

PREFACE TO THE SECOND REPRINT EDITION (1996)

It has been thirty-six years since *An Introduction to Statistical Communication Theory* was originally published, in 1960 by McGraw-Hill in its International Series in Pure and Applied Physics [1]. Nine years have also passed since its republication as a Reprint Edition by the Peninsula Publishing Co., of Los Altos, California, in 1987 [1a]. The present volume is the reincarnation of the previous editions, through the kind offices of the IEEE Press and under the welcome sponsorship of the Communication and Information Theory Societies of the IEEE, with selected changes and additions described below. From a historical viewpoint, *An Introduction* follows, at an advanced level of application, along the path, primarily, of the earlier work of Lawson and Uhlenbeck in the MIT Radiation Laboratory Series [2] and of Davenport and Root [3]. Because so much of the original material of *An Introduction* appears still to be pertinent and useful today and because almost three full generations of engineers and scientists have arrived since the book's inception, it seems appropriate once more to make it available to the concerned technical community.

Unlike the first Reprint Edition (1987–1995), for which no “updating” was attempted (save for the brief list of references cited in the Preface thereof), this edition does provide selected references which connect the earlier work of the original book's period with many new concepts, methods, and results which have appeared since 1960 and which may broadly be considered to follow from and extend the basic ideas and techniques of statistical communication theory (SCT), as explicated in [1, 1a]. These include, for example, (1) developments in such areas as detection and estimation in non-Gaussian noise environments involving propagation and scattering in nonhomogeneous channels [4–9, 14–15]; (2) state-space, model-based signal processing (estimation and control) [10]; (3) digital signal processing [11]; (4) spectrum analysis and array processing [12, 13]; as well as (5) other extensions of classical signal detection and estimation [14–16], including fuzzy logic [17] and neural networks [18]; (6) the rôle of information theory in physical science [20]; and of course, (7) the essential developments in computing technology which have made the practical implementation of SCT more fully possible.

In addition to these important topics, selectively referenced below, a rather extensive list of some 125 references has been added at the end of this Edition, with few exceptions, on a chapter-by-chapter basis. These represent mainly books which in the author's opinion describe the technical progress in the greatly expanded, interdisciplinary field of SCT from 1960 to the present. By and large, they themselves also remain vital and useful today, as well as of historical interest, and serve to connect the still active past with the ongoing present. This list is not intended to be, nor could it be, complete in any practical sense, nor is any slight to omitted worthy work intended. In any case, many useful references are also to be sought and found in these volumes.

Finally, the author has taken the opportunity of this Reprint Edition to correct the errors and misprints (known to him), fortunately few since the many printings (1960–1972) of the original work [1]. The author is deeply indebted to Professor Poor (Princeton University) for his support and interest in the republication of this book and for his generous foreword thereto. The author also wishes to extend his appreciation to the various reviewers and to Mr. Dudley Kay and the excellent editorial staff at the IEEE Press for their essential efforts in bringing this work to publication again.

David Middleton
New York, 1996

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- [15] D. Middleton, "Space-Time Processing for Weak-Signal Detection in Non-Gaussian and Non-Uniform Electromagnetic Interference (EMI) Fields", Contractor Report 86-36 (NTIS: PB-86-193406), ITS (NTIA), U.S. Department of Commerce, 325 Broadway, Boulder, CO 80303), February 1986.
- [16] C. W. Helstrom, *Elements of Signal Detection and Estimation*, Prentice Hall, Englewood Cliffs, NJ, 1995.
- [17] L. A. Zadeh, *Fuzzy Sets and Applications: Selected Papers by L. A. Zadeh*, eds. R. R. Yager et al., John Wiley, New York, 1987.
- [18] S. Haykin, *Neural Networks*, IEEE Press (Macmillan College Publishing Co.), New York, 1994.
- [19] R. Young, *Wavelet Theory and Its Applications*, Kluwer Academic Publishers, Boston, 1993.
- [20] L. Brillouin, *Science and Information Theory*, 2nd edition, Academic Press, New York, 1962.

See also the earlier selected references at the end of the Preface to the First Reprint Edition following. Additional references, by chapter, are given at the end of this book.

