PREFACE

This book is intended for a second course on finite elements. It is assumed that the readers are familiar with the basic concepts of finite element analysis that are covered in the author's book *Fundamental Finite Element Analysis and Applications: with Mathematica and MATLAB Computations*, John Wiley & Sons, 2005.

As was the case with that book, the goal of the present book is to keep an appropriate balance between the theory, generality, and the practical applications of the finite element method. The word *advanced* in the title of the book refers primarily to the subject matter, not to its presentation. Starting with the governing differential equations, a consistent weighted residual approach is used to formulate all elements. In multifield formulations this approach clearly distinguishes between equations that are being satisfied exactly and those that are satisfied in an approximate weak sense. The same approach is used even for nonlinear problems. This is in contrast to most existing literature, which relies on using established variational principles without showing their derivations. To present concepts as clearly as possible, most derivations use explicit matrix notation instead of compact, but sometimes difficult to follow, indicial notation. All computational details are fully explained, and numerous examples are included showing all calculations. To overcome the tedious nature of calculations associated with finite elements, extensive use of MATLAB and *Mathematica* is made in the book. All necessary computations are readily apparent from these implementations.

An obvious consequence of this detailed approach to presenting the material is that it is not possible to cover all topics in a single book. Thus the book is restricted to topics relevant to structural applications. Even within the structures area topics are covered that are more of a fundamental nature. No attempt is made to cover all possible variations of the basic ideas that have been proposed in the literature.

Topics Covered

1. *Essential Background.* To minimize references to the fundamentals book, this chapter provides a quick review of the steps in a finite element solution. Other topics included are a summary of commonly used interpolation function, integration by parts, numerical integration, and computations for mapped elements.

2. *Analysis of Elastic Solids.* Finite element analysis of general three-dimensional solids is presented in this chapter. Both tetrahedral and mapped solid elements are considered. Computational issues such as integration over volume, faces, and edges of solid elements are explained in detail through numerical examples. Other topics considered in this chapter include computation of stresses, nodal averaging, substructuring and the *patch test* used to verify whether or not an element is conforming. To show implementation of the finite element method in a conventional programming language, a FORTRAN computer program called fe2Quad is presented.

3. *Solids of Revolution.* This chapter considers analysis of three-dimensional solids that can be generated by revolving a plane figure about an axis. Formulation is presented for the axisymmetric analysis when both the material properties and loading are symmetric about the axis of revolution. The solution to the unsymmetric loading on a solid of revolution is also considered.

4. *Multifield Formulations for Beam Elements.* The purpose of this chapter is to present basic concepts related to extending the standard displacement-based finite element formulation to include additional variables. Beam elements provide an excellent opportunity to understand ideas used in various multifield formulations and the associated issues of shear locking and reduced integration. A thorough understanding of these concepts is crucial to successful development of refined elements for plane stress-strain and plate and shell problems.

5. *Multifield Formulations for Analysis of Elastic Solids.* The chapter presents multifield formulations for analysis of general elastic solids. The standard displacement formulation is reviewed and a possible stress-based formulation is presented. Details of general assumed stress mixed formulation are presented along with strategies for choosing appropriate stress interpolations. Finally, the mixed formulation ideas are used to present a practical formulation for analysis of nearly incompressible solids.

6. *Plates and Shells.* This chapter presents several elements for analysis of plate and shell structures. The governing differential equations for thin plates are presented along with their finite element formulations. Generalization of a plate element to a shell element is presented. The basic equations of Mindlin's plate theory are presented. Some elements based on this theory are presented, and their behavior as the plate thickness is reduced is discussed.

7. *Introduction to Nonlinear Problems.* This chapter starts with a brief review of solution procedures for nonlinear problems. The general steps in the solution of nonlinear finite element analysis are discussed. Solution details are explained by considering a second-order nonlinear ordinary differential equation. An important concept of *directional derivative* is introduced in this chapter. This concept is useful for deriving linearized forms for arbitrary nonlinear functions.

8. *Material Nonlinearity.* The basic concepts of finite element analysis involving material nonlinearities are explained by considering detailed examples involving axially loaded bars and trusses. This is followed by formulation for material nonlinearity in general solids. Incremental stress-strain equations, state determination procedures, and complete finite element formulations are discussed for elastic-perfectly plastic solids and solids with isotropic and kinematic hardening.

