

FUNDAMENTALS

COPYRIGHTED MATERIAL

FLUID PROPERTIES

1.1 INTRODUCTION

This text addresses the science and technology of controllably transmitting power using a pressurized fluid. As such, it is entirely appropriate to begin by considering the fundamental properties that characterize fluids that are typically used within hydraulic control systems. In this chapter the fluid properties that describe the condition of a liquid will be presented. Since imparting velocity to a pressurized fluid is the means for transmitting power hydraulically, it is important to consider the physical mechanisms that describe the pressurization of the fluid and the fluid's resistance to flow. In the presentation that follows, the mass density of a liquid will be discussed with the bulk coefficients that characterize the equation of state. These coefficients are expanded on in subsequent sections dealing with the fluid bulk modulus of elasticity and the coefficient of thermal expansion. Another important fluid property of consideration is viscosity. This parameter is discussed because it provides insight into the impedance of fluid flow and the ultimate generation of heat associated with shearing hydraulic fluid. Other fluid property topics of chemical composition, thermal conductivity, and fluid vapor pressure are discussed. This chapter concludes with a summary of fluid types and offers practical suggestions for the selection of various hydraulic fluids.

1.2 FLUID MASS DENSITY

1.2.1 Equation of State

The equation of state for any substance is used to describe the mass density of that substance as it varies with exposed conditions such as pressure and

temperature. For many substances, the equation of state is extremely complex and difficult to describe exactly. An example of a relatively simple equation of state is that of an ideal gas. In this case, the *ideal gas law* is used to relate the pressure and temperature of the gas to the mass density using a determined gas constant. This result is given by

$$\rho = \frac{P}{RT}, \quad (1.1)$$

where ρ is the mass density of the gas, T is the absolute temperature of the gas (expressed in Kelvin or Rankine), P is the absolute gas pressure, and R is the *universal gas constant* that has been determined for the specific gas in question. Unfortunately, for the case of liquids, the equation of state is not so simple. However, since liquids are fairly incompressible, it may be assumed that the mass density of a liquid will not change significantly with the exposed conditions of temperature and pressure. In this case, a first-order *Taylor series* approximation may be written to describe the small variations in density that occur due to changes in pressure and temperature.* This result is given by

$$\rho = \rho_o + \left. \frac{\partial \rho}{\partial P} \right|_o (P - P_o) + \left. \frac{\partial \rho}{\partial T} \right|_o (T - T_o), \quad (1.2)$$

where ρ_o , P_o , and T_o represent a reference density, pressure, and temperature, respectively. The equation of state may be more meaningfully expressed by making the following definitions:

$$\frac{1}{\beta} \equiv \frac{1}{\rho} \frac{\partial \rho}{\partial P} \quad \text{and} \quad \alpha \equiv -\frac{1}{\rho} \frac{\partial \rho}{\partial T}, \quad (1.3)$$

where β is the *isothermal fluid bulk modulus* and α is the *isobar fluid coefficient of thermal expansion*. Using Equation (1.3) with Equation (1.2) yields the following equation of state for a liquid:

$$\rho = \rho_o \left[1 + \frac{1}{\beta_o} (P - P_o) - \alpha_o (T - T_o) \right]. \quad (1.4)$$

If the nearly incompressible assumption of this equation is correct, one may infer that the fluid bulk modulus β is large and that the thermal coefficient of expansion α is small. Indeed, this is the case for hydraulic fluids, as will be shown in subsequent sections of this chapter.

*The Taylor series is discussed in Section 3.3.2 of Chapter 3.

1.2.2 Density-Volume Relationship

To evaluate the relationship between fluid mass density and fluid volume, consider a fluid element of mass M . This mass may be described as

$$M = \rho V, \quad (1.5)$$

where ρ is the fluid mass density, and V is the volume of the fluid element. An infinitesimal change in this mass would be described by

$$dM = \rho dV + V d\rho. \quad (1.6)$$

Since the mass itself cannot be diminished or increased, the left-hand-side of Equation (1.6) must be zero. Therefore, the following differential relationship between the fluid density and the fluid volume may be expressed:

$$-\frac{1}{\rho} d\rho = \frac{1}{V} dV. \quad (1.7)$$

Integrating both sides of this equation between, say, condition 1 and condition 2, yields the following relationship between the fluid mass density and the fluid volume:

$$\frac{\rho_1}{\rho_2} = \frac{V_2}{V_1}. \quad (1.8)$$

With Equation (1.5) it can be seen that this result simply states that the mass at condition 1 must equal the mass at condition 2. These density and volume relationships will be useful when considering the following discussion of the fluid bulk modulus.

1.3 FLUID BULK MODULUS

1.3.1 Definitions

The isothermal fluid bulk modulus describes the elasticity, or “stretchiness,” of the fluid at a constant temperature. This property is determined experimentally using a stress-strain test in which the volume of fluid is decreased while keeping the mass constant. During this process, the stress of the fluid is measured by measuring the fluid pressure. A plot of the fluid pressure versus the fluid strain is then generated, and the slope of this plot is used to describe the elasticity of the fluid. This slope generally is referred to as the *fluid bulk modulus*. Figure 1-1 shows a plot of the stress-strain curve for a

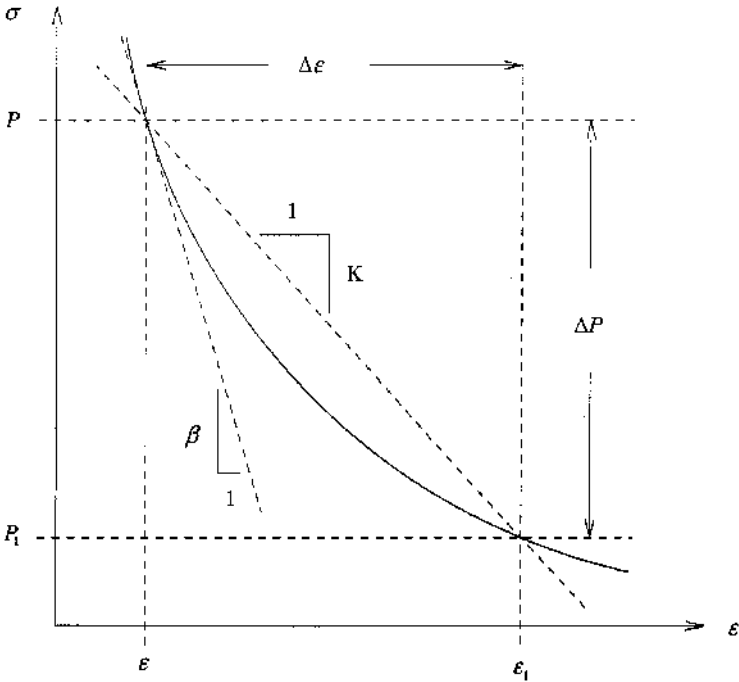


Figure 1-1. The stress-strain curve for a liquid showing the secant bulk modulus K and the tangent bulk modulus β .

liquid. As shown in this figure, the stress-strain curve is not linear because its slope is shown to vary in magnitude with pressure. In Figure 1-1, both the secant bulk modulus K and the tangent bulk modulus β are shown.

Using Figure 1-1, it can be seen that the secant bulk modulus is defined as

$$K = \frac{\Delta P}{\Delta \epsilon}, \tag{1.9}$$

where P is the fluid pressure, and ϵ is the fluid strain. For the secant bulk modulus, the fluid strain is defined by

$$\epsilon \equiv \frac{\Delta V}{V_o} = \frac{V_o - V}{V_o}, \tag{1.10}$$

where V_o is the fluid volume at atmospheric pressure, and V is the fluid volume at another point of interest. From Figure 1-1 and Equation (1.10) it may be shown that

$$\Delta P = P - P_1 \quad \text{and} \quad \Delta \varepsilon = \frac{V_1 - V}{V_o}. \quad (1.11)$$

Substituting Equation (1.11) into Equation (1.9) yields the following result for the secant bulk modulus:

$$K = \frac{V_o(P - P_1)}{V_1 - V}. \quad (1.12)$$

By convention, the reference pressure and volume for the calculation of the secant bulk modulus is given by $P_1 = 0$ and $V_1 = V_o$. Notice that with this convention, gauge pressures are assumed. Substituting these reference values into Equation (1.12) yields the following conventional expression for the secant fluid bulk modulus:

$$K = \frac{V_o P}{V_o - V} = \frac{\rho P}{\rho - \rho_o}. \quad (1.13)$$

In this result, we have used the relationship between the fluid volume V and the fluid density ρ , as shown in Equation (1.8).

The tangent fluid bulk modulus is defined by the slope of a line that is anywhere tangent to the stress-strain curve shown in Figure 1-1. This quantity is expressed mathematically by the following limit:

$$\beta = \lim_{\Delta \varepsilon \rightarrow 0} \frac{\Delta P}{\Delta \varepsilon} = \frac{dP}{d\varepsilon}. \quad (1.14)$$

The fluid strain for the calculation of the tangent bulk modulus is defined by

$$\varepsilon \equiv -\ln\left(\frac{V}{V_o}\right). \quad (1.15)$$

At this point the reader may note that the fluid strain for the tangent bulk modulus has been defined differently than it was for the secant bulk modulus. See Equation (1.10). In fact, one might argue that the strain definition for the secant bulk modulus is more typical and expected than the strain definition presented in Equation (1.15). This, of course, would be true especially from a perspective of solid mechanics, where the definition of material strain is very similar in form to that of the secant bulk modulus. The use of Equation (1.15) for describing the strain of the tangent bulk modulus is more an issue of convention rather than principle; however, it may give the reader comfort to recognize that for values of $V/V_o \approx 1$, Equation (1.15) may be linearly approximated as the secant bulk modulus just as it has been presented in

Equation (1.10). Since liquid is fairly incompressible, this approximation is easily justified, and the two strain definitions can be viewed as essentially the same. Using Equation (1.15), it may be shown that

$$d\varepsilon = -\frac{1}{V} dV. \quad (1.16)$$

Therefore, the tangent fluid bulk modulus may be expressed more explicitly as

$$\beta = -V \frac{dP}{dV} = \rho \frac{dP}{d\rho}. \quad (1.17)$$

Note: In this result we have used the relationship between the fluid volume V and the fluid density ρ , as shown in Equation (1.7).

1.3.2 Effective Bulk Modulus

General Equations. The fluid bulk modulus has been used to describe the elasticity of a fluid as it undergoes a volumetric deformation. This elasticity describes a spring effect that is often attributed to high-frequency resonance within hydraulic control systems. High-frequency resonance can create irritating noise problems and premature failures of vibrating parts; however, it usually does not present a control problem because the resonance occurs at a much higher frequency than the dominant natural frequency of the typical device being controlled. If, on the other hand, the effective spring rate of the hydraulic system becomes soft due to entrained air within the system or a large volume of compressed fluid or an overly compliant fluid container, the resonating frequency of the hydraulic system will become much lower, and a potential for control difficulties will exist.

