PART I

CMOS FUNDAMENTALS

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CHAPTER 1

ELECTRICAL CIRCUIT ANALYSIS

1.1 INTRODUCTION

We understand complex integrated circuits (ICs) through simple building blocks. CMOS transistors have inherent parasitic structures, such as diodes, resistors, and capacitors, whereas the whole circuit may have inductor properties in the signal lines. We must know these elements and their many applications since they provide a basis for understanding transistors and whole-circuit operation.

Resistors are found in circuit speed and bridge-defect circuit analysis. Capacitors are needed to analyze circuit speed properties and in power stabilization, whereas inductors introduce an unwanted parasitic effect on power supply voltages when logic gates change state. Transistors have inherent diodes, and diodes are also used as electrical protective elements for the IC signal input/output pins. This chapter examines circuits with resistors, capacitors, diodes, and power sources. Inductance circuit laws and applications are described in later chapters. We illustrate the basic laws of circuit analysis with many examples, exercises, and problems. The intention is to learn and solve sufficient problems to enhance one's knowledge of circuits and prepare for future chapters. This material was selected from an abundance of circuit topics as being more relevant to the later chapters that discuss how CMOS transistor circuits work and how they fail.

1.2 VOLTAGE AND CURRENT LAWS

Voltage, current, and resistance are the three major physical magnitudes upon which we will base the theory of circuits. Voltage is the potential energy of a charged particle in an electric field, as measured in units of volts (V), that has the physical units of Newton ·

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m/coulomb. Current is the movement of charged particles and is measured in coulombs per second or amperes (A). Electrons are the charges that move in transistors and interconnections of integrated circuits, whereas positive charge carriers are found in some specialty applications outside of integrated circuits.

Three laws define the distribution of currents and voltages in a circuit with resistors: Kirchhoff's voltage and current laws, and the volt–ampere relation for resistors defined by Ohm's law. Ohm's law relates the current and voltage in a resistor as

$$V = R \times I \tag{1.1}$$

This law relates the voltage drop (V) across a resistor R when a current I passes through it. An electron loses potential energy when it passes through a resistor. Ohm's law is important because we can now predict the current obtained when a voltage is applied to a resistor or, equivalently, the voltage that will appear at the resistor terminals when forcing a current.

An equivalent statement of Ohm's law is that the ratio of voltage applied to a resistor to subsequent current in that resistor is a constant R = V/I, with a unit of volts per ampere called an ohm (Ω). Three examples of Ohm's law in Figure 1.1 show that any of the three variables can be found if the other two are known. We chose a rectangle as the symbol for a resistor as it often appears in CAD (computer-aided design) printouts of schematics and it is easier to control in these word processing tools.

The ground symbol at the bottom of each circuit is necessary to give a common reference point for all other nodes. The other circuit node voltages are measured (or calculated) with respect to the ground node. Typically, the ground node is electrically tied through a building wire called the common to the voltage generating plant wiring. Battery circuits use another ground point such as the portable metal chassis that contains the circuit. Notice that the current direction is defined by the positive charge with respect to the positive terminal of a voltage supply, or by the voltage drop convention with respect to a positive charge. This seems to contradict our statements that all current in resistors and transistors is due to negative-charge carriers. This conceptual conflict has historic origins. Ben Franklin is believed to have started this convention with his famous kite-in-a-thunderstorm experiment. He introduced the terms positive and negative to describe what he called electrical fluid. This terminology was accepted, and not overturned when we found out later that current is actually carried by negative-charge carriers (i.e., electrons). Fortunately, when we calculate voltage, current, and power in a circuit, a positive-charge hypothesis gives the same results as a negative-charge hypothesis. Engineers accept the positive convention, and typically think little about it.



Figure 1.1. Ohm's law examples. The battery positive terminal indicates where the positive charge exits the source. The resistor positive voltage terminal is where positive charge enters.

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An electron loses energy as it passes through a resistance, and that energy is lost as heat. Energy per unit of time is power. The power loss in an element is the product of voltage and current, whose unit is the watt (W):

$$P = VI \tag{1.2}$$

1.2.1 Kirchhoff's Voltage Law (KVL)

This law states that "the sum of the voltage drops across elements in a circuit loop is zero." If we apply a voltage to a circuit of many serial elements, then the sum of the voltage drops across the circuit elements (resistors) must equal the applied voltage. The KVL is an energy conservation statement allowing calculation of voltage drops across individual elements: energy input must equal energy dissipated.

Voltage Sources. An ideal voltage source supplies a constant voltage, no matter the amount of current drawn, although real voltage sources have an upper current limit. Figure 1.2 illustrates the KVL law where $V_{\rm BB}$ represents a battery or bias voltage source. The polarities of the driving voltage $V_{\rm BB}$ and resistor voltages are indicated for the clockwise direction of the current.

