



The Language of Algebra (Using Whole Numbers)

Every discipline has its own language. Soccer has its goals, figure skating has its triple axels, and geometry has its polygons. Specialized terminology makes it possible to understand the discipline. How could someone describe baseball without mentioning the words “innings,” “strikes,” and “outs”?

The fifteen activities of this section focus on the basic terminology of algebra. Understanding this language will help your students to understand numerical relationships, describe operations, and reason algebraically. By becoming fluent in the language of algebra, your students will be prepared to move on to more advanced skills and concepts.

Teaching Suggestions for the Activities

I-1 Using the Order of Operations I

This activity requires your students to simplify an expression. The expressions of the activity focus on the four basic operations—addition, subtraction, multiplication, and division. No grouping symbols or exponents are included. All numerical values are positive whole numbers. The symbols \cdot and \times are used to denote multiplication.

Introduce this activity by discussing the rules for the Order of Operations, which are noted on the worksheet. Go over the example on the worksheet together, noting the steps that are taken to arrive at the numerical value.

Review the instructions on the worksheet with your students. Explain that the first column lists the expression and the second column the steps necessary to simplify an expression. Point out that each step on the right can be paired with only one expression on the left and that each step can be used only once. Remind your students to include the numerical values of the expressions in their answers.

I-2 Using the Order of Operations II

This activity builds upon the skills addressed in Activity 1-1 and provides more practice with the Order of Operations. It includes the use of grouping symbols.

Begin the activity by reviewing the Order of Operations with your students. Emphasize that parentheses, braces, brackets, and the fraction bar are all considered to be grouping symbols. Operations within these symbols must be done first. Remind your students that a number directly to the left of a grouping symbol implies multiplication.

Go over the instructions for the activity with your students. Note that, along with identifying the incorrect answer, they are to explain why the answer is wrong.

I-3 Using the Order of Operations III

For this activity your students are to simplify various expressions using the Order of Operations. Grouping symbols, exponents, and fractions appear in the expressions. Unless your students are experienced with simplifying expressions, you should assign Activities 1-1 and 1-2 before assigning this one.

Start the activity by reviewing the Order of Operations. You should also explain the concept of a base and an exponent. For example, in the expression 2^3 , 2 is called the base and 3 is the exponent. The exponent means that 2 is used as a factor three times. Thus, 2^3 means $2 \times 2 \times 2$ or 8. Point out that a common error students make when working with exponents is to multiply the base times the exponent, for example, in this case, 2×3 , obtaining the incorrect answer of 6.

Depending on the abilities and backgrounds of your students, you may also wish to introduce the concept of a square number, which is a number raised to the second power. For example, $5^2 = 25$ shows that 25 is a square number. Some other examples of square numbers are 9, 16, 36, 49, 64, and 81.

Make sure your students understand the instructions for the activity. Remind them to follow the directions closely when placing their answers in the Code Box.

I-4 Using Square Numbers

The purpose of this activity is to familiarize your students with square numbers. Review the examples of square numbers included on the worksheet. Depending on the needs of your students, you may wish to expand the list.

Start this activity by drawing squares on the board or an overhead projector. Draw: a 1×1 -inch square, a 2×2 -inch square, a 3×3 -inch square, and a 4×4 -inch square. Emphasize to your students that the number that represents the area in each square is a square number.

Next, explain the Square Number Theorem, referring to the examples on the worksheet. Remind your students to find the square numbers first, then add. You might mention that this procedure is the same as following the Order of Operations.

Go over the instructions on the worksheet, and emphasize that the last five problems require students to find the four square numbers that add up to the number. This is the opposite of what they have to do in problems 1 through 10. Suggest that guess and check is a good strategy to use in solving the last four problems.

I-5 Translating Algebraic Expressions into Phrases

This activity requires students to complete a crossword puzzle with a word omitted from a phrase. It provides practice in writing expressions.

Introduce the activity by explaining that an algebraic expression is a combination of a variable (or variables) and a number (or numbers). Note the examples on the worksheet. Emphasize that order matters when subtracting and dividing. For instance, $n - 3$ means 3 less than a number and not 3 minus a number. Incorrect order is a common mistake when writing expressions.

Review the instructions on the worksheet with your students. Remind them that this is a crossword puzzle, and encourage them to focus their attention on the clues.

I-6 Writing Phrases as Algebraic Expressions I

This activity is a follow-up to Activity 1-5. For this activity, your students are given a phrase containing an expression. They are to determine if the expression is stated correctly. If it is incorrect, they are to correct the expression.

Note the common errors that many students make with these types of expressions. For example, they may overlook order. $n - 8$ is not the same as $8 - n$. They may also overlook grouping symbols. For example,

three times the sum of a number and 10 is $3(n + 10)$ or $3(10 + n)$, but not $3n + 10$.

Review the instructions on the worksheet with your students. Remind them that half of the problems on the worksheet are correct. Students must correct the incorrect problems.

I-7 Writing Phrases as Algebraic Expressions II

This activity provides your students with more practice using algebraic expressions. In each problem of this activity, your students are to think of a number, do a series of numerical operations, and obtain an answer that the teacher can predict. Your students, with the aid of algebra, are to explain these problems as well as create problems of their own. To complete this activity successfully, your students must be able to translate phrases into algebraic expressions.

