1

POLARIZED LIGHT

1.1 INTRODUCTION

A normal beam of light in isotropic material consists of many individual waves, each vibrating in a direction perpendicular to its path. Measurable intensities therefore refer to a superposition of many millions of waves. Normally, the vibrations of each ray have different orientations with no favored direction. In some cases, however, all the waves in a beam vibrate in parallel planes in the same direction. Such light is said to be polarized, that is, to have a directional characteristic. More specifically, it is said to be linearly polarized, to distinguish it from circularly and elliptically polarized light (to be discussed later). Light from familiar sources such as a light bulb, the Sun, or a candle flame is unpolarized, but can easily become polarized as it interacts with matter. Such light is called natural light. Reflection, refraction, transmission, and scattering all can affect the state of polarization of light. The human eye cannot easily distinguish polarized from natural light. This is not true for all animals (Horváth, 2004). In fact, light from the sky is considerably polarized (Minnaert, 1954) as a result of scattering, and some animals, such as bees, are able to sense the polarization and use it as a directional aid. A major industrial use of polarized light is in photoelastic stress analysis (Föppl, 1972; Rohrbach, 1989; Dally, 1991). Models of mechanical parts are made of a transparent plastic, which becomes birefringent when stressed. Normal forces are applied to the model, which is then examined between polarizers. Between crossed polarizers, unstressed regions remain dark; regions under stress change the polarization of light so that light can be transmitted. Figure 1.1 shows an example of such stressed plastic parts
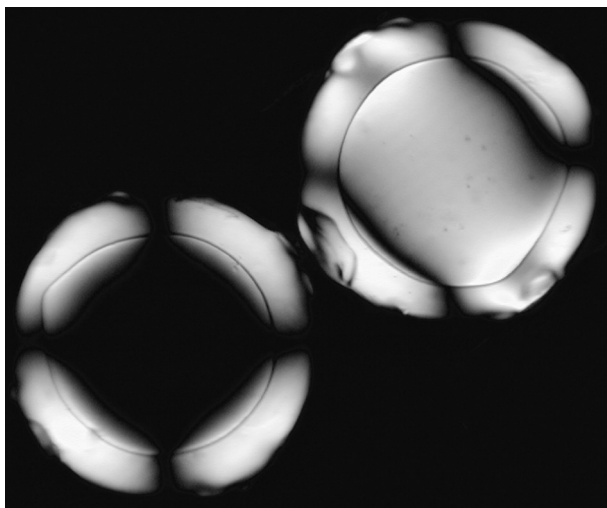
Figure 1.1   Plastic disks of 1 mm diameter arranged between crossed polarizers. The disks are microfabricated by photolithography in a photo-polymerizable isotropic material (SU8). The polarizer and analyzer are horizontally and vertically oriented respectively. The left disk is not stressed, but the right is pressed together from top left and bottom right. The stress produces birefringence that cerates interference colors.

between crossed polarizers. Disks of 1 mm diameter are shown. One is compressed and shows characteristic interference fringes due to stress. When white light is used, each wavelength is affected differently; the result is a highly colored contour map showing the magnitude and direction of the stresses. In Figure 1.1, the points where mechanical force is applied become visible as bright spots on the top-left and bottom-right positions. Another way to influence the polarization of light is optical activity. Many compounds are optically active; that is, they have the ability to rotate the plane of plane-polarized light. This property can be a molecular property and may be used to measure the concentrations of such compounds in solutions. More examples can be found in work by Pye (2001), Shurcliff (1962) and Minnaert (1954). Today, with advanced methods for measuring light fields now being available, polarized light still attracts a lot of attention. Recently, a discussion on singularities of light fields has led to a more intense discussion on polarization states and their propagations (Berry, 1999, 2003).

1.2   CONCEPT OF LIGHT POLARIZATION

Polarization is a property that is common to all types of vector waves. Electromagnetic waves also possess this property (Weller, 1979). For all types of vector waves, polarization refers to the behavior with time of one of the field vectors appropriate to that wave, observed at a fixed point in space. Light waves are electromagnetic in nature and require four basic field vectors for their complete description: the

electrical field strength **E**, the electric displacement density **D**, the magnetic field strength **H**, and the magnetic flux density **B**. Of these four vectors the electric field strength **E** is chosen to define the state of polarization of light waves (Born, 1993). This choice is based on the fact that, when light interacts with matter, the force exerted on the electrons by the electric field of the light waves is much greater than the force exerted on these electrons by the magnetic field of the wave. In general, once the polarization of **E** has been determined, the polarization of the three remaining vectors **D**, **H**, and **B** can be found, because the field vectors are interrelated by Maxwell's field equations and the associated constitutive (material) relations. In the following we will focus our attention on the propagation of light as defined by the behavior of the electric field vector $\mathbf{E}(\mathbf{r}, t)$ observed at a fixed point in space, **r**, and at time, t. In general the following statement holds: The change of polarization properties of light is initiated by a symmetry break while light propagates in a certain direction. This symmetry break can be made simply by the geometry of obstacles in the propagation path, by high dimensional order, or by anisotropy on a molecular or atomic level. Imagine a wave traveling in space (vacuum) in a direction described by a wave vector **k** as shown in Figure 1.2. If such a wave hits a surface of an isotropic body of different refractive index at normal incidence, the symmetry with respect to the propagation direction is preserved and the state of polarization is not changed. The situation becomes different when the incidence is no longer normal to the surface. Now, a projection of the different vector components of the electromagnetic field has to be performed. This leads to different equations for reflection and transmission coefficients and finally to the Fresnel equations (see Chapter 3 for details), which allow the calculation of the change of the polarization state. Examples for highly ordered systems with anisotropy are crystals, liquid crystalline phases, and ordered polymers. Here, even for normal incidence, the anisotropy of the material can lead to a different interaction

