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INTRODUCTION TO OPTIMIZATION

This text is an introduction to optimization theory and its application to problems arising in engineering. In the most general terms, optimization theory is a body of mathematical results and numerical methods for finding and identifying the best candidate from a collection of alternatives without having to explicitly enumerate and evaluate all possible alternatives. The process of optimization lies at the root of engineering, since the classical function of the engineer is to design new, better, more efficient, and less expensive systems as well as to devise plans and procedures for the improved operation of existing systems.

The power of optimization methods to determine the best case without actually testing all possible cases comes through the use of a modest level of mathematics and at the cost of performing iterative numerical calculations using clearly defined logical procedures or algorithms implemented on computing machines. The development of optimization methodology will therefore require some facility with basic vector–matrix manipulations, a bit of linear algebra and calculus, and some elements of real analysis. We use mathematical concepts and constructions not simply to add rigor to the proceedings but because they are the language in terms of which calculation procedures are best developed, defined, and understood.

Because of the scope of most engineering applications and the tedium of the numerical calculations involved in optimization algorithms, the techniques of optimization are intended primarily for computer implementation. However, although the methodology is developed with computers in mind, we do not delve into the details of program design and coding. Instead, our emphasis is on the ideas and logic underlying the methods, on the factors involved in selecting the appropriate techniques, and on the considerations important to successful engineering application.

1.1 REQUIREMENTS FOR THE APPLICATION OF OPTIMIZATION METHODS

To apply the mathematical results and numerical techniques of optimization theory to concrete engineering problems, it is necessary to clearly delineate the boundaries of the engineering system to be optimized, to define the quantitative criterion on the basis of which candidates will be ranked to determine the "best," to select the system variables that will be used to characterize or identify candidates, and to define a model that will express the manner in which the variables are related. This composite activity constitutes the process of *formulating* the engineering optimization problem. Good problem formulation is the key to the success of an optimization study and is to a large degree an art. It is learned through practice and the study of successful applications and is based on the knowledge of the strengths, weaknesses, and peculiarities of the techniques provided by optimization theory. For these reasons, this text is liberally laced with engineering applications drawn from the literature and the experience of the authors. Moreover, along with presenting the techniques, we attempt to elucidate their relative advantages and disadvantages wherever possible by presenting or citing the results of actual computational tests.

In the next several sections we discuss the elements of problem formulation in a bit more detail. In Section 1.2 we follow up this discussion by examining a few application formulations.

1.1.1 Defining the System Boundaries

Before undertaking any optimization study, it is important to clearly define the boundaries of the system under investigation. In this context a system is the restricted portion of the universe under consideration. The system boundaries are simply the limits that separate the system from the remainder of the universe. They serve to isolate the system from its surroundings, because, for purposes of analysis, all interactions between the system and its surroundings are assumed to be frozen at selected representative levels. Nonetheless, since interactions always exist, the act of defining the system boundaries is the first step in the process of approximating the real system.

In many situations it may turn out that the initial choice of boundary is too restrictive. To fully analyze a given engineering system, it may be necessary to expand the system boundaries to include other subsystems that strongly affect the operation of the system under study. For instance, suppose a manufacturing operation has a paint shop in which finished parts are mounted on an assembly line and painted in different colors. In an initial study of the paint shop, we may consider it in isolation from the rest of the plant. However, we may find that the optimal batch size and color sequence we deduce for this system are strongly influenced by the operation of the fabrication department that produces the finished parts. A decision thus has to be made whether to expand the system boundaries to include the fabrication system. An expansion of the system boundaries certainly increases the size and complexity of the composite system and thus may make the study much more difficult. Clearly, to make our work as engineers more manageable, we would prefer as much as possible to break down large complex systems into smaller subsystems that can be dealt with individually. However, we must recognize that such a decomposition may constitute a potentially misleading simplification of reality.

1.1.2 Performance Criterion

Given that we have selected the system of interest and have defined its boundaries, we next need to select a criterion on the basis of which the performance or design of the system can be evaluated so that the best design or set of operating conditions can be identified. In many engineering applications, an economic criterion is selected. However, there is a considerable choice in the precise definition of such a criterion: total capital cost, annual cost, annual net profit, return on investment, cost–benefit ratio, or net present worth. In other applications a criterion may involve some technological factors—for instance, minimum production time, maximum production rate, minimum energy utilization, maximum torque, maximum weight, and so on. Regardless of the criterion selected, in the context of optimization the best will always mean the candidate system with either the *minimum* or *maximum* value of the performance index.

It is important to note that within the context of the optimization methods discussed in this book, only *one* criterion or performance measure can be used to define the optimum. It is not possible to find a solution that, say, simultaneously minimizes cost and maximizes reliability and minimizes energy utilization. This again is an important simplification of reality, because in many practical situations it would be desirable to achieve a solution that is best with respect to a number of different criteria.

One way of treating multiple competing objectives is to select one criterion as primary and the remaining criteria as secondary. The primary criterion is then used as an optimization performance measure, while the secondary criteria are assigned acceptable minimum or maximum values and are treated as problem constraints. For instance, in the case of the paint shop study, the following criteria may well be selected by different groups in the company:

1. The shop foreman may seek a design that will involve long production runs with a minimum of color and part changes. This will maximize the number of parts painted per unit time.
2. The sales department would prefer a design that maximizes the inventory of parts of every type and color. This will minimize the time between customer order and order dispatch.

3. The company financial officer would prefer a design that will minimize inventories so as to reduce the amount of capital tied up in parts inventory.

These are clearly conflicting performance criteria that cannot all be optimized simultaneously. A suitable compromise would be to select as the primary performance index the minimum annual cost but then to require as secondary conditions that the inventory of each part not be allowed to fall below or rise above agreed-upon limits and that production runs involve no more than some maximum acceptable number of part and color changes per week.

In summary, for purposes of applying the methods discussed in this text, it is necessary to formulate the optimization problem with a single performance criterion. Advanced techniques do exist for treating certain types of multicriteria optimization problems. However, this new and growing body of techniques is quite beyond the scope of this book. The interested reader is directed to recent specialized texts [1, 2].

1.1.3 Independent Variables

The third key element in formulating a problem for optimization is the selection of the independent variables that are adequate to characterize the possible candidate designs or operating conditions of the system. There are several factors to be considered in selecting the independent variables.

First, it is necessary to distinguish between variables whose values are amenable to change and variables whose values are fixed by external factors, lying outside the boundaries selected for the system in question. For instance, in the case of the paint shop, the types of parts and the colors to be used are clearly fixed by product specifications or customer orders. These are specified system parameters. On the other hand, the order in which the colors are sequenced is, within constraints imposed by the types of parts available and inventory requirements, an independent variable that can be varied in establishing a production plan.

Furthermore, it is important to differentiate between system parameters that can be treated as fixed and those that are subject to fluctuations influenced by external and uncontrollable factors. For instance, in the case of the paint shop, equipment breakdown and worker absenteeism may be sufficiently high to seriously influence the shop operations. Clearly, variations in these key system parameters must be taken into account in the formulation of the production planning problem if the resulting optimal plan is to be realistic and operable.

Second, it is important to include in the formulation all the important variables that influence the operation of the system or affect the design definition. For instance, if in the design of a gas storage system we include the height, diameter, and wall thickness of a cylindrical tank as independent variables but exclude the possibility of using a compressor to raise the storage pressure, we may well obtain a very poor design. For the selected fixed pressure, we would certainly find the least-cost tank dimensions. However, by including the storage pressure as an independent variable and adding the compressor cost to our performance criteria, we could obtain a design with a lower overall cost because of a reduction in the required tank volume. Thus, the independent variables must be selected so that all important alternatives are included in the formulation. In general, the exclusion of possible alternatives will lead to suboptimal solutions.

