

Numbers, History, Society, and People

IN WHICH WE ENCOUNTER RELIGIOUS MATHEMATICIANS, MAD MATHEMATICIANS, famous mathematicians, mathematical savants, quirky questions, fun trivia, brief biographies, mathematical gods, historical oddities, numbers and society, gossip, the history of mathematical notation, the genesis of numbers, and “What if?” questions.

Mathematics is the hammer that shatters
the ice of our unconscious.

❓ **Ancient counting.** Let's start the book with a question. What is the earliest evidence we have of humans counting? If this question is too difficult, can you guess whether the evidence is before or after 10,000 B.C.—and what the evidence might be? (See Answer 1.1.)

🌀 **Mathematics and beauty.** I've collected mathematical quotations since my teenage years. Here's a favorite: "Mathematics, rightly viewed, possesses not only truth, but supreme beauty—a beauty cold and austere, like that of sculpture" (Bertrand Russell, *Mysticism and Logic*, 1918).

❗ **The symbols of mathematics.** Mathematical notation shapes humanity's ability to efficiently contemplate mathematics. Here's a cool factoid for you: The symbols + and −, referring to addition and subtraction, first appeared in 1456 in an unpublished manuscript by the mathematician Johann Regiomontanus (a.k.a. Johann Müller). The plus symbol, as an abbreviation for the Latin *et* (and), was found earlier in a manuscript dated 1417; however, the downward stroke was not quite vertical.

❓ **Mathematics and reality.** Do humans invent mathematics or discover mathematics? (See Answer 1.2.)

❗ **Mathematics and the universe.** Here is a deep thought to start our mathematical journey. Do you think humanity's long-term fascination with mathematics has arisen because the universe is constructed from a mathematical fabric? We'll approach this question later in the chapter. For now, you may enjoy knowing that in 1623, Galileo Galilei echoed this belief in a mathematical universe by stating his credo: "Nature's great book is written in mathematical symbols." Plato's doctrine was that God is a geometer, and Sir James Jeans believed that God experimented with arithmetic. Isaac Newton supposed that the planets were originally thrown into orbit by God, but even after God decreed the law of gravitation, the planets required continual adjustments to their orbits.

🌀 **Math beyond humanity.** "We now know that there exist true propositions which we can never formally prove. What about propositions whose proofs require arguments beyond our capabilities? What about propositions whose proofs require millions of pages? Or a million, million pages? Are there proofs that are possible, but beyond us?" (Calvin Clawson, *Mathematical Mysteries*).

❗ **The multiplication symbol.** In 1631, the multiplication symbol × was introduced by the English mathematician William Oughtred (1574–1660) in his book *Keys to Mathematics*, published in London. Incidentally, this Anglican minister is also famous for having invented the slide rule, which was used by generations of scientists and mathematicians. The slide rule's doom in the mid-1970s, due to the pervasive influx of inexpensive pocket calculators, was rapid and unexpected.

❗ **Math and madness.** Many mathematicians throughout history have had a trace of madness or have been eccentric. Here's a relevant quotation on the subject by the British mathematician John Edensor Littlewood (1885–1977), who suffered from depression for most of his life: “Mathematics is a dangerous profession; an appreciable proportion of us goes mad.”

❓ **Mathematics and murder.** What triple murderer was also a brilliant French mathematician who did his finest work while confined to a hospital for the criminally insane? (See Answer 1.3.)

🌀 **Creativity and madness.** “There is a theory that creativity arises when individuals are out of sync with their environment. To put it simply, people who fit in with their communities have insufficient motivation to risk their psyches in creating something truly new, while those who are out of sync are driven by the constant need to prove their worth.

❗ **Mathematicians and religion.** Over the years, many of my readers have assumed that famous mathematicians are not religious. In actuality, a number of important mathematicians were quite religious. As an interesting exercise, I conducted an Internet survey in which I asked respondents to name important mathematicians who were also religious. Isaac Newton and Blaise Pascal were the most commonly cited religious mathematicians.

In many ways, the mathematical quest to understand infinity parallels mystical attempts to understand God. Both religion and mathematics struggle to express relationships between humans, the universe, and infinity. Both have arcane symbols and rituals, as well as impenetrable language. Both exercise the deep recesses of our minds and stimulate our imagination. Mathematicians, like priests, seek “ideal,” immutable, nonmaterial truths and then often venture to apply these truths in the real world. Are mathematics and religion the most powerful evidence of the inventive genius of the human race? In “Reason and Faith, Eternally Bound” (December 20, 2003, *New York Times*, B7), Edward Rothstein notes that faith was the inspiration for Newton and Kepler, as well as for numerous scientific and mathematical triumphs. “The conviction that there is an order to things, that the mind can comprehend that order and that this order is not infinitely malleable, those scientific beliefs may include elements of faith.”

In his *Critique of Pure Reason*, Immanuel Kant describes how “the light dove, cleaving the air in her free flight and feeling its resistance against her wings, might imagine that its flight would be freer still in empty space.” But if we were to remove the air, the bird would plummet. Is faith—or a cosmic sense of mystery—like the air that allows some seekers to soar? Whatever mathematical or scientific advances humans make, we will always continue to swim in a sea of mystery.

They have less to lose and more to gain” (Gary Taubes, “Beyond the Soapsuds Universe,” 1977).

🌀 **Pascal's mystery.** “There is a God-shaped vacuum in every heart” (Blaise Pascal, *Pensées*, 1670).

? Leaving mathematics and approaching God. What famous French mathematician and teenage prodigy finally decided that religion was more to his liking and joined his sister in her convent, where he gave up mathematics and social life? (See Answer 1.4.)

! Ramanujan's gods. As mentioned in this book's introduction, the mathematician Srinivasa Ramanujan (1887–1920) was an ardent follower of several Hindu deities. After receiving visions from these gods in the form of blood droplets, Ramanujan saw scrolls that contained very complicated mathematics. When he woke from his dreams, he set down on paper only a fraction of what the gods showed him.

Throughout history, creative geniuses have been open to dreams as a source of inspiration. Paul McCartney said that the melody for the famous Beatles' song "Yesterday," one of the most popular songs ever written, came to him in a dream. Apparently, the tune seemed so beautiful and haunting that for a while

he was not certain it was original. The Danish physicist Niels Bohr conceived the model of an atom from a dream. Elias Howe received in a dream the image of the kind of needle design required for a lock-stitch sewing machine. René Descartes was able to advance his geometrical


methods after flashes of insight that came in dreams. The dreams of Dmitry Mendeleyev, Friedrich August Kekulé, and Otto Loewi inspired scientific breakthroughs. It is not an exaggeration to suggest that many scientific and mathematical advances arose from the stuff of dreams.


! Blaise Pascal (1623–1662), a Frenchman, was a geometer, a probabilist, a physicist, a philosopher, and a combinatorist. He was also deeply spiritual and a leader of the Jansenist sect, a Calvinistic quasi-Protestant group within the Catholic Church. He believed that it made sense to become a Christian. If the person dies, and there is no God, the person loses nothing. If there is a God, then the person has gained heaven, while skeptics lose everything in hell.


Legend has it that Pascal in his early childhood sought to prove the existence of God. Because Pascal could not simply command God to show Himself, he tried to prove the existence of a devil so that he could then infer the existence of God. He drew a pentagram on the ground, but the exercise scared him, and he ran away. Pascal said that this experience made him certain of God's existence.

One evening in 1654, he had a two-hour mystical vision that he called a "night of fire," in which he experienced fire and "the God of Abraham, Isaac, and Jacob . . . and of Jesus Christ." Pascal recorded his vision in his work "Memorial." A scrap of paper containing the "Memorial" was found in the lining of his coat after his death, for he carried this reminder about with him always. The three lines of "Memorial" are


Complete submission to Jesus Christ and to my director.
Eternally in joy for a day's exercise on the earth.
May I not forget your words. Amen.


 **Transcendence.** “Much of the history of science, like the history of religion, is a history of struggles driven by power and money. And yet, this is not the whole story. Genuine saints occasionally play an important role, both in religion and science. For many scientists, the reward for being a scientist is not the power and the money but the chance of catching a glimpse of the transcendent beauty of nature” (Freeman Dyson, in the introduction to *Nature’s Imagination*).


 **The value of eccentricity.** “That so few now dare to be eccentric, marks the chief danger of our time” (John Stuart Mill, nineteenth-century English philosopher).


 **Counting and the mind.** I quickly toss a number of marbles onto a pillow. You may stare at them for an instant to determine how many marbles are on the pillow. Obviously, if I were to toss just two marbles, you could easily determine that two marbles sit on the pillow. What is the largest number of marbles you can


quantify, at a glance, without having to individually count them? (See Answer 1.5.)

 **Circles.** Why are there 360 degrees in a circle? (See Answer 1.6.)

 **The mystery of Ramanujan.** After years of working through Ramanujan’s notebooks, the mathematician Bruce Berndt said, “I still don’t understand it all. I may be able to prove it, but I don’t know where it comes from and where it fits into the rest of mathematics. The enigma of Ramanujan’s creative process is still covered by a curtain that has barely been drawn” (Robert Kanigel, *The Man Who Knew Infinity*, 1991).