Naming V_1 the voltage drop across resistor R_1 , V_2 that across resistor R_2 , and, subsequently, V_5 for R_5 , the KVL states that

$$V_{\rm BB} = V_1 + V_2 + V_3 + V_4 + V_5 \tag{1.3}$$

Note that the resistor connections in Figure 1.2 force the same current I_{BB} through all resistors. When this happens, i.e., when the same current is forced through two or more resistors, they are said to be connected in series. Applying Ohm's law to each resistor of Figure 1.2, we obtain $V_i = R_i \times I_{BB}$ (where *i* takes any value from 1 to 5). Applying Ohm's law to each voltage drop at the right-hand side of Equation (1.3) we obtain

$$V_{BB} = R_1 I_{BB} + R_2 I_{BB} + R_3 I_{BB} + R_4 I_{BB} + R_5 I_{BB}$$

= $(R_1 + R_2 + R_3 + R_4 + R_5) I_{BB}$
= $R_{eq} I_{BB}$ (1.4)

where $R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5$. The main conclusion is that when a number of resistors are connected in series, they can be reduced to an equivalent single resistor whose value is the sum of the resistor values connected in series.



Figure 1.2. KVL seen in series elements.

EXAMPLE 1.1

Figure 1.3(a) shows a 5 V source driving two resistors in series. The parameters are referenced to the ground node. Show that the KVL holds for the circuit.



Figure 1.3. (a) Circuit illustrating KVL. (b) Equivalent circuit. The power supply cannot tell if the two series resistors or their equivalent resistance are connected to the power source terminals.

The voltage drop across R_A is

$$V_{\rm A} = R_{\rm A} \times I = 12 \text{ k}\Omega \times 156.25 \text{ }\mu\text{A} = 1.875 \text{ V}$$

Similarly, the voltage across $R_{\rm B}$ is 3.125 V. The applied 5 V must equal the series drops across the two resistors or 1.875 V + 3.125 V = 5 V. The last sentence is a verification of the KVL.

The current in Figure 1.3(a) through the 12 k Ω in series with a 20 k Ω resistance is equal to that of a single 32 k Ω resistor (Figure 1.3(b)). The voltage across the two resistors in Figure 1.3(a) is 5 V and, when divided by the current (156.25 μ A), gives an equivalent series resistance of (5 V/156.25 μ A) = 32 k Ω . Figure 1.3(b) is an equivalent reduced circuit of that in Figure 1.3(a).

Current Sources. We introduced voltage power sources first since they are more familiar in our daily lives. We buy voltage batteries in a store or plug computers, appliances, or lamps into a voltage socket in the wall. However, another power source exists, called a current source, that has the property of forcing a current out of one terminal that is independent of the resistor load. Although not as common, you can buy current power sources, and they have important niche applications.

Current sources are an integral property of transistors. CMOS transistors act as current sources during the crucial change of logic state. If you have a digital watch with a micro-controller of about 200k transistors, then about 5% of the transistors may switch during a clock transition, so 10k current sources are momentarily active on your wrist.

Figure 1.4 shows a resistive circuit driven by a current source. The voltage across the current source can be calculated by applying Ohm's law to the resistor connected between the current source terminals. The current source as an ideal element provides a fixed current value, so that the voltage drop across the current source will be determined by the element or elements connected at its output. The ideal current source can supply an infinite voltage, but real current sources have a maximum voltage limit.

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Figure 1.4. A current source driving a resistor load.

1.2.2 Kirchhoff's Current Law (KCL)

The KCL states that "the sum of the currents at a circuit node is zero." Current is a mass flow of charge. Therefore the mass entering the node must equal the mass exiting it. Figure 1.5 shows current entering a node and distributed to three branches. Equation (1.5) is a statement of the KCL that is as essential as the KVL in Equation (1.3) for computing circuit variables. Electrical current is the amount of charge (electrons) Q moving in time, or dQ/dt. Since current itself is a flow (dQ/dt), it is grammatically incorrect to say that "current flows." Grammatically, charge flows, but current does not.

$$I_0 = I_1 + I_2 + I_3 \tag{1.5}$$

The voltage across the terminals of parallel resistors is equal for each resistor, and the currents are different if the resistors have different values. Figure 1.6 shows two resistors connected in parallel with a voltage source of 2.5 V. Ohm's law shows a different current in each path, since the resistors are different, whereas all have the same voltage drop.

$$I_{\rm A} = \frac{2.5 \text{ V}}{100 \text{ k}\Omega} = 25 \text{ }\mu\text{A}$$

$$I_{\rm B} = \frac{2.5 \text{ V}}{150 \text{ }k\Omega} = 16.675 \text{ }\mu\text{A}$$
(1.6)

Applying Equation (1.5), the total current delivered by the battery is

$$I_{\rm BB} = I_{\rm A} + I_{\rm B} = 25 \ \mu \text{A} + 16.67 \ \mu \text{A} = 41.67 \ \mu \text{A} \tag{1.7}$$



Figure 1.5. KCL and its current summation at a node.