Start this activity by reviewing the Distributive Property. Go over the instructions on the worksheet, then do the first problem as a class exercise. Instruct your students to write the number they begin with in the blank in Column I, recording each successive number in the blanks provided. (You may wish to suggest that for problems 1 through 3 students choose a number between 1 and 9 to keep the math simple.) Complete Column II (for the first problem) as a class. The steps are n , which represents the first number the students record, n^2 , $n^2 - 4$, $2n^2 - 8$, $2n^2$, n . Point out that in this problem your students end with the number with which they started.

Note that your students should use a variable to represent a number in Column II. If they are asked to use another number, they should choose a different variable.

I-8 Simplifying Expressions by Combining Like Terms

This activity is designed to provide an introduction to combining similar terms. Begin the activity by explaining basic notation: $3 \times 4n$ can also be written as $3 \cdot 4n$ or $3(4n)$, all of which equal $12n$. Also, n can be written as $1n$. $0 \times n$ equals 0. Review the vocabulary on the worksheet and make sure that your students understand the Distributive Property.

Go over the instructions on the worksheet. Remind your students to simplify each expression completely.

I-9 Simplifying and Evaluating Expressions

In this activity your students will simplify expressions by combining like terms and then evaluate the expressions. The problems on the worksheet do not contain negative numbers or exponents.

Begin the activity by reviewing how to simplify expressions. Depending on the abilities of your students, you may also find it helpful to discuss the Distributive Property. Remind your students that expressions such as $2a$ can be expressed as $2 \times a$ and that b can be expressed as $1 \times b$. Note that to substitute a number for a variable, your students should write the number in place of the variable. For example, if $a = 3$, then $2a = 2 \times 3$ or 6.

Go over the instructions on the worksheet with your students. Caution them to pay close attention to grouping symbols and remind them to always multiply before adding or subtracting.

I-10 Evaluating Expressions Using Exponents

This activity requires your students to evaluate expressions using exponents. The answers will be positive whole numbers.

Begin this activity by discussing 0 and 1 as exponents. Note that $x^1 = x$ and that $x^0 = 1$. Depending on the background and abilities of your students, you may also find it useful to review grouping symbols.

Go over the instructions with your students and discuss the examples on the worksheet. Remind your students to be sure to follow the Order of Operations where necessary.

I-11 Writing Equations

In this activity your students are provided with information that they are to express in terms of an equation. Although they are not required to solve the equation, they may be curious to find the solution. The equations and their solutions are provided in the Answer Key.

Begin this activity by writing some equations on the board or an overhead projector. Two examples are $P = 4s$, for finding the perimeter of a square, and $A = l \times w$, for finding the area of a rectangle. Explain that the equal sign means that the number on the left of the equation has the same value as the number on the right. Encourage your students to volunteer examples of other equations, which you may list on the board or an overhead projector. As you do, emphasize the equality and the meaning of the variables. Also review key words such as “variable,” “more than,” and “product.”

Go over the instructions on the worksheet with your students. Caution them to be as accurate as possible in writing equations.

I-12 Writing Equations and Inequalities I

For this activity your students must recall, find, research, and synthesize various facts. (Most facts fall within the category of general knowl-

edge.) Your students are then required to compare numbers and write an equation or inequality.

Since your students may need to conduct minor research to find some of the information necessary to complete this activity successfully, you may prefer to assign this activity as homework. If you have access to the Internet from your classroom, the activity can easily be conducted there. Another option is to reserve time in your school's library so that students may use reference sources. This is a nice approach, because it provides an example to students of how math is linked to other areas.

Start this activity by reviewing the meaning of an equation. Note that the values on both sides of an equal sign are the same, and emphasize that in an inequality the numbers are not equal. If necessary, explain the meanings of the symbols $>$ and $<$, which appear on the worksheet.

Review the instructions on the worksheet with your students. Encourage them to concentrate on the phrases as they work to complete the activity.

I-13 Writing Equations and Inequalities II

This activity builds on the skills covered in Activity 1-12 and provides more practice with equations and inequalities. Along with the symbols $>$ and $<$, the symbols \geq and \leq are used in many of the equations.

Start the activity by discussing the symbols on the worksheet, then go over the instructions. Depending on the abilities of your students, you may find it helpful to do the first problem together. Remind students to read each problem carefully before writing an equation.

I-14 Identifying the Solution of an Equation

In this activity your students are required to match solutions to equations. They are to substitute a given value and determine whether or not it is a solution to the equation. Obtaining the correct answers will enable them to complete a statement at the bottom of the worksheet.

Begin the activity by reviewing the Order of Operations. Because there are no grouping symbols, your students should substitute, then multiply and divide in order from left to right, then add and subtract.

Go over the instructions on the worksheet. Note that of the five possible solutions above the four equations, only one is not a solution for any of the equations. Your students are to write the letter of each solution in the blank before the equation. After determining the correct solutions, they are to write the letters in order to complete the statement at the bottom of the worksheet.

I-15 Determining the Solutions of Equations and Inequalities

This activity builds on the skills covered in Activity 1-14. For this activity your students are given twenty equations and inequalities for which they are to choose the solutions.

