Section 1

SYMMETRY

Activity	Concept	Knowledge Level
1-1. Turn, Flip, or Slide	Turn, Flip, Slide	Learning
1-2. Draw the Ways	Turn, Flip, Slide	Learning
1-3. Turning and Turning	Turn, Rotational Symmetry	Learning
1-4. Turn Around	Turn	Challenging
1-5. Flip Me Over	Flip	Challenging
1-6. Flip Me to Read Me	Flip	Challenging
1-7. Mirror Lines	Bilateral Symmetry	Learning
1-8. Mirror, Mirror	Bilateral Symmetry	Challenging
1-9. Pentomino Search	Turn, Flip, Bilateral Symmetry	Challenging
1-10. How Many Times?	Rotational Symmetry	Learning
1-11. Rotate and Color	Rotational Symmetry	Challenging
1-12. Alpha Time	Bilateral and Rotational Symmetry	Challenging
1-13. Will It?	Tessellation	Learning
1-14. Flip Rows	Tessellation	Challenging

Teacher Directions

■ 1-1. TURN, FLIP, OR SLIDE (Learning Level)

Objectives: To determine how a figure was moved
To draw the resulting figure after a move

Materials: Book with a colorful cover
Plain sheets of paper
Crayons or markers
Turn, Flip, or Slide activity sheet

Directions: "When figures are moved there are usually three ways it can happen. One is a flip." (Write "Flip" on the board.) "Another is a turn." (Write "Turn" on the board.) A third is a slide." (Write "Slide" on the board.) Have the students describe each move as best as they can. (*Flip—to turn a figure over. Turn—to rotate a figure. Slide—to move a figure in one direction.*)

Hold the book so all can see. "A flip occurs when a figure is turned over. There can be a horizontal flip." (Flip the book horizontally to demonstrate. Move the book back to its original position.) "Or there can be a vertical flip." (Flip the book vertically to demonstrate.) Repeat the horizontal and vertical flips of the book several times to reinforce the different positions. Have the students identify if the flip is horizontal or vertical as you move the book.

"A turn occurs when a figure is rotated." Turn the book ¼ of the way around. "We can turn a little, like this one-fourth turn." (Move the book back to the original position and then turn it ¾ of the way around.) "Or we can turn it a lot, like this three-fourths turn. As we turn it, the book looks a little bit different until we get back to where we started."

"A slide occurs when a figure is moved in a straight path. It could move horizontally." (Move the book horizontally.) "Or vertically." (Move the book vertically.) "Or diagonally." (Move the book diagonally.) "Each time, the figure moves in a straight path."

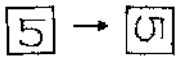
Distribute a sheet of paper to each student and have each write the numeral "3" on the piece of paper. Then have the students turn the paper over and trace the 3 from the front onto the back of the paper. Have them turn the paper over again and place it on the desk so the numeral "3" is shown correctly. Call out movements ("Flip your paper. Now turn your paper," etc.) and have the students demonstrate the movement. It will be easy to observe the students, since all of the papers should have the same orientation.

Review the concepts of vertical flip, horizontal flip, and ¼ turn. Continue to call out directions as you circulate around the room observing the students. ("Flip the figure horizontally. Turn the figure one-fourth of the way around. Flip the figure vertically," etc.) Since all of the students began in the same position and are making the same moves, it will again be easy to spot a student who has made an incorrect move.

Distribute the Turn, Flip, or Slide activity sheet. The students can use their paper as an aid if they wish as they identify how figures have moved and then interpret the moves and draw the resulting figures.

Flip

Turn



Slide

Answer Key: 1. horizontal turn; 2. slide; 3. vertical flip; 4. horizontal flip or a turn; 5. turn or a vertical flip; 6. slide; 7. Я; 8. Ƨ; 9. Ɔ; 10. X, X, X, X

Related Activity: Pair the students and provide each with paper, pencils, and stencils of letters. Each student will choose a letter and will trace the letter on the paper. Then, without their partners seeing, the students will make five turns, flips, or slides, tracing each move and numbering the moves as 1, 2, 3, 4, and 5. The

partners will exchange papers, then interpret and label each other's moves (1 slide, 2 flip, etc.). If the move can be accomplished two ways, the students will label both moves.

■ 1-2. DRAW THE WAYS (Learning Level)

Objective: To draw the resulting figure after specified moves

Materials: Trapezoid pattern block for each student (if not enough trapezoids are available, use a variety of pattern blocks)
Scissors, paper, and pencil
Draw the Ways activity sheet

Directions: Place a trapezoid on the overhead and use it to review the concepts of flip, turn, and slide, especially horizontal and vertical flips. Distribute the trapezoids. Review and model ¼ turns and ½ turns as the students follow along. Emphasize that turns could go in either of two directions, but for now all turns will be made in a clockwise manner, or to the right from the original position ( ). Have the students place their trapezoids on their paper with the longest edge of the trapezoid at the bottom, and trace around it. "Flip the block horizontally." The students will complete the move with their trapezoids and draw the new position by tracing around the block. Each student can check with a neighboring classmate to determine if he or she moved the block correctly.

Instruct the students to place the trapezoid below the drawings with the longest edge still on the bottom, and trace it again. "Move the block one-fourth turn." The students will move their block, trace it, and then check with a neighbor for agreement. Continue the procedure with "Flip the block vertically" and "Move the block one-half turn." Finally, give a two-step direction, "Flip the block horizontally, then flip it vertically. Draw its new position."

Distribute the Draw the Ways activity sheet and scissors. The students will follow the directions and draw what the figure looks like in its new position.

Answer Key: 1.  2.  3.  4.  5.  6. 

Related Activity: Pair the students. Provide a cutout of the letter L (approximately 3 inches) for each student and a die for each pair. The partners will sit back-to-back and place their L in front and to the left of where they are sitting. The students in each pair will take turns rolling the die and following these moves, without looking at what their partners are doing. Each number on the die determines how both students should move their Ls, but only one student at a time will be rolling the die and calling out the number.

1 = ¼ turn	4 = flip vertically
2 = ½ turn	5 = slide horizontally
3 = flip horizontally	6 = the student calls a move

After each student in the pair rolls the die three times and the moves are made, the students will pick up their Ls and sit *next* to their partners (not opposite). One partner's L should be in the same position as the other partner's. If not, one or both of the pair made an incorrect move. They can go back and recreate their moves, checking each other. Then the students can change partners with another pair and perform the activity again.

■ 1-3. TURNING AND TURNING (Learning Level)

Objective: To draw a symmetrical pattern using turns

Materials: Triangle pattern block and square pattern block
Overhead transparency of a Symmetrical Pattern and an Asymmetrical Pattern (see appendix)
2 blank transparencies
Paper and pencil
Turning and Turning activity sheet

Directions: Write "symmetry" on the board and ask the students to define the word. (*A figure or design that has balance. When the figure is balanced, it appears to be orderly and pleasant.*) The students may respond with the idea of one half of an object looking like the other half, which is bilateral symmetry. That type of symmetry will be covered later. Write "rotational symmetry." "Rotational symmetry is the result of turning, or rotating a motif (a design) around a point. The result is a pattern, and that pattern can be balanced, or symmetrical." Display the Symmetrical Pattern transparency. "Here we see balance. A motif (point to the top ellipse of the pattern) was turned around the circle and drawn again. Each turn was equal. This gives it the balanced and pleasant look. It is symmetrical." Display the Asymmetrical Pattern. "This pattern is not balanced. The motif was not turned equally. It is not symmetrical."

Place a blank transparency onto the overhead. Position the triangle pattern block at the top of the transparency with the square below it and touching it, making a "house" motif: ⏢ Trace around the motif and draw a point approximately ½ inch below the square. Turn the motif approximately ⅛ turn (clockwise) around the point and trace it. Then turn it so it shows about a ⅔ turn and trace the motif again. "Is this a balanced pattern?" (*no*) "Is it symmetrical?" (*no*)

Place another transparency on the overhead. Position the motif the same as before and trace it. This time move it ¼ turn and trace it. Move it another ¼ turn and trace it, followed by a last ¼ turn and trace. "I moved the motif one-fourth turn, then another one-fourth turn, then another one-fourth turn. I moved it the same each time. This pattern has balance, or symmetry. It is symmetrical." Place both patterns on the overhead at the same time to use as an example of symmetrical and nonsymmetrical patterns.

Instruct the students to write a manuscript capital C on their paper and draw a point under it. Have them picture what the C would look like after a ¼ turn around a point and draw it. (∩)

Use other letters (not the letters B, F, A, or X, which are on the activity sheet) to practice the orientation of ¼ and ½ turns around a point. Distribute the Turning and Turning activity sheet. The students will create patterns using turns and explain the orientation of the motif.

Answer Key: 1. B; 2. F; 3. A; 4. X; 5. straight up, on its right side, upside down, on its left side; 6. looks like an X in every position, an X looks like an X when it is moved ½ turn; 7. a design or figure

Related Activity: Pair the students and provide each student with a geoboard and geobands. Without the partner seeing, one of the pair will create an irregular figure on the geoboard with a geoband. The creator will then turn the geoboard either a ¼, ½, or ¾ turn, show the figure to the partner, and state, "I moved the figure ___ turn." The partner will then attempt to interpret the move, use his or her mind to picture the figure in its original position, and show the original figure on the geoboard. The partners will then switch roles.