 **The world’s most forgettable license plate?** Today, mathematics affects society in the funniest of ways. I once read an article about someone who claimed to have devised the most forgettable license plate, but the article did not divulge the secret sequence. What is the most forgettable license plate? Is it a random sequence of eight letters and numbers—for example, 6AZL4QO9 (the maximum allowed in New York)? Or perhaps a set of visually confusing numbers or letters—for example, MWNNMWWM? Or maybe a binary number like 01001100. What do you think? What would a *mathematician* think? (See Answer 1.7.)

 **Calculating π .** Which nineteenth-century British boarding school supervisor spent a significant portion of his life calculating π to 707 places and died a happy man, despite a sad error that was later found in his calculations? (See Answer 1.8.)

 **The special number 7.** In ancient days, the number 7 was thought of as just another way to signify “many.” Even in recent times, there have been tribes that used no numbers higher than 7.

In the 1880s, the German ethnologist Karl von Steinen described how certain South American Indian tribes had very few words for numbers. As a test, he repeatedly asked them to count ten grains of corn. They counted “slowly

but correctly to six, but when it came to the seventh grain and the eighth, they grew tense and uneasy, at first yawning and complaining of a headache, then finally avoided the question altogether or simply walked off.” Perhaps *seven* means “many” in such common phrases as “seven seas” and “seven deadly sins.” (These interesting facts come from Adrian Room, *The Guinness Book of Numbers*, 1989.)

❗ **Carl Friedrich Gauss** (1777–1855), a German, was a mathematician, an astronomer, and a physicist with a wide range of contributions. Like Ramanujan, after Gauss proved a theorem, he sometimes said that the insight did not come from “painful effort but, so to speak, by the grace of God.” He also once wrote, “There are problems to whose solution I would attach an infinitely greater importance than to those of mathematics, for example, touching ethics, or our relation to God, or concerning our destiny and our future; but their solution lies wholly beyond us and completely outside the province of science.”

❗ **Isaac Newton** (1642–1727), an Englishman, was a mathematician, a physicist, an astronomer, a coinventor of calculus, and famous for his law of gravitation. He was also the author of many books on biblical subjects, especially prophecy.

Perhaps less well known is the fact that Newton was a creationist who wanted to be known as much for his theological writings as for his scientific and mathematical texts. Newton believed in a Christian unity, as opposed to a trinity. He developed calculus as a means of describing motion, and perhaps for understanding the nature of God through a clearer understanding of nature and reality. He respected the Bible and accepted its account of Creation.

“**Genius and eccentricity.** “The amount of eccentricity in a society has been proportional to the amount of genius, material vigor and moral courage which it contains” (John Stuart Mill, *On Liberty*, 1869).

“**Mathematics and God.** “The Christians know that the mathematical principles, according to which the corporeal world was to be created, are co-eternal with God. Geometry has supplied God with the models for the creation of the world. Within the image of God it has passed into man, and was certainly not received within through the eyes” (Johannes Kepler, *The Harmony of the World*, 1619).

❗ **James Hopwood Jeans** (1877–1946) was an applied mathematician, a physicist, and an astronomer. He sometimes likened God to a mathematician and wrote in *The Mysterious Universe* (1930), “From the intrinsic evidence of his creation, the Great Architect of the Universe now begins to appear as a pure mathematician.” He has also written, “Physics tries to discover the pattern of events which controls the phenomena we observe. But we can never know what this pattern means or how it originates; and even if some superior intelligence were to tell us, we should find the explanation unintelligible” (*Physics and Philosophy*, 1942).

! **Leonhard Euler** (1707–1783) was a prolific Swiss mathematician and the son of a vicar. Legends tell of Leonhard Euler’s distress at being unable to mathematically prove the existence of God. Many mathematicians of his time considered mathematics a tool to decipher God’s design and codes. Although he was a devout Christian all his life, he could not find the enthusiasm for the study of theology, compared to that of mathematics. He was completely blind for the last seventeen years of his life, during which time he produced roughly half of his total output.

Euler is responsible for our common, modern-day use of many famous mathematical notations—for example, $f(x)$ for a function, e for the base of natural logs, i for the square root of -1 , π for pi, Σ for summation. He tested Pierre de Fermat’s conjecture that numbers of the form $2^n + 1$ were always prime if n is a power of 2. Euler verified this for $n = 1, 2, 4, 8$, and 16 , and showed that the next case $2^{32} + 1 = 4,294,967,297 = 641 \times 6,700,417$, and so is not prime.

! **George Boole** (1815–1864), an Englishman, was a logician and an algebraist. Like Ramanujan and other mystical mathematicians, Boole had “mystical” experiences. David Noble, in his book *The Religion of Technology*, notes, “The thought flashed upon him suddenly as he was walking across a field that his ambition in life was to explain the logic of human thought and to delve analytically into the spiritual aspects of man’s nature [through] the expression of logical relations in symbolic

or algebraic form. . . . It is impossible to separate Boole’s religious beliefs from his mathematics.”

Boole often spoke of his almost photographic memory, describing it as “an arrangement of the mind for every fact and idea, which I can find at once, as if it were in a well-ordered set of drawers.”

Boole died at age forty-nine, after his wife mistakenly thought that tossing buckets of water on him and his bed would cure his flu. Today, Boolean algebra has

found wide applications in the design of computers.

 ? **The value of puzzles.** “It is a wholesome plan, in thinking about logic, to stock the mind with as many puzzles as possible, since these serve much the same purpose as is served by experiments in physical science” (Bertrand Russell, *Mind*, 1905).

 ? **A mathematical nomad.** What legendary mathematician, and one of the most prolific mathematicians in history, was so devoted to math that he lived as a nomad with no home and no job? Sexual contact revolted him; even an accidental touch by anyone made him feel uncomfortable. (See Answer 1.9.)

 ! **Marin Mersenne** (1588–1648) was another mathematician who was deeply religious. Mersenne, a Frenchman, was a theologian, a philosopher, a number theorist, a priest, and a monk. He argued that God’s majesty would not be diminished had

🔍 Mirror phobia. What brilliant, handsome mathematician so hated mirrors that he covered them wherever he went? (See Answer 1.10.)

🔍 What is a mathematician? “A mathematician is a blind man in a dark room looking for a black cat which isn’t there” (Charles Darwin).

🔍 Animal math. Can animals count? (See Answer 1.11.)

He created just one world, instead of many, because the one world would be infinite in every part. His first publications were theological studies against atheism and skepticism.

Mersenne was fascinated by prime numbers (numbers like 7 that were divisible only by themselves and 1), and he tried to find a formula that he could use to find all primes. Although he did not find such a formula, his work on “Mersenne numbers” of the form $2^p - 1$, where p is a prime number, continues to interest us today. Mersenne numbers are the easiest type of number to prove prime, so they are usually the largest primes of which humanity is aware.


Mersenne himself found several prime numbers of the form $2^p - 1$, but he underesti-


mated the future of computing power by stating that all eternity would not be sufficient to decide if a 15- or 20-digit number were prime. Unfortunately, the prime number values for p that make $2^p - 1$ a prime number seem to form no regular sequence. For example, the Mersenne number is prime when $p = 2, 3, 5, 7, 13, 17, 19, \dots$. Notice that when p is equal to the prime number 11, $M_{11} = 2,047$, which is not prime because $2,047 = 23 \times 89$.


The fortieth Mersenne prime was discovered in 2003, and it contained 6,320,430 digits! In particular, the Michigan State University graduate student Michael Shafer discovered that $2^{20,996,011} - 1$ is prime. The number is so large that it would require about fifteen

hundred pages to write on paper using an ordinary font. Shafer, age twenty-six, helped find the number as a volunteer on a project called the Great Internet Mersenne Prime Search. Tens of thousands of people volunteer the use of their personal computers in a worldwide project that harnesses the power of hundreds of thousands of computers, in effect creating a supercomputer capable of performing trillions of calculations per second. Shafer used an ordinary Dell computer in his office for nineteen days. What would Mersenne have thought of this large beast?


In 2005, the German eye surgeon Martin Nowak, also part of the Great Internet Mersenne Prime Search, discovered the forty-second Mersenne prime number, $2^{25,964,951} - 1$, which has over seven million digits. Nowak’s 2.4-GHz Pentium-4 computer spent roughly fifty days analyzing the number before reporting the find. The Electronic Frontier Foundation, a U.S. Internet campaign group, has promised to give \$100,000 to whoever finds the first ten-million-digit prime number.


 **Mathematics and God.** “Before creation, God did just pure mathematics. Then He thought it would be a pleasant change to do some applied” (John Edensor Littlewood, *A Mathematician’s Miscellany*, 1953).


 **The division symbol.** The division symbol \div first appeared in print in Johann Heinrich Rahn’s *Teutsche Algebra* (1659).

 **Donald Knuth (1938–)** is a computer scientist and a mathematician. He is also a fine example of a mathematician who is interested in religion. For example, he has been an active Lutheran and a Sunday school teacher. His attractive book titled *3:16* consists entirely of commentary on chapter 3, verse 16, of

each of the books in the Bible. Knuth also includes calligraphic renderings of the verses. Knuth himself has said, “It’s tragic that scientific advances have caused many people to imagine that they know it all, and that God is irrelevant or nonexistent. The fact is that everything we learn reveals more things that we do not understand. . . . Reverence for God comes naturally if we are honest about how little we know.”


