CHAPTER 1

The Phenomenon of Light

THE ORIGIN of the special theory of relativity lies in a dilemma concerned with the nature and velocity of light. Appreciation of this dilemma adds purpose and meaning to relativity, and it is for this reason that the present chapter is concerned with light and its properties. The first two sections trace the evolution of thought with respect to whether light is corpuscular or wavelike, and whether its velocity is finite or infinite; present-day views of these properties culminate both developments. Light and sound (the latter being representative of wave phenomena requiring a *tangible* medium) are compared in the third section and their essential similarities and differences are highlighted; the resulting contrast prepares the way for the introduction, in Chapter 2, of the aforementioned dilemma.

More than usual space is given in this chapter to the historical aspects of the subject. An explanation of the decision to do this may be found in the Preface. The reader wishing to concentrate his efforts on the technical development may prefer to limit his attention to the Bradley aberration experiment in Section 1.2 and the comparison of light and sound in Section 1.3.

1.1* HISTORICAL SURVEY—THE NATURE OF LIGHT

Speculation about the *nature* of light can be traced back to antiquity. The Sicilian Empedocles (c.490–c.435 B.C.) was credited with the view[1] that light consists of small particles emitted from a visible body. These particles were presumed to enter the eyes and were then returned to the visible body (a conservation law!) with the resulting streams of particles being responsible for the sensations of shape and color. Unfortunately, only fragments of the writings of this extraordinary man have survived, and the direct evidence of his view is merely suggestive, being contained in the lyrical passage[2]

> As when a man, about to sally forth,
> Prepares a light and kindles him a blaze
> Of flaming fire against the wintry night,

* Throughout this book the content of sections marked with an asterisk is primarily historical. The reading of these sections can be omitted without materially affecting the technical exposition.

[1] Plato, *Meno*. (See, e.g., the W. R. M. Lamb translation, Vol. 165 of the Loeb Classical Library, p. 285, Harvard University Press, 1962.)

[2] W. E. Leonard, *The Fragments of Empedocles*, pp. 42–43, The Open Court Publishing Company, Chicago, 1908.

In horny lantern shielding from all winds;
Though it protect from breath of blowing winds,
Its beam darts outward, as more fine and thin,
And with untiring rays lights up the sky:
Just so the Fire primeval once lay hid
In the round pupil of the eye, enclosed
In films and gauzy veils, which through and through
Were pierced with pores divinely fashioned,
And thus kept off the watery deeps around,
Whilst Fire burst outward, as more fine and thin.

Empedocles was a close observer of nature, the apparent originator of the long-standing and influential notion that all things are composed of the four elements: air, fire, water, and earth. He was a poet of stature whose wide-ranging opinions exerted a strong influence on later Greek scholars. Aristotle (384–332 B.C.) quotes him frequently, often contentiously, and in *De Sensu* says[3]

> Empedocles at times seems to hold that vision is to be explained as above-stated, by light issuing forth from the eye; e.g., in the following passage: [The 13 lines given above are then quoted.] Sometimes he accounts for vision thus, but at other times he explains it by emanations from the visible objects.

Aristotle states his own opinion about the nature of light in *De Anima:*[4]

> Now there clearly is something which is transparent, and by "transparent" I mean what is visible, and yet not visible in itself, but rather owing its visibility to the color *of something else;* of this character are air, water, and many solid bodies. Neither air nor water is transparent because it is air or water; they are transparent because each of them has contained in it a certain substance which is the same in both and is also found in the eternal body which constitutes the uppermost shell of the physical Cosmos. Of this substance light is the activity—the activity of what is transparent so far forth as it has in it the determinate power of becoming transparent; where this power is present, there is also the potentiality of the contrary, viz. darkness. Light is as it were the proper color of what is transparent, and exists whenever the potentially transparent is excited to actuality by the influence of fire or something resembling "the uppermost body"; for fire too contains something which is one and the same with the substance in question.
>
> We have now explained what the transparent is and what light is; light is neither fire nor any kind whatsoever of body nor an efflux from any kind of body (if it were, it would again itself be a kind of body)—it is the presence of fire or something resembling fire in what is transparent. It is certainly not a body, for two bodies cannot be present in the same place. The opposite of light is darkness; darkness is the absence from what is transparent of the corresponding positive state above characterized; clearly therefore, light is just the presence of that.

Aristotle's influence was greater with later cultures than with his own, and thus one finds most ancient Greek scholars preferring to accept a simpler view similar to that of

[3] Aristotle, *De Sensu*, 437^{b}, 23, English translation under editorship of W. D. Ross, Oxford at the Clarendon Press, 1931.

[4] Aristotle, *De Anima*, 418^{b}, 4, English translation under editorship of W. D. Ross, Oxford at the Clarendon Press, 1931.

Empedocles; for example, both Euclid and Ptolemy held the opinion that light consists of rays which originate in the eye, illuminate the object seen, and then return to the eye.

In contrast to the richness of Greek speculation about light, Roman scholars do not appear to have been interested in this problem. Indeed, all of Roman science was essentially derivative in character and distinctly low order, contributing little that was original, and nothing worthy of note in the present survey. Arabic science, on the other hand, while also being derivative, was of a rather high order, being based on the finest products of Greek scientific achievement. The successors of Mohammed evinced a great interest in the ideas of the western people whom they conquered, and far from being the destroyers of Western literature, they were its chief preservers. The Arabs came into contact with the Greeks in Egypt as well as western Asia, and became their virtual successors in carrying forward the torch of learning. Although inclined to be conservative and traditional, thus accepting most Greek ideas as authoritative, the Arabian scholars did make several independent discoveries of significance. An important example is the Arabic numbering system in use today, which evolved during this period.

In the specific field of light, many accomplishments can be credited to Ibn al-Haitham (c.965–c.1039), known to the Western world by the Latin name Alhazen. He was the *true* physicist of medieval Islam, just as Archimedes had been in the Grecian period, for he combined with rare skill both the experimental investigation of natural phenomena and the analysis of results by mathematics.[5] Alhazen was one of the ablest students of optics of all times and published a seven-volume treatise on this subject which had great celebrity throughout the medieval period and strongly influenced Western thought, notably that of Roger Bacon and Kepler.[6] This treatise discussed concave and convex mirrors in both cylindrical and spherical geometries, anticipated Fermat's law of least time, and considered refraction and the magnifying power of lenses. It contained a remarkably lucid description of the optical system of the eye, which study led Alhazen to the belief that light consists of rays which originate in the object seen, and not in the eye, a view contrary to that of Euclid and Ptolemy.

Ibn Sina, or Avicenna (980–1037), the most famous of the Islamic scientists, whose immense medical encyclopedia, the *Quanun*, made him the greatest name in medicine for four centuries, was also a perceptive student of various physical questions —motion, contact, force, vacuum, infinity, light, and heat. He shared Alhazen's view that light originated in the luminous source and felt that it must consist of some type of particles.[7]

Roger Bacon (1214–1294), a learned scholar who stressed the value of reading works in their original languages, was well-versed in the teaching of Aristotle, St. Augustine, and the Muslim scientists Alhazen and Avicenna. During a sojourn in Paris, he so impressed the future Clement VI that the latter, upon elevation to the Papacy in 1265, requested Bacon to transmit copies of all his writings without delay. Up to that time, Bacon had written but little; however, in the span of one year, he composed the *Opus Majus*, the *Opus Minor*, and the *Opus Tertium*, a stupendous undertaking, the fruits of which exerted a great influence on Western thought for centuries. In his masterpiece,

[5] H. J. J. Winter, *Eastern Science*, John Murray Publishers, Ltd., London, 1952.

[6] G. Sarton, *Introduction to the History of Science*, Vol. 1, p. 721, Williams and Wilkins Company, Baltimore, Md., 1927.

[7] *Ibid.*, Vol. 1, p. 710.

the *Opus Majus*, Bacon appears to endow Alhazen and Avicenna with an ambivalent position by saying[8]

> If, moreover, Alhazen and Avicenna, in the third book on the Soul . . . are cited as opposed to this view, I reply that they are not opposed to the generation of the species of vision, nor to the part it plays in producing sight; but they are opposed to those who have maintained that some material substance as a visible or similar species is extended from the sight to the object, in order that vision may perceive the object itself, and that it may seize upon the species of the object seen and carry it back to the sight.

Bacon's own view coincided with the opinion of many of the ancients, that light consists of emanations which originate in the eye, and he defends this view in the passage[9]

> The reason for this position is that everything in nature completes its action through its own force and species alone, as, for example, the sun and the other celestial bodies through their forces sent to the things of the world cause the generation and corruption of things; and in a similar manner inferior things, as, for example, fire by its own force dries and consumes and does many things. Therefore vision must perform the act of seeing by its own force. But the act of seeing is the perception of a visible object at a distance, and therefore vision perceives what is visible by its own force multiplied to the object . . . it is clear to him who gives it due consideration that vision must take place by means of its species emitted to the visible object.

As for the species of light itself, Bacon says, in an explanation which has the interesting tinge of wave motion, that[10]

> . . . the species is not a body, nor is it changed as regards itself as a whole from one place to another, but that which is produced in the first part of the air is not separated from that part, since form cannot be separated from the matter in which it is, unless it be soul, but the species forms a likeness to itself in the second position of the air, and so on. Therefore it is not a motion as regards place, but is a propagation multiplied through the different parts of the medium; nor is it a body which is there generated, but a corporeal form, without, however, dimensions *per se*, but it is produced subject to the dimensions of the air . . . .

The passage of three centuries marks the interval between the death of Roger Bacon and the birth of René Descartes (1596–1650), whose intellect and creative genius were to stir scientific imagination, and whose prolific pen was to prove even more influential than Bacon's. Descartes lived at a time in which the world was ripe for a new conception of the nature of things. Major changes in attitude about man's surroundings were being culminated; Galileo and Kepler were advocating the overthrow of the geocentric hypothesis of Ptolemy, the Magellan expedition had circumnavigated the globe, the invention of the telescope was leading to expanded knowledge of the skies, and Aristotelian scholasticism was under attack at all its weakest points. Montaigne's skepticism had paved the way for a break with tradition, and Descartes set for himself the task of erecting a new structure to replace the old. In the words of

[8] R. Bacon, *Opus Majus*, Part 5, 7th Distinction, Chap. 3, the R. B. Burke translation, University of Pennsylvania Press, Philadelphia, 1928.
[9] *Ibid.*, 7th Distinction, Chap. 4; 9th Distinction, Chap. 1.
[10] *Ibid.*, 9th Distinction, Chap. 4.

Whittaker,[11] "His aim was nothing less than to create from the beginning a theory of the universe, worked out as far as possible in every detail."

To understand Descartes' position on the particular subject of the nature of light, one must first appreciate the major features of his grand design of the universe and the attitudes which shaped this design. His philosophy was essentially dualistic; he believed the physical world to be mechanistic and divorced from the mind, the only connection between the two being through God's intervention. In science, he supported the inductive method of Francis Bacon, but with emphasis on rationalization and logic, rather than upon experiences. Mathematics was Descartes' greatest interest and he is widely called the father of analytic geometry. Under Kepler's influence, he became convinced that the precision and universality of mathematics set it apart from all other fields of study. This admiration of the clarity of mathematical expression serves to explain why, as the first rule in the *Discourse on Method,* Descartes vowed

> never to accept anything as true if I had not evident knowledge of its being so; that is, to accept only what presented itself to my mind so clearly and distinctly that I had no occasion to doubt it.

This attitude led Descartes to the decision that, since effects produced by means of contacts and collisions were the simplest and most comprehensible phenomena in the physical world, *he would accept no other causes.* Such a decision implies that bodies can act on each other only when they are contiguous, and thus Descartes ruled out action at a distance. To account for such phenomena as the lunar influence on tides, Descartes assumed that space is not a void but is a plenum,† being populated by transparent particles capable of transmitting force. He actually went further than this, postulating that all matter was in one of three distinct forms, the luminous matter of the sun, the transparent matter of interplanetary space, and the opaque matter of the earth, giving as his reason,[12]

> For, seeing that the sun and the fixed stars emit light, that the heavens transmit it, and that the earth, the planets, and the comets reflect it, it appears to me that there is ground for using these three qualities of luminosity, transparency, and opacity to distinguish the three elements of the visible world.

Descartes assumed that the luminous matter of the sun consisted of particles which were in continuous motion. Since there was no empty space for the particles to move into, he argued that they took the places vacated by other particles which were also in motion, and thus developed the notion of closed chains of moving particles. The motions of these closed chains constituted *vortices,* an important concept in his explanation of the universe. Thus, according to Descartes' theory,[13] the sun consists of an enormous vortex composed of the first or subtlest kind of matter. The luminous particles of this vortex, due to centrifugal action, constantly strain away from their centers of rotation and thus press against the transparent particles of the ether. The ether

† Thus did the concept of an *ether* enter science for the first time. The word is of Greek extraction and originally meant blue sky.

