b) 0.693 km/min
(c) 0.4 radians/min

55 (a) $h(t) = 300 - 30t$
$0 \le t \le 10$
(b) $\theta = \arctan(\frac{200-30t}{150})$
$d\theta/dt =$
$-\left(\frac{1}{5}\right)\left(\frac{150^2}{(150^2+(200-30t)^2)}\right)$
(c) When the elevator is at the level of the observer.

## Section 3.7

1 $dy/dx = -x/y$

3 $1/25$

5 $(1-y)/(x-3)$

7 $ax/by$

9 $-2xye^{x^2}$

11 $\frac{dy}{dx} = \frac{y(1-x\ln y)}{x(x+3y^3)}$

13 $dy/dx = -y^{1/3}/x^{1/3}$

15 $dy/dx = 0$

17 Slope is infinite

19 $-5/4$

21 $y = e^2x$

23 $y = 0$

25 (a) $dy/dx = -9x/25y$
(b) The slope is not defined anywhere along the line $y = 0$.

27 (a) $\frac{dy}{dx} = \frac{y^2-3x^2}{3y^2-2xy}$
(c) $y \approx 1.9945$
(d) Horizontal:
$(1.1609, 2.0107)$
and
$(-0.8857, 1.5341)$
Vertical:
$(1.8039, 1.2026)$
and
$(\sqrt[3]{5}, 0)$

29 $(-1/3, 2\sqrt{2}/3)$,
$(7/3, 4\sqrt{2}/3)$

## Section 3.8

1 The particle moves on straight lines from $(0,1)$ to $(1,0)$ to $(0,-1)$ to $(-1,0)$ and back to $(0,1)$.

3 The particle moves on straight lines from $(-1,1)$ to $(1,1)$ to $(-1,-1)$ to $(1,-1)$ and back to $(-1,1)$.

5 Clockwise for all $t$.

7 Clockwise: $t < 0$,
Counter-clockwise: $t > 0$.

9 Counterclockwise: $t > 0$.

11 $x = 3\cos t$, $y = -3\sin t$, $0 \le t \le 2\pi$

13 $x = 2 + 5\cos t$, $y = 1 + 5\sin t$,
$0 \le t \le 2\pi$

15 $x = t$, $y = -4t + 7$

17 $x = -3\cos t$, $y = -7\sin t$,
$0 \le t \le 2\pi$

19 $y = -(4/3)x$

21 Speed $= |t| \cdot \sqrt{(4+9t^2)}$,
Particle stops when $t = 0$.

23 Speed $= \sqrt{4\sin^2(2t) + \cos^2 t}$,
Particle stops when
$t = (2n+1)\pi/2$, for any integer $n$

25 $x = 4 + 4(t-2)$
$y = 8 + 12(t-2)$

27 (a) The part of the line with $x < 10$ and $y < 0$.
(b) The line segment between $(10,0)$ and $(11,12)$.

29 (b) $v \approx 2.2$
(c) $v = 2.2363$

33 (a) $a = b = 0, k = 5$ or $-5$
(b) $a = 0, b = 5, k = 5$ or $-5$
(c) $a = 10, b = -10, k = \sqrt{200}$ or $-\sqrt{200}$

35 (b) For example:
$R = 12$, $t = \pi$

## Section 3.9

1 $\sqrt{1+x} \approx 1 + x/2$

3 $1/x \approx 2 - x$

7 (b) This estimate is about right.
(c) Below

9 (c) 0

11 (c) $-0.135\%$

13 (a) $\Delta T \approx -T\Delta g/(2g)$
(b) 0.5% decrease

15 (a) $f'(5)$
(b) 0
(c) $f(5) + f'(5)\Delta x$

## Section 3.10

1 Negative

3 Negative

5 0

7 0

9 $0.1x^7$

11 $x^{0.2}$

13 0

15 0.909297

17 Does not exist

19 $-2$

## Chapter 3 Review

1 $2e^t + 2te^t + 1/(2t^{3/2})$

3 $3/(y\ln(2y^3))$

5 $kx^{k-1} + k^x\ln k$

7 $-6\cos(3t+5)\cdot\sin(3t+5)$

9 $6\cos(3\theta-\pi)\sin(3\theta-\pi)$

11 $-\theta^{-3}(\theta\cos(5-\theta) + 2\sin(5-\theta))$

13 $x^{-\frac{1}{2}}/2 - x^{-2} - 3x^{-\frac{5}{2}}/2$

15 $e^{(e^\theta+e^{-\theta})}(e^\theta - e^{-\theta})$

17 $e^{\tan(\sin\alpha)}\cos\alpha/\cos^2(\sin\alpha)$

19 $e(\tan 2 + \tan r)^{e-1}/\cos^2 r$

21 $e^{\tan x} + xe^{\tan x}/\cos^2 x$

23 $6x/\left(9x^4 + 6x^2 + 2\right)$

25 $2^{\sin x}\left((\ln 2)\cos^2 x - \sin x\right)$

27 $a$

29 $e^{-4kt}(\cos t - 4k \sin t)$

31 $-4a^2x/(a^2+x^2)^2$

33 $(-3a^2s - s^3)/(a^2+s^2)^{3/2}$

35 $-(\sin\theta\cos\theta)/(\sqrt{a^2 - \sin^2\theta})$

37 $\cos(t/k)/[k\sin(t/k)]$

39 $20w/(a^2-w^2)^3$

41 $4a/(e^{ax}+e^{-ax})^2$

43 $6(3\theta-\pi)\cos[(3\theta-\pi)^2]$

45 $k$

47 $(-4-6x)(6x^e - 3\pi) + (2-4x-3x^2)(6ex^{e-1})$

49 0

51 $4x - 2 - 4x^{-2} + 8x^{-3}$

53 $2/3$

55 $-2xy/(x^2-2)$

57 $(y + b\sin(bx))/(a\cos(ay) - x)$

59 Proportional to $r^2$

61 (a) $H'(2) = 11$
(b) $H'(2) = -1/4$
(c) $H'(2) = r'(1)\cdot 3$ (we don't know $r'(1)$)
(d) $H'(2) = -3$

63 (a) $y = 20x - 48$
(b) $y = 11x/9 - 16/9$
(c) $y = -4x + 20$
(d) $y = -24x + 57$
(e) $y = 8.06x - 15.84$
(f) $y = -0.94x + 6.27$

65 1.909 radians ($109.4°$) or 1.231 radians ($70.5°$)

67 Not perpendicular; $x \approx 1.3$

69 (a) $dg/dr = -2GM/r^3$
(b) $dg/dr$ is rate of change of acceleration due to pull of gravity.
The further away from the earth's center, the weaker gravity's pull.
(c) $-3.05 \times 10^{-6}$
(d) It is reasonable because the magnitude of $\frac{dg}{dr}$ is so small (compared to $g = 9.8$) that for $r$ near 6400 km, $g$ is not varying much at all.

71 (a) $v(t) = 10e^{t/2}$
(b) $v(t) = s(t)/2$

73 (a) Falling, 0.38 m/hr
(b) Rising, 3.76 m/hr
(c) Rising, 0.75 m/hr
(d) Falling, 1.12 m/hr

75 (a) $v = -2\pi\omega y_0 \sin(2\pi\omega t)$
$a = -4\pi^2\omega^2 y_0 \cos(2\pi\omega t)$
(b) Amplitudes: different
$(y_0,\ 2\pi\omega y_0,\ 4\pi^2\omega^2 y_0)$
Periods $= 1/\omega$

77 $100\pi$ cm$^3$/sec

79 (a) Angular velocity
(b) $v = a(d\theta/dt)$

81 $-k/V^2$

83 (a) $y^{(n)} = (-1)^{n+1}(n-1)!x^{-n}$
(b) $y^{(n)} = xe^x + ne^x$

85 $(f/g)'/(f/g) = (f'/f) - (g'/g)$

87 (a) 1
(b) 1
(c) $\sin(\arcsin x) = x$

89 (a) 0
(b) 0
(c) $\ln(1-1/t) + \ln(t/(t-1)) = \ln 1$

## Ch. 3 Understanding

1 True

3 True

5 True

7 False

9 True

11 False; $f(x) = |x|$

13 False; $\cos t + t^2$

15 False; $f(x) = 6, g(x) = 10$

17 False; $f(x) = 5x + 7, g(x) = x + 2$

19 False; $f(x) = x^2, g(x) = x^2 - 1$

21 False; $f(x) = e^{-x}, g(x) = x^2$

23 (a) Not a counterexample
(b) Not a counterexample
(c) Not a counterexample
(d) Counterexample

## Section 4.1

5 (a) Critical point $x \approx 0$;
Inflection points between $-1$ and 0 and between 0 and 1
(b) Critical point at $x = 0$,
Inflection points at $x = \pm 1/\sqrt{2}$

7 Local maximum $x = 1$

9 (a) $x \approx 2.5$ (or any $2 < x < 3$)
$x \approx 6.5$ (or any $6 < x < 7$)
$x \approx 9.5$ (or any $9 < x < 10$)
(b) $x \approx 2.5$: local max;
$x \approx 6.5$: local min;
$x \approx 9.5$: local max

13 Local max: $(-1.4, 6.7)$
Local min: $(1.4, -4.7)$

15 Local max: $(-1, 2)$
Local min: $(1, -2)$
Crit. pt. (not max/min): (0,0)

17 Local min: $(2.3, -13.0)$

19 Local min: $(-0.71, -0.43)$
Local max: $(0.71, 0.43)$
Horizontal asymptote: $y = 0$

23 (a) Local maxima at $x \approx 1, x \approx 8$,
Local minimum at $x \approx 4$
(b) Local maxima at $x \approx 2.5, x \approx 9.5$
Local minimum at $x \approx 6.5$,

25 $a = -1/3$

35 (b) At most four zeros.
(c) Possibly no zeros.
(d) Two inflection points.
(e) Degree four
(f) $\frac{-2}{15}(x+1)(x-1)(x-3)(x-5)$

37 $B = f, A = f', C = f''$

39 III even; I, II odd
I is $f''$, II is $f$, III is $f'$

## Section 4.2

1 $y = -(x-2)^2 + 5$

3 $y = 5(1 - e^{-bx})$

5 $y = -5x/(x-2)$

7 $y = 0.25\sin(\pi x/10) + 1.75$

9 $y = -x^4 + 2x^2 + 3$

11 $y = 3x^{-1/2}\ln x$

13 (a) Local maximum:
$p(-\sqrt{\frac{a}{3}}) = +\frac{2a\sqrt{a}}{3\sqrt{3}}$
Local minimum:
$p(\sqrt{\frac{a}{3}}) = -\frac{2a\sqrt{a}}{3\sqrt{3}}$
(b) Further from $x$- and $y$-axis

15 $a = 200{,}000$
$k = -\ln 0.9 \approx 0.105$

19 $(1/b, 1/be)$

21 (a) $f'(x) = abe^{-bx} > 0$
(b) $f''(x) = -ab^2e^{-bx} < 0$

23 (a) $x = e^a$
(c) Max at $(e^{a-1}, e^{a-1})$ for any $a$

27 (a) Local max: $x = \frac{1}{b}$
No local minima
Inflection point: $x = \frac{2}{b}$
(b) Varying $a$ stretches or flattens the graph vertically.
Incr $b$ shifts critical, inflection points to left; lowers max

31 (a) Intercept: $x = a$
Asymptotes: $x = 0$, $U = 0$
(b) Local min: $(2a, -b/4)$
Local max: none