Finally, another consideration in the selection of variables is the level of detail to which the system is considered. While it is important to treat all key independent variables, it is equally important not to obscure the problem by the inclusion of a large number of fine details of subordinate importance. For instance, in the preliminary design of a process involving a number of different pieces of equipment—pressure vessels, towers, pumps, compressors, and heat exchanges—one would normally not explicitly consider all the fine details of the design of each individual unit. A heat exchanger may well be characterized by a heat transfer surface area as well as shell-side and tube-side pressure drops. Detailed design variables such as number and size of tubes, number of tube and shell passes, baffle spacing, header type, and shell dimensions would normally be considered in a separate design study involving that unit by itself. In selecting the independent variables, a good rule is to include only those variables that have a significant impact on the composite system performance criterion.

1.1.4 System Model

Once the performance criterion and the independent variables have been selected, the next step in problem formulation is to assemble the model that describes the manner in which the problem variables are related and the way in which the performance criterion is influenced by the independent variables. In principle, optimization studies may be performed by experimenting directly with the system. Thus, the independent variables of the system or process may be set to selected values, the system operated under those conditions, and the system performance index evaluated using the observed performance. The optimization methodology would then be used to predict improved choices of the independent variable values and the experiments continued in this fashion. In practice, most optimization studies are carried out with the help of a simplified mathematical representation of the real system, called a *model*. Models are used because it is too expensive or time consuming or risky to use the real system to carry out the study. Models are typically used in engineering design because they offer the cheapest and fastest way of studying the effects of changes in key design variables on system performance.

In general, the model will be composed of the basic material and energy balance equations, engineering design relations, and physical property equations that describe the physical phenomena taking place in the system. These equations will normally be supplemented by inequalities that define allowable operating ranges, specify minimum or maximum performance requirements, or set bounds on resource availabilities. In sum, the model consists of all elements that normally must be considered in calculating a design or in predicting the performance of an engineering system. Quite clearly, the assembly of a model is a very time consuming activity and one that requires a thorough understanding of the system being considered. In later chapters we will have occasion to discuss the mechanics of model development in more detail. For now, we simply observe that a model is a collection of equations and inequalities that define how the system variables are related and that constrain the variables to take on acceptable values.

From the preceding discussion, we observe that a problem suitable for the application of optimization methodology consists of a performance measure, a set of independent variables, and a model relating the variables. Given these rather general and abstract requirements, it is evident that the methods of optimization can be applied to a very wide variety of applications. In fact, the methods we will discuss have been applied to problems that include the optimum design of process and structures, the planning of investment policies, the layout of warehouse networks, the determination of optimal trucking routes, the planning of heath care systems, the deployment of military forces, and the design of mechanical components, to name but a few. In this text our focus will be on engineering applications. Some of these applications and their formulations are discussed in the next section.

1.2 APPLICATIONS OF OPTIMIZATION IN ENGINEERING

Optimization theory finds ready application in all branches of engineering in four primary areas:

1. Design of components or entire systems
2. Planning and analysis of existing operations
3. Engineering analysis and data reduction
4. Control of dynamic systems

In this section we briefly consider representative applications from each of the first three areas. The control of dynamic systems is an important area to which the methodology discussed in this book is applicable but which requires the consideration of specialized topics quite beyond the scope of this book.

In considering the application of optimization methods in design and operations, keep in mind that the optimization step is but one step in the overall process of arriving at an optimal design or an efficient operation. Generally, that overall process will, as shown in Figure 1.1, consist of an iterative cycle

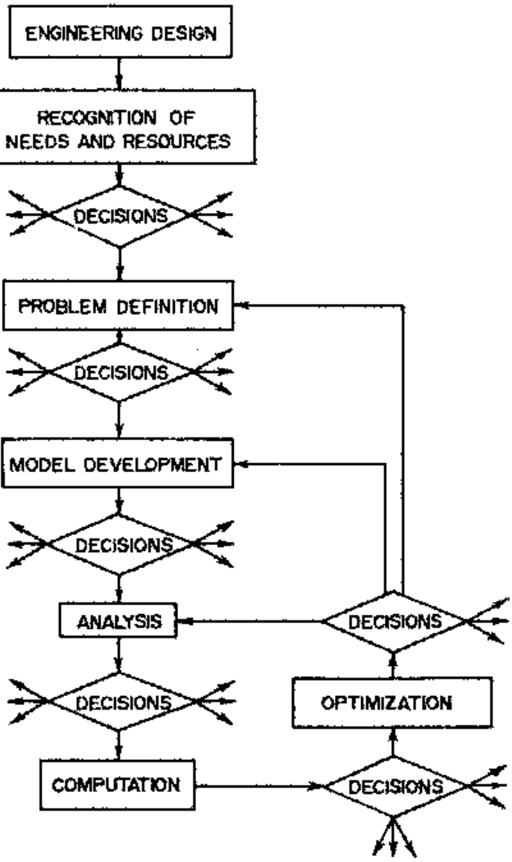

Figure 1.1. Engineering design process.

involving synthesis or definition of the structure of the system, model formulation, model parameter optimization, and analysis of the resulting solution. The final optimal design or new operating plan will be obtained only after solving a series of optimization problems, the solution to each of which will serve to generate new ideas for further system structures. In the interests of brevity, the examples in this section show only one pass of this iterative cycle and deal mainly with preparations for the optimization step. This focus should not be interpreted as an indication of the dominant role of optimization methods in the engineering design and systems analysis process. Optimization theory is a very powerful tool, but to be effective, it must be used skillfully and intelligently by an engineer who thoroughly understands the system under study. The primary objective of the following examples is simply to illustrate the wide variety but common form of the optimization problems that arise in the process of design and analysis.

1.2.1 Design Applications

Applications in engineering design range from the design of individual structural members to the design of separate pieces of equipment to the preliminary design of entire production facilities. For purposes of optimization, the shape or structure of the system is assumed to be known, and the optimization problem reduces to that of selecting values of the unit dimensions and operating variables that will yield the best value of the selected performance criterion.

Example 1.1 Design of an Oxygen Supply System

Description. The basic oxygen furnace (BOF) used in the production of steel is a large fed-batch chemical reactor that employs pure oxygen. The furnace is operated in a cyclical fashion. Ore and flux are charged to the unit, treated for a specified time period, and then discharged. This cyclical operation gives rise to a cyclically varying demand rate for oxygen. As shown in Figure 1.2, over each cycle there is a time interval of length t_1 of low demand rate D_0 and a time interval $t_2 - t_1$ of high demand rate D_1. The oxygen used in the BOF is produced in an oxygen plant in a standard process in which oxygen is separated from air by using a combination of refrigeration and distillation. Oxygen plants are highly automated and are designed to deliver oxygen at a fixed rate. To mesh the continuous oxygen plant with the cyclically operating BOF, a simple inventory system (Figure 1.3) consisting of a compressor and a storage tank must be designed. A number of design possibilities can be considered. In the simplest case, the oxygen plant capacity could be selected

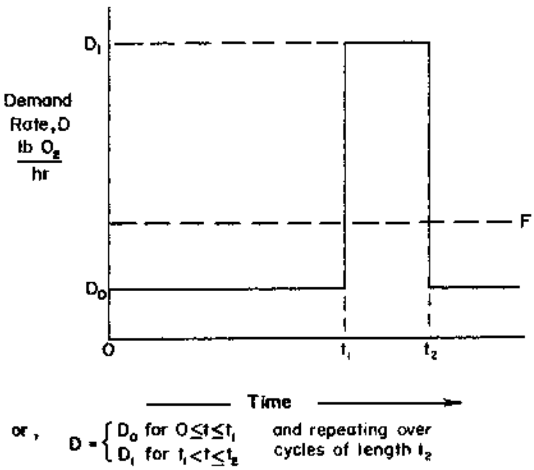

Figure 1.2. Oxygen demand cycle, Example 1.1.