Figure 1.6. Parallel resistance example.

and

$$\frac{V_{\rm BB}}{I_{\rm BB}} = \frac{2.5 \,\rm V}{41.67 \,\mu\rm A} = 60 \,\rm k\Omega \tag{1.8}$$

Notice that the sum of the currents in each resistor branch is equal to the total current from the power supply, and that the resistor currents will differ when the resistors are unequal. The equivalent parallel resistance R_{eq} in the resistor network in Figure 1.6 is $V_{BB} = R_{eq}(I_A + I_B)$. From this expression and using Ohm's law we get

$$\frac{V_{\rm BB}}{R_{\rm eq}} = I_{\rm A} + I_{\rm B}$$

$$I_{\rm A} = \frac{V_{\rm BB}}{R_{\rm A}}$$

$$I_{\rm B} = \frac{V_{\rm BB}}{R_{\rm B}}$$
(1.9)

and

$$\frac{V_{\rm BB}}{R_{\rm eq}} = \frac{V_{\rm BB}}{R_{\rm A}} + \frac{V_{\rm BB}}{R_{\rm B}}$$
$$\frac{1}{R_{\rm eq}} = \frac{1}{R_{\rm A}} + \frac{1}{R_{\rm B}}$$
(1.10)
$$R_{\rm eq} = \frac{1}{\frac{1}{R_{\rm A}} + \frac{1}{R_{\rm B}}} = \frac{1}{\frac{1}{100 \text{ k}\Omega} + \frac{1}{150 \text{ k}\Omega}} = 60 \text{ k}\Omega$$

This is the expected result from Equation (1.8). R_{eq} is the equivalent resistance of R_A and R_B in parallel, which is notationly expressed as $R_{eq} = R_A ||R_B$. In general, for *n* resistances in parallel,

$$R_{\rm eq} = (R_1 ||R_2|| \cdots ||R_n) = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$$
(1.11)

The following examples and self-exercises will help you to gain confidence on the concepts discussed.

Self-Exercise 1.1

Calculate V_0 and the voltage drop V_p across the parallel resistors in Figure 1.7. *Hint:* Replace the 250 k Ω and 180 k Ω resistors by their equivalent resistance and apply KVL to the equivalent circuit.



Figure 1.7.

Self-Exercise 1.2

Calculate R_3 in the circuit of Figure 1.8.



Figure 1.8. Equivalent parallel resistance.

When the number of resistors in parallel is two, Equation (1.11) reduces to

$$R_{\rm p} = \frac{R_{\rm A} \times R_{\rm B}}{R_{\rm A} + R_{\rm B}} \tag{1.12}$$

EXAMPLE 1.2

Calculate the terminal resistance of the resistors in Figures 1.9(a) and (b).

$$R_{eq} = 1 \text{ M}\Omega || 2.3 \text{ M}\Omega$$
$$= \frac{(10^6)(2.3 \times 10^6)}{10^6 + 2.3 \times 10^6}$$
$$= 697 \text{ k}\Omega$$



Figure 1.9. Parallel resistance calculations and equivalent circuits.

The equivalent resistance at the network in Figure 1.9(b) is found by combining the series resistors to 185 k Ω and then calculating the parallel equivalent:

$$R_{eq} = 75 \text{ k}\Omega || 185 \text{ k}\Omega$$
$$= \frac{(75 \times 10^3)(185 \times 10^3)}{75 \times 10^3 + 185 \times 10^3}$$
$$= 53.37 \text{ k}\Omega$$

Self-Exercise 1.3

Calculate the resistance at the voltage source terminals R_{in} , I_{BB} , and V_0 at the terminals in Figure 1.10. If you are good, you can do this in your head.



Figure 1.10.

Self-Exercise 1.4

Use Equations (1.11) or (1.12) and calculate the parallel resistance for circuits in Figures 1.11(a)–(d). Estimates of the terminal resistances for circuits in (a) and (b) should be done in your head. Circuits in (c) and (d) show that the effect of a large parallel resistance becomes negligible.





Self-Exercise 1.5

Calculate R_{in} , I_{BB} , and V_0 in Figure 1.12. Estimate the correctness of your answer in your head.





Self-Exercise 1.6

(a) In Figure 1.13, find I_3 if $I_0 = 100 \ \mu\text{A}$, $I_1 = 50 \ \mu\text{A}$, and $I_2 = 10 \ \mu\text{A}$. (b) If $R_3 = 50 \ \text{k}\Omega$, what are R_1 and R_2 ?



Self-Exercise 1.7

Calculate R_1 and R_2 in Figure 1.14.



Self-Exercise 1.8

If the voltage across the current source is 10 V in Figure 1.15, what is R_1 ?



Resistance Calculations by Inspection. A shorthand notation for the terminal resistance of networks allows for quick estimations and checking of results. The calculations are defined before computing occurs. Some exercises below will illustrate this. Solutions are given in the Appendix.

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Self-Exercise 1.9

Write the shorthand notation for the terminal resistance of the circuits in Figures 1.16(a) and (b).



Figure 1.16.

Self-Exercise 1.10

Write the shorthand notation for the terminal resistance of the three circuits in Figure 1.17.



Figure 1.17. Terminal resistance using shorthand notation.

