Chapter 1

Tackling Technical Trig

In This Chapter

- Acquainting yourself with angles
- Identifying angles in triangles
- Taking apart circles

Angles are what trigonometry is all about. This is where it all started, way back when. Early astronomers needed a measure to tell something meaningful about the sun and moon and stars and their relationship between man standing on the earth or how they were positioned in relation to one another. Angles are the input values for the trig functions.

This chapter gives you background on how angles are measured, how they are named, and how they relate to one another in two familiar figures, including the triangle and circle. A lot of this material is terminology. The words describe things very specific, but this is a good thing, because they're consistent in trigonometry and other mathematics.

Getting Angles Labeled by Size

An angle is formed where two *rays* (straight objects with an endpoint that go on forever in one direction) have a common endpoint. This endpoint is called the *vertex.* An angle can also be formed when two segments or lines intersect. But, technically, even if it's formed by two segments, those two segments can be extended into rays to describe the angle. Angle measure is sort of *how far apart* the two sides are. The measurement system is unique to these shapes.

Angles can be classified by their size. The measures given here are all in terms of degrees. *Radian measures* (measures of angles that use multiples of π and relationships to the circumference) are covered in Chapter 4, so you can refer to that chapter when needed.

- **Acute angle:** An angle measuring less than 90 degrees.
- **Right angle:** An angle measuring exactly 90 degrees; the two sides are perpendicular.
- **Obtuse angle:** An angle measuring greater than 90 degrees and less than 180 degrees
- **Straight angle:** An angle measuring exactly 180 degrees.

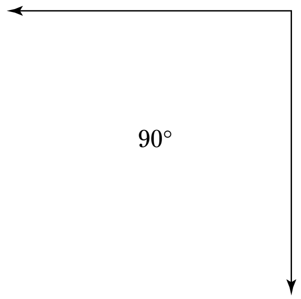
Q. Is an angle measuring 47 degrees acute, right, obtuse, or straight?

A. An angle measuring 47 degrees is acute.

Q. Is an angle measuring 163 degrees acute, right, obtuse, or straight?

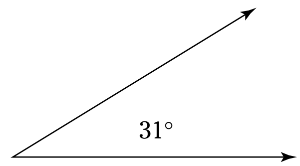
A. An angle measuring 163 degrees is obtuse.

1. What type of angle is shown in the figure?



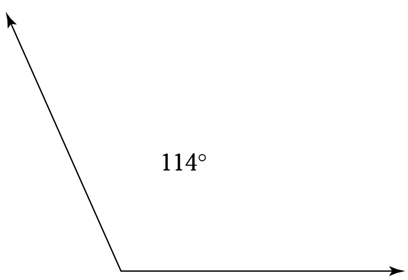
Solve It

2. What type of angle is shown in the figure?



Solve It

3. What type of angle is shown in the figure?



Solve It

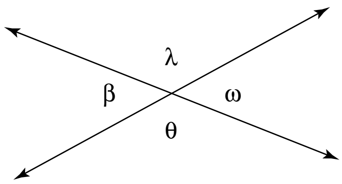
4. What type of angle is shown in the figure?

Solve It

Naming Angles Where Lines Intersect

When two lines cross one another, four angles are formed, and there's something special about the pairs of angles that can be identified there. Look at Figure 1-1. The two lines have intersected, and I've named the angles by putting Greek letters inside them to identify the angles.

Figure 1-1: Two intersecting lines form four angles.



The angles that are opposite one another, when two lines intersect, are called *vertical angles.* The special thing they have in common, besides the lines they share, is that their measures are the same, too. There are two pairs of vertical angles in Figure 1-1. Angles β and ω are vertical. So are angles λ and θ.