PREFACE TO THE FIRST REPRINT EDITION (1987–1995)

It has been over a quarter of a century since *An Introduction to Statistical Communication Theory* was first published, in 1960, by McGraw-Hill (New York) in its International Series in Pure and Applied Physics. Since then almost two generations of scientists and engineers have appeared and many specialized works within the broad domain of Statistical Communication Theory (SCT) have been produced. Today, what is often referred to as “signal processing” has become a standard engineering discipline in all areas and applications which require the transmission and reception of “information.” Signal processing itself has become an ensemble of techniques whose foundations were introduced and broadly described in the original edition of the present book.

Because so much of the material of *An Introduction* appears to be continuously useful and because this work has been out-of-print since 1972, it seems particularly appropriate to reintroduce the book at this time. Indeed, inasmuch as the concepts and methods, and many of the examples described therein, are *canonical*, that is, have a general, functional *form* independent of specific physical models and analytical detail, they remain relevant to current and future applications. It is in this spirit that this Reprint Edition (Peninsula Publishing) is presented.

No attempt to “update” the book itself has been made here. To do so with the same emphasis on comprehensiveness and detail covered in the original would now require a whole *series* of books. Apart from losing the advantages of self-containment in a single volume, such a task does not appear attractive, in the face of the many excellent books which have subsequently appeared and which treat in more detail many of the topics originally discussed in the author’s treatise.

Of course, new methods and ideas in Statistical Communication Theory (SCT) have appeared since the early sixties. In addition to the topics treated generally in the original edition, such as *noise theory*, *noise physics*, *information theory*, *statistical decision theory (SDT)*, and *data processing*, one should add within the domain of SCT important new developments: (1) *nongaussian noise and interference models* [10], [12], [18]; (2) *threshold signal detection and estimation* [11] - [13], [15], [16]; (3) *spatial processing*, as well as temporal sampling, including general arrays and apertures [17]; and (4) *statistical-physical approaches* [10], [17] - [20], to describe the channel itself, including scattering and various types of propagation encountered in real-world environments. Moreover, as predicted at the time (1960), cf. Sec. 23.5, [1], the many areas of then future study noted therein have indeed been (and are being) investigated, as well as others not then known. For example, whole areas of current importance such as Electromagnetic Compatibility (EMC) and Spectrum Management [11], [12], [13], [16], quantum optical signal processing [8], generalized arrays and spatial sampling [17] - [19], as well as the development of powerful and economical computers and programs for handling the vast data loads incurred in practice, were either unknown or barely beginning to emerge. In fact, the recognition (slow

even today) that one lives in a “nongaussian” world [10], [12] is perhaps one of the more significant features of the current evolution of SCT and its specific applications.

Finally, to assist the reader in relating the SCT fundamentals of 1960 with the developing practicalities of the present, the author has appended below his own brief, highly personalized list of books and papers. No attempt at completeness is made and no slight to worthy work not mentioned here is intended.

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New York, 1987

I. Books

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- [2]. L. A. Wainstein and V. D. Zubakov, *Extraction of Signals from Noise* (translated from the Russian by R. L. Silverman), Prentice Hall (New Jersey), 1962.
- [3]. D. Middleton, *Topics in Communication Theory*, McGraw-Hill (New York), 1965.
- [3a]. Д. Миддлтон ОЧЕРКИ ТЕОРИИ СВЯЗИ
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- [6]. H. Van Trees, *Detection, Estimation, and Modulation Theory*, Part I (1968), Part II (1971), Part III (1971), John Wiley (New York).
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II. Papers

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- [16]. A. D. Spaulding, "Locally Optimum and Suboptimum Detector Performance in a Non-Gaussian Interference Environment," *IEEE Trans. Commun.*, Vol. COM-33, No. 6, June 1985, pp. 509-517.
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- [18]. D. Middleton, "Second-Order Non-Gaussian Probability Distributions and Their Application to 'Classical' Nonlinear Processing Problems in Communication Theory," *Proceedings* of 1986 Conf. on Information Sciences and Systems, Princeton University, March 19-21, 1986.
- [19]. D. Middleton, "A Statistical Theory of Reverberation and Similar First-Order Scattered Fields," Parts I, II, *IEEE Trans. Information Theory*, Vol. *IT-13*, 372-392; 393-414 (1967); Parts III, IV, *ibid.*, Vol. *IT-18*, 35-67; 68-90 (1972).
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PREFACE TO THE FIRST EDITION (1960)