Self-Exercise 1.11

In the lower circuit of Figure 1.17, $R_1 = 20 \text{ k}\Omega$, $R_2 = 15 \text{ k}\Omega$, $R_3 = 25 \text{ k}\Omega$, $R_4 = 8 \text{ k}\Omega$, $R_5 = 5 \text{ k}\Omega$. Calculate R_{eq} for these three circuits.

Dividers. Some circuit topologies are repetitive and lend themselves to analysis by inspection. Two major inspection techniques use *voltage divider* and *current divider* concepts that take their analysis from the KVL and KCL. These are illustrated below with derivations of simple circuits followed by several examples and exercises. The examples have slightly more elements, but they reinforce previous examples and emphasize analysis by inspection.

Figure 1.18 shows a circuit with good visual voltage divider properties that we will illustrate in calculating V_3 .

The KVL equation is

$$V_{\rm BB} = I_{\rm BB}(R_1) + I_{\rm BB}(R_2) + I_{\rm BB}(R_3) = I_{\rm BB}(R_1 + R_2 + R_3)$$

$$I_{\rm BB} = \frac{V_{\rm BB}}{R_1 + R_2 + R_3} = \frac{V_3}{R_3}$$
(1.13)

and

$$V_3 = \frac{R_3}{R_1 + R_2 + R_3} V_{\rm BB} \tag{1.14}$$



Figure 1.18. Voltage divider circuit.

Equation (1.14) is a shorthand statement of the voltage divider. It is written by inspection, and calculations follow. The voltage dropped by each resistor is proportional to their fraction of the whole series resistance. Figure 1.18 is very visual, and you should be able to write the voltage divider expression by inspection for any voltage drop. For example, the voltage from node V_2 to ground is

$$V_2 = \frac{R_2 + R_3}{R_1 + R_2 + R_3} V_{\rm BB}$$
(1.15)

Self-Exercise 1.12

Use inspection and calculate the voltage at V_0 (Figure 1.19). Verify that the sum of the voltage drops is equal to $V_{\rm BB}$. Write the input resistance $R_{\rm in}$ by inspection and calculate the current $I_{\rm BB}$.

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Figure 1.19. Voltage divider analysis circuit.

Self-Exercise 1.13

Write the expression for $R_{\rm in}$ at the input terminals, V_0 , and the power supply current (Figure 1.20).



Current divider expressions are visual, allowing you to see the splitting of current as it enters branches. Figure 1.21 shows two resistors that share total current $I_{\rm BB}$.

KVL gives

$$V_{\rm BB} = (R_1 || R_2) I_{\rm BB} = \frac{R_1 \times R_2}{R_1 + R_2} I_{\rm BB} = (I_1)(R_1) = (I_2)(R_2)$$
(1.16)

then

$$I_1 = \frac{R_2}{R_1 + R_2} I_{BB}$$
 and $I_2 = \frac{R_1}{R_1 + R_2} I_{BB}$ (1.17)



Figure 1.21. Current divider.

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Currents divide in two parallel branches by an amount proportional to the opposite leg resistance divided by the sum of the two resistors. This relation should be memorized, as was done for the voltage divider.

Self-Exercise 1.14

Write the current expression by inspection and solve for currents in the 12 k Ω and 20 k Ω paths in Figure 1.22.





Self-Exercise 1.15

(a) Write the current expression by inspection and solve for currents in all resistors in Figure 1.23, where $I_{BB} = 185.4 \mu A$. (b) Calculate V_{BB} .





Self-Exercise 1.16

(a) Solve for current in all resistive paths in Figure 1.24 using the technique of inspection. (b) Calculate a new value for the 20 k Ω resistor so that its current is 5 μ A.



Figure 1.24.

In Figure 1.25, calculate V_0 , I_2 , and I_9 .





Self-Exercise 1.18

(a) Write R_{in} between the battery terminals by inspection and solve (Figure 1.26). (b) Write the $I_{1.5k}$ expression by inspection and solve. This is a larger circuit, but it presents no problem if we adhere to the shorthand style. We write R_{in} between battery terminals by inspection, and calculate $I_{1.5k}$ by current divider inspection.



1.3 CAPACITORS

Capacitors appear in CMOS digital circuits as parasitic elements intrinsic to transistors or with the metals used for interconnections. They have an important effect on the time for a transistor to switch between on and off states, and also contribute to propagation delay between gates due to interconnection capacitance. Capacitors also cause a type of noise called cross-talk. This appears especially in high-speed circuits, in which the voltage at one interconnection line is affected by another interconnection line that is isolated but located close to it. Cross-talk is discussed in later chapters.

The behavior and structure of capacitors inherent to interconnection lines are significantly different from the parasitic capacitors found in diodes and transistors. We introduce ideal parallel plate capacitors that are often used to model wiring capacitance. Capacitors inherent to transistors and diodes act differently and are discussed later.