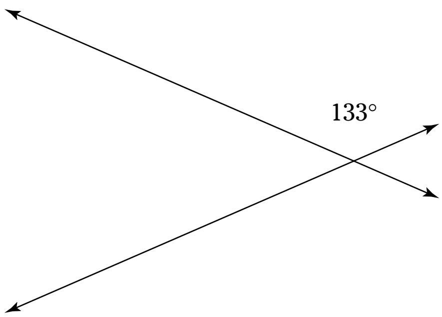
The other special angles that are formed are pairs of *supplementary angles.* Two angles are supplementary when their sum is 180 degrees. The supplementary angles in Figure 1-1 are those that lie along the same straight line with a shared ray between them. The pairs of supplementary angles are: λ and ω, ω and θ, θ and β, and β and λ.

EXAMPLE

Q. If one angle in a pair of supplementary angles measures 80 degrees, what does the other angle measure?

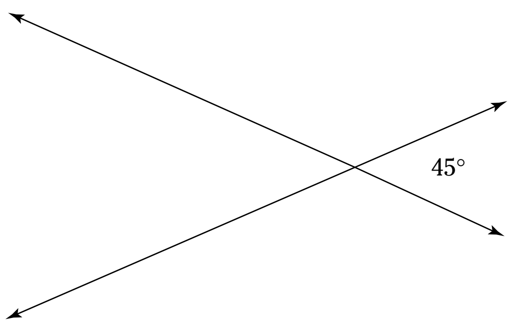
A. The other measures 180 – 80 = 100 degrees.

5. Give the measure of the angles that are *supplementary* to the angle shown in the figure.



Solve It

6. Give the measure of the angle that is *vertical* to the angle shown in the figure.



Solve It

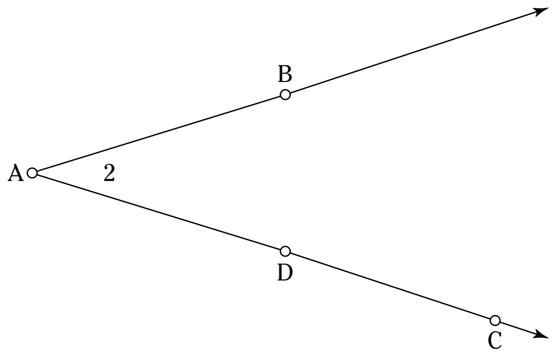
Writing Angle Names Correctly

An angle can be identified in several different ways:

- Use the letter labeling the point that's the *vertex* of the angle. Points are labeled with capital letters.
- Use three letters that label points — one on one ray of the angle, then the vertex, and the last on the other ray.
- Use a letter or number in the inside of the angle. Usually, the letters used are Greek or lowercase.

EXAMPLE

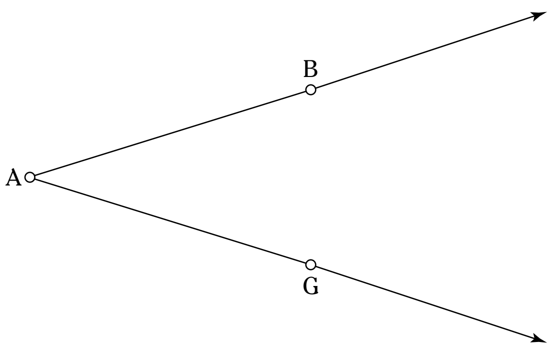
Q. Give all the different names that can be used to identify the angle shown in the figure.



A. The names for this angle are:

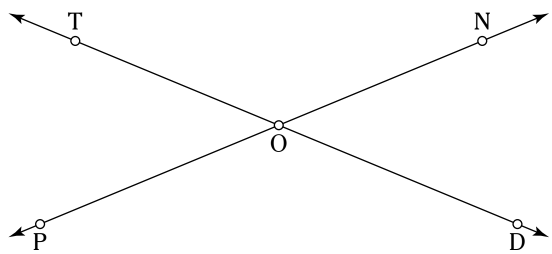
- Angle *A* (just using the label for the vertex)
- Angle *BAD* (using *B* on the top ray, the vertex, and *D* on the bottom ray)
- Angle *BAC* (using *B* on the top ray, the vertex, and *C* on the bottom ray)
- Angle *DAB* (using *D* on the bottom ray, the vertex, and *B* on the top ray)
- Angle *CAB* (using *C* on the bottom ray, the vertex, and *B* on the top ray)
- Angle *2* (using the number inside the angle)