33 (a) Zero: $r = B/A$
Vertical asymptote: $r = 0$
Horizontal asymptote: $f(r) = 0$
(b) Minimum: $\left(3B/(2A), -4A^3/(27B^2)\right)$
Point of inflection: $\left(2B/A, -A^3/(8B^2)\right)$

## Section 4.3

3 (a) $f(1)$ local minimum;
$f(0), f(2)$ local maxima
(b) $f(1)$ global minimum
$f(2)$ global maximum

5 (a) $f(\frac{2\pi}{3})$ local maximum
$f(0)$ and $f(\pi)$ local minima
(b) $f(\frac{2\pi}{3})$ global maximum
$f(0)$ global minimum

7 44.1 feet

9 $r = \frac{2}{3}R$

11 (a) $0 \le y \le a$
(b) $y = 0$

13 $r = 3B/(2A)$

15 $x = L/2$

17 Minimum: $0.148mg$ newtons
Maximum: $1.0mg$ newtons

19 Minimum: $-2$ amps
Maximum: 2 amps

21 $0.91 < y \le 1.00$

23 $0 \le y < 1.61$

25 $0 \le y \le 2\pi$

27 (b) 2 hours
(c) Equal

29 (b) $f(v) = v \cdot a(v)$
(c) When $a(v) = f'(v)$
(d) $a(v)$

31 (a) $g(v) = f(v)/v$
(b) 220 mph
(c) 300 mph

33 (b) Yes, at $x = 0$
(c) Max: $x = -2$, Min: $x = 2$
(d) $5 > g(0) > g(2)$

## Section 4.4

1 \$5000, \$2.40, \$4

3 $\pi(q) = 490q - q^2 - 150$
Max at $q = 245$

5 \$1.1 m, 70, \$1.2 m

7 (a) $\pi(q)$ max when $R(q) > C(q)$ and $R$ and $Q$ are farthest apart
(b) $C'(q_0) = R'(q_0) = p$

9 (a) No
(b) Yes

11 (a) 150; \$750
(b) \$44,500; 130; \$850

13 (b) (i) $N'(x) = 20$
(ii) $\frac{N(x)}{x} = \frac{100}{x} + 20$

17 (b) $q = [Fa/(K(1-a))]^a$

## Section 4.5

1 $2000 - (1200/\sqrt{5})$

3 Min $v = \sqrt{2}k$; no max

5 (a) $V = Ax/4 - x^3/2$
(c) $(A/6)^{3/2}$

7 $w = 34.64$ cm, $h = 48.99$ cm

9 40 feet by 80 feet

11 $h = \sqrt{50}$ meters

13 $(1, 1)$

15 When the rectangle is a square.

17 15 miles/hour

19 (a) The arithmetic mean unless $a = b$, in which case the two means are equal.
(b) The arithmetic mean unless $a = b = c$, in which case the two means are equal.

21 0.8 miles from Town 1

23 65.1 meters

25 (a) $T = \sqrt{a^2 + (c-x)^2}/v_1 + \sqrt{b^2 + x^2}/v_2$

## Section 4.6

1 $2\sinh(2x)$

3 $\sinh(\sinh t) \cdot \cosh t$

5 $2\cosh t \cdot \sinh t$

7 $3\cosh(3y) \cdot \cosh(\sinh(3y))$

13 $\sinh(2x) = 2\sinh x \cosh x$

15 (a) 0
(b) Positive for $x > 0$
Negative for $x < 0$
Zero for $x = 0$
(c) Increasing everywhere
(d) $1, -1$
(e) Yes; derivative positive everywhere

17 (b) $A = 6.325$
(stretch factor)
$c = 0.458$
(horizontal shift)

19 (b) U-shaped
(c) Incr ($A > 0$)
or Decr ($A < 0$)
(d) Max: $A < 0, B < 0$
Min: $A > 0, B > 0$

21 $y \approx 715 - 100\cosh(x/100)$

## Section 4.7

1 False

3 False

5 False

7 True

9 No; no

11 No; no

19 Racetrack

21 Mean Value

23 $21 \le f(2) \le 25$

## Chapter 4 Review

3 (a) Increasing for $x > 0$
Decreasing for $x < 0$
(b) Local and global min: $f(0)$

5 (a) Increasing for $0 < x < 4$
Decreasing for $x < 0$ and $x > 4$
(b) Local max: $f(4)$
Local min: $f(0)$

7 (a) $f'(x) = 3x(x-2)$
$f''(x) = 6(x-1)$
(b) $x = 0$
$x = 2$
(c) Inflection point: $x = 1$
(d) Endpoints:
$f(-1) = -4$
$f(3) = 0$
Critical Points:
$f(0) = 0$
$f(2) = -4$
Global max:
$f(0) = 0$ and $f(3) = 0$
Global min:
$f(-1) = -4$ and
$f(2) = -4$
(e) $f$ increasing:
for $x < 0$ and $x > 2$
$f$ decreasing:
for $0 < x < 2$
$f$ concave up:
for $x > 1$
$f$ concave down:
for $x < 1$.

9 (a) $f'(x) = -e^{-x}\sin x + e^{-x}\cos x$
$f''(x) = -2e^{-x}\cos x$
(b) Critical points:
$x = \frac{\pi}{4}$ and $\frac{5\pi}{4}$
(c) Inflection points:
$x = \frac{\pi}{2}$ and $\frac{3\pi}{2}$
(d) Endpoints:
$f(0) = 0$
$f(2\pi) = 0$
Global max:
$f(\frac{\pi}{4}) = (e^{-\frac{\pi}{4}})(\frac{\sqrt{2}}{2})$
Global min:
$f(\frac{5\pi}{4}) = -e^{\frac{-5\pi}{4}}(\frac{\sqrt{2}}{2})$
(e) $f$ increasing:
$0 < x < \frac{\pi}{4}$ and
$\frac{5\pi}{4} < x < 2\pi$
$f$ decreasing:
$\frac{\pi}{4} < x < \frac{5\pi}{4}$
$f$ concave down:
for $0 \le x < \frac{\pi}{2}$
and $\frac{3\pi}{2} < x \le 2\pi$
$f$ concave up:
for $\frac{\pi}{2} < x < \frac{3\pi}{2}$

11 $\lim_{x\to\infty} f(x) = \infty$
$\lim_{x\to-\infty} f(x) = -\infty$
(a) $f'(x) = 6(x-2)(x-1)$
$f''(x) = 6(2x-3)$
(b) $x = 1$ and $x = 2$
(c) $x = \frac{3}{2}$
(d) Critical points:
$f(1) = 6, f(2) = 5$
Local max: $f(1) = 6$
Local min: $f(2) = 5$
Global max and min: none
(e) $f$ increasing: $x < 1$ and $x > 2$
Decreasing: $1 < x < 2$
$f$ concave up: $x > \frac{3}{2}$
$f$ concave down: $x < \frac{3}{2}$

13 $\lim_{x\to-\infty} f(x) = -\infty$
$\lim_{x\to\infty} f(x) = 0$
(a) $f'(x) = (1-x)e^{-x}$
$f''(x) = (x-2)e^{-x}$
(b) Only critical point is at $x = 1$.
(c) Inflection point: $f(2) = \frac{2}{e^2}$
(d) Global max: $f(1) = \frac{1}{e}$
Local and global min: none
(e) $f$ increasing: $x < 1$
$f$ decreasing: $x > 1$
$f$ concave up: $x > 2$
$f$ concave down: $x < 2$

15 Local max: $f(-3)$
Local min: $f(3)$
Inflection pts: $x = 0$, $x = \pm\frac{3}{\sqrt{2}}$
Global max and min: none

17 Local max: $f(-\frac{2}{5})$
Global and local min: $f(0)$
Inflection pts: $x = \frac{-2\pm\sqrt{2}}{5}$
Global max: none

19 Local maxima: none
Global and local min:
$f(0) = 0$
Inflection pts: $x = \pm\frac{1}{\sqrt{3}}$

21 $a = 1.63, b = -2$

23 (a) $x_3$
(b) $x_1, x_5$
(c) $x_2$
(d) 0

27 $x = y = \sqrt[3]{V}$

29 $r \approx 2.7$ cm

31 (a) $2\sqrt{x^2 + 300^2} + (1000 - x)$
(b) 173.21 miles south of Boise on the coast

33 $-4.81 \le f(x) \le 1.82$

37 (a) $g(e)$ is a global maximum.
There is no minimum.
(b) There are exactly two solutions.
(c) $x = 5$ and $x \approx 1.75$

39 $a = 363.23$, and $b \approx 4.7665$

43 (a) $1/\sqrt{1+x^2}$

45 (a) Min at $x = a$
(c) 4.6477

47 (b) No

## Ch. 4 Understanding

1 True

3 False

5 False

7 False

9 True

11 False

13 One possibility:
$f(x) = ax^2, a \ne 0$

15 Possible answer: $f(x) = |x|$

17 Possible answer
$$f(x) = \begin{cases} x^2 & \text{if } 0 \le x < 1 \\ 1/2 & \text{if } x = 1 \end{cases}$$

19 (a) Not implied
(b) Not implied
(c) Implied

21 Impossible

23 Impossible

25 Impossible

## Section 5.1

1 (a) $570\ \text{m}^3/\text{hr}$
(b) Every 2 minutes

3 (a) Lower estimate $= 5.25$ mi
Upper estimate $= 5.75$ mi
(b) Lower estimate $= 11.5$ mi
Upper estimate $= 14.5$ mi
(c) Every 30 seconds

5 (a) Car $A$
(b) Car $A$
(c) Car $B$

7 (a) 430 ft
(b) (ii)

9 Upper estimate $= 0.75$ m;
Lower estimate $= 0.65$ m;
Average $= 0.70$ m

11 Between 140 and 150 meters

13 (a) Upper estimate $= 34.16$ m/sec
Lower estimate $= 27.96$ m/sec
(b) 31.06 m/sec;
It is too high

## Section 5.2

1 (a) 224
(b) 96
(c) 200
(d) 136

3 20

5 6375

7 205.5

9 10.0989

11 1.4936

13 limit $= 1$
True value is between
1.00314 and 0.99686.

15 2.545

17 24.7

19 4.39

21 0.0833

23 (a) 78
(b) 46; underestimate
(c) 118; overestimate

27 0.80

29 $2/125$

31 (a) $-4$
(b) 0
(c) 8

33 $a = 2$, $b = 6$, $f(x) = x^2$; other answers possible

## Section 5.3

1 2

3 15

5 Dollars

7 (a) Car 1: 1031.25 ft
Car 2: 562.5 ft
(b) 1.6 minutes

9 (b) Twice
At each intersection point, the distance between the two vehicles is at a local extremum.

11 (a) $\int_0^2 R(t)\,dt$
(c) lower estimate: 2.81
upper estimate: 3.38

13 (a) $-4$
(b) (ii)

15 \$2392.87

17 About \$13,800

19 (b) 0.64

21 Units of $f$

23 \$6080

25 \$485.80

27 (a) 0.375 thousand/hour
(b) 1.75 thousands

29 12 newton · meters

31 (a) III
(b) I
(c) II and IV

## Section 5.4

3 (a) $f(1), f(2)$
(b) $2, 2.31, 2.80, 2.77$

5 $8c$

7 $-52$

9 8

11 (a) 2
(b) $12, 2$
(c) $0, 0$

15 $f(3) - f(2)$,
$[f(4) - f(2)]/2$,
$f(4) - f(3)$.