Begin the activity by reviewing the meaning of the equal sign and the four inequality symbols: $>$, $<$, \geq , and \leq . Go over the instructions with your students. Emphasize that they are to record the letter of every solution. Some problems have more than one. If no solution is given, they are to record the letter that precedes “none.”

Answer Key for Section I

I-1. 1. $8 - 6 + 5 = 7$ 2. $3 + 8 + 4 = 15$ 3. $3 + 2 + 4 = 9$ 4. $21 - 14 + 6 = 13$
 5. $2 + 9 - 1 = 10$ 6. $48 + 2 - 7 = 43$ 7. $3 + 25 - 2 = 26$ 8. $4 + 12 - 16 = 0$
 9. $5 + 2 - 4 = 3$ 10. $60 - 60 + 2 = 2$ 11. $6 + 18 + 4 = 28$ 12. $1 + 6 - 2 = 5$

I-2. 1. $15 - 2 \times 3 = 39$ is incorrect. Subtraction was done first, then multiplication was done. 2. $16 + 8 \div 2 = 12$ is incorrect. Addition was done first, then division was done. 3. $3[2 + 4] = 10$ is incorrect. Grouping symbols were ignored; multiplication was done first, then 4 was added. 4. $50 \div (5 \times 10) = 100$ is incorrect. Grouping symbols were ignored; division was done, then multiplication was done. 5. $9 + 3 \div 3 + 4 = 8$ is incorrect. Addition was done first, then division was done, followed by addition. 6. $\frac{6 \cdot 4}{2 + 4} = 16$ is incorrect. The fraction bar was ignored. Multiplication was done first, then division by 2 was done, and finally 4 was added. 7. $19 - [3 + 5] = 21$ is incorrect. Brackets were ignored. Three was subtracted from 19, then 5 was added. 8. $\frac{3 + 7 + 12}{3 + 20 - 1} = 33$ is incorrect. $3 + 7$ were added to $12 \div 3$. Then 20 was added and 1 was subtracted. 9. $3 + 12 \div 3 \times 2 = 10$ is incorrect. Addition, division, and multiplication were completed from left to right. 10. $6 \times 10 - 3 \times 4 = 228$ is incorrect. Multiplication, subtraction, and multiplication were done from left to right.

I-3. 1. 9 2. 144 3. 384 4. 325 5. 57 6. 19 7. 104 8. 24,389 9. 22
 10. 300 11. 72 12. 30 13. 1 14. 1,296 The numbers remaining are the first thirteen square numbers.

I-4. 1. 177 2. 37 3. 54 4. 146 5. 494 6. 189 7. 245 8. 142 9. 246
 10. 269. For problems 11–15 answers may vary; possible answers follow.
 11. $1 + 4 + 9 + 9$ 12. $1 + 4 + 9 + 16$ 13. $9 + 9 + 16 + 25$ 14. $9 + 16 + 25 + 36$
 15. $25 + 25 + 25 + 25$

[illegible]

-

A. $6b + 28$, 40 Y. $7a + 2d + 5$, 36 N. $a + 6b$, 15 E. $4d + 6$, 26 H. $18a + 6$, 60 S. $4b + 4d + 4$, 32 I. $20a + 12b + 6d + 8$, 122 V. $3a + 1$, 10
It is easier to simplify first and then evaluate.

I-10. 1. I, 1 2. N, 9 3. H, 49 4. I, 1 5. S, 32 6. B, 100 7. O, 64 8. O, 64
9. K, 512 10. L, 81 11. A, 21 12. G, 29 13. E, 19 14. O, 64 15. M,
218 16. E, 19 17. T, 1,728 18. R, 432 19. I, 1 20. E, 19 21. P, 4
22. U, 22 23. B, 100 24. L, 81 25. I, 1 26. S, 32 27. H, 49 28. E, 19
29. D, 11 René Descartes (1596–1650), a French philosopher and math-
ematician, first used the modern notation for exponents in his book *La*
Geometrie, published in 1637.

I-11. Equations may vary. (Solutions were not required, but answers are pro-
vided.) 1. $s = 31 - 1$; $s = 30$ 2. $c = 7 - 3$; $c = 4$ 3. $l = 44 + 50$; $l = 94$
4. $2p = 16$; $p = 8$ 5. $\frac{1}{6}w = 4$; $w = 24$ 6. $l = 3,787,319 - 251,041$;
 $l = 3,536,278$ 7. $r = \frac{1}{2} \times 70$; $r = 35$ 8. $l = 2 \times 7 + 1$; $l = 15$ 9. $p = 1.9 \times$
 $86,560,000$; $p = 164,464,000$ 10. $f = 2 \times 17 + 9$; $f = 43$ 11. $b = 350 - 206$;
 $b = 144$ 12. $g = 2,800 - 400$; $g = 2,400$ 13. $p = 9 \times 8$; $p = 72$ 14. $t =$
 $1965 - 77 - 21$; $t = 1867$ 15. $a = 23 \times 13 + 12$; $a = 311$