7. Find all the names for the angle shown in the figure.



Solve It

8. Find all the names for the angle that's vertical to the angle *POT* in the figure.



Solve It

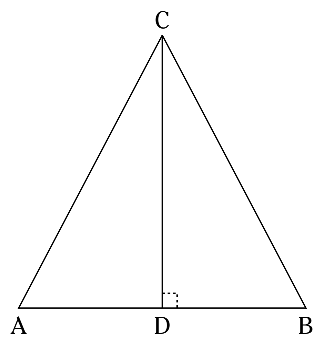
Finding Missing Angle Measures in Triangles

Triangles are probably one of the most familiar forms in geometry and trigonometry. They're studied and restudied and gone over for the minutest of details. One thing that stands out, is always true, and is often used, is the fact that the sum of the measures of the angles of any triangle is 180 degrees. It's always that sum — never more, never less. This is a good thing. It allows you to find missing measures — when they go missing — for angles in a triangle.

Q. If the measures of two of the angles of a triangle are 16 degrees and 47 degrees, what is the measure of the third angle?

A. To solve this, add 16 + 47 = 63. Then subtract 180 – 63 = 117 degrees.

Q. An *equilateral triangle* has three equal sides and three equal angles. If you draw a segment from the vertex of an equilateral triangle perpendicular to the opposite side, then what are the measures of the angles in the two new triangles formed? Look at the figure to help you visualize this.



A. Because the triangle is equilateral, the angles must each be 60 degrees, because $3 \times 60 = 180$. That means that angles *A* and *B* are each 60 degrees. If the segment *CD* is perpendicular to the bottom of the triangle, *AB,* then angle *ADC* and angle *BDC* must each measure 90 degrees. What about the two top angles? Because angle *A* is 60 degrees and angle *ADC* is 90 degrees, and because 60 + 90 = 150, that leaves 180 – 150 = 30 degrees for angle *ACD.* The same goes for angle *BCD.*

9. Triangle *SIR* is isosceles. An *isosceles triangle* has two sides that are equal; the angles opposite those sides are also equal. If the vertex angle, *I,* measures 140 degrees, what do the other two angles measure?

Solve It

10. A triangle has angles that measure n degrees, $n + 20$ degrees, and $3n - 15$ degrees. What are their measures?

Solve It

Determining Angle Measures along Lines and outside Triangles

Angles can be all over the place and arbitrary, or they can behave and be predictable. Two of the situations in the predictable category are those where a transversal cuts through two parallel lines (a *transversal* is another line cutting through both lines), and where a side of a triangle is extended to form an exterior angle.

When a transversal cuts through two parallel lines, the acute angles formed are all equal and the obtuse angles formed are all equal (unless the transversal is perpendicular to the line — in that case, they're all right angles). In Figure 1-2, on the left, you can see how creating acute and obtuse angles comes about. Also, the acute and obtuse angles are supplementary to one another.

An *exterior angle* of a triangle is an angle that's formed when one side of the triangle is extended. The exterior angle is supplementary to the interior angle it's adjacent to. Also, the exterior angle's measure is equal to the sum of the two nonadjacent interior angles.

