

Tartaglia versus Cardano

Solving Cubic Equations

In 1545, Girolamo Cardano, an Italian physician and mathematician, set the world of mathematics abuzz with a book on algebra. Referred to today as *Ars Magna* or *The Rules of Algebra*, it is still considered by many scholars to be one of the scientific masterpieces of the Renaissance.

What was so important about an algebra book?

Ars Magna began with some introductory material, including standard solutions to linear and quadratic equations. But then it jumped into uncharted territory and laid out for the first time a complete procedure for solving cubic and biquadratic (third-degree and fourth-degree) algebraic equations.

The book was in truth a stunning achievement and was to play an important role in stimulating the growth of algebra in Europe during

most of the remainder of the 16th century. It was not until the arrival of mathematicians at the level of François Viète (1540–1603) and René Descartes (1596–1650) that the book's contributions were superseded.

But its impact didn't stop at mathematics, for the Renaissance was also a formative period in the world of science, and Cardano's book played a role there as well. As the eminent mathematician and scholar Morris Kline explains, "Many people credit the rise of modern science to the introduction of experimentation on a large scale and believe that mathematics served only occasionally as a handy tool. The true situation . . . was actually quite the reverse. The Renaissance scientist approached the study of nature as a mathematician. . . . There was to be little or no assistance from experimentation. He then expected to deduce new laws from these principles."¹

By energizing a long-dormant mathematical field, then, Cardano also provided fuel for the advance of science. The mathematics historian Ronald Calinger, for example, sees Cardano as one of the architects of the new science of the Renaissance. As a result, the *Ars Magna* has been compared to Vesalius's *On the Structure of the Human Body* and to Copernicus's *On the Revolutions of the Heavenly Spheres*, both of which appeared at about the same time.

Vesalius was a Belgian, however, and Copernicus was Polish. Surely, Italian mathematicians swelled with pride that one of their own had made such an extraordinary contribution to the advancement of their discipline.

Certainly, they did—with one major exception. Almost immediately after the publication of *Ars*, an Italian mathematician generally known by a single name, Tartaglia, began attacking Cardano. Though Cardano had stated very clearly in the text, and in several places, that credit for the solution to one of the basic cubic equations belonged to Tartaglia, this, said Tartaglia, was not the point. Filled with rage at what he saw as Cardano's treachery, Tartaglia maintained that when he had shown Cardano that solution, Cardano had promised faithfully—as a Christian and a gentleman—that he would not reveal it until Tartaglia published it first.

To understand Tartaglia's objections and the strange outcome of the resulting dispute, we must travel back to the beginning of the 16th century.

Rebirth

The European Renaissance, which actually began during the 14th century, was a rebirth, a reawakening of European mind and culture after a thousand years of sleep. Artists and scholars, especially in Italy, were rediscovering the riches of the past and were adding to them. More slowly, but just as surely, science, technology, and finally mathematics began to awaken as well.

The first stirrings in mathematics began in algebra toward the end of the 15th century. As with other aspects of the Renaissance, these were largely a rediscovery of earlier work, in this case the remarkable achievements of the earlier Greek, Arab, and Hindu mathematicians, who had solved linear and quadratic equations (equations of the form $ax + b = c$ and $ax^2 + bx + c = d$) many centuries earlier.

Arab mathematicians had solved some cubics as long ago as the 9th century and perhaps earlier, but these were geometrical solutions, or even guessed solutions, for specific numerical problems. Still badly needed, and actively sought, was a solution for the general cubic ($ax^3 + bx^2 + cx + d = 0$). Luca Pacioli, the author of the most influential book in Italian mathematics prior to *Ars*, maintained (1494) that such a solution could not be found.

Then, sometime between 1510 and 1515, Scipione del Ferro (1465–1526), a professor of mathematics at Bologna, came up with the first algebraic solution to a cubic equation. He had developed an algebraic formula for solving the “depressed cubic,” a specific third-degree equation that lacks its second-degree term. In other words, he had come up with a general solution for $x^3 + ax = b$, with a and b positive. It was a real breakthrough, but he kept it a virtual secret for a dozen years and maybe more! What could explain such surprising behavior?