A capacitor has two conducting plates separated by an insulator, as represented in Figure 1.27(a). When a DC voltage is applied across the conducting plates (terminals) of the capacitor, the steady-state current is zero since the plates are isolated by the insulator. The effect of the applied voltage is to store charges of opposite sign at the plates of the capacitor.

The capacitor circuit symbol is shown in Figure 1.27(b). Capacitors are characterized by a parameter called capacitance (C) that is measured in Farads. Strictly, capacitance is defined as the charge variation ∂Q induced in the capacitor when voltage is changed by a quantity ∂V , i.e.,

$$C = \frac{\partial Q}{\partial V} \tag{1.18}$$

This ratio is constant in parallel plate capacitors, independent of the voltage applied to the capacitor. Capacitance is simply the ratio between the charge stored and the voltage applied, i.e., C = Q/V, with units of Coulombs per Volt called a Farad. This quantity can also be computed from the geometry of the parallel plate and the properties of the insulator used to construct it. This expression is

$$C = \frac{\varepsilon_{\rm ins}A}{d} \tag{1.19}$$

where ε_{ins} is an inherent parameter of the insulator, called permittivity, that measures the resistance of the material to an electric field; *A* is the area of the plates used to construct the capacitor; and *d* the distance separating the plates.

Although a voltage applied to the terminals of a capacitor does not move net charge through the dielectric, it can displace charge within it. If the voltage changes with time, then the displacement of charge also changes, causing what is known as displacement current, that cannot be distinguished from a conduction current at the capacitor terminals. Since this current is proportional to the rate at which the voltage across the capacitor changes with time, the relation between the applied voltage and the capacitor current is



Figure 1.27. (a) Parallel plate capacitor. (b) Circuit symbol.

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$$i = C \frac{dV}{dt} \tag{1.20}$$

If the voltage is DC, then dV/dt = 0 and the current is zero. An important consequence of Equation (1.20) is that the voltage at the terminals of a capacitor cannot change instantaneously, since this would lead to an infinite current. That is physically impossible. In later chapters, we will see that any logic gate constructed within an IC has a parasitic capacitor at its output. Therefore, the transition from one voltage level to another will always have a delay time since the voltage output cannot change instantaneously. Trying to make these output capacitors as small as possible is a major goal of the IC industry in order to obtain faster circuits.

1.3.1 Capacitor Connections

Capacitors, like resistors, can be connected in series and in parallel. We will show the equivalent capacitance calculations when they are in these configurations.

Capacitors in parallel have the same terminal voltage, and charge distributes according to the relative capacitance value differences (Figure 1.28(a)). The equivalent capacitor is equal to the sum of the capacitors:

$$C_{1} = \frac{Q_{1}}{V}, \qquad C_{2} = \frac{Q_{2}}{V}$$

$$C_{1} + C_{2} = \frac{Q_{1}}{V} + \frac{Q_{2}}{V} = \frac{Q_{1} + Q_{2}}{V}$$

$$C_{eq} = \frac{Q_{eq}}{V}$$
(1.21)

where $C_{eq} = C_1 + C_2$, and $Q_{eq} = Q_1 + Q_2$. Capacitors connected in parallel simply add their values to get the equivalent capacitance.

Capacitors connected in series have the same charge stored, whereas the voltage depends on the relative value of the capacitor (Figure 1.28(b)). In this case, the expression for the equivalent capacitor is analogous to the expression obtained when connecting resistors in parallel:



Figure 1.28. Capacitance interconnection. (a) Parallel. (b) Series.

$$C_{1} = \frac{Q}{V_{1}}, \qquad C_{2} = \frac{Q}{V_{2}}$$

$$\frac{1}{C_{1}} + \frac{1}{C_{2}} = \frac{V_{1}}{Q} + \frac{V_{2}}{Q} = \frac{V_{1} + V_{2}}{Q}$$

$$C_{eq} = \frac{Q}{V_{eq}}$$
(1.22)

where $V_{eq} = V_1 + V_2$, and

$$C_{\rm eq} = \frac{C_1 C_2}{C_1 + C_2}$$

EXAMPLE 1.3

In Figures 1.28(a) and (b), $C_1 = 20$ pF and $C_2 = 60$ pF. Calculate the equivalent capacitance seen by the voltage source.

(a)
$$C_{eq} = C_1 + C_2 = 20 \text{ pF} + 60 \text{ pF} = 80 \text{ pF}$$

(b)
$$C_{\rm eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{1}{1/20 \text{ pF} + 1/60 \text{ pF}} = 15 \text{ pF}$$

Self-Exercise 1.19

Calculate the terminal equivalent capacitance for the circuits in Figure 1.29.



1.3.2 Capacitor Voltage Dividers

There are open circuit defect situations in CMOS circuits in which capacitors couple voltages to otherwise unconnected nodes. This simple connection is a capacitance voltage di-

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Figure 1.30. Capacitance voltage divider.

vider circuit (Figure 1.30). The voltage across each capacitor is a fraction of the total voltage $V_{\rm DD}$ across both terminals.

EXAMPLE 1.4

Derive the relation between the voltage across each capacitor C_1 and C_2 in Figure 1.30 to the terminal voltage V_{DD} .