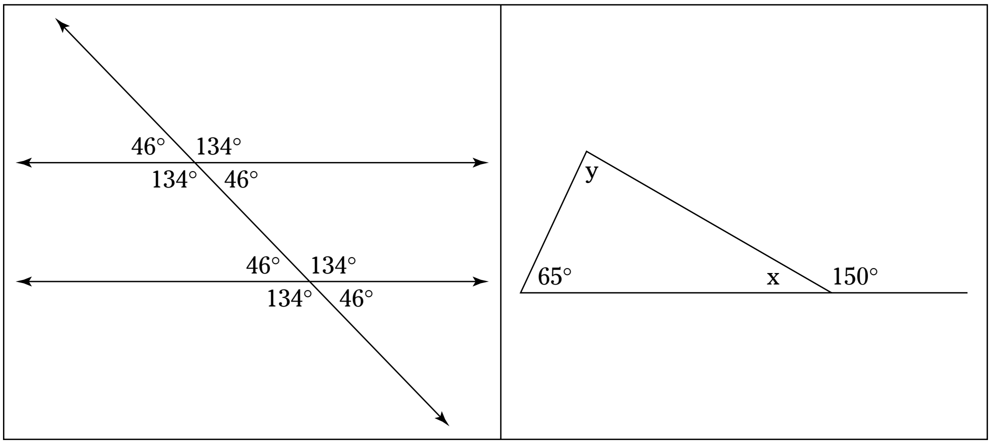
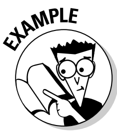


Figure 1-2: Parallel lines with a transversal and a triangle with an exterior angle.



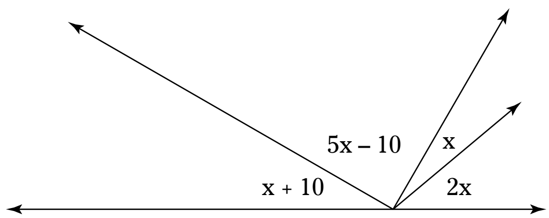
Q. In Figure 1-2, on the right, what are the measures of angles *x* and *y?*

A. The angle *x* is supplementary to an angle of 150 degrees, so its measure is 180 – 150 = 30 degrees. The measure of angle *y* plus the 65-degree angle must equal 150 degrees (the exterior angle's measure). Subtract 150 – 65 = 85. So angle *y* is 85 degrees. To check this, add up the measures of the interior angles: 65 + 85 + 30 = 180. This is the sum of the measures of the angles of any triangle.

11. Find the measures of the acute and obtuse angles formed when a transversal cuts through two parallel lines if the obtuse angles are three times as large as the acute angles.

Solve It

12. Find the measures of the four angles shown in the figure, if one is two times the size of the smallest angle, one is 10 degrees less than five times the smallest angle, and the last is 10 degrees larger than the smallest angle.



Solve It

Dealing with Circle Measurements

A circle is determined by its center and its radius. The *radius* is the distance, shown by a segment, from the center of the circle to any point on the circle. The *diameter* of a circle is a segment drawn through the center, which has its endpoints on the circle. A diameter is the longest segment that can be drawn within a circle.

The measure of the diameter of a circle is equal to twice that of the radius. The diameter and radius are used when determining the *circumference* (the distance around the outside of a circle) and the *area* of a circle.

The circumference of a circle is $C = \pi d$ or $C = 2\pi r$. Circumference equals π times diameter, or circumference equals two times π times radius.

The area of a circle is $A = \pi r^2$. Area equals π times radius squared.

Q. If a circle has a diameter of 30 inches, find its radius, circumference, and area.

A. If the diameter is 30 inches, then the radius is half that, or 15 inches. The circumference is equal to π times the diameter, so $C = \pi(30) = 30\pi \approx 94.2$ inches. (The squiggly equal sign is a way of showing that the measure is "about" that much, not exactly equal to that much.) The approximation was obtained letting $\pi \approx 3.14$. And the area is equal to $A = \pi(15)^2 = 225\pi \approx 706.5$ square inches.