First, the turn of the 16th century was not a time of “publish or perish.” There were no peer-reviewed journals; there was no Internet. In fact, the more likely scenario was for the discoverer of a new solution to keep it close to his chest and to use it publicly, if he did at all, only when it could somehow prove advantageous.

For example, the idea of tenure lay long in the future, so academic appointments in mathematics could be tenuous. Chairs were held by

virtue of eminence and reputation, and public challenges might come at any time. Contests sometimes resulted in public disputations that could be large, contentious affairs, often attended by the disputants' students and supporters. In some cases, the contests attracted large crowds and even passionate betting. Del Ferro apparently believed that if challenged with a list of problems to be solved, he could always use his method as a powerful counterpunch.

History does not tell us whether del Ferro ever used the solution in this way, but we do know that upon his death in 1526, his papers, with the solution, passed on to his son-in-law and successor, Annibale della Nave, and, more important, to one of his pupils, Antonio Maria Fior.²

Fior felt he was now in possession of a valuable treasure, and he returned to his native city of Venice with the objective of establishing himself as a teacher of mathematics. He let it be known that he had a special ability regarding cubic equations. Yet he kept hearing that maybe it wasn't so special, that someone else had this ability, too. The name he heard was that of Tartaglia, a teacher of mathematics in Venice and Verona who was making a name for himself in public debates and who had also made some claims regarding cubic equations.

Fior thought about issuing a public challenge to Tartaglia. If Tartaglia's claims were exaggerated, which was quite possible, it would be a good way to build up his own reputation while tearing down that of this pretender.

Tartaglia

The likelihood that Tartaglia would later have anything to offer mathematics had seemed small when he was born in Brescia, in northern Italy, in 1499. His father was a mailman, and the family was poor. Whatever Tartaglia learned of mathematics and science, he picked up on his own.

He was not always called Tartaglia; he was christened Niccolò Fontana. But this was a dangerous time, and about dozen years after his birth the town was sacked by the French. Young Niccolò took a

slashing wound to the mouth and the palate and came very close to death. Although his mother brought him through with careful, tender care, the wound caused permanent damage to his speaking apparatus, with the result that he was nicknamed Tartaglia, meaning the Stutterer. The name stuck.

Tartaglia eventually settled in Venice and made his living as a teacher of mathematics. As with other such teachers, he did the best he could to keep his name before the public by participating in public contests and disputes, and he seems to have had some success in these contests. The 19th-century biographer Henry Morley wrote of Tartaglia that he may “fairly enough be said to have become wholly by his own exertions a distinguished mathematician, as it is also certain that he grew up to be like many other self-taught men, rugged and vain.”³

Tartaglia had implied to a colleague that he could solve a numerical equation of the form $x^3 + cx^2 = d$. This was enough to act as a direct challenge to Fior. Early in 1535, Fior challenged Tartaglia to a public contest. They came to an agreement: each would propose 30 problems to the other. Whoever should solve the most problems after 30 days would be the winner. There was little fear of collusion, for there was nowhere else to turn for help.

No one knows how much Tartaglia really knew at the start of the contest, but by the evening of February 13, 1535, he was able to solve both types of numerical cubic (with and without the x^2 term), a tremendous accomplishment. This meant he could solve all 30 of Fior’s problems. On the other hand, Tartaglia had apparently been aware that Fior was capable of solving only the depressed cubic, and he had designed his questions around this form. Fior therefore had little success with Tartaglia’s questions.

Tartaglia was the clear winner, and little was heard of Fior after that.

Tartaglia’s fame grew, and the numbers of his students grew apace. Again, with our modern mindset, we would expect Tartaglia to have published his newfound, or newly developed, technique for solving the cubic, but no, he, too, kept it close to his chest. Morley would later complain, “His new rules concerning cubic equations he maintained as his private property, cherishing them as magic arms

which secured to him a constant victory in algebraic tilts, and caused him to be famed and feared. . . . That,” he continued, “was a selfish use to make of scientific acquisitions, with which no scholar of the present day [1854] would sympathize, and which, also, in the sixteenth century, would have been thought illiberal . . . even by our erratic and excitable Cardan.”⁴

As we shall see, he might better have said, “*especially* by our erratic and excitable Cardan.”