The charge across the plates of the series capacitors is equal so that $Q_1 = Q_2$. The capacitance relation C = Q/V allows us to write

$$Q_1 = Q_2$$
 $C_1 V_1 = C_2 V_2$

or

$$V_2 = \frac{C_1}{C_2} V_1$$

Since

$$V_{\rm DD} = V_1 + V_2$$

then

$$V_2 = V_{\rm DD} - V_1 = \frac{C_1}{C_2} V_1$$

Solve for

$$V_1 = \frac{C_2}{C_1 + C_2} V_{\rm DD}$$

and get

 $V_2 = \frac{C_1}{C_1 + C_2} V_{\rm DD}$

The form of the capacitor divider is similar to the resistor voltage divider except the numerator term differs.

Self-Exercise 1.20

Solve for V_1 and V_2 in Figure 1.31.



Figure 1.31.

Self-Exercise 1.21

If $V_2 = 700$ mV, what is the driving terminal voltage V_D in Figure 1.32?



1.3.3 Charging and Discharging Capacitors

So far we have discussed the behavior of circuits with capacitors in the steady state, i.e., when DC sources drive the circuit. In these cases, the analysis of the circuit is done assuming that it reached a stationary state. Conceptually, these cases are different from situations in which the circuit source makes a sudden transition, or a DC source is applied to a discharged capacitor through a switch. In these situations, there is a period of time during which the circuit is not in a stationary state but in a transient state. These cases are important in digital CMOS ICs, since node state changes in ICs are transient states determining the timing and power characteristics of the circuit. We will analyze charge and discharge of capacitors with an example.

EXAMPLE 1.5

In the circuit of Figure 1.33, draw the voltage and current evolution at the capacitor with time starting at t = 0 when the switch is closed. Assume $V_{in} = 5$ V and that the capacitor is initially at 0 V.



Figure 1.33.

The Kirchoff laws for current and voltage can be applied to circuits with capacitors as we did with resistors. Thus, once the switch is closed, the KVL must follow at any time:

$$V_{\rm in} = V_{\rm R} + V_{\rm C}$$

The Kirchoff current law applied to this circuit states that the current through the resistor must be equal to the current through the capacitor, or

$$\frac{V_{\rm R}}{R} = C \frac{dV_{\rm C}}{dt}$$

Using the KVL equation, we can express the voltage across the resistor in terms of the voltage across the capacitor, obtaining

$$\frac{V_{\rm in} - V_{\rm C}}{R} = C \frac{dV_{\rm C}}{dt}$$

This equation relates the input voltage to the voltage at the capacitor. The solution gives the time evolution of the voltage across the capacitor,

$$V_{\rm C} = V_{\rm in}(1 - e^{-t/RC})$$

The current through the capacitor is

$$I_{\rm C} = I_{\rm R} = \frac{V_{\rm in} - V_{\rm C}}{R}$$
$$I_{\rm C} = \frac{V_{\rm in}}{R} e^{-t/RC}$$

At t = 0, the capacitor voltage is zero (it is discharged) and the current is maximum (the voltage drop at the resistor is maximum), whereas in DC (for $t \rightarrow \infty$) the capacitor voltage is equal to the source voltage and the current is zero. This example shows that the voltage evolution is exponential when charging a capaci-





tor through a resistor. The time constant is defined for t = RC, that is, the time required to charge the capacitor to $(1 - e^{-1})$ of its final value, or 63%. This means that the larger the value of the resistor or capacitor, the longer it takes to charge/discharge it (Figure 1.34).

1.4 DIODES

A circuit analysis of the semiconductor diode is presented below; later chapters discuss its physics and role in transistor construction. Diodes do not act like resistors; they are non-linear. Diodes pass significant current at one voltage polarity and near zero current for the opposite polarity. A typical diode nonlinear current–voltage relation is shown in Figure 1.35(a) and its circuit symbol in Figure 1.35(b). The positive terminal is called the anode, and the negative one is called the cathode. The diode equation is

$$I_{\rm D} = I_{\rm S} (e^{\frac{qV_{\rm D}}{kT}} - 1)$$
(1.23)

where k is the Boltzmann constant ($k = 1.38 \times 10^{-23}$ J/K), q is the charge of the electron ($q = 1.6 \times 10^{-19}$ C), and $I_{\rm S}$ is the reverse biased current. The quantity kT/q is called the thermal voltage ($V_{\rm T}$) whose value is 0.0259 V at T = 300 K; usually, we use $V_{\rm T} = 26$ mV at that



Figure 1.35. (a) Diode *I*–*V* characteristics and (b) symbol.