13. Find the radius, circumference, and area of a circle that has a diameter of $2\sqrt{3}$ yards.

Solve It

14. Find the diameter, radius, and area of a circle that has a circumference of 18π centimeters.

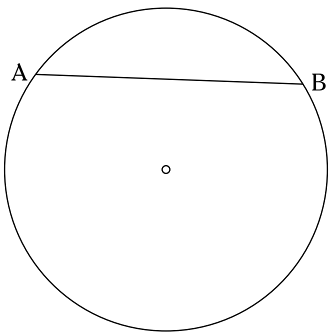
Solve It

Tuning In with the Right Chord

A *chord* is a segment that's drawn from one point on a circle to another point on the same circle. The longest chord of a circle is its diameter. The two endpoints of a chord divide a circle into two arcs — the major arc and the minor arc. The major arc is, of course, the larger of the two. A circle has a total of 360 degrees, so the sum of those two arcs must equal 360.

EXAMPLE

Q. The chord *AB,* shown in the figure, divides the circle into two arcs, one of which is 100 degrees greater than the other. What is the measure of the major arc?

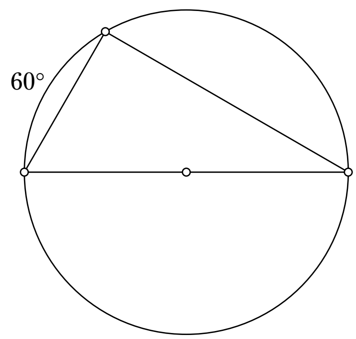


A. Let the measure of the minor arc be x. Then the larger arc is 100 greater than that, or $x + 100$. The sum of the two is 360. Write that as $x + x + 100 = 360$. This simplifies to $2x + 100 = 360$. Subtract 100 from each side to get $2x = 260$. Divide by 2, and $x = 130$. This is the measure of the minor arc. Add 100 to that, and the major arc measures 230 degrees.

15. A chord divides a circle into two arcs, one of which is 15 degrees less than 14 times the other. What are the measures of the two arcs?

Solve It

16. Three chords are drawn in a circle to form a triangle, as shown in the figure. One of the chords is drawn through the center of the circle. If the minor arc determined by the shortest chord is 60 degrees, what are the measures of the other two arcs determined by the vertices of the triangle?



Solve It

Sectioning Off Sectors of Circles

A sector of a circle is a wedge or slice of it. Look at Figure 1-3, showing a sector of a circle that has an arc that measures 70 degrees.

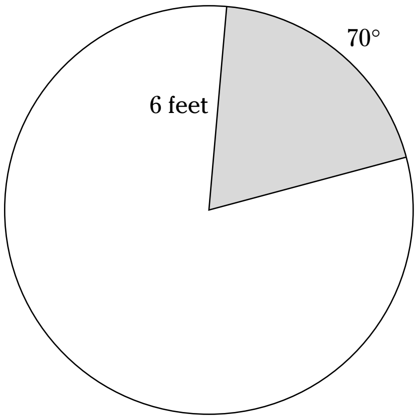
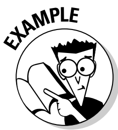


Figure 1-3: A sector of a circle.



Q. Find the area of the sector, shown in Figure 1-3, that has an arc of 70 degrees and a radius of 6 feet.

A. The area of the entire circle is $A = \pi(6)^2 = 36\pi \approx 113.04$ square feet. The arc of 70 degrees is $\frac{70}{360} = \frac{7}{36}$ of the entire circle. So multiply the area of the entire circle by that fraction to get the area of the sector. $36\pi\left(\frac{7}{36}\right) = \pi(7) \approx 21.98$ square inches.

17. Find the area of the sector of a circle that has an arc measuring 120 degrees and a radius of 2.4 meters.

Solve It

18. A pizza is being divided into three unequal slices (sectors). The largest slice has an arc measuring 1 less than three times that of the smallest slice's arc, and the middle-sized piece has an arc that's 1 more than twice the smallest slice's arc. If this is an 18-inch pizza, what is the area of each of the pieces?

Solve It

Answers to Problems on Tackling Technical Trig

The following are the solutions to the practice problems presented earlier in this chapter.

