PREFACE

Classical signal processing is dominated by linear filtering. There are many reasons for this, but perhaps these can best be summarized by noting that many tasks can be accomplished with an acceptable level of performance by linear filters, and the mathematical analysis of linear operators is much easier than the analysis of nonlinear operators. The problem is that performance loss is often beyond what is acceptable. A filter is a transformation between random processes, the goal being to estimate a desired output from the input. If one considers the entire class of operators between two random processes, then restriction to linearity can result in being forced to choose a filter from a class of filters that is inappropriate to the task at hand. While an optimal linear filter may perform well, rarely will the optimal filter be linear, and very often the optimal linear filter will perform poorly compared to some nonlinear filters. In the latter case, one needs to enlarge, or change, the class from which filters are to be selected.

Two fundamental issues arise at once: from what class should we select the filter, and, given a class of nonlinear filters, how do we choose a good one? There are three factors implicit in the problem of choosing a class: understanding the mathematical properties of the filter class, having suitable filter representations, and quantifying filter performance over random processes of interest. For linear filters, these problems have been addressed for decades. Regarding selection of a good filter from a given class, one needs to have design tools that lead to optimal or acceptable suboptimal performance relative to relevant error measures. For linear filters under the criterion of mean-square error, filter optimization can be carried out in the framework of Hilbert spaces. We have analytic characterization of optimality via an integral equation, analytic formulation of the mean-square error, and, given a canonical expansion of the input random process, analytic expression of the optimal filter. Moreover, all of this probabilistic analysis involves only second moments of the random processes. Not only does this greatly simplify the analysis, it also reduces statistical estimation to second moments. The price for these conveniences is heavy: not only can linearity represent a stringent constraint on optimality, it also only involves second-order information. These restrictions are often unacceptable for image processing.

The two main thrusts of nonlinear image filtering have been addressed to the two aforementioned fundamental issues. First, operator classes have been defined, mathematical properties studied, representations developed, and effects on images investigated; second, methods for optimal filter design have been developed. As would naturally be expected, properties and representations have come before optimization, and prior to the recent decade the bulk of the research was deterministic. But the balance has shifted during the present decade and in recent years both deterministic and random analyses have made significant progress.

As it applies to image processing, nonlinear filtering has two historical roots. First, beginning in the 1960's, there is mathematical morphology, in its early application in the context of random sets. The basic erosion and opening representations of nonlinear filtering go back to G. Matheron's classic 1975 work, *Random Sets and Integral Geometry*. One can hardly overemphasize the impact of this book, not only on image processing, but on the theory of random sets. The basic operator representations formulated by Matheron have provided the seed for the algebraic representations of nonlinear filtering, which in turn have provided the framework for filter optimization. But probability, not algebraic representation, is Matheron's central issue. Concerning his endeavors, he writes, "The formulation and the very choice of problems for solution are directly inspired by experimental techniques of texture analysis that currently permit the thorough study of such [porous] media...." Matheron is not talking about the historical problems of signal estimation; rather, he is laying the foundations for the random analysis of geometric forms and texture. Today, his study remains the fundament for both random geometry and texture analysis.

The second historical root for nonlinear filtering is classical digital signal processing. Here, signal values in a window are treated as a random sample and the value at the window center is estimated by some nonlinear estimator. The effort was to move away from linear operators, especially in the case of noise suppression, where linear filters have serious detrimental effects on edge information, which is crucial for image processing. Early studies involved the median, beginning with the median filtering of time series by J. Tukey in the early 1970's, and then stack filters, which are increasing filters that commute with thresholding, of which the median is a particular case. Medians are attractive from the perspective of robust statistics, and are readily viewed in terms of maximum-likelihood estimation.

In fact, the two roots are not distinct. Morphological representation provides the link. Matheron's representation of increasing filters applied only to binary operators. This was extended to increasing gray-scale operators, to nonincreasing binary images, and finally to all translation-invariant lattice operators, and in all cases special effort was attached to minimal representations (see Section 2.12). But the whole matter goes quite far back, with the erosion representation of the binary median and commutation of the gray-scale median with thresholding being recognized by 1980. In any event, the latter half of the last decade and the first part of the present saw a large effort at operator representation.

To use the representations, there need to be ways of synthesizing operators in accord with desired outcomes and the probabilistic characteristics of the random processes or sets upon which they are acting. Perhaps we can paraphrase R. Haralick, who, at a talk during the *1986 IEEE CVPR Conference*, stated that the near future would see the publication of a large number of papers on the design of optimal morphological filters to parallel the developments of the Wiener-Kolmogorov theory of linear filtering. Haralick's prediction proved prophetic because shortly thereafter came early attempts to design optimal increasing filters, including the special cases of stack filters, and optimal openings. Henceforth, the subject would find itself more in line with classical signal processing, where the algebraic structures (in that case, linear spaces) are applied in conjunction with the theory of random processes.

Currently there appears to be a split that is reflective of the dual roots of nonlinear filtering, and perhaps more substantive than the original differences. The movement in the direction of statistics has become so pronounced that nonlinear filtering may be becoming more associated with nonlinear statistical estimation and inverse problems, whereas mathematical morphology may be becoming associated with the theory of lattice operators, nonlinear equations of evolution, and other mathematical issues. Random set theory, from which the basics sprung, is a subject in its own right with applications to various physical sciences, as well as to image processing. Such fragmentation in conjunction with maturation is natural, if sometimes disheartening.

The present text remains, for the most part, in the unified tradition, no doubt because the editors prefer it that way. We continue to believe that there remains an integral whole involving nonlinear estimation, mathematical morphology, and random processes (sets), and that this whole has both discrete and continuous components. Indeed, *fundamental separation is impossible because nonlinear filtering involves nonlinear estimation of random processes using operators that possess morphological representations.* The goal of the book is to provide a collection of chapters on topics of contemporary interest, in addition to sufficient basic material to keep the book self-contained and connected to its roots in translation-invariant set operators, stack filters, and random sets. The mathematical treatment is intended to be rigorous, but not so abstract as to make it inhospitable to engineers possessing good mathematical training. We hope you enjoy the book and find it useful for your studies.

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