Yet mathematics, remember, can be a useful tool, as well as a fascinating puzzle, and with many of the Western powers fighting for control of Italy, the application of mathematics to ballistics was a hot topic. Tartaglia applied himself to it. So it was that in 1537, Tartaglia’s eye was not on cubic equations but on ballistics, and he published a successful book on the subject; his *Nova Scientia* described both new methods and new instruments.

His star, clearly, was rising, but it was to take a most unexpected turn. For just as Fior was falling out of the ring, Girolamo Cardano, a far more dangerous opponent, was stepping in.

Cardano, Renaissance Man

Though Girolamo Cardano was close in age to Tartaglia, his early life had been very different. Born in Pavia in 1501, he later recalled, “My father, in my earliest childhood, taught me the rudiments of arithmetic. . . . After I was twelve years old he taught me the first six books of Euclid, but in such a manner that he expended no effort on such parts as I was able to understand by myself.”⁵ Cardano’s father was a well-educated lawyer, a lecturer in geometry, and a friend of Leonardo da Vinci, who was himself interested in mathematics.

Although Cardano’s father wanted him to study law, Cardano, despite showing clear ability in mathematics, leaned toward medicine as a career. He began his university training at age 19 at the University of Pavia. By age 21, he was debating in public and lecturing on Euclid. After transferring to the University of Padua, he received his medical degree there at age 25.

Øystein Ore, one of his biographers, writes that “he was quick

tempered and vindictive and often unable to control his anger. At times this involved him in brawls of the most serious kind.”⁶ So we should not be surprised when Cardano tells us in his autobiography that from the years 1524 through 1547, he was engaged almost constantly in lawsuits—and, he claims, won them all. Perhaps he should have gone into law?

Apparently not. Smart and personable, he was at the same time building a strong reputation in medicine and, by the late 1530s, had become possibly the most sought-after physician in northern Italy. It seems, however, that medicine at the time presented different career choices than it does today. In 1537, he was invited to teach medicine at Pavia, but he refused, “for there seemed little hope of receiving pay for the work.”⁷ His medical income came basically from private patients and from patrons.

Cardano was a most unusual character, though, and the term “Renaissance man” could have been invented to describe him. For in addition to medicine, he made his mark in several other, quite different, fields. He was, for example, an inveterate gambler, and he published a very popular and useful handbook of gambling, which included some advanced work on probability, as well as detailed information on cheating.

He also cast horoscopes for the rich and powerful. This was a common and widespread practice among people skilled in mathematics and astronomy, and although Cardano was proud of his capability, it got him into various kinds of trouble. In one case he cast a horoscope for Edward VI, the boy king of England, that was just plain wrong, and later he did one for Jesus, which turned out to be a very bad idea.

Cardano was born into a superstitious family and carried on the tradition. At the same time, he made some good observations in medicine and in natural history (what we know today as science). As Cardano himself put it in his autobiography: “If then you place the number of important branches of learning at thirty-six, from . . . any acquaintance with [twenty-six of] them I have refrained.” He then modestly admitted, “To ten I have devoted myself.”⁸

We are concerned with his mathematical work, and specifically with his *Ars Magna*. Certainly, he brought the algebraic solutions for

cubic and biquadratic equations into the open, which was a tremendous accomplishment. How much of this did he owe to Tartaglia and others? Let's take a closer look at how the book came into being.

Preliminaries

After hearing of Tartaglia's success over Fior, Cardano had asked Tartaglia for permission to publish his solution to the cubic equation in his (Cardano's) own forthcoming book on mathematics, promising to give full credit to Tartaglia.

Tartaglia's initial answer was that he was planning to write a book himself in which he would spell out the rule. When? He couldn't say, for he was occupied with other things at the moment, including, first, his ballistics work, and then a translation of Euclid. Not being easily dissuaded, and convinced of the importance of the solution, Cardano kept after Tartaglia, using a variety of entreaties.