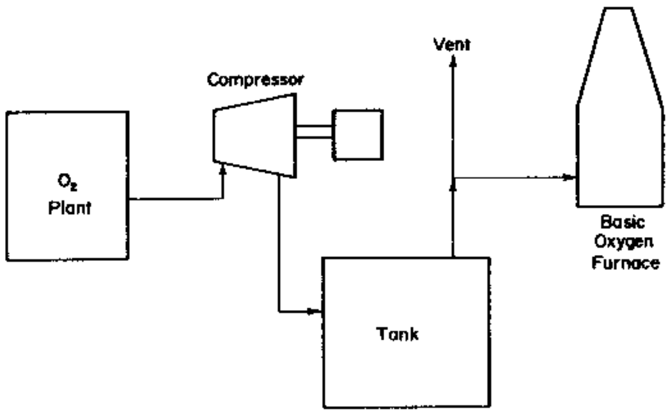

Figure 1.3. Design of oxygen production system, Example 1.1.

to be equal to D_1, the high demand rate. During the low-demand interval the excess oxygen could just be vented to the air. At the other extreme, the oxygen plant capacity could be chosen to be just enough to produce the amount of oxygen required by the BOF over a cycle. During the low-demand interval, the excess oxygen produced would then be compressed and stored for use during the high-demand interval of the cycle. Intermediate designs could use some combination of venting and storage of oxygen. The problem is to select the optimal design.

Formulation. The system of concern will consist of the O_2 plant, the compressor, and the storage tank. The BOF and its demand cycle are assumed fixed by external factors. A reasonable performance index for the design is the total annual cost, which consists of the oxygen production cost (fixed and variable), the compressor operating cost, and the fixed costs of the compressor and storage vessel. The key independent variables are the oxygen plant production rate F (lb O_2/hr), the compressor and storage tank design capacities, H (HP) and V (ft^3), respectively, and the maximum tank pressure p (psia). Presumably the oxygen plant design is standard so that the production rate fully characterizes the plant. Similarly, we assume that the storage tank will be of a standard design approved for O_2 service.

The model will consist of the basic design equations that relate the key independent variables.

If $I_{\max}$ is the maximum amount of oxygen that must be stored, then using the corrected gas law we have

$$V = \frac{I_{\max}}{M}\frac{RT}{p}z \tag{1.1}$$

where R = gas constant
T = gas temperature (assume fixed)
z = compressibility factor
M = molecular weight of O_2

From Figure 1.2, the maximum amount of oxygen that must be stored is equal to the area under the demand curve between t_1 and t_2 and D_1 and F. Thus,

$$I_{max} = (D_1 - F)(t_2 - t_1) \tag{1.2}$$

Substituting (1.2) into (1.1), we obtain

$$V = \frac{(D_1 - F)(t_2 - t_1)}{M} \frac{RT}{p} z \tag{1.3}$$

The compressor must be designed to handle a gas flow rate of $(D_1 - F)(t_2 - t_1)/t_1$ and to compress the gas to the maximum pressure p. Assuming isothermal ideal gas compression [3],

$$H = \frac{(D_1 - F)(t_2 - t_1)}{t_1} \frac{RT}{k_1 k_2} \ln\left(\frac{p}{p_0}\right) \tag{1.4}$$

where k_1 = unit conversion factor
k_2 = compressor efficiency
p_0 = O_2 delivery pressure

In addition to (1.3) and (1.4), the O_2 plant rate F must be adequate to supply the total oxygen demand, or

$$F \geqslant \frac{D_0 t_1 + D_1(t_2 - t_1)}{t_2} \tag{1.5}$$

Moreover, the maximum tank pressure must be greater than the O_2 delivery pressure,

$$p \geqslant p_0 \tag{1.6}$$

The performance criterion will consist of the oxygen plant annual cost,

$$C_1 \text{ (\$/yr)} = a_1 + a_2 F \tag{1.7}$$

where a_1 and a_2 are empirical constants for plants of this general type and include fuel, water, and labor costs.

The capital cost of storage vessels is given by a power law correlation,

$$C_2\ (\$) = b_1 V^{b_2} \tag{1.8a}$$

where b_1 and b_2 are empirical constants appropriate for vessels of a specific construction.

The capital cost of compressors is similarly obtained from a correlation:

$$C_3\ (\$) = b_3 H^{b_4} \tag{1.8b}$$

The compressor power cost will, as an approximation, be given by $b_5 t_1 H$, where b_5 is the cost of power. The total cost function will thus be of the form

$$\text{Annual cost} = a_1 + a_2 F + d(b_1 V^{b_2} + b_3 H^{b_4}) + N b_5 t_1 H \tag{1.9}$$

where N is the number of cycles per year and d is an appropriate annual cost factor.

The complete design optimization problem thus consists of the problem of minimizing (1.9) by the appropriate choice of F, V, H, and p subject to Eqs. (1.3) and (1.4) as well as inequalities (1.5) and (1.6).

The solution of this problem will clearly be affected by the choice of the cycle parameters (N, D_0, D_1, t_1, and t_2), the cost parameters (a_1, a_2, b_1–b_5, and d), and the physical parameters (T, p_0, k_2, z, and M).

In principle we could solve this problem by eliminating V and H from (1.9) using (1.3) and (1.4), thus obtaining a two-variable problem. We could then plot the contours of the cost function (1.9) in the plane of the two variables F and p, impose the inequalities (1.5) and (1.6), and determine the minimum point from the plot. However, the methods discussed in subsequent chapters allow us to obtain the solution with much less work. For further details and a study of solutions for various parameter values, the reader is invited to consult Jen et al. [4].

Example 1.1 presented a preliminary design problem formulation for a system consisting of several pieces of equipment. The next example illustrates a detailed design of a single structural element.

Example 1.2 Design of a Welded Beam

Description. A beam A is to be welded to a rigid support member B. The welded beam is to consist of 1010 steel and is to support a force F of 6000 lb. The dimensions of the beam are to be selected so that the system cost is minimized. A schematic of the system is shown in Figure 1.4.

Formulation. The appropriate system boundaries are quite self-evident. The system consists of the beam A and the weld required to secure it to B. The independent or design variables in this case are the dimensions h, l, t, and b, as shown in Figure 1.4. The length L is assumed to be specified at 14 in. For

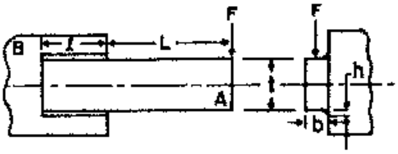

Figure 1.4. Welded beam, Example 1.2.

notational convenience we redefine these four variables in terms of the vector of unknowns $\mathbf{x}$:

$$\mathbf{x} = [x_1, x_2, x_3, x_4,]^{\mathrm{T}} = [h, l, t, b]^{\mathrm{T}}$$

The performance index appropriate to this design is the cost of a weld assembly. The major cost components of such an assembly are (1) setup labor cost, (2) welding labor cost, and (3) material cost:

$$F(x) = c_0 + c_1 + c_2 \tag{1.10}$$

where $F(x)$ = cost function
c_0 = setup cost
c_1 = welding labor cost
c_2 = material cost

Setup Cost c_0. The company has chosen to make this component a weldment, because of the existence of a welding assembly line. Furthermore, assume that fixtures for setup and holding of the bar during welding are readily available. The cost c_0 can therefore be ignored in this particular total-cost model.