 **Mathematics and sex.** “A well-known mathematician once told me that the great thing about liking both math and sex was that he could do either one while thinking about the other” (Steven E. Landsburg, in a 1993 post to the newsgroup sci.math).


 **Brain limitation.** “Our brains have evolved to get us out of the rain, find where the berries are, and keep us from getting killed. Our brains did not evolve to help us grasp really large numbers or to look at things in a hundred thousand dimensions” (Ronald Graham, a prior director of Information Sciences Research at AT&T Research, quoted in Paul Hoffman’s “The Man Who Loves Only Numbers,” *Atlantic Monthly*, 1987).

 **Mystery mathematician.** Around A.D. 500, the Greek philosopher Metrodorus gave us the following puzzle that describes the life of a famous mathematician:

A certain man’s boyhood lasted $\frac{1}{6}$ of his life; he married after $\frac{1}{7}$ more; his beard grew after $\frac{1}{12}$ more, and his son was born 5 years later; the son lived to half his father’s final age, and the father died 4 years after the son.

Tell me the mystery man’s name or his age at death. (See Answer 1.12.)

 **Mathematician starves.** What famous mathematician deliberately starved himself to death in 1978? (Hint: He was perhaps the most brilliant logician since Aristotle.) (See Answer 1.13.)

 **Understanding brilliance.** “Maybe the brilliance of the brilliant can be understood only by the nearly brilliant” (Anthony Smith, *The Mind*, 1984).

! **Georg Friedrich Bernhard Riemann** (1826–1866) was a German mathematician who made important contributions to geometry, number theory, topology, mathematical

physics, and the theory of complex variables. He also attempted to write a mathematical proof of the truth of the Book of Genesis, was a student of theology and bibli-

cal Hebrew, and was the son of a Lutheran minister.

The Riemann hypothesis, published by Riemann in 1859, deals with the zeros of a very wiggly function, and the hypothesis still resists modern mathematicians' attempts to prove it. Chapter 3 describes the hypothesis further.

! **Calculating prodigy has plastic brain.** Rüdiger Gamm is shocking the world with his calculating powers and is changing the way we think about the human brain. He did poorly at mathematics in school but is now a world-famous human calculator, able to access regions of his brain that are off limits to most of us. He is not autistic but has been able to train his brain to perform lightning calculations. For example, he can calculate 53 to the ninth power in his head. He can divide prime numbers and calculate the answer to 60 decimal points and more. He can calculate fifth roots.

Amazing calculating powers such as these were previously thought to be possible only by “autistic savants.” (Autistic savants often have severe developmental disabilities but, at the same time, have special skills and an incredible memory.) Gamm’s talent has attracted the curiosity of European researchers, who have imaged his brain with PET scans while he performed math problems. These breathtaking studies reveal that Gamm is now able to use areas of his brain that ordinary humans can use for *other* purposes. In particular, he can make use of the areas of his brain that are normally responsible for long-term memory, in order to perform his rapid calculations. Scientists hypothesize that Gamm temporarily uses these areas to “hold” digits in so-called “working memory,” the brain’s temporary holding area. Gamm is essentially doing what computers do when they extend their capabilities by using swap space on the hard drive to increase their capabilities. Scientists are not sure how Gamm acquired this ability, considering that he became interested in mathematical calculation only when he was in his twenties. (You can learn more in Steve Silberman’s “The Keys to Genius,” *Wired*, no. 11.12, December 2003.)

6 **A dislike for mathematics.**

“I’m sorry to say that the subject I most disliked was mathematics. I have thought about it. I think the reason was that mathematics leaves no room for argument. If you made a mistake, that was all there was to it” (Malcolm X, *The Autobiography of Malcolm X*, 1965).

6 **Mathematics and humanity.**

“Anyone who cannot cope with mathematics is not fully human. At best he is a tolerable subhuman who has learned to wear shoes, bathe, and not make messes in the house” (Robert A. Heinlein, *Time Enough for Love*, 1973).

❗ **Kurt Gödel (1906–1978)** is an example of a mathematical genius obsessed with God and the afterlife. As discussed in Answer 1.13 about the mathematician who starved himself to death, Gödel was a logician, a mathematician, and a philosopher who was famous for having shown that in any axiomatic system for mathematics, there are propositions that cannot be proved or disproved within the axioms of the system.

Gödel thought it was possible to show the logical necessity for life after death and the existence of God. In four long letters to his mother, Gödel gave reasons for believing in a next world.

❓ **Math and madness.**

“Cantor’s work, though brilliant, seemed to move in half-steps. The closer he came to the answers he sought, the further away they seemed. Eventually, it drove him mad, as it had mathematicians before him” (Amir D. Aczel, *The Mystery of the Aleph: Mathematics, the Kabbalah, and the Search for Infinity*, 2000).

❗ **Gottfried Wilhelm von Leibniz (1646–1716)**, a German, was an analyst, a combinatorist, a logician, and the coinventor of calculus who also passionately argued for the existence of God. According to Leibniz, God chooses to actualize this world out of an infinite number of possible worlds. In other words, limited only by contradiction, God first conceives of every possible world, and then God simply chooses which of them to create.

Leibniz is also famous for the principle of “preestablished harmony,” which states that God constructed the universe in such a way that corresponding mental and physical events occur simultaneously. His “monad theory” states that the universe consists of an infinite number of substances called monads, each of which has its own individual identity but is an expression of the whole universe from a particular unique viewpoint.

❗ **Greater-than symbol.** The greater-than and less-than symbols ($>$ and $<$) were introduced by the British mathematician Thomas Harriot in his *Artis Analyticae Praxis*, published in 1631.

❗ **Greater-than or equal-to symbol.** The symbol \geq (greater than or equal to) was first introduced by the French scientist Pierre Bouguer in 1734.

❓ **Mathematician murdered.** Why was the first woman mathematician murdered? (See Answer 1.14.)

❓ **Going to the movies.** What was the largest number ever used in the title of an Ameri-

can movie? Name the movie! (See Answer 1.15.) What is the largest number less than a billion ever used in a major, full-length movie title? (Hint: The song was popular in the late 1920s and the early 1930s.) (See Answer 1.16.)

🔍 Math and madness. Many mathematicians were depressed and religious at the same time. Which famous mathematician invented the concept of “transfinite numbers” (essentially, different “levels” of infinity), believed that God revealed mathematical ideas to him, and was a frequent guest of sanitariums? (See Answer 1.17.)

🔍 Mathematics and diapering. “A human being should be able to change a diaper, plan an invasion, butcher a hog, conn a ship, design a building, write a sonnet, balance accounts, build a wall, set a bone, comfort the dying, take orders, give orders, cooperate, act alone, solve equations, analyze a new problem, pitch manure, program a computer, cook a tasty meal, fight efficiently, die gallantly. Specialization is for insects” (Robert A. Heinlein, *Time Enough for Love*, 1973).

🔍 Mathematicians and God. “Mathematicians, astronomers, and physicists are often religious, even mystical; biologists much less often; econ-

omists and psychologists very seldom indeed. It is as their subject matter comes nearer to man himself that their antireligious bias hardens” (C. S. Lewis, *The Grand Miracle: And Other Selected Essays on Theology and Ethics from God in the Dock*, 1983).

🔍 The Number Pope. As I write this book, I realize that a thousand years ago, the last Pope-mathematician died. Gerbert of Aurillac (c. 946–1003) was fascinated by mathematics and was elected to be Pope Sylvester II in 999. His advanced knowledge of mathematics convinced

some of his enemies that he was an evil magician.

In Reims, he transformed the floor of the cathedral into a giant abacus. That must have been a sight to see! The “Number Pope” was also important because he adopted Arabic numerals (1, 2, 3, 4, 5, 6, 7, 8, 9) as a replacement for Roman numerals. He contributed to the invention of the pendulum clock, invented devices that tracked planetary orbits, and wrote on geometry. When he realized that he lacked knowledge of formal logic, he studied under German logicians. He said, “The just man lives by faith; but it is good that he should combine science with his faith.”

🔍 Hardy’s six wishes. In the 1920s, the British mathematician G. H. Hardy wrote a postcard to his friend, listing six New Year’s wishes:

1. prove the Riemann hypothesis
2. score well at the end of an important game of cricket
3. find an argument for the nonexistence of God that convinces the general public
4. be the first man at the top of Mount Everest
5. be the first president of the USSR, Great Britain, and Germany
6. murder Mussolini

(*The London Mathematical Society Newsletter*, 1994)

❗ **Charles Babbage** (1792–1871), an Englishman, was an analyst, a statistician, and an inventor who was also interested in religious miracles. He once wrote, “Miracles are not a breach of established laws, but . . . indicate the existence of far higher laws.” Babbage argued that miracles could occur in a mechanistic world. Just as Babbage could program strange behavior on his calculating machines, God could program similar irregularities in nature. While investigating biblical miracles, he assumed that the chance of a man rising from the dead is one in 10^{12} .

Babbage is famous for conceiving an enormous hand-cranked mechanical calculator, an early progenitor of our modern computers. Babbage thought the device would be most useful in producing mathematical tables, but he worried about mistakes that would be made by humans who transcribed the results from its thirty-one metal output wheels. Today, we realize that Babbage was a hundred years ahead of his time and that the politics and the technology of his era were inadequate for his lofty dreams.