(a)   (b)   (c)
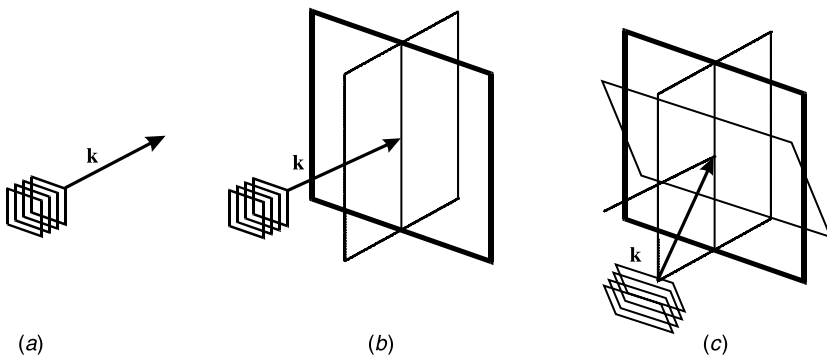
Figure 1.2   A plane wave traveling in direction **k**. (a) The planes of constant phases (wavefronts) are indicated with the small layers. If the wave hits an interface normal as in (b), the symmetry of the problem is maintained. For isotropic materials no change of the polarization state of light is expected. If the light train has an oblique incidence as in (c) a change will be observed that leads to different reflection and transmission properties for differently polarized light trains.

of the vector components of the incident light: Projection of the electric field vectors on the symmetry axes of the system is needed to describe light propagation correctly. Matrix methods are convenient in this case to describe the change of polarization, for example the Jones matrix formalism (Chapter 4). Nanostructured materials with structures smaller than the wavelengths of light also fall into this category. An example is zero-order gratings with their particular polarization properties (Herzig, 1997). Light scattering also leads to polarization effects. For example, according to Rayleigh's classical scattering laws, any initially unpolarized beam becomes polarized when scattered. For this reason, the diffuse scattered light from the Sun in the atmosphere is partially polarized (Minnaert, 1954). On the molecular level, optical activity is an exciting case of interaction of polarized light with anisotropic molecules. Such a phenomenon is observed in sugar in solutions. The description is made by using a particular reference frame that is based on special states of light polarization (circularly polarized light). Optical activity is found naturally in crystals and can also be induced with electrical and magnetic fields and through mechanical stress.

Polarized light interaction happens on every length scale and is therefore responsible for a multitude of different effects. A description is particular difficult if all kinds of mechanisms overlap.

1.3   DESCRIPTION OF THE STATE OF POLARIZATION

To describe a general radiation field, four parameters should be specified: intensity, degree of polarization, plane of polarization, and ellipticity of the radiation at each point and in any given direction. However, it would be difficult to include such diverse quantities as intensity, a ratio, an angle, and a pure number in any symmetrical way in formulating the equation of propagation. A proper parametric representation of polarized light is therefore a matter of some importance. The most convenient representation of polarized light uses a set of four parameters, introduced by Sir George Stokes in 1852. One standard book on polarization optics (Goldstein, 2003) is based on this formulation and offers a deep insight into the formalisms by giving examples. It seems advantageous to use a description of light polarization that is linked to a measurement scheme. That is the case for the Stokes formalism. Assume for a moment that one has tools to separate the linear polarized (of different directions) and circular polarized light (of different sense of polarizations) from an incident light beam. If we know the direction of propagation of the light we are able to determine the properties related to polarization. Four values have to be measured to identify the state of polarization (including the ellipticity), the direction of the ellipse, and the degree of polarization (Gerrard, 1994). The scheme introduced by Stokes is based on the measurement of intensities by using ideal polarization components. Measurement of the total intensity I is performed without any polarizing component. Next, three intensities have to be measured when passing through an ideal polarizer (100% transmission for linear polarized light, 0% for extinction) at $0°$ $45°$ and $90°$ orientation, respectively. The coordinate system is fixed with respect to the direction of propagation. The last measurement uses a

circular polarizer. All these measurements, together, allow the determination of the Stokes parameters S_0, S_1, S_2, S_3. The degree of polarization is given by comparing the total intensity with the sum of the ones measured with polarizers. The direction of the polarization ellipse can be found by analyzing the measurement with linear polarizers, carried out under different angles of the polarizer. The sense of rotation and the ellipticity is accessible when all measurements with polarizers are considered. To obtain this in a more quantitative manner, we start with a description of a transversal wave and the polarization states following the description in the work of Chandrasekhar (1960). We assume propagation in an isotropic material. The polarization state description is closely related to the propagation direction. In a ray model the propagation direction is easily defined as the vector of the ray direction. To have easy access to the main parameters of light polarization one assumes that the propagation direction is known and defines a coordinate system. Let z be the direction of propagation and the **k** vector. Then the two components of the electrical field can be assigned with a phase and amplitude such that

$$E_x = E_{x0} \sin(\omega t + \varphi_x) \qquad \text{and} \qquad E_y = E_{y0} \sin(\omega t + \varphi_y), \qquad (1.1)$$

where E_x and E_y are the components of the vibration along directions x and y, at right angles to each other, ω is the angular frequency of vibration, and E_{x0}, E_{y0}, φ_x, and φ_y are constants. Figure 1.3 presents the geometrical arrangement. The field components vary in time with a certain phase shift and describe an ellipse. If the principal axes of the ellipse described by (E_x, E_y) are in directions making an angle Θ and $\Theta + \pi/2$ to the direction x, the equations representing the vibration take the simplified forms