Figure 1-2 shows a schematic of a flexible container filled with a fluid mixture of liquid and air. A piston is moved to the left to compress the fluid while the structural volume of the container also is expanded in a radial direction. For the analysis that follows, container deflection in the axial direction is neglected. The total volume of the chamber is given by

$$V = V_o + V_\delta - Ax = V_l + V_a, \quad (1.18)$$

where V_o is the initial volume of the container, V_δ is an additional volume that results from expanding the chamber, A is the cross-sectional area of the piston, x is the piston displacement, V_l is the volume of liquid within the chamber, and V_a is the volume of air within the chamber. By subtracting the deflection volume V_δ from each side of Equation (1.18), an effective volume may be calculated as

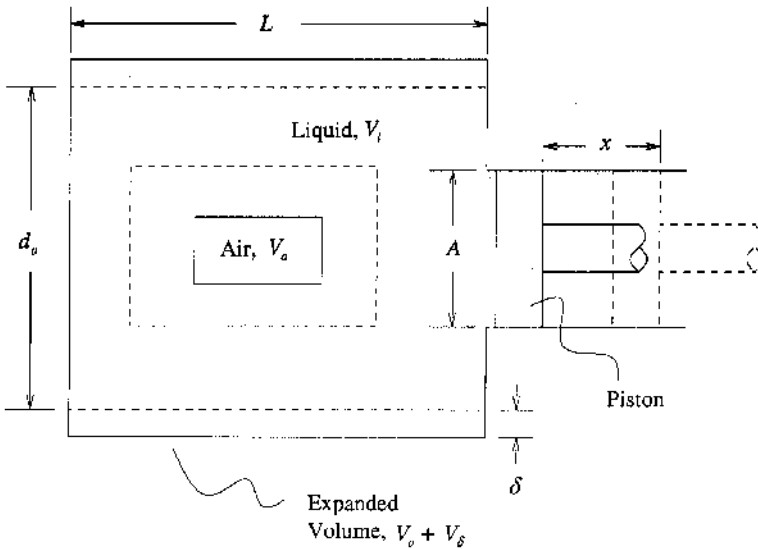


Figure 1-2. A pressurized flexible container filled with a fluid mixture of liquid and air.

$$V_e = V_o - Ax = V_l + V_a - V_\delta. \quad (1.19)$$

This definition for the effective volume is useful because it represents a quantity that may be calculated easily without knowing the deformation characteristics of the chamber. The change in the effective volume is expressed differentially as

$$dV_e = dV_l + dV_a - dV_\delta. \quad (1.20)$$

Using the general form of Equation (1.17), the effective fluid bulk modulus for the device shown in Figure 1-2 may be expressed as

$$\frac{1}{\beta_e} = -\frac{1}{V_e} \frac{dV_e}{dP}, \quad (1.21)$$

where β_e is the effective fluid bulk modulus, V_e is the effective volume that undergoes deformation, and P is the fluid pressure within the hydraulic system. Substituting Equation (1.20) into Equation (1.21) produces the following result for the effective fluid bulk modulus:

$$\frac{1}{\beta_e} = \frac{V_l}{V_e} \left(-\frac{1}{V_l} \frac{dV_l}{dP} \right) + \frac{V_a}{V_e} \left(-\frac{1}{V_a} \frac{dV_a}{dP} \right) + \frac{1}{V_e} \frac{dV_\delta}{dP}. \quad (1.22)$$

By definition, the bulk moduli for liquid and air, respectively, are given as

$$\frac{1}{\beta_l} = -\frac{1}{V_l} \frac{dV_l}{dP} \quad \text{and} \quad \frac{1}{\beta_a} = -\frac{1}{V_a} \frac{dV_a}{dP}. \quad (1.23)$$

It is also useful to define the bulk modulus of the container with respect to the effective fluid volume as

$$\frac{1}{\beta_c} = \frac{1}{V_e} \frac{dV_\delta}{dP}. \quad (1.24)$$

Notice that this definition is a different form from that of Equation (1.23) because the volume and differential volume terms are based on different volume quantities; however, the details of these differences will be addressed later. Substituting Equations (1.23) and (1.24) into Equation (1.22) yields the following result for the effective fluid bulk modulus of the system shown in Figure 1-2:

$$\frac{1}{\beta_e} = \frac{V_l}{V_e} \frac{1}{\beta_l} + \frac{V_a}{V_e} \frac{1}{\beta_a} + \frac{1}{\beta_c}. \quad (1.25)$$

The volumetric ratios within this expression describe the fractional volume content of liquid and air. Equation (1.19) may be used with Equation (1.25) to show that

$$\frac{1}{\beta_e} = \left(1 + \frac{V_\delta}{V_e}\right) \frac{1}{\beta_l} + \left(1 - \frac{\beta_a}{\beta_l}\right) \frac{V_a}{V_e} \frac{1}{\beta_a} + \frac{1}{\beta_c}. \quad (1.26)$$

Recognizing that $V_e \gg V_\delta$ and $\beta_l \gg \beta_a$, Equation (1.26) may be closely approximated as

$$\frac{1}{\beta_e} = \frac{1}{\beta_l} + \frac{V_a}{V_e} \frac{1}{\beta_a} + \frac{1}{\beta_c}. \quad (1.27)$$

Equation (1.27) is a useful expression for describing the effective fluid bulk modulus within the flexible container shown in Figure 1-2; however, there are several unknowns in this equation that must be discussed further. In particular, useful expressions for the bulk moduli must be developed, and the fractional content of air must be discussed.

An Equivalent Spring System. Figure 1-3 shows a spring system that is equivalent to the pressurized chamber shown in Figure 1-2. The equivalent spring system is useful for showing the effective spring rate of the hydraulic system and for lending insight into the reduction of natural frequencies of oscillation. In the top portion of Figure 1-3, a cylinder is shown with internal pistons that are separated by springs. The right-hand spring is intended to

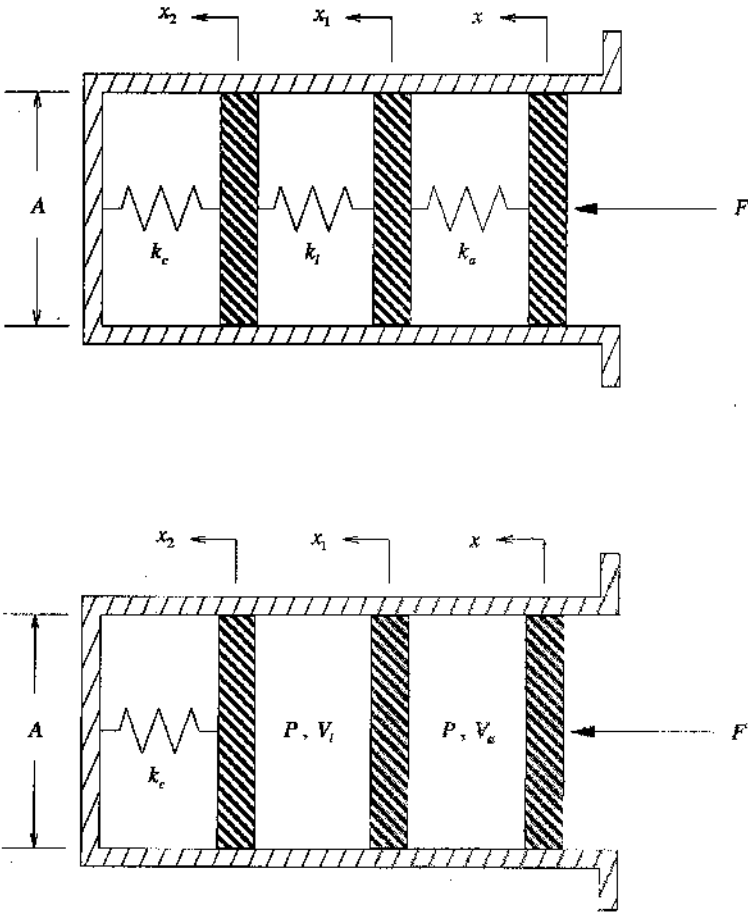


Figure 1-3. An equivalent spring system illustrating the compressibility effects of the liquid, the air, and the container.

model the spring rate of the air within the system, the middle spring is used to model the spring rate of the liquid, and the left-hand spring is used to simulate the spring rate of the container. The pistons are free to slide within the cylinder and may change their location depending on the input force F .

From an overall system view, the input force may be described as

$$F = xk_e, \tag{1.28}$$

where x is the displacement of the first piston, and k_e is the effective spring rate of the overall system. From a static analysis of the spring system it may be shown that the input force is also described by

$$F = (x - x_1)k_a = (x_1 - x_2)k_l = x_2k_c, \quad (1.29)$$

where k_a , k_l , and k_c are the spring rates of the air, the liquid, and the container, respectively. From Equation (1.29) it may be shown that

$$\begin{aligned} x_2 &= \frac{F}{k_c}, \\ x_1 &= x_2 + \frac{F}{k_l} = \frac{F}{k_c} + \frac{F}{k_l}, \\ x &= x_1 + \frac{F}{k_a} = \frac{F}{k_c} + \frac{F}{k_l} + \frac{F}{k_a}. \end{aligned} \quad (1.30)$$

From Equation (1.28) it can be seen that $x = F/k_e$. Substituting this expression into the bottom result of Equation (1.30) yields the following equation for the effective spring rate of the system:

$$\frac{1}{k_e} = \frac{1}{k_c} + \frac{1}{k_l} + \frac{1}{k_a}. \quad (1.31)$$

This is the classic expression that is used to describe the effective spring rate for a group of springs placed in series with respect to one another.