A series of letters has come down to us that alternates between sharp and friendly. Initially, for example, Cardano labeled Tartaglia as greedy and unwilling to help mankind.⁹ Then Cardano sharply criticized some work in Tartaglia's book on ballistics. Tartaglia fired back an answer that included "But, in believing that you can demonstrate miraculously by your ridiculous opposition that I am wrong, you have only demonstrated, I will not say, that you are a great ignoramus, but that you are a person of poor judgment."¹⁰

Cardano shifted gears: "You should not imagine that my sharp words were caused by enmity. . . . I really wrote that abuse to excite you to a reply."¹¹

As part of his campaign he invited Tartaglia over for a friendly visit to his home, figuring, correctly, that he would have greater leverage that way. Cardano proclaimed that he was interested in the solution for purely academic reasons. What finally did the trick was his use of an important name, that of his patron Alphonso d'Avalos, Marchese del Vasto. D'Avalos—the Spanish governor of Lombardy, whose capital is Milan, and commander of the imperial army stationed in the area—was one of the most powerful men in Italy. In a letter to Tartaglia, Cardano wrote, "I must in the first place state that

I have held you in good esteem, and as soon as your book on Artillery appeared, I bought two copies, of which I gave one to Signor the Marchese.”¹²

In another letter, dated March 13, 1539, Cardano wrote that his excellency “commanded me at once to write the present letter to you with great urgency in his name, to advise you that on receipt of the same you should come to Milan without fail, for he desires to speak with you.” Tartaglia, well aware that friendship with d’Avalos could be very useful, finally assented: “I go thither unwillingly: however, I will go.”¹³

As it turned out, d’Avalos was not in Milan when Tartaglia arrived. Deliberate deceit? Or just the result of the busy schedule of an important man? It’s hard to say. Øystein Ore, in his biography of Cardano, evaluates the former possibility and points out that this “would have been a very complicated and dangerous scheme. The ruse could readily have backfired on Cardano if Tartaglia on the strength of the invitation had written directly to d’Avalos.”¹⁴

Nevertheless, Cardano did manage to pry Tartaglia’s secret out of him, but Tartaglia was not so foolish as to just hand over the solution. What he gave Cardano was his “rule” for solving the depressed cubic but not his “demonstration,” which would be the general method, or in modern terms the proof that the rule produced the solution. In addition, he gave it in the form of a cryptic verse, though he may later have clarified it for Cardano.

In May 1539, Cardano’s *Practica Arithmeticae Generalis* appeared—without Tartaglia’s solution. It contained some errors, which Tartaglia was happy to point out. In fact, he made fun of both Cardano and the book, which Cardano revised in future editions. Then Tartaglia began hearing rumors about a new book on algebra. Cardano denied the rumors, and things were quiet for a while, but he was indeed working on such a book.

Cardano was in fact a prolific writer. By the end of his life, he had published thousands of pages in various disciplines. *Ars* was to have been volume 10 in an encyclopedia of mathematics—which he never completed and of which not much remains. *Ars Magna* is a shortened version of his original title (*Artis Magnae Sive de Regulis Algebraicis*). In English, it means *The Great Art*, to distinguish it from other, more

elementary works, such as his own earlier one on arithmetic. He was also well aware that the solution to the cubic equation would be of great importance to its success. So, along with his very able assistant, Ludovico (also spelled Lodovico) Ferrari, he put in several years puzzling out the meaning of the verse and expanding the implications when he began to understand it. For, as we'll see, the *Ars* presentation was no simple restatement of Tartaglia's rule.

Ars Magna

The material on cubic equations first appears in chapter 11, which is titled "On the Cube and First Power Equal to the Number." This is interesting on several counts. The rule Tartaglia gave Cardano covered the three basic forms of the depressed cubic. In modern terms, these would be: $x^3 + bx = c$, $x^3 = bx + c$, and $x^3 + c = bx$. The three forms were necessary because mathematicians at the time did not use negative coefficients, and this precluded use of the single, general form $x^3 + ax + b = 0$. In addition, our modern algebraic notation still lay in the future, and most of the mathematical statements were verbal. The chapter title, for example, refers to the specific form that we would today write as $x^3 + bx = c$.