$$E_\Theta = E_0 \cos \beta \sin \omega t \qquad \text{and} \qquad E_{\Theta+\pi/2} = E_0 \sin \beta \cos \omega t, \qquad (1.2)$$

where β denotes an angle whose tangent is the ratio of the axis of the ellipse traced
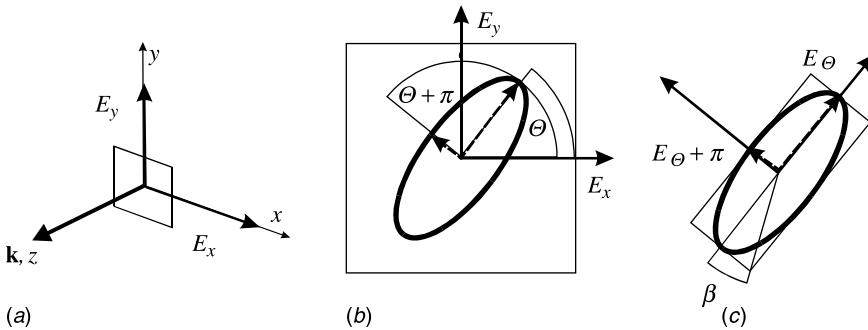
Figure 1.3   To a plane wave traveling in the *z*-direction with the wave vector **k**, one can assign a plane with **k** as the normal vector, which contains the field components of the electric field E_x and E_y.

by the end point of the electric vector as shown in Figure 1.3. We shall suppose that the numerical value of β lies between 0 and $\pi/2$ and that the sign of β is positive or negative according to whether the polarization is right-handed or left-handed. E_0 denotes a quantity proportional to the mean of the electric field vector, whose square is equal to the intensity $\hat{I}$ of the beam:

$$\hat{I} = E_0^2 = E_{x0}^2 + E_{y0}^2 = \hat{I}_x + \hat{I}_y. \tag{1.3}$$

Note that the intensity $\hat{I}$ introduced here is not the intensity measured by a detector. A detector measures time averaged values of the electric fields that is half of $\hat{I}$ because of the time average of the trigonometric functions in Equation (1.1).

The formulas connecting the representations of polarized light with four intensities S_0, S_1, S_2, and S_3 are important and can be combined in the following manner. The electrical field components of the field amplitude E_Θ and $E_{\Theta+\pi/2}$ in the x- and y-directions are summed (see Fig. 1.3). Starting from the representation in Equation (1.2) we obtain for the vibrations in the x- and y-direction the expressions

$$E_x = E_0(\cos\beta\cos\Theta\sin\omega t - \sin\beta\sin\Theta\cos\omega t) \tag{1.4a}$$

$$E_y = E_0(\cos\beta\sin\Theta\sin\omega t + \sin\beta\cos\Theta\cos\omega t). \tag{1.4b}$$

These equations can be reduced to the form of Equation (1.1) by letting

$$E_{x0} = E_0\sqrt{\cos^2\beta\cos^2\Theta + \sin^2\beta\sin^2\Theta} \tag{1.5a}$$

$$E_{y0} = E_0\sqrt{\cos^2\beta\sin^2\Theta + \sin^2\beta\cos^2\Theta} \tag{1.5b}$$

and

$$\tan\varphi_x = \tan\beta\tan\Theta \quad \text{and} \quad \tan\varphi_y = -\tan\beta/\tan\Theta. \tag{1.6}$$

The intensities $\hat{I}_x$ and $\hat{I}_y$ in the directions x and y are therefore given by

$$\hat{I}_x = E_{x0}^2 = I(\cos^2\beta\cos^2\Theta + \sin^2\beta\sin^2\Theta) \tag{1.7a}$$

$$\hat{I}_y = E_{y0}^2 = I(\cos^2\beta\sin^2\Theta + \sin^2\beta\cos^2\Theta). \tag{1.7b}$$

Further, according to Equations (1.5) and (1.6) we may readily verify that

$$2E_{x0}E_{y0}\cos(\varphi_x - \varphi_y) = 2E_0^2(\cos^2\beta - \sin^2\beta)\cos\Theta\sin\Theta = \hat{I}\cos 2\beta\sin 2\Theta. \tag{1.8}$$

Similarly,

$$2E_{x0}E_{y0}\sin(\varphi_x - \varphi_y) = \hat{I}\sin 2\beta. \tag{1.9}$$

From the foregoing equations (1.3–1.7) it follows that whenever the regular vibrations representing an elliptically polarized beam can be expressed in the form of Equation (1.1), we can at once write the relations

$$S_0 = \hat{I} = E_{x0}^2 + E_{y0}^2 = \hat{I}_x + \hat{I}_y \tag{1.10a}$$

$$S_1 = E_{x0}^2 - E_{y0}^2 = I \cos 2\beta \cos 2\Theta = \hat{I}_x - \hat{I}_y \tag{1.10b}$$

$$S_2 = 2E_{x0}E_{y0}\cos(\varphi_x - \varphi_y) = \hat{I} \cos 2\beta \sin 2\Theta = (\hat{I}_x - \hat{I}_y)\tan 2\Theta \tag{1.10c}$$

$$S_3 = 2E_{x0}E_{y0}\sin(\varphi_x - \varphi_y) = \hat{I} \sin 2\beta = (\hat{I}_x - \hat{I}_y)\frac{\tan 2\beta}{\cos 2\Theta}. \tag{1.10d}$$