The bottom portion of Figure 1-3 is shown with certain springs removed, and now the spaces between pistons have been filled with liquid and air. The spring associated with the container remains because this is the best model for the effect of deforming a solid material. From the bottom portion of Figure 1-3 it can be seen that the input force is statically equivalent to the pressure of the air (or liquid) times the cross-sectional area of a single piston. This force is simply expressed by

$$F = PA. \quad (1.32)$$

From the definition of the effective fluid bulk modulus given in Equation (1.21), it can be seen that the pressure within the system is described by the differential expression

$$dP = -\beta_e \frac{1}{V_e} dV_e, \quad (1.33)$$

where β_e is the effective fluid bulk modulus, and V_e is the effective volume of the fluid. Solving Equation (1.33) produces the following result for the fluid pressure:

$$P = \beta_e \ln\left(\frac{V_o}{V_e} - 1\right) \approx \beta_e \left(\frac{V_o}{V_e} - 1\right), \quad (1.34)$$

where V_o is the volume of the fluid when the pressure is zero. *Note:* The right-hand side of Equation (1.34) assumes that $V_o \approx V_e$, which means that the changes in the fluid volume are small. Using Equation (1.34) with Equation (1.32) yields the following result for the input force to the equivalent spring system:

$$F = \beta_e \left(\frac{V_o}{V_e} - 1\right) A. \quad (1.35)$$

Setting Equation (1.35) equal to Equation (1.28) yields the following result for the effective spring rate of the system:

$$\frac{1}{k_e} = \frac{1}{\beta_e} \frac{V_e}{A^2}. \quad (1.36)$$

In this result it has been recognized from Equation (1.19) that $V_e = V_o - Ax$. Since the definitions of the effective fluid bulk modulus and the bulk modulus for air and liquid are similar in form [compare Equations (1.21) and (1.23)], a rerun of the previous analysis may be done for the columns of air and liquid shown in Figure 1-3. This analysis produces the following results for the spring rate of air and the spring rate of liquid:

$$\frac{1}{k_a} = \frac{1}{\beta_a} \frac{V_a}{A^2} \quad \text{and} \quad \frac{1}{k_l} = \frac{1}{\beta_l} \frac{V_l}{A^2}. \quad (1.37)$$

From the static analysis of Figure 1-3, it may be shown that

$$k_c x_2 = PA. \quad (1.38)$$

Equation (1.24) presents the definition of the container bulk modulus and can be used to develop an equivalent expression for the fluid pressure within the system. This expression is determined by rearranging Equation (1.24) as follows:

$$dP = \beta_c \frac{1}{V_e} dV_\delta. \quad (1.39)$$

Solving this equation yields

$$P = \beta_c \frac{1}{V_e} V_\delta = \beta_c \frac{1}{V_e} Ax_2, \quad (1.40)$$

where it has been recognized that $V_\delta = Ax_2$. Substituting Equation (1.40) into Equation (1.38) yields the following result for the spring rate of the container:

$$\frac{1}{k_c} = \frac{1}{\beta_c} \frac{V_e}{A^2}. \quad (1.41)$$

By substituting the results of Equations (1.36), (1.37), and (1.41) into Equation (1.31), the effective fluid bulk modulus for the system may be expressed as

$$\frac{1}{\beta_e} = \frac{V_l}{V_e} \frac{1}{\beta_l} + \frac{V_a}{V_e} \frac{1}{\beta_a} + \frac{1}{\beta_c}, \quad (1.42)$$

which is the exact same expression presented in Equation (1.25). *Note:* Making the appropriate simplifications, this result may be reduced further to the form of Equation (1.27).

All this discussion has been used to show that the compressibility effects within a hydraulic system may be considered as a series of springs that describe the stiffness of the liquid, the air, and the container itself. Since these springs are arranged in series, the overall stiffness of the system will never exceed the stiffness of any one spring. The spring rate of each substance depends on geometry and the bulk modulus property. In the paragraphs that follow, the bulk modulus of the liquid, the air, and the container will be considered in their turn.

Bulk Modulus of Liquid. From experiments it has been determined that over a limited pressure range the secant bulk modulus of all liquids increases linearly with pressure. This is to say,

$$K = K_o + mP, \quad (1.43)$$

where K_o is the secant bulk modulus of the liquid at zero gauge pressure, and m is the slope of increase. For any one liquid, the value of m is practically the same at all temperatures; however, the value of K_o carries a temperature dependency with it. Equation (1.43) is valid for mineral oils and most other hydraulic fluids up to about 800 bar. This same equation is valid for water up to about 3000 bar. Consequently, if the appropriate values for K_o and m are known for any liquid, the secant bulk modulus for that liquid can be calculated easily using Equation (1.43). By setting Equation (1.43) equal to Equation (1.13), it may be shown that

$$V = V_o \left(1 - \frac{P}{K_o + mP} \right) \quad \text{and} \quad \frac{dP}{dV} = -\frac{(K_o + mP)^2}{K_o V_o}. \quad (1.44)$$

Substituting this result into Equation (1.17) produces the following expression that may be used to evaluate the tangent bulk modulus of a liquid as a function of the liquid parameters K_o and m :

$$\beta_t = K_o \left(1 + \frac{(m-1)P}{K_o} \right) \left(1 + \frac{mP}{K_o} \right). \quad (1.45)$$

Table 1-1 presents typical fluid properties for liquids that are used commonly within hydraulic systems.

Bulk Modulus of Air. There are two methods for determining the fluid bulk modulus of air. One method assumes that the temperature of the air remains constant (*isothermal*), and the other method assumes that no heat transfer occurs in or out of the volume of air during the expansion and compression of the fluid (*adiabatic*). While the isothermal assumption is more consistent with our definition of the fluid bulk modulus, the adiabatic assumption is used more commonly and is recommended [1].

To develop an expression for the isothermal bulk modulus of air, we use the ideal gas law of Equation (1.1) and enforce the assumption of a constant temperature T . A convenient representation of this equation is given by

Table 1-1. Fluid Bulk Modulus Properties K_o and m (Values for K_o are in kbar.)

Temperature, °C	Mineral Oil	Water	Water Glycol	Water in Oil Emulsion	Phosphate Ester*
0	20.7	19.7	32.0	20.8	29.7
10	19.8	20.9	31.8	20.2	28.1
20	19.0	21.8	31.5	19.6	26.5
30	18.1	22.4	31.1	19.0	25.0
40	17.3	22.6	30.5	18.4	23.6
50	16.4	22.7	29.9	17.8	22.3
60	15.6	22.5	29.1	17.2	21.1
70	14.7	22.2	28.2	16.6	19.9
80	13.9	21.6	27.2	16.0	18.8
90	13.0	21.1	26.0	15.4	17.8
100	12.2	20.4	24.8	14.8	16.9
m (for all temperatures)	5.6	3.4	4.5	5.0	5.5

* Viscosity greater than 50 cSt at 22°C.

$$\frac{\rho}{P} = \frac{1}{RT} = \text{constant}, \quad (1.46)$$

where ρ is the density of air, P is the air pressure, R is the gas constant, and T is the constant air temperature. Taking the derivative of Equation (1.46) yields the following result:

$$\frac{1}{P} d\rho - \frac{\rho}{P^2} dP = 0. \quad (1.47)$$

Rearranging this equation yields an expression for the isothermal bulk modulus of air. This result is given by

$$\frac{1}{\beta_a} = \frac{1}{\rho} \frac{d\rho}{dP} = \frac{1}{P}. \quad (1.48)$$

In other words, the isothermal bulk modulus of air is simply equal to the fluid pressure itself.

The adiabatic bulk modulus of air may be determined by assuming that no heat transfer occurs between the air and the surrounding liquid or container material. Using the first law of thermodynamics and the ideal gas law, it can be shown that

$$PV_a^\gamma = \text{constant}, \quad (1.49)$$

where P is the air pressure, V_a is the air volume, and γ is the ratio of the constant-pressure specific heat to the constant-volume specific heat. Note that $\gamma = 1.4$ for air. Taking the derivative of Equation (1.49), it may be shown that

$$V_a^\gamma dP + P\gamma V_a^{(\gamma-1)} dV_a = 0. \quad (1.50)$$

Dividing this expression through by V_a^γ and rearranging terms produces the following expression for the bulk modulus of entrained air within the hydraulic system:

$$\frac{1}{\beta_a} = -\frac{1}{V_a} \frac{dV_a}{dP} = \frac{1}{P\gamma}. \quad (1.51)$$

A comparison of Equations (1.48) and (1.51) shows that the adiabatic bulk modulus of air differs from the isothermal bulk modulus of air by a factor of 1.4. Although the isothermal bulk modulus is more compatible with the equation of state, which has been used to define the bulk modulus, the adiabatic bulk modulus is used more often and therefore will be applied in the examples that follow.

Bulk Modulus of the Container. To consider the bulk modulus of a container, it will be instructive to examine the cylindrical container shown in Figure 1-2. The volume of this container is given by

$$V_c = \frac{\pi}{4} d^2 L, \quad (1.52)$$

where d is the diameter of the container, and L is the length of the container. If it is assumed that the container expands only in the radial direction, then the expanded volume of the container may be expressed as

$$V_c = \frac{\pi}{4} (d_o + 2\delta)^2 L = \frac{\pi}{4} d_o^2 \left[1 + 4 \left(\frac{\delta}{d_o} \right) + 4 \left(\frac{\delta}{d_o} \right)^2 \right] L, \quad (1.53)$$

where d_o is the original diameter of the container, and δ is the radial deflection, as shown in Figure 1-2. If it is assumed that $\delta/d_o \ll 1$, then the container volume may be closely approximated as

$$V_c = V_o + V_\delta, \quad (1.54)$$

where the original volume and the deformed volume are given, respectively, by

$$V_o = \frac{\pi}{4} d_o^2 L \quad \text{and} \quad V_\delta = \pi d_o \delta L. \quad (1.55)$$

Notice that these are more explicit expressions for the volume terms than were used in Equation (1.18) for describing the total volume of the container. From a strength-of-materials textbook [2] we learn that the inside radial deflection of a thick-walled cylinder (without capped ends) is given by

$$\delta = \frac{d_o P}{2 E} \left(\frac{D_o^2 + d_o^2}{D_o^2 - d_o^2} + \nu \right), \quad (1.56)$$

where P is the internal pressure, E is the tensile modulus of elasticity for the cylinder material, ν is Poisson's ratio, D_o is the original outside diameter of the cylinder, and d_o is the original inside diameter of the cylinder. *Note:* The result of Equation (1.56) is valid for both thick- and thin-walled cylinders. Using Equations (1.55) and (1.56) together, it may be shown that the deformed volume of the cylinder is given by

$$V_\delta = \frac{\pi}{2} d_o^2 \frac{P}{E} \left(\frac{D_o^2 + d_o^2}{D_o^2 - d_o^2} + \nu \right) L. \quad (1.57)$$

The derivative of this expression with respect to the pressure P is then

$$\frac{dV_\delta}{dP} = \frac{\pi d_o^2 L}{2E} \left(\frac{D_o^2 + d_o^2}{D_o^2 - d_o^2} + \nu \right). \quad (1.58)$$

If we assume that the ratio of the displaced volume in the chamber to the original volume is much less than unity, that is, $Ax/V_o \ll 1$, then Equation (1.19) may be used to show that the effective volume of the chamber is given by

$$V_e \approx V_o = \frac{\pi}{4} d_o^2 L. \quad (1.59)$$

Equations (1.58) and (1.59) now may be substituted into Equation (1.24) to express the container bulk modulus as

$$\frac{1}{\beta_c} = \frac{2}{E} \left(\frac{D_o^2 + d_o^2}{D_o^2 - d_o^2} + \nu \right). \quad (1.60)$$

Recognizing that the inside diameter of the container is given by $d_o = D_o - 2t$, where t is the container wall thickness, an equivalent expression for the container bulk modulus may be written as

$$\frac{1}{\beta_c} = \frac{2}{E} \left[\frac{D_o}{2t} \left(1 + \frac{t}{D_o - t} \right) - (1 - \nu) \right]. \quad (1.61)$$

Two special cases of these results are instructive. For very thick walls it may be assumed that $D_o/d_o \gg 1$. In this case, Equation (1.60) may be approximated as

$$\frac{1}{\beta_c} = \frac{2(1 + \nu)}{E}. \quad (1.62)$$

On the other hand, for a thin-walled cylinder, it may be assumed that $D_o/t \gg 1$. In this case, Equation (1.61) may be used to show that

$$\frac{1}{\beta_c} = \frac{D_o/t}{E}. \quad (1.63)$$

Equations (1.62) and (1.63) may be used to describe the range for the bulk modulus of the container as

$$\frac{2(1 + \nu)}{E} < \frac{1}{\beta_c} < \frac{D_o/t}{E}. \quad (1.64)$$

Table 1-2 presents the modulus of elasticity and Poisson's ratio for common materials that are used to construct hydraulic containers.