Cardano's book also employs considerable geometric material. In fact, as William Dunham puts it in his fine book *Journey through Genius*, "His argument was purely geometrical, involving literal cubes and their volumes. Actually, the surprise here is minimized when we recall the primitive state of algebraic symbolism and exalted position of Greek geometry among Renaissance mathematicians."¹⁵

In each chapter, then, Cardano first gives a geometrical demonstration of a specific numerical cubic equation, then a verbal rule for solving that general type of equation, then one or more sample problems and solutions using the rule. Because the use of zero and negative coefficients still lay in the future, Cardano is forced into spelling out 13 different cubic equations, all with positive coefficients, and with a separate chapter for each type.

Furthermore, these geometric solutions are by their nature both roundabout and somewhat cumbersome, and because the notation at the time was primitive, the book makes for difficult reading today,

and we needn't go through any of his demonstrations. Yet it's worth showing how his rule works for a specific example of the depressed cubic, which he gives in chapter 11.

In the book, Cardano first presents a general statement of the rule for each chapter, which would work for any numerical example of this type; then he gives a specific example and shows it at work in that example. I'll combine them. I'll state his rule and, to save space, will simply insert the results of this particular example in square brackets as we go.

In modern notation, the example is $x^3 + 6x = 20$ and his rule, in translation, begins:

"Cube one-third the coefficient of x [$\{1/3(6)\}^3 = 2^3 = 8$]; add to it the square of one-half the constant of the equation [$10^2 = 100$]; and take the square root of the whole [$\sqrt{108}$]. You will duplicate this, and to one of the two you add one half the number you have already squared and from the other you subtract one-half the same. You will then have a *binomium* [$\sqrt{108} + 10$] and its *apotome* [$\sqrt{108} - 10$]. Subtract [the cube root of the] apotome from that of the binomium and you will have the value of x :

$$^3\sqrt{\{\sqrt{(108)} + 10\}} - ^3\sqrt{\{\sqrt{(108)} - 10\}}."$$

Cardano doesn't bother to spell out the answer, but the mathematicians among you will realize that the solution to this complicated expression is nothing more than the number 2.

Not all of his examples ended with whole number answers. In some examples, he found himself with imaginary roots. Though he was baffled by them, he did acknowledge their existence.

A Sacred Promise?

There is no question that Cardano's contribution to the field was considerable. The question is, just how perfidious was his treatment of Tartaglia? The answer remains as elusive as ever. First, Ore points out that none of Cardano's contemporaries expressed their displeasure at the time, even though the details of the affair were widely discussed. The negative points of view seem to have arisen later, in the 18th and the beginning of the 19th centuries.¹⁶

Was there a sacred promise of secrecy? Many, perhaps most, writers say yes, but this is mainly based on Tartaglia's claim. In the year following publication of *Ars*, Tartaglia published his own work *Questi et Inventioni Diverse* (*New Problems and Inventions*), which included what he maintained were word-by-word accounts of their meetings.¹⁷ Many writers depend on this publication and quote the following promise that Tartaglia says Cardano made to him: "I swear to you by the sacred Gospel, and on the faith of a gentleman, not only never to publish your discoveries, if you will tell them to me, but also I promise and pledge my faith as a true Christian to put them down in cipher, so that after my death nobody shall be able to understand them."¹⁸

Others point to the fact that Ludovico Ferrari, Cardano's secretary and assistant, was also present when Cardano and Tartaglia met, and that Ferrari later swore, just as vociferously, that Cardano never made such a promise. In fact, Ferrari claimed that in general, Tartaglia's accounts of the earlier proceedings, made in the heat of his rage, were doctored.

Alan Wykes, a modern biographer, goes even further. He argues that Cardano had figured out the algebraic equations by himself, or at least without help from Tartaglia. In *Ars*, says Wykes, "by a slip of pen or memory, he [Cardano] wrote that Tartaglia had communicated the discovery to him and given him permission to use it." But, Wykes argues, "It may perhaps not have been a slip but a muddled attempt at a generous gesture on Cardano's part."¹⁹

Yet between the time of Tartaglia's visit to Cardano and the appearance of *Ars*, six years had elapsed. During that period, Cardano and Ferrari, having heard rumors of the existence elsewhere of such a solution, had traveled to Bologna in 1543 and visited their colleague Annibale della Nave. There they were given permission to examine the papers of Scipione del Ferro, and they learned that del Ferro and not Tartaglia had been the first to solve such an equation algebraically. In such case, they reasoned, even if Cardano had been sworn to secrecy, that promise was no longer valid.