If this proves too difficult for some students, they can copy the figure on their geoboards after they have been turned, and then turn their boards back to the original position. This still allows them to see the result of turning a figure.

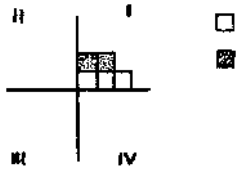
■ 1-4. TURN AROUND (Challenging Level)

Objective: To draw or create a symmetrical design using turns

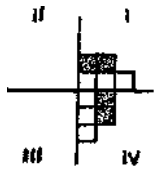
Materials: 16 red, 8 blue, and 4 yellow (or any three colors) 1-inch-square overhead counters
Overhead transparency of Four Regions (see appendix)
4 square, 4 blue rhombus, and 4 white rhombus pattern blocks, or cutouts of square and blue and white rhombus pattern blocks for each student
Turn Around activity sheet

Directions: This activity can be completed in two different ways depending on the abilities of the students. The students can follow the directions on the activity sheet and glue pattern block cutouts to show the turns, or they can use pattern blocks to reproduce the pattern, turn the pattern, and then trace its new position. When finished, the students can then color the design the corresponding colors. The second method would probably be more difficult for most of the students.

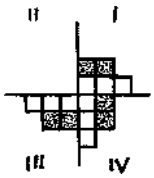
If necessary, quickly review the concepts of turn, flip, and slide. Display the Four Regions overhead. Add the counters as shown.



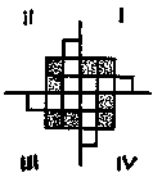
Point out the four regions, the horizontal and vertical lines, and the pattern. Ask the students to try to picture moving the pattern ¼ turn from region I to region IV. Ask several students to describe what that might look like. Place three red counters and two blue counters on top of the red and blue counters in region I. Move the counters one by one, moving the pattern ¼ turn into region IV, leaving the original counters in region I.



Have the students describe why this correctly shows a ¼ turn. Ask the students to describe what the pattern in region IV would look like if it was moved ¼ turn into region III. Place counters in region III to show the turn as individual students describe the move.



The final step is to describe a ¼ turn from region III to region II.



Have the students describe the completed design and the procedure used to develop the design.

Remove the counters and create a new pattern in region I by adding a yellow counter to the design. The task is still to move the pattern ¼ turn through the regions. Ask individuals to come to the overhead and make the different turns as they describe the process.

Decide if it will be better for the students to use cutouts or the actual pattern blocks. Distribute the Turn Around activity sheet and the materials. The cutouts are provided on the activity sheet. The students will turn a pattern to create a design. Save the completed Turn Around activity sheets for use in Section 1-5, "Flip Me Over."

Answer Key: 

Related Activity: Provide additional copies of Four Regions (see appendix). The students will choose pattern blocks or other materials in the room (base-10 blocks, Cuisenaire rods, etc.), create a pattern that encompasses most of region I, and then turn the pattern through regions IV, III, and II. The designs can be colored accordingly and displayed throughout the school.

■ 1-5. FLIP ME OVER (Challenging Level)

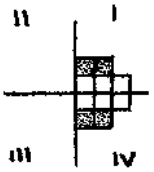
Objective: To draw or create a symmetrical design using flips

Materials: 16 red, 8 blue, and 4 yellow (or any three colors) 1-inch-square overhead counters
Overhead transparency of Four Regions (see appendix)
4 square, 4 blue rhombus, and 4 white rhombus pattern blocks, or cutouts of square, blue rhombus, and white rhombus pattern blocks for each student
Flip Me Over activity sheet

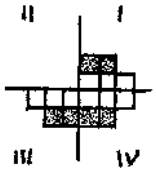
Directions: This activity, like the previous Turn Around activity, can be completed in two different ways depending on the abilities of the students. The students can follow the directions on the activity sheet and glue pattern block cutouts to show the flips, or they can use pattern blocks to reproduce the pattern, flip the pattern, and then trace its new position. When finished, the students can then color the design the corresponding colors. The second method would probably be more difficult for most of the students.

If necessary, quickly review the concepts of turn, flip, and slide. Display the Four Regions overhead. Add the counters as shown.

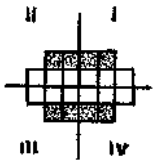
Point out the four regions, the horizontal and vertical lines, and the pattern. Ask the students to try to picture flipping the pattern vertically from region I to region IV. Ask several students to describe what that might look like. Place three red counters and two blue counters on top of the red and blue counters in region I. Move the counters one by one, flipping vertically into region IV, leaving the original counters in region I.



Have the students describe why this correctly shows a flip. Ask the students to describe what the pattern in region IV would look like if it was flipped horizontally into region III. Place counters in region III to show the turn and ask the students to describe the move.



The final step is to describe a horizontal flip from region III to region II.



Have the students describe the completed design and the procedure used to develop the design.

Remove the counters and create a new pattern in region I by adding a yellow counter to the design. The task is still to move the pattern by flipping through regions IV, III, and II. Ask individuals to come to the overhead and make the different flips as they describe the process.

Decide if it will be better for the students to use cutouts or the actual pattern blocks. Distribute the Flip Me Over activity sheet and the materials. The students will flip a pattern to create a design. Make the students' Turn Around activity sheets available. They will be asked to compare the design resulting from turning the motif to the design resulting from flipping the motif.

Answer Key:  The resulting designs are the same.

Related Activity: Provide additional copies of Four Regions (see appendix). The students will choose pattern blocks or other materials in the room (base-10 blocks, Cuisenaire rods, etc.), create a pattern that encompasses most of region I, and then flip the pattern through regions IV, III, and II. The designs can be colored accordingly and displayed throughout the school.

■ 1-6. FLIP ME TO READ ME (Challenging Level)

Objective: To interpret words that have been flipped vertically

Materials: Blank transparencies and overhead pens
1 copy of Flipped Words (see appendix) for each student
1 mirror for each student
Flip Me to Read Me activity sheet

Directions: Write the word "Hello" on a transparency, flip it horizontally, and place it on the overhead. Turn on the overhead and see if the students can identify the word. Then ask how the word was moved from the normal way that we write. (*flipped horizontally*) Show "Hello" as it is normally written and demonstrate the horizontal flip for all to see. Turn the overhead off, put "Hello" in its normal position and move it ¼ turn. Ask the students to identify the move. (*¼ turn*) Continue the process using a ½ turn and flipping vertically.

Write "number" on another transparency, flip it vertically, and place it on the overhead. Ask the students to identify the word and how it was moved. (*vertical flip*) Ask how the students were able to identify the word, and take several explanations. Distribute a copy of Flipped Words to each student along with a mirror. Have the students determine how to use the mirrors to identify the words and then write the words correctly. (*rectangle, covering, rhombus, vertically*) Some may flip the paper and hold it up to the light. This will work, but also encourage them to use the mirrors.

Distribute the Flip Me to Read Me activity sheet. The students will use the mirrors to read the questions and then answer the questions. The answer to question 1 should be written flipped vertically.

Answer Key: 1. their name flipped vertically; 2. answers will vary; 3. to rotate a figure; 4. to turn a figure over; 5. to move a figure in one direction

Related Activity: Pair the students and have them sit next to each other. Provide each student in the pair with approximately eight assorted one-inch cubes, the same variety of colors for each student in the pair. One student will use the cubes to create a motif. The other student will mentally flip the motif vertically and then show the flip using her or his cubes. Once the two students agree that the flip is accurately shown, they can switch roles and do the activity again.

■ 1-7. MIRROR LINES (Learning Level)

Objective: To determine the number of mirror lines for different figures

Materials: 1 copy of Mirror Lines and Figures (see appendix) for each student
Mirrors and scissors for each student
Mirror Lines activity sheet

Directions: Write "bilateral symmetry" on the board and ask students to define the term. "Bilateral symmetry is also called mirror symmetry. Bilateral symmetry occurs when a design or figure has two or more regions that are equal, but flipped. We call it mirror symmetry because it is like looking at one of the regions with a mirror."

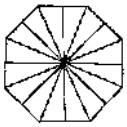
Distribute a copy of Mirror Lines and Figures to each student along with a mirror. "Some figures have bilateral symmetry. Look at figure 1. This figure has bilateral symmetry. We can tell by using our mirrors. Place your mirror along the vertical line. You should be able to see one-half of the original figure plus a mirror image of the other half. Do the halves of the figure and the mirror image look the same?" (*yes*) "Together do they look like the original figure?" (*yes*) "The vertical line is called a mirror line. It divides the figure into two equal or identical regions. Use a pencil and the edge of the mirror to finish drawing the mirror line on the figure."

Continue, "Now look at figure 2. Place your mirror along the horizontal line." The halves of the figure and the mirror image should be identical. Instruct the students to complete the mirror line by using the bottom edge of the mirror as a straightedge. "You have drawn one mirror line for figure 2. Some figures have more than one mirror line. See if you can find another mirror line." There is also a vertical mirror line. Have the students make sure that the region of the figure and the mirror image are identical and then draw the vertical mirror line. Then have the students explore figure 3 and draw the two mirror lines. (*horizontal and vertical*)

Distribute the scissors and instruct the students to cut out figure 3. "Another way to determine mirror lines is to fold the figure into equal regions." Demonstrate folding on the mirror line and unfolding the figure to reveal two equal and identical regions. The students will fold their figure on both mirror lines, one

at a time, to show the equal regions. Have the students cut out figures 4 and 5, and find their mirror lines by folding. Remind them that the regions must be identical on both sides of the mirror line. Give them a little time to explore and fold the figure. Then stop them and stress that not every figure will have a mirror line. Some figures do not have bilateral symmetry, such as the parallelogram they had been exploring.