Case Study

It is instructive to consider typical calculations of the effective bulk modulus for the purposes of illustrating the wide variation that may be expected for this parameter. Let us consider a petroleum-based fluid (mineral oil) that is pressurized to 20 MPa (0.20 kbar) at a temperature of 70°C. Furthermore, let us consider a case where the fluid is contained within a cylindrical container with an outside diameter equal to six times the wall thickness ($D_o/t = 6$). From Equation (1.45) and Table 1-1 it may be shown that the tangent bulk modulus for this liquid is 15.82 kbar. From Equation (1.51), the bulk modulus of the air is 0.28 kbar. Calculate the effective bulk modulus of this system for the following two cases:

1. When the container is made of steel and the liquid has no entrained air
2. When the container is made of high-pressure hose material and the liquid has 1% entrained air by volume ($V_a/V_e = 0.01$)

For the first case, the bulk modulus of the container may be calculated using Equation (1.61) and the material properties for steel that are shown in Table 1-2. This calculation shows that $\beta_c = 504.63$ for the steel container. Using this result with Equation (1.27), the effective bulk modulus may be calculated as

Table 1-2. Material Properties for Common Hydraulic Containers (Modulus of Elasticity E is reported in kbar.)

Property	Ductile			High-Pressure			
	Steel	Cast Iron	Copper	Brass	Aluminum	Hose*	TPE†
E	2069	1655	1103	1034	724	59	0.0393
ν	0.30	0.28	0.30	0.34	0.33	0.47	0.47

*Flexible and reinforced with stainless steel braids (consult manufacturers for more accurate numbers).

†Thermoplastic elastomer (melt-processible rubber).

$$\begin{aligned}
 \beta_e &= \left(\frac{1}{\beta_t} + \frac{V_a}{V_e} \frac{1}{\beta_a} + \frac{1}{\beta_c} \right)^{-1} \\
 &= \left(\frac{1}{15.82 \text{ kbar}} + 0 \times \frac{1}{0.28 \text{ kbar}} + \frac{1}{504.63 \text{ kbar}} \right)^{-1} \\
 &= 15.82 \text{ kbar}.
 \end{aligned}$$

Similarly, for the second case, the bulk modulus of the container may be calculated using Equation (1.61) and the material properties for high-pressure hose that are shown in Table 1-2. This calculation shows that $\beta_c = 13.82$ for the hose. Using this result with Equation (1.27), the effective bulk modulus may be calculated as

$$\begin{aligned}
 \beta_e &= \left(\frac{1}{\beta_t} + \frac{V_a}{V_e} \frac{1}{\beta_a} + \frac{1}{\beta_c} \right)^{-1} \\
 &= \left(\frac{1}{15.82 \text{ kbar}} + 0.01 \times \frac{1}{0.28 \text{ kbar}} + \frac{1}{13.82 \text{ kbar}} \right)^{-1} \\
 &= 5.84 \text{ kbar}.
 \end{aligned}$$

A comparison of these two results reveals a 63% difference in the effective bulk modulus for fluids that are presumably very similar, and much of this difference is a result of simply putting the fluid in a different container.

Summary. The preceding work has shown that the effective bulk modulus is a result of various physical considerations. These considerations include the liquid properties, the amount of entrained air within the liquid, and the flexibility of the device that is used to contain the fluid. In summary, it may be noted that a wide variation in the effective bulk modulus can be expected from one hydraulic system to the next. As mentioned previously, this variation in the effective bulk modulus depends on the air content of the fluid, which also may depend on the circulation system for the fluid. For instance, one may suspect that an open-circuit system where fluid is circulated through a ventilated reservoir may contain more entrained air than a closed-circuit system where most of the fluid is contained within the working components of the system. Other issues related to the reservoir design may have an impact on the turbulent mixing that occurs within the reservoir, which also may have an impact on the amount of entrained air that is introduced by each application. Since the uncertainty of the fluid bulk modulus is significant from one application to the next, it is worth considering techniques that may be used to measure the fluid bulk modulus within an actual system. This will be the topic of the next subsection.

1.3.3 Measuring the Fluid Bulk Modulus

Figure 1-4 shows a closed container with a piston that is supported by a fluid generally consisting of a mixture of liquid and air. The piston has a cross-sectional area given by A . When the fluid pressure is at zero gauge pressure, the height of the fluid column is given by the dimension l_o . When the force F is applied to the piston, the fluid column is decreased in height by the dimension x . Obviously, to support the applied force, the fluid pressure within the container must increase.

From the definition of the fluid bulk modulus, the fluid pressure within the closed container may be expressed differentially as

$$dP = \beta \frac{1}{V} dV, \quad (1.65)$$

where β is the effective fluid bulk modulus, and V is the effective fluid volume. By treating β as a constant, Equation (1.65) may be solved and rearranged to show that the effective fluid bulk modulus is given by

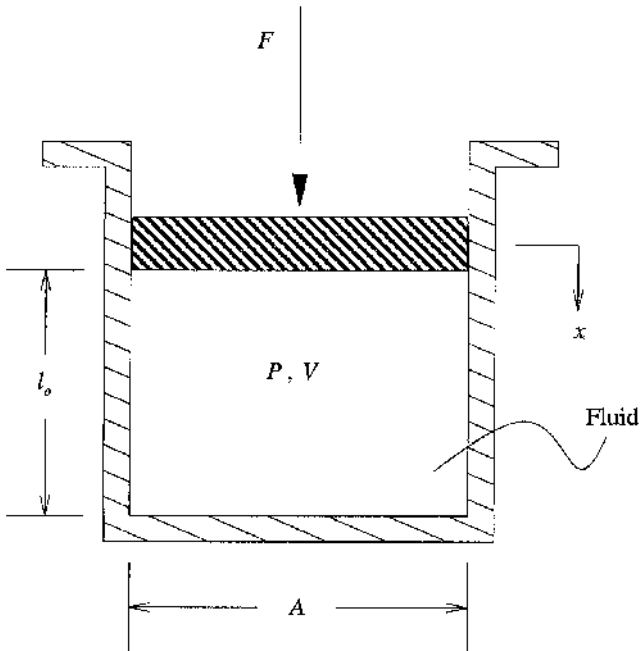


Figure 1-4. Geometry of a test device used to measure the fluid bulk modulus within a closed container.

$$\beta = \frac{P}{\ln(V_o/V)}, \quad (1.66)$$

where V_o is the fluid volume in the chamber when the pressure is zero. This volume would be given more explicitly as Al_o . From geometry, the effective fluid volume after compression takes place may be written as

$$V = V_o - Ax = A(l_o - x). \quad (1.67)$$

Substituting this result into Equation (1.66) yields the following result for the effective fluid bulk modulus of the device shown in Figure 1-4:

$$\beta = \frac{P}{\ln[l_o/(l_o - x)]}. \quad (1.68)$$

From this equation it can be seen that the parameters that must be measured for determining the fluid bulk modulus are the fluid pressure P , the original height of the fluid column l_o , and the displaced distance of the piston x .

As all experimentalists know, the measurements of the parameters shown in Equation (1.68) will always deviate slightly from the actual physical values. If we denote these measured parameters using primed notation, the measured value for the fluid bulk modulus may be written as

$$\beta' = \frac{P'}{\ln[l'_o/(l'_o - x')]}. \quad (1.69)$$

Using this expression, a dimensionless uncertainty in our measurements may be defined as

$$\varepsilon = \frac{\beta - \beta'}{\beta} \quad (1.70)$$

Using Equation (1.68), the dimensionless uncertainty may be written more explicitly as

$$\varepsilon = 1 - \beta' \frac{\ln[l_o/(l_o - x)]}{P}, \quad (1.71)$$

where the unprimed values of P , l_o , and x are the unknown true values of these parameters.

If it is assumed that the measurement uncertainty is small, a first-order Taylor series expansion of Equation (1.71) may be written as

$$\varepsilon = \left(\frac{\partial \varepsilon}{\partial P}\right)' (P - P') + \left(\frac{\partial \varepsilon}{\partial l_o}\right)' (l_o - l'_o) + \left(\frac{\partial \varepsilon}{\partial x}\right)' (x - x'). \quad (1.72)$$

Using Equation (1.71) and assuming that $x'/l'_o \ll 1$, it can be shown that

$$\left(\frac{\partial \varepsilon}{\partial P}\right)' = \frac{1}{P'}, \quad \left(\frac{\partial \varepsilon}{\partial l_o}\right)' \approx \frac{1}{l'_o}, \quad \left(\frac{\partial \varepsilon}{\partial x}\right)' \approx -\frac{1}{x'}. \quad (1.73)$$

Substituting these results back into Equation (1.72) yields the following result for the uncertainty associated with measuring the fluid bulk modulus in the closed container:

$$\varepsilon = \frac{P}{P'} + \frac{l_o}{l'_o} - \frac{x}{x'} - 1. \quad (1.74)$$

The reader will recall that the primed variables are the measured values, whereas the unprimed variables are the true values. The accuracies for instruments that are used for measuring pressures and displacements typically are specified in terms of some percentage of the full-scale capabilities of the instrument itself. This means that the true values of the measured parameters are somewhere within a range that has been identified by the manufacturer of the instrument. Mathematically, this maximum range of uncertainty for each measurement may be described as

$$(P - P') = \pm \xi_P P_{\max}, \quad (l_o - l'_o) = \pm \xi_l l_{\max}, \quad (x - x') = \pm \xi_x x_{\max}, \quad (1.75)$$

where the values of ξ_P , ξ_l , and ξ_x are supplied by the instrument manufacturers, and P_{\max} , l_{\max} , and x_{\max} are the maximum measurement ranges for these instruments. Using the results of Equations (1.72), (1.73), and (1.75), an expression for the maximum measurement uncertainty may be given as

$$\varepsilon_{\max} = \pm \left(\xi_P \frac{P_{\max}}{P'} + \xi_l \frac{l_{\max}}{l'_o} + \xi_x \frac{x_{\max}}{x'} \right). \quad (1.76)$$

Equation (1.76) describes the maximum range of uncertainty associated with the measurement of the fluid bulk modulus for the mechanical device shown in Figure 1-4. If a strain-gauged pressure transducer is used, it is common to be able to measure pressures accurately within $\pm 1.5\%$ of the full-scale reading. This means that $\xi_P = 0.015$. For measuring linear dimensions such as l_o , the method used may vary from a highly accurate coordinate-measuring machine (CMM) to a simple visual inspection of a calibrated scale. Therefore, the range of accuracies for making the linear measurement of the original height of the fluid column may be anywhere between $\pm 1 \times 10^{-4}\%$

and $\pm 1\%$ of the full-scale reading [3]. This means that $1 \times 10^{-6} < \xi_l < 0.01$. In practice, it is quite common to use a linear variable differential transformer (LVDT) for making linear displacement measurements. These devices typically are accurate within $\pm 0.5\%$ of full-scale readings, and therefore, $\xi_x = 0.005$.