Even before publication of *Ars*, there are suggestions that something of this sort was afoot. For example, after one of Tartaglia's refusals, Cardano wrote again with another request but added, "I

should like to save you from the illusion that you are the first man in the world . . . , I want to write to you amiably to dissolve the fantasy that you are so great. I will lovingly let you know even through your own words that in knowledge you are rather in the valley than near the summit of the mountain.”²⁰

Was he here suggesting that Tartaglia had not been the original possessor of the solution? Why not come right out and say so? This was 400 years ago, and things may have been done differently.

In any case, Cardano was careful not to claim credit for discovering the rule. In three places, he included citations for earlier work on the cubic equation. Near the beginning of chapter 1, for example, he writes, “In our own days Scipione del Ferro of Bologna has solved the case of the cube and first power equal to a constant, a very elegant and admirable accomplishment. Since this art surpasses all human subtlety and the perspicuity of men’s minds, whoever applies himself to it will believe that there is nothing that he cannot understand. In emulation of him, my friend Niccolò Tartaglia of Brescia, wanting not to be outdone, solved the same case when he got into a contest with his [Scipione’s] pupil, Antonio Maria Fior, and, moved by my many entreaties, gave it to me.”²¹

Cardano repeats almost the same words at the beginning of chapter 11: “He [Tartaglia] gave it to me in response to my entreaties,” but adds that Tartaglia withheld the demonstration: “Armed with this assistance [the rule], I sought out its demonstration in [various] forms. This was very difficult. My version of it follows.”

And his version is indeed different, and much expanded, from what Tartaglia had given him—or from whatever he had gotten from any other source. More specifically, with the help of his secretary/assistant, Ludovico Ferrari, he uses the solution he started with (for the depressed cubic, obtained from Tartaglia or whoever) as a stepping-stone. By employing appropriate substitutions, which reduced them to the known case, he had found solutions to the three additional cubic equations. Then, again with the help of Ferrari, to whom he gives appropriate credit, Cardano also shows how these cubic solutions could be used as a foundation on which to build solutions for biquadratics, or equations of the fourth degree. In addition, Cardano points out that cubic equations should have three roots.

A Battle Not-So-Royal

Despite Cardano's acknowledgments, Tartaglia was in a rage. The following year he published his book *New Problems and Inventions*, mentioned earlier. The first half does indeed contain solutions to problems that had been put to him over the years, but the latter part is devoted entirely to a full-out attack on Cardano and *Ars*. It includes reproductions of their correspondence, along with his comments. It is a bitter, and powerful, attack. He publicly heaps scorn on Cardano's mathematical abilities. Wykes describes it as a "swingeing attack . . . denying ever having given Cardano permission and accusing him of theft."²²

As Tartaglia thought would happen, he got a response, but not quite what he was expecting. For it seems we are dealing here with not just two, but with three quite disputatious types. Ore writes, "The Renaissance abounds in impulsive and hotheaded geniuses and Ferrari ran true to form. He had such a temper that even Cardano at times was afraid to speak to him, and one day when he was seventeen years old he came home from a brawl missing the fingers on his right hand."²³

On February 10, 1547, Ferrari, rather than Cardano, responded with a printed cartello, a challenge to Tartaglia to meet him in a dispute on almost any scientific topic, maintaining that Tartaglia had "written things which falsely and unworthily slander the above-mentioned Signor Gerolamo [*sic*] (Cardano), compared to whom you are hardly worth mentioning."²⁴ Ferrari sent the cartello to a variety of scholars and dignitaries all across Italy, so that Tartaglia could hardly refuse. Ferrari's attack was strong; he argued that Tartaglia had built up his reputation by attacking others; that, ironically, he had published a proof in his book that was stolen and for which he had not given credit; and that the book was full of errors.

There was an interchange. Tartaglia complained that he wanted to meet Cardano, not Ferrari, the pupil. Ferrari, however, acting on Cardano's behalf, insulted Tartaglia strongly enough that Tartaglia had little choice but to acquiesce. Ferrari also made the point that Cardano attributed the solution to del Ferro and Fior, both of whom knew it before Tartaglia, and that there was no oath of secrecy.