I-12. 1. $3 < 4$ 2. $5 > 3$ 3. $12 < 13$ 4. $16 = 16$ 5. $100 > 50$ 6. $42 < 64$ 7. 9
 < 22 8. $48 > 40$ 9. $5 < 10$ 10. $89 = 89$ 11. $4 = 4$ 12. $12 < 36$ 13. 8
 $= 8$ 14. $4 < 7$ 15. $16 = 16$ 16. $32 > 25$ 17. $4 > 2$ 18. $100^\circ \text{C} = 212^\circ \text{F}$
19. $2 = 2$ 20. $9 \text{ sq. ft.} > 3 \text{ sq. ft.}$ 21. $\$20 < \100 22. $100 = 100$ 23. 360
 > 180 24. $6 < 8$ 25. $7 = 7$

I-13. 1. $p \geq 10$ 2. $a \geq 18$ 3. $h > 4$ 4. $a < 12$ 5. $t \geq 25$ 6. $c = \$7$ 7. $p \leq 350$
8. $t = \$120$ 9. $c = \$240$ 10. $p \leq 10$ 11. $t = 15$ 12. $t = 10$ 13. $n \geq 3$
14. $d = 390$ 15. $d > 3$

I-14. Set 1. B, 5 Y, 8 A, 52 L, 54 Set 2. K, 12 H, 4 W, 36 A, 8 Set 3. R, 7
I, 9 Z, 30 M, 18 I, 85 Algebra comes from the Arabic word “al-jabr,”
which appeared in the title of a book written around 825 by al-
Khwarizmi, an Islamic mathematician.

I-15. 1. none 2. $\{10, 20\}$ 3. $\{0\}$ 4. $\{1, 2\}$ 5. $\{12, 14, 16\}$ 6. $\{2\}$ 7. none
8. $\{9, 34\}$ 9. $\{3, 6, 9\}$ 10. $\{4, 8\}$ 11. $\{1, 2\}$ 12. $\{6, 7\}$ 13. $\{8, 12\}$
14. none 15. $\{1, 2, 3\}$ 16. none 17. $\{3, 6, 9\}$ 18. $\{2, 3\}$ 19. $\{24, 36\}$
20. none. It is impossible to list all of the solutions.

NAME _____

DATE _____

SECTION _____



Using the Order of Operations I

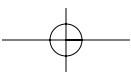
Use the following rules to simplify an expression:

- 1. Simplify products and quotients in order from left to right.
- 2. Simplify sums and differences in order from left to right.

For example, $12 \div 2 + 2 \times 2 - 1$ can be written as $6 + 4 - 1$ or 9.

Directions: Each expression can be simplified using the Order of Operations. Choose the steps from the list on the right and write these steps below the appropriate expression. Then simplify the expression.

Expression	Steps
1. $72 \div 9 - 3 \times 2 + 5 =$ _____	$48 + 2 - 7$
2. $3 + 2 \cdot 4 + 4 =$ _____	$1 + 6 - 2$
3. $18 \div 6 + 6 \div 3 + 4 =$ _____	$3 + 25 - 2$
4. $21 - 7 \times 2 + 6 =$ _____	$60 - 60 + 2$
5. $14 \div 7 + 18 \div 2 - 1 =$ _____	$3 + 8 + 4$
6. $6 \times 8 + 4 \div 2 - 7 =$ _____	$8 - 6 + 5$
7. $3 + 75 \div 3 - 2 =$ _____	$3 + 2 + 4$
8. $4 + 6 \times 2 - 4 \times 4 =$ _____	$6 + 18 + 4$

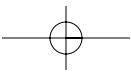


NAME _____ DATE _____ SECTION _____

I - I
(continued)

Using the Order of Operations I

Expression	Steps
9. $15 \div 3 + 2 - 4 \cdot 1 =$ _____	$21 - 14 + 6$
10. $6 \cdot 10 - 4 \times 15 + 2 =$ _____	$5 + 2 - 4$
11. $6 + 12 \div 2 \times 3 + 4 =$ _____	$2 + 9 - 1$
12. $1 + 2 \times 3 - 6 \div 3 =$ _____	$4 + 12 - 16$



NAME _____

DATE _____

SECTION _____

Using the Order of Operations II

The Order of Operations is listed below.

1. Simplify expressions that are set within grouping symbols such as parentheses, braces, brackets, and the fraction bar.
2. Simplify products and quotients in order from left to right.
3. Simplify sums and differences in order from left to right.

Directions: Each expression below has been simplified in two ways: one by following the Order of Operations and the other by not following the Order of Operations. Circle the equation with the *incorrect* answer, then explain how the incorrect answer was obtained.

1. $15 - 2 \times 3 = 9$

$15 - 2 \times 3 = 39$

2. $16 + 8 \div 2 = 12$

$16 + 8 \div 2 = 20$

3. $3[2 + 4] = 10$

$3[2 + 4] = 18$

4. $50 \div (5 \times 10) = 100$

$50 \div (5 \times 10) = 1$

5. $9 + 3 \div 3 + 4 = 14$

$9 + 3 \div 3 + 4 = 8$

I-2
(continued)