21 (a) 0.1574
(b) 0.9759

23 $30/7$

## Chapter 5 Review

1 (a) 260 ft
(b) Every 0.5 sec

3 $\approx$ 455 feet or 0.086 miles

5 396

7 3.4

9 36.00

11 10.67

13 5.1666

15 486.15 quadrillion BTU
$\int_0^{30} f(t)dt$

17 65 km from home
3 hours
90 km

19 About 13,500 liters

21 (b) Largest to smallest:
$n = 1$, $n = 3$,
$n = 4$, and $n = 2$.

23 V < IV < II < III < I
I, II, III positive
IV, V negative

25 (a) $\int_0^5 f(x)\,dx - \frac{1}{2}\int_{-2}^2 f(x)\,dx$
(b) $\int_{-2}^5 f(x)\,dx - 2\int_{-2}^0 f(x)\,dx$
(c) $\frac{1}{2}\left(\int_{-2}^5 f(x)\,dx - \int_2^5 f(x)\,dx\right)$

27 45.8°C.

29 741.6 liters

31 (a) V
(b) IV
(c) III
(d) II
(e) III
(f) I

33 (a) $300\ \text{m}^3$/sec
(b) $250\ \text{m}^3$/sec
(c) 1996: $1250\ \text{m}^3$/sec
1957: $3500\ \text{m}^3$/sec
(d) 1996: 10 days
1957: 4 months
(e) $10^9$ meter$^3$
(f) $2 \cdot 10^{10}$ meter$^3$

35 (a) At $t = 17, 23, 27$ seconds
(b) Right: $t = 10$ seconds
Left: $t = 40$ seconds
(c) Right: $t = 17$ seconds
Left: $t = 40$ seconds
(d) $t = 10$ to 17 seconds,
20 to 23 seconds, and
24 to 27 seconds
(e) At $t = 0$ and $t = 35$

37 9 years

39 (a) $\sum_{i=1}^n i^5/n^6$
(b) $(2n^4 + 6n^3 + 5n^2 - 1)/12n^4$
(c) $1/6$

41 (a) $\sum_{i=0}^{n-1}(n+i)^2/n^3$.
(b) $7/3 + 1/(6n^2) - 3/(2n)$.
(c) 7/3
(d) 7/3

43 (a) $\cos(ac)/c - \cos(bc)/c$
(b) $-\cos(cx)/c$

## Ch. 5 Understanding

1 True

3 False

5 False

7 False

9 False

11 True

13 True

15 False

17 True

## Section 6.1

5 128, 169, 217

7 (a) $-19$
(b) 6

9 (a) $x = 1$, $x = 3$
(b) Local min at $x = 1$, local max at $x = 3$

11 $x_1$ local max;
$x_2$ inflection pt;
$x_3$ local min

13 $x_2, x_3$ inflection pts.

15 (a) $f(3) = 1$; $f(7) = 0$
(b) $x = 0, 5.5, 7$

17 Critical points: $(0, 5), (2, 21), (4, 13), (5, 15)$

19 Acceleration is zero at points $A$ and $C$.

21 (a) (I) volume; (II) flow rate
(b) (I) is an antiderivative of (II)

23 (a) $f(x)$ is greatest at $x_1$
(b) $f(x)$ is least at $x_5$
(c) $f'(x)$ is greatest at $x_3$
(d) $f'(x)$ is least at $x_5$
(e) $f''(x)$ is greatest at $x_1$
(f) $f''(x)$ is least at $x_5$

25 (b) Maximum in July 1993
Minimum in Jan 1994
(c) Increasing fastest in May 1993
Decreasing fastest in Oct 1993

## Section 6.2

1 $5x$

3 $x^3/3$

5 $\sin t$

7 $\ln|z|$

9 $-1/2z^2$

11 $-\cos t$

13 $t^4/4 - t^3/6 - t^2/2$

15 $\sin t + \tan t$

17 $-\cos 2\theta$

19 $(t+1)^3/3$

21 $5x^2/2 - 2x^{3/2}/3$

23 $x^4/4 - x^2/2 + C$

25 $t^4/4 + 5t^2/2 - t + C$

27 $2t^{3/2}/3 + C$

29 $x^4 - 7x + C$

31 $2t^{1/2} + C$

33 $-1/x + C$

35 $F(x) = x^2$
(only possibility)

37 $F(x) = x^2/8$
(only possibility)

39 $F(x) = \frac{2}{3}x^{3/2}$
(only possibility)

41 $F(x) = -\cos x + 1$
(only possibility)

43 $x^4/4 + C$

45 $x^4/4 - 2x + C$

47 $8w^{3/2}/3 + C$

49 $-4/t + C$

51 $\sin\theta + C$

53 $x^2/2 + 2x^{1/2} + C$

55 $\pi x + x^{12}/12 + C$

57 $\sin(x+1) + C$

59 $-e^{-z} + C$

61 36

63 $-(\sqrt{2}/2) + 1 = 0.293$

65 $\frac{609}{4} - 39\pi \approx 29.728$

67 $\ln 2 + \frac{3}{2} \approx 2.193$

69 1

71 $2e - 2 \approx 3.437$

73 $3/(2\ln 2) \approx 2.164$

75 36

77 $e - 1 - \sin 1$

79 $c = 3$

81 $c = 6$

83 $C(x) = 5x^2 + 4000x + 1{,}000{,}000$

## Section 6.3

1 $x^4/4 + 5x + C$

3 $8t^{3/2}/3 + C$

7 $10e^t + 15$

9 $2z - \cos z + 6$

11 (a) $a(t) = -9.8$ m/sec$^2$
$v(t) = -9.8t + 40$ m/sec
$h(t) = -4.9t^2 + 40t + 25$ m
(b) 106.6 m; 4.08 sec
(c) 8.75 sec

13 $y = 2kt^{3/2}/3$

15 19.55 ft/sec$^2$

17 (c) 200 ft
(d) 200 ft

19 (a) 6 seconds
(b) Left sum: 97.5 ft
overestimate
Right sum: 82.5 ft
underestimate
(c) 90 ft
(d) $s(t) = 30t - \frac{5}{2}t^2$; s(6) = 90 ft
Distance by antidiff = Average of left and right hand sums

21 (a) 4 seconds
(b) 576 ft
(c) 10 seconds
(d) 192 ft/sec downward

23 (a) 80 ft/sec
(b) 640 ft

25 $-33.56$ ft/sec$^2$

## Section 6.4

7 $f(x) = 7 + \int_0^x \sin(t^2)\,dt$

9 $f(x) = 2 + \int_0^x \mathrm{Si}(t)\,dt$

11 500

13 $-3.905$

15 $(1+x)^{200}$

17 $-\cos(t^3)$

19 $(2\sin x^2)/x$

21 (a) 0
(c) $F(x) \geq 0$ everywhere.
$F(x) = 0$ only at integer multiples of $\pi$.

23 $\mathrm{erf}(x) + \frac{2}{\sqrt{\pi}}xe^{-x^2}$

25 $3x^2e^{-x^6}$

## Section 6.5

1 (a) 1.5 m
(b) 7 m/sec
(c) 9.8 m/sec$^2$ downward

3 $t = 5$; $v = -160$ ft/sec

5 400 feet

9 (a) First second: $-g/2$
Second: $-3g/2$
Third: $-5g/2$
Fourth: $-7g/2$
(b) Galileo seems to have been correct.

## Chapter 6 Review

1 $\frac{5}{2}x^2 + 7x + C$

3 $2t + \sin t + C$

5 $3e^x - 2\cos x + C$

7 $16\sqrt{x} + C$

9 $e^x + 5x + C$

11 $\tan x + C$

13 $(x+1)^3/3 + C$

15 $\frac{1}{10}(x+1)^{10} + C$

17 $\frac{1}{2}x^2 + x + \ln|x| + C$

19 $3\sin x + 7\cos x + C$

21 $2e^x - 8\sin x + C$

23 $\sin x + C$

25 $-\cos x + C$

27 $5\ln|t| + C$

29 $e^x - x + C$

31 $x^4/4 + +2x^3 - 4x + 4.$

33 $e^x + 3$

35 $\sin x + 4$

37 9

39 $\sqrt{3} - \pi/9$

41 (a) 253/12
(b) $-125/12$

43 $c = 3/4$

45 2

47 (a) $t = 2, t = 5$
(b) $f(2) \approx 55$, $f(5) \approx 40$
(c) $-10$

51 Global max: at $x = \pi$
Global min: at $x = \pi/2$

53 (a) $f(x) = 3 + \int_0^x e^{t^2}\,dt$
(b) $f(x) = 5 + \int_{-1}^x e^{t^2}\,dt$

57 (a) 14,000 revs/min$^2$
(b) 180 revolutions

59 (b) Highest pt: $t = 2.5$ sec
Hits ground: $t = 5$ sec
(c) Left sum:
136 ft (an overest.)
Right sum:
56 ft (an underest.)
(d) 100 ft

61 (b) $t = 6$ hours
(c) $t = 11$ hrs

63 Positive, zero, negative, positive, zero

65 (a) $(1/2)e^{2x}$, $(1/3)e^{3x}$, $(1/3)e^{3x+5}$
(b) $(1/a)e^{ax+b}$

67 (a) $x - \ln|x-1|$, $x - 2\ln|x-1|$, $x + \ln|x-2|$
(b) $x + (b-a)\ln|x-b|$

## Ch. 6 Understanding

1 True

3 True

5 False

7 True

9 True

11 True

13 False

15 False

## Section 7.1

1 (a) $2x\cos(x^2+1)$;
$3x^2\cos(x^3+1)$
(b) (i) $\frac{1}{2}\sin(x^2+1) + C$
(ii) $\frac{1}{3}\sin(x^3+1) + C$
(c) (i) $-\frac{1}{2}\cos(x^2+1) + C$
(ii) $-\frac{1}{3}\cos(x^3+1) + C$

3 $(1/3)e^{3x} + C$

5 $-0.5\cos(2x) + C$

7 $\frac{1}{18}(y^2+5)^9 + C$

9 $\frac{1}{5}y^5 + \frac{1}{2}y^4 + \frac{1}{3}y^3 + C$

11 $\frac{1}{9}(x^2-4)^{9/2} + C$

13 $-2\sqrt{4-x}+C$
15 $\frac{1}{148}(2t-7)^{74}+C$
17 $-\frac{1}{8}(\cos\theta+5)^8+C$
19 $-\frac{1}{2}e^{-x^2}+C$
21 $\frac{1}{3}e^{x^3+1}+C$
23 $\frac{1}{4}\sin^4\alpha+C$
25 $\frac{1}{3}(\ln z)^3+C$
27 $\frac{1}{2}\ln(y^2+4)+C$
29 $2e^{\sqrt{y}}+C$
31 $\ln(2+e^x)+C$
33 $\frac{1}{6}\ln(1+3t^2)+C$
35 $t+2\ln|t|-\frac{1}{t}+C$
37 $\sinh x+C$
39 $e^{\cosh z}+C$
41 $(\pi/4)t^4+2t^2+C$
43 $\sin x^2+C$
45 $\frac{1}{5}\cos(2-5x)+C$
47 $\frac{1}{2}\ln(x^2+1)+C$
49 0
51 $1-(1/e)$
53 $3(e^2-e)$
55 $2(\sin 2-\sin 1)$
57 40
59 $\ln 3$
61 $14/3$
63 (a) Yes; $-0.5\cos(x^2)+C$
(b) No
(c) No
(d) Yes; $-1/(2(1+x^2))+C$
(e) No
(f) Yes; $-\ln|2+\cos x|+C$
65 $\ln 3$
67 $4\ln 2$
69 $\frac{1}{2}\ln 3$
71 (a) $\frac{\sin^2\theta}{2}+C$
(b) $-\frac{\cos^2\theta}{2}+C$
(c) $-\frac{\cos 2\theta}{4}+C$
(d) Functions differ by a constant
73 (a) 5.3 billion, 6.1 billion
(b) 5.7 billion
75 (a) $E(t)=1.4e^{0.07t}$
(b) $0.2(e^7-1)\approx 219$ million megawatt-hours
(c) 1972
(d) Graph $E(t)$ and estimate $t$ such that $E(t)=219$
77 $-\frac{1}{k}\ln\left(\left(e^{t\sqrt{gk}}+e^{-t\sqrt{gk}}\right)/2\right)+h_0$