Case Study

The test device shown in Figure 1-4 is used to measure the effective fluid bulk modulus. While taking this measurement, each instrument is being used at half its maximum capacity, and the instrument accuracies are given by $\xi_p = 0.015$, $\xi_l = 0.005$, and $\xi_x = 0.005$. Calculate the uncertainty in the measurement of the bulk modulus.

To calculate the uncertainty in this measurement, Equation (1.76) will be used. This calculation is given by

$$\begin{aligned} \varepsilon_{\max} &= \pm \left(\xi_p \frac{P_{\max}}{P'} + \xi_l \frac{l_{\max}}{l'_o} + \xi_x \frac{x_{\max}}{x'} \right) \\ &= \pm (0.015 \times 2 + 0.005 \times 2 + 0.005 \times 2) \\ &= \pm 0.05. \end{aligned}$$

This result shows a measurement uncertainty of $\pm 5\%$.

As shown by the preceding case study, measurements of the effective fluid bulk modulus may be taken with the device shown in Figure 1-4 while maintaining a fair degree of confidence in the results. Of course, the uncertainty in this measurement can get better or worse depending on the quality of the instruments, their operating range, and the operating point of the measurement. In any event, the range of expected uncertainty should be checked before conducting the experiment to see whether or not the measurement is worthwhile or to design an acceptable test device. Equation (1.76) provides the tool for checking this uncertainty as it pertains to the device shown in Figure 1-4. It also should be mentioned that the device in Figure 1-4 is a laboratory device that may or may not represent an actual hydraulic control system adequately. In situ measurements of the fluid bulk modulus have been considered in previous research [4]; however, these measurements have been shown to be expensive and highly uncertain, thus rendering these methods somewhat doubtful. In summary, the reader should be impressed with the range of variability that the effective fluid bulk modulus may undergo and with the difficulty in obtaining confidence in its value even from an experimental point of view. Indeed, when simulating hydraulic control systems, the specification for the fluid bulk modulus becomes one of the most arbitrary aspects of the model, which is highly unfortunate because this parameter is

often the deciding factor between a stable and an unstable hydraulic component or system. This influence of the fluid bulk modulus will be shown in subsequent chapters of this text.

1.4 THERMAL FLUID PROPERTIES

1.4.1 Coefficient of Thermal Expansion

Definition. The isobar coefficient of thermal expansion for a hydraulic fluid was presented in Equation (1.3) and is rewritten here for convenience in terms of both density and fluid volume:

$$\alpha = -\frac{1}{\rho} \frac{d\rho}{dT} = \frac{1}{V} \frac{dV}{dT}, \quad (1.77)$$

where T is the fluid temperature. The coefficient of thermal expansion α describes the change in fluid density (or volume) as the temperature of the fluid increases or diminishes. Since the fluid consists of air and liquid, and since the fluid container also exhibits expansion and contraction with changes in temperature, it is of use to consider the effective coefficient of thermal expansion for the entire fluid system.

The Effective Coefficient of Thermal Expansion. The effective coefficient of thermal expansion can be evaluated by considering the device in Figure 1-2 and the effective volume of the fluid V_e as given in Equation (1.19). This volume consists of air and liquid and is adjusted for small deflections of the container itself. Using the effective volume, we may calculate the effective coefficient of thermal expansion as

$$\alpha_e = \frac{1}{V_e} \frac{dV_e}{dT}. \quad (1.78)$$

Substituting the differential volume dV_e of Equation (1.20) into this result yields the following expression for the effective coefficient of thermal expansion:

$$\alpha_e = \frac{V_l}{V_e} \left(\frac{1}{V_l} \frac{dV_l}{dT} \right) + \frac{V_a}{V_e} \left(\frac{1}{V_a} \frac{dV_a}{dT} \right) - \frac{1}{V_e} \frac{dV_\delta}{dT}, \quad (1.79)$$

where V_l is the volume of the liquid, V_a is the volume of the air, and V_δ is the deflected volume of the container shown in Figure 1-2. By definition, the coefficient of thermal expansion for liquid and air are given respectively as

$$\alpha_l = \frac{1}{V_l} \frac{dV_l}{dT} \quad \text{and} \quad \alpha_a = \frac{1}{V_a} \frac{dV_a}{dT}, \quad (1.80)$$

and a convenient definition for the coefficient of thermal expansion for the container is

$$\alpha_c = \frac{1}{V_e} \frac{dV_\delta}{dT}. \quad (1.81)$$

Again, the reader will observe that the definition for the coefficient of thermal expansion for the container α_c is somewhat different in form compared with that of α_l and α_a . Substituting Equations (1.80) and (1.81) into Equation (1.79) yields the following result for the effective coefficient of thermal expansion:

$$\alpha_e = \frac{V_l}{V_e} \alpha_l + \frac{V_a}{V_e} \alpha_a - \alpha_c. \quad (1.82)$$

The volumetric ratios in this expression describe the fractional content of liquid and air within the container. Using Equation (1.19) with Equation (1.82), it may be shown that

$$\alpha_e = \left(1 + \frac{V_\delta}{V_e}\right) \alpha_l + \left(1 - \frac{\alpha_l}{\alpha_a}\right) \frac{V_a}{V_e} \alpha_a - \alpha_c. \quad (1.83)$$

Since $V_e \gg V_\delta$, Equation (1.83) may be closely approximated as

$$\alpha_e = \alpha_l + \left(1 - \frac{\alpha_l}{\alpha_a}\right) \frac{V_a}{V_e} \alpha_a - \alpha_c. \quad (1.84)$$

Table 1-3 shows a partial list of data that are currently available for estimating the coefficient of thermal expansion of mineral oil and water as it varies with temperature. More extensive data related to the coefficient of thermal expansion for hydraulic fluids have been published by the U.S. Army

Table 1-3. Volumetric Coefficient of Thermal Expansion for Various Liquids Used within Hydraulic Systems [Reported in units of $10^{-4}/(^{\circ}\text{C}$ or $^{\circ}\text{K}$)]

	Temperature, $^{\circ}\text{C}$						
	0	20	40	60	80	100	120
Mineral oil	7.00	7.00	7.00	7.00	7.00	7.00	7.00
Water	-0.68	1.73	3.60	5.04	6.24	7.28	8.40

and may be consulted for more accurate information [8]. In the following paragraphs, the coefficient of thermal expansion for air and the hydraulic container will be discussed.

The isobar coefficient of thermal expansion for air may be determined using the ideal gas law of Equation (1.1). A convenient form of this equation is given by

$$\rho T = \frac{P}{R} = \text{constant}, \quad (1.85)$$

where the pressure P is fixed to satisfy the isobar requirement. Differentiating this equation yields the following result:

$$\rho dT + T d\rho = 0. \quad (1.86)$$

Rearranging this expression yields the following result for the coefficient of thermal expansion for air:

$$\alpha_a = -\frac{1}{\rho} \frac{d\rho}{dT} = \frac{1}{T}. \quad (1.87)$$

The reader will recall that this temperature T is given in the absolute scale; therefore, the coefficient of thermal expansion for air is given more familiarly as

$$\alpha_a = \frac{1}{(T_C + 273.15) \text{ }^\circ\text{C or }^\circ\text{K}} = \frac{1}{(T_F + 459.67) \text{ }^\circ\text{F or }^\circ\text{R}}, \quad (1.88)$$

where T_C and T_F are the fluid temperatures measured in the Celsius and Fahrenheit scales, respectively.

An expression for the coefficient of thermal expansion for the container is given in Equation (1.81). If we use the cylindrical container geometry of Figure 1-2, the instantaneous volume of the container may be described using Equations (1.54) and (1.55). Differentiation of V_δ as shown in Equation (1.55) shows that the change in the deformed volume of the container with respect to temperature is given by

$$\frac{dV_\delta}{dT} = \pi d_o L \frac{d\delta}{dT}, \quad (1.89)$$

where d_o is the original inside diameter of the container, L is the length of the container, and δ is the inside radial growth of the container that occurs due to an increase in the temperature. If we use the approximation of Equation (1.59) for the effective volume of the container, then it may be shown that

$$\alpha_c = \frac{1}{V_e} \frac{dV_\delta}{dT} = 4 \frac{1}{d_o} \frac{d\delta}{dT}. \quad (1.90)$$

By definition, the linear coefficient of thermal expansion is given by

$$\gamma = \frac{1}{l} \frac{dl}{dT}, \quad (1.91)$$

where l is the linear dimension that grows due to an increase in temperature. Recognizing that the linearly growing dimension in this problem is the diameter of the container, it may be shown that

$$l = d_o + 2\delta \approx d_o \quad \text{and} \quad \frac{dl}{dT} = 2 \frac{d\delta}{dT}. \quad (1.92)$$

Using this result with Equations (1.90) and (1.91), the volumetric coefficient of thermal expansion for the container may be approximated by

$$\alpha_c = 2\gamma, \quad (1.93)$$

where γ is the linear coefficient of thermal expansion of the container material, which may be found in material handbooks. Using material handbooks and Equation (1.93), the volumetric coefficients of thermal expansion for typical hydraulic containers are given in the Table 1-4.

Case Study

To consider the possible variation that may be expected in the coefficient of thermal expansion, let us once again consider a petroleum-based fluid (mineral oil) that is at a temperature of 70°C within a cylindrical container. From Table 1-3 it may be shown that the coefficient of thermal expansion for this liquid is $7 \times 10^{-4}/^\circ\text{C}$. Using Equation (1.88), the coefficient of

Table 1-4. Volumetric Coefficients of Thermal Expansion for Materials Commonly Used to Make Hydraulic Containers

α_c	Ductile					High-Pressure Hose*	TPE†
	Steel	Cast Iron	Copper	Brass	Aluminum		
$10^{-6}/^\circ\text{C}$ or $^\circ\text{K}$	22.0	26.4	34.0	38.0	48.0	447	460
$10^{-6}/^\circ\text{F}$ or $^\circ\text{R}$	12.2	14.7	18.8	21.2	26.6	248	256

*Flexible and reinforced with stainless steel braids (consult manufacturers for more accurate numbers).