In any case, a public contest did take place the following August. The details are vague, but Tartaglia seems to have withdrawn after a brief but possibly vituperative battle, and Ferrari was declared the victor. Cardano didn't even attend.

Though few details of the actual contest have come down to us, the apparent outcome is supported by the later results. Tartaglia lost a promised position in his hometown of Brescia, whereas Ferrari received a variety of good offers: he was invited to lecture in Venice, Tartaglia's home territory, and went on to become a professor of mathematics in Bologna. Tartaglia now had additional fuel for his bitterness.

It seems odd that Cardano, so proud of his learning and having done some solid work in the mathematics field, was apparently content to have Ferrari act as his champion. Ore points out, "Cardano began his university disputes in his student days, apparently with much success. . . . Cardano must have been well equipped for the debates; he had a quick wit, a good memory, and a sharp tongue. According to his own account he became so proficient in these mental duels that his opponents had little chance of victory, or even defeat with honor."²⁵ Ore is referring mainly to medical contests; did Cardano think less of his mathematical abilities?

As had happened much earlier with Fior, Tartaglia lost some public esteem and some reputation. This, however, only ramped up his bitterness, which was aimed directly at Cardano, and he retreated into a corner of the ring—fuming, waiting, watching, and plotting.

A First Attempt

What Tartaglia saw, though, was an opponent whose star continued to rise, and who was therefore not an easy target. Wykes writes, "The name of Doctor Cardano rang through the halls of philosophy, astrology, mathematics, science and medicine. Books, tracts, and treatises by the score came from the press of Petrieus [his publisher]."²⁶ How this must have galled Tartaglia.

He first tried to entrap Cardano via a complicated scheme that involved his [Cardano's] friendship with Gonazaga, the governor of

Milan and an opponent of the pope, and the old pope's addiction to astrology. Tartaglia had an idea that he could make trouble by engineering an offer to Cardano to enter the service of the pope as an astrologer and a physician. Perhaps he thought he could generate some sort of charge of political intrigue.

Cardano was riding high, however, and in no need of patronage. He turned down the offer for personal reasons. Tartaglia then tried to implant the idea that Cardano had intended to offend His Holiness by his refusal. Tartaglia also produced a copy of Cardano's earlier horoscope of Jesus in an attempt to convince the pope that it was blasphemous. The situation was briefly troubling to Cardano, but this first attempt at revenge by Tartaglia petered out early in the 1550s. He retreated and began plotting another campaign.

Cardano's star, however, was still shining bright. He held the chair of medicine at Pavia, which provided material success, time to write, and considerable prestige. Offers of all kinds arrived constantly. Tartaglia held himself in check until, eventually, his patience was rewarded. Fate stepped in and dealt him a full hand.

Cardano was blessed in many ways, but not with his children. Three of them, two sons and a daughter, were to pave the way to his undoing. Thanks to Cardano's eminence, his daughter Chiara had made a good marriage to Bartolomeo Sacco, who came from a noble lineage. Unfortunately, Chiara was a promiscuous young woman, and Sacco not only sought an annulment but wanted recompense from Cardano for this "worthless baggage."²⁷ During the years 1557 and 1558, Cardano found himself enmeshed in both legal and church battles that began to tear at his once-illustrious name.

Though he managed to continue his work as a doctor and a philosopher/scholar/writer, he was deeply enmeshed in his daughter's problems when trouble came from another direction. In December 1557, his elder and favorite son, Giambattista, married. Cardano would have expected the boy to marry well, but he chose the daughter of a down-and-out family, which Cardano was expected to support and for which he ended up supplying most of the funds.

The details are not clear, but it must have been a marriage made in hell, for two years later, Giambattista's wife was dead of arsenic poisoning, and Cardano's son was arrested for her murder. Cardano

did all he could to prove his son's innocence, but Giambattista confessed and was executed. Cardano never got over this, and it may well have affected his mind, for he started to become suspicious of attempts on his life.

It is, of course, possible that some of these were actual attempts to destroy him in one way or another. Envy of the famous is not uncommon. Add to the pot his cantankerous character, however, and it's easy to believe that there were some people who did what they could to pull him down.