NAME _____

DATE _____

SECTION _____

Using the Order of Operations II

6. $\frac{6 \cdot 4}{2 + 4} = 4$

$\frac{6 \cdot 4}{2 + 4} = 16$

7. $19 - [3 + 5] = 21$

$19 - [3 + 5] = 11$

8. $\frac{3 + 7 + 12}{3 + 20 - 1} = 1$

$\frac{3 + 7 + 12}{3 + 20 - 1} = 33$

9. $3 + 12 \div 3 \times 2 = 11$

$3 + 12 \div 3 \times 2 = 10$

10. $6 \times 10 - 3 \times 4 = 228$

$6 \times 10 - 3 \times 4 = 48$

I-3

NAME _____

DATE _____

SECTION _____

Using the Order of Operations III

Sometimes an expression may contain a number raised to a power such as 2^3 , which is read as 2 to the third power, or 8. These expressions must be simplified after the expressions within grouping symbols but before multiplying or dividing. Follow these steps:

1. Simplify expressions within grouping symbols.
2. Simplify powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

Directions: Simplify each expression in problems 1 through 14. Find each answer in order in the Code Box at the end of the activity, starting in the upper left and moving across each line. When you find an answer, cross it out. (The answer to Problem 1 appears before the answer to Problem 2 and so on.) Each answer will be separated by a number or group of numbers. Some numbers will not be crossed out. Describe the numbers that remain.

- | | |
|---------------------------------------|-------------------------------------|
| 1. $6 \times 3 - 9 =$ _____ | 2. $12^2 =$ _____ |
| 3. $6 \times 8^2 =$ _____ | 4. $(8 + 5)(2 + 3)^2 =$ _____ |
| 5. $2^3 \times 7 + 1 =$ _____ | 6. $1 + 2(4 - 1)^2 =$ _____ |
| 7. $2(3 + 7^2) =$ _____ | 8. $(4 + 5^2)^3 =$ _____ |
| 9. $6 + 12 \div 3 \times 2^2 =$ _____ | 10. $3 \times 10^2 =$ _____ |
| 11. $2^3(7 + 2) =$ _____ | 12. $\frac{1+3+8}{4} + 3^3 =$ _____ |
| 13. $\frac{3^2+4^2}{5^2} =$ _____ | 14. $(3^2 \times 2^2)^2 =$ _____ |

Code Box

9	1	1	4	4	4	3	8	4	9	3	2	5	1	6	5	7	2	5	1	9
3	6	1	0	4	4	9	2	4	3	8	9	6	4	2	2	8	1	3	0	0
1	0	0	7	2	1	2	1	3	0	1	4	4	1	1	6	9	1	2	9	6

Describe the numbers that remain. _____

NAME _____

DATE _____

SECTION _____

Using Square Numbers

A *square number* is a number raised to the second power. Some square numbers are listed below.

$$\begin{array}{ll} 1 = 1 \times 1 = 1^2 & 9 = 3 \times 3 = 3^2 \\ 4 = 2 \times 2 = 2^2 & 16 = 4 \times 4 = 4^2 \end{array}$$

Pierre de Fermat (1601–1665), a French mathematician, stated that every natural number can be expressed as the sum of four square numbers. This was proven to be true in 1770 by another mathematician, Joseph Louis Lagrange.

Directions: Find the number represented by the sum of the numbers below.

1. $2^2 + 3^2 + 8^2 + 10^2 =$ _____

2. $1^2 + 2^2 + 4^2 + 4^2 =$ _____

3. $2^2 + 3^2 + 4^2 + 5^2 =$ _____

4. $1^2 + 3^2 + 6^2 + 10^2 =$ _____

5. $2^2 + 11^2 + 12^2 + 15^2 =$ _____

6. $2^2 + 6^2 + 7^2 + 10^2 =$ _____

7. $1^2 + 6^2 + 8^2 + 12^2 =$ _____

8. $2^2 + 5^2 + 7^2 + 8^2 =$ _____

9. $5^2 + 6^2 + 8^2 + 11^2 =$ _____

10. $3^2 + 4^2 + 10^2 + 12^2 =$ _____

Now find the square numbers that have been added to express the numbers below.

11. _____ + _____ + _____ + _____ = 23

12. _____ + _____ + _____ + _____ = 30

13. _____ + _____ + _____ + _____ = 59

14. _____ + _____ + _____ + _____ = 86

15. _____ + _____ + _____ + _____ = 100

I-5

NAME _____

DATE _____

SECTION _____

Translating Algebraic Expressions into Phrases

This crossword puzzle is like most crossword puzzles. The only difference is that all of the missing words relate to expressions.

Directions: Use the clues to complete the crossword puzzle.

Down

1. $5n$ means 5 _____ a number.
2. $\frac{1}{3}n$ means $\frac{1}{3}$ _____ a number.
4. 4 more than 5 is _____.
5. $8n$ means a number _____ by 8.
6. $n + 7$ means 7 _____ than a number.
7. $n - 9$ means _____ 9 from a number.
8. $2(3 + 4)$ equals _____.
10. 10^2 equals one _____.
12. $n - 2$ means a number _____ by 2.
16. The product of 6 and 2 is _____.
19. $\frac{n}{4}$ is the _____ of a number and 4.
20. 4^2 equals _____.
23. $\frac{n}{7}$ means a number _____ by 7.
24. $n - 8$ means 8 _____ than a number.
25. $n + 2$ means a number _____ 2.

Across

1. 3 less than 5 is _____.
3. $2 \times 3 + 4$ equals _____.
5. $n - 5$ means a number _____ 5.
9. $\frac{1}{2}n$ means one _____ of a number.
11. $n + 10$ means the _____ of a number and 10.
13. n is a variable that stands for a _____.
14. $2n$ means _____ a number.
15. $6n$ means the _____ of a number and 6.
17. 12 minus 4 equals _____.