†Thermoplastic elastomer (melt-processible rubber).

thermal expansion for the air is $3 \times 10^{-3}/^{\circ}\text{C}$. Calculate the effective coefficient of thermal expansion of the fluid for the following two cases:

1. When the container is made of steel and the liquid has no entrained air
2. When the container is made of high-pressure hose material and the liquid has 1% entrained air by volume ($V_a/V_e = 0.01$)

For the first case, the coefficient of thermal expansion for the steel container may be determined using Table 1-4. This table shows that $\alpha_c = 22 \times 10^{-6}/^{\circ}\text{C}$ for the steel container. Using this result with Equation (1.84), the effective coefficient of thermal expansion may be calculated as

$$\alpha_e = \alpha_l - \alpha_c = \frac{7 \times 10^{-4}}{^{\circ}\text{C}} - \frac{22 \times 10^{-6}}{^{\circ}\text{C}} = \frac{6.78 \times 10^{-4}}{^{\circ}\text{C}}.$$

Similarly, for the second case, the coefficient of thermal expansion for the high-pressure hose may be determined using Table 1-4. This table shows that $\alpha_c = 447 \times 10^{-6}/^{\circ}\text{C}$ for the hose. Using this result with Equation (1.84), the effective coefficient of thermal expansion may be calculated as

$$\begin{aligned} \alpha_e &= \alpha_l + \left(1 - \frac{\alpha_l}{\alpha_a}\right) \frac{V_a}{V_e} \alpha_a - \alpha_c = \frac{7 \times 10^{-4}}{^{\circ}\text{C}} + \left(1 - \frac{7 \times 10^{-4}}{3 \times 10^{-3}}\right) \\ &\quad \times 0.01 \times \frac{7 \times 10^{-4}}{^{\circ}\text{C}} - \frac{447 \times 10^{-6}}{^{\circ}\text{C}} \\ &= \frac{2.58 \times 10^{-4}}{^{\circ}\text{C}}. \end{aligned}$$

A comparison of these two results reveals a 62% difference in the calculated effective coefficient of thermal expansion for fluids that are presumably very similar. Again, most of this difference is a result of simply putting the fluid in a different container.

In the preceding case study it has been shown that the effective coefficient of thermal expansion can vary significantly for fluids that are used within hydraulic control systems. As shown by this example, the coefficient of thermal expansion is not greatly influenced by reasonable percentages of entrained air, and since this is the only parameter within our model that varies with temperature, we can say that the effective coefficient of thermal expansion does not vary much with temperature. On the other hand, this example has shown that the effective coefficient of thermal expansion for the fluid is

influenced significantly by the container properties, which will vary for the fluid as it flows through various conduit materials within the same hydraulic circuit.

Measuring the Coefficient of Thermal Expansion. Since temperature variations within hydraulic control systems are not nearly as pronounced as, say, pressure variations, an accurate value for the coefficient of thermal expansion is not nearly as important as having an accurate value for the fluid bulk modulus. For this reason, we do not include an extensive discussion on measurement techniques (and the associated uncertainty) for the effective coefficient of thermal expansion. The reader should recognize, however, that these techniques do not change in principle from the techniques that were described for measuring the fluid bulk modulus. Similar types of experiments as those described in Section 1.3.3 may be used, and the uncertainty in each measurement may be calculated as well.

1.4.2 Thermal Conductivity

The rate at which heat is transferred through a material (fluid or solid) is governed by Fourier’s law. This law is based on observed phenomena and states that the heat flux (heat per unit area) is directly proportional to the temperature gradient in a direction normal to an isothermal surface. Figure 1-5 illustrates this phenomenon. By definition, the thermal conductivity of a material is given by

$$k = -\frac{q_x}{A} \frac{1}{(\partial T/\partial x)} \approx \frac{q_x}{A} \frac{\Delta x}{\Delta T}, \tag{1.94}$$

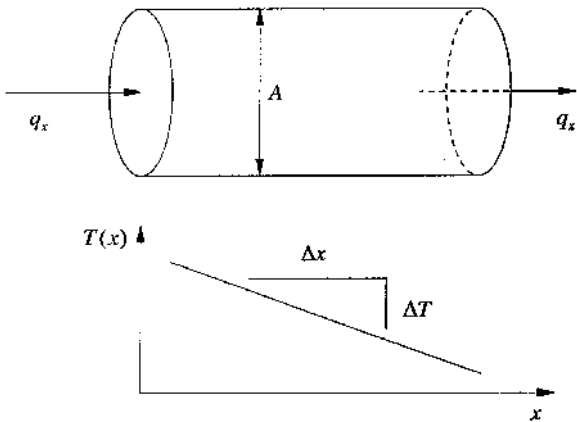


Figure 1-5. The relationship between the coordinate system, heat flow direction, and temperature gradient in one dimension.

where q_x is the heat being transferred through the material, A is the cross-sectional area normal to the heat flow, x is the dimension in the direction of the heat flow, and T is the material temperature. Table 1-5 provides values of thermal conductivity for mineral oil and water as it varies with temperature.

1.4.3 Specific Heat

The specific heat is a thermodynamic property that is useful for determining the change in internal energy of a substance for small increases in temperature. As such, the specific heat for a substance that maintains a constant volume is given by

$$c_v \equiv \left. \frac{\partial u}{\partial T} \right|_v, \quad (1.95)$$

where u is the specific internal energy of the substance. If the internal energy is considered as a measure of added heat to the substance, we can see that the specific heat c_v describes the amount of additional heat that is required to raise the temperature of the substance by a unit of temperature (e.g., 1°C or °K). The thermodynamic property of enthalpy is given by the expression

$$h \equiv u + Pv, \quad (1.96)$$

where u is the specific internal energy, P is pressure, and v is the specific volume. By definition, the specific heat for a substance that maintains a constant pressure is given by

$$c_P \equiv \left. \frac{\partial h}{\partial T} \right|_P. \quad (1.97)$$

By differentiating Equation (1.96) with respect to temperature, it can be shown that

Table 1-5. Values of Thermal Conductivity k for Petroleum-Based Fluid and Water Reported in 10^{-3} W/(m °C)

	Temperature, °C						
	0	20	40	60	80	100	120
Mineral oil	147	145	145	141	138	137	135
Water	569	598	628	650	668	679	686

$$c_p = \left. \frac{\partial h}{\partial T} \right|_P = \left. \frac{\partial u}{\partial T} \right|_P + P \left. \frac{\partial v}{\partial T} \right|_P = \left. \frac{\partial u}{\partial T} \right|_P + Pv\alpha, \tag{1.98}$$

where α is the isobar coefficient of thermal expansion. Since α is very small for liquids (see Table 1-3), it can be shown that the change in internal energy is approximately equal to the change in enthalpy, and therefore,

$$c_v = c_p \tag{1.99}$$

for liquids that are used commonly within hydraulic control systems. Table 1-6 shows the specific heat for mineral oil and water as it varies with fluid temperature. These results will be used in the next chapter to calculate the temperature rise within a fluid due to a pressure drop across a flow passage.

1.5 FLUID VISCOSITY

1.5.1 Definitions

From solid mechanics one may recall that the shear stress within a solid is proportional to the shear strain within a region of linear deformation. In the case of solids, this constant of proportionality relating the shear stress to the shear strain is called the *shear modulus of elasticity* and is often noted by the symbol G . When a shear stress is applied to a solid, a fixed distortion of the material occurs, and until the shear stress is released, the material remains in a fixed distorted shape while storing potential energy much like a mechanical spring. Fluids do not behave this way. When a shear stress is applied to a fluid, the fluid will not deform into a fixed shape while storing potential energy. Rather, for a constantly applied shear stress, a fluid will deform continuously while dissipating energy in the form of heat. In this case it has been observed experimentally that the shear stress within the fluid is proportional to the rate of shear strain, and the constant of proportionality in this relationship is called the *absolute viscosity of the fluid* and is noted by the symbol μ . Mathematically, we describe this relationship as

Table 1-6. Specific Heat c_p for Petroleum-Based Fluid and Water [Reported in kJ/(kg °C)]

	Temperature, °C						
	0	20	40	60	80	100	120
Mineral oil	1.796	1.868	1.951	2.035	2.118	2.206	2.294
Water	4.217	4.184	4.178	4.184	4.195	4.214	4.239

$$\tau = \mu \dot{\gamma}, \quad (1.100)$$

where τ is the shear stress within the fluid, and $\dot{\gamma}$ is the rate of shear strain. *Note:* Equation (1.100) has been shown to be valid for both liquids and gases.

Figure 1-6 shows two parallel plates separated by a thin film of fluid. The bottom plate is stationary, whereas the top plate is being pulled across the fluid medium at a velocity given by U . From experiments, it has been observed that the fluid “sticks” to the surfaces of both plates so that the fluid velocity at the bottom plate is zero and the fluid velocity at the top plate is given by U . Between the two plates, the fluid velocity u varies as a function of y . Two particles p_1 and p_2 are shown to be moving within the fluid medium. At one instant in time these particles are perfectly in line with each other, whereas at the next instant in time one particle is ahead of the other. The second instant in time is shown in Figure 1-6 using the primed notation. The angle $\delta\gamma$ formed by the triangular relationship of these particles is called the *shear strain of the fluid*, and this definition is common with the language of solid mechanics as well. From geometry, it can be shown that

$$\delta\gamma \approx \tan(\delta\gamma) = \frac{\delta x}{\delta y}. \quad (1.101)$$

From the motion of the fluid particles, it is clear that the position of each particle at the second instant in time is given by

$$x'_1 = u_1 \delta t \quad \text{and} \quad x'_2 = u_2 \delta t, \quad (1.102)$$

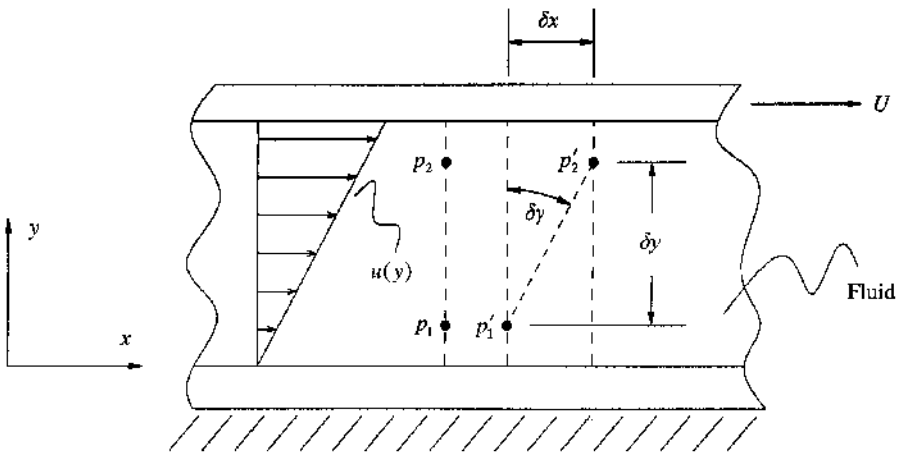


Figure 1-6. A schematic illustrating the shear strain within a fluid.