Statistical communication theory may be broadly described as a theory which applies probability concepts and statistical methods to the communication process. In the wide sense, this includes not only the transmission and reception of messages, the measurement and processing of data, measures of information, coding techniques, and the design and evaluation of decision systems for these purposes, but also the statistical study of language, the bio- and psychophysical mechanisms of communication, the human observer, and his role in the group environment. Here, however, we consider statistical communication theory from the narrower viewpoint of the physical systems that are specifically involved, e.g., radio, radar, etc. For this purpose, it is convenient to regard statistical communication theory as consisting of the following contiguous and, to varying degrees, overlapping areas of interest: (1) *noise theory*, which embraces the mathematical description of random processes and their properties under various linear and nonlinear transformations; (2) *noise physics*, which is concerned mainly with the underlying physical mechanisms of the noise processes encountered in applications; (3) *information theory*, defined here in the strict sense as a theory of information measures and coding; and (4) *statistical decision theory*, as modified and extended to system design, evaluation, and the comparison of optimum and suboptimum systems, where the desired end product is some definite decision—a “yes” or “no” or a measurement. With (3) and (4), it is natural to consider also *data processing*, in the course of transmission and reception, the *physics and the technology of the components*, e.g., tubes, transistors, etc., by which actual systems for the above purposes are to be realized, and, of course, the various concepts and methods of *probability* and *statistics* that are required for a quantitative treatment, subject now to the different constraints imposed by the communication process itself.

The present book is addressed principally to engineers, physicists, and applied mathematicians. Its aims are threefold: (1) to outline a systematic approach to the design of optimal communication systems of various fundamental types, including an evaluation of performance and a comparison with nonoptimum systems for similar purposes; (2) to incorporate within a framework of a unified theoretical approach the principal results of earlier work, as well as to indicate some of the more important new ones; and finally (3) to be used as a text at various levels, as an instrument for the

research worker in current problems, and, it is hoped, as a starting point for new developments. The emphasis here is mainly on (1) and (2), although (3) has not been neglected: method as well as results are stressed. About 300 problems have been included, not only as exercises but also as a source of additional results which could not otherwise be presented. While the bibliography is necessarily selective, 500 references (including the supplement) are available, from which more specialized interests may in turn be served. The mathematical exposition is for the most part heuristic, and although a detailed, rigorous treatment is outside the scope and intent here, in this respect reference is made to the appropriate literature. Moreover, little space is devoted to the probabilistic and statistical background, the elements of which the reader is assumed to possess. A knowledge of Fourier- and Laplace-transform methods, contour integration, matrices, simple integral equations, and the usual techniques of advanced calculus courses is also required, along with the elements of circuit theory and the principles of radio, radar, and other types of electronic communication systems.

The book is divided into four main parts. Part 1 introduces and describes some of the statistical techniques required in the analysis of communication systems and concludes with an introductory chapter on information theory. Part 2 considers the normal process and some of the processes derived from it and gives a short account of the physical models of shot and thermal noise. Part 3 is concerned mainly with various nonlinear operations that are common in transmission and reception, such as modulation and demodulation, and the calculation of signal-to-noise ratios. Linear measurement, filtering, and prediction and more general distribution problems, the results of which are needed in the general analysis of Part 4, are also examined here. Finally, Part 4 gives a detailed development of a statistical communication theory for the basic single-link communication system, consisting of message or signal source, transmitter, medium of propagation (or channel), and receiver and decision-making elements. The attention here is on optimization and evaluation of receiver performance, e.g., signal detection and extraction, with a short introduction (in Chap. 23) to the still more general question of simultaneous optimization of both the reception and transmission operations. About a third of the material of Parts 1 to 3 coincides with that in recent publications on noise theory. Part 4 has not appeared in book form before, and much of the rest, Part 3 particularly, is also believed to be new in this respect. While about three-quarters of the subject matter of the book as a whole may be found scattered through the original technical literature, many of the results mentioned in Chaps. 12, 17, 19 to 21, and 23 have not been presented previously.