## Section 7.2

1 $x\cdot\arctan x-\frac{1}{2}\ln(1+x^2)+C$
3 $\frac{1}{5}t^2e^{5t}-\frac{2}{25}te^{5t}+\frac{2}{125}e^{5t}+C$
5 $-t\cos t+\sin t+C$
7 $\frac{x^4}{4}\ln x-\frac{x^4}{16}+C$
9 $-t^2\cos t+2t\sin t+2\cos t+C$
11 $-\frac{1}{2}\sin\theta\cos\theta+\frac{\theta}{2}+C$
13 $\frac{1}{6}q^6\ln 5q-\frac{1}{36}q^6+C$
15 $t(\ln t)^2-2t\ln t+2t+C$
17 $-(\theta+1)\cos(\theta+1)+\sin(\theta+1)+C$
19 $-x^{-1}\ln x-x^{-1}+C$
21 $-2t(5-t)^{1/2}-\frac{4}{3}(5-t)^{3/2}-14(5-t)^{1/2}+C$
23 $w\arcsin w+\sqrt{1-w^2}+C$
25 $\frac{1}{2}x^2\arctan x^2-\frac{1}{4}\ln(1+x^4)+C$
27 $\frac{1}{3}x^3\sin x^3+\frac{1}{3}\cos x^3+C$
29 $\cos 5+5\sin 5-\cos 3-3\sin 3\approx -3.944$
31 $\frac{9}{2}\ln 3-2\approx 2.944$
33 $6\ln 6-5\approx 5.751$
35 $\frac{1}{2}(\frac{\pi}{2}-1)\approx 0.285$
37 $\pi$
39 Integration by parts gives:
$\frac{1}{2}\sin\theta\cos\theta+\frac{1}{2}\theta+C$
The identity for $\cos^2\theta$ gives:
$\frac{1}{2}\theta+\frac{1}{4}\sin 2\theta+C$
41 $\frac{1}{2}e^{\theta}(\sin\theta+\cos\theta)+C$
43 $\frac{1}{2}\theta e^{\theta}(\sin\theta+\cos\theta)-\frac{1}{2}e^{\theta}\sin\theta+C$
45 Integrate by parts choosing $u=x^n$, $v'=\cos ax$.
47 Integrate by parts choosing $u=\cos^{n-1}x$, $v'=\cos x$.
49 Approximately 77
51 (a) $-a^2e^{-a}-2ae^{-a}-2e^{-a}+2$
(b) Increasing
(c) Concave up
53 (a) $V_0$: increases,
$\omega$: none, $\phi$:none
(b) $V_0$: increases,
$\omega$: increases, $\phi$:none
(c) $V_0$: increases,
$\omega$: none, $\phi$:none
55 (a) $C_1=\sqrt{2}$
(b) $C_n=\sqrt{2}$

## Section 7.3

1 $\frac{1}{10}e^{(-3\theta)}(\sin\theta-3\cos\theta)+C$
3 $-\frac{1}{5}x^3\cos 5x+\frac{3}{25}x^2\sin 5x+\frac{6}{125}x\cos 5x-\frac{6}{625}\sin 5x+C$
5 $\frac{1}{7}x^7+\frac{5}{2}x^4+25x+C$
7 $-\frac{1}{4}\sin^3 x\cos x-\frac{3}{8}\sin x\cos x+\frac{3}{8}x+C$
9 $\frac{1}{2}\frac{\sin x}{\cos^2 x}+\frac{1}{4}\ln\left|\frac{\sin x+1}{\sin x-1}\right|+C$
11 $\frac{5}{16}\sin 3\theta\sin 5\theta+\frac{3}{16}\cos 3\theta\cos 5\theta+C$
13 $(\frac{1}{3}x^2-\frac{2}{9}x+\frac{2}{27})e^{3x}+C$
15 $(\frac{1}{3}x^4-\frac{4}{9}x^3+\frac{4}{9}x^2-\frac{8}{27}x+\frac{8}{81})e^{3x}+C$
17 $t+\ln|t-1|-\ln|t+1|+C$
19 $\frac{1}{45}(7\cos 2y\sin 7y-2\sin 2y\cos 7y)+C$
21 $\frac{1}{34}e^{5x}(5\sin 3x-3\cos 3x)+C$
23 $-\frac{1}{2\tan 2\theta}+C$
25 $\frac{1}{21}\frac{\tan 7x}{\cos^2 7x}+\frac{2}{21}\tan 7x+C$
27 $\frac{1}{3}\frac{\sin x}{\cos^3 x}-\frac{4}{3}\frac{\sin x}{\cos x}+x+C$
29 $-\frac{1}{4}(\ln|y-2|-\ln|y+2|)+C$
31 $\arctan(y+2)+C$
33 $-\cos x+\frac{1}{3}\cos^3 x+C$
35 $\frac{1}{8}e^{2z^2}(\cos 2z^2+\sin 2z^2)+C$
39 (a) $NA+2\pi B(1-e^{-N})/(1+4\pi^2)$
(b) $A+2\pi B\left(\frac{1-e^{-N}}{N}\right)/(1+4\pi^2)$
(c) $A$
(d) $R(t)\approx A$ for large $t$
(e) No, a real oil well will eventually dry out.

## Section 7.4

1 $2/(5-x)+2/(5+x)$
3 $-2/y+1/(y-2)+1/(y+2)$
5 $-2\ln|5-x|+2\ln|5+x|+C$
7 $-2\ln|y|+\ln|y-2|+\ln|y+2|+C$
9 (a) Yes; $x=3\sin\theta$
(b) No
11 $-\ln|x|+x^{-1}+\ln|x-1|+K$
13 $x^2/2+\ln|x|+\ln|x+1|+2\ln|x+11|+K$
15 $(1/2)\arcsin(2x/3)+C$
17 $\arcsin(x-2)+C$
19 $x=(4\tan\theta)-3$
21 $x=(\tan\theta)-1$
23 $z=(\sin\theta)+1$
25 $w=(t+2)^2+3$
27 $(\ln|x-5|-\ln|x-3|)/2+C$
29 $\ln|x|-(1/2)\ln\left|x^2+1\right|+\arctan x+K$
31 $-x\sqrt{9-x^2}/2+(9/2)\arcsin(x/3)+C$
33 $-(\sqrt{1+t^2})/t+C$
35 $-\ln|x-1|+2\ln|x-2|+C$
37 $5\ln 2$
39 $\pi/12-\sqrt{3}/8$
41 $\ln(1+\sqrt{2})$
43 (b) $-\sqrt{5-y^2}/(5y)+C$
45 (a) $(k/(b-a))\ln|(2b-a)/b|$
(b) $T\to\infty$

## Section 7.5

1 (a) Underestimate
(b) Overestimate
(c) Overestimate
(d) Underestimate
3 (a) Underestimate
(b) Overestimate
(c) Underestimate
(d) Overestimate
5 (a) 27
(b) 135
(c) 81
(d) 67.5
7 (a) MID(2)= 24;
TRAP(2)= 28
(b) MID(2) underestimate;
TRAP(2) overestimate

9 $\text{LEFT}(6) = 31$
$\text{RIGHT}(6) = 39$
$\text{TRAP}(6) = 35$

11 (a) $0.664 = \text{LEFT}$
$0.633 = \text{TRAP}$
$0.632 = \text{MID}$
$0.601 = \text{RIGHT}$
(b) Between
0.632 and 0.633

13 MID: over; TRAP: under

15 TRAP: over; MID: under

17 (a) TRAP(4); 1027.5
(b) Underestimate

19 (b) $\text{LEFT}(5) \approx 1.32350$
error $\approx -0.03810$
$\text{RIGHT}(5) \approx 1.24066$
error $\approx 0.04474$
$\text{TRAP}(5) \approx 1.28208$
error $\approx 0.00332$
$\text{MID}(5) \approx 1.28705$
error $\approx -0.001656$

25 $\text{RIGHT}(10) = 5.556$
$\text{TRAP}(10) = 4.356$
$\text{LEFT}(20) = 3.199$
$\text{RIGHT}(20) = 4.399$
$\text{TRAP}(20) = 3.799$

## Section 7.6

1 72

5 (a) $\approx 53.598$
(b) LEFT(2)= 16.778;
error= 36.820
RIGHT(2)= 123.974;
error= −70.376
TRAP(2)= 70.376;
error= −16.778
MID(2)= 45.608;
error= 7.990
SIMP(2)= 53.864;
error= −0.266
(c) LEFT(4)= 31.193;
error=22.405
RIGHT(4)= 84.791;
error=−31.193
TRAP(4)= 57.992;
error=−4.394
MID(4)= 51.428;
error=2.170
SIMP(4)= 53.616;
error=−0.018

7 (a) 4 places: 2 seconds
8 places: ≈ 6 hours
12 places: ≈ 6 years
20 places: ≈ 600 million years
(b) 4 places: 2 seconds
8 places: ≈ 3 minutes
12 places: ≈ 6 hours
20 places: ≈ 6 years

9 0.272

## Section 7.7

3 (a) 0.9596, 0.9995, 0.99999996
(b) 1.0

5 Diverges

7 Converges to $1/2$

9 1

11 $\ln 2$

13 $\pi/2$

15 Does not converge

17 $\pi/4$

19 Does not converge

21 Does not converge

23 0.01317

25 $2 - 2e^{-\sqrt{\pi}}$

27 $\pi/2$

29 $\frac{1}{2}\ln\frac{5}{3}$

31 Converges; 0

33 1

35 $\sqrt{b\pi}$

37 Converges for $p > -1$
to $pe^{p+1}/(p+1)^2$

39 (b) $t = 2$
(c) 4000 people

## Section 7.8

1 Converges; behaves like $1/x^2$

3 Diverges; behaves like $1/x$

5 Diverges; behaves like $1/x$

7 Converges; behaves like $5/x^3$

9 Converges; behaves like $1/x^2$

11 Does not converge

13 Converges

15 Does not converge

17 Converges

19 Converges

21 Converges

23 Converges

25 Does not converge

27 0.606

29 Converges for $p > 1$
Diverges for $p \le 1$

31 $a = 0.399$

33 (a) $\int_3^\infty e^{-x^2}\,dx \le \frac{e^{-9}}{3}$
(b) $\int_n^\infty e^{-x^2}\,dx \le \frac{1}{n}e^{-n^2}$