1. What type of angle is shown in the figure? **Right angle.**

   The angle shown in the figure is a right angle, because it measures exactly 90 degrees.

2. What type of angle is shown in the figure? **Acute angle.**

   The angle shown in the figure is an acute angle, because 31 is between 0 and 90 degrees.

3. What type of angle is shown in the figure? **Obtuse angle.**

   The angle shown in the figure is an obtuse angle, because 114 is between 90 and 180 degrees.

4. What type of angle is shown in the figure? **Straight angle.**

   The angle shown in the figure is a straight angle, because it measures exactly 180 degrees.

5. Give the measure of the angles that are *supplementary* to the angle shown in the figure. **47 degrees.**

   In the figure, the measure of the angles that are *supplementary* to the 133-degree angle is 47 degrees, because $180 - 133 = 47$.

6. Give the measure of the angle that is *vertical* to the angle shown in the figure. **45 degrees.**

   In the figure, the measure of the angle that is *vertical* to the 45-degree angle is also 45 degrees, because vertical angles always have the same measure.

7. Find all the names for the angle shown in the figure. ***A, BAG,* and *GAB.***

8. Find all the names for the angle that's vertical to the angle *POT* in the figure. ***NOD* and *DON.***

   WARNING!

   In the figure, you can't use the letter *O* to name an angle. This is a case where just using the letter for the point at the vertex doesn't give enough information to identify which angle you're talking about.

9. Triangle *SIR* is isosceles. An *isosceles triangle* has two sides that are equal; the angles opposite those sides are also equal. If the vertex angle, *I,* measures 140 degrees, what do the other two angles measure? **20 degrees each.**

   If the vertex angle, *I,* measures 140 degrees, the other two angles have to be the angles that are equal. The reason for this is that, if the 140-degree angle were one of the pair of equal angles, the sum of it and its pair would be 280 degrees, which is already too much for the sum of the angles in a triangle. So, to find the measure of the two equal angles, first subtract $180 - 140 = 40$. That leaves a total of 40 degrees for the two equal angles; they're 20 degrees each.

10. A triangle has angles that measure n degrees, $n + 20$ degrees, and $3n - 15$ degrees. What are their measures? **35 degrees, 55 degrees, and 90 degrees.**

    The angles have to add up to 180 degrees: $n + (n + 20) + (3n - 15) = 180$. Simplifying on the left, $5n + 5 = 180$. Subtract 5 from each side to get $5n = 175$. Divide each side by 5, and $n = 35$. And $n + 20 = 55$. Lastly, $3n - 15 = 90$. Adding the three angles: $35 + 55 + 90 = 180$.

11 Find the measures of the acute and obtuse angles formed when a transversal cuts through two parallel lines if the obtuse angles are three times as large as the acute angles. **45 degrees and 135 degrees.**

Let the sum of the acute and obtuse angles be 180. This is true because the angles are supplementary. Let the acute angle's measure be x. Then the obtuse angle measures $3x$. Adding, $x + 3x = 180$, $4x = 180$. Dividing by 4, $x = 45$. If the acute angles are 45 degrees, then the obtuse angles are three times that, or 135 degrees.

12 Find the measures of the four angles shown in the figure, if one is two times the size of the smallest angle, one is 10 degrees less than five times the smallest angle, and the last is 10 degrees larger than the smallest angle. **20 degrees, 40 degrees, 90 degrees, and 30 degrees.**

The sum of the four angles is 180 degrees — the measure of a straight angle. Let the smallest measure be x degrees. Then the others are $2x$, $5x - 10$, and $x + 10$. Adding them, $x + 2x + 5x - 10 + x + 10 = 180$. Simplifying on the left, $9x = 180$. Dividing by 9, $x = 20$. The angles are: 20, 40, 90, and 30 degrees.