[11] E. Whittaker, *A History of the Theories of Aether and Electricity,* Vol. 1, p. 4, Thomas Nelson and Sons, Ltd., London, 1951.

[12] R. Descartes, *Principes de la Philosphie,* 4th ed., Part 3, Sec. 52, Chez Théodore Girard, Paris, 1681.

[13] *Ibid.,* Sec. 55–64.

Descartes imagined to consist of a closely packed assemblage of globules, of a size intermediate between that of the luminous matter of the sun and the opaque matter of the earth. The pressure of the vortex against these ether particles causes them to tend to move, thus exerting a pressure on their neighbors, which in turn tend to move, and in this manner the force exerted by the vortex is passed along through the ether particles, from layer to layer. In Descartes' view, the transmission of this pressure constitutes light, a thought he summarizes in the passage[14]

> . . . the force of light . . . does not consist in the duration of some motion but *only in the fact* that these small globules (of the ether) are pressed and tend to move toward some new location, although they do not actually move.

Descartes also provided the first theoretical derivation of the law of refraction, discovered experimentally somewhat earlier (1621) by Willebrord Snell. This derivation is important because it contains a consequence which later loomed as a decisive factor in settling the controversy as to the true nature of light. In the Descartes derivation, a light ray is assumed to be incident on a plane interface between two media at an angle i with respect to the normal, traveling at a velocity v_i in the first medium, and departing from the interface at a velocity v_r in the second medium, in a direction making an angle r with respect to the normal. Descartes then assumed that the component of velocity parallel to the interface was unaffected, obtaining

$$v_i \sin i = v_r \sin r$$

from which Snell's law

$$\frac{\sin i}{\sin r} = \frac{v_r}{v_i} = n$$

follows immediately. However, if the second medium is denser, so that $i > r$, it follows that $v_r > v_i$. Thus Descartes' derivation leads to the conclusion that light must travel *faster* in a denser medium, a conclusion which was later shown to be in contradiction with experiment.

Descartes' opinions were vigorously attacked by Robert Hooke (1635–1703), whose views mark a significant turning point in conjectures about the nature of light. Noted for Hooke's law, he was an able mechanician who devised many improvements in clocks and astronomical instruments, and was the first to formulate a theory of planetary movements as a mechanical problem. He was responsible for the development of microscopy as a science in England, and his interest in this subject led him to many experiments concerned with light itself. Hooke became convinced that light was an undulatory phenomenon, and his reasons are lucidly expressed in the passage[15]

> And first for Light, it seems very manifest, that there is no luminous Body but has the parts of it in motion more or less . . . . It would be somewhat too long . . . to examine, and positively to prove, what particular kind of motion it is that must be the efficient of Light . . . . I found it ought to be exceeding *quick* . . . that in all extreamly hot shining bodies, there is a very quick motion that causes Light, as well as a more robust that causes Heat, may be argued from the celerity wherewith the bodies are dissolv'd.

[14] *Ibid.*, Sec. 63.

[15] R. Hooke, *Micrographia, or Some Physiological Descriptions of Minute Bodies Made by Magnifying Glasses*, 1st ed., pp. 54–56, published by the Royal Society of London, reproduced by Dover Publications, Inc., New York, 1961.

Next, it must be a *Vibrative motion.* And for this the newly mention'd *Diamond* affords us a good argument; since if the motion of the parts did not return, the Diamond must after many rubbings decay and be wasted . . . .

And Thirdly, That it is a very *short vibrating* motion, I think the instances drawn from the shining of Diamonds will also make probable. For a Diamond being the hardest body we yet know in the World, and consequently the least apt to yield or bend, must consequently also have its *vibrations* exceeding short.

Having proposed an explanation for the sources of light, Hooke then suggested

That the motion is propagated every way through an *Homogeneous medium* by *direct* or *straight* lines extended every way like Rays from the center of a sphere . . . in an *Homogeneous medium* this motion is propagated every way with *equal velocity,* whence necessarily every *pulse* or *vibration* of the luminous body will generate a Sphere, which will continually increase, and grow bigger, just after the same manner (though indefinitely swifter) as the waves or rings on the surface of the water do swell into bigger and bigger circles about a point of it, where, by the sinking of a stone the motion was begun, whence it necessarily follows, that all the parts of these Spheres undulated through an *Homogeneous medium* cut the Rays at right angles.

Thus Hooke paralleled Descartes in postulating a medium as the vehicle of light. However, he replaced Descartes' notion that light was a statical pressure in the medium with the notion that it is a rapid undulatory motion of small amplitude. Hooke then went on to replace the Descartes analysis of refraction with one of his own, based on the tilting of a wavefront at the interface of two media, but he failed to notice that it would be necessary to assume the velocity to be *slower* in the denser medium in order to be consistent with Snell's law.

The issue of whether light was wavelike or particlelike was firmly joined with the emergence on the scientific scene of Isaac Newton (1642–1727). Renowned for his discoveries in mechanics, Newton also made many significant contributions in the field of light. His most notable discovery was that white light is made up of the spectral colors, which led him to propound a theory of prismatic colors directly opposed to an earlier theory put forward by Hooke. This precipitated a bitter controversey in which Hooke displayed considerable vexation and accused Newton of favoring the doctrine that light is a material substance. Newton gave his answer in a communication to the Royal Society in 1675 in which he said[16]

Were I to assume an hypothesis, it should be this, if propounded more generally, so as not to determine what light is, farther than that it is something or other capable of exciting vibrations in the aether: for thus it will become so general and comprehensive of other hypotheses, as to leave little room for new ones to be invented. And therefore, because I have observed the heads of some great virtuosos to run much upon hypotheses, as if my discourses wanted an hypothesis to explain them by, and found, that some, when I could not make them take my meaning, when I spake of the nature of light and colours abstractedly, have readily apprehended it, when I illustrated my discourse by an hypothesis; for this reason I have here thought fit to send you a description of the circumstances of this hypothesis as much tending to the illustration of the papers I herewith send you. And though I shall not assume either this or any other hypothesis, not thinking it necessary to concern myself,

[16] I. Newton, *Papers and Letters on Natural Philosophy,* edited by I. Bernard Cohen, p. 179, Harvard University Press, 1958.

> whether the properties of light, discovered by me, be explained by this, or Mr. Hooke's, or any other hypothesis capable of explaining them; yet while I am describing this, I shall sometimes, to avoid circumlocution, and to represent it more conveniently, speak of it, as if I assumed it, and propounded it to be believed. This I thought fit to express, that no man may confound this with my other discourses, or measure the certainty of one by the other, or think me obliged to answer objections against this script: for I desire to decline being involved in such troublesome and insignificant disputes.

Newton's lifelong distaste for controversy is clearly evident here, but equally evident is his refreshing lack of dogmatism about rigid hypotheses. He thoroughly disliked highly imaginative suppositions, such as Descartes had invoked for his grand scheme of the universe, and was much more interested in the formulation of the laws which govern natural phenomena. Despite this, he found it impossible to give coherence to the observed facts about light without resorting to some speculation about its nature. Thus in this same communication, after an exhaustive and detailed discussion of the possible composition of an ether, Newton goes on to suppose that

> Light is neither aether, nor its vibrating motion, but something of a different kind propagated from lucid bodies. They, that will, may suppose it an aggregate of various peripatetic qualities. Others may suppose it multitudes of unimaginable small and swift corpuscles of various sizes, springing from shining bodies at great distances one after another; but yet without any sensible interval of time, and continually urged forward by a principle of motion, which in the beginning accelerates them, till the resistance of the aethereal medium equal the force of that principle, much after the manner that bodies let fall in water are accelerated till the resistance of the water equals the force of gravity.

In Newton's lifetime, all the facts known about light could not be harmonized with either the corpuscular or wave theories then being proposed. However, he leaned toward a corpuscular hypothesis, and near the end of his life summed up his objections to the wave theory in a query at the conclusion of a revised edition of his *Opticks*[17]

> Are not all Hypotheses erroneous, in which Light is supposed to consist in Pression or motion, propagated through a fluid Medium? . . . If Light consisted only in Pression propagated without actual Motion, it would not be able to agitate and heat the Bodies which refract and reflect it . . . . And if it consisted in Pression or Motion, propagated either in an instant or in time, it would bend into the Shadow. For Pression or Motion cannot be propagated in a Fluid in right Lines, beyond an obstacle which stops part of the Motion, but will bend and spread every way into the quiescent Medium which lies beyond the Obstacle . . . . The Waves on the Surface of stagnating Water, passing by the sides of a broad Obstacle which stops part of them, bend afterwards . . . . But Light is never known to follow crooked Passages nor to bend into the Shadow.

Newton goes on, in this query, to add the further objection that the wave theory (as it then existed) could not account for the recently discovered phenomenon of the polarization of light.

The discoverer of this phenomenon of polarization was Christiaan Huygens (1629–1695), a contemporary of both Hooke and Newton, who sided with Hooke in favoring a wave theory of light. Inventor of the pendulum clock, perceptive and influential critic of Descartes' cosmological theories, Huygens is known principally for his work in optics.

[17] I. Newton, *Opticks*, 4th ed., pp. 362–370, William Innys, Publisher, London, 1730. (Reprinted by Whittlesey House, McGraw-Hill Book Company, New York, 1931.)

He greatly extended and improved the wave theory first enunciated by Hooke and subscribed wholeheartedly to Hooke's hypothesis that light consists of some form of motion. Witness the passage[18]

> It is inconceivable to doubt that light consists in the motion of some sort of matter. For whether one considers its production, one sees that here upon the Earth it is chiefly engendered by fire and flame which contain without doubt bodies that are in rapid motion, since they dissolve and melt many other bodies, even the most solid; or whether one considers its effects, one sees that when light is collected, as by concave mirrors, it has the property of burning as a fire does, that is to say it disunites the particles of bodies. This is assuredly the mark of motion, at least in the true Philosophy, in which one conceives the causes of all natural effects in terms of mechanical motions. This, in my opinion, we must necessarily do, or else renounce all hopes of ever comprehending anything in Physics.
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> And as, according to this Philosophy, one holds as certain that the sensation of sight is excited only by the impression of some movement of a kind of matter which acts on the nerves at the back of our eyes, there is here yet one reason more for believing that light consists in a movement of the matter which exists between us and the luminous body.

Huygens next addresses himself to the question as to whether the motion is that of a medium, as assumed by Hooke, or whether it is a stream of particles, as favored by Newton. He says

> Further, when one considers the extreme speed with which light spreads on every side, andhow, when it comes from different regions, even from those directly opposite, the rays traverse one another without hindrance, one may well understand that when we see a luminous object, it cannot be by any transport of matter coming to us from this object, in the way in which a shot or an arrow traverses the air; for assuredly that would too greatly impugn these two properties of light, especially the second of them.

Huygens shared with Newton the inclination to picture an ethereal medium in which light propagated. Whereas Newton favored the idea that this medium was set into vibration by the passage of light corpuscles through it, Huygens preferred to imagine a process analogous to sound, in which the vibrating particles of the luminous source would excite the contiguous portion of the medium into vibration, which would in turn transfer this excitation on to the next portion, etc. This mechanical model of light propagation led him to his most important contribution, ever since known as Huygen's principle, and explained in the passage[19]

> There is the further consideration in the emanation of these waves, that each particle of matter in which a wave spreads, ought not to communicate its motion only to the next particle which is in the straight line drawn from the luminous point, but that it also imparts some of it necessarily to all the others which touch it and which oppose themselves to its movement. So it arises that around each particle there is made a wave of which that particle is the centre.

Using this principle, Huygens was able to show how all the points in one wavefront could be treated as secondary sources which created the next wavefront, and thus provided satisfactory explanations for propagation and reflection. By assuming that the velocity of light was *slower* in a denser medium he was also able to explain refraction.

[18] C. Huygens, *Traité de la Lumière*, pp. 3–4, first published in Leyden in 1690; English translation by S. P. Thompson, London, 1912; reprinted by University of Chicago Press.
[19] *Ibid.*, p. 19.

This proved to be a pivotal point a century and a half later in deciding between a corpuscular or wave theory, since it has already been observed that the corpuscular theory requires a faster velocity in a denser medium in order to be consistent with the law of refraction.