❗ **Tinkertoy computer.** In the early 1980s, the computer geniuses Danny Hillis, Brian Silverman, and friends built a Tinkertoy computer that played tic-tac-toe. The device was made from 10,000 Tinkertoy pieces.

❗ **Fantasy meeting of Pythagoras, Cantor, and Gödel.** I often fantasize about the outcome of placing mathematicians from different eras in the same room. For example, I would be intrigued to

gather Pythagoras, Cantor, and Gödel in a small room with a single blackboard to debate their various ideas on mathematics and God. What profound knowledge might we gain if we had the power to bring together great thinkers of various ages for a conference on mathematics? Would a roundtable discussion with Pythagoras, Cantor, and Gödel produce less interesting ideas than one with Newton and Einstein?

Could ancient mathematicians contribute *any* useful ideas to modern mathemati-

cians? Would a meeting of time-traveling *mathematicians* offer more to humanity than a meeting of other scientists—for example, biologists or sociologists? These are all fascinating questions to which I don’t yet have answers.

🗨️ **Mathematicians as God’s messengers.** “Cantor felt a duty to keep on, in the face of adversity, to bring the insights he had been given as God’s messenger to mathematicians everywhere” (Joseph Dauben, *Georg Cantor*, 1990).

❗ **Power notation.** In 1637, the philosopher René Descartes was the first person to use the superscript notation for raising numbers and variables to powers—for example, as in x^2 .

❓ **Numerical religion.** What ancient mathematician established a numerical religion whose main tenets included the transmigration of souls and the sinfulness of eating beans? (See Answer 1.18.)

? Modern mathematical murderer. Which modern mathematician murdered and maimed the most people from a distance? (See Answer 1.19.)

? The ∞ symbol. Most high school students are familiar with the mathematical symbol for infinity (∞). Do you think this symbol was used a hundred years ago? Who first used this odd symbol? (See Answer 1.20.)

? Mathematics of tic-tac-toe. In how many ways can you place Xs and Os on a standard tic-tac-toe board? (See Answer 1.21.)

! The square root symbol. The Austrian mathematician Christoff Rudolff was the first to use the square root symbol $\sqrt{\quad}$ in print; it was published in 1525 in *Die Coss*.

! Mathematics and poetry. “It is impossible to be a mathematician without being a poet in soul” (Sofia Kovalevskaya, quoted in *Agnesi to Zeno* by Sanderson Smith, 1996).

? Chickens and tic-tac-toe. The mathematics of tic-tac-toe have been discussed for decades, but can a chicken actually learn to play well? In 2001, an Atlantic City casino offered its patrons a tic-tac-toe “chicken challenge” and offered cash prizes of up to \$10,000. The chicken gets the first entry, usually by pecking at X or O on a video display inside a special henhouse set up in the casino’s main concourse. Gamblers standing outside the booth then get to make the next move by pressing buttons on a separate panel. There is no prize for a tie. A typical game lasts for about a minute, and the chicken seems to be trained to peck at an X or an O, depending on the human’s moves.


Supposedly, the tic-tac-toe-playing chickens work in shifts of one to two hours to avoid stressing the animals. Various animal-rights advocates have protested the use of chickens in tic-tac-toe games. Can a chicken actually learn to play tic-tac-toe? (See Answer 1.22.)


! Mathematics and God. “God exists since mathematics is consistent, and the devil exists since we cannot prove the consistency” (Morris Kline, *Mathematical Thought from Ancient to Modern Times*, 1990).


! Lunatic scribbles and mathematics. “If a lunatic scribbles a jumble of mathematical symbols it does not follow that the writing means anything merely because to the inexpert eye it is indistinguishable from higher mathematics” (Eric Temple Bell, quoted in J. R. New-


man’s *The World of Mathematics*, 1956).


! God’s perspective. “When mathematicians think about algorithms, it is usually from the God’s-eye perspective. They are interested in proving, for instance, that *there is* some algorithm with some interesting property, or that *there is no* such algorithm, and in order to prove such things you needn’t actually locate the algorithm you are talking about . . . ” (Daniel Dennett, *Darwin’s Dangerous Idea: Evolution and the Meaning of Life*, 1996).

 **Erdős contemplates death.** Once, while pondering his own death, the mathematician Paul Erdős (1913–1996) remarked, “My mother said, ‘Even you, Paul, can be in only one place at one time.’ Maybe soon I will be relieved of this disadvantage. Maybe, once I’ve left, I’ll be able to be in many places at the same time. Maybe then I’ll be able to collaborate with Archimedes and Euclid.”


 **Mathematics and the infinite.** “Mathematics is the only infinite human activity. It is conceivable that humanity could eventually learn everything in physics or biology. But humanity certainly won’t ever be able to find out everything in mathematics, because the subject is infinite. Numbers themselves are infinite” (Paul Erdős, quoted in Paul Hoffman’s *The Man Who Loved Only Numbers*, 1998).

 **First female doctorate.** Who was the first woman to receive a doctorate in mathematics, and in what century do you think she received it? (See Answer 1.23.)

 **Creativity and madness.** “Creativity and genius feed off mental turmoil. The ancient Greeks, for instance, believed in divine forms of madness that inspired mortals’ extraordinary creative acts” (Bruce Bower, *Science News*, 1995).

 **Science, Einstein, and God.** “The scientist’s religious feeling takes the form of a rapturous amazement at the

harmony of natural law, which reveals an intelligence of such superiority that, compared with it, all the systematic thinking and acting of human beings is an utterly insignificant reflection. This feeling is the guiding principle of his life and work. . . . It is beyond question closely akin to that which has possessed the religious geniuses of all ages” (Albert Einstein, *Mein Weltbild*, 1934).


 **Math in the movies.** In the movie *A Beautiful Mind*, Russell Crowe scrolls the following formulas on the blackboard in his MIT class:

$$V = \{F : \mathbb{R}^3 - X \rightarrow \mathbb{R}^3 \text{ smooth}\}$$

$$W = \{F = \nabla g\}$$

$$\dim(V/W) = ?$$

Movie directors were told by advisers that this set of formulas was subtle enough to be out of reach for most undergraduates, but accessible enough so that Jennifer Connelly’s character might be able to dream up a possible solution.

 **Mathematics and homosexuality.** Which brilliant mathematician was forced to become a human guinea pig and was subjected to drug experiments to reverse his homosexuality? (Hint: He was a 1950s computer theorist whose mandatory drug therapy made him impotent and caused his breasts to enlarge. He also helped to break the codes of the German Enigma code machines during World War II.) (See Answer 1.24.)

❗ **A famous female mathematician.** Maria Agnesi (1718–1799) is one of the most famous female mathematicians of the last few centuries and is noted for her work in differential calculus. When she was seven years old, she mastered the Latin, the Greek, and the Hebrew languages, and at age nine she published a Latin discourse defending higher education for women. As an adult, her clearly written textbooks condensed the diverse research writings and methods of a number of mathematicians. They also contained many of her own original contributions to the field, including a discussion of the cubic curve that is now known as the “Witch of Agnesi.” However, after the death of her father, she stopped doing scientific work altogether and devoted the last forty-seven years of her life to caring for sick and dying women.

🌀 **Einstein’s God.** “It was, of course, a lie what you read about my religious convictions, a lie which is being systematically repeated. I do not believe in a personal God and I have never denied this but have expressed it clearly. If something is in me which can be called religious then it is the unbounded admiration for the structure of the world so far as our science can reveal it” (Albert Einstein, personal letter to an atheist, 1954).

❓ **Mathematician cooks.** What eighteenth-century French mathematician cooked himself to death? (See Answer 1.25.)

❗ **Women and math.** Despite horrible prejudice in earlier times, several women have fought against the establishment and persevered in mathematics. Emmy Amalie Noether (1882–1935) was described by Albert Einstein as “the most significant creative mathematical genius thus far produced since the higher education of women began.” She is best known for her contributions to abstract algebra and, in particular, for her study of “chain conditions on ideals of rings.” In 1933, her mathematical achievements counted for nothing when the Nazis caused her dismissal from the University of Göttingen because she was Jewish.

🌀 **Science and religion.** “A contemporary has said, not unjustly, that in this materialistic age of ours the serious scientific workers are the only profoundly religious people” (Albert Einstein, *New York Times Magazine*, 1930).

❓ **Mathematics and money.** What effect would doubling the salary of every mathematics teacher have on education and the world at large? (See Answer 1.26.)

❗ **A famous female mathematician.** Sophie Germain (1776–1831) made major contributions to number theory, acoustics, and elasticity. At age thirteen, Sophie read an account of the death of Archimedes at the hands of a Roman soldier. She was so moved by this story that she decided to become a mathematician. Sadly, her parents felt that her interest in mathematics was inappropriate, so at night she secretly studied the works of Isaac Newton and the mathematician Leonhard Euler.

? Mad mom tortures mathematician daughter. What brilliant, famous, and beautiful woman mathematician died in incredible pain because her mother withdrew all pain medication? (Hint: The woman is recognized for her contributions to computer programming. The mother wanted her daughter to die painfully so that her daughter's soul would be cleansed.) (See Answer 1.27.)