Welding Labor Cost c_1. Assume that the welding will be done by machine at a total cost of \$10/hr (including operating and maintenance expense). Furthermore, suppose that the machine can lay down a cubic inch of weld in 6 min. The labor cost is then

$$c_1 = \left(10\,\frac{\$}{\text{hr}}\right)\left(\frac{1}{60}\,\frac{\text{hr}}{\text{min}}\right)\left(6\,\frac{\text{min}}{\text{in.}^3}\right)V_w = 1\left(\frac{\$}{\text{in.}^3}\right)V_w$$

where V_w = weld volume, in.^{3}

Material Cost c_2

$$c_2 = c_3 V_w + c_4 V_B$$

where c_3 = cost per volume of weld material, \$/in.^{3}, =(0.37)(0.283)
c_4 = cost per volume of bar stock, \$/in.^{3}, =(0.17)(0.283)
V_B = volume of bar A, in.^{3}

From the geometry,

$$V_w = 2(\tfrac{1}{2}h^2 l) = h^2 l$$

and

$$V_B = tb(L + l)$$

so

$$c_2 = c_3 h^2 l + c_4 tb(L + l)$$

Therefore, the cost function becomes

$$F(x) = h^2 l + c_3 h^2 l + c_4 tb(L + l) \tag{1.11}$$

or, in terms of the x variables,

$$F(x) = (1 + c_3)x_1^2 x_2 + c_4 x_3 x_4 (L + x_2) \tag{1.12}$$

Not all combinations of x_1, x_2, x_3, and x_4 can be allowed if the structure is to support the load required. Several functional relationships between the design variables that delimit the region of feasibility must certainly be defined. These relationships, expressed in the form of inequalities, represent the design model. Let us first define the inequalities and then discuss their interpretation. The inequalities are

$$g_1(x) = \tau_d - \tau(x) \geqslant 0 \tag{1.13}$$

$$g_2(x) = \sigma_d - \sigma(x) \geqslant 0 \tag{1.14}$$

$$g_3(x) = x_4 - x_1 \geqslant 0 \tag{1.15}$$

$$g_4(x) = x_2 \geqslant 0 \tag{1.16}$$

$$g_5(x) = x_3 \geqslant 0 \tag{1.17}$$

$$g_6(x) = P_c(x) - F \geqslant 0 \tag{1.18}$$

$$g_7(x) = x_1 - 0.125 \geqslant 0 \tag{1.19}$$

$$g_8(x) = 0.25 - \delta(x) \geqslant 0 \tag{1.20}$$

where τ_d = design shear stress of weld
$\tau(x)$ = maximum shear stress in weld; a function of x
σ_d = design normal stress for beam material
$\sigma(x)$ = maximum normal stress in beam; a function of x
$P_c(x)$ = bar buckling load; a function of x
$\delta(x)$ = bar end deflection; a function of x

To complete the model, it is necessary to define the important stress states.

Weld Stress $\tau(x)$. After Shigley [5], the weld shear stress has two components, τ' and τ'', where τ' is the primary stress acting over the weld throat area and τ'' is a secondary torsional stress:

$$\tau' = \frac{F}{\sqrt{2}x_1x_2} \qquad \text{and} \qquad \tau'' = \frac{MR}{J}$$

with

$$M = F\left(L + \frac{x_2}{2}\right)$$

$$R = \left[\frac{x_2^2}{4} + \left(\frac{x_3 + x_1}{2}\right)^2\right]^{1/2}$$

$$J = 2\left\{0.707x_1x_2\left[\frac{x_2^2}{12} + \left(\frac{x_3 + x_1}{2}\right)^2\right]\right\}$$

where M = moment of F about center of gravity of weld group
J = polar moment of inertia of weld group

Therefore, the weld stress τ becomes

$$\tau(x) = [(\tau')^2 + 2\tau'\tau'' \cos\theta + (\tau'')^2]^{1/2}$$

where

$$\cos\theta = \frac{x_2}{2R}$$

Bar Bending Stress $\sigma(x)$. The maximum bending stress can be shown to be equal to

$$\sigma(x) = \frac{6FL}{x_4x_3^2}$$

Bar Buckling Load $P_c(x)$. If the ratio $t/b = x_3/x_4$ grows large, there is a tendency for the bar to buckle. Those combinations of x_3 and x_4 that will cause this buckling to occur must be disallowed. It has been shown [6] that for narrow rectangular bars a good approximation to the buckling load is

$$P_c(x) = \frac{4.013\sqrt{EI\alpha}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{EI}{\alpha}}\right)$$

where E = Young's modulus, $=30 \times 10^6$ psi
$I = \frac{1}{12}x_3x_4^3$
$\alpha = \frac{1}{3}Gx_3x_4^3$
G = shearing modulus, $=12 \times 10^6$ psi

Bar Deflection $\delta(x)$. To calculate the deflection, assume the bar to be a cantilever of length L. Thus,

$$\delta(x) = \frac{4FL^3}{Ex_3^3x_4}$$

The remaining inequalities are interpreted as follows: Inequality g_3 states that it is not practical to have the weld thickness greater than the bar thickness, Inequalities g_4 and g_5 are nonnegativity restrictions on x_2 and x_3. Note that the nonnegativity of x_1 and x_4 are implied by g_3 and g_7. Constraint g_6 ensures that the buckling load is not exceeded. Inequality g_7 specifies that it is not physically possible to produce an extremely small weld.

Finally, the two parameters τ_d and σ_d in g_1 and g_2 depend on the material of construction. For 1010 steel, $\tau_d = 13{,}600$ psi and $\sigma_d = 30{,}000$ psi are appropriate.

The complete design optimization problem thus consists of the cost function (1.12) and the complex system of inequalities that results when the stress formulas are substituted into (1.13)–(1.20). All of these functions are expressed in terms of four independent variables.

This problem is sufficiently complex that graphical solution is patently infeasible. However, the optimum design can readily be obtained numerically by using the methods of subsequent chapters.

For a further discussion of this problem and its solution, see reference 7.

1.2.2 Operations and Planning Applications

The second major area of engineering application of optimization is found in the tuning of existing operations and development of production plans for multiproduct processes. Typically an operations analysis problem arises when an existing production facility designed under one set of conditions must be adapted to operate under different conditions. The reasons for doing this might be as follows:

1. To accommodate increased production throughout
2. To adapt to different feedstocks or a different product slate
3. To modify the operations because the initial design is itself inadequate or unreliable

The solution to such problems might require the selection of new temperature, pressure, or flow conditions; the addition of further equipment; or the definition of new operating procedures. Production planning applications arise from the need to schedule the joint production of several products in a given plant or to coordinate the production plans of a network of production facilities. Since in such applications the capital equipment is already in place, only the variable costs need to be considered. Thus, this type of application can often be formulated in terms of linear or nearly linear models. We will illustrate this class of applications using a refinery planning problem.