Huygens was unsuccessful in explaining interference effects, such as the colored rings of thin films and sharp shadows past obstacles, partly because it was not then appreciated how short the wavelengths of visible light are. He also confessed his inability to explain his own discovery of polarization, but this is easily understood when one remembers that in 1700 it was not recognized that light consisted of *transverse* vibrations. Similarly, Newton had difficulty in explaining the colors of thin films under the corpuscular theory and the noninterference of beams of light whose paths crossed. Although neither theory was adequate, the esteem in which Newton was held by his contemporaries and followers was so great that the wave theory was rejected and allowed to remain unnourished for over a century. If the fact that Newton found the corpuscular hypothesis more acceptable retarded the growth of the theory of light, as some have claimed, the fault lay with those who blindly espoused all his views. It has already been noted that Newton himself did not hold rigidly to any one hypothesis but rather gave tentative acceptance to that theory which appeared to him to fit most of the facts.

Although most scientists of the eighteenth century accepted the corpuscular hypothesis, the wave theory was not totally without advocates. Franklin (1706–1790) favored it, and Euler (1707–1783) took the same position, being persuaded by the notion that particle emission from a luminous source would cause a diminution in its mass, an effect not observed, whereas the emission of waves did not involve such a consequence. However, the wave theory did not make any serious headway until a new champion arose when Thomas Young (1773–1829) turned his attention to the subject. A man of diverse and considerable talent, Young was a practicing physician on the staff of St. George's Hospital. He was also a physicist, whose lectures at the Royal Institution of London introduced the modern physical concept of energy. He was a prodigy at two, an accomplished linguist while still in his boyhood, a musician, and an archeologist who participated in the deciphering of the Rosetta stone. He made contributions to the theory of tides, explained capillarity, and established the coefficient of elasticity known as Young's modulus.

Drawing upon an earlier explanation by Newton in connection with tides, Young introduced the concept of interference by saying[20]

> Suppose a number of equal waves of water to move upon the surface of a stagnant lake, with a certain constant velocity, and to enter a narrow channel leading out of the lake. Suppose then another similar cause to have excited another equal series of waves, which arrive at the same channel, with the same velocity, and at the same time with the first. Neither series of waves will destroy the other, but their effects will be combined: if they enter the channel in such a manner that the elevations of one series coincide with those of the other, they must together produce a series of greater joint elevations; but if the elevations of one series are so situated as to correspond to the depressions of the other, they must exactly fill up those depressions, and the surface of the water must remain smooth; at least I can discover no alternative, either from theory or from experiment.

[20] T. Young, *Miscellaneous Works*, edited by George Peacock, Vol. 1, pp. 202–203, John Murray Publishers, Ltd., London, 1855.

> Now I maintain that similar effects take place whenever two portions of light are thus mixed; and this I call the general law of the interference of light.

Young demonstrated this concept in an experiment performed before the Royal Society of London in 1803. Using a distant source of a single color, he permitted light to pass through two tiny holes placed close together in one screen, and to fall on a second screen. The second screen showed a pattern of fine bands, alternately light and dark. Young explained this pattern by recourse to a law he had enunciated[21] in 1802:

> Wherever two portions of the same light arrive at the eye by different routes, either exactly or very nearly in the same direction, the light becomes most intense when the differences of the routes is any multiple of a certain length, and least intense in the intermediate state of the interfering portions; and this length is different for light of different colours.

He also used this law to give the first satisfactory explanation of the colors of light reflected from thin plates, arguing that the incident light causes two beams to reach the eye: the first of these beams has been reflected from the first surface of the thin plate, and the other from the second. These two beams produce the colors in the reflected light due to their interference. Indeed, Young used the measured thickness of thin plates to determine for the first time the characteristic lengths, or wavelengths, of the various colors of visible light, publishing[22] a table of values which is remarkably accurate by today's standards.

Despite a bitter attack on Young by the followers of Newton, support for the wave theory accumulated rapidly. Fresnel (1788–1827) satisfactorily explained diffraction past a sharp edge in terms of mutual interference of the secondary "Huygens" waves generated by those portions of the original wavefront not obstructed by the diffracting obstacle. Sharp shadows beyond obstacles big in terms of wavelengths thus became understood, a point about the wave theory which had always bothered Newton. Fresnel also demonstrated light interference by employing two mirrors, and in a brilliant experiment confirmed an hypothesis by Young that light consisted of transverse vibrations by showing that two cross-polarized beams of light do not interfere with each other. This permitted an explanation under the wave theory of the phenomenon of light polarization in crystals, which had earlier been a stumbling block for Huygens. Kirchhoff (1824–1887), starting from the wave equation, developed a diffraction formula in which Huygens' secondary sources were revealed, thus putting that principle on a much firmer foundation.[23]

Finally, the coup de grace was delivered to the corpuscular theory in 1850 when Foucault[24] (1819–1868) and Fizeau[25] (1819–1896) measured the velocity of light in

[21] T. Young, "An Account of Some Cases of the Production of Colours," *Phil Trans Roy Soc (London)*, **92,** 387–397; July 1802.

[22] T. Young, "On the Theory of Light and Colours," *Phil Trans Roy Soc (London)*, **92,** 12–48; November 1801.

[23] Kirchoff summarized his work in the textbook *Vorlesungen über mathematische Optik*, Zweite Vorlesung, Sec. 2, Berlin, 1891.

[24] M. L. Foucault, "General Method for Measuring the Speed of Light in Air and Transparent Media. Relative Speeds of Light in Air and Water," *Compt Rend*, **30,** 551–560; May 1850.

[25] H. Fizeau and L. Brequet, "Note on an Experiment Relative to the Comparative Velocities of Light in Air and in Water," *Compt Rend*, **30,** 562–563; May 1850.

air and water, finding that it was slower in the latter. This result was consistent with the wave theory, whereas the reverse had been predicted by the corpuscular hypothesis. With this experiment, all sensible objection to the wave theory of light had disappeared.

At about this time Maxwell (1831–1879) began formulating his theory of electromagnetism, culminating in the celebrated equations which bear his name. Wavelike solutions to these equations indicated that electromagnetic fields would propagate through a vacuum at the same speed as light. This led Maxwell to the important conjecture that light is an electromagnetic phenomenon and further strengthened the belief that light is basically wavelike in nature.

In 1887, Heinrich Hertz (1857–1894) provided the first successful demonstration of the generation and propagation of electromagnetic waves, using separate spark gap coils to transmit and receive. This achievement was hailed immediately by his contemporaries as the crowning victory of physics, the first experimental verification of the validity of Maxwell's theory. Ironically, a side effect of this experiment was destined to contribute to a great revolution in scientific thought. Hertz noticed that the sparks produced in the gap of his receiving coil were influenced by the light falling on this gap from the sparks in the transmitting coil. Further investigation led Hertz to conclude that it was the ultraviolet portion of the light which was responsible for the effect, and that the effect was greatest if the light were incident on the negative point of the gap. Hertz reported these observations but carried the investigation no further. However, his discovery intrigued many others, and significant contributions were made by Hallwachs, who showed that the photoelectric effect, as it came to be called, consisted of the emission of negative charges, and by Lenard, who measured the charge to mass ratio of the emitted charges and concluded that they were electrons.

A variety of materials was found to be photosensitive, but the characteristics of the emission were surprising. The number of electrons emitted per unit time was proportional to the intensity of the incident light, which seemed reasonable. However, the *maximum* kinetic energy of the emitted electrons was dependent on the *frequency* of the light used, but independent of its *intensity.* A classical argument, assuming a collision-like process, would anticipate that the greater the intensity of the incident wave, the greater would be the energy of the electrons which were torn loose from the surface.

Albert Einstein (1879–1955) offered an explanation of the photoelectric effect in 1905, the same year he received his doctorate from Zurich and published his first paper on relativity. Drawing on an hypothesis made several years earlier by Planck, who had been concerned with the spectral distribution of black-body radiation, Einstein assumed[26]

> . . . that the incident light is composed of quanta of energy $(R/N_A)\beta\nu$ . . . . The quanta of energy penetrate the surface of the material and their respective energies are at least in part changed into the kinetic energy of electrons. The simplest process conceivable is that a quantum of light gives up all its energy to a single electron . . . . Upon reaching the surface, an electron originally inside the body will have lost a part of its kinetic energy. Furthermore, one may assume that each electron in leaving the body does an amount of work W, which is characteristic of the material. Those electrons which are ejected normal to and from the immediate surface will have the greatest velocities. The kinetic energy of these

[26] A. Einstein, "An Heuristic Viewpoint Concerned with the Generation and Transformation of Light," *Ann Phys*, **322,** 132–148; 1905.

electrons is

$$\frac{R}{N_A}\beta\nu - W$$

Einstein thus hypothesized that the incident light was composed of quanta, or photons, whose energy was proportional to the frequency ν of the light. His proportionality factor consisted of a parameter β multiplied by the Boltzmann constant, $k = R/N_A$, with R the ideal gas constant and N_A Avogadro's number. Einstein then argued that, if the photoelectric material were raised to a potential V above a surrounding grounded electrode, then even the most energetic emitted electrons would not reach the grounded electrode if V were of such magnitude that

$$\mathrm{V}e = \frac{R}{N_A}\beta\nu - W$$

in which e is the electronic charge. He then went on to say

> If the formula derived is correct, it would follow that V, if plotted in cartesian coordinates as a function of the frequency of the exciting photons, would yield a straight line whose slope is independent of the material under investigation . . . . If each quantum of light were to give its energy to the electrons independently of all the others then the velocity distribution . . . will be independent of the intensity of the exciting radiation; on the other hand the numbers of electrons leaving the body under equal conditions will be directly proportional to the intensity of the incident radiation.

Einstein's formula and explanation are notable for their simplicity and fit all the observed facts. At the time he proposed this explanation he had at his disposal only qualitative data, but his equation received final and thorough experimental verification through the precise work of Millikan in 1916.[27] Working with a circuit shown in simplified form in Figure 1.1, Millikan varied the reverse bias until it reached a value V such that the ammeter read no current. Since this voltage was just enough to prevent the most energetic electrons from reaching the second electrode, one could argue that $\mathrm{V}e$ was the maximum kinetic energy any of the electrons had upon being emitted from the photosensitive electrode. When Millikan varied ν, the frequency of the incident light, and recorded V for each frequency, he obtained a curve such as shown in Figure 1.2. This experimental result was consistent with Einstein's equation $\mathrm{V}e = (R/N_A)\beta\nu - W$, and the experimental significance of the intercept ν_0 is that light at a lower frequency cannot cause photoelectric emission from the metal concerned. The quantity ν_0 was found to be characteristic of the photosensitive material forming the electrode, but the *slope* of the curve was the same for all electrodes. The slope, which is Einstein's proportionality constant $(R/N_A)\beta$ proved to be identical with the constant h which Planck employed to explain black-body radiation. Thus Einstein's quantum of light, or photon, was found to have an energy $E = h\nu$.

However, the concept that light consists of discrete energy bundles, or photons, smacks strongly of the earlier corpuscular theories. Is light wavelike or corpuscular? The best current answer appears to be that it has a dual personality, exhibiting one set of characteristics or the other, depending on how it is interacting with its environment. If the process being considered is at the microscopic level, the quantized nature

[27] R. A. Millikan, "A Direct Photoelectric Determination of Planck's *h*," *Phys Rev*, **7**, 355–388; 1916.

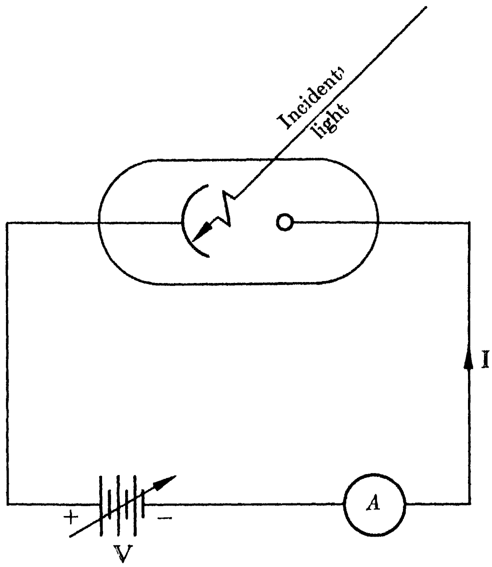

FIGURE 1.1 *Photoelectric diode.*

of light will most likely have to be considered; if it is a macroscopic process, the wave nature of light should account successfully for the interaction.

It would seem that just about everybody was right all along.