These are the Stokes parameters representing an elliptically polarized beam. We observe that among the quantities S_0, S_1, S_2, and S_3 defined as in Equations (1.10) there exists the relation

$$S_0^2 = S_1^2 + S_2^2 + S_3^2. \tag{1.11}$$

Further, the plane of polarization and the ellipticity follow from the equations

$$\tan 2\Theta = \frac{S_2}{S_1} \qquad \text{and} \qquad \sin 2\beta = \frac{S_3}{\sqrt{S_1^2 + S_2^2 + S_3^2}}. \tag{1.12}$$

For the representation given in Equation (1.1) one considers the amplitude and the phases to be constant. This is not realistic. The high angular frequency ω of the electromagnetic oscillation of light allows us to suppose that the phases and amplitudes may be constant for millions of vibrations and yet changes irregularly millions of times a second. In an polarized light beam these irregular variations must be such that the ratio of amplitudes and the difference of phases are absolute constant. One is able to measure the mean intensity in any direction in the transverse plane. The apparent intensities $\hat{I}_x$ and $\hat{I}_y$ will be given by their mean values of electric field components E_x and E_y. Equations (1.10) can be rewritten by using mean intensities and allows to apply the formalism to measurements.

1.4 THE STOKES CONCEPT

Using Equations (1.10), the so-called Stokes column **S** for completely polarized light is defined as

$$\mathbf{S} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} E_{x0}^2 + E_{y0}^2 \\ E_{x0}^2 - E_{y0}^2 \\ 2E_{x0}E_{y0}\cos(\varphi_x - \varphi_y) \\ 2E_{x0}E_{y0}\sin(\varphi_x - \varphi_y) \end{pmatrix}. \tag{1.13}$$
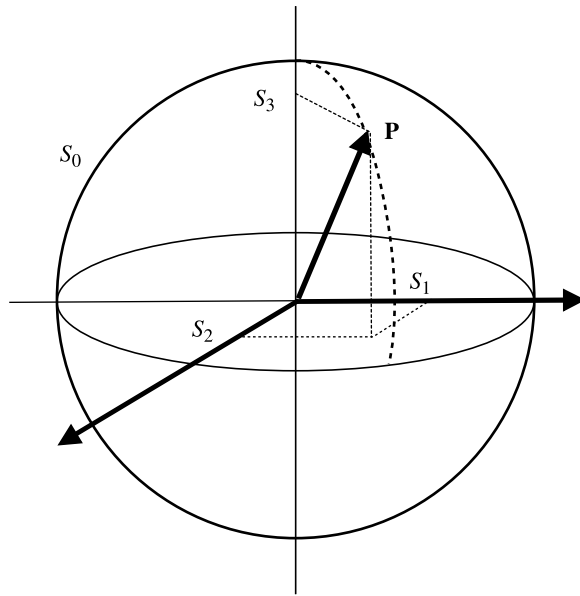
Figure 1.4   Poincaré sphere representation of the polarization states of a monochromatic wave. In most definitions the sphere has a radius of 1. One can introduce the parameter S_0 as the sphere radius to consider partially polarized light or different intensities.

The four elements of **S** are directly obtained from intensity measurements and are related by the expression

$$S_0^2 = S_1^2 + S_2^2 + S_3^2, \tag{1.14}$$

which was shown above. S_0 is the intensity of the beam, S_1, S_2, and S_3 can have any real value between $-S_0$ and $+S_0$. Equation (1.14) allows a representation of the polarization states on the surface of a sphere, which is called a Poincaré sphere as shown in Figure 1.4. Here, S_1, S_2, and S_3 may be regarded as Cartesian coordinates of a point P on a sphere of radius S_0. Every possible state of polarization of a mono-chromatic plane wave corresponds to a point on the Poincaré sphere and vice versa. The right-handed circular polarization is represented by the north pole, the left-handed polarization by the south pole, the linear polarizations by points in the equatorial plane, and the elliptical states by the points between the poles and the equatorial plane. The right-handed polarization points lie above the equatorial plane and the left-handed lie below. According to a description of the polarization by complex numbers, the ensemble of the polarization states is mapped onto the complex plane; here it is mapped onto the surface of the Poincaré sphere and it exists as a unique projection between them.

Stokes columns can also describe partially polarized light. This can be thought of as combinations of several mutually incoherent beams of different polarizations. The combination can be obtained by addition of the intensities, which are represented by the individual elements of the Stokes column. These are directly related to the

so-called coherency matrix (Goodman, 2000). For partially polarized light the condition of Equation (1.14) will no longer be fulfilled. A measure for the validity of this condition can be the parameter p, given by

$$p = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0},$$ (1.15)

which is called the degree of polarization. The degree of polarization p is equal to 1 for fully polarized light (mathematically, this is valid for all Stokes columns that are transformed directly from Maxwell columns) and equal to 0 for nonpolarized (natural) light. Stokes columns with a degree of polarization between 0 and 1 represent partially polarized light beams. Stokes vectors of partially polarized light beams can also be represented with the help of the Poincaré sphere. One can establish a matrices formalism that relates the Stokes vector of a light beam leaving an optical device with the Stokes vectors of the input beam. This matrix is called a Mueller matrix after its inventor. It is a 4×4 matrix with real elements. The Stokes vectors **S** are then transformed by

$$\mathbf{S}' = \begin{pmatrix} S_0' \\ S_1' \\ S_2' \\ S_3' \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix} \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix}.$$ (1.16)

Not every real 4×4 matrix can be a Mueller matrix **M**. There exist conditions for testing the consistence of a matrix to be a Mueller matrix (Brosseau, 1998). These conditions can be formulated as inequalities. Mueller matrices can also be specified for nondepolarizing devices and then are called Mueller–Jones matrices.