Example 1.3 Refinery Production Planning

Description. A refinery processes crude oils to produce a number of raw gasoline intermediates that must subsequently be blended to make two grades of motor fuel, regular and premium. Each raw gasoline has a known performance rating, a maximum availability, and a fixed unit cost. The two motor fuels have a specified minimum performance rating and selling price, and their blending is achieved at a known unit cost. Contractual obligations impose minimum production requirements of both fuels. However, all excess fuel production or unused raw gasoline amounts can be sold in the open market at known prices. The optimal refinery production plan is to be determined over the next specified planning period.

Formulation. The system in question consists of the raw gasoline intermediates, the blending operation, and the fluid motor fuels, as shown schematically in Figure 1.5. Excluded from consideration are the refinery processes involved in the production of the raw gasoline intermediates as well as the inventory and distribution subsystems for crudes, intermediates, and products. Since equipment required to carry out the blending operations is in place, only variable costs will be considered.

The performance index in this case will be the net profit over the planning period. The net profit will be composed of motor fuel and intermediate sales minus blending costs minus the charged costs of the intermediates. The independent variables will simply be the flows depicted as directed arcs in Figure 1.5. Thus, each intermediate will have associated with it a variable that represents the amount of that intermediate allocated to the production of

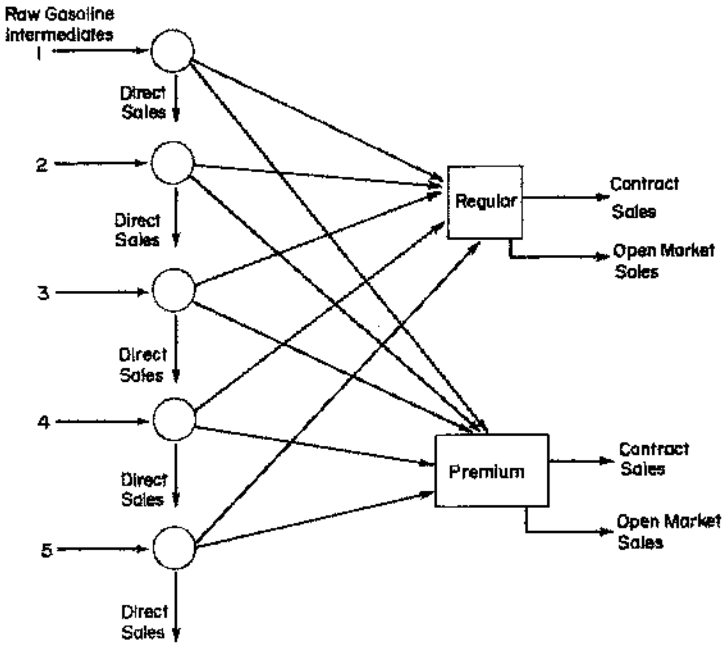

Figure 1.5. Schematic of refinery planning problem, Example 1.3.

regular-grade gasoline, another that represents the amount used to make premium, and a third that represents the amount sold directly.

Thus, for each intermediate i,

$$x_i = \text{amount used for regular, bbl/period}$$

$$y_i = \text{amount used for premium, bbl/period}$$

$$z_i = \text{amount sold directly, bbl/period}$$

Each product will have two variables associated with it: one to represent the contracted sales and one to represent the open-market sales.

Thus, for each product j,

$$u_j = \text{amount allocated to contracts, bbl/period}$$

$$v_j = \text{amount sold in open market, bbl/period}$$

The model will consist of material balances on each intermediate and product, blending constraints that ensure that product performance ratings are met, and bounds on the contract sales:

1. Material balances on each intermediate i:

$$x_i + y_i + z_i \leqslant \alpha_i \tag{1.21}$$

where α_i is the availability of intermediate i over the period, in bbl/period.

2. Material balances on each product:

$$\sum_i x_i = u_1 + v_1 \qquad \sum_i y_i = u_2 + v_2 \tag{1.22}$$

3. Blending constraints on each product:

$$\sum_i \beta_i x_i \geqslant \gamma_1(u_1 + v_1) \qquad \sum_i \beta_i y_i \geqslant \gamma_2(u_2 + v_2) \tag{1.23}$$

where β_i is the performance rating of intermediate i and γ_j is the minimum performance rating of product j.

4. Contract sales restrictions for each product j.

$$u_j \geqslant \delta_j \tag{1.24}$$

where δ_j is the minimum contracted production, in bbl/period.

The performance criterion (net profit) is given by

$$\sum c_j^{(1)} u_j + \sum c_j^{(2)} v_j + \sum c_i^{(3)} z_i - \sum_i c_i^{(4)}(x_i + y_i + z_i) - \sum_i c_i^{(5)}(x_i + y_i)$$

where $c_i^{(1)}$ = unit selling price for contract sales of j
$c_j^{(2)}$ = unit selling price for open-market sales of j
$c_i^{(3)}$ = unit selling price of direct sales of intermediate i
$c_i^{(4)}$ = unit charged cost of intermediate i
$c_i^{(5)}$ = blending cost of intermediate i

Using the data given in Table 1.1, the planning problem reduces to

$$\begin{aligned}\text{Maximize } & 40u_1 + 55u_2 + 46v_1 + 60v_2 + 6z_1 + 8z_2 + 7.50z_3 \\ & + 7.50z_4 + 20z_5 - 25(x_1 + y_1) - 28(x_2 + y_2) \\ & - 29.50(x_3 + y_3) - 35.50(x_4 + y_4) - 41.50(x_5 + y_5)\end{aligned}$$

Subject to
Constraints of type (1.21):

Table 1.1 Data for Example 1.3

Raw Gasoline Intermediate	Availability, α_i (bbl/period)	Performance Rating, β_i	Selling Price, $c_i^{(3)}$	Charged Cost, $c_i^{(4)}$	Blending Cost, $c_i^{(5)}$
1	2×10^5	70	30.00	24.00	1.00
2	4×10^5	80	35.00	27.00	1.00
3	4×10^5	85	36.00	28.50	1.00
4	5×10^5	90	42.00	34.50	1.00
5	5×10^5	99	60.00	40.00	1.50

Product Type	Minimum Contract Sales δ_j	Minimum Performance Rating	Selling Price ($/bbl) Contract, $c_j^{(1)}$	Selling Price ($/bbl) Open Market, $c_j^{(2)}$
Regular (1)	5×10^5	85	$40.00	$46.00
Premium (2)	4×10^5	95	$55.00	$60.00

$$x_1 + y_1 + z_1 \leqslant 2 \times 10^5$$

$$x_2 + y_2 + z_2 \leqslant 4 \times 10^5$$

$$x_3 + y_3 + z_3 \leqslant 4 \times 10^5$$

$$x_4 + y_4 + z_4 \leqslant 5 \times 10^5$$

$$x_5 + y_5 + z_5 \leqslant 5 \times 10^5$$

Constraints of type (1.22):

$$x_1 + x_2 + x_3 + x_4 + x_5 = u_1 + v_1$$

$$y_1 + y_2 + y_3 + y_4 + y_5 = u_2 + v_2$$

Constraints of type (1.23):

$$70x_1 + 80x_2 + 85x_3 + 90x_4 + 99x_5 \geqslant 85(u_1 + v_1)$$

$$70y_1 + 80y_2 + 85y_3 + 90y_4 + 99y_5 \geqslant 95(u_2 + v_2)$$

Constraints of type (1.24):

$$u_1 \geqslant 5 \times 10^5 \qquad u_2 \geqslant 4 \times 10^5$$

In addition, all variables must be greater than or equal to zero to be physically realizable. The composite optimization problem involves 19 variables and 11 constraints plus the nonnegativity conditions. Note that all model functions are linear in the independent variables.