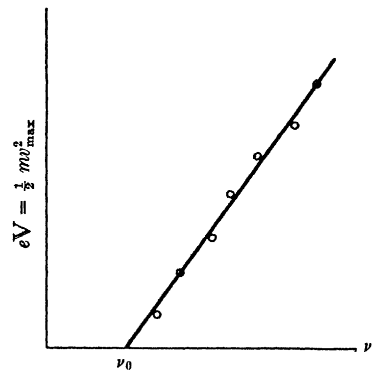

FIGURE 1.2 *Maximum electron energy vs. light frequency.*

1.2* HISTORICAL SURVEY—THE VELOCITY OF LIGHT

Whereas a determination of the *nature* of light is not totally decisive, such ambivalence does not exist when the discussion turns to the conception of the *velocity* of light. Whether light is thought of as a stream of photons or a propagating wave, the transfer

* The reader solely interested in the technical presentation may wish to omit this section except for the discussion of Bradley's experiment.

of energy occurs at a speed which, today, can be measured with extraordinary precision. Yet this speed is so great that it is not surprising to find earlier debates as to whether the velocity of light is finite or infinite.

The direct evidence is lost to us, but Empedocles apparently felt that the velocity is finite, for Aristotle disputes with him in the passage[28]

> Empedocles (and with him all others who used the same forms of expression) was wrong in speaking of light as 'traveling' or being at a given moment between the earth and its envelope, its movement being unobservable by us; that view is contrary both to the clear evidence of argument and to the observed facts; if the distance traversed were short, the movement might have been unobservable, but where the distance is from extreme East to extreme West, the draught upon our powers of belief is too great.

Heron of Alexandria, whose life span has variously been placed in the period from the second century B.C. to the third century A.D., and who is noted for his invention of many contrivances operated by water, steam, or compressed air, believed with Euclid and Ptolemy that light rays originated in the eye. This belief led him to an interesting argument as proof that the velocity of light is infinite:[29]

> That the sight rays emanating from our eyes move with infinite velocity can also be seen from the following. Namely if, after having closed our eyes, we look again upward to the heavens, these rays reach the heavens without any time interval having elapsed (i.e., immediately). For in the same instant in which we open our eyes, we see the stars, even though we may say that the distance is practically infinite. Also, if this distance were even greater, the same occurrence would be repeated in any case, and thus it results that the rays emanating from our eyes propagate with infinite velocity. They therefore suffer in their propagation no interruption in their motion, nor do they make a detour, nor follow a broken-line path, but rather move along the shortest line, namely the straight one.

Alhazen believed otherwise, and in his treatise on optics stated:[30]

> And we shall see that color will not be perceived in that which is color by the sight, nor light in that which is light, except in time . . . the arrival of the sensation (of light) to the hollow of the optic nerve is like the arrival of light from holes . . . the passing of light from a hole to an object opposite the hole will not be possible except in time, even though this fact is concealed from the mind.
>
> The passing of light from a hole to an object opposite the hole cannot escape being in one of the two following ways, namely, that *either* light will come to that part of the air which is near the hole, before it can arrive to another following point, and thereafter it will come to another point, and so to another, until it arrives at the object opposite the hole, *or* light will arrive at the entire intermediate atmosphere between the hole and the object opposite the hole, and to the very object, all at the same time. If the air received light in a successive fashion, the light would not arrive at the object opposite the hole, except through movement. But movement does not exist except in time; thus, if the whole atmosphere receives light at the same time, even the arrival of light to the atmosphere does not exist, since it was not in the atmosphere before . . . .

[28] Aristotle, *De Anima*, 418^b, 20, English translation under editorship of W. D. Ross, Oxford at the Clarendon Press, 1931.

[29] Heronis Alexandrini, *Catoptrica*, Vol. 2, pp. 320–323, translated into German by L. Nix and W. Schmidt, von B. G. Teubner, Leipzig, 1900. (Private English translation.)

[30] Alhazen, *Opticae Thesaurus*, edited by Risner, Vol. 2, Chap. 2, Article 21, Basel, 1572. (Private translation.)

> If the hole through which the light enters becomes blocked, and then the blockage is removed, the instant during which the blockage is removed . . . is different from the instant during which the light reaches the contiguous atmosphere . . . . Therefore this is done by a movement; but a movement does not exist except in time . . . . However, this time element is strongly concealed from the mind due to the rapidity of the perception of the sensation of light by the air.

Avicenna agreed with Alhazen, basing his opinion on the belief that light consisted of the motion of finite particles which therefore could not have an infinite velocity. Roger Bacon also sided with Alhazen, although he did not like the reasons advanced above and preferred the argument Alhazen put forth in his seventh volume that "from the same terminus the perpendicular ray reaches more quickly the terminus of the space than the ray that is not perpendicular." However, Bacon was very gentle in his disagreement with Aristotle, drawing a fine distinction between perceptible and imperceptible intervals of time. His principal reason for believing in a finite velocity is contained in the passage[31]

> . . . an instant has the same relation to time as a point to a line. Therefore, interchanging terms, an instant has the same relation to a point as time has to a line; but the passage through a point is in an instant. Therefore the passage through the whole line is in time. Therefore species [of light] passing through linear space, however small, will pass through in time . . . . If, therefore, the multiplication of light is instantaneous, and not in time, there will be an instant without time; because time does not exist without motion. But it is impossible that there should be an instant without time, just as there cannot be a point without a line. It remains, then, that light is multiplied in time, and likewise all species of a visible thing and of vision. But nevertheless the multiplication does not occupy a sensible time and one perceptible by vision, but an imperceptible one, since any one has experience that he himself does not perceive the time in which light travels from east to west.

Francis Bacon (1561–1626), an English philosopher credited with the formulation and introduction of the inductive method of modern science, struggled with the question of the velocity of light in the absence of experimental information, as is evident in this excerpt.[32]

> Even in sight, whereof the action is most rapid, it appears that there are required certain moments of time for its accomplishment . . . . (It is not surprising that we do not see the actual passage of light, for there are things which by reason of the velocity of their motion cannot be seen—as when a ball is discharged from a musket . . . .) This fact, with others like it, has at times suggested to me a strange doubt, viz. whether the face of a clear and star-light sky be seen at the instant at which it really exists, and not a little later; and whether or not, as regards our sight of heavenly bodies, [there is] a real time and an apparent time, just like the real place and apparent place which is taken account of by astronomers in the correction for parallaxes . . . [whether or not] the images or rays of heavenly bodies . . . take a perceptible time in travelling to us. But this suspicion as to any considerable interval between the real time and the apparent afterwards vanished entirely . . . what had most weight of all with me was, that if any perceptible interval of time were interposed between

[31] R. Bacon, *Opus Majus*, Part 5, 9th Distinction, Chap. 3, the R. B. Burke translation, University of Pennsylvania Press, Philadelphia, 1928.

[32] Francis Bacon, *Philosophical Works*, edited by J. M. Robertson from the edition of Ellis and Spedding, p. 363, London, 1905. (As quoted in I. B. Cohen, *Roemer*, p. 11, The Burndy Library, Inc., New York, 1944.)

the reality and the sight, it would follow that the images would oftentimes be intercepted and confused by clouds rising in the meanwhile, and similar disturbances of the medium.

A contrast to all this metaphysical speculation is found in the attitude of Galileo Galilei (1564–1642). Widely regarded as the father of modern physics, Galileo was a champion of the experimental method. At the age of twenty-six, while professor of mathematics at Pisa, he began a systematic investigation of the mechanical doctrines of Aristotle. Having convinced himself by experiment of the error in many of Aristotle's assertions, Galileo invoked the enmity of the Church by loudly proclaiming his dissensions. These included the question of whether or not a heavy body falls faster than a light one, and later the profound question of whether the Ptolemaic or Copernican view of the universe was the proper one.

Galileo was the first to observe that a simple pendulum has a natural period. He properly deduced the formulas of uniformly accelerated motion, and his contributions to mechanics were an important precursor to the generalizations made by Newton a century later. He constructed the first astronomical telescope and with it discovered the satellites of Jupiter, the crescent phases of Venus, sunspots and the rotation of the sun, and the libration of the moon. Galileo became interested in the question of light velocity and, believing it to be finite, undertook to establish this experimentally. His approach was logical but doomed to failure because of the great velocity involved. In the famous *Dialogues*, published in Leyden in 1638, Galileo proposed that[33]

> Each of two persons take a light contained in a lantern, or other receptacle, such that by the interposition of the hand, the one can shut off or admit the light to the vision of the other. Next let them stand opposite each other at a distance of a few cubits and practice until they acquire such skill in uncovering and occulting their lights that the instant one sees the light of his companion he will uncover his own. After a few trials the response will be so prompt that without sensible error the uncovering of one light is immediately followed by the uncovering of the other, so that as soon as one exposes his light he will instantly see that of the other. Having acquired skill at this short distance let the two experimenters, equipped as before, take up positions separated by a distance of two or three miles and let them perform the same experiment at night, noting carefully whether the exposures and occultations occur in the same manner as at short distances; if they do, we may safely conclude that the propagation of light is instantaneous; but if time is required at a distance of three miles which, considering the going of one light and the coming of the other, really amounts to six, then the delay ought to be easily observable. If the experiment is to be made at still greater distances, say eight or ten miles, telescopes may be employed, each observer adjusting one for himself at the place where he is to make the experiment at night; then although the lights are not large and are therefore invisible to the naked eye at so great a distance, they can readily be covered and uncovered since by aid of the telescopes, once adjusted and fixed, they will become easily visible . . . .

Later he comments,

> In fact I have tried the experiment only at a short distance, less than a mile, from which I have not been able to ascertain with certainty whether the appearance of the opposite light was instantaneous or not; but if not instantaneous it is extraordinarily rapid—I should call it momentary; . . . .

[33] Galileo Galilei, *Dialogues Concerning Two New Sciences*, p. 43, reprinted by Dover Publications, Inc., New York.

Galileo's experiment was repeated by scientists of the Florentine Academy but with inconsistent results. The human reaction times were much too great, the separation of the lanterns was only a few miles, and the timepieces of that era were extremely crude.

In the continuing absence of decisive experimental results, the speculation continued. Kepler (1571–1630) held an Aristotelian view,[34] maintaining that light can be propagated an infinite distance in zero time. He based this view on the argument that light is not matter and thus cannot offer resistance to the force which moves it. In Aristotelian mechanics, this requires that light attain an infinite velocity.

Descartes, as has already been noted, believed that light consisted of a transmission of pressure through the tightly packed globules of the ether. However, in his conception, light was not a motion because the globules only tended to move, being restrained in position by their neighbors. Thus each globule was capable of transmitting force instantaneously, which led Descartes to conclude[35]

> Thus, we shall have no trouble in realizing why such an effect, which I attribute to light, extends in a spherical fashion all around the sun . . . and why such light propagates instantaneously to all distances.

It is interesting to observe that Descartes could believe both that the velocity of light was infinite and that the velocity of light was not the same in different media, an assumption he made in deriving Snell's law (see Section 1.2).

In a correspondence with the Dutch physicist Beekman (1570–1637), Descartes was hard pressed to defend his metaphysical arguments in favor of an infinite light velocity, and hit upon an argument which is scientifically sound, and which seemed to him to be a complete proof that his position was the only correct one. Descartes proposed consideration of a lunar eclipse, caused by the earth being interposed between the sun and the moon. He then supposed that it requires an hour for light to travel from the earth to the moon, which would mean that the moon did not grow dark until an hour after the instant of collinearity of the three bodies. People on earth would not be aware of this darkening for an additional hour, or until the earth and moon had moved in their orbits an additional two hours beyond the position of collinearity. But, argued Descartes, this is clearly contrary to experience, for the eclipsed moon is always observed at a point in the ecliptic opposite to the sun. Thus the light must travel instantaneously.

Huygens challenged this proof at its only weak point, saying[36]

> But it must be noted that the speed of light in this argument has been assumed such that it takes a time of one hour to make the passage from here to the Moon. If one supposes that for this . . . it requires only ten seconds of time . . . then it will not be easy to perceive anything of it in observations of the Eclipse; nor, consequently, will it be permissible to deduce from it that the movement of light is instantaneous.
>
> It is true that we are here supposing a strange velocity that would be a hundred thousand times greater than that of Sound . . . . But this supposition ought not to seem to be an impossibility; since it is not a question of the transport of a body with so great a speed, but of a successive movement which is passed on from some bodies to others. I have then

[34] J. Kepler, *Ad Vitellionem paralipomena quibus astronomiae pars optica traditur*, Frankfurt, 1604.

[35] R. Descartes, *Principes de la Philosophie*, 4th ed., Part 3, Sec. 64, Chez Théodore Girard, Paris, 1681.

[36] C. Huygens, *Traité de la Lumière*, pp. 6–7, first published in Leyden in 1690; English translation by S. P. Thompson, London, 1912; reprinted by University of Chicago Press.

made no difficulty, in meditating on these things, in supposing that the emanation of light is accomplished with time . . . .