## Chapter 7 Review

1 $-\cos t + C$

3 $(1/5)e^{5z} + C$

5 $(-1/2)\cos 2\theta + C$

7 $2x^{5/2}/5 + 3x^{5/3}/5 + C$

9 $(r+1)^4/4 + C$

11 $x^2/2 + \ln|x| - x^{-1} + C$

13 $\frac{1}{2}e^{t^2} + C$

15 $(\frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{4})e^{2x} + C$

17 $\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$

19 $x(\ln x)^2 - 2x\ln x$
$+ 2x + C$

21 $-e^{0.5-0.3t}/0.3 + C$

23 $-\frac{1}{3}(4-x^2)^{3/2} + C$

25 $2\sin\sqrt{y} + C$

27 $\frac{1}{4}\sin 2\theta + \frac{1}{2}\theta + C$

29 $-\frac{1}{2}\ln|\cos(2x-6)| + C$

31 $e - 2 \approx 0.71828$

33 $-11e^{-10} + 1$

35 0

37 $\pi/4$

39 $\ln|t| - 4/t - 2/t^2 + C$

41 $\ln|t| - 1/t + C$

43 $-\ln|\cos\theta| + C$

45 $(1/2)\ln|x^2+1| + C$

47 $\frac{1}{2}\arctan 2z + C$

49 $(-1/20)\cos^4 5\theta + C$

51 $(1/12)(t-10)^{12}$
$+ (10/11)(t-10)^{11} + C$

53 $xe^x - e^x + C$

55 $(1/142)(10^{71} - 2^{71})$

57 $(1/3)\arctan(u/3) + C$

59 $-\ln(|\cos(\ln x)|) + C$

61 $(1/2)\arcsin(2x) + C$

63 $y - (1/2)e^{-2y} + C$

65 $\ln|\ln x| + C$

67 $\sin\sqrt{x^2+1} + C$

69 $ue^{ku}/k - e^{ku}/k^2 + C$

71 $(1/\sqrt{2})e^{\sqrt{2}+3} + C$

73 $(1/2)e^{2x} + 2xe^x - 2e^x$
$+ (x^3/3) + C$

75 $\frac{5}{2}\ln|x^2+4| + 3\arctan(x/2) + C$

77 $\frac{1}{20}[\ln|r-10| - \ln|r+10|]$
$+ C$

79 $-e^{-ct}(k\cos kt + c\sin kt)/(k^2+c^2) + C$

81 $(x^{\sqrt{k}+1}/\sqrt{k}+1)$
$+ (\sqrt{k}^x/\ln\sqrt{k}) + C$

83 $(\ln|x+1| - \ln|x+4|)/3 + C$

85 $x^2/2 - 3x - \ln|x+1|$
$+ 8\ln|x+2| + C$

87 $\frac{1}{b}(\ln|x| - \ln|x+b/a|) + C$

89 $\ln|z| - \ln|z+1| + C$

91 $(1/\ln 2)\ln|2^t+1| + C$

93 $(1/7)x^7 + 3x^5 + 25x^3 + 125x + C$

95 $\frac{1}{2}x\sqrt{4-x^2} + 2\arcsin(x/2) + C$

97 $-(1/2)\ln|1+\cos^2 w| + C$

99 $x\tan x + \ln|\cos x| + C$

101 $(2/3)(\sqrt{x+1})^3 - 2\sqrt{x+1} + C$

103 $(1/2)\ln|e^{2y}+1| + C$

105 $-1/(z-5) - 5/(2(z-5)^2)$
$+ C$

107 $e^{x^2-x} + C$

109 $-\frac{2}{9}(2+3\cos x)^{3/2} + C$

111 $\sin^3(2\theta)/6 - \sin^5(2\theta)/10 + C$

113 $(x+\sin x)^4/4 + C$

115 $(\ln|x-2| - \ln|x+2|)/4 + C$

117 $(\ln|x| - \ln|x+5|)/5 + C$

119 $(1/3)\arcsin(3x) + C$

121 $2\ln|x-1| - \ln|x+1| - \ln|x| + C$

123 $\arctan(x+1) + C$

125 $-4\ln|x-1| + 7\ln|x-2| + C$

127 Converges
$\int_4^\infty t^{-3/2}\,dt = 1$

129 Converges
$\int_0^\infty we^{-w}\,dw = 1$

131 Converges
$\int_{-\pi/4}^{\pi/4}\tan\theta\,d\theta = 0$

133 Converges
$\int_{10}^\infty \frac{1}{z^2-4}\,dz = (\ln(3/2))/4$

135 Does not converge

137 Does not converge

139 Does not converge

141 $5/6$

143 $11/3$

145 (a) (i) 0
(ii) $\frac{2}{\pi}$
(iii) $\frac{1}{2}$
(b) Smallest to largest:
Average value of $f(t)$
Average value of $k(t)$
Average value of $g(t)$

147 error for $\mathrm{TRAP}(10)$
$\approx 0.0078$

149 (a) 0.5 ml
(b) 99.95%

151 (a) $(\ln x)^2/2$, $(\ln x)^3/3$, $(\ln x)^4/4$
(b) $(\ln x)^{n+1}/(n+1)$

153 (a) $(-9\cos x + \cos(3x))/12$
(b) $(3\sin x - \sin(3x))/4$

155 (a) $x + x/(2(1+x^2)) - (3/2)\arctan x$
(b) $1 - (x^2/(1+x^2)^2) - 1/(1+x^2)$

## Ch. 7 Understanding

1 False

3 False

5 True

7 False

9 False

11 False

13 True

15 False

17 True

19 True

21 False

23 False

25 True

27 False.

## Section 8.1

1 15

3 15/2

5 $(5/2)\pi$

7 1/6

9 $\int_0^9 4\pi\,dx = 36\pi$ cm$^3$

11 $\int_0^5 (4\pi/25)y^2\,dy = 20\pi/3$ cm$^3$

13 $\int_0^5 \pi(5^2 - y^2)\,dy = 250\pi/3$ mm$^3$

15 5 to 1

17 Triangle; $b, h = 1, 3$

19 Quarter circle $r = \sqrt{15}$

21 Hemisphere, $r = 12$

23 Cone, $h = 6, r = 3$

25 $V = (4\pi r^3)/3$

27 (a) $3\Delta x$;
$\int_0^4 3\,dx = 12$ cm$^3$
(b) $8(1 - h/3)\Delta h$;
$\int_0^3 8(1 - h/3)\,dh = 12$ cm$^3$

29 $\int_0^{150} 1400(160 - h)\,dh = 1.785 \cdot 10^7$ m$^3$

## Section 8.2

1 $\pi/5$

3 $\pi(e^2 - e^{-2})/2$

5 $256\pi/15$

7 $\pi^2/4$

9 $\pi((e^6/6) - (e^2/2) + 1/3)$

11 3.526

13 $e - 1$

15 (a) $16\pi/3$
(b) 1.48

17 $V = (16/7)\pi \approx 7.18$

19 $V = (\pi^2/2) \approx 4.935$

21 $V \approx 42.42$

23 $V = (e^2 - 1/2) \approx 3.195$

25 (a) Volume $\approx 152$ in$^3$
(b) About 15 apples

27 $40,000LH^{3/2}/(3\sqrt{a})$

29 (a) $dh/dt = -6/\pi$
(b) $t = \pi/6$

31 (a) $4\int_0^r \sqrt{1 + (-\frac{x}{y})^2}\,dx$
(b) $2\pi r$

33 $e - e^{-1}$

## Section 8.3

1 $1 - e^{-10}$ gm

3 (a) $\displaystyle\sum_{i=1}^{N}(2 + 6x_i)\Delta x$
(b) 16 grams

5 (b) $\sum_{i=1}^{N}[600 + 300\sin(4\sqrt{x_i + 0.15})]\frac{20}{N}$
(c) $\approx 11513$

7 2 cm to right of origin

9 1 gm

11 (a) $\int_0^5 2\pi r(0.115e^{-2r})dr$
(b) 181 cubic meters

13 (a) $\pi r^2 l/2$
(b) $2klr^3/3$

15 $\int_0^{60} \frac{1}{144}g(t)\,dt$ ft$^3$

17 $x = 2$

19 $\pi/2$

21 (a) Right
(b) $2/(1 + 6e - e^2) \approx 0.2$

23 (a) $10/3$ gm
(b) $\overline{x} = 3/5$ cm; $\overline{y} = 3/8$ cm

25 1.25 cm from center of base

27 (a) $16000\delta/3$ gm
(b) 2.5 cm above center of base

## Section 8.4

1 9/2 joules

3 (a) 1.5 joules,
13.5 joules
(b) For $x = 4$ to $x = 5$
Force larger

5 $1.489 \cdot 10^{10}$ joules

7 11,000 ft-lbs

9 1,404,000 ft-lb

11 2,822,909.50 ft-lb

13 354,673 ft-lb

15 (a) Force on dam
$\approx \sum_{i=0}^{N-1} 1000(62.4h_i)\Delta h$
(b) $\int_0^{50} = 1000(62.4h)\,dh =$
78,000,000 pounds

17 Bottom: 1497.6 lbs
Front and back: 499.2 lbs
Both sides: 374.4 lbs

19 (a) 21,840 lb/ft$^2$; 151.7 lb/in$^2$
(b) (i) 546,000 pounds
(ii) 542,100 pounds

21 $9800\int_0^{100} h(3600 - 6h)\,dh = 1.6 \cdot 10^{11}$ newtons

23 60 joules

25 $(GmM)/(a(a + l))$

27 $GMmy/(a^2 + y^2)^{3/2}$ toward center

## Section 8.5

1 \$15,319.30.

3 \$8,242, \$12,296

7 (a) \$5820 per year
(b) \$36,787.94

9 Installments

11 \$46,800

13 (a) 10.6 years
(b) 624.9 million dollars

15 \$85,750,000

19 (a) Less
(b) Can't tell
(c) Less

## Section 8.6

5 pdf; $1/2$

7 pdf; $2/3$

9 pdf; 2

11 (a) 0.9 m–1.1 m

15 (a) Cumulative distribution increasing
(b) Vertical 0.2, horizontal 2

17 (a) 22.1%
(b) 33.0%
(c) 30.1%
(d) $C(h) = 1 - e^{-0.4h}$

19 (b) About $3/4$

## Section 8.7

5 (a) 0.684 : 1
(b) 1.6 hours
(c) 1.682 hours

7 (a) $P(t)$ = Fraction of population who survive up to $t$ years after treatment
(b) $S(t) = e^{-Ct}$
(c) 0.178

9 (a) $p(x) = \left(e^{-\frac{1}{2}\left(\frac{x-100}{15}\right)^2}\right) / \left(15\sqrt{2\pi}\right)$
(b) 6.7% of the population

11 (c) $\mu$ represents the mean of the distribution, while $\sigma$ is the standard deviation.