In general, refineries will involve many more intermediate streams and products than were considered in this example. Moreover, in practice it may be important to include further variables, reflecting the material in inventory, as well as to expand the model to cover several consecutive planning periods. In the latter case, a second subscript could be added to the set of variables, for example,

x_{ik} = amount of intermediate i used for regular grade in planning period k

The resulting production planning model can then become very large. In practice, models of this type with over a thousand variables are solved quite routinely.

1.2.3 Analysis and Data Reduction Applications

A further fertile area for the application of optimization techniques in engineering can be found in nonlinear regression problems as well as in many analysis problems arising in engineering science. A very common problem arising in engineering model development is the need to determine the parameters of some semitheoretical model given a set of experimental data. This data reduction or regression problem inherently transforms to an optimization problem, because the model parameters must be selected so that the model fits the data as closely as possible.

Suppose some variable y assumed to be dependent upon an independent variable x and related to x through a postulated equation $y = f(x, \theta_1, \theta_2)$ that depends upon two parameters θ_1 and θ_2. To establish the appropriate values of θ_1 and θ_2, we run a series of experiments in which we adjust the independent variable x and measure the resulting y. As a result of a series of N experiments covering the range of x of interest, a set of y and x values (y_i, x_i), $i = 1, \ldots, N$, is available. Using these data, we now try to "fit" our function to the data by adjusting θ_1 and θ_2 until we get a "good fit." The most commonly used measure of a good fit is the *least-squares criterion,*

$$L(\theta_i, \theta_2) = \sum_{i=1}^{N} [y_i - f(x_i, \theta_1, \theta_2)]^2 \tag{1.25}$$

The difference $y_i - f(x_i, \theta_1, \theta_2)$ between the experimental value y_i and the predicted value $f(x_i, \theta_1, \theta_2)$ measures how close our model prediction is to the data and is called the *residual.* The sum of the squares of the residuals at all the experimental points gives an indication of goodness of fit. Clearly, if $L(\theta_1, \theta_2)$ is equal to zero, then the choice of θ_1, θ_2 has led to a perfect fit; the data points fall exactly on the predicted curve. The data-fitting problem can thus be viewed as an optimization problem in which $L(\theta_1, \theta_2)$ is minimized by appropriate choice of θ_1 and θ_2.

Example 1.4 Nonlinear Curve Fitting

Description. The pressure–molar volume–temperature relationship of real gases is known to deviate from that predicted by the ideal gas relationship,

$$Pv = RT$$

where P = pressure, atm
v = molar volume, $cm^2/g \cdot mol$
T = temperature, K
R = gas constant, 82.06 $atm \cdot cm^3/g \cdot mol \cdot K$

The semiempirical Redlich–Kwong equation [3]

$$P = \frac{RT}{v - b} - \frac{a}{T^{1/2}v(v + b)} \tag{1.26}$$

is intended to correct for the departure from ideality, but it involves two empirical constants a and b whose values are best determined from experimental data. A series of PvT measurements, listed in Table 1.2, are made for CO_2, from which a and b are to be estimated using nonlinear regression.

Formulation. Parameters a and b will be determined by minimizing the least-squares function (1.25). In the present case, the function will take the form

$$\sum_{i=1}^{8} \left[P_i - \frac{RT_i}{v_i - b} + \frac{a}{T_i^{1/2}v_i(v_i + b)} \right]^2 \tag{1.27}$$

Table 1.2 PVT Data for CO_2

Experiment Number	P, atm	v, $cm^3/g \cdot mol$	T, K
1	33	500	273
2	43	500	323
3	45	600	373
4	26	700	273
5	37	600	323
6	39	700	373
7	38	400	273
8	63.6	400	373

where P_i is the experimental value at experiment i and the remaining two terms correspond to the value of P predicted from Eq. (1.26) for the conditions of experiment i for some selected values of the parameters a and b. For instance, the term corresponding to the first experimental point will be

$$\left(33 - \frac{82.06(273)}{500 - b} + \frac{a}{(273)^{1/2}(500)(500 + b)}\right)^2$$

Function (1.27) is thus a two-variable function whose value is to be minimized by appropriate choice of the independent variables a and b. If the Redlich–Kwong equation were to precisely match the data, then at the optimum the function (1.27) would be exactly equal to zero. In general, because of experimental error and because the equation is too simple to accurately model the CO_2 nonidealities, Eq. (1.27) will not be equal to zero at the optimum. For instance, the optimal values of $a = 6.377 \times 10^7$ and $b = 29.7$ still yield a squared residual of 9.7×10^{-2}.

In addition to regression applications, a number of problems arise in engineering science that can be solved by posing them as optimization problems. One rather classical application is the determination of the equilibrium composition of a chemical mixture [3]. It is known that the equilibrium composition of a closed system at fixed temperature and pressure with specified initial composition will be that composition that minimizes the Gibbs free energy of the system. As shown by White et al. [8], the determination of the equilibrium composition can thus be posed as the problem of minimizing a nonlinear function subject to a set of linear equations in nonnegative variables.

Another classical engineering problem that can be posed and solved as an optimization problem is the determination of the steady-state current flows in an electrical resistance network [9]. Given a network with specified arc resistances and a specified overall current flow, the arc current flows can be determined as the solution of the problem of minimizing the total I^2R power loss subject to a set of linear constraints that ensure that Kirchhoff's current law is satisfied at each arc junction in the network.

Example 1.5 Metal Cutting

One of the engineering applications of mathematical programming is the problem of determining the optimal machining parameters in metal cutting. A detailed discussion of different optimization models in metal cutting with illustrations is given in a survey paper by Philipson and Ravindran [10]. Here we shall discuss a machining problem in which a single cutting tool turns a diameter in one pass.

The decision variables in this machining problem are the cutting speed v and the feed per revolution f. Increasing the speed and feed reduces the actual machining time and hence the machining cost, but it has an adverse effect on the life of the cutting tool and results in a higher tooling cost. In addition, the optimal values of v and f will also depend on labor and overhead costs of nonproductive time and tool-changing time. Hence, minimization of the total cost per component is the criterion most often used in selecting the optimal machining parameters, feed and speed.

The cost per component c for a part produced in one pass is given by Armarego and Brown [11]:

$$\begin{aligned} c &= (\text{cost of nonproductive time/component}) + (\text{machining time cost}) \\ &\quad + (\text{cost of tool-changing time/component}) + (\text{tool cost/component}) \end{aligned}$$

Material costs are not considered. The third and fourth terms may be stated more specifically as

Cost of tool-changing time per component

$$= \frac{\text{cost rate} \times \text{tool-changing time}}{\text{number of parts produced between tool changes}}$$

and

$$\text{Tool cost per component} = \frac{\text{tool cost per cutting edge}}{\text{number of parts produced between tool changes}}$$

The cost equation can be expressed mathematically as

$$c = xT_L + xT_c + xT_d\left(\frac{T_{ac}}{T}\right) + y\left(\frac{T_{ac}}{T}\right) \qquad \text{(dollars)} \tag{1.28}$$

where x = labor plus overhead cost rate, \$
T_L = nonproductive time (loading, unloading, and inspection time), min
T_c = machining time, including approach time, min
T_{ac} = actual cutting time (approximately equal to T_c), min
T = tool life, min [given by Eq. (1.30)]
T_d = tool-changing time, min
y = tool cost per cutting edge, \$
T/T_{ac} = number of parts produced between tool changes

The equation for machining time T_c is

$$T_c = \frac{l}{\lambda v f} \qquad \text{(min)} \tag{1.29}$$

where l = distance traveled by the tool in making a turning pass, in.
$\lambda = 12/\pi D$, where D is the mean workpiece diameter, in.
v = cutting speed, surface feet/min
f = feed, in./rev

It has been found [11] that tool life, cutting speed, and feed are related as follows:

$$T = \frac{A}{v^{1/n} f^{1/n_1}} \qquad \text{(min)} \tag{1.30}$$

where A, n, and n_1 are constants.