Mathematics and relationships. “‘No one really understood music unless he was a scientist,’ her father had declared, and not just a scientist, either, oh, no, only the real ones, the theoreticians, whose language is mathematics. She had not understood mathematics until he had explained to her that it was the symbolic language of relationships. ‘And relationships,’ he had told her, ‘contained the essential meaning of life’” (Pearl S. Buck, *The Goddess Abides*, 1972).

? Mathematician pretends. What important eleventh-century mathematician pretended he was insane so that he would not be put to death? (Hint: He was born in Iraq and made contributions to mathematical optics.) (See Answer 1.28.)

Mathematical greatness. “Each generation has its few great mathematicians, and mathematics would not even notice the absence of the others. They are useful as teachers, and their research harms

no one, but it is of no importance at all. A mathematician is great or he is nothing” (Alfred Adler, “Reflections: Mathematics and Creativity,” *The New Yorker*, 1972).

Mathematics, mind, universe. “If we wish to understand the nature of the Universe we have an inner hidden advantage: we are ourselves little portions of the universe and so carry the answer within us” (Jacques Boivin, *The Single Heart Field Theory*, 1981).

Christianity and mathematics. “The good Christian should beware of mathematicians, and all those who make empty prophecies. The danger already exists that the mathematicians have made a covenant with the devil to darken the spirit and to confine man in the bonds of Hell” (St. Augustine, *De Genesi Ad Litteram*, Book II, c. 400).

? Mathematician believes in angels. What famous English mathematician had not the slightest interest in sex and was also a biblical fundamentalist, believing in the reality of angels, demons, and Satan? (Hint: According to most scholars, he is the most influential scientist and mathematician to have ever lived.) (See Answer 1.29.)

? History's most prolific mathematician. Who was the most prolific mathematician in history? (If you are unable to answer this, can you guess in what century he lived?) (See Answer 1.30.)

? Suicidal mathematician. What mathematician accepted a duel, knowing that he would die? (Hint: He spent the night before the duel feverishly writing down his mathematical ideas, which have since had a great impact on mathematics.) (See Answer 1.31.)

? Marry a mathematician? Would you rather marry the best mathematician in the world or the best chess player? (See Answer 1.32.)

Mathematics and truth. “We who are heirs to three recent centuries of scientific development can hardly imagine a state of mind in which many mathematical objects were regarded as symbols of spiri-

tual Truth” (Philip Davis and Reuben Hersh, *The Mathematical Experience*, 1981).

Mathematics and lust. “I tell them that if they will occupy themselves with the study of mathematics they will find in it the best remedy against the lusts of the flesh” (Thomas Mann, *The Magic Mountain*, 1924).

Newton’s magic. “Had Newton not been steeped in alchemical and other magical learning, he would never have proposed forces of attraction and repulsion between bodies as the major feature of his physical system” (John Henry, “Newton, Matter, and Magic,” in John Fauvel’s *Let Newton Be!*, 1988).

! Earliest known symbols. The Egyptian Rhind Papyrus (c. 1650 B.C.) contains the earliest known symbols for mathematical operations. “Plus” is denoted by a pair of legs walking toward the number to be added.

Mathematics and the divine. “Mathematical inquiry lifts the human mind into closer proximity with the divine than is attainable through any other medium” (Hermann Weyl, quoted in Philip Davis and Reuben Hersh, *The Mathematical Experience*, 1981).

! π and the law. In 1896, an Indiana physician promoted a legislative bill that made π equal to 3.2, exactly. The Indiana House of Representatives approved the bill unanimously, 67 to 0. The Senate, however, deferred debate about the bill “until a later date.”

The mathematical life. “The mathematical life of a mathematician is short. Work rarely improves after the age of twenty-five or thirty. If little has been accomplished by

? Mathematical corpse. You come home and see a corpse on your foyer floor. Would you be more frightened if (1) scrawled on the floor is the Pythagorean theorem: $a^2 + b^2 = c^2$, or (2) scrawled on the floor is the following complicated formula:

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k!)(1103 + 26390k)}{(k!)^4 396^{4k}}$$

(See Answer 1.33.)

then, little will ever be accomplished” (Alfred Adler, “Mathematics and Creativity,” *The New Yorker*, 1972).

❗ **The genesis of $x^0 = 1$.**

Ibn Yahya al-Maghribi Al-Samawal in 1175 was the first to publish

$$x^0 = 1$$

In other words, he realized and published the idea that any number raised to the power of 0 is 1. Al-Samawal’s book was titled *The Dazzling*. His father was a Jewish scholar of religion and literature from Baghdad.

❗ **Euler’s one-step proof of**

God’s existence. In mathematical books too numerous to mention, we have heard the story of the mathematician Leonhard Euler’s encounter with the French encyclopedist Denis Diderot. Diderot was a devout atheist, and he challenged the religious Euler to mathematically prove the existence of God. Euler replied, “Sir $(a + b^n)/n = x$; hence, God exists. Please reply!”

Supposedly, Euler said this in a public debate in St.

❓ **Blind date.** You are single and going on a blind date. The date knocks on your door and is extremely attractive. However, you note that the person has the following formulas tattooed on the right arm:

$$\frac{dR}{dt} = -k_B B(t), R(0) = R_0$$

$$\frac{dB}{dt} = -k_R R(t), B(0) = B_0$$

From this little information, do you think you would enjoy the evening with this person? (See Answer 1.34.)

❓ **Fundamental Anagram of Calculus.** Probably most of you have never heard of Newton’s “Fundamental Anagram of Calculus”:


6accdae13eff7i3l9n4o4qrr4s8t12ux


Can you think of any possible reason Newton would want to code aspects of his calculus discoveries? (See Answer 1.35.)


Petersburg and embarrassed the freethinking Diderot with this simple algebraic proof of God’s existence. Diderot was shocked and fled. Was Euler deliberately demonstrating how lame these kinds of arguments can be?


Today we know that there is little evidence that the encounter ever took place. Dirk J. Struik, in his book *A Concise History of Mathematics*, third revised edition (New York: Dover, 1967, p.

129), says that Diderot was mathematically well versed and wouldn’t have been shocked by the formula. Moreover, Euler wasn’t the type of person to make such a zany comment. While people of the time did seek simple mathematical proofs of God, the “Euler versus Diderot” story was probably fabricated by the English mathematician De Morgan (1806–1871).

 **Science and religion.** “I have always thought it curious that, while most scientists claim to eschew religion, it actually dominates their thoughts more than it does the clergy” (Fred Hoyle, astrophysicist, “The Universe: Past and Present Reflections,” *Annual Review of Astronomy and Astrophysics*, 1982).

 **Parallel universes and mathematics.** In theory, it is possible to list or “enumerate” all rational numbers. How has this mathematical fact helped certain cosmologists to “prove” that there is an infinite number of universes alongside our own? (As you will learn in the next chapter, “rational numbers” are numbers like $\frac{1}{2}$, which can be expressed as fractions.) (See Answer 1.36.)

 **The mysterious 0^0 .** Students are taught that any number to the zero power is 1, and zero to any power is 0. But serious mathematicians often consider 0^0 undefined. If you try to make a graph of x^y , you’ll see it has a discontinuity at the point (0,0). The

 **π savants.** In 1844, Johann Dase (a.k.a., Zacharias Dahse) computed π to 200 decimal places in less than two months. He was said to be a calculating prodigy (or an “idiot savant”), hired for the task by the Hamburg Academy of Sciences on Gauss’s recommendation. To compute


$\pi = 3.14159\ 26535\ 89793\ 23846\ 26433\ 83279$
 $50288\ 41971\ 69399\ 37510\ 58209\ 74944\ 59230$
 $78164\ 06286\ 20899\ 86280\ 34825\ 34211\ 70679$
 $82148\ 08651\ 32823\ 06647\ 09384\ 46095\ 50582$
 $23172\ 53594\ 08128\ 48111\ 74502\ 84102\ 70193$
 $85211\ 05559\ 64462\ 29489\ 54930\ 38196$


Dase supposedly used $\pi/4 = \arctan(1/2) + \arctan(1/5) + \arctan(1/8) \dots$ with a series expansion for each arctangent. Dase ran the arctangent job in his brain for nearly sixty days.

Not everyone believes the legend of Dase. For example, Arthur C. Clarke recently wrote to me that he simply doesn’t believe the story of Dase calculating pi to 200 places in his head. Clarke says, “Even though I’ve seen fairly well authenticated reports of other incredible feats of mental calculation, I think this is totally beyond credibility.”

I would be interested in hearing from readers who can confirm or deny this story.

discussion of the value of 0^0 is very old, and controversy raged throughout the nineteenth century.

 **One-page proof of God’s existence.** Which famous German mathematician “proved” God’s existence in a proof that fit on just one page of paper? (See Answer 1.37.)

 **Greek numerals.** Did you know that the ancient Greeks had two systems of numerals? The earlier of these was based on the initial letters of the names of numbers: the number 5 was indicated by the letter pi; 10 by the letter delta; 100 by the antique form of the letter H; 1,000 by the letter chi; and 10,000 by the letter mu.