13 (b)

15 (a) $p(r) = 4r^2e^{-2r}$
(b) Mean: 1.5 Bohr radii
Median: 1.33 Bohr radii
Most likely:
1 Bohr radius

## Chapter 8 Review

1 $\int_0^b h\,dx = hb$

3 $\int_0^b \frac{hx}{b}\,dx = \frac{hb}{2}$

5 $\int_0^{12} \pi h\,dh = 72\pi$

7 $\int_0^{\pi} \sqrt{1+\cos^2 x}\,dx$

9 (b) $\sum_{i=1}^{N} \pi x_i \Delta x$
(c) Volume $= \pi/2$

11 $V = \pi$

13 (a) $a = b/l$
(b) $(1/3)\pi b^2 l$

15 Volume $= 6\pi^2$

17 (a) $2/3$ gm
(b) Less than $1/2$
(c) $\overline{x} = \overline{y} = 9/20$ cm

19 1000 ft-lb

21 1,170,000 lbs

23 (a) $\sum_{i=0}^{n-1} (2000 - 100t_i) \times e^{-0.1t_i}\,\Delta t$
(b) $\int_0^M e^{-0.10t}(2000 - 100t)\,dt$
(c) After 20 years
$\$11{,}353.35$

25 (a) $\sum_{i=1}^{N} \pi \left(\frac{3.5\cdot 10^5}{\sqrt{h+600}}\right)^2 \Delta h$
(b) $1.05 \cdot 10^{12}$ cubic feet

27 $\int_0^R 2\pi N r S(r)\,dr$

29 (a) $\pi P R^4/(8\eta l)$

31 (a) $\pi h^2/(2a)$
(b) $\pi h/a$
(c) $dh/dt = -k$
(d) $h_0/k$

33 The thin spherical shell.

35 (c) $\pi^2 a^3/8$

37 (a) $(1/2)\sqrt{t}\sqrt{1+4t} + (1/4)\text{arcsinh}\,(2\sqrt{t})$
(b) $t$

## Ch. 8 Understanding

1 True

3 False

5 False

7 False

9 True

11 True

13 False

15 False

17 False

19 True

21 False

23 False

25 False

27 True

29 False

## Section 9.1

1 Yes, $a = 1$, ratio $= -1/2$

3 Yes, $a = 5$, ratio $= -2$

5 No

7 Yes, $a = 1$, ratio $= -x$

9 No

11 $y^2/(1-y)$, $|y| < 1$

13 $1/(1+y^2)$, $|y| < 1$

15 $-4/3$

17 $1/54$

19 260.42 mg

21 (a) $P_n = 250(0.04) + 250(0.04)^2 + 250(0.04)^3 + \cdots + 250(0.04)^{n-1}$
(b) $P_n = 250 \cdot 0.04(1-(0.04)^{n-1})/(1-0.04)$
(c) $\lim_{n\to\infty} P_n \approx 10.42$
We'd expect the difference between them to be 250 mg.

23 (a) $h_n = 10(3/4)^n$
(b) $D_1 = 10$ feet
$D_2 = h_0 + 2h_1 = 25$ feet
$D_3 = h_0 + 2h_1 + 2h_2 = 36.25$ feet
$D_4 = h_0 + 2h_1 + 2h_2 + 2h_3 \approx 44.69$ feet
(c) $D_n = 10 + 60\left(1 - (3/4)^{n-1}\right)$

25 (a) \$1250
(b) 12.50

27 \$900 million

## Section 9.2

1 0

3 0

5 Does not exist

7 $4/7$

9 Converges

11 Converges

13 Diverges

15 Diverges

17 Converges

19 Diverges

21 Not convergent

25 For all $\epsilon > 0$, there is an $N$ such that $|S_n - L| < \epsilon$ for all $n \geq N$

31 12

## Section 9.3

5 Does not converge

7 Converges

9 Converges

11 Converges

13 Converges

15 Converges

17 Converges

19 Does not converge

21 Converges

23 Diverges

25 Converges

27 Diverges

## Section 9.4

1 Yes

3 No

5 $1 \cdot 3 \cdot 5 \cdots (2n-1)x^n/(2^n \cdot n!);\ n \geq 1$

7 $(-1)^k (x-1)^{2k}/(2k)!;\ k \geq 0$

9 $(x-a)^n/(2^{n-1} \cdot n!);\ n \geq 1$

11 $1/5$

13 2

15 1

17 $\infty$

19 $1/4$

21 1

23 Diverges when $x = 1/2$, converges when $x = -1/2$

## Chapter 9 Review

3 Converges

5 Converges

7 Diverges

9 Diverges

11 Diverges

13 $1/4$

15 $\infty$

17 (a) $\approx 1\%$
(b) $Q_1 = 250$
$Q_2 = 250 + 250(0.01)$
$Q_3 = 250 + 250(0.01) + 250(0.01)^2$
$Q_4 = 250 + 250(0.01) + 250(0.01)^2 + 250(0.01)^3$
(c) $Q_3 = \frac{250(1-(0.01)^3)}{1-0.01} \approx 252.5$
$Q_4 = \frac{250(1-(0.01)^4)}{1-0.01} \approx 252.5$
(d) $Q_n = 250 + 250(0.01) + 250(0.01)^2 + \cdots + 250(0.01)^{n-1} = \frac{250(1-(0.01)^n)}{1-0.01}$
(e) $Q = \lim_{n\to\infty} Q_n = \frac{250}{1-0.01} = 252.5$

19 \$25,503

21 \$926.40

23 (a) \$1000
(b) When the interest rate is 5%, the present value equals the principal.
(c) The value of the bond.
(d) Because the present value is more than the principal.

27 (b) Coefficient of $x$ stays same
coefficient of $x^2$ stays same after $T_2(S_2(x))$
(c) Coefficient of $x^3$ is 31/6
(d) Coefficient of $x^k$ stays same after $T_k(S_k(x))$

## Ch. 9 Understanding

1 True

3 True

5 True

7 True

9 False

11 False

13 True

15 False

17 True

19 True

21 False

23 False

25 False

## Section 10.1

1 $P_4(x) = 1 - x + x^2 - x^3 + x^4$
$P_6(x) = 1 - x + x^2 - x^3 + x^4 - x^5 + x^6$
$P_8(x) = 1 - x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8$

3 $P_2(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2$
$P_3(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$
$P_4(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4$

5 $P_3(x) = P_4(x) = x - \frac{1}{3}x^3$

7 $P_2(x) = 1 - \frac{1}{3}x - \frac{1}{9}x^2$
$P_3(x) = 1 - \frac{1}{3}x - \frac{1}{9}x^2 - \frac{5}{81}x^3$
$P_4(x) = 1 - \frac{1}{3}x - \frac{1}{9}x^2 - \frac{5}{81}x^3 - \frac{10}{243}x^4$

9 $P_2(x) = 1 - \frac{1}{2}x + \frac{3}{8}x^2$
$P_3(x) = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3$
$P_4(x) = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \frac{35}{128}x^4$

11 $P_4(x) = 1 - \frac{1}{2!}\left(x - \frac{\pi}{2}\right)^2 + \frac{1}{4!}\left(x - \frac{\pi}{2}\right)^4$

13 $P_4(x) = e[1 + (x-1) + \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 + \frac{1}{24}(x-1)^4]$

15 $c < 0, b > 0, a > 0$

17 $a < 0, b > 0, c > 0$

19 (a) 0
(b) 3
(c) $-24$
(d) 0
(e) 3600

21 $P_2(x) = 4x^2 - 7x + 2$
$f(x) = P_2(x)$

23 (a) If $f(x)$ is a polynomial of degree $n$, then $P_n(x)$, the $n^{\text{th}}$ degree Taylor polynomial for $f(x)$ about $x = 0$, is $f(x)$ itself.

27 (a) 3/7
(b) 0

29 (a) $P_4(x) = 1 + x^2 + \frac{1}{2}x^4$
(b) If we substitute $x^2$ for $x$ in the Taylor polynomial for $e^x$ of degree 2, we will get $P_4(x)$, the Taylor polynomial for $e^{x^2}$ of degree 4.
(c) $P_{20}(x) = 1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \cdots + \frac{x^{20}}{10!}$
(d) $e^{-2x} \approx 1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4 - \frac{4}{15}x^5$

31 (a) $0.94444\cdots$
(b) $0.94611\cdots$

## Section 10.2

1 $f(x) = 1 + x + x^2 + x^3 + \cdots$

3 $f(x) = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \cdots$

5 $\sin x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(x - \frac{\pi}{4}\right)^3 - \cdots$

7 $\sin\theta = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(\theta + \frac{\pi}{4}\right) + \frac{\sqrt{2}}{4}\left(\theta + \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(\theta + \frac{\pi}{4}\right)^3 + \cdots$

9 $\frac{1}{x} = 1 - (x-1) + (x-1)^2 - (x-1)^3 + \cdots$

11 $\frac{1}{x} = -1 - (x+1) - (x+1)^2 - (x+1)^3 - \cdots$

13 $(-1)^n x^n;\ n \geq 0$

15 $(-1)^{n-1} x^n / n;\ n \geq 1$

17 $(-1)^k x^{2k+1}/(2k+1);\ k \geq 0$

19 $(-1)^k x^{2k+2}/(2k)!;\ k \geq 0$

21 (a) $\ln(1+2x) = 2x - 2x^2 + \frac{8}{3}x^3 + \cdots$
(b) To get the expression for $\ln(1+2x)$ from the series for $\ln(1+x)$, substitute $2x$ for $x$ in the series.
(c) $-\frac{1}{2} < x < \frac{1}{2}$

23 $-1 < x < 1$

25 1

27 $e^2$

29 $4/3$

31 $\ln(3/2)$

33 $e^3$

35 $e^{-0.1}$

37 $e^{0.2} - 1$

39 $\frac{d}{dx}(x^2 e^{x^2})|_{x=0} = 0$
$\frac{d^6}{dx^6}(x^2 e^{x^2})|_{x=0} = \frac{6!}{2} = 360$

## Section 10.3

1 $\sqrt{1-2x} = 1 - x - \frac{x^2}{2} - \frac{x^3}{2} - \cdots$

3 $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$

5 $\ln(1-2y) = -2y - 2y^2 - \frac{8}{3}y^3 - 4y^4 - \cdots$

7 $\frac{1}{\sqrt{1-z^2}} = 1 + \frac{1}{2}z^2 + \frac{3}{8}z^4 + \frac{5}{16}z^6 + \cdots$

9 $\frac{z}{e^{z^2}} = z - z^3 + \frac{z^5}{2!} - \frac{z^7}{3!} + \cdots$

11 $e^t \cos t = 1 + t - \frac{t^3}{3} - \frac{t^4}{6} + \cdots$

13 $1 + 3x + 3x^2 + x^3$
$0 \cdot x^n$ for $n \geq 4$

15 $1 + \frac{1}{2}y^2 + \frac{3}{8}y^4 + \cdots + \frac{(1/2)(3/2)\cdots(\frac{1}{2}+n-1)\,y^{2n}}{n!} + \cdots;$
$n \geq 1$

17 $\frac{a}{\sqrt{a^2+x^2}} = 1 - \frac{1}{2}(\frac{x}{a})^2 + \frac{3}{8}(\frac{x}{a})^4 - \frac{5}{16}(\frac{x}{a})^6 + \cdots$

19 Smallest: $\cos\theta$
Largest: $1 + \sin\theta$

21 (a) I
(b) IV
(c) III
(d) II

23 (a) $1 + x^2/2! + x^4/4! + x^6/6! + x^8/8!$
(b) 1.54308
(c) $x + x^3/3! + x^5/5! + x^7/7!$

25 $E \approx \frac{kQ}{R^2}\left(\frac{4}{R} + \frac{8}{R^3}\right)$

29 (a) $((l_1+l_2)/c) \cdot (v^2/c^2 + (5/4)v^4/c^4)$
(b) $v^2, (l_1+l_2)/c^3$

31 (a) $\frac{GMm}{h^2}\left(1 - \frac{3}{4}\frac{a^2}{h^2}\right)$
(b) $F \approx (GMm)/(h^2)$
(c) 0.03%

33 (a) Left $= 0.8076$
Right $= 0.6812$
(b) $1 - x^2 + x^4/2 - x^6/6$
(c) 0.74286

## Section 10.4

1 $|E_3| \leq 0.033$

3 $|E_3| \leq 4.375$

5 (a) Underestimate
(b) 1

7 (a) Overestimate: $0 < \theta \leq 1$
Underestimate: $-1 \leq \theta < 0$
(b) $|E_2| \leq 0.17$

9 (a) $4, 0.2$
(b) 1
(c) $-4 \leq x \leq 4$

11 Four decimal places: $n = 7$
Six decimal places: $n = 9$

## Section 10.5

1 Not a Fourier series

3 Fourier series

5 $F_1(x) = F_2(x) = \frac{4}{\pi}\sin x$
$F_3(x) = \frac{4}{\pi}\sin x + \frac{4}{3\pi}\sin 3x$

7 99.942% of the total energy

9 $H_n(x) = \frac{\pi}{4} + \sum_{i=1}^{n} \frac{(-1)^{i+1}\sin(ix)}{i} + \sum_{i=1}^{[n/2]} \frac{-2}{(2i-1)^2\pi}\cos((2i-1)x)$,
where $[n/2]$ denotes the biggest integer smaller than or equal to $n/2$.