where u_1 and u_2 are the linear velocities of particles 1 and 2, respectively, and δt is the separation of time between the two instances. Subtracting the second position of particle 1 from the second position of particle 2 yields the following result for their relative displacement at the second instant in time:

$$\delta x = \delta u \delta t, \quad (1.103)$$

where δu is given by $u_2 - u_1$. Substituting Equation (1.103) into Equation (1.101) yields the following relationship between the fluid shear strain, time, fluid velocity, and the dimensional variable y :

$$\frac{\delta \gamma}{\delta t} = \frac{\delta u}{\delta y}. \quad (1.104)$$

In the limit as δt and δy go to zero, it can be shown that

$$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{du}{dy}, \quad (1.105)$$

where $\dot{\gamma}$ is the rate of shear strain, and du/dy is the velocity gradient in the y direction. Using Equations (1.100) and (1.105), it may now be shown that the shear stress within the fluid is given by

$$\tau = \mu \frac{du}{dy}. \quad (1.106)$$

From this result we define the absolute fluid viscosity as

$$\mu \equiv \frac{\tau}{\partial u / \partial y}. \quad (1.107)$$

In accordance with this definition, a plot of shear stress versus the rate of shear strain should be linear where the slope is equal to the viscosity. The actual value of the viscosity depends on the type of fluid, and for a particular fluid, it is also highly dependent on the temperature. Fluids for which the shear stress is linearly related to the rate of shear strain are called *Newtonian fluids*. Fluids for which the shear stress is not linearly related to the rate of shear strain are designated as *non-Newtonian fluids*. The kinematic viscosity is defined as $\nu = \mu/\rho$, where ρ is the fluid density. Physically speaking, the viscosity of the fluid is a type of friction coefficient associated with the motion of the fluid itself.

1.5.2 Viscous Drag Coefficient

The viscous drag coefficient is a lumped parameter that is used commonly in the modeling of dynamic systems to describe energy dissipation effects. This model is based on the principle of fluid shear and therefore is appropriately discussed in this section. If we consider the two plates shown in Figure 1-6, it is apparent that a certain force is required to drag the top plate across the fluid film that separates the two plates from each other. This drag force may be written as

$$F = \tau A, \quad (1.108)$$

where τ is the fluid shear stress at the contact surface of the top plate, and A is the surface area of contact at this plate. Using Equation (1.106), it may be shown that the shear stress at the top plate is given by $\mu U/h$, where μ is the fluid viscosity, U is the sliding velocity of the top surface, and h is the fluid film thickness that exists between the two plates. Using this expression for the fluid shear, the following expression may be written for the viscous drag force on the top plate:

$$F = cU, \quad (1.109)$$

where the viscous drag coefficient is given by

$$c = \frac{\mu A}{h}. \quad (1.110)$$

From this expression, it may be seen that the viscous drag coefficient increases with fluid viscosity and the surface contact area A . Furthermore, this coefficient decreases as the fluid film thickness h increases. Within the machinery that is used to operate hydraulic control systems, the fluid film thickness between moving parts is often on the order of microns; therefore, in a number of applications, the viscous drag coefficient becomes an important parameter for describing viscous energy dissipation.

1.5.3 Viscosity Charts and Models

Viscosity is a bulk property of a fluid that generally is measured rather than being predicted analytically. A chart showing this measured data for typical SAE oils is given in Figure 1-7. A very common hydraulic fluid is an SAE 10W fluid. Units of absolute viscosity are shown in Figure 1-7 in pascal-seconds (Pa s). It is also common to report the absolute viscosity in the English system in micro-Reyn (μReyn) and in the SI system as centipoise (cP). These units are related to each other according to the following definitions:

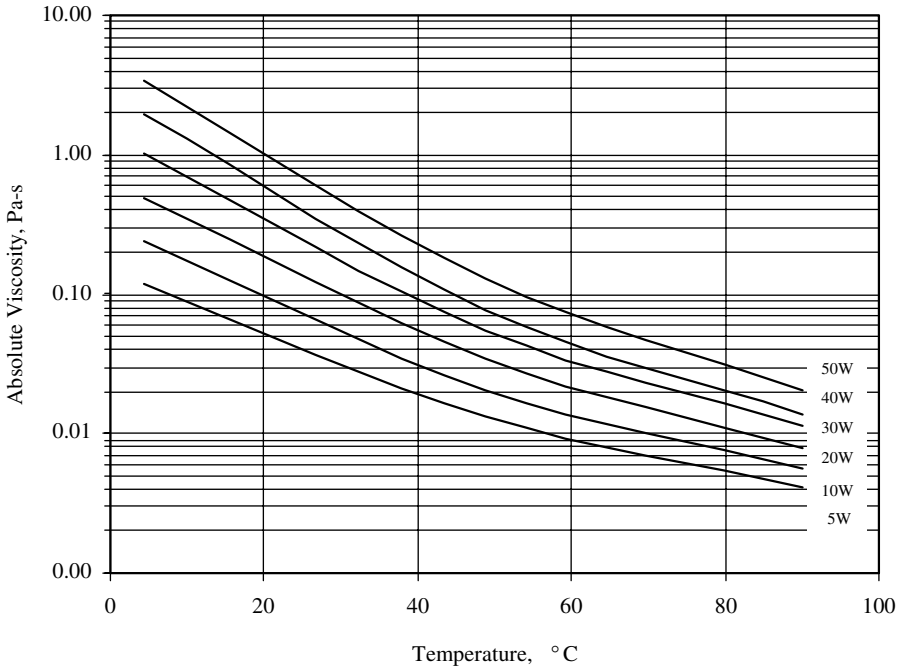


Figure 1-7. Fluid viscosity for SAE grade fluids.

$$1 \times 10^6 \mu\text{Reyn} = 1 \text{ psi-s}, \quad 1 \times 10^3 \text{ cP} = 1 \text{ Pa-s}. \quad (1.111)$$

As with all bulk properties of hydraulic fluids, the viscosity of the fluid varies significantly with exposed conditions. For instance, the absolute viscosity of a liquid tends to decrease markedly with temperature and increase mildly with pressure. Both these variations occur exponentially, and their combined effects may be written mathematically as

$$\mu = \mu_o \exp[\alpha(P - P_o) - \lambda(T - T_o)], \quad (1.112)$$

where μ_o is the fluid viscosity at a pressure and temperature given by P_o and T_o , and α and λ are constants for a particular fluid. *Note:* α in this expression is not the coefficient of thermal expansion. For mineral oils, it has been shown empirically that the pressure coefficient is related to the viscosity at zero gauge pressure by the dimensional relationship

$$\alpha = [0.6 + 0.965 \log_{10}(\mu_o)] \times 10^{-8}, \quad (1.113)$$

where μ_o is measured in centipoise, and the units of α are in pascals^{-1} [5]. Figure 1-7 shows a chart of the fluid viscosities for SAE grade fluids as they vary with temperature at standard atmospheric pressure. The viscosity index

is used to describe how quickly the viscosity drops with temperature. If the viscosity does not decrease rapidly with temperature, the fluid is said to have a *high* viscosity index. On the other hand, if the fluid viscosity drops off rapidly with temperature, the fluid is said to have a *low* viscosity index.

This subsection has focused on the viscosity of mineral oils because they are used predominantly to operate hydraulic control systems. However, other fluids may be used as well, and viscosity data for those fluids may be obtained from the fluid manufacturers. Regardless of the fluid type that is used, it is recommended that fluid viscosities remain within the following range for the continuous operation of a hydraulic control system:

$$7.8 \text{ cP} < \mu < 95.7 \text{ cP} \quad \text{and} \quad 1.1 \mu\text{Reyn} < \mu < 13.8 \mu\text{Reyn}. \quad (1.114)$$

The optimal operating viscosity is given by $\mu = 11.3 \text{ cP} = 1.6 \mu\text{Reyn}$. Maximum cold-start viscosities are recommended to be less than 1390 cP (201 μReyn), and minimum intermittent viscosities are recommended to be greater than 5.0 cP (0.7 μReyn).

1.6 VAPOR PRESSURE

There is a combination of pressure and temperature at which all liquids will tend to vaporize and change phase. The pressure at which this occurs is called the *vapor pressure* of the liquid and must be specified at a given temperature. For instance, water tends to vaporize (boil) at atmospheric pressure when its temperature reaches 100°C. This is to say, the vapor pressure of water at 100°C is 101 kPa absolute. If the pressure drops below 1 atm (101 kPa), then water will vaporize at a lower temperature. Similarly, if the pressure increases above 1 atm, water will vaporize only at some temperature above 100°C. This discussion of vapor pressure is important because vapor bubbles do not tend to dissolve in solution like air; rather, vapor bubbles collapse. When a vapor bubble collapses near a surface, it can cause erosion and extreme damage to mechanical parts of hydraulic machinery. This phenomenon is familiarly referred to as *cavitation*, a term coined by R. E. Froude (1810–1879). Due to the localized reduction in pressure that tends to occur in high-velocity jet streams, vapor bubbles are formed at moderate temperatures within hydraulic systems. Once these bubbles are exposed to a higher pressure, say, when the fluid velocity is reduced, the bubbles collapse with violent motions that create large transient pressures. If the collapse occurs near a surface, these transient pressures can be large enough to fail the surface by inducing high stresses in the solid material. As a result of repeated exposures to cavitating conditions, the surface eventually will erode, and before long, the structural part may fail completely. The vapor pressure for petroleum-based fluids operating near 65°C is extremely low, i.e., less than 350 Pa absolute [6]. Due to the high percentage of water contained in water-glycol solutions and water-in-oil emul-

sions, these fluids have a much higher vapor pressure than that of petroleum-based fluids.

1.7 CHEMICAL PROPERTIES

In this section various terms that are used commonly to describe fluid chemical properties are defined. These terms do not have quantitative definitions as given in the previous discussions; however, their qualitative meanings are of use and therefore are included here [1,7,9].

Emulsivity. This quality describes a fluid's ability to form emulsions. An emulsion is a fluid that is formed by suspending oily liquid in another liquid, often by means of a gummy substance.

Lubricity. This quality of a fluid describes the "oiliness" of the fluid and refers to its adequacy when used as a lubricant. Many oils naturally contain some molecular species with boundary lubricating properties. Some vegetable oils such as castor oil and rapeseed oil contain more natural boundary lubricants than mineral oils. Additives therefore usually are incorporated into mineral oils for the purpose of improving the lubricity. Lack of adequate lubrication properties promotes wear and shortens the life of hydraulic components.