Now add to his real and imagined difficulties new problems with his other son, Aldo. Aldo turned out to be a drinker and a gambler. He lived with Cardano for a while, but the two did not get along. Early in the 1560s, Aldo moved out, but, having gotten deeply into debt, he broke into Cardano's house and was caught. Cardano wanted nothing further to do with him. Here was yet another disaster with his offspring.

Again he lost himself in his writing, but by now attempts were being made to eject him from his chair of medicine at Pavia, and then at the even more prestigious college in Milan. In 1563, his name was removed from the list of scholars qualified to lecture, and he was accused of various crimes. He was actually exiled from Milan, which included Pavia, and he left there at the end of 1563 in the depths of despair, his fortune gone, his books impounded.

A Shadowy Hand

The rest of the decade saw no improvement for Cardano. His country, dominated by Spain and decimated by wars, labored under heavy taxation. The Spanish Inquisition, well underway, was a potent force. Scholars of all sorts were under suspicion, but somehow Tartaglia had managed to place himself satisfactorily. Cardano could find no employment, and, according to Wykes, "it was Tartaglia who was the instigator of most of the refusals that met him in College and University. It was simple enough, with the network of the Inquisition flourishing in city, vineyard, village and public square, to keep a shadowy hand on the shoulder of any citizen, great or small."²⁸

This was just the warmup, though. On October 13, 1570, almost a quarter of a century after publication of *Ars Magna*, Tartaglia served up a double blow. Using Cardano's own son Aldo as an informant as to Cardano's whereabouts, Tartaglia handed him to the Inquisition. Tartaglia had been collecting evidence against Cardano for years. Among this "evidence" was Cardano's rejection of the pope's invitation that he become the pope's astrologer and physician. Tartaglia pointed to the "sarcasm" evident in Cardano's comment that "His Holiness by his study of astrology has surely raised himself among the greatest of such scientists and has no need of help from such as myself."²⁹

Cardano's horoscope of the life of Jesus was also damning, as were a variety of other statements that, taken out of context, could be construed as blasphemous. In one of his publications, for example, he had suggested that God is a universal spirit whose benevolence is not restricted to holders of the Christian faith. Today he might be admired for such an ecumenical statement; at the time, it was apparently a dangerous idea.

And so it went. Cardano, fortunately, was not subjected to torture or put to death, but he was thrown into jail. He sought desperately for help and was able to reach out to an official in the church, Archbishop Hamilton, who had in the past asked to be called upon if need be. The archbishop came through for Cardano, who was released a few months later. It was just in time, for not long after, the archbishop's own fortunes changed; he was captured by the forces of Mary, Queen of Scots, and beheaded.

Tartaglia finally had had his revenge. Cardano lived on in obscurity in Rome, where he worked on his autobiography, which is one of the works that has come down to us in full. He probably never knew, and just as well, that his daughter Chiara had died of syphilis, and that it was Aldo who had betrayed him to the Inquisition and who was rewarded with an appointment as official torturer and executioner in Bologna.

Cardano died on September 20, 1576. Less than a year later, Tartaglia followed him to the grave.

Who Won?

Ore, who has studied Cardano carefully, argues, “His originality in other fields has sometimes been questioned, but *De Ludo Aleae*, his obscure and somewhat disreputable book on how to win at cards and in dice games, contains indisputable proofs of his genius.”³⁰ In addition, Wilhelm Gottfried Leibniz (see chapter 3 in this book) maintains, “Cardano was a great man with all his faults; without them he would have been incomparable.”³¹

As for Tartaglia, Ore believes that “had Tartaglia never existed, the science of mathematics would not have been deprived of a single great or fertile idea.”³² When Tartaglia died, an attempt was made to assemble and publish his unpublished papers. Oddly, none could be found that even mentioned the solution to the cubic equation.

Nevertheless, says Ore, Tartaglia was no slouch. Under other circumstances, his star might well have shown brighter. As Ore puts it: “His great tragedy was the head-on collision with the only two opponents in the world [Cardano and Ferrari] who could be ranked above him.”³³

While both Tartaglia and Cardano paid dearly in this battle, there is no question that mathematics came out a winner.