Distribute the Mirror Lines activity sheet. The students will determine the number of different mirror lines for the various figures.

Answer Key: 1. 2; 2. 4; 3. 3; 4. 5; 5. 6; 6. 8, it has eight sides 

Related Activity: Display an overhead transparency of What Comes Next? (see appendix). Place the students in small groups and provide a copy of What Comes Next? to each student. Each group will study the pattern and draw the next figure. When the group correctly identifies the next figure, have them draw the next five figures to correctly complete the pattern. (The next figure in the pattern is 4.) There is a vertical mirror line in each figure. On the right side are the numerals 1, 2, 3, and 4. The left side is flipped horizontally. The next five figures are 5, 6, 7, 8, and 9.

■ 1-8. MIRROR MIRROR (Challenging Level)

Objectives: To interpret words that have been flipped vertically
To create a symmetrical design

Materials: Blank transparencies and overhead pens
Overhead pattern blocks
1 mirror for each student
Mirror Mirror activity sheet
Crayons

Directions: "Bilateral symmetry is also known as mirror symmetry. The mirror line divides the design into two regions. You see both regions, but one is flipped. People show bilateral symmetry very nicely. Look at my face." Use your finger to draw an imaginary line vertically down your face. "My finger is showing where a mirror line would be. The mirror line would divide my face into two regions. They are the same but one side is flipped. One eyebrow is not identical to the other. One is flipped." Emphasize the difference by tracing your eyebrows with your finger or using two fingers to show how they are opposite. "One side of my mouth is like the other side except that it is flipped. In fact, we could draw a mirror line down the center of the torso. One side would be the same as the other, only flipped." Use hand gestures to emphasize the mirror line and the opposite sides of the torso, arms, hands, etc.

"The flip can be horizontal or vertical. With either kind, if the design is symmetrical, there would be at least one mirror line." Place a transparency on the overhead and four square pattern blocks in the middle. Have the students identify the mirror lines. For now, work with only the vertical mirror line. Place a different pattern block on one side of the squares so that it touches the squares, for instance, . Ask a student to add a pattern block to the other side to keep the design symmetrical across the vertical mirror line. Add another block and ask a student to add a block to keep the design symmetrical. Repeat the process several times until you feel sure that the students understand the concept.

Distribute the Mirror Mirror activity sheet and mirrors to the students. The students will use the mirrors to read the directions and add regions to the design so that it remains symmetrical.

Answer Key: Answers will vary.

Related Activity: Provide each student with an 8½-inch by 11-inch piece of white paper. Have the students fold the paper into four regions. In the upper left region have the students use black chalk or charcoal to draw several lines that go from edges to folds and overlap some. Make sure they use pressure to produce a dark line. Fold the paper vertically and rub the paper to transfer the lines to the next region. Open the paper to observe the bilateral symmetry. Trace the lines in the mirror region with black chalk or charcoal and then fold the paper horizontally. Rub the paper again to transfer the lines. Open the paper to observe the symmetry. A motif was produced in the upper left region. It was flipped horizontally across the vertical mirror line. The new motif was then flipped vertically across the horizontal mirror line. Instruct the students to color the first region and then color the other regions to show the flips. The finished product will be an example of bilateral symmetry.

■ 1-9. PENTOMINO SEARCH (Challenging Level)

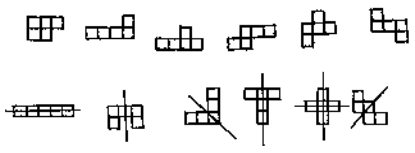
Objectives: To show the 12 different pentomino nets
To identify the 6 pentomino nets that have bilateral symmetry

Materials: 60 1-inch-square cutouts or 60 1-inch counters for each pair of students
20 1-inch-square overhead counters
Pentomino Search activity sheet

Directions: Write "pentomino" on the board and ask the students to define the term. (*five squares joined at their edges to create a figure*  *or*) If students are not familiar with the term, give clues such as "It has the same prefix as pentagon." Place five counters on the overhead and explain the rules about determining pentomino nets. The five squares must be joined together. The full edges must join one another. This is allowable  , this is not  , and this is not  . Once a net has been created, a flip, a turn, or a slide of that net is not a different net. It is the same net with a flip, a turn, or a slide.

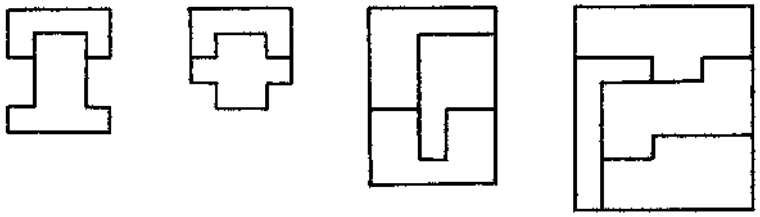
Arrange the five counters into a net such as  . Arrange five more counters into the same net and turn the net. "Is this a different net?" (*no*) "Why not?" (*It is the same net, only turned.*) Rearrange the counters to look like the original net, and this time flip the net. "Is this a different net?" (*no*) "Why not?" (*It is the same net, only flipped.*) Repeat the process using a slide. It is not a different net. Keep the  and place five additional counters on the overhead. Arrange the squares into the net . Turn the net, flip the net, and slide the net, each time determining that it is not a different net. It is still the same net in spite of the movement.

Leave the two nets on the overhead and add five more squares. Invite a student to come to the overhead and attempt to create a third net. Then invite a different student to create a fourth net. As each net is created, review the rules for forming a pentomino net to make sure it is in compliance. Pair the students. Distribute the Pentomino Search activity sheet to each student and 60 square cutouts to each pair. The students will form the squares into 12 different pentomino nets, each containing 5 squares. Each net must follow the rules concerning the edges of the squares and may not be a turn, a flip, or a slide of any previously formed net. As the students discover each new net, they will draw and shade it on their activity sheet. After they have found all 12 pentomino nets, they will identify the six nets that have bilateral symmetry and draw the mirror lines on those nets.

Answer Key: 

Related Activity: Pair the students. Provide each pair with a set of pentomino pieces, some yarn or string, and scissors. The pairs of students will choose two or more pentomino pieces and join them so that the resulting figure has at least one line of symmetry. The rules for joining the pieces are the same as for making pentominoes: the edges must join fully and the pieces may not be joined at the vertices. The students will then cut a piece of yarn and place the yarn on the figure to show the mirror line. They will then join other pentomino pieces so the resulting figures have at least one mirror line, and show the mirror line using a piece of yarn. After some time, the students can tour the room to see what figures other pairs have created.

Here are some solutions:

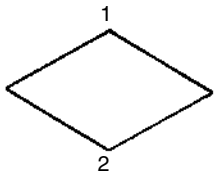


■ 1-10. HOW MANY TIMES? (Learning Level)

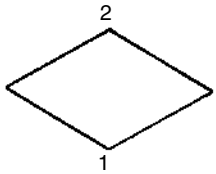
Objective: To identify and draw the number of times a figure can be turned and still look like its original position

Materials: Blank transparency
Overhead pattern blocks
1 triangle, square, trapezoid, and hexagon pattern block for each student, or pattern block cutouts
How Many Times? activity sheet

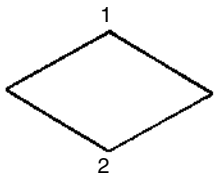
Directions: Place a blue rhombus pattern block on the overhead in the same position as shown in the following figure and have the students place a blue rhombus on their desks in the same position. "Sometimes figures can be moved, and you cannot tell if you did not see it move." Turn the overhead off and move the figure ½ turn but without the students seeing what you do. Then turn the overhead back on. "Was the figure moved or is it still in its original position?" To either answer: "How can you tell? You cannot really tell unless you saw what happened. Look at your blue rhombus and use your imagination to see the numbers 1 and 2."



"Now slowly turn the rhombus and stop when it looks exactly like the way that it started. It looks the same, but something is different. What is different?" (*The numbers have changed places.*) "Correct, the rhombus still looks like it did in the original position but it was actually turned. Vertex 2 is where vertex 1 was, and vertex 1 is where vertex 2 was."



"Now turn the rhombus again and stop when it looks exactly like the original position. We are back to where we started. Vertex 1 is back in its original position and so is vertex 2."



Write, "A rhombus can be turned _____ times and still look like its original position" on the board. Ask a student to fill in the answer (*2*) and explain why it is correct.

Distribute the How Many Times? activity sheet. Instruct the students to place a green triangle pattern block on top of the triangle on the page. Place a blank transparency on the overhead and display a green triangle. Write the numbers "1," "2," and "3" on the transparency in the same order as they are on the activity sheet. "Slide your pattern block to the right so it is not on the drawn triangle. Use your

imagination to follow the numbers. Turn your pattern block and stop when it looks like the drawn triangle, its original position. Trace the triangle and show the numbers of the turned triangle for each vertex." Slide the triangle to the right, turn it until it looks like the original, trace it, and show the numbers for each vertex. Now slide the triangle, turn it until it looks like its original position, trace it, and number the vertices. How many times did we turn the equilateral triangle and still have it look like its original position?" (*3*) Some students may answer *4* since they see that many triangles. Allow students to state why they believe their answer is correct. They may see 4 triangles but there were only 3 turns. Instruct the students to fill in the answer for number 1 (*3*) and then determine the number of turns for each of the other figures. They should draw and number the vertices for each turn.