🌀 Our mathematical perceptions. “The three of you stare at the school of fish and watch them move in synchrony, despite their lack of eyes. The resulting patterns are hypnotic, like the reflections from a hundred pieces of broken glass. You imagine that the senses place a filter on how much humans can perceive of the mathematical fabric of the universe. If the universe is a mathematical carpet, then all creatures are looking at it through imperfect glasses. How might humanity perfect those glasses? Through drugs, surgery, or electrical stimulation of the brain? Probably our best chance is through the use of computers” (Cliff Pickover, *The Loom of God*, 1997).


❗ Pierre de Fermat. In the early 1600s, Pierre de Fermat, a French lawyer, made brilliant discoveries in number theory. Although he was an “amateur” mathematician, he created mathematical challenges such as “Fermat’s Last Theorem,” which was not solved until 1994. Fermat’s Last Theorem states that $x^n + y^n = z^n$ has no nonzero integer solutions for x , y , and z when $n > 2$.


Fermat was no ordinary lawyer indeed. He is considered, along with Blaise Pascal, a founder of probability theory. As the coinventor of analytic geometry, he is considered, along with René Descartes, one of the first modern mathematicians.


❗ The Beal reward. In the mid-1990s, the Texas banker Andrew Beal posed a perplexing mathematical problem and offered \$5,000 for the solution of this problem. In particular, Beal was curious about the equation $A^x + B^y = C^z$. The six letters represent integers, with x , y , and z greater than 2. (Fermat’s Last Theorem involves the special case in which the exponents x , y , and z are the same.) Oddly enough, Beal noticed that for any solution of this general equation he could find, A , B , and C have a common factor. For example, in the equation $3^6 + 18^3 = 3^8$, the numbers 3, 18, and 3 all have the factor 3. Using computers at his bank, Beal checked equations with exponents up to 100 but could not discover a solution that didn’t involve a common factor.

❗ Ancient number notation lets humans “think big.” The earliest forms of number notation, which used straight lines for grouping 1s, were inconvenient when dealing with large numbers. By 3400 B.C. in Egypt, and 3000 B.C. in Mesopotamia, a special symbol was adopted for the number 10. The addition of this second number symbol made it possible to express the number 11 with 2 symbols instead of 11, and the number 99 with 18 symbols instead of 99.


❗ Caterpillar vehicle. Many mathematicians were creative inventors, although not all of their inventions were practical. For example, Polish-born Josef Hoëné-Wronski (1778–1853), an analyst, a philosopher, a combinatorialist, and a physicist, developed a fantastical design for caterpillar-like vehicles that he intended to replace railroad transportation. He also attempted to build a perpetual motion machine and to build a machine to predict the future (which he called the *prognometre*).


 **Progress in mathematics.** “In most sciences, one generation tears down what another has built and what one has established another undoes. In mathematics alone, each generation adds a new story to the old structure” (Hermann Hankel, 1839–1873, who contributed to the theory of functions, complex numbers, and the history of mathematics, quoted in Desmond MacHale, *Comic Sections*, 1993).

 **Hilbert’s problems.** In 1900, the mathematician David Hilbert submitted twenty-three important mathematical problems to be targeted for solution in the twentieth century. These twenty-three problems extend over all fields of mathematics. Because of Hilbert’s prestige, mathematicians spent a great deal of time tackling the problems, and many of the problems have been solved. Some, however, have been solved only very recently, and still others continue to daunt us. Hilbert’s twenty-three wonderful problems were designed to lead to the furthering of various disciplines in mathematics.

 **God’s math book.** “God has a transfinite book with all the theorems and their best proofs. You don’t really have to believe in God as long as you believe in the book” (Paul Erdős, quoted in Bruce Schechter, *My Brain Is Open*:

The Mathematical Journeys of Paul Erdős, 1998).

 **Song lyrics.** What is the largest number ever used in the lyrics to a popular song? (See Answer 1.38.)

 **The magnificent Erdős.** It is commonly agreed that Paul Erdős is the second-most prolific mathematician of all times, being surpassed only by Leonhard Euler, the great eighteenth-century mathematician whose name is spoken with awe in mathematical circles. In addition to Erdős’s roughly 1,500 published papers, many are yet to be published after his death. Erdős was still publishing a paper a week in his seventies. Erdős undoubtedly had the greatest number of coauthors (around 500) among mathematicians of all times.

In 2004, an eBay auction offered buyers an opportunity to link their names, through five degrees of separation, with Paul Erdős. In particular, the mathematician William Tozier presented bidders with the chance to collaborate on a research paper. Tozier was linked to Erdős through a string of coauthors. In particular, he had collaborated with someone who had collaborated with someone who had collaborated with someone who had collaborated with Paul Erdős. A mathematician who has published a paper with Erdős has an Erdős number of 1. A mathematician who has published a paper with someone who has published a paper with Erdős has an Erdős number of 2, and so on. Tozier has an Erdős number of 4, quite a respectable ranking in the mathematical community. This means that the person working with Tozier would have an Erdős number of 5. During the auction, Tozier heard from more than a hundred would-be researchers. (For more information, see Erica Klarreich, “Theorems for Sale: An Online Auctioneer Offers Math Amateurs a Backdoor to Prestige,” *Science News* 165, no. 24 (2004): 376–77.)

❗ **Rope and lotus symbols.** The Egyptian hieroglyphic system evolved special symbols (resembling ropes, lotus plants, etc.) for the numbers 10, 100, 1,000, and 10,000.

❗ **Strange math title.** I love collecting math papers with strange titles. These papers are published in serious math journals. For example, in 1992, A. Granville published an article with the strange title “Zaphod Beeblebrox’s Brain and the Fifty-Ninth Row of Pascal’s Triangle,” in the prestigious *The American Mathematical Monthly* (vol. 99, no. 4 [April]: 318–31).

❗ **Urantia religion and numbers.** In the modern-day Urantia religion, numbers have an almost divine quality. According to the sect, headquartered in Chicago, we live on the 606th planet in a system called Satania, which includes 619 flawed but evolving worlds. Urantia’s grand universe number is 5,342,482,337,666. Urantians believe that human minds are created at birth, but the soul does not develop until about age six. They also believe

❗ **A calculating prodigy and 365,365,365,365,365,365.** When he was ten years old, the calculating prodigy Truman Henry Safford (1836–1901) of Royalton, Vermont, was once asked to square, in his head, the number 365,365,365,365,365,365. His church leader reports, “He flew around the room like a top, pulled his pantaloons over the tops of his boots, bit his hands, rolled his eyes in their sockets, sometimes smiling and talking, and then seeming to be in agony, until in not more than a minute said he, 133,491,850,208,566,925,016,658,299,941, 583,225!”


Incidentally, I had first mentioned this large number in my book *Wonders of Numbers* and had misprinted one of the digits. Bobby Jacobs, a ten-year-old math whiz from Virginia, wrote to me with the corrected version that you see here. He was the only person to have discovered my earlier typographical error.


that when we die, our souls survive. Incidentally, Jesus Christ is number 611,121 among more than 700,000 Creator Sons.


🌀 **Mathematics and God.** “Philosophers and great religious thinkers of the last century saw evidence of God in the symmetries and harmonies around them—in the beautiful equations of classical physics that describe such phenomena as electricity and magnetism. I don’t see the simple patterns underlying nature’s complexity as evidence of God. I believe *that* is God. To behold

[mathematical curves], spinning to their own music, is a wondrous, spiritual event” (Paul Rapp, in Kathleen McAuliffe, “Get Smart: Controlling Chaos,” *Omni* [1989]).

❗ **The discovery of calculus.** The English mathematician Isaac Newton (1642–1727) and the German mathematician Gottfried Wilhelm Leibniz (1646–1716) are generally credited with the invention of calculus, but various earlier mathematicians explored the concept of rates and limits, starting with the ancient Egyptians, who

 **Newton's giants.** “If I have seen further than others, it is by standing upon the shoulders of giants” (Isaac Newton, personal letter to Robert Hooker, 1675 [see next quotation]).


 **Abelson's giants.** “If I have not seen as far as others, it is because giants were standing on my shoulders” (Hal Abelson, MIT professor).

 **Mathematical marvel.** “Even stranger things have happened; and perhaps the strangest of all is the marvel that mathematics should be possible to a race akin to the apes” (Eric T. Bell, *The Development of Mathematics*, 1945).


developed rules for calculating the volume of pyramids and approximating the areas of circles.


In the 1600s, both Newton and Leibniz puzzled over problems of tangents, rates of change, minima, maxima, and infinitesimals (unimaginably tiny quantities that are almost but not quite zero). Both men understood that differentiation (finding tangents to curves) and integration (finding areas under curves) are inverse processes. Newton's discovery (1665–1666) started with his interest in infinite sums; however, he was slow to publish his findings. Leibniz published his discovery of differential calculus in 1684 and of


integral calculus in 1686. He said, “It is unworthy of excellent men, to lose hours like slaves in the labor of calculation. . . . My new calculus . . . offers truth by a kind of analysis and without any effort of imagination.” Newton was outraged. Debates raged for many years on how to divide the credit for the discovery of calculus, and, as a result, progress in calculus was delayed.

 **The notations of calculus.** Today we use Leibniz's symbols in calculus, such as $\frac{df}{dx}$ for the derivative and the \int symbol for integration. (This integral symbol was actually a long letter *S* for *summa*, the

Latin word for “sum.”) The mathematician Joseph Louis Lagrange (1736–1813) was the first person to use the notation $f'(x)$ for the first derivative and $f''(x)$ for the second derivative. In 1696, Guillaume François de L'Hôpital, a French mathematician, published the first textbook on calculus.