Hooke also appreciated the weakness in Descartes' argument, and in speaking of the propagation of light through a transparent body or medium, he asserted[37] that the light

> . . . may be communicated or propagated through it to the greatest imaginable distance in the least imaginable time; though I see no reason to affirm, that it must be in an instant: For I know not any one Experiment or observation that does prove it. And, whereas it may be objected, That we see the Sun risen at the very instant when it is above the sensible Horizon, and that we see a Star hidden by the body of the Moon at the same instant, when the Star, the Moon, and our Eye are all in the same line; and the like Observations, or rather suppositions, may be urg'd. I have this to answer, That I can as easily deny as they affirm; for I would fain know by what means any one can be assured any more of the Affirmative, than I of the Negative. If indeed the propagation were very slow, 'tis possible something might be discovered by Eclypses of the Moon; but though we should grant the progress of the light from the Earth to the Moon, and from the Moon back to the Earth again to be full two Minutes in performing, I know not any possible means to discover it . . . .

The distinction for having performed the first decisive determination of the velocity of light goes to Ole Roemer (1644–1710). Born in Denmark, and educated under the Bartholins at the University of Copenhagen, he then went to Paris as a young astronomer for the Académie Royale des Sciences, which at that time was undertaking a project to prepare more accurate maps. A technique had been proposed whereby the longitude of any place could be determined relative to the longitude of Paris by simultaneous observation of an astronomical phenomenon from the two positions. What was needed was a celestial occurrence of reasonable frequency, and a tentative selection was made of the eclipses of the satellites of Jupiter, a phenomenon which had been discovered earlier in the same century by Galileo.

In choosing Roemer to work on this project, the Académie picked a man who was to prove to be one of the greatest practical astronomers of all time. He built the first good transit instrument and the earliest transit circle, greatly improved on the construction of micrometers, and showed that the epicycloid is the best shape for gear teeth, incorporating this discovery into the design of all his astronomical instruments; in his later years he supervised the erection of an excellent observatory near Copenhagen.

While in Paris at the beginning of his career, and upon launching into a study of the eclipses of Jupiter's moons, Roemer was struck by a surprising observation. Since one would expect that the period of a moon would remain constant, knowing the time at which one eclipse occurred, it was then a simple matter to predict a sequence of later times at which a given moon would be eclipsed by Jupiter. But when Roemer did this, he predicted a time sequence which did not agree with later eclipse measurements. He attributed this disparity to the changed distance between Earth and Jupiter, which, if the velocity of light were finite, would explain the irregularity in eclipse occurrences.

Accordingly, in September 1676, Roemer announced to members of the Paris Académie that the next eclipse of the innermost satellite of Jupiter, expected on

[37] R. Hooke, *Micrographia*, 1st ed., p. 56, published by the Royal Society of London, reproduced by Dover Publications, Inc., New York, 1961.

November 9, would occur exactly ten minutes later than the time computed on the basis of previous eclipses. When observation had confirmed this startling prediction, Roemer again addressed the Académie, saying[38]

> The necessity of this new equation of the retardation of light, is established by all the observations that have been made by the Académie Royale and by the Observatory during the last eight years, and it has been confirmed anew by the emersion of the first satellite, observed at Paris last November 9th at $5^h\ 35^m\ 45^s$ at night, 10 minutes later than had been expected . . . .

From his knowledge of the relative positions of the earth and Jupiter, Roemer deduced that this retardation was such that light should take 22 minutes to cross the diameter of the earth's orbit, which translates into a velocity of light of approximately 140,000 mi/sec. Roemer's value was thus about 25 percent low,† but his accomplishment was nevertheless impressive. For the first time in history man had been able to measure a velocity which was so great that many had thought it to be infinite.

Roemer's assertion was accepted promptly by Huygens and Newton, and many of his colleagues were quick to rectify the error in his calculations. Thus Newton, in the first edition of his *Opticks* (1704), introduces the proposition that[39]

> Light is propagated from luminous Bodies in time, and spends about seven or eight minutes of an Hour in passing from the Sun to the Earth.

adding that this effect was first observed by Roemer. However, no such acceptance was found among the Cartesians, and such was the influence of Descartes' ideas that the Continent remained unconvinced until the brilliant confirming experiments of Bradley a half century later.

Bradley (1693–1762) was born in Gloucestershire and educated at Oxford. His interest in astronomy was aroused early by an uncle whose home contained an excellent amateur observatory, and he became an acute observer through having engaged in a regular series of observations extending from boyhood. He was elected a member of the Royal Society in 1718 and three years later was appointed Savilian Professor of Astronomy at Oxford. He succeeded Halley as Astronomer Royal in 1742 and devoted the remainder of his life to the Greenwich observatory.

In addition to the discovery of stellar aberration, to be discussed below, Bradley's minute observations led him to the detection of the nutation of the earth's axis. In an action so characteristic of his painstaking nature, Bradley refrained from announcing the discovery of nutation until February 1748, after he had assured himself of its certainty by careful measurements extending over an entire revolution (18.6 years).

Bradley's discovery and interpretation of the phenomenon of stellar aberration came

† His principal source of error was an oversight. Roemer had used eclipse data from the years 1671–1673 to predict the retardation time, because he had at his disposal many observations from that period, and also because Jupiter at that time had been making an aphelion passage and thus was at a nearly constant distance from the sun. However, in 1676 Jupiter was no longer in such a position, and Roemer failed to account for its changed distance from the sun between eclipses, thus obtaining an incorrect value for the change in the distance between Earth and Jupiter.

[38] O. Roemer, "Demonstration Concerning the Movement of Light," *J des Scavans*, 233–236; December 7, 1676. (Reprinted in *Phil Trans Roy Soc (London)*, **12**, 893–894; June 25, 1677.)

[39] I. Newton, *Opticks*, 4th ed., Book 2, Part 3, Proposition 11, William Innys, Publisher, London, 1730. (Reprinted by Whittlesey House, McGraw-Hill Book Company, New York, 1931.)

as the result of an effort to detect stellar parallax, which he began in 1725. The absence of any measurable parallax had long been a stumbling block for adherents of the Copernican system. Tycho (1546–1601) had recognized earlier that, when viewed from opposite sides of the earth's orbit, stars should show a displacement in direction, but his careful observations convinced him that no such displacement so great as one minute of arc existed. Later observers also had sought this effect in vain, and stellar parallax had become one of the outstanding problems in astronomy.

Working with improved instruments, Bradley attacked this problem by systematically recording the position of γ Draconis, a bright star in the constellation Draco, at various times during the year. As shown in Figure 1.3*a*, what he was seeking was a difference in the angles α and β, which certainly should be evident if r_1 and r_2 were not too much greater than the diameter of the earth's orbit. It is obvious from the figure that this parallax effect should be greatest for stars near the ecliptic pole,† and thus γ Draconis was an ideal choice. The plane containing the ecliptic axis and γ Draconis cuts the earth's orbit in points the earth occupies in June and December. Thus Bradley expected to find γ Draconis making its smallest angle to the ecliptic plane in December and its greatest angle in June. To his surprise, he found that γ Draconis lies closest to the ecliptic in March and is most elevated in September, the difference in these angles being about 40 sec of arc.

Bradley checked his findings by observing other stars over a three-year period, always with similar results. Finally satisfied that the effect was real, he reported[40] his observations in 1728. After carefully eliminating other possible explanations for the effect, he said

> At last I conjectured, that all the *Phenomena* hitherto mentioned, proceeded from the progressive Motion of Light and the Earth's annual Motion in its Orbit. For I perceived, that, if Light was propagated in Time, the apparent Place of a fixt Object would not be the same when the Eye is at Rest, as when it is moving in any other Direction, than that of the Line passing through the Eye and Object; and that, when the Eye is moving in different Directions, the apparent Place of the Object would be different.

Bradley then proceeded to explain the apparent shift in position of the stars under this hypothesis. His reasoning can be understood with reference to Figure 1.3*b*, in which Cartesian axes have been chosen *fixed in the sun*, with the Z axis pointing toward the ecliptic pole and γ Draconis in the XZ plane, close to the Z axis. In March the orbital velocity of the earth is toward γ Draconis, whereas in September it is away from γ Draconis. Neglecting the diurnal rotational motion of the earth (which is only about 1 percent of the orbital motion), Bradley reasoned in effect that in March the velocity components of the light entering his telescope from γ Draconis were $(c_x + v, 0, c_z)$, whereas in September they were $(c_x - v, 0, c_z)$, with v the orbital speed and c_x, c_z the velocity components of the light relative to the sun. Thus in March he needed to point his telescope at an angle α above the ecliptic plane given by $\tan \alpha = c_z/(c_x + v)$, and in September he needed to point his telescope at a slightly higher angle β above the

† The earth's orbit lies in the plane of the ecliptic, and the ecliptic pole is the axis perpendicular to this plane and piercing it at the center of the earth's orbit.

[40] J. Bradley, "An Account of a New Discovered Motion of the Fix'd Stars," *Phil Trans Roy Soc (London)*, **35**, 637–660; December 1728.

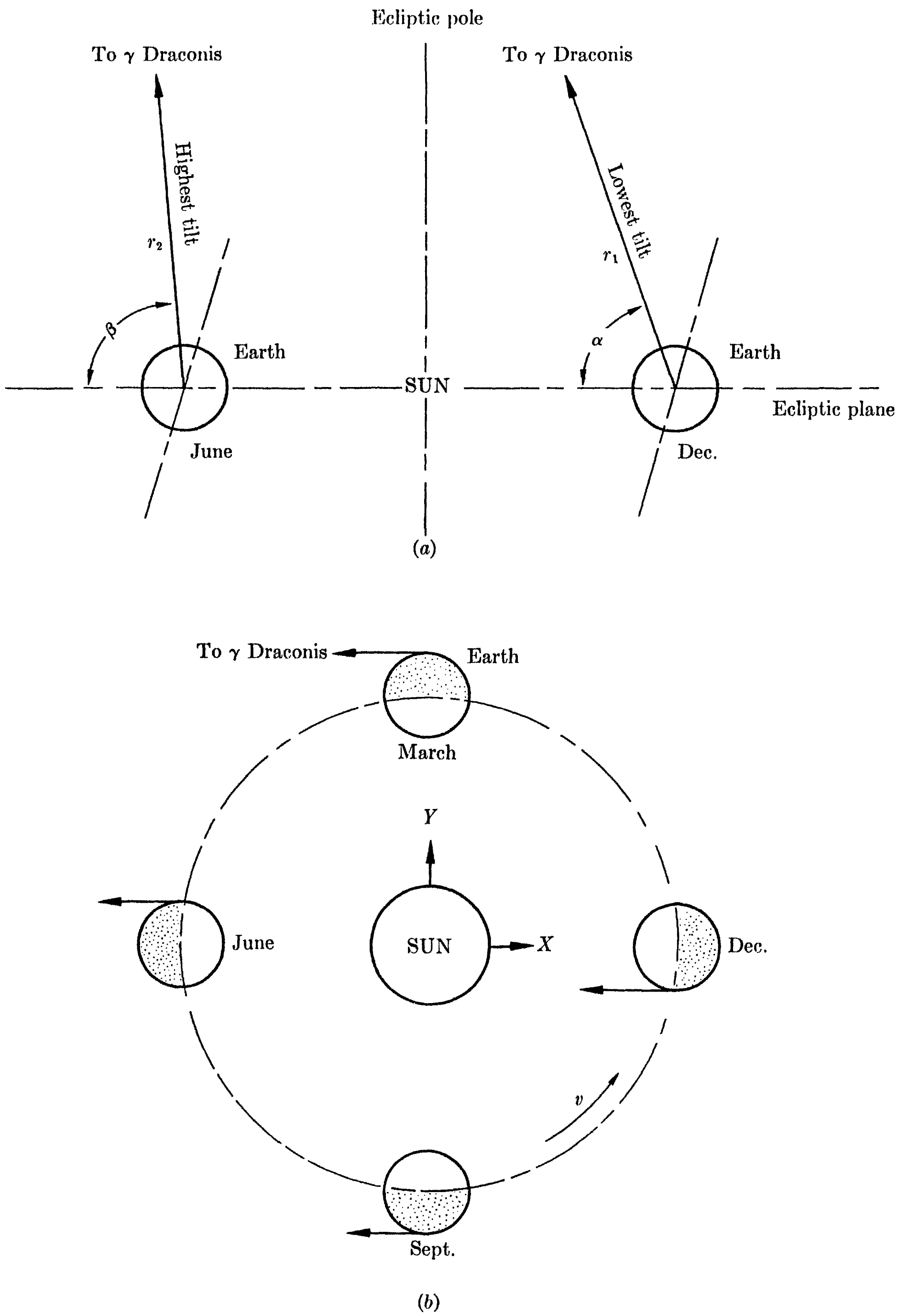

FIGURE 1.3 *Stellar aberration.*

ecliptic given by $\tan \beta = c_z/(c_x - v)$. Since $\beta - \alpha$ is small,

$$\frac{\tan \beta - \tan \alpha}{1 + \tan \beta \tan \alpha} = \frac{2vc_z}{c^2 - v^2} = \tan (\beta - \alpha) \simeq \beta - \alpha$$

from which, because $v \lll c$, it follows that

$$\beta - \alpha \simeq 2\frac{v}{c}\frac{c_z}{c} \tag{1.1}$$

Upon inserting measured values for α, β, and v into Equation (1.1), Bradley was able to deduce a value for the velocity of light c, since he knew the direction cosine c_z/c. In his own words,

> . . . the Velocity of Light [is] to the Velocity of the Eye (which in this Case may be supposed the same as the Velocity of the Earth's annual Motion in its Orbit) as 10,210 to One, from whence it would follow, that Light moves, or is propagated as far as from the Sun to the Earth in 8′12″.
>
> It is well known, that Mr. *Romer*, who first attempted to account for an apparent Inequality in the Times of the Eclipses of *Jupiter's* Satellites, by the Hypothesis of the progressive Motion of Light, supposed that it spent about 11 Minutes of Time in its Passage from the Sun to us: but it hath since been concluded by others from the like Eclipses, that it is propagated as far in about 7 Minutes. The Velocity of Light therefore deduced from the foregoing Hypothesis, is as it were a *Mean* betwixt what had at different times been determined from the Eclipses of *Jupiter's* Satellites.