Answer Key: 1. 3; 2. 4; 3. 1; 4. 6

Related Activity: Distribute the Find Those That Match sheet (see appendix). There are five pairs of identical figures, except that one has been turned. The students will determine and record the matching pairs of figures. (a matches e; b matches f; c matches j; d matches g; h matches i)

■ 1-11. ROTATE AND COLOR (Challenging Level)

Objective: To color a turned figure and demonstrate rotational symmetry

Materials: Transparency of Rotational Symmetry (see appendix)
Overhead color pens
Crayons
Rotate and Color activity sheet

Directions: Write "Rotational Symmetry" on the board. "Rotational symmetry is also known as turning symmetry. A motif is turned around a point. If each turn is the same as all of the rest of the turns, there is balance, or symmetry. It is important to picture the motif turning and how each part of the motif would look when it is turned."

Display the Rotational Symmetry transparency. Color each circle in the motif (figure 1) according to the diagram (substitute colors as necessary). In figure 2 the motif has been moved ¼ turn. Have the students state the color of each circle in figure 2 so it would show what figure 1 would look like if it was moved ¼ turn. If the students experience difficulty, move the transparency ¼ turn to let them see what the result would look like. Then move the transparency to the original position. Color the circles in figure 2 as the students name the correct color.

Repeat the process as the students name the colors in figure 3 to show what figure 2 would look like moved ¼ turn.

Then move figure 3 ¼ turn to figure 4.

The result should be a balanced design. A motif was rotated around a point (show the point) ¼ turn each time. The result is a symmetrical design.

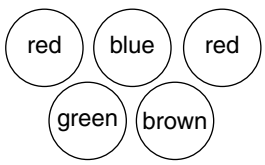


Figure 1

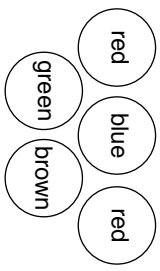


Figure 2

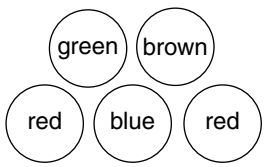


Figure 3

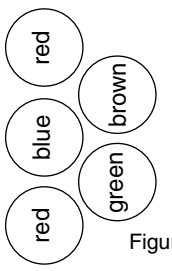


Figure 4

Distribute the crayons and the Rotate and Color activity sheet. The students will color the motif (figure 1) according to the directions and then color the other figures to show the motif moving through ¼ turns.

Answer Key:

Related Activity: On a computer with a drawing program (AppleWorks or Microsoft Word are two that can be used), have the students create a motif. Highlight the motif, duplicate it, rotate the duplicate 60 degrees, and move it to show a turn around an imaginary point. Highlight that motif, duplicate it, rotate the duplicate 60 degrees, and move it to show a turn around an imaginary point. Continue the process until a symmetrical design is created from 60-degree turns. The students can also explore 90-degree and 120-degree turns to create a symmetrical design. The completed designs can be categorized by the number of turns and can then be displayed in the room.

■ 1-12. ALPHA TIME (Challenging Level)

Objective: To demonstrate bilateral and rotational symmetry

Materials: Alpha Time activity sheet

Directions: Review the concepts of bilateral and rotational symmetry by having the students define each. (*bilateral, mirror, or flip symmetry; rotational, turning symmetry*) Ask several students to show examples of each type of symmetry either with materials or drawing on the board. Write the numerals "0, 1, 2, 3, 4, 5, 6, 7, 8, 9" on the board. Ask the students to identify the numerals that have one or more lines of bilateral symmetry and come to the board to draw the lines. (~~0~~, ~~3~~, ~~8~~) Ask which of the numerals has rotational symmetry and have the students demonstrate why they believe it is correct. (*0, 8*) Finally, have the students determine which numerals have both bilateral *and* rotational symmetry. (*0, 8*)

Write "808" on the board and draw a mirror line to the right of the number. "The motif is 808. What would this motif look like on the other side of the mirror line?" (*808*) Have a student write the number on the right side of the mirror line. (*808 | 808*) "The number eight hundred eight has bilateral symmetry. It looks exactly the same on both sides of the mirror line. Let's try three hundred three." 303 | Ɛ0Ɛ "Does three hundred three have bilateral symmetry?" (*no*) "What other numbers besides eight hundred eight show bilateral symmetry?" Allow the students to try different numbers either at their desks or at the board. The only numbers that will work are combinations of 8 and 0. (*8008*, *80808*, etc.) If you show the numeral one as having a line of symmetry (|), then you can add it to the numerals that have bilateral symmetry, rotational symmetry, and both. You can also use it in numbers that have bilateral symmetry. (101, 181, 818, etc.)

Distribute the Alpha Time activity sheet. The students will identify rotational and bilateral symmetry.

Answer Key: 1. A, B, C, D, E, H, I, M, N, O, T, U, V, W, X, Y, Z; 2. H, I, O, X; 3. answers will vary

Related Activity: The students will, in their own words, answer the following questions and use illustrations to add to their descriptions:

- What is symmetry?
- How can you tell if a figure is symmetrical? (Describe two types of symmetry.)

■ 1-13. WILL IT? (Learning Level)

Objective: To cover an area with a figure

Materials: Several regular octagon cutouts approximately 1 or 1½ inches in size
Overhead transparency of Will It? Frame (see appendix) and overhead pen
Figures for Will It? activity sheet (see appendix) copied onto card stock for each student
Will It? activity sheet

Directions: Place the Will It? Frame transparency onto the overhead. "I want to cover the area of this square and I want to do it using only one figure. I will place a figure in the middle and then add more of those figures until I cover the area. There can be no gaps and no overlapping pieces." Place an octagon cutout in the middle of the square. "I am going to place several more octagons in the square. Do you think I will be able to cover the area with no gaps and no overlaps?" Take several responses and ask the students to justify their answers, but do not acknowledge the correctness of the answer yet.

Place another octagon in the square so the two are touching along one side. "Would anyone like to change their answer or give an answer?" Allow students to give their opinions and justifications. Add another octagon so all three octagons are touching one another. "Are there any different opinions on whether I can cover this area?" Continue adding octagons so they cover the area with no gaps and no overlapping pieces. It is important to emphasize that the figures may extend past the edge of the square, and that is fine. The area of the square will be covered with no gaps and no overlaps.

Write "tessellate" on the board. "To tesselate means to cover an area completely." Write "tessellation" on the board. "The figure used to cover the area completely is a tessellation. Some figures tessellate and some do not. We can see that a regular (all sides and angles are equal) octagon will cover the area, or tessellate. We can also show that figures tessellate using a different method." Remove the octagons but keep the transparency on the overhead. Place one octagon in the middle of the square. Trace around the octagon, move the octagon to a position where it is touching the traced one along a side, and trace around the octagon. Move the octagon so that it is touching both traced octagons and trace it again. Continue moving and tracing until the area is covered. Again emphasize that any parts of figures that go beyond the area of the square are not drawn. Only draw the parts of the figure up to the edge of the overhead glass.

"Once again we see that a regular octagon will tessellate. An octagon is a tessellation. Now it is your turn to work with other figures to see if they will tessellate. Remember, the figure must cover the area with no gaps or overlaps." Distribute the Will It? activity sheet. The students may cut out the figures, or you may have the figures precut to save time and to keep the integrity of the figure. The students will determine which figures tessellate.

Answer Key: 1. yes; 2. yes; 3. no; 4. yes; 5. yes; 6. no, answers will vary

Related Activity: Provide each student with a 12-inch by 18-inch sheet of paper. Each student will choose a pentomino piece and predict if it will tessellate. The students will then verify their predictions by tracing the pentomino piece, moving it, tracing it, and moving it until the sheet of paper is covered or the piece will not tessellate.

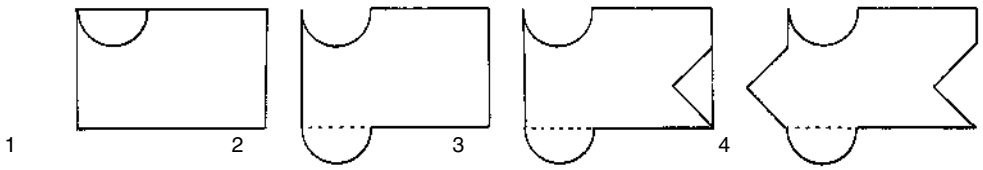
■ 1-14. FLIP ROWS (Challenging Level)

Objective: To tessellate by flipping an irregular figure

Materials: 2-inch by 2-inch piece of index card
3-inch by 5-inch index cards
Scissors and tape
12-inch by 18-inch sheets of paper
Flip Rows activity sheet

Directions: "We have found that many figures will tessellate, or cover an area. Some have been regular figures like squares, equilateral triangles, and hexagons, and others have been irregular figures like the pentomino pieces. Now I want to test your prediction skills. I am going to create a figure and you will predict whether or not it will tessellate."