13 Find the radius, circumference, and area of a circle that has a diameter of $2\sqrt{3}$ yards. $\boldsymbol{\frac{1}{2}\left(2\sqrt{3}\right) = \sqrt{3}}$**,** $\boldsymbol{C = \pi\left(2\sqrt{3}\right) = 2\pi\sqrt{3}}$ **yards, and** $\boldsymbol{3\pi}$ **square yards.**

The radius is half the diameter, so half of $2\sqrt{3}$ is $\frac{1}{2}\left(2\sqrt{3}\right) = \sqrt{3}$. The circumference is π times the diameter, so $C = \pi\left(2\sqrt{3}\right) = 2\pi\sqrt{3}$ yards. The area is π times the square of the radius, so $A = \pi\left(\sqrt{3}\right)^2 = \pi(3) = 3\pi$ square yards.

14 Find the diameter, radius, and area of a circle that has a circumference of 18π centimeters. **18 centimeters, 9 centimeters, and** $\boldsymbol{81\pi}$ **square centimeters.**

The circumference is π times the diameter, so the diameter must be 18 centimeters. That means that the radius is half that, or 9 centimeters. The area is π times the square of the radius or $A = \pi(9)^2 = \pi(81) = 81\pi$ square centimeters.

15 A chord divides a circle into two arcs, one of which is 15 degrees less than 14 times the other. What are the measures of the two arcs? **25 degrees and 335 degrees.**

Start by finding the measures of the two arcs. Let one arc measure x degrees. Then the other measures $14x - 15$ degrees. Their sum is 360 degrees. So $x + 14x - 15 = 360$. Simplify on the left to get $15x - 15 = 360$. $15x = 375$. Dividing by 15, $x = 25$. The minor arc is 25 degrees, and the major arc is $14(25) - 15 = 335$ degrees.

16 Three chords are drawn in a circle to form a triangle, as shown in the figure. One of the chords is drawn through the center of the circle. If the minor arc determined by the shortest chord is 60 degrees, what are the measures of the other two arcs determined by the vertices of the triangle? **180 degrees and 120 degrees.**

The diameter divides the circle into two equal arcs, so they're each 180 degrees. That leaves 180 degrees for the top half. Subtract $180 - 60$, and the other arc on the top is 120 degrees.

17 Find the area of the sector of a circle that has an arc measuring 120 degrees and a radius of 2.4 meters. **1.92**$\boldsymbol{\pi}$ **square meters.**

The sector is $\frac{120}{360} = \frac{1}{3}$ of the entire circle. Multiply that times the area of the entire circle, which is found by multiplying π times the square of the radius: $\frac{1}{3}\pi(2.4)^2 = \frac{5.76}{3}\pi = 1.92\pi$ square meters.

18 A pizza is being divided into three unequal slices (sectors). The largest slice has an arc measuring 1 less than three times that of the smallest slice's arc, and the middle-sized piece has an arc that's 1 more than twice the smallest slice's arc. If this is an 18-inch pizza, what is the area of each of the pieces? ≈ **42.39 square inches,** ≈ **126.46 square inches, and** ≈ **85.49 square inches.**

Let x represent the measure of the arc cut by the smallest piece. Then the other two arcs are $3x - 1$ and $2x + 1$. Add all three together to get $x + 3x - 1 + 2x + 1 = 360$. Simplifying on the left, $6x = 360$. Dividing by 6, $x = 60$. The other two measures are then 179 and 121. The three different areas can be found by multiplying their fraction of the pizza by the area of the whole pizza, which is determined by multiplying π times the square of the radius. An 18-inch pizza has a radius of 9 inches.

$$\frac{60}{360}\pi(9)^2 = \frac{4860}{360}\pi \approx 42.39 \text{ square inches}$$

$$\frac{179}{360}\pi(9)^2 = \frac{14499}{360}\pi \approx 126.46 \text{ square inches}$$

$$\frac{121}{360}\pi(9)^2 = \frac{9801}{360}\pi \approx 85.49 \text{ square inches}$$