Bradley's value for the time of passage of light from the sun to the earth translates into a light velocity of 189,000 mi/sec, a value in close agreement with modern measurements.

Bradley termed this effect which shifts the apparent position of a star *aberration.* When his findings became widely known, all sensible objection to the view that the velocity of light is great, but finite, ceased to exist.

The first attempt to measure the velocity of light using a purely terrestrial method was made by Fizeau in 1849. He employed a large toothed wheel as a light chopper and selective receiver, sending light pulses to a remote mirror at a known distance. Upon their return, the pulses would be unable to get past a tooth which had moved over to replace a space, if the rotational speed of the wheel were a critical value; this fact was used to deduce the time taken for a pulse to travel from the wheel to the distant mirror and back, from which the velocity of light followed immediately.

A lifelong resident of Paris, Fizeau (1819–1896) devoted his long and productive career to scientific research. With Foucault, he conducted an extensive series of experiments on interference of both light rays and heat rays. He explained the Doppler effect, made valuable discoveries related to the polarization of light, and applied the principle of light interference to the measurement of the dilatation of crystals. He is best remembered for determinations of the velocity of light in air and in moving water. The latter determination played a significant role in the development of the special theory of relativity and will be discussed in Chapter 2. Fizeau's determination of the velocity of light in air was accomplished earlier, in 1849, with an apparatus which is suggested in simplified form by Figure 1.4.

In this experiment, light from a source S was focused at f by means of the lens L_1

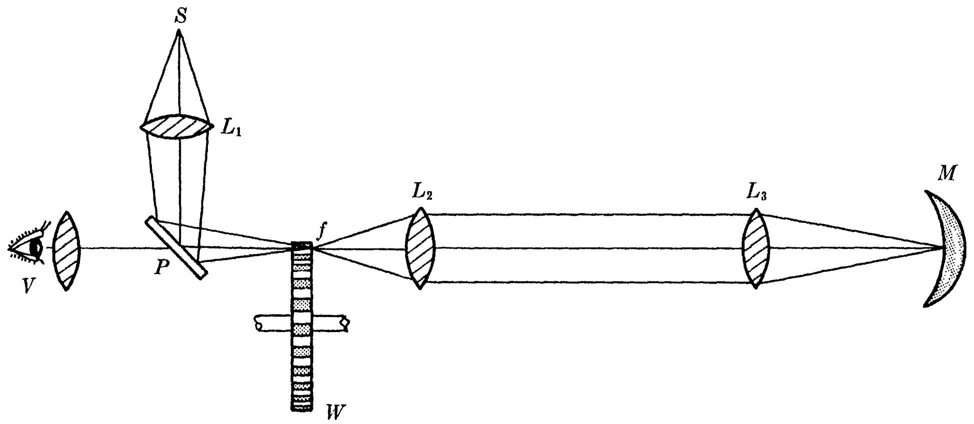

FIGURE 1.4 *Fizeau's apparatus.*

and the half-silvered mirror P. The principal focus of the lens L_2 was made to coincide with f so that a parallel beam of light emerged from the apparatus and traveled to a distant station consisting of the lens L_3 and the spherical mirror M. This beam was focused by L_3 on M, whose center of curvature was chosen to lie in L_3. Thus the reflected beam emerged from L_3 in a parallel pencil and was brought to a focus at f, from whence it diverged to fall upon the half-silvered mirror P and be partially transmitted to the eyepiece V.

When a toothed wheel W was inserted in the light path at f, an image of the source S could be seen at V unless f were blocked by the presence of a tooth. Fizeau used a wheel with 720 teeth separated by spaces congruent to the teeth, and connected the wheel to a clockwork driven by weights, thus using the wheel to pulse the light. With the wheel rotating very slowly, the image of S would appear and disappear successively as the spaces and teeth passed before f. However, if the speed were increased to the point that several teeth per second passed f, the persistence of vision would render a permanent image at half the intensity which had been seen with the wheel at rest and two teeth straddling f.

When the speed of the toothed wheel was increased further, because of the finite velocity of light, a sensible part of the light transmitted through a space toward M would, upon returning, fall upon the adjacent tooth and be intercepted, thus decreasing the intensity of the image. If the rotational speed became great enough so that, when the light returned, the tooth had just moved into the position previously occupied by the space, then all the returning light was intercepted and the image at V was totally extinguished.

What occurred, therefore, was that at first a bright image was observed, which faded away as the rotational speed increased to a value just sufficient to replace a space by a tooth in the time τ it took light to travel from f to M and back. When the rotational speed was increased further, the image returned, increasing in brightness until a maximum was reached corresponding to one space replacing another in time τ. Having thus reached a maximum, the image would fade away again, and so on in succession for higher and higher speeds.

From his knowledge of the wheel geometry and a measurement of the rotational speed during image eclipse, Fizeau was able to deduce τ and thus the velocity of light, since he knew the distance from f to M. In reporting this experiment[41] he said

> . . . the result turned out very well, and one was able to observe, depending on whether the speed of rotation was more or less, a bright point of light or a total eclipse. Under the conditions in which the experiment was performed, the first eclipse occurred for 12.6 rotations per second. For double that speed, a new bright point; for triple, a second eclipse . . . and so forth.
>
> The first station was placed in the belvedere of a house situated at Suresnes, the second on the top of Montmartre, at a distance of approximately 8633 meters . . . .
>
> These first attempts furnished a value for the velocity of light which differs but little from that which has been obtained by astronomers. The mean deduced from twenty-eight observations made so far give for its value 70,948 leagues† . . . .

Fizeau's technique was limited in its accuracy because it was difficult to judge just when the image had reached maximum or minimum intensity. Foucault devised a modification of the apparatus which overcame this limitation by replacing the toothed wheel with a rotating mirror. This mirror caused a measurable displacement of the image, thus providing a determination of the velocity of light. In 1850 Foucault used this apparatus to measure the relative velocities of light in air and water, and in 1862 he used an improved version to make an absolute determination of the velocity of light in air.

Foucault (1819–1868) was also a Parisian, the son of a publisher. He originally studied for a medical career but then abandoned it for physical science. With Fizeau he carried on a series of investigations on the intensity of the light of the sun, as well as the above-mentioned interference experiments. He established that the velocity of light is inversely proportional to the refractive index of the medium, thus contributing to the overthrow of the corpuscular theory. In 1851 he demonstrated the diurnal motion of the earth via what has come to be known as the Foucault pendulum, and in 1852 he invented the gyroscope; for these two achievements he received the Copley medal in 1855.

The 1862 determination of the velocity of light was achieved with the apparatus shown in Figure 1.5. Foucault let solar light, transmitted from a rectangular aperture S, pass through a half-silvered mirror P and fall upon the achromatic lens L. The light then proceeded to a rotatable plane mirror R, which was initially fixed at the proper angular position to bring the rays to a focus at the point M. A concave mirror fixed at M, with a radius of curvature equal to RM, then reflected the light along a return path such that half of the light came to a focus at A, to be viewed by a micrometer eyepiece. A fine grating was stretched over the slit at S, so that the image at A was crossed by dark lines, above which a cross-hair of the eyepiece could be positioned accurately.

When the mirror R was rotated, it acted as a light chopper, in that only when R

† The league is an itinerary measure of distance which varies from country to country but is usually estimated at about 3 mi. Fizeau used it in a precise sense such that his result was equivalent to a light velocity of 3.13×10^8 m/sec or 194,000 mi/sec.

[41] A. H. Fizeau, "On an Experiment Relative to the Speed of Propagation of Light," *Compt Rend*, **29**, 90–92; July 1849.

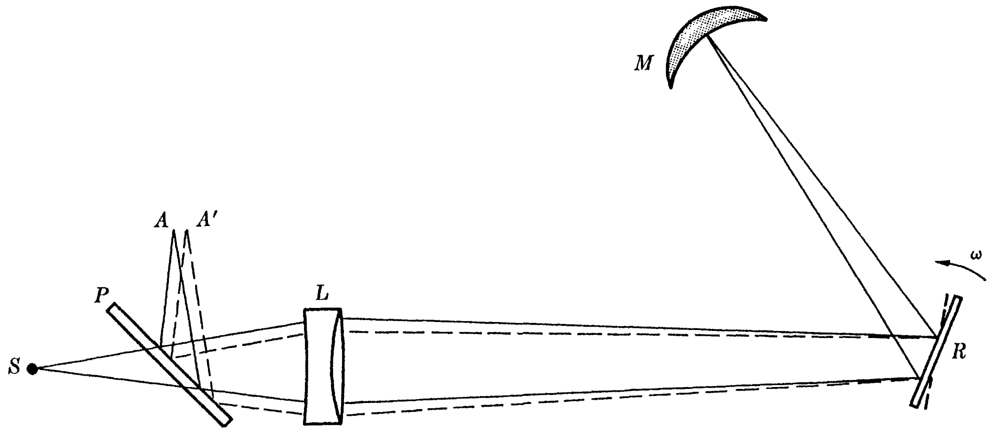

FIGURE 1.5 *Foucault's apparatus.*

was in the proper angular position to deliver light to M would an image be seen at the eyepiece. However, during the time τ light takes to travel from R to M and back, the mirror would rotate an additional angular amount $\alpha = \omega\tau$ in which ω was the angular velocity of the mirror. This caused the reflected beam to be deflected an angle 2α, thus shifting the image from A to A'. By measuring the displacement AA' and the rotational speed ω, since he knew the relative positions of the components of his apparatus, Foucault was able to determine τ and thus the velocity of light.

Foucault placed the mirrors R and M an equivalent distance of 20 m apart through the use of multiple reflections, and turned the mirror R at speeds up to 1,000 revolutions per second, obtaining image displacements in the order of 1 mm. Of his results he said[42]

> Definitively, the velocity of light has been found to be noticeably diminished. Earlier data had indicated that the velocity was 308 millions of meters per second, and this new experiment with the turning mirror gives a value, in round numbers, of 298 millions.
>
> One is able, it seems to me, to count on the exactness of this number, in the sense that the corrections it would have to suffer should not change its value more than 500,000 meters.

Despite the confidence expressed by Foucault in this determination, his apparatus also suffered from a serious limitation. The distance RM could not be increased significantly without diminishing the intensity of the image at A', since the intensity of the light reflected from M was attenuated as $(RM)^2$ before returning to R. But with RM at 20 m and extremely high speeds for the rotating mirror, the displacement AA' was still small enough to be subject to considerable error.

Michelson eliminated this drawback by placing the lens L between R and M so that S lay at its principal focus, thus providing a parallel beam to travel to M. The mirror M could then be made plane and placed at a much larger distance from R, thus enhancing the displacement AA'; indeed, Michelson was able to achieve such great image displacements that he eliminated the half-silvered mirror P. His simplified version of the

[42] J. B. L. Foucault, "Experimental Determination of the Velocity of Light," *Compt Rend,* **55,** 501–503; September 1862.