Take the 2-inch by 2-inch piece of index card. Start in the upper left corner and make a curved cut (1). Slide the piece directly to the bottom of the card and tape it into place (2). Go to the lower right corner, cut a triangle (3), slide it directly to the lower left corner, and tape it into place (4). Place the card on the overhead. "Will this figure tessellate?" Take several responses, asking students to justify their prediction. "Let me add one more idea and see if it will change your prediction. Will this figure tessellate if I sometimes flip the figure?" Flip the card horizontally on the overhead to emphasize the point. Again take several responses and the reasons for the predictions. "I am going to let you answer that question."



Distribute the 3-inch by 5-inch index cards, scissors, tape, paper, and the Flip Rows activity sheet. The students will make a figure and tessellate it over the 12-inch by 18-inch paper. After the students have traced a horizontal row across the paper, they will flip their figure horizontally and fit it into the row above what they have drawn. For each new row, the figure should be flipped horizontally. When the tessellation is completed, the students can give it a title and their artwork can be displayed throughout the school.

Answer Key: The figures will tessellate.

Related Activity: Obtain books or other resources about M. C. Escher (1898–1972). Introduce the students to Escher's works by looking for tessellations and exploring how Escher used them. The students can continue to make tessellations, exploring the use of turns and flips.

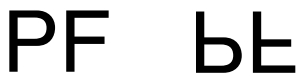
Name _____ Date _____

1-1. Turn, Flip, or Slide

Part One: For each figure below, decide if the second figure was moved by a turn, a flip, or a slide. Some could have been moved two ways. If so, identify all of the ways.

1.  Moved by a _____.

2.  Moved by a _____.

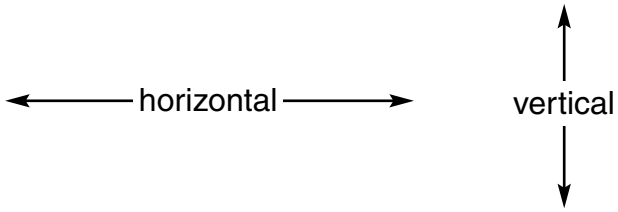
3.  Moved by a _____.

4.  Moved by a _____ or a _____.

5.  Moved by a _____ or a _____.

6.  Moved by a _____.

Part Two: Draw each figure according to the directions. These figures may be helpful to you.



The letter G flipped horizontally would look like this: 

The letter G flipped vertically would look like this: 

Name _____ Date _____

1-1. Turn, Flip, or Slide *(continued)*

7. 

Draw what this figure would look like if flipped horizontally: _____

8.

Draw what this figure would look like if flipped vertically: _____

9. 

Draw what this figure would look like if turned halfway around: _____

10. 

Draw what this figure would look like if flipped horizontally: _____

Draw what this figure would look like if flipped vertically: _____

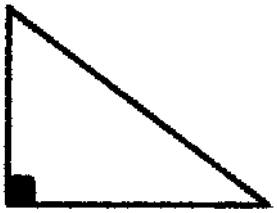
Draw what this figure would look like if turned halfway around: _____

Draw what this figure would look like if moved by a slide: _____

Name Date

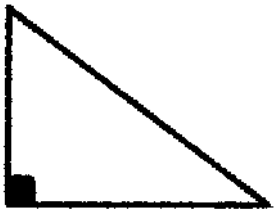
1-2. Draw the Ways

Trace the triangle below onto another piece of paper and cut it out. Place the cutout on top of this triangle. Are the two figures congruent? _____ If they are not, retrace and cut out the figure again.

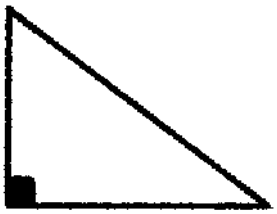


Use your cutout to follow the moves described below and draw the new position of the figure next to the original.

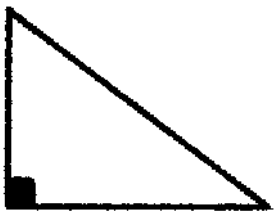
1. Move the figure ¼ turn and draw its new position.



2. Flip the figure horizontally and draw its new position.



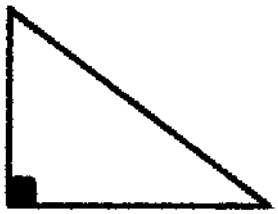
3. Move the figure ½ turn and draw its new position.



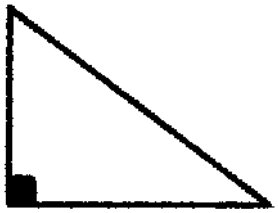
Name Date

1-2. Draw the Ways *(continued)*

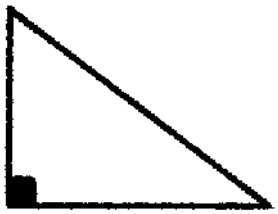
4. Flip the figure vertically, then flip it vertically again. Draw its new position.



5. Flip the figure horizontally, then make a ¼ turn. Draw its new position.



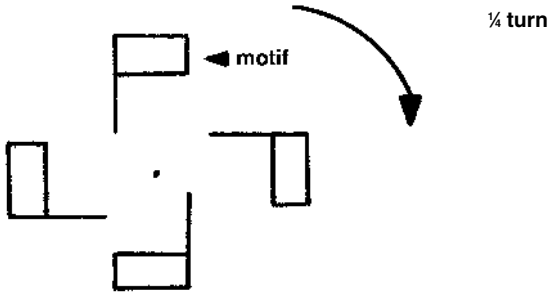
6. Slide the figure, flip it horizontally, flip it vertically, then flip it horizontally. Draw its new position.



Name Date

1-3. Turning and Turning

Rotational symmetry is the result of turning or rotating a motif (a design) around a point. The motif can be turned as much or as little as we want. If each of the turns is equal, then the pattern looks balanced, or symmetrical. Look at the figure below. The motif has been turned, or rotated, ¼ of the way around and then drawn, then rotated ¼ of the way and drawn, and finally repeated one more time.



The resulting pattern is symmetrical due to turning. For each motif below, follow the directions and draw the motif in each of the positions.

1. Turn the motif halfway around and draw it as it would look in the new position.

B

•

2. Turn the motif ¼ way around (¼ turn) as many times as you can, drawing what the motif would look like each time.

F

•

Name _____ Date _____

1-3. Turning and Turning *(continued)*

3. Rotate the motif ¼ turn as many times as you can, drawing the motif each time.

4. Rotate the motif ½ turn as many times as you can, drawing the motif each time.

5. Describe the motif in each of its positions in question 3.

_____

_____

_____

6. Describe the motif in each of its positions in question 4.

_____

_____

Why did that happen? _____

_____

7. Define motif. _____

_____

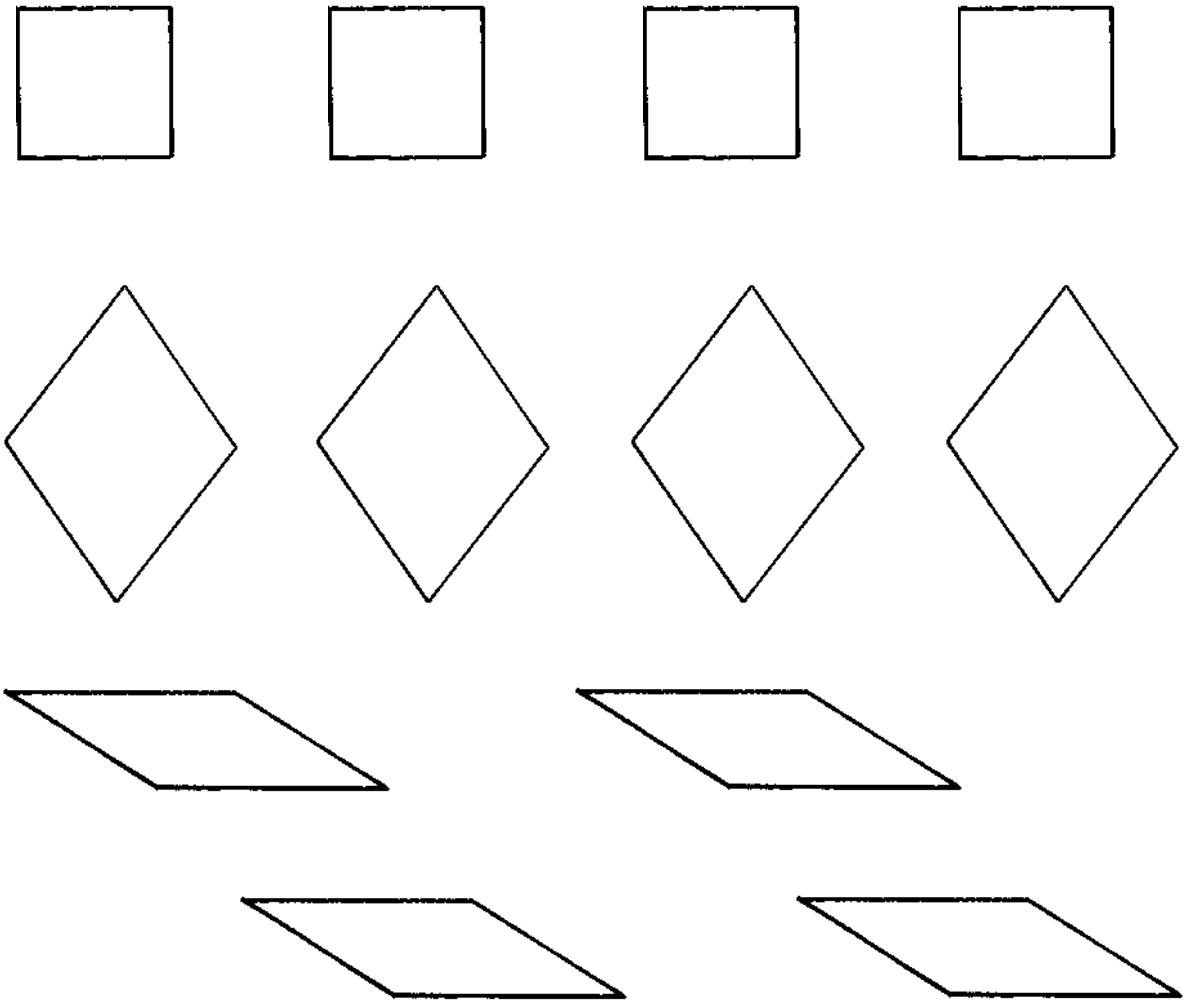
Name Date

1-4. Turn Around

Turn the pattern according to the directions to complete the design. Glue the cutouts on the vertical and horizontal lines so the pieces in one region touch the pieces in the next region.