1.5   THE JONES CONCEPT

In order to characterize the propagation of plane waves of light and the effect of anisotropic optical devices, it is convenient to use matrix methods. Several matrix methods have been developed (Jones, 1941, 1942, 1947, 1948, 1956; McMaster, 1961; Born, 1993). Under the condition of completely polarized beams a 2×2 matrix method is sufficient for the description of the state of polarization. The light is represented only by its electric field and we examine a monochromatic plane wave of angular frequency ω, which propagates in direction z, perpendicularly to the x–y-plane. Then, the amplitude and phase behavior of the plane wave is determined by the two electric field vectors

$$E_x = E_{x0}e^{i(\omega t + \varphi_x)} \qquad \text{and} \qquad E_y = E_{y0}e^{i(\omega t + \varphi_y)},$$ (1.17)

from which we obtain

$$\text{Re}(E_x) = E_{x0}\cos(\omega t + \varphi_x) \qquad \text{and} \qquad \text{Re}(E_y) = E_{y0}\cos(\omega t + \varphi_y).$$ (1.18)

E_{x0} and E_{y0} are the amplitudes of the two electric fields, φ_x and φ_y their phases with a phase difference $(\varphi_y - \varphi_x)$. Note that here the definition of the phase difference is consistent with that in Section 1.3, although the absolute phases are different.

The column with the elements E_x and E_y is called the Maxwell column, and the corresponding time-independent vector

$$\mathbf{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} E_{x0}\mathrm{e}^{\mathrm{i}\varphi_x} \\ E_{y0}\mathrm{e}^{\mathrm{i}\varphi_y} \end{pmatrix} \tag{1.19}$$

is called the Jones vector. The polarization of the plane wave described by Equation (1.19) can be defined by using a complex number notation. The intensity of $\mathbf{E}$ is $\mathbf{EE}^*$ (* means the complex conjugate). The vector in Equation (1.19) has the same polarization and intensity as

$$\mathbf{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} E_{x0} \\ E_{y0}\mathrm{e}^{\mathrm{i}(\varphi_y - \varphi_x)} \end{pmatrix}. \tag{1.20}$$

There is a phase shift φ_x relative to Equation (1.19).

A polarization-dependent device can be characterized by a 2×2 matrix $\mathbf{J}$ that connects the incoming and outcoming waves in the Jones vector description by

$$\mathbf{E}_{\mathrm{out}} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}. \tag{1.21}$$

Note that the elements of $\mathbf{J}$ are in general complex. The Jones matrix describes the linear transformation of the Jones vector of a plane wave by reflection or transmission. The Jones matrix of a device that is composed of several devices in series can be obtained as the product of the Jones matrices of the individual devices. Jones matrices can be measured with special setups by analyzing the outgoing intensities for incoming beams of different polarizations.

For nondepolarizing devices these exists a mathematical correspondence between the Mueller–Jones and Jones matrices, namely

$$\mathbf{M} = \mathbf{A}(\mathbf{J} \otimes \mathbf{J}^*)\mathbf{A}^{-1}, \tag{1.22}$$

where $\mathbf{J}$ is the corresponding Jones matrix, $\mathbf{M}$ the corresponding Mueller matrix, and $\mathbf{A}$ is defined by

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix}, \tag{1.23}$$

where $\otimes$ designates the so-called Kronecker product (or outer product), which transforms two 2×2 matrices into a 4×4 matrix by multiplication of sub-

matrices component for component. The form of $\mathbf{M}$ in terms of the Jones matrix elements is

$$
\left(
\begin{array}{cc}
\frac{1}{2}\left(|J_{11}|^2 + |J_{12}|^2 + |J_{21}|^2 + |J_{22}|^2\right) & \frac{1}{2}\left(|J_{11}|^2 - |J_{12}|^2 + |J_{21}|^2 - |J_{22}|^2\right) \\
\frac{1}{2}\left(|J_{11}|^2 + |J_{12}|^2 - |J_{21}|^2 - |J_{22}|^2\right) & \frac{1}{2}\left(|J_{11}|^2 - |J_{12}|^2 - |J_{21}|^2 + |J_{22}|^2\right) \\
\mathrm{Re}(J_{11}^*J_{21} + J_{12}^*J_{22}) & \mathrm{Re}(J_{11}^*J_{21} - J_{12}^*J_{22}) \\
\mathrm{Im}(J_{11}^*J_{21} + J_{12}^*J_{22}) & \mathrm{Im}(J_{11}^*J_{21} - J_{12}^*J_{22})
\end{array}
\right.
$$
$$
\left.
\begin{array}{cc}
\mathrm{Re}(J_{11}^*J_{12} + J_{21}^*J_{22}) & -\mathrm{Im}(J_{11}^*J_{12} + J_{21}^*J_{22}) \\
\mathrm{Re}(J_{11}^*J_{12} - J_{21}^*J_{22}) & -\mathrm{Im}(J_{11}^*J_{12} - J_{21}^*J_{22}) \\
\mathrm{Re}(J_{11}^*J_{22} + J_{12}^*J_{21}) & -\mathrm{Im}(J_{11}^*J_{22} - J_{12}^*J_{21}) \\
\mathrm{Im}(J_{11}^*J_{22} + J_{12}^*J_{21}) & \mathrm{Re}(J_{11}^*J_{22} - J_{12}^*J_{21})
\end{array}
\right).
$$

$$(1.24)$$

Mueller matrices of optical devices can be determined by Stokes vector measurements for different polarizations of the incident light, also for partially polarized light. Examples for Jones and Mueller matrices of particular devices and their measurement schemes can be found in the works of Gerrard (1994) and Brosseau (1998).