Assuming that $T_{ac} \simeq T_c$ and inserting Eqs. (1.29) and (1.30) into Eq. (1.28), we obtain

$$c = xT_L + \frac{xl}{\lambda v f} + \left(xT_d \frac{l}{\lambda A} + \frac{yl}{\lambda A} \right) v^{(1/n)-1} f^{(1/n_1)-1} \qquad \text{(dollars)} \tag{1.31}$$

The constraints imposed on v and f by the machine tool and by process and part requirements are as follows:

(i) *Maximum and Minimum Available Cutting Speed*

$$v_{\min} \leqslant v \leqslant v_{\max}$$

(ii) *Maximum and Minimum Available Feed*

$$f_{\min} \leqslant f \leqslant f_{\max}$$

(iii) *Maximum Allowable Cutting Force* ($F_{t,\max}$). This constraint is necessary to limit tool work deflections and their effect upon the accuracy of the turned part. Armarego and Brown [11] give the following expression for the tangential cutting force F_t:

$$F_t = c_f f^{\alpha} d_c^{\gamma} \tag{1.32}$$

where c_t, α, and γ are constants and d_c is the depth of cut, which is held constant at a given value. This constraint on the cutting force results in the following feed constraint:

$$f \leqslant \left[\frac{F_{t,\max}}{c_t d_c^{\gamma}} \right]^{1/\alpha}$$

(iv) *Maximum Available Horsepower.* The horsepower consumed in cutting can be determined from the equation

$$\text{HP} = \frac{F_t v}{33{,}000}$$

where F_t is obtained from Eq. (1.32). If $P_{\max}$ is the maximum horsepower available at the spindle, then

$$v f^{\alpha} \leqslant \frac{P_{\max}(33{,}000)}{c_t d_c^{\gamma}}$$

For a given $P_{\max}$, c_t, γ, and d_c, the right-hand side of the above inequality will be a constant.

(v) *Stable Cutting Region.* Certain combinations of v and f values are likely to cause chatter vibration, adhesion, and built-up edge formation. To avoid this problem, the following stable cutting region constraint is used:

$$v^{\delta} f \geqslant \beta$$

where δ and β are given constants.

As an illustration [10], consider the case where a single diameter is to be turned in one pass using the feed rate and cutting speed that will minimize costs. The bar is 2.75 in. in diameter by 12.00 in. long. The turned bar is 2.25 in. in diameter by 10.00 in. long. In the cutting speed calculations, a mean diameter of 2.50 in. will be used. The lathe has a 15-HP motor and a maximum speed capability of 1500 rpm. The minimum speed available is 75 rpm. The cost rate, tool costs, ideal time, tool-changing time, and tool life parameters are given below:

$x = \$0.15/\text{min}$ | $T_L = 2.00$ min
$l = 10.00$ in. | $D = 2.50$ in.
$\lambda = 1.528$ | $T_d = 1.00$ min
$y = \$0.50$ | $A = 113{,}420$
$n = 0.30$ | $n_1 = 0.45$
$d_c = 0.25$ in. | $N_{\min} = 75$ rpm
$N_{\max} = 1500$ rpm | $c_t = 344.7$
$F_{t,\max} = 1583.0$ lb | $\gamma = 0.9$
$\alpha = 0.78$ | Machine drive efficiency $= 0.8$
$\delta = 2.0$ | Machine HP $= 15.0$
$\beta = 380{,}000$

When the fixed values are inserted into Eq. (1.28), the cost function becomes

$$\text{Minimize} \quad c = 0.30 + \frac{982.0}{vf} + 8.1 \times 10^{-9} v^{2.333} f^{1.222}$$

(***Note:*** For ease of calculations, f is expressed in thousandths of an inch per revolution rather than in inches per revolution.)

The constraints on v and f are given by

$$\begin{aligned}
v &\leqslant 982.0 \\
v &\geqslant 49.1 \\
f &\leqslant 35.0 && \text{(cutting force)} \\
f &\geqslant 1.0 \\
vf^{0.78} &\leqslant 4000.0 && \text{(horsepower)} \\
v^{2.0}f &\geqslant 380{,}000.0 && \text{(stable cutting region)} \\
v, f &\geqslant 0
\end{aligned}$$

1.2.4 Classical Mechanics Applications

The methods described in this book can be useful in a number of application areas, including engineering design, and various forms of data approximation. A student of this field learns that design is the essence of engineering and optimization is the essence of design. On the other hand, it is now clear that the optimization philosophy given in this text is also applicable to problems in classical mechanics and many other fields not considered by the authors.

Ragsdell and Carter proposed the "energy method" [12] as an alternative to more traditional approaches to the solution of conservative, linear and nonlinear initial-value problems. The energy method employs a minimum principle and avoids most of the difficulties associated with the solution of nonlinear differential equations by formulation and iterative solution of an appropriate sequence of optimization problems. It is interesting to note that solutions to almost any predetermined accuracy level are possible and that the energy method appears to be especially useful for very nonlinear problems with solutions that progress over long periods of time and/or space. The method has been used to predict the position of spacecraft over long distances and the position of the robot arm on the Space Shuttle.

Carter and Ragsdell [13] have given a direct solution to the optimal column problem of Lagrange. This problem is of interest because it defied solution for so long. The problem statement is simple: Given an amount of material with known physical properties (e.g., 1010 mild steel) that is to be formed into a column and pinned at each end, what is the optimal tapering function if we assume uniform, solid cross sections? (See Figure 1.6.)

Lagrange worked on this problem most of his life and published [14] his conclusions shortly before his death. He was led to conclude that the optimal

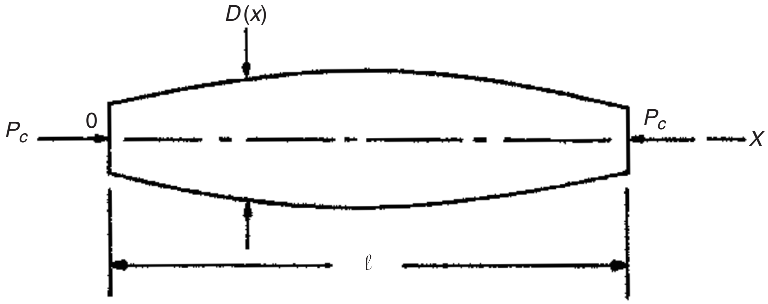

Figure 1.6. Optimal column.

column was a right circular cylinder. Lagrange suspected and many later proved that this could not be so. Carter and Ragsdell later showed that a direct optimization approach employing Fourier approximating functions would produce a valid solution to the optimal column problem of Lagrange and to many related minimum-weight structural design problems. The reader is referred to the references for additional details.