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temperature. When the diode applied voltage is positive and well beyond the thermal voltage ($V_{\rm D} \gg V_{\rm T} = kT/q$), Equation (1.23) becomes

$$I_{\rm D} = I_{\rm S} e^{\frac{qV_{\rm D}}{kT}} \tag{1.24}$$

The voltage across the diode can be solved from Equation (1.23) as

$$V_{\rm D} = \frac{kT}{q} \ln\left(\frac{I_{\rm D}}{I_{\rm S}} + 1\right) \tag{1.25}$$

For forward bias applications $I_D/I_S \gg 1$ and this reduces to

$$V_{\rm D} = \frac{kT}{q} \ln \frac{I_{\rm D}}{I_{\rm S}} \tag{1.26}$$

Self-Exercise 1.22

(a) Calculate the forward diode voltage if $T = 25^{\circ}$ C, $I_{\rm D} = 200$ nA, and $I_{\rm S} = 1$ nA. Compute from Equation (1.25). (b) At what current will the voltage drop be 400 mV?

Diode Equations (1.23)–(1.26) are useful in their pure form only at the temperature at which I_S was measured. These equations predict that I_D will exponentially drop as temperature rises which is not so. I_S is more temperature-sensitive than the temperature exponential and doubles for about every 10°C rise. The result is that diode current markedly increases as temperature rises.

1.4.1 Diode Resistor Circuits

Figure 1.36 shows a circuit that can be solved for all currents and node element voltages if we know the reverse bias saturation current I_S .

EXAMPLE 1.6

If $I_s = 10$ nA at room temperature, what is the voltage across the diode in Figure 1.36 and what is I_D ? Let kT/q = 26 mV.



Figure 1.36. Forward-biased diode analysis.

Write KVL using the diode voltage expression:

$$2 \text{ V} = I_{\text{D}}(10 \text{ k}\Omega) + (26 \text{ mV}) \ln\left(\frac{I_{\text{D}}}{I_{\text{S}}} + 1\right)$$

This equation has one unknown (I_D) , but it is difficult to solve analytically, so an iterative method is easiest. Values of I_D are substituted into the equation, and the value that balances the LHS and RHS is a close approximation. A starting point for I_D can be estimated from the upper bound on I_D . If $V_D = 0$, then $I_D = 2 \text{ V}/10 \text{ k}\Omega = 200 \text{ }\mu\text{A}$. I_D cannot be larger than 200 μA . A close solution is $I_D = 175 \text{ }\mu\text{A}$.

The diode voltage is

$$V_{\rm D} = \frac{kT}{q} \ln \frac{I_{\rm D}}{I_{\rm S}}$$
$$= 26 \text{ mV} \times \ln \frac{175 \text{ uA}}{10 \text{ nA}} = 244.2 \text{ mV}$$

Self-Exercise 1.23

Estimate I_D and V_D in Figure 1.37 for $I_S = 1$ nA.



EXAMPLE 1.7

Figure 1.38 shows two circuits with the diode cathode connected to the positive terminal of a power supply ($I_{\rm S} = 100$ nA). What is V_0 in both circuits?



Figure 1.38.

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Figure 1.38(a) has a floating node at V_0 so there is no current in the diode. Since $I_D = 0$, the diode voltage drop $V_D = 0$ and

$$V_0 = V_D + 2 \text{ V} = 2 \text{ V}$$

Figure 1.38(b) shows a current path to ground. The diode is reversed-biased and $I_{\rm BB} = -I_{\rm D} = 100$ nA. Then

 $V_0 = I_{BB} \times 1 \text{ M}\Omega = 100 \text{ nA} \times 1 \text{ M}\Omega = 100 \text{ mV}$

Both problems in Example 1.7 can be analyzed using Equations (1.23) to (1.26) or observing the process in the I-V curve of Figure 1.35. In Figure 1.38(a), the operating point is at the origin. In Figure 1.38(b), it has moved to the left of the origin.

Self-Exercise 1.24

The circuit in Figure 1.39 is similar to IC protection circuits connected to the input pins of an integrated circuit. The diodes protect the logic circuit block when input pin (pad) voltages are accidentally higher than the power supply voltage (V_{DD}) or lower than the ground voltage. If $V_{PAD} > 5$ V, then diode D_2 turns on and bleeds charge away from the input pin. The same process occurs through diode D_1 if the input pad voltage becomes less than ground (0 V). An integrated circuit tester evaluates the diodes by forcing current (100 μ A) and measuring the voltage. If the protection circuit is damaged, an abnormal voltage is usually read at the damaged pin.

(a) If diode reverse bias saturation current is $I_{\rm S} = 100$ nA, what is the expected input voltage measured if the diodes are good and R_1 and R_2 are small? Apply $\pm 100 \ \mu$ A to assess both diodes.

(b) If the upper diode has a dead short across it, what is V_{IN} when the test examines the upper diode?



Figure 1.39.

Self-Exercise 1.25

Calculate V_0 and V_{D1} in Figure 1.40, where $I_S = 100 \mu A$ and T = 25 °C.



1.4.2 Diode Resistance

Although diodes do not obey Ohm's law, a small signal variation in the forward bias can define a resistance from the slope of the I-V curve. The distinction with linear elements is important as we cannot simply divide a diode DC voltage by its DC current. That result is meaningless.