1.6   COHERENCE AND POLARIZED LIGHT

One particular beauty of experiments with polarized light is the appearing of colors appearing between crossed polarizers. This is due to interference effects in polarized light. However, interference is only possible under certain circumstances. This chapter will explain conditions when interference occurs and what kind of description is adequate for handling such problems. Three topics will be discussed: the effect of the spectra on interference, the effect of the quality of the source on interference, and the conditions for interference with polarized light.

1.6.1   Spectra and Coherence

A great majority of optical sources emit light by means of spontaneous emission from a collection of excited atoms or molecules, as is the case for the Sun, incandescent bulbs, and gas discharge lamps, for example. Such radiation, consisting of a large number of independent contributions, is referred to as thermal light. This kind of light must ultimately be treated as a random process (Goodman, 2000). In the preceding chapter it was assumed that light is deterministic or "coherent." An exception was the description of partially polarized light with Stokes columns. An example of completely coherent light is a monochromatic wave with a description like $\mathbf{U} = \mathrm{Re}(\mathbf{U}_0 e^{i\omega t})$, for which the complex amplitude $\mathbf{U}_0$ is a deterministic complex valued function, for example, $\mathbf{U}_0 = \mathbf{E}_0 e^{i\mathbf{k}\mathbf{r}}$ in the case of a plane wave. The dependence of the wave function on time and space is perfectly periodic and

therefore predictable. If the light train is composed of several frequencies forming a continuous spectrum, the situation is more complicated. As a result, the temporal coherence of light influences the appearance of the interference effects. To quantify the influence one considers the spectrum of light as confined to a narrow band centered at a central frequency v_0 or wavelength λ_0. The spectral width, or linewidth, of light is the width of Δv of the spectral density $S(v)$ (Saleh, 1991). The coherence time is introduced as a parameter of characterization. A light source of broad spectrum has a short coherence time, whereas a light source with narrow linewidth has a long coherence time. For a given coherence time τ_c, the distance that light travels in this time is $l_c = c\ \tau_c$. An experiment that produces interference effects with different path differences around l_c provides information about the coherence of light. The coherence properties of light can be demonstrated by an experiment where a birefringent material of varying thickness is put between crossed polarizers. A possible experimental setup is as follows. A convex lens and a microscope slide are treated with a polyimide and rubbed to form a uniform parallel aligned liquid crystal cell of variable thickness. This cell is filled with a highly birefringent liquid crystal mixture, such as BL006 from Merck ($\Delta n = 0.28$, clearing temperature $T_{NI} = 110°C$). Such a sample is observed between crossed polarizers where the rubbing direction is set to $45°$ with respect to the analyzer. Figure 1.5
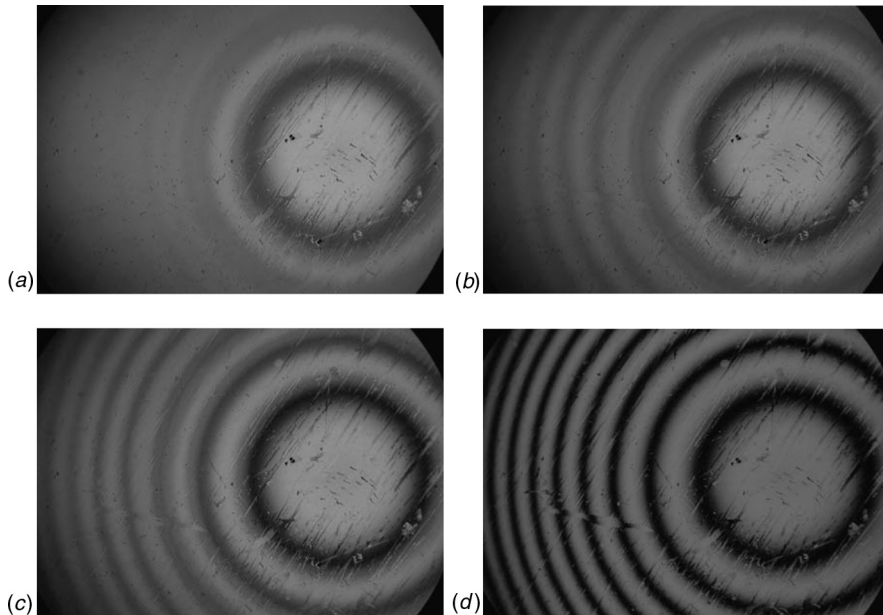

Figure 1.5 Interference effects of a varying thickness liquid crystal cell observed between crossed polarizers. The thickness increases from the center of the round fringe at the right to the left. The width of the spectra of illumination is changed. It can be observed that the contrast of the interference fringes varies for different types of spectra. The smaller the bandwidths of the spectra the higher the contrast. (*a*) White light, $\Delta\lambda = 400$ nm at $\lambda_0 = 550$ nm; (*b*) $\Delta\lambda = 200$ nm at $\lambda_0 = 550$ nm; (*c*) $\Delta\lambda = 100$ nm at $\lambda_0 = 550$ nm; (*d*) $\Delta\lambda = 5$ nm at $\lambda_0 = 550$ nm.

shows the interference fringe system that can be seen for different illumination conditions.

The coherence of a light source can be increased by using optical filters to reduce its spectral width. To illustrate this effect, Figure 1.5 shows the result of increased coherence on the interference fringes. For white light, only the central fringe appears in good contrast. If the spectrum is narrowed to 200 nm, the contrast increases remarkably and additional fringes becomes visible. Decreasing the spectral width to 100 nm and finally to 5 nm leads to pronounced fringes at the highest contrast. The visibility of fringes is a measure of the coherence of light. A plot of the intensity over position for two different spectra is shown in Figure 1.6. The contrast of fringes is highest for the bandwidth of 5 nm and much smaller for the larger spectrum of 200 nm bandwidth. The positions of the peaks of maximum intensity are not the same for the different illuminations.