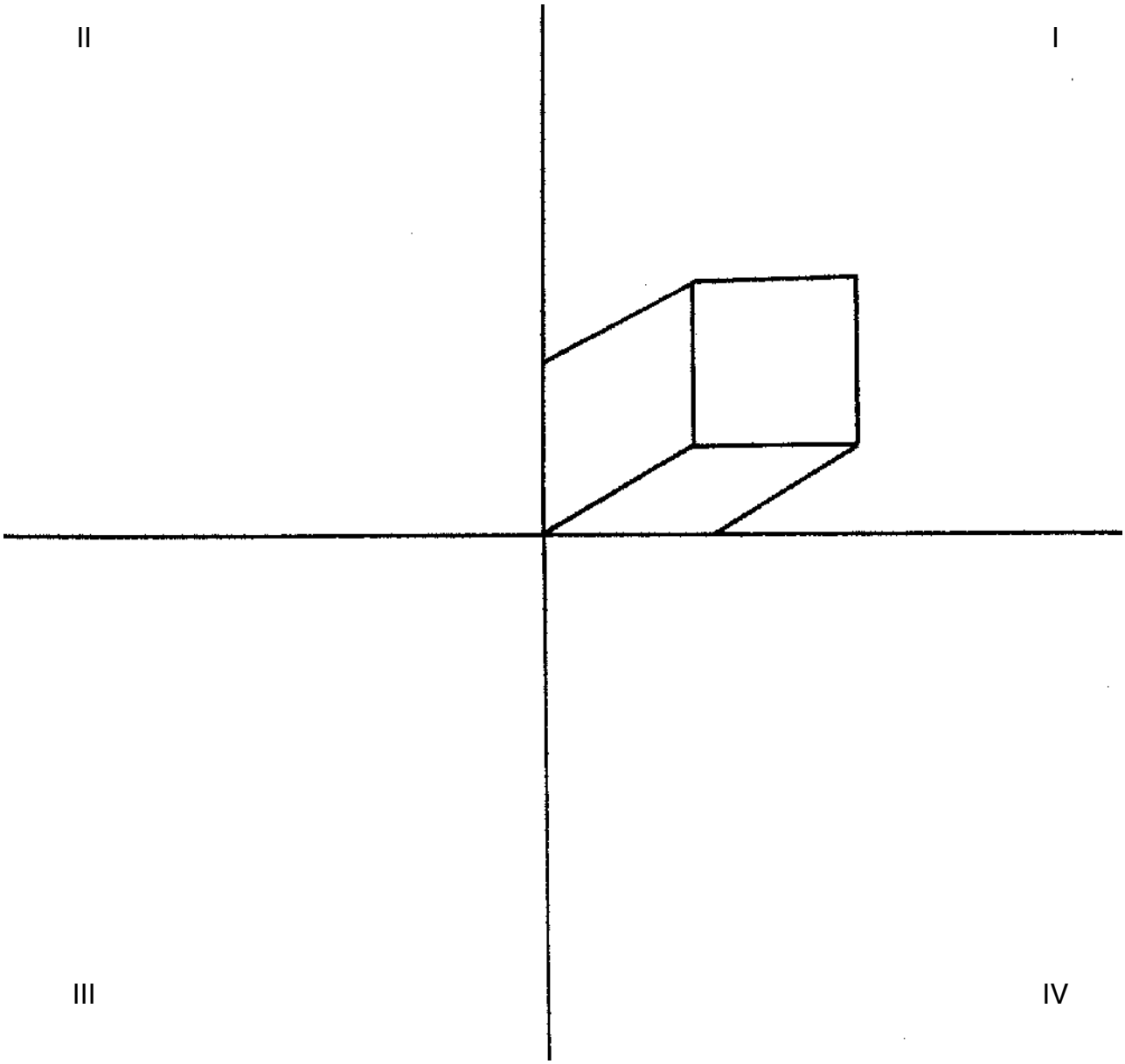
- Start by gluing the cutouts onto the pattern in region I.
- Make a ¼ turn of the pattern into region IV and glue those cutouts.
- Make a ¼ turn of the pattern in region IV into region III and glue those cutouts.
- Make a ¼ turn of the pattern in region III into region II and glue those cutouts.

You have now completed a pattern block design by using turns.



Name Date

1-4. Turn Around *(continued)*



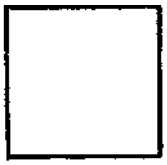
Name Date

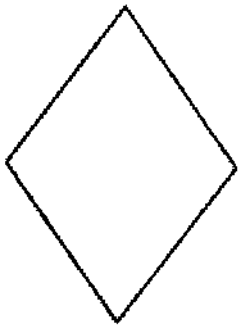
1-5. Flip Me Over

Flip the pattern according to the directions to complete the design. Glue the cutouts on the vertical and horizontal lines so the pieces in one region touch the pieces in the next region.

- Start by gluing the cutouts onto the pattern in region I.
- Flip the pattern vertically into region IV and glue those cutouts.
- Flip the pattern in region IV horizontally into region III and glue those cutouts.
- Flip the pattern in region III vertically into region II and glue those cutouts.

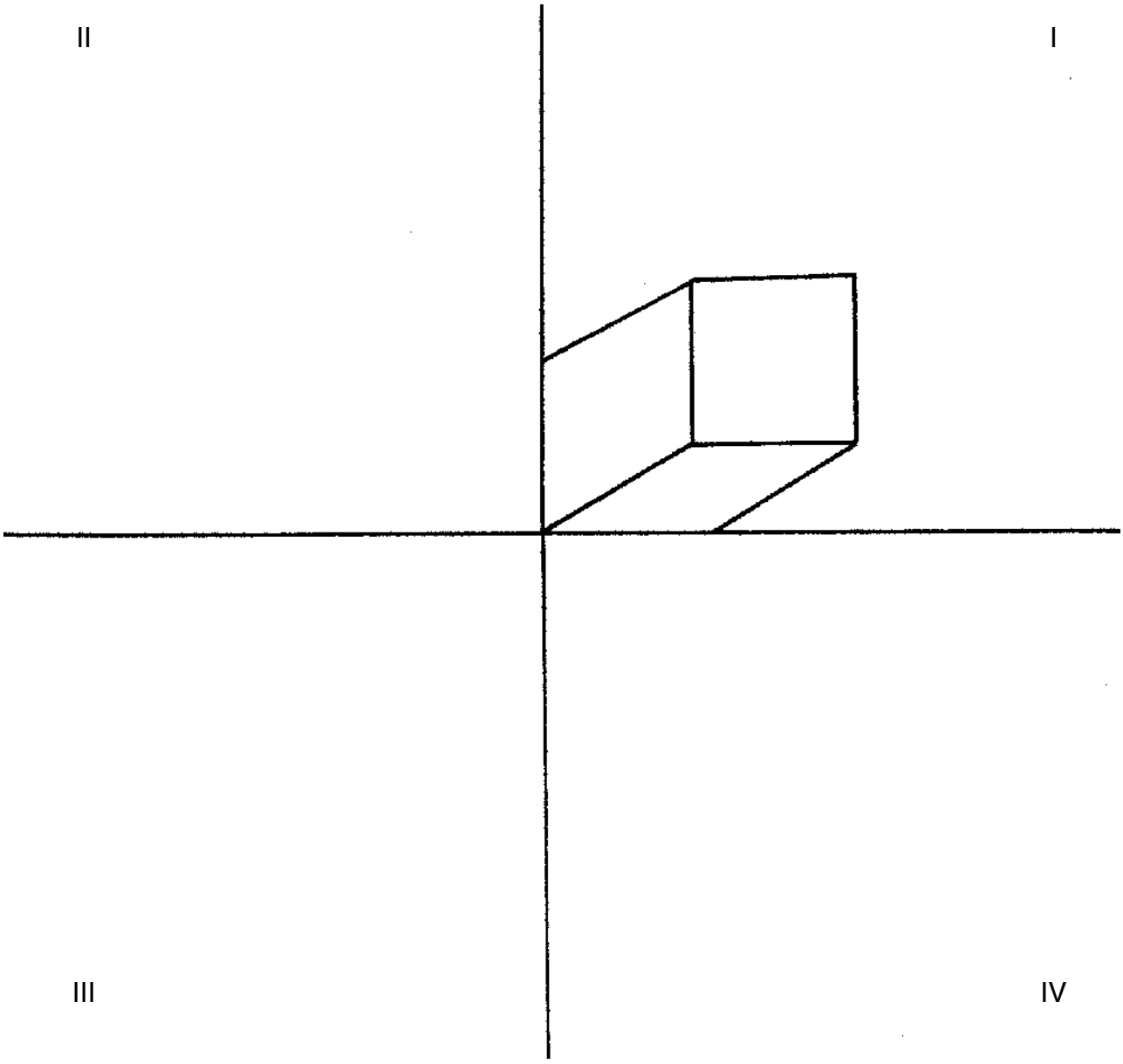
You have now completed a pattern block design by using flips.