In spite of its size, this work must still be considered as an introduction, for it has not been possible to treat more than a few important topics, such as detection theory, with any great degree of completeness and still maintain

the attempt at a broad coverage which is one aim of the book. In fact, a number of important subjects have been omitted, because they require a separate volume for an adequate treatment, or because they are already so handled in the literature, or in some instances because no theory of any stature is as yet available. Among the topics which have been regrettably dropped are linear and nonlinear communication feedback (and related control) systems, coding methods, sequential detection and estimation, various types of noise-measuring systems, and many more specialized results in the theory of noise, such as the probability distributions of zeros and maxima of random waves. Problems involving large-scale, i.e., many-link, communication networks, game-theory applications, and their relationship to the single-link systems examined in Part 4 are likewise omitted. Some attention is given to Markoff processes, the Fokker-Planck and related equations, with applications to shot and thermal noise, and, later, to questions of optimum linear measurements, their statistical errors, filtering, prediction, and the like. However, a more extensive coverage of the second-moment theory (involving spectra and signal-to-noise ratios) of AM and FM transmission and reception is provided. The same is true for spectra and covariance functions of linearly filtered random and mixed processes, including random pulse trains. The treatment of the normal process, and those processes derived from it, including gaussian and nongaussian functionals, is intermediate, sufficient to satisfy the needs of subsequent chapters. The discussion of optimum systems for detecting and extracting signals from noise backgrounds, on the other hand, is comparatively detailed, though by no means complete, since this whole subject is still in a state of rapid development. Finally, besides the analytical features of system optimization and evaluation, considerable attention is given to the interpretation of system structure in such cases: the representation of optimum systems in terms of ordered sets of physically realizable elements. Threshold or weak-signal systems receive particular emphasis in the present study, since they represent the furthest extent to which performance can be pushed under a given environment. Moreover, optimum design for threshold operation, while not usually optimum for strong signals, is nevertheless almost always satisfactory in the latter instances, so that a system designed to make the most of threshold operation will also function acceptably when the interference is weak or ignorable. Threshold performance is important whenever we are required to operate at the limit of system capabilities. Terrestrial, satellite, and space communications offer many conspicuous examples.

As a text, the selection of material may be made rather arbitrarily, depending on the level and scope of the intended course. For example, one might use Part 1 followed by Chaps. 7, 9, 11 and parts of Chaps. 12 to 16. A more advanced program might use portions of Part 1, all of Part 2, most of Part 3, and, as an introduction to decision-theory methods in communica-

tion, Chaps. 18 and 19. Specific topics, such as system optimization, might be based on Chap. 17 and Part 4, with Chap. 6 and selected portions of earlier chapters. A perusal of the table of contents may suggest still other programs. An excellent introduction to Parts 1 to 3, with a more detailed coverage in various instances, may be found in the book by Davenport and Root.† The principal references on which the material of each chapter is based are listed at the chapter ends, with a supplementary bibliography at the end of the book. For a critique of the methods discussed in Part 4 and some possible avenues of future development, see Chap. 23. A glossary of principal symbols is included, with two appendixes on special functions and integral equations that occur frequently in the analysis.

Portions of Parts 1, 2, 3 were developed from material presented by the author during 1949 to 1954 as a series of graduate courses in the Division of Applied Science at Harvard University; also, some of the author's published work referred to in the text stemmed from research originally supported at Harvard and at the Massachusetts Institute of Technology under Department of Defense contracts with those institutions. The book itself was begun in 1949, the final draft of Part 1 was completed during 1955–1956, those of Parts 2 and 3 in 1956–1957, and Part 4 in 1957–1958.