Thermal stability. This quality of a fluid describes the fluid's ability to resist chemical reactions and decomposition at high temperatures. Fluids react more vigorously as temperature is increased and may form solid reaction products.

Oxidative stability. This quality of a fluid describes an ability to resist reactions with oxygen-containing materials, especially air. Again, these reactions may form solid by-products within the fluid.

Hydrolytic stability. This quality of a fluid describes an ability to resist reactions with water. Undesirable formation of solids may result, or a stable water-in-oil emulsion may be formed that degrades lubricating ability and promotes rusting and corrosion. Demulsifier additives are used often to inhibit emulsion formations.

Compatibility. This quality of a fluid is a "catch all" that describes the fluid's ability to resist chemical reactions with any material that may be used in the system to which it is exposed. For instance, some fluids tend to soften seals and gaskets, which may cause them to be incompatible. Water is incompatible with steel because it causes the steel to corrode.

Foaming. This term is used to describe a fluid's ability to combine with gases, principally air, and to form emulsions. Entrained air reduces the lubricating ability and bulk modulus of a liquid. A reduction in the bulk modulus can severely limit dynamic performance, and for this reason,

fluids should have the ability to release air without forming emulsions. Antifoamant additives are used to encourage this ability.

Flash point. This is the lowest temperature at which the vapor of a volatile oil will ignite with a flash.

Pour point. This is the lowest temperature at which a fluid will flow.

Handling properties. These properties refer to the toxicity, odor, color, and storage characteristics of a fluid. These characteristics can be dangerous or annoying, thereby making the handling or use of the fluid somewhat undesirable.

1.8 FLUID TYPES AND SELECTION

1.8.1 Petroleum-Based Fluids

Petroleum-based fluids are by far the most common fluids used in hydraulic systems. They are a complex mixture of hydrocarbons that must be refined to produce a fluid with the appropriate characteristics that are suitable for hydraulic control systems. Various additive packages are sold for petroleum-based fluids. These packages include inhibitors against oxidation, foaming, and corrosion. Additives can be used to increase the viscosity index and to improve the lubricity of a fluid as well.

1.8.2 Synthetic Fluids

Synthetic fluids are used to provide a fire-resistant alternative to petroleum based fluids and are named after their base stock, which is the predominant material used to make them. Examples of such fluids are phosphate esters and silicate esters.

1.8.3 Biodegradable Fluids

Biodegradation is the ability of a substance to be broken down into innocuous products by the action of living things. Due to environmental concerns, biodegradable fluids have become an important alternative for use in hydraulic control systems. This is especially true in the mobile hydraulic industry, where an unintentional spill of mineral oil may result in long-term soil and water contamination in the vicinity of the spill itself. Various types of biodegradable fluids are available for use. The most basic forms of biodegradable fluids are the vegetable oils, especially those extracted from rapeseed. Other biodegradable fluids with higher performance are ester-based synthetic fluids. While the ester-based synthetic fluids provide a wider and more robust range of performance (especially at cold temperatures), they are very costly, which prohibits their use in many applications.

1.8.4 Water

Water hydraulics is a growing topic of discussion among the engineering community. The reasons for this are somewhat obvious. Water is available in abundance and therefore is very inexpensive. Water is also very friendly to the environment and therefore alleviates the concerns for contamination in the event of a hydraulic failure or spill. On the other hand, water has several disadvantages that must be overcome before it is used widely as a medium for transmitting power hydraulically. Among these disadvantages, water has the ability to sustain life in the form of bacterial growth. This growth causes inherent contamination within the hydraulic system, which, in turn, may cause a failure. Another disadvantage of water is that it has a very narrow temperature range between phase changes. Water freezes at 0°C and boils at 100°C (at standard atmospheric pressure). This narrow range of temperature over which water will remain liquid is not easily overcome by many applications. Water also exhibits poor lubricity and low viscosity. These characteristics make water a poor lubricating medium which is often necessary for maintaining a longer life for hydraulic systems. Another disadvantage of water is that it is corrosive to the ferrous materials that are used commonly to build hydraulic machinery. In order to use water as the working medium, machine parts must be coated with polymer-type materials, which means that they must be subject to low surface stresses to keep the lower-strength polymers from failing. This drives the hydraulic system toward low-pressure applications and causes the power density of the system to be sacrificed.

1.8.5 Fluid Selection

When selecting a hydraulic fluid, one must keep in mind the following characteristics:

1. The fluid must exhibit good lubricity with compatible materials that are used for bearings and sealing surfaces.
2. The fluid must exhibit a high viscosity index over a wide range of operating temperatures.
3. The fluid must provide a long service life (at least 5000 hours). This means that it must be stable against heat, water, oxidation, and shear.
4. The fluid must be compatible with environmental requirements. This may or may not require the use of biodegradable fluids.
5. The fluid must have a high bulk modulus for a satisfactory dynamic response of the hydraulic system. Generally, this means that the fluid should resist absorption of air and exhibit a low tendency to foam.
6. The fluid generally must be low cost and highly available.
7. The fluid must exhibit a resistance to flammability.
8. The fluid should exhibit a high thermal conductivity for the purpose of transferring heat away from the system.

9. The fluid should exhibit a low vapor pressure and a high boiling temperature to avoid cavitation.

1.9 CONCLUSION

In conclusion, this chapter has considered the various physical properties that are commonly discussed for hydraulic fluids. These properties have been related to the mass density of the fluid, its thermal characteristics, the fluid viscosity, and other important topics of cavitation potential and chemical compatibilities. It must be mentioned that this brief overview is far from exhaustive, and the reader is referred to sundry texts that have fluid property data scattered throughout. For the remainder of this text, the fluid properties mentioned here will be referred to and used for describing the overall performance of hydraulic control systems. The following chapter on fluid mechanics will assume a working knowledge of the terms and definitions that have been presented here.

1.10 REFERENCES

- [1] Merritt, H. E. 1967. *Hydraulic Control Systems*. John Wiley & Sons, New York.
- [2] Boresi A. P., R. J. Schmidt, and O. M. Sidebottom. 1993. *Advanced Mechanics of Materials*, 5th ed. John Wiley & Sons, New York.
- [3] Doebelin, E. O. 1990. *Measurement Systems: Application and Design*, 4th ed. McGraw Hill, New York, NY.
- [4] Manring, N. D. 1997. The effective fluid bulk-modulus within a hydrostatic transmission. *ASME Journal of Dynamic Systems, Measurement, and Control* 119:462–66.
- [5] Hutchings, I. M. 1992. *Tribology: Friction and Wear of Engineering Materials*. CRC Press, London.
- [6] Esposito, A. 2000. *Fluid Power with Applications*, 5th ed. Prentice-Hall, Upper Saddle River, New Jersey.
- [7] Yeaple, F. 1996. *Fluid Power Design Handbook*, 3d ed. Marcel Dekker, New York.
- [8] United States Army Material Command. 2000. *Engineering Design Handbook: Hydraulic Fluids*. University Press of Hawaii, Honolulu.
- [9] Radhakrishnan, M. 2003. *Hydraulic Fluids: A Guide to Selection, Test Methods and Use*. American Society of Mechanical Engineers, New York.

1.11 HOMEWORK PROBLEMS

1.11.1 Fluid Mass Density

- 1.1 The density of water at the standard reference state (pressure = 1 atm., temperature = 25°C) is 1000 kg/m³. Calculate the water's fluid density

when the pressure is increased to 20 MPa and the temperature is increased to 80°C. *Note:* At the reference state, the bulk modulus of water is 22 kbar, and the coefficient of thermal expansion is $200 \times 10^{-6}/^\circ\text{C}$.

- 1.2** The density of hydraulic fluid at atmospheric pressure and a temperature of 100°C is 842 kg/m³. Calculate the percent change in the fluid's density when the pressure is increased to 40 MPa and the temperature is reduced to 0°C. *Note:* At the original state, the bulk modulus of the hydraulic fluid is 12 kbar, and the coefficient of thermal expansion is $7 \times 10^{-4}/^\circ\text{C}$.
- 1.3** An open container contains 1 gal of hydraulic fluid weighing 7 lbf. The temperature of the hydraulic fluid is increased by 100°F, and the volume of fluid increases to 1.01 gal. What are the original and final densities of the hydraulic fluid? What is the coefficient of thermal expansion for this fluid?

1.11.2 Fluid Bulk Modulus

- 1.4** It is shown from experiments that for a fixed mass of water-glycol solution, the fluid pressure varies according to the following equation:

$$P = \frac{K_o(V_o - V)}{V_o - m(V_o - V)},$$

where K_o and m are experimental constants, V_o is the fluid volume when the pressure is zero, and V is the fluid volume associated with the pressure P . Using the definition of for the secant bulk modulus K and the tangent bulk modulus β , develop an expression for each. What is the percent difference between these two results for $m = 4.5$ and $V = 0.98V_o$?

- 1.5** Calculate the effective fluid bulk modulus for mineral oil with 1% of entrained air (by volume) inside a brass tube with an outside diameter of 20 mm and a wall thickness of 2.5 mm. Assume that the fluid pressure is 2100 kPa and that the operating temperature is 90°C. All processes are assumed to be isothermal.
- 1.6** A measurement is taken in the laboratory to determine the effective fluid bulk modulus of a fluid within a container similar to what is shown in Figure 1-4. A 6000-psi pressure transducer (accurate within $\pm 1.5\%$ of its full scale reading) is used with a 1/2-in. linear variable differential transformer (LVDT) (accurate within $\pm 1.5\%$ of its full scale reading) to make this measurement. The original depth of the container is 1 ft \pm 0.1 in. The experimental records are given in the following table:

Fluid Pressure P , psi	Displacement x , in.	Bulk Modulus β , psi	Maximum Error ε_{\max}
1500	0.203		
3000	0.274		
4500	0.336		
6000	0.389		

Using these data, fill in the remainder of the table for the measured fluid bulk modulus and the uncertainty associated with the measurement. At what point would you begin to consider the measurements to be useful?

1.11.3 Thermal Fluid Properties

- 1.7** Calculate the effective coefficient of thermal expansion for a hydraulic fluid with 3% entrained air within a high-pressure hose. The operating temperature is 40°C.
- 1.8** A heat flux of 550 W/m² is measured across a stagnant film of hydraulic fluid 2.54 mm thick. The temperature drop across the fluid film is 10°C. Calculate the coefficient of thermal conductivity for the hydraulic fluid

1.11.4 Fluid Viscosity

- 1.9** Using Figure 1-7, determine the fluid viscosity of a 10W petroleum-based fluid at an operating temperature of 50°C. Using Equation (1.111), convert this result to the μReyn and the centipoise scales.
- 1.10** A 10W hydraulic fluid is nominally used at 70°F and atmospheric pressure. Using Equation (1.112) and Figure 1-7, calculate the fluid property λ if the temperature is increased to 200°F. If the pressure is also increased to 6000 psi, what is the new viscosity? Does the pressure have much effect in your opinion?