Name Date

1-5. Flip Me Over *(continued)*



The motif was turned in the Turn Around activity and the motif was flipped in the Flip Me Over activity. Compare the design resulting from turning the motif to the design resulting from flipping the motif.

_____

_____

_____

Name ______________________________ Date ____________

1-6. Flip Me to Read Me

The words below have been flipped vertically. Use a mirror to read each question. Write your answer for number 1 flipped vertically. Write normally for the rest of the questions.

1. What is your name? ______________________________

2. What is your favorite school subject? ______________________________

____

3. There are three ways that a figure can be moved. In your own words, describe moving a figure using a turn.

____

____

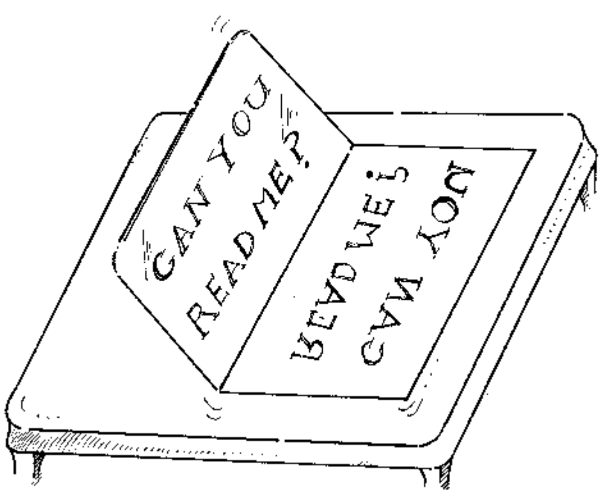
4. In your own words, describe moving a figure using a flip. ______________________________

____

5. In your own words, describe moving a figure using a slide. ______________________________

____

____



Name Date

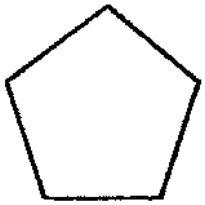
1-7. Mirror Lines

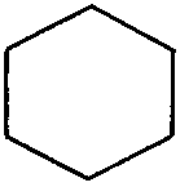
Mirror (bilateral) symmetry is often shown by a line of symmetry within a figure. A line of symmetry divides a figure into two equal parts that are mirror images of each other. For each figure below, determine the number of different lines of symmetry each has. Then draw all of the lines of symmetry and complete the statement for each figure. You may draw each figure on another piece of paper, cut the figure out, and fold it in different ways to determine all of the lines of symmetry.

1.  A rectangle has _____ lines of symmetry.

2.  A square has _____ lines of symmetry.

3.  An equilateral triangle has _____ lines of symmetry.

4.  A pentagon has _____ lines of symmetry.

5.  A hexagon has _____ lines of symmetry.

6. An octagon would have _____ lines of symmetry. How do you know? _____

_____

_____

Draw an octagon on a separate piece of paper and show all of its lines of symmetry.

Name ______________________________ Date ______________

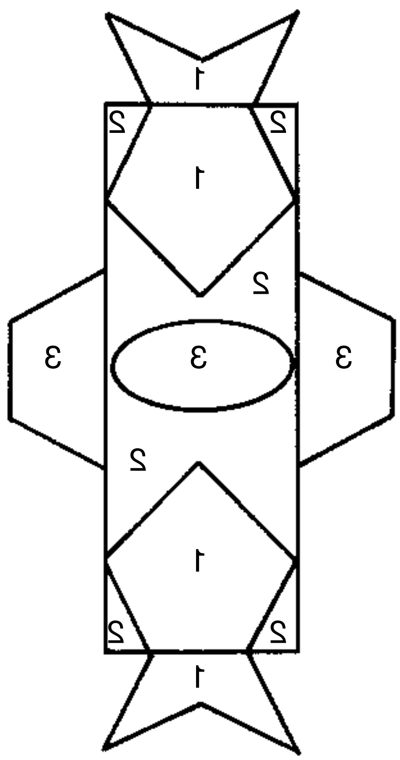
1-8. Mirror, rorriM

Color this pattern by following the directions.
You may use a mirror to read the directions.

1. Color the regions marked with the number 1 red.

2. Color the regions marked with the number 2 green.

3. Color the regions marked with the number 3 orange.



4. Add several figures to one side of the design. Then use the mirror to draw the figures on the other side so the design is still symmetrical.

5. Do you like writing backwards? Write your answer backwards. _______ Why?

____

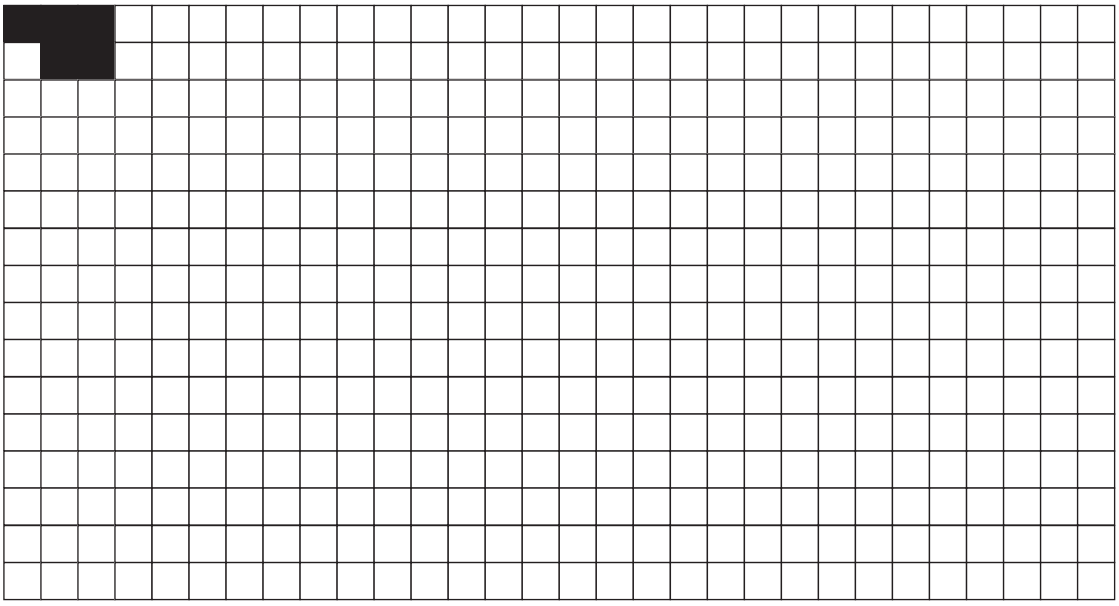
____

Name Date

1-9. Pentomino Search

Pentomino nets are made from 5 equal squares that are joined along their edges. The edges must join for the entire length and the pentomino nets cannot be joined at vertices. It is possible to make 12 different nets of 5 equal squares, but flips and turns of the nets are not allowed. A flipped or turned net is still the same net.

Use your square cutouts to find the 12 different nets and shade each net on the grid. Remember, when joining the edges, this is okay:  . But this is not allowed:  . One net has been filled in for you.



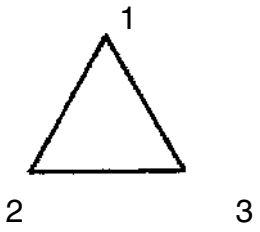
Six of the nets have bilateral symmetry. Draw them below and show the line of symmetry for each.

Name Date

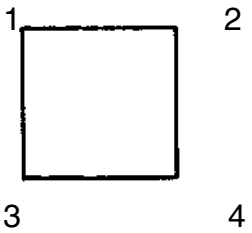
1-10. How Many Times?

Antonio and Christi were exploring pattern blocks. They found out that some of the blocks can be turned a number of times and still look the same. This is one type of rotational symmetry. Use pattern blocks or cutouts to trace and show how many times each figure can be turned and still look like the same position. Use the numbers to show each time the figure is turned.

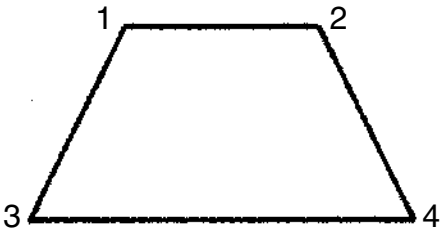
1. An equilateral triangle can be turned ___ times and still look like its original position.



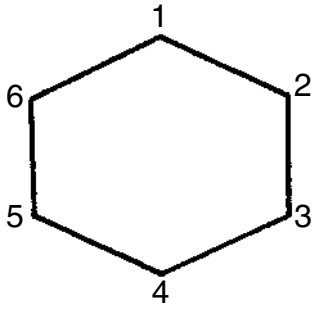
2. A square can be turned ___ times and still look like its original position.