9. *Geometric Nonlinearity.* This chapter starts with a review of important continuum mechanics concepts needed for large displacement formulations. Concepts such as deformation gradient, Green-Lagrange strains, and Cauchy and Piola-Kirchhoff stresses are illustrated through detailed numerical examples. A general form of incremental finite element equations using total Lagrangian formulation is presented. To develop a clear understanding of the ideas, specific equations and numerical examples are presented for planar problems.

10. *Contact Problems.* Several practical problems involve situations in which one or more bodies can come in contact during loading. The contact problems are formulated in a variety of ways. The simplest situation is the frictionless, small displacement, elastic contact problem. The other extreme is the contact problem involving friction and large strain inelastic behavior. From the finite element point of view, all formulations involve the use of some form of constraint equation. This chapter introduces useful finite element formulations for contact problems.

Unique Features

1. Numerous numerical examples are included to clearly show all computations involved. Often these examples are supplemented with *Mathematica* and MATLAB implementations to help with tedious calculations. Few of these implementations are actually shown in the printed book. All others can be found on the companion book web site. This is done to keep these implementations from distracting the reader not familiar with these systems and also to keep the size of the printed book to a reasonable length.

2. Chapter 2: This chapter shows all computational details of mapped three-dimensional solid elements. It contains specific numerical examples illustrating general procedures for computing surface and edge integrals involving 8 and 20 node solids.

3. Chapter 3: Most textbooks do not include treatment of general unsymmetric loading on solids of revolution.

4. Chapters 4 and 5 on multifield formulations are unique. No existing book offers the breadth of coverage included in these chapters. Furthermore, the treatment in the book does not employ ad hoc variational principles without any systematic derivation. Instead, the formulations are derived directly from the appropriate governing equations using a Galerkin weighted residual approach in essentially the same way that the first formulation is derived in Chapter 2 of the fundamentals book.

5. Instead of reviewing the vast literature on the subject of plate and shell elements, Chapter 6 presents few elements commonly used in practice for both thin and thick plates. Newer elements based on multifield formulations are presented concisely, since most related issues are already dealt with in a simpler context in Chapter 4.

6. The last four chapters tackle the difficult subject of nonlinear analysis. To make the presentation accessible to students who may not have taken the advanced plasticity and continuum mechanics courses, these chapters derive all the basic concepts needed from these areas. Numerical examples, geared directly toward finite element applications, are included to show the relevance of the concepts. All computational steps are laid out clearly. When dealing with geometric nonlinearity, the algebra becomes messy. However, the overall finite element formulation is still based on simple familiar concepts. Chapter 10 carries this approach through to contact problems for which the solution algorithms are still evolving.

7. All chapters contain problems for homework assignment. Most chapters also contain problems suitable for computer labs and projects. The accompanying Web site contains all text examples, MATLAB and *Mathematica* functions, and ANSYS and Abaqus files in electronic form. Some lengthy examples are not fully reproduced in the printed textbook. To see all intermediate results full computational details for these examples are posted on the Web. I plan to keep the book up to date by posting new material on the Web as the need arises.

Acknowledgments

Generally, the material presented in the book is part of established finite element literature. However, for the most part, detailed derivations, as presented here, are not available from a single source. Thus, it is difficult to acknowledge contributions of specific individuals. I am indebted to the authors of all books listed in the bibliography and numerous authors of journal papers on the subject. I have obviously benefited from their contributions.

The book grew out of my lecture notes for the second finite element course that I have taught at the University of Iowa typically every other year for the past 20 years. I am grateful to the students whose questions led me to dig deeper and find consistent and logical explanations to some difficult concepts. I am particularly indebted to my former graduate student Ryan Vignes, who read through at least two different drafts and provided valuable feedback. My colleague, Professor Jia Lu, has provided valuable comments to improve the book. I want to thank Professor David Forkenbrock, Director Public Policy Center, and Professor Robert Ettema, Chairman, Department of Civil and Environmental Engineering at the University of Iowa, for providing great work environment for completing this work.

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