I-5

(continued)

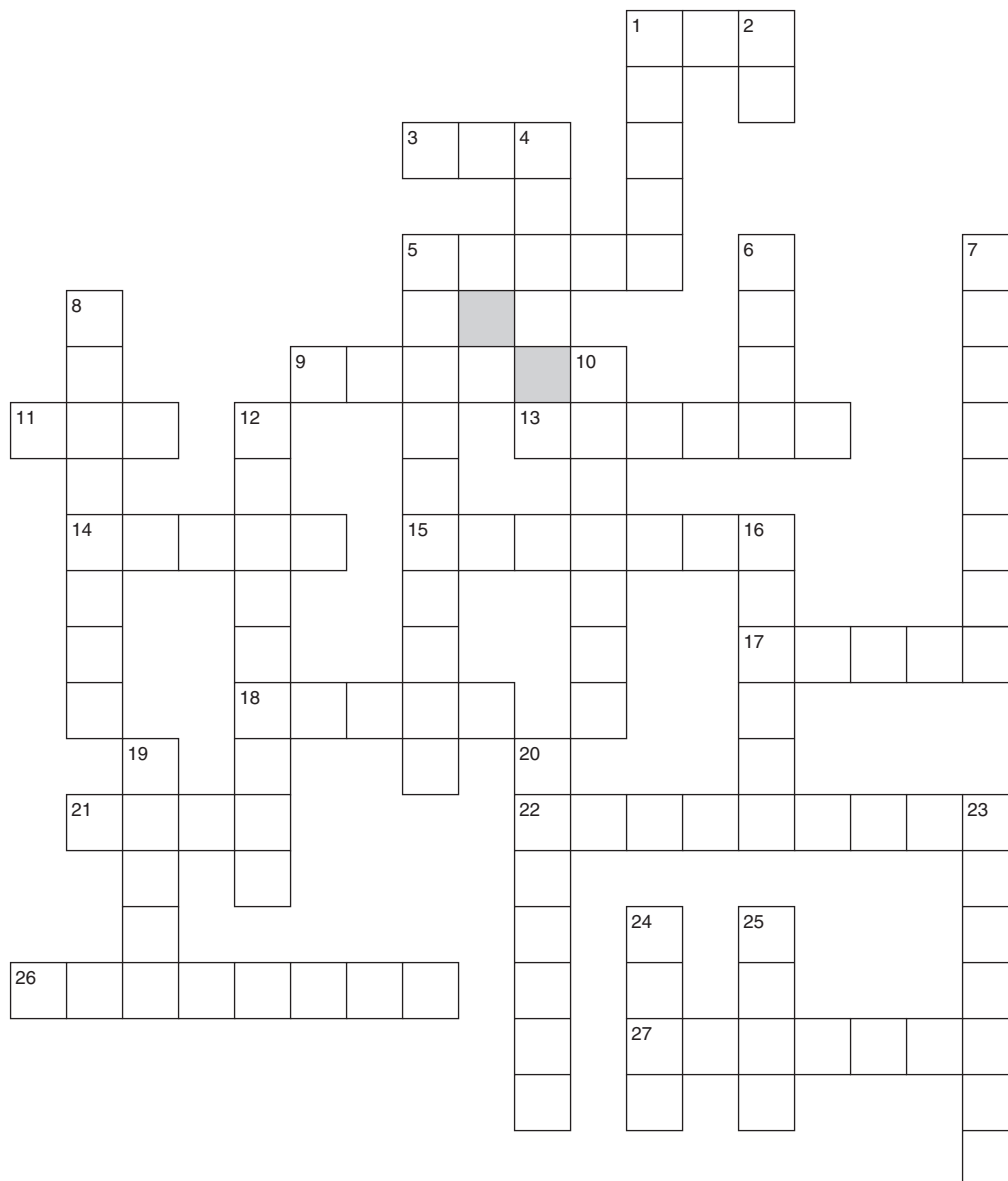
NAME _____

DATE _____

SECTION _____

Translating Algebraic Expressions into Phrases

18. $n + 1$ means a number _____ to 1.
21. $n - 3$ means _____ 3 from a number.
22. $n + 10$ means a number _____ by 10.
26. $n \div 7$ means the _____ of a number and 7.
27. 3^2 means 3 _____.



I-6

NAME _____

DATE _____

SECTION _____

Writing Phrases as Algebraic Expressions I

The foundation for solving problems in algebra as well as in life is being able to express ideas clearly. In algebra, for example, a number can be expressed simply as n .

In an algebraic expression, order does not matter in addition or multiplication. For example, 2 more than a number can be expressed as $n + 2$ or $2 + n$. Three times a number can be expressed as $3n$ or $n \times 3$.

Order does matter in subtraction and division. For example, 4 less than a number is $n - 4$. The quotient of 10 and 2 is $10 \div 2$ and not $2 \div 10$.

Grouping symbols, such as parentheses and fraction bars, must be included if numbers are to be grouped and then added, subtracted, multiplied, divided, or squared. For example, 3 times the sum of a number and 5 is represented as $3(n + 5)$. The sum is found first, then multiplied by 3. The sum of a number and 8, divided by 16 is written as $\frac{n+8}{16}$ or $(n + 8) \div 16$.