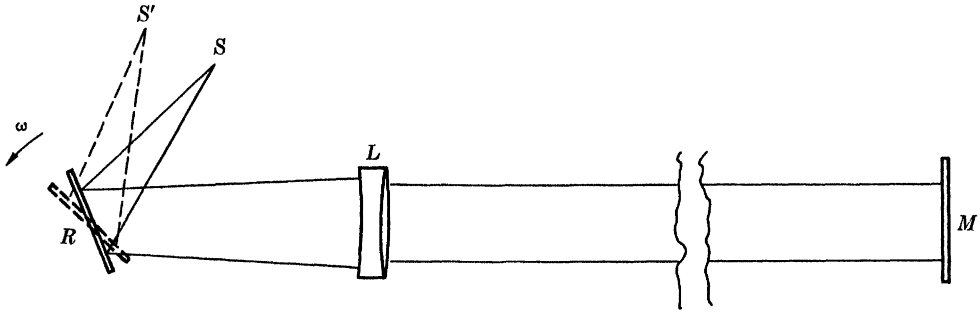

FIGURE 1.6 *Michelson's apparatus.*

apparatus is shown in Figure 1.6. About this apparatus and his measurements, Michelson said[43]

> In the following experiments the distance between the mirrors was nearly 2000 feet . . . and the speed of the mirror was about 257 revolutions per second. The deflection exceeded 133 millimeters, being about 200 times as great as that obtained by Foucault. If it were necessary it could be still further increased. This deflection was measured within three or four hundredths of a millimeter in each observation; and it is safe to say that the result, so far as it is affected by this measurement, is correct to within one ten-thousandth part.
>
> The revolving mirror was actuated by a current of air . . . . To regulate and measure the speed of rotation a tuning fork, bearing on one prong a steel mirror, was employed. This was kept in vibration by a current of electricity. The fork was so placed that the light from the revolving mirror was reflected to a piece of plane glass in front of the eye-piece, and thence reflected to the eye. When fork and mirror are both at rest, an image of the revolving mirror is perceived. When the fork vibrates, this image is drawn out into a band of light. When the mirror commences to revolve, this band breaks up into a number of moving images of the mirror; and when, finally the mirror makes as many turns as the fork makes vibrations, or any multiple . . . of this number, the images become stationary . . . .
>
> The electric fork made about 128 vibrations per second. No dependence was placed upon this rate, however, but at each set of observations it was compared with a standard Ut_3 fork, the temperature being noted at the time.

Being thus assured of great accuracy in both of the critical measurements—image displacement and mirror velocity—Michelson listed 200 data points, each of which was the mean of 10 separate observations, and concluded that the velocity of light in air was 299,740 km/sec, being thus 299,820 km/sec *in vacuo*. In 1882 he repeated the experiment and announced a new value for the velocity of light *in vacuo*, 299,853 km/sec. This was to remain the accepted figure for forty-five years, and when it was replaced by a more precise figure, Michelson was once again involved in the determination.

Albert A. Michelson (1852–1931) was born in Poland but emigrated to America with his parents at the age of two. They settled in the West following the gold rush and he was raised in a mining town. A rare presidential appointment as midshipman at the Naval Academy insured his college education and stimulated his interest in science. Upon graduation he became an instructor at Annapolis and embarked on his

[43] A. A. Michelson, "Experimental Determination of the Velocity of Light," *Am J Sci*, **18**, 390–393; November 1879.

first determination of the velocity of light, described above. There followed a period of study in Europe during which he invented the interferometer and with it performed the first ether drift experiment. Upon returning to the United States, he teamed with Professor Morley to improve the interferometer and repeat this celebrated experiment which has so influenced the subject of relativity. They also collaborated in a precise repetition of Fizeau's moving-water experiment and in the establishment of the wavelength of sodium light as a standard of length.

Michelson's ingenuity at optical instrumentation also led to the development of an echelon spectroscope, to a determination of the rigidity of the earth, and to measurements of the distances and diameters of giant stars. In recognition of his many contributions to physics, he was awarded the Nobel prize in 1907, the first American scientist so honored.

In 1923 Michelson was asked to go to Pasadena to make another determination of the speed of light, and this he accomplished with the apparatus shown in Figure 1.7.

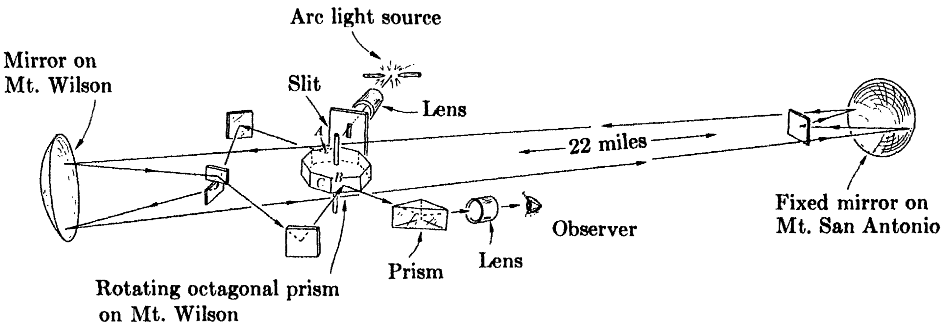

FIGURE 1.7 *Michelson's improved apparatus.* [From *Michelson and the Speed of Light* by Bernard Jaffe. (Science Study Series). Copyright 1960 by Educational Services Incorporated. Reprinted by permission of Doubleday & Company, Inc.]

The principle of operation was still the same, although many refinements of the original apparatus are evident. An eight-sided rotating prism of nickel-steel, with its mirror surfaces polished true to one part in a million, was used in place of the single rotating mirror. Once again, an air blast was used to actuate the mirror system, and a tuning-fork stroboscope to measure its rotational speed. The two stations were considerably farther apart, being placed on Mt. Wilson and Mt. San Antonio. The United States Coast and Geodetic Survey established the distance between these stations within a fraction of an inch in 22 miles. The intensity of the image was enhanced by using large parabolic mirrors at both stations. Many observations yielded a mean value for the velocity of light of 299,798 km/sec.

But Michelson was not yet through. He wanted to measure the velocity of light in as near perfect a vacuum as possible, free from the obstruction of haze or smoke. A mile-long tube of corrugated steel was constructed and evacuated down to a pressure of $\frac{1}{2}$ mm, with a version of the apparatus of Figure 1.7 enclosed. Unfortunately, Michelson did not live to see the end of this experiment, succumbing two years before its completion. His colleagues made almost 3,000 independent observations, reporting[44] a mean figure for the velocity of light in vacuum to be 299,774 km/sec.

The value 299,792.5 km/sec *in vacuo* has been adopted as the velocity of light by the International Union of Geodesy and Geophysics and by the International Scientific Radio Union. This fundamental constant is within the limits of error of Michelson's final figure.

1.3 SOUND WAVES AND LIGHT WAVES

The previous two sections have indicated that light as a wave phenomenon has characteristics common to those of all other types of waves. These include a wavelength, a frequency, and their product the wave velocity, as well as a variety of interference effects. However, light has one characteristic which makes it unique—it can propagate in the absence of a tangible medium. This feature will prove to be of fundamental significance.

It is instructive to contrast the properties of light with those of other wave phenomena. A comparison of the behavior of sound waves and light waves in air is a good illustrative example, because the air can be permitted to become increasingly rarefied, approaching in the limit the absence of a tangible medium.

The Acoustic Wave Equation. Sound waves in air consist of longitudinal molecular vibrations, resulting in alternate compression and rarefaction of the air. If one considers the case in which sound is propagating in the positive X direction, the molecules which (on the average) lie in a plane x = constant will (on the average) oscillate in the X direction. As seen in Figure 1.8, their instantaneous average position will be $x + \xi(x,t)$ in which $\xi(x,t)$ is the time-varying displacement around the average position x. Similarly, the average position of molecules at an adjacent cross section will be $x + dx + \xi(x + dx, t)$. For unit transverse area, the instantaneous volume between these two planes of molecules is

$$[x + dx + \xi(x + dx, t)] - [x + \xi(x,t)] = \left(1 + \frac{\partial \xi}{\partial x}\right) dx \tag{1.2}$$

and thus the fractional change in volume is $\partial\xi/\partial x$. Since the average number of molecules in this volume is a constant, it follows that the density is fluctuating. If the instantaneous density is designated by $\rho_0 + \rho_1(x,t)$, then

$$[\rho_0 + \rho_1(x,t)]\left(1 + \frac{\partial \xi}{\partial x}\right) dx = \text{constant} = \rho_0\, dx \tag{1.3}$$

When it is assumed that the density fluctuation $\rho_1(x,t)$ is small compared to the average value ρ_0 and that the fractional change in volume $\partial\xi/\partial x$ is small compared to unity, Equation (1.3) yields the first-order result

$$\frac{\partial \xi}{\partial x} = -\frac{\rho_1(x,t)}{\rho_0} \tag{1.4}$$

[44] A. A. Michelson, F. G. Pease, and F. Pearson, "Measurement of the Velocity of Light in a Partial Vacuum," *Astrophys J*, **82**, 26–61; July 1935.

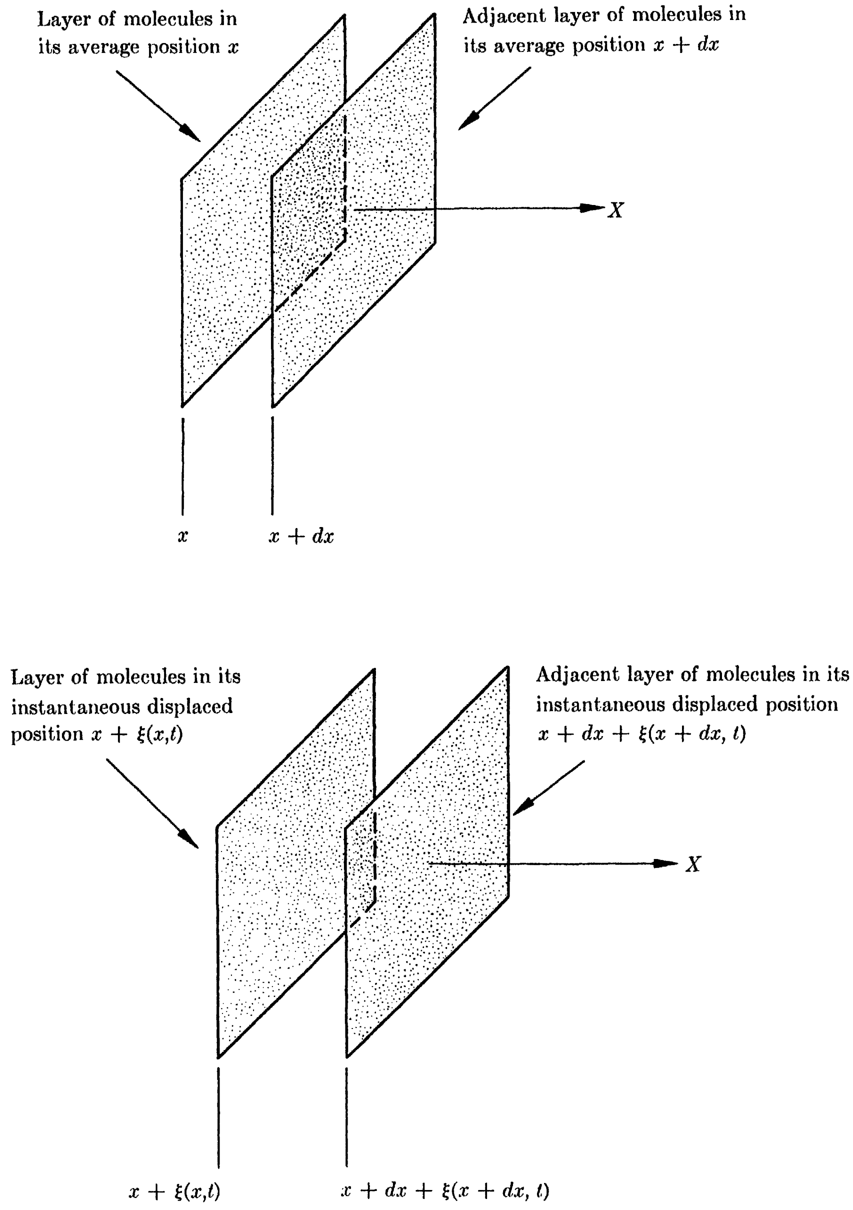

FIGURE 1.8 *Average behavior of layers of air molecules in presence of sound waves.*

The fluctuations in density of the air as the sound waves pass through are so rapid that the air does not transfer heat. The compressions and rarefactions are thus adiabatic, and the process conforms to the gas law equation

$$pV^{\gamma} = \text{constant} \tag{1.5}$$

in which p is the pressure, V the volume, and γ is the ratio of specific heats at constant pressure and constant volume.