3. A trapezoid can be turned ___ times and still look like its original position.



4. A hexagon can be turned _____ times and still look like its original position.



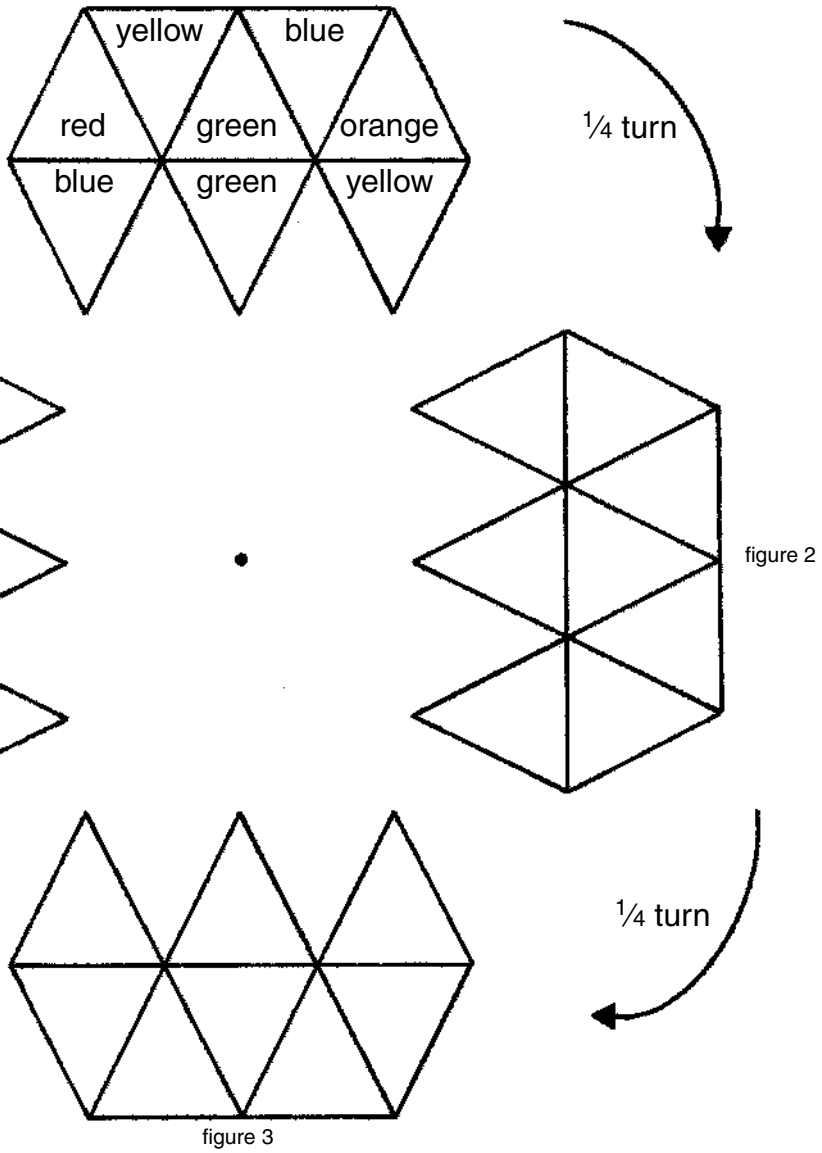
Name Date

1-11. Rotate and Color

Color figure 1 according to the directions. The pattern has been moved using a ¼ turn to the right. Color figure 2 to show what the original figure 1 would look like if it had been moved ¼ turn. Then color figure 3 to show what figure 2 would look like if it had been moved ¼ turn. This would be the same as moving figure 1 ½ turn.

Finally, color figure 4 to show what figure 3 would look like after a ¼ turn. Be sure to picture the turned figures correctly before coloring.

figure 1



Name _____ Date _____

1-12. Alpha Time

A B C D E F G H I J K L M N
O P Q R S T U V W X Y Z

1. There are 17 letters that have one or more lines of symmetry. Write those letters below and draw the line, or lines, of symmetry for each.

_____ _____ _____

_____ _____ _____

_____ _____ _____

_____ _____ _____

_____ _____ _____

_____ _____ _____

_____ _____ _____

2. There are 4 letters that have *both* rotational symmetry and one or more lines of symmetry. Which letters are they?

_____ _____ _____ _____

3. Some words have mirror or flip symmetry. One example is MOM. If you flip the word horizontally you get:

MOM | MOM

mirror

The word DAD does not work. When you flip the word horizontally you get:

DAD | ꓷAD

mirror

Think of three words that have mirror or flip symmetry. Show the words and their mirror images.

_____|_____ _____|_____ _____|_____

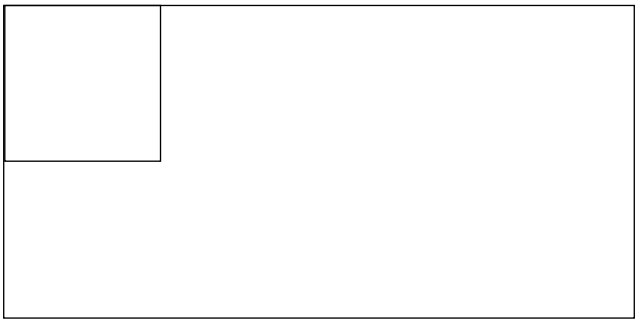
mirror mirror mirror

Name Date

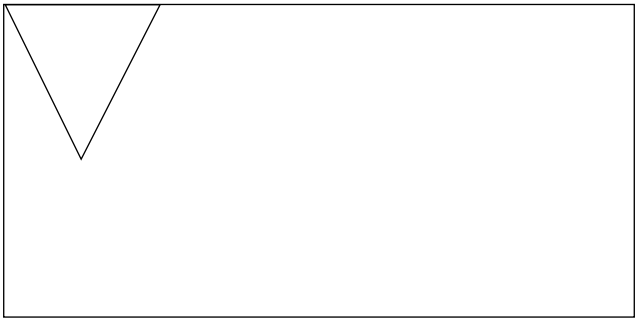
1-13. Will It?

To tessellate means to cover an area with no gaps or overlaps. You are going to determine if different figures tessellate. For each given figure, trace that figure in the space provided and then keep moving and tracing the figure, trying to cover the space. Then answer the question about each figure.

1. Will a square cover the area (tessellate)? _____



2. Will an equilateral triangle cover the area (tessellate)? _____



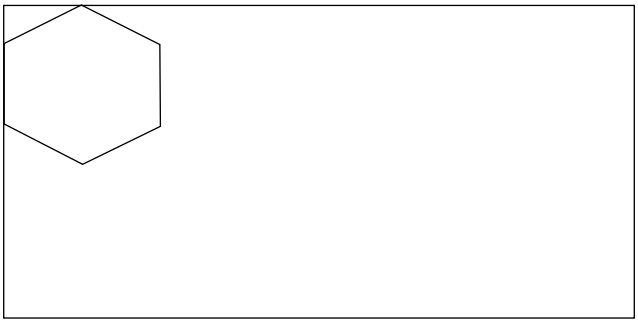
3. Will a pentagon tessellate? _____

Name Date

1-13. Will It? *(continued)*

4. Will a rhombus tessellate? _____

5. Will a hexagon tessellate? _____



6. Did all of the figures tessellate? _____ Why do you think some figures will tessellate and some figures will not tessellate?

_____

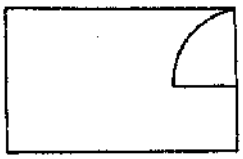
_____

_____

Name Date

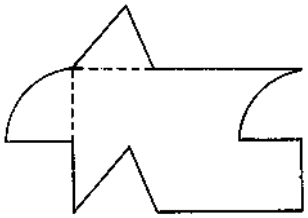
1-14. Flip Rows

A tessellation is a motif that will cover an area with no gaps or overlaps. Many times, even an odd-shaped motif will tessellate. Take an index card, and start in any corner and make a cut.

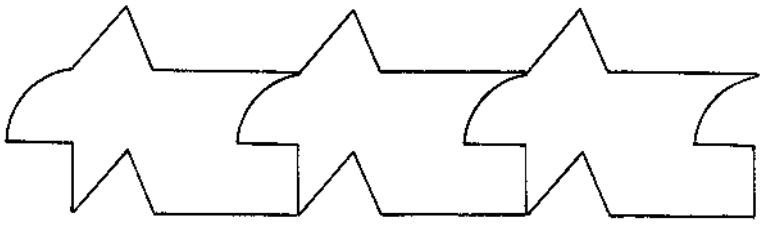


Slide the cut piece either from one side to the other, or from top to bottom, or from bottom to top. Be sure not to flip or turn the cut piece. Tape it into place.

Move to another corner and make another cut. Once again, slide the piece across, up, or down, and tape it into place. If you desire, move to another corner and make a third cut. Move the cut piece across, up, or down, and tape it into place.



When you have finished, place the tessellation in the middle of a large piece of paper and trace around it. Then slide the tessellation to the right or left until it fits, and trace again. Keep sliding and tracing until you have completed one horizontal row across the paper.



Then flip the tessellation horizontally and fit it in above the row. Keep moving and tracing until you have a second row that is opposite from your first row. Then flip the tessellation horizontally and add another row above the two rows. Once you have covered the top of the paper, work on the bottom of the paper. When finished, name your picture and share it with your classmates.