Directions: Each phrase is paired with an algebraic expression. If the algebraic expression is correct, write C (for correct) on the line following the equation. If it is incorrect, correct it. Hint: Half of the problems are incorrect.

- | | | |
|--|-----------------|-------|
| 1. The sum of a number and 8 | $n + 8$ | _____ |
| 2. 2 less than a number | $2 - n$ | _____ |
| 3. Twice a number | $2n$ | _____ |
| 4. The sum of twice a number and 4 times the same number | $2(n + 4n)$ | _____ |
| 5. A number divided by 3 more than the same number | $n \div n + 3$ | _____ |
| 6. The sum of a number and 6 divided by 12 | $n + 6 \div 12$ | _____ |
| 7. The square of the quotient of a number and 7 | $(n \div 7)^2$ | _____ |
| 8. 10 less than a number | $n - 10$ | _____ |

I-6
(continued)

NAME _____

DATE _____

SECTION _____

Writing Phrases as Algebraic Expressions I

- | | | |
|---|----------------------|-------|
| 9. The square of 4 more than a number | $n + 4^2$ | _____ |
| 10. A number multiplied by 8 less than itself | $n \times n - 8$ | _____ |
| 11. Half the sum of a number and 5 | $\frac{1}{2}(n + 5)$ | _____ |
| 12. 4 more than a number | $n + 4$ | _____ |
| 13. 7 decreased by a number | $n - 7$ | _____ |
| 14. The sum of a number and 5 divided by 6 | $(n + 5) \div 6$ | _____ |
| 15. 6 more than twice a number | $2n + 6$ | _____ |
| 16. A number decreased by 15 | $n - 15$ | _____ |
| 17. The product of 5 and a number squared | $5 + 2n^2$ | _____ |
| 18. One-half of a number | $\frac{n}{2}$ | _____ |
| 19. One less than 8 times a number | $8(n - 1)$ | _____ |
| 20. The product of a number and 22 | $22n$ | _____ |
| 21. 6 times the sum of a number and 3 | $6n + 3$ | _____ |
| 22. The sum of 3 times a number and 2 | $3(n + 2)$ | _____ |
| 23. The product of a number squared and 8 | $8n^2$ | _____ |
| 24. The quotient of 3 times a number and 3 | $3n \div 3$ | _____ |

I-7

NAME _____

DATE _____

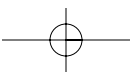
SECTION _____

Writing Phrases as Algebraic Expressions II

Algebra can be used to explain some “mysterious tricks” about numerical operations. Some people call this “mathemagic,” although no magic is involved. Algebra can be used to explain the trick.

Directions: Follow the instructions for each problem. Write the result in Column I. Then complete Column II by using a variable for the number you are thinking of. Be sure to simplify expressions in Column II before you go to the next step.

	Column I	Column II
Problem 1		
1. Think of a number	_____	_____
2. Square the number	_____	_____
3. Subtract 4	_____	_____
4. Multiply by 2	_____	_____
5. Add 8	_____	_____
6. Divide by twice the original number	_____	_____
Problem 2		
1. Think of a number	_____	_____
2. Add 5	_____	_____
3. Multiply by 2	_____	_____
4. Add 6	_____	_____
5. Subtract 12	_____	_____
6. Divide by 2	_____	_____
7. Subtract the original number	_____	_____
Problem 3		
1. Think of a number	_____	_____
2. Add 7	_____	_____
3. Multiply by 10	_____	_____
4. Divide by 5	_____	_____
5. Subtract 14	_____	_____
6. Divide by 2	_____	_____



I-7
(continued)

NAME

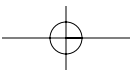
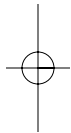
DATE

SECTION

Writing Phrases as Algebraic Expressions II

	Column I	Column II
Problem 4		
1. Start with the grade you are currently in		
2. Multiply by 10		
3. Add 6		
4. Multiply by 10		
5. Subtract 12		
6. Add your age		
7. Subtract 31		
8. Subtract 17		

Problem 5
Create a problem so that the answer is twice the original number. Share your problem with another student in class.



I-8

NAME _____

DATE _____

SECTION _____

Simplifying Expressions by Combining Like Terms

Expressions are sometimes made up of *terms*, which are the product of numbers and variables. (A *variable* is a letter that represents a number.)

Like terms are also called *similar terms*. They have the same variables. For example, $3a$ and $2a$ are like terms, but $3a$ and $2b$ are not.

Only like terms can be combined by addition or subtraction. The result is an expression in simplest form. Examples:

$$3a + 2a = 5a$$

$$3a + 2b \text{ cannot be simplified}$$

$$3a + 10 + 3b + 2a - 2b + 13 = 5a + b + 23$$

You must follow the rules for multiplication and the Distributive Property to simplify expressions. Examples:

$$2 \times 6a = 12a$$

$$3(4a + 2b) = 12a + 6b$$

Now go to the next page of this activity.

NAME

DATE

SECTION

Simplifying Expressions by Combining Like Terms

Directions: Simplify each expression. Find your answer in the Answer Bank at the right, then write the letter of your answer in the blank before the problem number. Write the letter above the problem number in the message at the end of the activity to learn an important fact about mathematics. The first problem is done for you.