 **Jews, π , the movie.** The 1998 cult movie titled π stars a mathematical genius who is fascinated by numbers and their role in the cosmos, the stock market, and Jewish mysticism. According to the movie, what is God's number? (See Answer 1.39.)

 **Mathematics and romance.** What romantic comedy has the most complicated mathematics ever portrayed in a movie? (See Answer 1.40.)

 **Greek death.** Why did the ancient Greeks and other cultures believe 8 to be a symbol of death? (See Answer 1.41.)

❗ **The mechanical Pascaline.** One example of an early computing machine is Blaise Pascal's wheel computer called a Pascaline. In 1644, this French philosopher and mathematician built a calculating machine to help his father compute business accounts. Pascal was twenty years old at the time. The machine used a series of spinning numbered wheels to add large numbers.

The wonderful Pascaline was about the size of a shoe-box. About fifty models were made.

❓ **The Matrix.** What number is on Agent Smith's license plate in the movie *The Matrix Reloaded*? Why? (See Answer 1.42.)

❓ **At the movies.** What famous book and movie title contains a number that is greater than 18,000 and less than 38,000? (See Answer 1.43.)

🌀 **The mathematical life.** "The mathematician lives long and lives young; the wings of the soul do not early drop off, nor do its

pores become clogged with the earthly particles blown from the dusty highways of vulgar life" (James Joseph Sylvester, 1814–1897, a professor of mathematics at Johns Hopkins University, 1869 address to the British Mathematical Association).

🌀 **Mathematical progress.** "More significant mathematical work has been done in the latter half of this century than in all previous centuries combined" (John Casti, *Five Golden Rules*, 1997).

❗ **Simultaneity in science.** The simultaneous discovery of calculus by Newton and Leibniz makes me wonder why so many discoveries in science were made at the same time by people working independently. For example, Charles Darwin (1809–1882) and Alfred Wallace (1823–1913) both independently developed the theory of evolution. In fact, in 1858, Darwin announced his theory in a paper presented at the same time as a paper by Wallace, a naturalist who had also developed the theory of natural selection.

As another example of simultaneity, the mathematicians János Bolyai (1802–1860) and Nikolai Lobachevsky (1793–1856) developed hyperbolic geometry independently and at the same time (both perhaps stimulated indirectly by Carl Friedrich Gauss). Most likely, such simultaneous discoveries have occurred because the time was "ripe" for such discoveries, given humanity's accumulated knowledge at the time the discoveries were made. On the other hand, mystics have suggested that there is a deeper meaning to such coincidences. The Austrian biologist Paul Kammerer (1880–1926) wrote, "We thus arrive at the image of a world-mosaic or cosmic kaleidoscope, which, in spite of constant shufflings and rearrangements, also takes care of bringing like and like together." He compared events in our world to the tops of ocean waves that seem isolated and unrelated. According to his controversial theory, we notice the tops of the waves, but beneath the surface there may be some kind of synchronistic mechanism that mysteriously connects events in our world and causes them to cluster.

? First mathematician. What is the name of the first human who was identified as having made a contribution to mathematics? (See Answer 1.44.)

? Game show. Why was the 1950's TV game show called *The \$64,000 Question*? Why not a rounder number like \$50,000? (See Answer 1.45.)

☯ God and the infinite. "Such as say that things infinite are past God's knowledge may just as well leap headlong into this pit of impiety, and

say that God knows not all numbers. . . . What madman would say so? . . . What are we mean wretches that dare presume to limit His knowledge?" (St. Augustine, *The City of God*, A.D. 412).

! Who was Pythagoras?

You can tell from some of the following ! factoids that I love trivia that relates to the famous ancient Greek mathematician Pythagoras. His ideas continue to thrive after three millennia of mathematical science. The philosopher Bertrand Russell once wrote

☯ Mathematical scope. "Mathematics is not a book confined within a cover and bound between brazen clasps, whose contents it needs only patience to ransack; it is not a mine, whose treasures may take long to reduce into possession, but which fill only a limited number of veins and lodes; it is not a soil, whose fertility can be exhausted by the yield of successive harvests; it is not a continent or an ocean, whose area can be mapped out and its contour defined: it is limitless as that space which it finds too narrow for its aspirations; its possibilities are as infinite as the worlds which are forever crowding in and multiplying upon the astronomer's gaze; it is as incapable of being restricted within assigned boundaries or being reduced to definitions of permanent validity, as the consciousness, the life, which seems to slumber in each monad, in every atom of matter, in each leaf and bud and cell, and is forever ready to burst forth into new forms of vegetable and animal existence" (James Joseph Sylvester, *The Collected Mathematical Papers of James Joseph Sylvester*, Volume III, address on Commemoration Day at Johns Hopkins University, February 22, 1877).

that Pythagoras was intellectually one of the most important men who ever lived, both when he was wise and when he was unwise. Pythagoras was the most puzzling mathematician of history because he founded a numerical religion whose main tenets were the transmigration of souls and the sinfulness of eating beans, along with a host of other odd rules and regulations. To the Pythagoreans, mathematics was an ecstatic revelation.

The Pythagoreans, like modern day fractalists, were akin to musicians. They created pattern and beauty as they discovered mathematical truths. Mathematical and theological blending began with Pythagoras and eventually affected all religious philosophy in Greece, played a role in religion of the Middle Ages, and extended to Kant in modern times. Bertrand Russell felt that if it were not for Pythagoras, theologians would not have sought logical proofs of God and immortality.

If you want to read more about Pythagoras, see my book *The Loom of God* and Peter Gorman's *Pythagoras: A Life*.

❗ **The secret life of numbers.** To Pythagoras and his followers, numbers were like gods, pure and free from material change. The worship of numbers 1 through 10 was a kind of polytheism for the Pythagoreans.

Pythagoreans believed that numbers were alive, independent of humans, but with a telepathic form of consciousness. Humans could relinquish their three-dimensional lives and telepathize with these number beings by using various forms of meditation. Meditation upon numbers was communing with the gods, gods who desired nothing from humans but their sincere admiration and contemplation. Meditation upon numbers was a form of prayer that did not ask any favors from the gods.

These kinds of thoughts are not foreign to modern mathematicians, who often debate whether mathematics is a creation of the human mind or is out there in the universe, independent of human thought. Opinions vary. A few mathematicians believe that mathematics is a form of human logic that is not necessarily valid in all parts of the universe.

❗ **Anamnesis and the number 216.** Was the ancient Greek mathematician Pythagoras once a plant? This is a seemingly bizarre question, but Pythagoras claimed that he had been both a plant and an animal in his past lives, and, like Saint Francis, he preached to animals. Pythagoras and his followers believed in *anamnesis*, the recollection of one's previous incarnations. During Pythagoras's time, most philosophers believed that only men could be happy. Pythagoras, on the other hand, believed in the happiness of plants, animals, and women.

In various ancient Greek writings, we are told the exact number of years between each of Pythagoras's incarnations: 216. Interestingly, Pythagoreans considered 216 to be a mystical number, because it is 6 cubed ($6 \times 6 \times 6$). Six was also considered a "circular number" because its powers always ended in 6. The fetus was considered to have been formed after 216 days.

The number 216 continues to pop up in the most unlikely of places in theological literature. In an obscure passage from *The Republic* (viii, 546 B–D), Plato notes that $216 = 6^3$. It is also associated with auspicious signs on the Buddha's footprint.

❗ **Shades of ghosts.** Pythagoras believed that even rocks possess a psychic existence. Mountains rose from the earth because of growing pains of the earth, and Pythagoras told his followers that earthquakes were caused by the shades of ghosts of the dead, which created disturbances beneath the earth.

❗ **Pythagorean sacrifice.** Although some historians report that Pythagoras joy-

fully sacrificed a hecatomb of oxen (a hundred animals) when he discovered his famous theorem about the right-angled triangle, this would have been scandalously un-Pythagorean and is probably not true. Pythagoras refused to sacrifice animals. Instead, the Pythagoreans believed in the theurgic construction of agalmata—statues of gods consisting of herbs, incense, and metals to attract the cosmic forces.

! Pythagoras and aliens. UFOs and extraterrestrial life are hot topics today. But who would believe that these same ideas enthralled the Pythagoreans a few millennia ago? In fact, Pythagoreans believed that all the planets in the solar system are inhabited, and humans dwelling on Earth were less advanced than these other inhabitants were. (The idea of advanced extraterrestrial neighbors curiously has continued, for some, to this day.) According to later Pythagoreans, as one travels farther from Earth, the beings on other planets and in other solar systems become less flawed.

Pythagoras went as far as to suggest that disembodied intelligences existed in the universe. These mind-creatures had very tenuous physical bodies. Many of Pythagoras's followers believed that Pythagoras himself had once been a superior being who inhabited the Moon or the Sun.

☉ Gods and sets. "The null set is also a set; the absence of a god is also a god" (A. Moreira).

? Famous epitaphs. Replace x and y with the names of two famous mathematicians in these two puzzles. In Puzzle 1, the following couplet of Alexander Pope is the engraved epitaph on the mystery person's sarcophagus in Westminster Abbey, in London:

Nature and Nature's laws lay hid in the night;
God said, "Let x be" and all was light.