11 (a) $F_3(x) = \frac{1}{2} + \frac{2}{\pi}\cos x - \frac{2}{3\pi}\cos 3x.$
(b) There are cosines instead of sines, but the energy spectrum remains the same

13 $F_4(x) = 1 - \frac{4}{\pi}\sin(\pi x) - \frac{2}{\pi}\sin(2\pi x) - \frac{4}{3\pi}\sin(3\pi x) - \frac{1}{\pi}\sin(4\pi x)$

19 (a) 15.9155%, 0.451808%
(b) $(4\sin^2\frac{k}{2})/(k^2\pi^2)$
(c) The constant term and the first five harmonics are needed.
(d) $F_5(x) = \frac{1}{2\pi} + \frac{2\sin(1/2)}{\pi}\cos x + \frac{\sin 1}{\pi}\cos 2x + \frac{2\sin(3/2)}{3\pi}\cos 3x + \frac{\sin 2}{2\pi}\cos 4x + \frac{2\sin(5/2)}{5\pi}\cos 5x$

21 (a) 31.83%, 76.91%
(b) 90.07%
(c) $F_3(x) = \frac{1}{\pi} + \frac{2\sin 1}{\pi}\cos x + \frac{\sin 2}{\pi}\cos 2x + \frac{2\sin 3}{3\pi}\cos 3x$

# Chapter 10 Review

1 $e^x \approx 1 + e(x-1) + \frac{e}{2}(x-1)^2$

3 $\sin x \approx -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}(x + \frac{\pi}{4}) + \frac{1}{2\sqrt{2}}(x + \frac{\pi}{4})^2$

5 $P_3(x) = 4 + 12(x-1) + 10(x-1)^2 + (x-1)^3$

7 $\sin t^2 = t^2 - \frac{t^6}{3!} + \frac{t^{10}}{5!} - \frac{t^{14}}{7!} + \cdots$

9 $\frac{1}{1-4z^2} = 1 + 4z^2 + 16z^4 + 64z^6 + \cdots$

11 $\sqrt{R-r} = \sqrt{R}(1 - \frac{1}{2}\frac{r}{R} - \frac{1}{8}\frac{r^2}{R^2} - \frac{1}{16}\frac{r^3}{R^3} - \cdots)$

13 $3/4$

15 $e^{-2}$

17 $3e$

19 1.45

21 $-1 < x < 1; R = 1$

23 (a) 2
(b) Near $\theta = 0$, we make the approximation $\frac{\sin 2\theta}{\theta} \approx 2 - \frac{4}{3}\theta^2$.

25 (a) $2 - (x^2/4) - (x^4/64)$
(b) 1.9135
(c) $(\sqrt{3}/2) + (\pi/3)$
(d) Three decimal places

27 (a) $V(x) \approx V(0) + V''(0)x^2/2$
$V''(0) > 0$
(b) Force $\approx -V''(0)\,x$
$V''(0) > 0$
Toward origin

29 (a) Set $\frac{dV}{dr} = 0$, solve for $r$. Check for max or min.
(b) $V(r) = -V_0 + 72V_0 r_0^{-2} \cdot (r - r_0)^2 \cdot \frac{1}{2} + \cdots$
(d) $F = 0$ when $r - r_0$

33 (a) $g(x) \approx P_n(x) = g(0) + \frac{g''(0)}{2!}x^2 + \frac{g'''(0)}{3!}x^3 + \cdots + \frac{g^{(n)}(0)}{n!}x^n$
(b) If $g''(0) > 0$:
0 is a local minimum
If $g''(0) < 0$:
0 is a local maximum.

37 (b) If the amplitude of the $k^{\text{th}}$ harmonic of $f$ is $A_k$, then the amplitude of the $k^{\text{th}}$ harmonic of $f'$ is $kA_k$.
(c) The energy of the $k^{\text{th}}$ harmonic of $f'$ is $k^2$ times the energy of the $k^{\text{th}}$ harmonic of $f$.

41 (a) $P_7(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$
$Q_7(x) = x - \frac{2x^3}{3} + \frac{2x^5}{15} - \frac{4x^7}{315}$
(b) For $n$ odd, ratio of coefficients of $x^n$ is $2^{n-1}$

43 (a) $P_{10}(x) = 1 + \frac{x^2}{12} - \frac{x^4}{720} + \frac{x^6}{30240} - \frac{x^8}{1209600} + \frac{x^{10}}{47900160}$
(b) All even powers
(c) $f$ even

# Ch. 10 Understanding

1 False

3 False

5 False

7 True

9 False

11 False

13 True

15 False

17 True

19 True

21 False

23 True

# Section 11.1

1 (a) (III)
(b) (V)
(c) (I)
(d) (II)
(e) (IV)

3 Yes.

5 $-2$

13 (b) 1

15 (a) (IV)
(b) None
(c) (V)
(d) (I), (II)
(e) (III)

# Section 11.2

3 (b) $y = -x - 1$

5 (b) $y = n\pi$

7 (c) Increasing:
$-1 < y < 2$
Decreasing:
$y > 2$ or $y < -1$
Horizontal:
$y = 2$ or $y = -1$

9 (a) II
(b) VI
(c) IV
(d) I
(e) III
(f) V

# Section 11.3

1 (a) $y(0.4) \approx 1.5282$
(b) $y(0.4) = -1.4$

3 (b) $y(x) = x^4/4$

7 (a) $\Delta x = 0.5, y(1) \approx 1.5$
$\Delta x = 0.25, y \approx 1.75$
(b) $y = x^2 + 1$, so $y(1) = 2$
(c) Yes

9 (a) $B \approx 1050$
(b) $B \approx 1050.63$
(c) $B \approx 1050.94$

# Section 11.4

1 $P = 20e^{0.02t}$

3 $P(t) = \sqrt{2t+1}$

5 $L(p) = 100e^{p/2}$

7 $m = 5e^{3t-3}$

9 $z = 5e^{5t-5}$

11 $z(y) = e^{y^2/2}$

13 $P = 104e^t - 4$

15 $m = 3000e^{0.1t} - 2000$

17 $y = 200 - 150e^{t/2}$

19 $R(r) = 1 - 0.9e^{1-r}$

21 $z = -\ln(1 - t^2/2)$

23 $y = -2/(t^2 + 2t - 4)$

25 $w = 2/(\cos\theta^2 + 1)$

27 $w = -2/(\sin\psi^2 - 2)$

29 $Q = Ae^{t/k}$

31 $Q = b - Ae^{-t}$

33 $P(t) = (1/a)(Ae^{at} - b)$

35 $y(t) = -1/\left(k(t + t^3/3) + C\right)$

37 $L(x) = b + Ae^{k((1/2)x^2 + ax)}$

39 $x = \arcsin(At^{\ln t + 1})$

41 $y = 2(2^{-e^{-t}})$

43 (c) $y(x) = Ae^{x^2/2}$

45 (c) $y^2 - x^2 = 2C$

# Section 11.5

1 (a) (I)
(b) (IV)
(c) (III)

3 (a) $y = 2$: stable; $y = -1$: unstable

5 $y = 3$ stable;
$y = 1$ unstable

7 (a) $dS/dt = kS$
(b) $S = Ce^{kt}$
(c) $C = 5$
(d) $S = 5e^{0.1576t}$

9 (a) 69,300 barrels/year
(b) 25.9 years

11 (a) $dH/dt = k(68 - H)$
(b) $H = 68 - Ce^{-kt}$
(c) $57.8^\circ$F

13 (a) $\frac{dB}{dt} = \frac{r}{100}B$
Constant $= \frac{r}{100}$
(b) $B = Ae^{\frac{r}{100}t}$

15 Michigan: 72 years
Ontario: 18 years

17 (b) $dQ/dt = -0.0187Q$
(c) 3 days

19 (a) $dT/dt = -k(T - 68)$
(b) $T = 68 + 22.3e^{-0.06t}$;
3:45 am.

21 About 2150 B.C.

23 (a) $\frac{dD}{dt} = kD$

# Section 11.6

1 $da/dt = -ka/m$
$a = ge^{-kt/m}$

3 (a) $dB/dt = 0.08B - 2000$
(b) $B = 25{,}000 + Ae^{0.08t}$
(c) (i) \$17,540.88
(ii) \$32,549.12

5 $dD/dt = -0.75(D - 4)$
Equilibrium = 4 g/cm$^2$

7 about 3 days

9 $y = A\sqrt{x}$

11 (a) $dy/dt = ky$
(b) 0.2486 grams

13 (a) $I = Ae^{-kl}$
(b) 20 feet: 75%
25 feet: 82.3%

15 (a) $dx/dt = k(a - x)(b - x)$
(b) $x = ab(e^{bkt} - e^{akt})/(be^{bkt} - ae^{akt})$

17 (b) $dc/dt = (43.2/35{,}000) - 0.082c$
(c) $c = 0.015(1 - e^{-0.082t})$
As $t \to \infty$,
$c \to 0.015$ mg/ml

19 (a) $dp/dt = -k(p - p^*)$

(b) $p = p^* + (p_0 - p^*)e^{-kt}$
(d) As $t \to \infty$, $p \to p^*$

21 (a) $\frac{dc}{dt} = \frac{0.0001}{60} - \frac{0.002}{60}c$
(b) $c = 0.05 - 0.05e^{-3\times10^{-5}t}$
(c) $c \to 0.05$

23 (a) $\frac{dS}{dt} = 600 - \frac{3S}{100{,}000}$
($S$ in grams)
(b) $S(t) =$
$2 \times 10^7(1 - e^{-\frac{3}{100{,}000}t})$
(c) As $t \to \infty$
$S(t) \to 2 \times 10^7$

## Section 11.7

1 $P = (6.6 \times 10^6)e^{0.002t}$

3 (a) 0.252
(b) All the available land will be used for farming
(c) In 1974
(d) In 1974

5 $dP/dt$ is largest in 1920.
Using this population to estimate the 1990 population, we get 211.4 million.

7 (a) $P = 5000/(1 + 499e^{-1.78t})$
(c) $t \approx 3.5$; $P \approx 2500$

9 (a) $dI/dt = k(M - I)$
$(k > 0)$
(b) $dI/dt = kI(M - I)$
$(k > 0)$

11 (a) $dp/dt = kp(B - p)$
$(k > 0)$
(b) Half of the tin

13 (c) $P = 0$ (stable)
$P = 4$ (unstable)

15 (b) $dP/dt < 0$ for $P < b/a$
$dP/dt > 0$ for $P > b/a$
(c) $P > b/a$: increase
$P < b/a$: extinction

## Section 11.8

3 $r = r_0e^{-t}$, $w = w_0e^{t}$

5 Worms decrease, robins increase. Long run: populations oscillate.

9 Robins increase;
Worms constant, then decrease;
Both oscillate in long run

11 $x$ and $y$ increase, about same rate

13 $x$ decreases quickly while $y$ increases more slowly.