In the measurements above we have used rectangle-shaped spectral functions. The spectral width is the difference in width of the smallest and highest frequencies in the spectra. But the shape of the spectral density can have a different form and a definition of the spectral width becomes necessary. There are several definitions of spectral widths. The most common is the full width at half maximum (FWHM) of the spectral density $S(\nu)$. The relation between the coherence time and spectral width depends on the spectral profile, as indicated in Table 1.1. A particular situation arises if the light has no continuous spectrum. In particular, in the case of discharge lamps with spectral lines, the definition of the coherence time needs special care. In general, it is possible to define an equivalent coherence time. In such an approach, actual coherence properties can be simulated by using weighted averages.
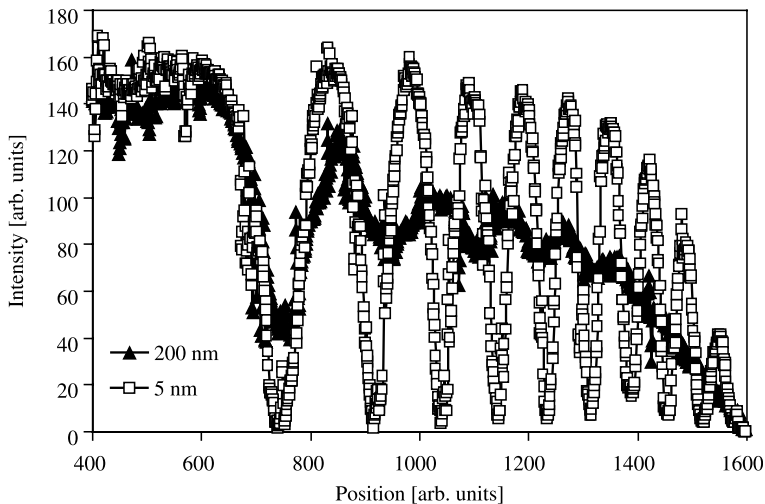
Figure 1.6 Measured intensity for two different spectral widths: 200 nm and 5 nm. The mean intensity is the same. The fringe contrast is reduced for broadband illumination with the 200 nm bandwidth.

Table 1.1   Relation between spectral width and coherence time. Saleh, Teich 1991.

Spectral Density Shape	Spectral Width $\Delta\nu_{\text{FWHM}}$
Rectangular	$\dfrac{1}{\tau_{\text{c}}}$
Lorentzian	$\dfrac{1}{\pi\tau_{\text{c}}} \approx \dfrac{0.32}{\tau_{\text{c}}}$
Gaussian	$\dfrac{(2\ln 2/\pi)^{1/2}}{\tau_{\text{c}}} \approx \dfrac{0.66}{\tau_{\text{c}}}$

1.6.2   Spatial Coherence

An additional important parameter, which determines the visibility of fringes in an interference experiment, is the spatial coherence. Light sources have in general a certain dimension. An area of coherent emission related to this is called the coherence area. The coherence area is an important parameter that characterizes random light and must be considered in relation to other dimensions of the optical system. For example, if the area of spatial coherence is greater than the size of the aperture through which light is transmitted, the light may be regarded as spatially coherent. Similarly, if the coherence area is much smaller than the resolution of the optical system, it has to be regarded as incoherent. Light radiated from an extended radiating hot surface has an area of coherence in the order of λ^2, where λ is the central wavelength. In most practical cases it must be regarded as incoherent. Interference occurs when coherent waves are superimposed. Those portions of the light field that produce interference have to be spatially coherent. The spatially coherent parts of light can be identified as spatial modes of the light field. The light field is considered as being composed of a number of such modes. These modes can have any shape. The shape depends on the boundary conditions imposed when light is generated or detected. Independent of the particular mode shape, there exists a relation between the cross-sectional area A_{M} and the solid angle Ω_{M}. For a mode at a given position this relation is always given by

$$A_{\text{M}}\Omega_{\text{M}} = \lambda^2, \tag{1.25}$$

where λ is the wavelength of the light. Equation (1.25) links properties of a spatially coherent emitting area λ^2 to parameters of a coherent light beam. As only light that is spatially coherent and belonging to the same spatial mode can interfere, the coherent power of the light is equal to the power P_{mode} in one spatial mode. P_{mode} is directly related to the radiance B,

$$B = \frac{\mathrm{d}P}{\mathrm{d}A\mathrm{d}\Omega}, \tag{1.26}$$

which is the power dP per unit area dA and unit solid angle dΩ, through

$$P_{\text{mode}} = B\Omega_M A_M = B\lambda^2. \tag{1.27}$$

The radiance B, as defined in Equation (1.26), measures the quality of light in terms of coherence. Liouville's theorem of optics states that the radiance B cannot be increased by passive optical elements such as lenses or mirrors. This means also that the coherent power of the power per mode P_{mode} cannot be increased by passive elements.