1.2.5 Taguchi System of Quality Engineering

The American, British, and European schools of thought on optimization and optimal design have developed along rather different paths than the work in Japan since the early work of George Dantzig [14] and others during and after World War II. The American, British, and European approach is essentially monolithic, whereas there are some important differences to be noted in the Japanese approach. It is not our purpose to judge whether one path is best, but simply to note that they appear to be significantly different. In the United States many optimization methods have been developed and used based on the assumption that an analytical (mathematical) model or simulation is available. This assumption is used much less in Japan. On the contrary, the common assumption seen is that the system performance will be available through direct measurement of the system being optimized. In addition, in Japan there seems to be much greater emphasis on stochastic formulations and a desire to understand the resulting variation propagation.

Genichi Taguchi is the father of what is now called quality engineering in Japan. He was asked to help with the poor telephone system performance in Japan after the war. It was difficult to have a phone conversation lasting more than a few minutes. It would have been easy to suggest higher quality components to achieve better reliability, but funding was poor, so he took another approach. Taguchi asked, “Is it possible to achieve good performance with inexpensive components?” He wondered, “Can we control the propagation of unwanted variation?” This led him to develop product parameter design, which is now known as the first of four actions in the Taguchi system of quality engineering (Figure 1.7).

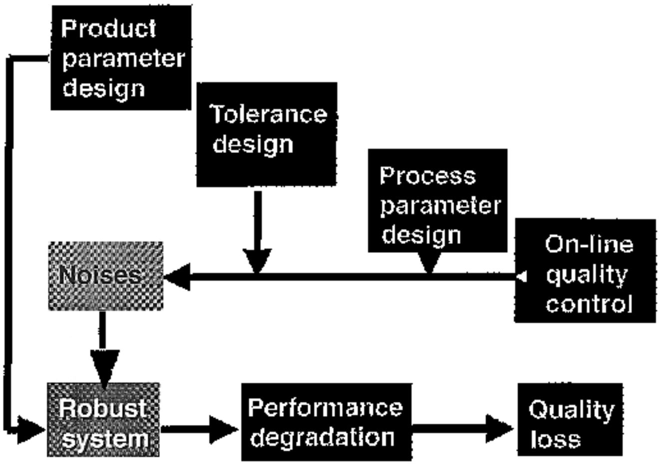

Figure 1.7. Taguchi's system of quality engineering.

Taguchi's approach is based on an additive model or linear independence of factor effects. He seeks to find the control factor levels, which will attenuate variation propagation, so that the product will continue to perform at target even in the presence of internal and external variation. The emphasis is on finding better, more stable designs, not necessarily optimal designs. The approach employs direct sampling of system performance typically using prototypes and very compact sampling strategies. Orthogonal arrays are used to estimate system sensitivities. Taguchi's work is beyond the scope of this book, but it is important work and the interested reader is advised to consult the many excellent references [15–18].

1.3 STRUCTURE OF OPTIMIZATION PROBLEMS

Although the application problems discussed in the previous section originate from radically different sources and involve different systems, at their root they have a remarkably similar form. All four can be expressed as problems requiring the minimization of a real-valued function $f(x)$ of an N-component vector argument $x = (x_1, x_2, \ldots, x_N)$ whose values are restricted to satisfy a number of real-valued equations $h_k(x) = 0$, a set of inequalities $g_j(x) \geqslant 0$, and the variable bounds $x_i^{(U)} \geqslant x_i \geqslant x_i^{(L)}$. In subsequent discussions we will refer to the function $f(x)$ as the *objective function,* to the equations $h_k(x) = 0$ as the *equality constraints,* and to the inequalities $g_j(x) \geqslant 0$ as the *inequality constraints.* For our purposes, these problem functions will always be assumed to be real valued, and their number will always be finite.

The general problem

$$
\begin{array}{lll}
\text{Minimize} & f(x) & \\
\text{Subject to} & h_k(x) = 0 & k = 1, \ldots, K \\
& g_j(x) \geq 0 & j = 1, \ldots, J \\
& x_i^{(U)} \geq x_i \geq x_i^{(L)} & i = 1, \ldots, N
\end{array}
$$

is called the *constrained* optimization problem. For instance, Examples 1.1, 1.2, and 1.3 are all constrained problems. The problem in which there are no constraints, that is,

$$J = K = 0$$

and

$$x_i^{(U)} = -x_i^{(L)} = \infty \qquad i = 1, \ldots, N$$

is called the *unconstrained* optimization problem. Example 1.4 is an unconstrained problem.

Optimization problems can be classified further based on the structure of the functions f, h_k, and g_j and on the dimensionality of x. Unconstrained problems in which x is a one-component vector are called *single-variable* problems and form the simplest but nonetheless very important subclass. Constrained problems in which the function h_k and g_j are all linear are called *linearly constrained* problems. This subclass can further be subdivided into those with a linear objective function f and those in which f is nonlinear. The category in which all problem functions are linear in x includes problems with continuous variables, which are called *linear programs,* and problems in integer variables, which are called *integer programs.* Example 1.3 is a linear programming problem.

Problems with nonlinear objective and linear constraints are sometimes called *linearly constrained nonlinear programs.* This class can further be subdivided according to the particular structure of the nonlinear objective function. If $f(x)$ is quadratic, the problem is a *quadratic program;* if it is a ratio of linear functions, it is called a *fractional* linear program; and so on. Subdivision into these various classes is worthwhile because the special structure of these problems can be efficiently exploited in devising solution techniques. We will consider techniques applicable to most of these problem structures in subsequent chapters.

1.4 SCOPE OF THIS BOOK

In this text we study the methodology applicable to constrained and unconstrained optimization problems. Our primary focus is on general-purpose techniques applicable to problems in continuous variables, involving real-valued constraint functions and a single real-valued objective function. Problems posed in terms of integer or discrete variables are considered only briefly. Moreover, we exclude from consideration optimization problems involving functional equations, non-steady-state models, or stochastic elements. These very interesting but more complex elements are the appropriate subject matter for more advanced, specialized texts. While we make every effort to be precise in stating mathematical results, we do not normally present detailed proofs of these results unless such a proof serves to explain subsequent algorithmic constructions. Generally we simply cite the original literature source for proofs and use the pages of this book to motivate and explain the key concepts underlying the mathematical constructions.

One of the goals of this text is to demonstrate the applicability of optimization methodology to engineering problems. Hence, a considerable portion is devoted to engineering examples, to the discussion of formulation alternatives, and to consideration of computational devices that expedite the solution of applications problems. In addition, we review and evaluate available computational evidence to help elucidate why particular methods are preferred under certain conditions.

In Chapter 2, we begin with a discussion of the simplest problem, the single-variable unconstrained problem. This is followed by an extensive treatment of the multivariable unconstrained case. In Chapter 4, the important linear programming problem is analyzed. With Chapter 5 we initiate the study of nonlinear constrained optimization by considering tests for determining optimality. Chapters 6–10 focus on solution methods for constrained problems. Chapter 6 considers strategies for transforming constrained problems into unconstrained problems, while Chapter 7 discusses direct-search methods. Chapters 8 and 9 develop the important linearization-based techniques, and Chapter 10 discusses methods based on quadratic approximations. Then, in Chapter 11 we summarize some of the methods available for specially structured problems. Next, in Chapter 12 we review the results of available comparative computational studies. The text concludes with a survey of strategies for executing optimization studies (Chapter 13) and a discussion of three engineering case studies (Chapter 14).

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