The diode curve is repeated and enlarged in Figure 1.41. If a small signal variation v(t) is applied in addition to the DC operating voltage V_{DC} , then the exponential current/voltage characteristic can be approximated to a line [given that v(t) is small enough] and an equivalent resistance for that bias operation can be defined. Bias point 1 has a current change with voltage that is larger than that of bias point 2; therefore, the dynamic resistance of the diode is smaller at bias point 1. Note that this resistance value depends strongly on the operating voltage bias value. Each point on the curve has a slope in the forward bias. Dividing the DC voltage by the current, V_1/I_1 , is not the same as V_2/I_2 . Therefore, these DC relations are meaningless. However, dynamic resistance concepts are important in certain transistor applications.

This concept is seen in manipulation of the diode equation, where the forward-biased dynamic resistance r_d is

$$\frac{1}{r_d} = \frac{dI_{\rm D}}{dV_{\rm D}} = \frac{d[I_{\rm S}e^{qV_{\rm D}/kT}]}{dV_{\rm D}} = \frac{qI_{\rm S}e^{qV_{\rm D}/kT}}{kT} = \frac{qI_{\rm D}}{kT}$$
(1.27)



Figure 1.41.

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The diode dynamic resistance is

$$r_d = \frac{dV_{\rm D}}{dI_{\rm D}} = \frac{kT}{qI_{\rm D}} \tag{1.28}$$

or at room temperature

$$r_d \approx \frac{26 \text{ mV}}{I_D} \tag{1.29}$$

Self-Exercise 1.26

Find the diode dynamic resistance at room temperature for $I_D = 1 \mu A$, 100 μA , 1 mA, and 10 mA.

How do we use the concept of dynamic diode resistance? A diode can be biased at a DC current, and small changes about that operating point have a resistance. A small sinusoid voltage (v_D) causes a diode current (i_D) change equal to v_D/r_D . The other important point is that you cannot simply divide DC terminal voltage by DC terminal current to calculate resistance. This is true for diodes and also for transistors, as will be seen later.

1.5 SUMMARY

This chapter introduced the basic analysis of circuits with power supplies, resistors, capacitors, and diodes. Kirchhoff's current and voltage laws were combined with Ohm's law to calculate node voltages and element currents for a variety of circuits. The technique of solving for currents and voltages by inspection is a powerful one because of the rapid insight into the nature of circuits it provides. Finally, the section on diodes illustrated analysis with a nonlinear element. The exercises at the end of the chapter should provide sufficient drill to prepare for subsequent chapters, which will introduce the MOSFET transistor and its simple configurations.

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EXERCISES

- 1.1. Write the shorthand expression for R_{eq} at the open terminals in Figure 1.42.
- 1.2. Write the shorthand expression for R_{eq} at the open terminals in Figure 1.43.



Figure 1.42.



Figure 1.43.

- 1.3. For the circuit in Figure 1.44, (a) calculate V_0 ; (b) calculate I_{2M} .
- 1.4. Calculate V_0 by first writing a voltage divider expression and then solving for V_0 (Figure 1.45a and b).
- 1.5. Write the shorthand notation for current I_2 in resistor R_2 in Figure 1.46 as a function of driving current *I*.
- 1.6. For the circuit in Figure 1.47, (a) solve for V_0 using a voltage divider expression; (b) solve for I_{2K} ; (c) solve for I_{900} .

EXERCISES 31











Figure 1.46.



Figure 1.47.

1.7. Use the circuit analysis technique by inspection, and write the shorthand expression to calculate I_{2K} for Figure 1.48.



1.8. Given the circuit in Figure 1.49, (a) write the expression for I_{450} and solve; (b) write the expression for V_{800} ; (c) show that $I_{800} + I_{400} = 2$ mA.



Figure 1.49.

- 1.9. Find I_{6k} in Figure 1.50. *Hint:* when we have two power supplies and a linear (resistive) network, we solve in three steps.
 - 1. Set one power supply to 0 V and calculate current in the 6 k Ω resistor from the nonzero power supply.
 - 2. Reverse the role and recalculate I_{6k} .
 - 3. The final answer is the sum of the two currents.

This is known as the superposition theorem and can be applied only for linear elements.



Figure 1.50.

1.10. Find the equivalent capacitance at the input nodes in Figure 1.51.



Figure 1.51.

- 1.11. Find C_1 in Figure 1.52.
- 1.12. Solve for I_D and V_D in Figure 1.53, where the diode has the value $I_S = 1 \mu A$.
- 1.13. Calculate V_0 in Figure 1.54, given that the reverse-bias saturation current $I_S = 1$ nA, and you are at room temperature.

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Figure 1.53.



Figure 1.54.



1.14. Diode D_1 in Figure 1.55 has a reverse-bias saturation current of $I_{01} = 1$ nA, and diode D_2 has $I_{02} = 4$ nA. At room temperature, what is V_0 ?



1.15. Calculate the voltage across the diodes in Figure 1.56, given that the reverse-bias saturation current in D_1 is $I_{01} = 175$ nA and $I_{02} = 100$ nA.



Figure 1.56.

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