—Alexander Pope (1688–1744)

In Puzzle 2, the epitaph " $S = k \ln W$ " is engraved on y 's tombstone. Who are x and y ? (See Answer 1.46.)

! Matrix prayers. Some of you will be interested in Underwood Dudley's *Mathematical Cranks* (Washington, D.C.: Mathematical Association of America, 1992). The book contains musings of slightly mad mathematicians. My favorite chapter is on the topic of "Matrix Prayers," designed by a priest of the Church of England. The priest regularly prayed to God in mathematical terms using matrices, and he taught the children in his church to pray and think of God in matrices. *Mathematical Cranks* goes into great detail regarding "revelation matrices," "Polite Request Operators," and the like. The priest finally derives a beautiful

prayer that he succinctly writes as

$$P < R\{S\} \rightarrow \{U\}, r > 0$$

He says, "This prayer should be sufficiently concise to be acceptable to Christ, yet every single Christian inhabitant of Northern Ireland has been separately included." The chapter concludes with information regarding the geometry of heaven.

? Secret mathematician. Who really was the famous and secretive French mathematician "N. Bourbaki"? (See Answer 1.47.)

? How much? How much mathematics can we know? (See Answer 1.48.)

! Mathematics and reality. From string theory to quantum theory, the deeper one goes in the study of physics, the closer one gets to pure mathematics. Mathematics is the fabric of reality. Some might even say that mathematics “runs” reality in the same way that Microsoft’s Windows runs your computer and shapes your interactions with the vast network beyond. Schrödinger’s wave equation—which describes basic reality and events in terms of wave functions and probabilities—is the evanescent substrate on which we all exist:

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}) \right] \psi(\vec{r}, t) = i\hbar \frac{\partial \psi}{\partial t}(\vec{r}, t)$$

Freeman Dyson, in the introduction to *Nature’s Imagination*, speaks highly of this formula: “Sometimes the understanding of a whole field of science is suddenly advanced by the discovery of a single basic equation. Thus it happened that the Schrödinger equation in 1926 and the Dirac equation in 1927 brought a miraculous order into the previously mysterious processes of atomic physics. Bewildering complexities of chemistry and physics were reduced to two lines of algebraic symbols.”

! Factorial symbol. In 1808 Christian Kramp (1760–1826) introduced the “!” as the factorial symbol as a convenience to the printer.

? Nobel Prize. Why is there no Nobel Prize for mathematics? (See Answer 1.49.)

? Roman numerals. Why don’t we use Roman numerals anymore? (See Answer 1.50.)

? Insights and analysis. “Einstein’s fundamental insights of space/matter relations came out of philosophical musings about the nature of the universe, not from rational analysis of observational data—the logical analysis, prediction, and testing coming only after the formation of the creative hypotheses” (R. H. Davis, *The Skeptical Inquirer*, 1995).

? Einstein on mathematics and reality. “At this point an enigma presents itself which in all ages has agitated inquiring minds. How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality? Is human reason, then, without experience, merely by taking thought, able to fathom the properties of real things? In my opinion the answer to this question is briefly this: As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality” (Albert Einstein’s address to the Prussian Academy of Science in Berlin, 1921).

! Ramanujan’s tongue. In the spring of 2003, the Aurora Theater Company of Berkeley performed Ira Hauptman’s play *Partition*, which focuses on the collaboration of the mathematicians Ramanujan and Hardy. In the play, Namagiri, Ramanujan’s personal deity and inspiration in real life, is seen literally writing equations on Ramanujan’s tongue with her finger.

Mathematics and reality.

“My complete answer to the late 19th century question ‘what is electrodynamics trying to tell us’ would simply be this: *Fields in empty space have physical reality; the medium that supports them does not.* Having thus removed the mystery from electrodynamics, let me immediately do the same for quantum mechanics: *Correlations have physical reality; that which they correlate does not*” (N. David Mermin, “What Is Quantum Mechanics Trying to Tell Us?” *American Journal of Physics* 66, [1998]: 753–67).

More mathematics and

reality. “[Much of frontier mathematics] confounds even mathematicians and physicists, as they use math to calculate the inconceivable, undetectable, nonexistent and impossible. So what does it mean when mainstream explanations of our physical reality are based on stuff that even scientists cannot comprehend? When nonscientists read about the strings and branes of the latest physics theories, or the Riemann surfaces and Galois fields

of higher mathematics, how close are we to a real understanding?” (Susan Kruglinski, “When Even Mathematicians Don’t Understand the Math,” *New York Times*, May 25, 2004).

Dyson on the infinite reservoir of mathematics.

“Gödel proved that the world of pure mathematics is inexhaustible; no finite set of axioms and rules of inference can ever encompass the whole of mathematics; given any finite set of axioms, we can find meaningful mathematical questions which the axioms leave unanswered. I hope that an analogous situation exists in the physical world. If my view of the future is correct, it means that the world of physics and astronomy is also inexhaustible; no matter how far we go into the future, there will always be new things happening, new infor-

mation coming in, new worlds to explore, a constantly expanding domain of life, consciousness, and memory” (Freeman Dyson, “Time without End: Physics and Biology in an Open Universe,” *Reviews of Modern Physics*, 1979).

More mathematics and

reality. “It is difficult to explain what math is, let alone what it says. Math may be seen as the vigorous structure supporting the physical world or as a human idea in development. [Dr. John Casti says that] ‘the criteria that mathematicians use for what constitutes good versus bad mathematics is much more close to that of a poet or a sculptor or a musician than it is to a chemist’” (Susan Kruglinski, “When Even Mathematicians Don’t Understand the Math,” *New York Times*, May 25, 2004).

Creativity and travel.

“One way of goosing the brain is traveling, particularly internationally. It helps shake up perspective and offers new experiences. Interviews with 40 MacArthur ‘genius’ award winners found 10 lived overseas permanently or temporarily, three traveled at least a few months each year, and at least two have a ‘horror of a home’” (Sharon McDonnell, “Innovation Electrified,” *American Way*).

❗ **Ramanujan redux.** How would mathematics have been advanced if Ramanujan had developed in a more nurturing early environment? Although he would have been a better-trained mathematician, would he have become such a unique thinker? Could he have discovered so many wonderful formulas if he had been taught the rules of mathematics early on and pushed to publish his results with rigorous proofs? Perhaps his relative isolation and poverty enhanced the greatness of his mathematical thought. For Ramanujan, equations were not just the means for proofs or calculations. The *beauty* of the equation was of paramount value for Ramanujan.

🌀 **The secret life of formulas.** “We cannot help but think that mathematical formulae have a life of their own, that they know more than their discoverers do and that they return more to us than we have invested in them” (Heinrich Hertz, German physicist, quoted in Eric Bell, *Men of Mathematics*, 1937).

❗ **The Fractal Murders.** In 2002, Mark Cohen, a lawyer and a judge, published *The Fractal Murders*, a novel in which three mathematicians, all of whom are experts in fractals, have died. Two were murdered. The third was an apparent suicide. In the novel, the math professor Jayne Smyers hires a private eye, Pepper Keane, to look into the three deaths, which seem to be related only because each victim was researching fractals.


❓ **Mathematical universe.** Why does the universe seem to operate according to mathematical laws? (See Answer 1.51.)

❓ **A universe of blind mathematicians.** Sighted mathematicians generally work by studying vast assemblages of numbers and symbols scribbled on paper. It would seem extremely difficult to do mathematics without being able to see, and to be forced to keep the information “all in one’s head.” Can a great mathematician be totally blind? (See Answer 1.52.)


🌀 **Einstein on comprehensibility and reality.** “The very fact that the totality of our sense experiences is such that by means of thinking . . . it can be put in order, this fact is one which leaves us in awe, but which we shall never

understand. One may say ‘the eternal mystery of the world is its comprehensibility.’ It is one of the great realizations of Immanuel Kant that this setting up of a real external world would be senseless without this comprehensibility” (Albert Einstein, “Physics and Reality,” 1936).

🌀 **Wigner on mathematics and reality.** “The miracle of appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it, and hope that it will remain valid for future research, and that it will extend, for better or for worse, to our pleasure even though perhaps also to our bafflement, to wide branches of learning” (Eugene Wigner, “The Unreasonable Effectiveness of Mathematics,” 1960).


 **Close to reality.** We've talked quite a bit about mathematics and reality. Who do you think is in more direct contact with reality, a mathematician or a physicist? What do famous twentieth-century mathematicians say on this subject? (See Answer 1.53.)


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 **All reality is mathematics.** "The Gedemondan chuckles. 'We read probabilities. You see, we see—perceive is a

better word—the math of the Well of Souls. We feel the energy flow, the ties and bands, in each and every particle of matter and energy. All reality is mathematics, all existence—past, present, and future—is equations'" (Jack Chalker, *Quest for the Well of Souls*, 1985).

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 **Pythagoras on mathematics and reality.** "All things are numbers."

 **Why learn mathematics?**

It has been estimated that much greater than 99.99 percent of all Americans will never use the quadratic formula or most of the other algebraic or geometrical relations they learn in school. Why teach or learn mathematics beyond the basic operations of addition, subtraction, multiplication, and division? (See Answer 1.54.)