15 (a) Symbiosis
(b) Both $\to \infty$ or both $\to 0$

17 (a) Predator-prey
(b) $x$, $y$ tend to $\approx 1$

19 (a) $dy/dx = bx/ay$

21 (b) $dx/dt = -xy, dy/dt = -x$
(c) $dy/dx = 1/y$
soln: $y^2/2 = x + C$
(d) If $C > 0$, $y$ wins
If $C < 0$, $x$ wins
If $C = 0$, mutual annihilation

23 (a) $\frac{dy}{dx} = \frac{y(x+3)}{x(y+2)}$
$y^2e^y = Ax^3e^x$
(d) $y^2e^y = x^3e^{x+4}$
(e) $y = x = e^{-4} \approx 0.0183$
(f) $y \approx e^{-13}$

## Section 11.9

1 (a) $\frac{dS}{dt} = 0$ where
$S = 0$ or $I = 0$.
$\frac{dI}{dt} = 0$ where
$I = 0$ or $S = 192$.
(b) Where $S > 192$,
$\frac{dS}{dt} < 0$ and $\frac{dI}{dt} > 0$.
Where $S < 192$,
$\frac{dS}{dt} < 0$ and $\frac{dI}{dt} < 0$.

3 (a) $(0, 0)$; $(5, 2)$
(b) $dx/dt$ and $dy/dt$ not both 0

5 Horizontal nullclines; $x = \frac{1}{2}$, $y = 0$
Vertical nullclines; $y = \frac{1}{3}(2 - x)$, $x = 0$
Equilibrium points; $(0, 0)$, $(\frac{1}{2}, \frac{1}{2})$, $(2, 0)$

7 Horizontal nullclines; $y = 0$, $y = 2(1 - x)$
Vertical nullclines; $x = 0$, $y = 1 - x/3$
Equilibrium points; $(0, 0)$, $(0, 2)$, $(3, 0)$, $(3/5, 4/5)$

9 $\frac{dx}{dt} = 0$ when
$x = 0$ or $y + \frac{x}{2} = 1$.
$\frac{dy}{dt} = 0$ when
$y = 0$ or $x + \frac{y}{3} = 1$.
Equilibrium points:
$(2, 0), (0, 3), (0, 0), (\frac{4}{5}, \frac{3}{5})$

11 (a) $\frac{dP_1}{dt} = 0$ where $P = 0$ or
$P_1 + 3P_2 = 13$.
$\frac{dP_2}{dt} = 0$ where $P = 0$ or
$P_2 + 0.4P_1 = 6$.

13 (a) $dx/dt = 0$ where
$x = 23.3$
$dy/dt = 0$ where
$8.2x - 0.8y - 142 = 0$
Equilibrium point:
(23.3,61.7)

## Section 11.10

7 (c) $A = 5$, $\phi = 0.93$

9 $\sqrt{58}$

11 $\omega = 2$, $A = 13$, $\psi = tan^{-1}12/5$

13 Highest point, at rest

15 Lowest point, at rest

17 (a) goes with (II)
(b) goes with (I) and (IV)
(c) goes with (III)
(I) $x = 2\sin 2t$
(II) $x = -\sin t$
(III) $x = \cos 4t$
(IV) $x = -3\sin 2t$

19 (a) $x = v_0\sqrt{l/g}\sin\sqrt{g/l}t$
(b) $x = x_0\cos\sqrt{g/l}t$

21 (a) $x'' + (g/2)x = 0, x(0) = 5, x'(0) = 0$
(b) $x = 5\cos((\sqrt{g/2})t)$

23 (a) $A\sin(\omega t + \phi) =$
$(A\sin\phi)\cos\omega t +$
$(A\cos\phi)\sin\omega t$

25 $C = \frac{1}{20}$ farads

## Section 11.11

1 $y(t) = C_1e^{-t} + C_2e^{-3t}$

3 $y(t) =$
$C_1e^{-2t}\cos t + C_2e^{-2t}\sin t$

5 $s(t) =$
$C_1\cos\sqrt{7}t + C_2\sin\sqrt{7}t$

7 $z(t) = C_1e^{-t/2} + C_2e^{-3t/2}$

9 $p(t) = C_1e^{-t/2}\cos\frac{\sqrt{3}}{2}t +$
$C_2e^{-t/2}\sin\frac{\sqrt{3}}{2}t$

11 $y(t) = A + Be^{-2t}$

13 $y(t) = -2e^{-3t} + 3e^{-2t}$

15 $y(t) = \frac{1}{5}e^{4t} + \frac{4}{5}e^{-t}$

17 $y(t) = \frac{5}{4}e^{-t} - \frac{1}{4}e^{-5t}$

19 $y(t) = 2e^{-3t}\sin t$

21 $y(t) = \frac{1}{1-e}e^{-2t} + \frac{-e}{1-e}e^{-3t}$

23 $p(t) = 20e^{(\pi/2)-t}\sin t$

25 (a) (IV)
(b) (II)
(c) (I)
(d) (III)

27 $k = 6$
$y(t) = C_1e^{2t} + C_2e^{3t}$

29 (iii)

31 (iv)

33 (ii)

35 Overdamped: $c < 2$
Critically damped: $c = 2$
Underdamped: $c > 2$

37 $z(t) = 3e^{-2t}$

39 (a) $Q(t) = \frac{2}{\sqrt{3}}(e^{(-1+\frac{\sqrt{3}}{2})t}$
$- e^{(-1-\frac{\sqrt{3}}{2})t})$
(b) $Q(t) =$
$\frac{1}{\sqrt{3}}((2 + \sqrt{3})e^{(-1+\frac{\sqrt{3}}{2})t} -$
$(2 - \sqrt{3})e^{(-1-\frac{\sqrt{3}}{2})t})$

41 (a) $Q(t) = 16e^{-\frac{1}{8}t}\sin\frac{t}{8}$
(b) $Q(t) =$
$2e^{-\frac{1}{8}t}\left(\sin\frac{t}{8} + \cos\frac{t}{8}\right)$
(c) Goes from overdamped to underdamped

43 No

## Chapter 11 Review

1 (a) Yes (b) No (c) Yes
(d) No (e) Yes (f) Yes
(g) No (h) Yes (i) No
(j) Yes (k) Yes (l) No

3 $y(x) = 40 + Ae^{0.2x}$

5 $H = Ae^{0.5t} - 20$

7 $P = 100Ae^{0.4t}/(1 + Ae^{0.4t})$

9 $P = \frac{40000}{3}(e^{0.03t} - 1)$

11 $y = \sqrt[3]{33 - 6\cos x}$

13 $\frac{1}{2}y - 4\ln|y| = 3\ln|x| - x$
$+ \frac{7}{2} - 4\ln 5.$

15 $z(t) = 1/(1 - 0.9e^t)$

17 $20\ln|y| - y =$
$100\ln|x| - x + 20\ln 20 - 19$

19 $y = \ln(e^x + e - 1)$

21 $z = \sin(-e^{\cos\theta} + \pi/6 + e)$

23 $y = -\ln(\frac{\ln 2}{3}\sin^2 t\cos t$
$2\ln 23\cos t - \frac{2\ln 2}{3} + 1)/\ln 2$

25 $y(t) = C_1e^{t/3} + C_2e^{-t/3}$

27 $y(t) = C_1e^{-2t} + C_2e^{-4t}$

29 $y(t) =$
$C_1e^{-t}\cos 3t + C_2e^{-t}\sin 3t$

31 (a) $y(1) \approx 3.689$
(b) Overestimate
(c) $y = 5 - 4e^{-x}$
$y(1) = 5 - 4e^{-1} \approx$
$3.528$
(d) $\approx 3.61$

33 Always overdamped

35 2.21 hours

37 5.7 days, 126.32 liters

39 (a) $dB/dt = 0.1B + 1200$
(b) $B = 12{,}000(e^{0.1t} - 1)$
(c) \$7784.66

41 $dI/dt = 0.001I(1 - 10(I/M))$; $M/10$

43 (b) $d^2r/dt^2 = -ABr$
$r(t) = C_1 \cos\sqrt{AB}t + C_2 \sin\sqrt{AB}t$
(c) $r(t) = \cos\sqrt{AB}t$
$j(t) = \sqrt{\frac{A}{B}} \sin\sqrt{AB}t$
(d) One quarter of the time.

45 (a) $P = 0, 1, 2$
(c) 1/2, 3/2, 0, 2

## Ch. 11 Understanding

1 False
3 False
5 False
7 False
9 True
11 False
13 True
15 True
17 True
19 True
21 False
23 False
25 True
27 False
29 False
31 True
33 True
35 $dy/dx = y - x^2$
37 $dy/dx = x/y$

## Appendix A

1 (a) $y \le 30$
(b) two zeros
3 $-1.05$
5 2.5
7 $x = 1.05$
9 0.45
11 1.3
13 (a) $x = -1.15$
(b) $x = 1, x = 1.41$, and $x = -1.41$
15 (a) $x \approx 0.7$
(b) $x \approx 0.4$
17 (a) 4 zeros
(b) $[0.65, 0.66]$, $[0.72, 0.73]$, $[1.43, 1.44]$, $[1.7, 1.71]$.
19 (b) $x \approx 5.573$
21 Bounded $-5 \le f(x) \le 4$
23 Not bounded.

## Appendix B

1 $(1, 0)$
3 $(\sqrt{2}, \pi/4)$
5 $(4.2, 5\pi/4)$
7 $(5, 0.92)$
9 (1,0)
11 $(-2, 0)$
13 $(\frac{5\sqrt{3}}{2}, -\frac{5}{2})$
15 $(\cos 1, \sin 1)$
17 Circle, radius 2, about origin
19 Line through origin, angle of $3\pi/4$
23 $r \le 4, -\pi/2 \le \theta \le \pi/2$
25 $r \le 1, 0 \le \theta \le \pi/4$
27 $3 \le r \le 7, \pi/3 \le \theta \le 2\pi/3$
29 $y = \sqrt{3}x$
31 $(x-1)^2 + y^2 = 1$

## Appendix C

1 $2e^{i\pi/2}$
3 $\sqrt{2}e^{i\pi/4}$
5 $0e^{i\theta}$, for any $\theta$.
7 $\sqrt{10}e^{i\arctan(-3)}$
9 $-3 - 4i$
11 $-5 + 12i$
13 $\frac{1}{4} - \frac{9i}{8}$
15 $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$
17 $-125i$
19 $\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$
21 $\frac{\sqrt{3}}{2} + \frac{i}{2}$
23 $-2^{50}$
25 $2i\sqrt[3]{4}$
27 $\frac{1}{\sqrt{2}}\cos(\frac{-\pi}{12}) + i\frac{1}{\sqrt{2}}\sin(\frac{-\pi}{12})$
29 $-i, -1, i, 1$
$i^{-36} = 1, i^{-41} = -i$
31 $A_1 = 1 + i$
$A_2 = 1 - i$
37 True
39 False
41 True

## Appendix D

1 (a) $f'(x) = 3x^2 + 6x + 3$
(b) At most one
(c) $[0, 1]$
(d) $x \approx 0.913$
3 $\sqrt[4]{100} \approx 3.162$
5 $x \approx 0.511$
7 $x \approx 1.310$
9 $x \approx 1.763$
11 $x \approx 0.682328$