1.6.3   Coherence and Polarization

There are additional conditions to be satisfied in order to observe interference effects with polarized light. They are referred to as the rules of Fresnel and Aarago (Goldstein, 2003):

1. Two waves linearly polarized in the same plane can interfere.
2. Two waves linearly polarized with perpendicular polarizations cannot interfere.
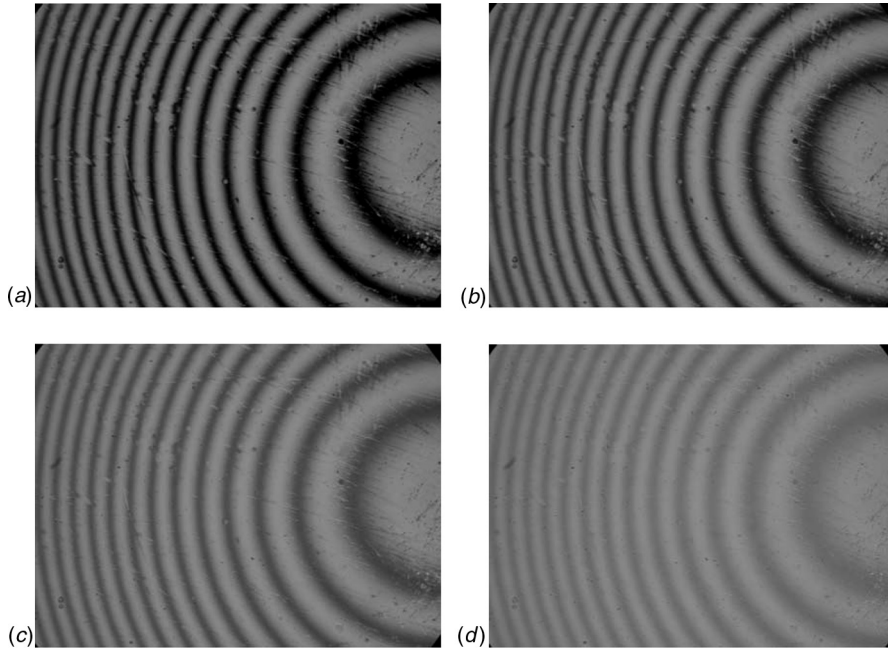
Figure 1.7   Interference fringes of a liquid crystal cell of varying thickness with planar alignment for different voltages: (*a*) 0 V; (*b*) 10 V; (*c*) 15 V; and (*d*) 30 V. The contrast of the fringes varies as a function of the degree of polarization of light. For ideal conditions, in 0 V, maximum contrast is found. Increasing the voltage reduces the degree of polarization of the incoming light and the contrast is reduced.

3. Two waves linearly polarized with perpendicular polarizations, if derived from the perpendicular components of unpolarized light and subsequently brought into the same plane, cannot interfere.
4. Two waves linearly polarized with perpendicular polarizations, if derived from the same linearly polarized wave and subsequently brought into the same plane, can interfere.

These conditions can be worked out with the Stokes formalism and it is interesting to see how this formalism applies (Goldstein, 2003). We will not treat the different cases here. We shall instead show the influence of the degree of polarization on interference with the example discussed above. Using liquid crystal display technology it is possible to build switchable polarizers with dichroic dyes diluted in liquid crystals. The performance, that is, the extinction ratio or pure transmission, of such a polarizer is not very good but still sufficient to perform the experiment. The dichroic liquid crystal cell in planar alignment with a positive dichroic dye absorbs electromagnetic energy for one polarization direction (Bahadur, 1990). As a result, the outgoing beam becomes polarized. If the absorption is not complete only a part of the concerned electrical field component is reduced in amplitude. This results in partial polarization. The degree of polarization can be changed by influencing the strength of absorption of the dichroic dye. With the dichroic liquid crystal cell this may be achieved by changing the orientation of the dichroic dye molecules in the liquid crystal host by applying a voltage (Bahadur, 1990). The result is an electrically switchable polarizer that allows the choice of degree of
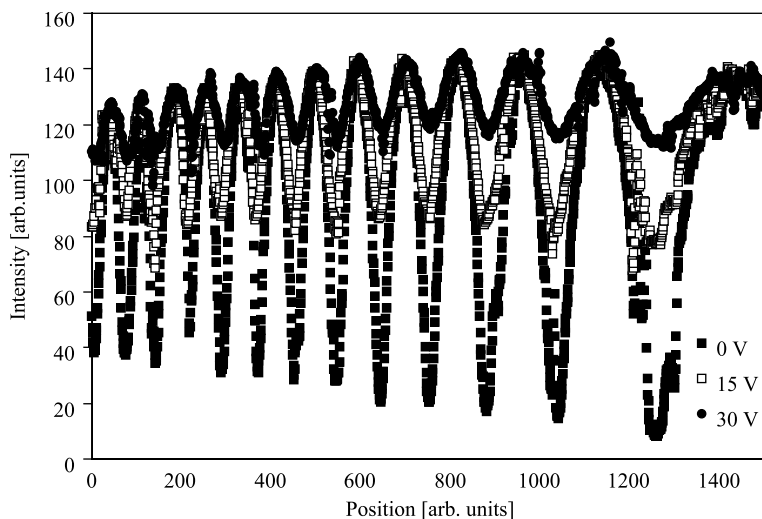
Figure 1.8   Intensity as a function of position for different voltages for the same measurement setup as described in the text. When the degree of polarization is changed, the contrast of fringes is varied but the positions of the maxima and minima remain the same. The contrast reduction from right to left is a result of scattering present in the system.

polarization. A microscope was equipped with such a polarizer to observe the inter-ference fringes of a variable thickness cell. Figure 1.7 shows the case when the cell of variable thickness is illuminated with green light at 550 nm (spectral width $\Delta\lambda = 5$ nm). For increasing voltage, the degree of polarization is reduced. In Figure 1.7 photographs of the interference fringes are shown for different voltages. Increasing the voltage leads to a decrease of contrast for the interference fringes. Only the polarized part can produce interference fringe contrast. The higher the voltage is the viewer the intensity of the polarized light becomes. The fringe contrast decreases but the overall intensity is maintained. To prove this, a plot of intensity as a function of position is given in Figure 1.8.

The positions of the maxima and minima stay the same as seen in Figure 1.8; only the contrast is changed. In comparison to Figure 1.6, the maximum intensity is always the same and the minimum changes with voltage.

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