Since it has been observed that the volume occupied by a fixed number of molecules is fluctuating, it follows from (1.5) that the total pressure is varying also. Thus one may write

$$p = p_0 + p_1(x,t) \tag{1.6}$$

in which $p_1(x,t)$ is the small fluctuation around the relatively large constant average pressure p_0.

Taking the total differential of (1.5) and then dividing by (1.5) itself, one obtains

$$\frac{dp}{p} = -\gamma \frac{dV}{V}$$

which yields the first-order result

$$\frac{p_1(x,t)}{p_0} = -\gamma \frac{\partial \xi}{\partial x} \tag{1.7}$$

because it has been noted, in connection with Equation (1.2), that $\partial\xi/\partial x$ is the fractional change in volume.

Newton's force law can be applied to the segment of air between the two adjacent cross sections. The net force per unit transverse area acting on the molecules is $-[p_1(x + dx, t) - p_1(x,t)]$. Since to first order the mass is $\rho_0\, dx$, one may write

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = - \frac{\partial p_1}{\partial x} \tag{1.8}$$

Combination of (1.8) with the spatial derivative of (1.7) yields the wave equation

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{c_s^2} \frac{\partial^2 \xi}{\partial t^2} \tag{1.9}$$

in which

$$c_s = \left(\gamma \frac{p_0}{\rho_0}\right)^{1/2} \tag{1.10}$$

The reader will have little difficulty convincing himself that the general solution of (1.9) is

$$\xi(x,t) = f(x - c_s t) + g(x + c_s t) \tag{1.11}$$

in which f and g are arbitrary functions. At a time t_1 the spatial distribution of f is $f(x - c_s t_1)$, as illustrated in Figure 1.9. At a later time t_2 it is

$$f(x - c_s t_2) = f(\{x - c_s(t_2 - t_1)\} - c_s t_1)$$

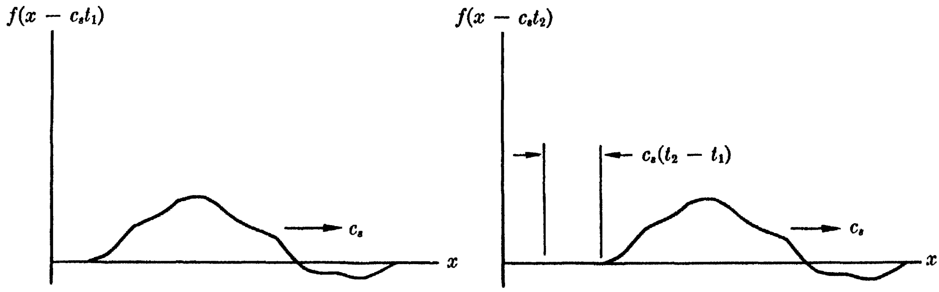

FIGURE 1.9 *Traveling sound waves.*

and is therefore the same spatial distribution as earlier, but shifted along the X axis a distance $c_s(t_2 - t_1)$. For this reason $f(x - c_st)$ represents a wave of arbitrary but constant spatial shape, traveling in the $+X$ direction at speed c_s. Similarly, $g(x + c_st)$ represents an arbitrary wave traveling in the $-X$ direction at speed c_s. The speed of these waves is seen, from Equation (1.10), to depend on the conditions of the medium, namely, the pressure and density of the air. If the air is sufficiently well approximated by the ideal gas law†

$$pV = \mathfrak{N}RT$$

in which $\mathfrak{N}$ is the number of moles, then

$$c_s = \left(\gamma \frac{\mathfrak{N}RT}{\rho_0 V}\right)^{1/2} \sim T^{1/2} \tag{1.12}$$

since $\mathfrak{N}/\rho_0 V$ is a constant. Therefore this first-order theory yields the result that the propagation velocity of sound waves in air depends only on the temperature of the air.

Propagation Independent of Source. A significant feature of Equation (1.10) is its suggestion that c_s is independent of the motion of the source of the sound waves and is governed solely by the properties of the medium. This suggestion is confirmed by experiment and is reasonable when one considers that only the air molecules in the proximity of the source make contact with it, all others depending for their excitation on somewhat-ordered collisions with their neighbors.

The fact that sound waves have a velocity controlled only by the medium and independent of the motion of the source can be used to explain the Doppler effect. This effect is familiar through the common example of an approaching locomotive. As shown in Figure 1.10, at an instant when the diaphragm of the locomotive's horn is in its most forward position, the air adjacent to the diaphragm suffers a compression, and this compression travels forward at a velocity c_s. If τ is the period of oscillation of the diaphragm, then τ seconds later the next compression of air is about to be launched from the horn. At this moment, the earlier compression is a distance $\lambda = (c_s - v)\tau$ in front of the horn, with v the speed of the locomotive. λ is the separation between points in the wave train representing positions of successive maximum compression and is thus the wavelength. The frequency of the sound wave is therefore

$$\nu = \frac{c_s}{\lambda} = \frac{c_s}{c_s - v}\,\nu_0 \tag{1.13}$$

in which $\nu_0 = 1/\tau$ is the frequency the sound wave would have if the locomotive were at rest (ν_0 is also the frequency of oscillation of the diaphragm). Equation (1.13) has been amply confirmed by experiment.

Thus the motion of the source of a sound wave affects both its frequency and wavelength but in such a way that their product remains constant at the value c_s given by (1.10).

Acoustic Power. The rate at which *energy* is being transmitted by the sound wave, per unit transverse area of the wavefront, is called the *intensity*, and will be denoted by Υ. Consider a column of air of unit cross section, extending to infinity from the layer of molecules whose average position is x. The net force on this column is $p_1(x,t)$ and

† This approximation becomes better as the air is rarefied.

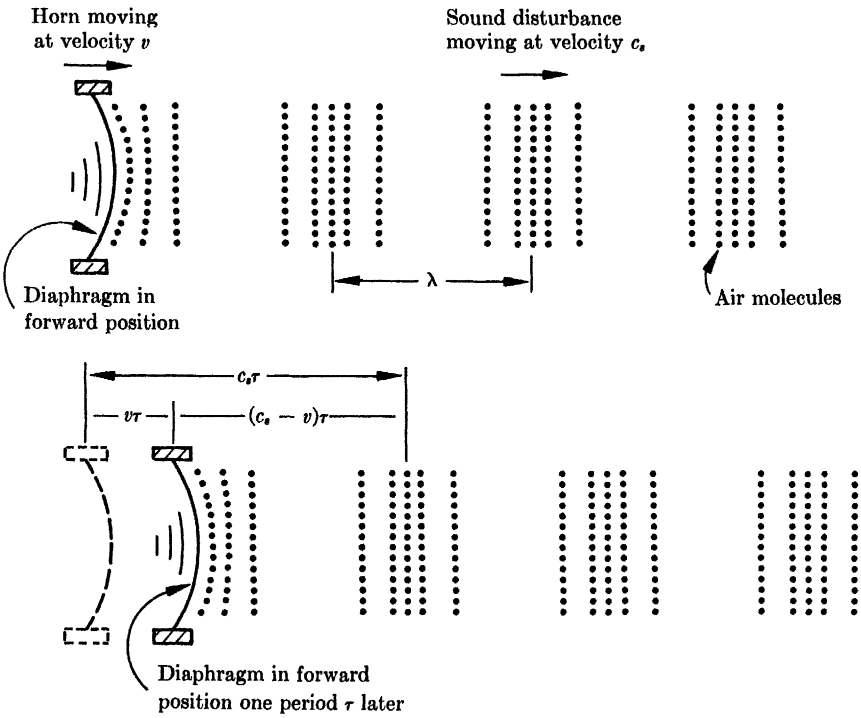

FIGURE 1.10 *The Doppler effect in sound waves.*

during a time interval dt the column is compressed an amount $(\partial\xi/\partial t)\,dt$ so that the work done on the column during this interval is $p_1(\partial\xi/\partial t)\,dt$. With the aid of (1.7), the rate of energy flow into the column can thus be written

$$\Upsilon = -\gamma p_0 \frac{\partial\xi}{\partial x}\frac{\partial\xi}{\partial t} = -\rho_0 c_s^2 \frac{\partial\xi}{\partial x}\frac{\partial\xi}{\partial t} \tag{1.14}$$

For a simple harmonic wave traveling in the positive X direction one can write

$$\xi = A \cos \frac{2\pi}{\lambda}(x - c_s t) \tag{1.15}$$

which is a special case of (1.11). In this equation, A is a constant (the amplitude of molecule oscillation) and λ is the wavelength of the sound disturbance. Since $c_s = \lambda\nu$, introducing the wave number $k = 2\pi/\lambda$ and the angular frequency $\omega = 2\pi\nu$ enables one to rewrite Equation (1.15) in the form

$$\xi = A \cos (\omega t - kx) \tag{1.16}$$

Substitution of (1.16) into (1.14) gives

$$\Upsilon = \rho_0 c_s \omega^2 A^2 \sin^2 (\omega t - kx) \tag{1.17}$$

At any cross section the time average flow is therefore

$$\Upsilon_{av} = \tfrac{1}{2}\rho_0 c_s \omega^2 A^2 \tag{1.18}$$

Equation (1.18) reveals that, if the air is increasingly rarefied, the intensity of a sound wave diminishes. This occurs because the density ρ_0 decreases, whereas, if the temperature remains constant, c_s is unaffected (cf. Equation (1.12)); the amplitude of molecule oscillation $\mathbb{A}$ is limited by the finite amplitude of oscillation of the source. In the limit, with no molecules to transfer the oscillations to their neighbors, no acoustic power can be transmitted, and the sound wave ceases to exist.

This discussion can be summarized by saying that sound waves cannot exist without the presence of a tangible medium, but that they are characterized by a wave velocity which depends on the properties of the medium but *not* on the motion of the source. These remarks are equally true of water waves, elastic waves in solids, etc.

Comparison. Does light share these characteristics? With respect to the requirement of a tangible medium, the answer is no. Light can propagate in gaseous, liquid, and solid media, but it does not require the presence of these media to exist. Indeed, it can propagate in the almost complete vacuum which separates the stars from each other, and many times has been shown to traverse man-made vacua with an intensity no less than it had when air was present. For example, Michelson's last experiments on the determination of the speed of light were performed in a huge evacuated tunnel. In this respect light† as a wave phenomenon is unique in not requiring a tangible medium for its existence.

Does light share the second characteristic, that is, does it possess a wave velocity which is independent of the motion of the source? An indication that it does was provided when Maxwell discovered that wavelike solutions to his equations described electromagnetic fields which would propagate through space at the velocity of light, leading him to assert that light is an electromagnetic phenomenon. But the equation he used to obtain these wavelike solutions was similar to (1.9), the wave equation for sound. Thus just as in the case of acoustic disturbances, Maxwell's analysis suggested that the velocity of light should be completely independent of its source.

There is also strong experimental evidence to support this view. W. de Sitter[45] has analyzed with great care the dynamics of eclipsing binary stars. Were the velocity of light dependent on the motion of the source, it is apparent that the time for light to reach the earth from the *approaching* star of a binary would be different than the time for light to reach the earth from the *receding* star. de Sitter deduced that this would introduce apparent eccentricities in their orbits as they circled each other, but such eccentricities have never been observed. Some binary stars are at such a distance from the earth and have sufficiently high orbital velocities that this effect could scarcely escape observation. Because of this evidence the postulate will be accepted that light, in common with all other wave phenomena, has a velocity which does not depend on the motion of the source. (Many successful Doppler radar systems have been built under this assumption.)

The Ether. It has been noted earlier, in Section 1.1, that light was not really accepted as being wavelike in nature until the middle of the nineteenth century. By that time many other wave phenomena were well understood. Since these other wave phenomena all required a medium for transmission, it was natural to believe that light

† The term "light" is used here in the broad sense to include the nonvisible portions of the electromagnetic spectrum.

[45] W. de Sitter, "An Astronomical Argument for the Constancy of the Velocity of Light," *Z Phys*, **14**, 429; May 15, 1913.

did also, even after it was appreciated that light could propagate in a vacuum. Thus an *intangible* medium was hypothesized to provide the support for light waves. The ether, as this medium was called, being intangible, was endowed with extraordinary properties not shared by any other known medium. These included the ability to pass through all substances without frictional resistance and the property of being mass-less and thus unaffected by gravitation. Despite the mystical aspects of this hypothesis, most nineteenth-century scientists firmly believed in the existence of the ether and many serious scientific experiments were undertaken to prove the validity of the ether concept. The quest for the ether served to sharpen a dilemma concerned with the velocity of light, a subject which will be explored in Chapter 2.

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