While it is regrettably impracticable to mention here the many who have contributed to this work over the last decade, they will find themselves for the most part recorded in the name index at the end of the book, as well as in the references throughout the text. However, the author wishes specifically to thank Prof. Leon Brillouin for his original encouragement of this project, Dr. Arthur Kohlenberg for his criticism of Chap. 4, and Prof. Peter Elias for his valuable comments concerning Chap. 6. The author also wishes to thank several of his former students for their assistance, and in particular Dr. David Van Meter for his careful reading of the entire final manuscript and Dr. Julian Bussgang for his detailed criticisms of the first 10 chapters. Thanks are also due Dr. James A. Mullen and Dr. D. B. Brick for assistance with proofs. In all instances, the errors of omission and commission remain the author's, as do the opinions and results therein, unless otherwise indicated. Finally, the author is indebted to Drs. Davenport and Root for the opportunity to examine their manuscript when both books were in the final stages of completion and to discuss with them various aspects of the several works. For their heroic efforts in coping with a difficult manuscript, the author's sincere appreciation is also extended to Mrs. Edith Nicholson, who typed the first third, and to Miss Anne Guest, who handled most of the remaining two-thirds.

Finally, the author wishes to thank the editors of various journals for permission to use certain material which appeared originally in their publications. In particular, the author is indebted to the editors of the *Journal*

† W. B. Davenport, Jr., and W. L. Root, "An Introduction to the Theory of Random Signals and Noise," McGraw-Hill, New York, 1958.

of the *Society of Industrial and Applied Mathematics* for a number of excerpts from the paper by Middleton and Van Meter, "Detection and Extraction of Signals in Noise from the Point of View of Statistical Decision Theory," which appeared in the journal of the society in December, 1955, and June, 1956 (see Chap. 1, Ref. 6). Specifically, most of Chap. 18, about 40 per cent of Chap. 19, a few pages of Chap. 20, most of Secs. 21.1-3, 21.1-4, 21.4, and 21.5, parts of Secs. 21.2-2, 21.2-3, 21.3-1, and all of Chap. 22, except for Secs. 22.1, 22.2, have been taken directly or with minor modifications from the aforementioned paper. The author is also grateful to the Institute of Radio Engineers for permission to excerpt and paraphrase portions of his earlier work, in particular, Middleton and Van Meter, *IRE Trans. on Inform. Theory*, **IT-1**: 1-7 (September, 1955), and to use figs. 1 to 3 of this paper (Figs. 23.3, 23.4, P 23.3 here); also, for figs. 4, 5 (Figs. 13.11, 13.12 here), *Proc. IRE*, **36**: 1467 (1948); for figs. 2, 3, 7 (Fig. 20.4a, b here), *IRE Trans. on Inform. Theory, Symposium*, **PGIT-4**: 145 (September, 1954) (with Van Meter); for fig. 4, *IRE Trans. on Inform. Theory*, **IT-3**: 86 (1957). Similarly, the author wishes to thank the *Quarterly of Applied Mathematics* for permission to use figs. 3 to 10, *Quart. Appl. Math.*, **5**: 445 (1948) (here embodied in Figs. 5.3, 5.5, 9.1, 9.3, 13.3, 13.5); for figs. 2.1, 2.2, *ibid.*, **9**: 337 (1952), and for fig. 3.2, *ibid.*, **10**: 38 (1952) (here Figs. 12.2, 12.3, 14.9); and for figs. 4, 5, 6, *ibid.*, **7**: 129 (1949) (here Figs. 15.14, 15.15a, b). The author is likewise indebted to the *Journal of Applied Physics* for permission to use figs. 1, 2, 6, 11, 13, *J. Appl. Phys.*, **17**: 778 (1946) (here embodied in Figs. 12.1, 13.17a, b, 13.18); for figs. 1, 3, 4, 8, *ibid.*, **20**, 334 (1949) (here as Figs. 15.5, 15.6, 15.8, 15.16); for figs. 1, 4, *ibid.*, **23**: 377 (1952) (with W. B. Davenport and R. A. Johnson) (here as Figs. 16.2, 16.3); and for figs. 4, 5, 7, *ibid.*, **24**: 379 (1953) (here as Figs. 20.10 to 20.12).

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नष्टो मोहः स्मृतिर्लब्धा त्वत्प्रसादान्मयाच्युत ॥

स्थितीऽस्मि गतसन्देहः करिष्ये वचनं तव ॥ ७६ ॥

Srimad-Bhagavad-Gita
Chapter XVIII, verse 73