1. \checkmark $4x + 7x =$
2. $\quad 4 \cdot 3x =$
3. $\quad 6x + 5y - 2x + 4 =$
4. $\quad x + 3y + 6x + 1 =$
5. $\quad 3 \cdot 5x =$
6. $\quad 4x + 3x - 7x =$
7. $\quad x + 2x + 3x - 4x =$
8. $\quad 5 + 2x + 9 =$
9. $\quad 2x - x =$
10. $\quad 2(x + 3y) + 4x =$
11. $\quad 4(2x + 3y) + 2(x + 3y) =$
12. $\quad 7(2x) + 3(x + 4) =$
13. $\quad 4(5x) + 2(3y) - 20x =$
14. $\quad 12x - 3x + 2y + 12 - 2y =$
15. $\quad 3 + 6(3x + 2y) =$

- Answer Bank
- R. $6y$
- S. $18x + 12y + 3$
- P. $4x + 5y + 4$
- I. $10x + 18y$
- K. $15x$
- V. $11x$
- N. $6x + 6y$
- U. $2x$
- O. 0
- Q. $12x$
- W. $2x + 14$
- T. $9x + 12$
- A. x
- E. $17x + 12$
- L. $7x + 3y + 1$

A French mathematician, Francois Viète (1540–1603), introduced the use of

\checkmark

16

13

7

2

6

12

10

9

8

15

12

14

11

14

6

10

12

15

I-9

NAME _____

DATE _____

SECTION _____

Simplifying and Evaluating Expressions

To *simplify* an expression means to replace it with its simplest name, being sure that like terms are combined. Example:

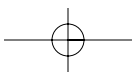
$$3a + 2a + 3b + 4 = (3a + 2a) + 3b + 4 = 5a + 3b + 4$$

To *evaluate* an expression means to substitute a given number for each variable and follow the Order of Operations. If $a = 3$ and $b = 2$, the above equation would be evaluated as $(5 \times 3) + (3 \times 2) + 4 = 15 + 6 + 4 = 25$.

Directions: Simplify each expression and write your answer in the first blank. Then evaluate each expression for the given values and write this answer in the second blank. After you have finished the problems, write the letter of the problem above its “evaluated” answer on the bottom of the page to complete the statement. The first problem is done for you.

$$a = 3 \quad b = 2 \quad c = 0 \quad d = 5$$

U. $3a + a$	<u>4a</u>	<u>12</u>
F. $3a + 12 + 2a$	_____	_____
L. $2a + b + 3d$	_____	_____
P. $4(a + b)$	_____	_____
R. $5(2a + 3)$	_____	_____
D. $a + 12 + c + 3$	_____	_____
M. $3c + 2(5c + 7)$	_____	_____
T. $3a + 6(3 + 2a)$	_____	_____
O. $12 + 3b + 2(3 + b)$	_____	_____
A. $2(3b + 14)$	_____	_____
Y. $a + 5 + 2(3a + d)$	_____	_____
N. $a + b(2 + 4)$	_____	_____



I-9

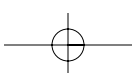
(continued)

NAME _____ DATE _____ SECTION _____

Simplifying and Evaluating Expressions

- E. $14 + 4(d - 2)$ _____
- H. $2(3a + 1) + 4(3a + 1)$ _____
- S. $6(b + d) - 2(b + d) + 4$ _____
- I. $8 + 6(3a + 2b + d) + 2a$ _____
- V. $4 + 3(a - 1)$ _____

122	63	122	32	26	40	32	122	26	45
63	28	32	122	14	20	23	122	27	36
27	122	45	32	63	40	15	18		
63	60	26	15	26	10	40	23	12	40
									63
									26



I-10

NAME _____

DATE _____

SECTION _____

Evaluating Expressions Using Exponents

An *exponent* is a number that appears in the upper right-hand corner of a number or an expression. It represents the number of times the base (which is the number or expression) is used as a factor. Examples:

$$\begin{aligned}3^2 &= 3 \times 3 \text{ or } 9 \\4 \times 3^2 &= 4 \times 3 \times 3 \text{ or } 36 \\(4 \times 3)^2 &= 4 \times 3 \times 4 \times 3 \text{ or } 144\end{aligned}$$

Note that the placement of grouping symbols makes a difference when using exponents.

Also note that any base to the zero power is equal to 1 and that any base to the first power is equal to itself. Examples:

$$3^0 = 1 \quad 3^1 = 3$$

Directions: Evaluate each expression. Write your answer in the blank that follows the problem. Then find your answer in the Answer Bank and write the letter of each answer in the blank space before each problem. Finally, write the letters of each problem in order, starting with the first problem, to complete the statement at the end of the activity.

1. _____ a^0 if $a = 5$ _____
2. _____ b^2 if $b = 3$ _____
3. _____ n^2 if $n = 7$ _____
4. _____ $(2a)^0$ if $a = 5$ _____
5. _____ a^5 if $a = 2$ _____
6. _____ m^2 if $m = 10$ _____
7. _____ c^2 if $c = 8$ _____
8. _____ h^3 if $h = 4$ _____
9. _____ $(2h)^3$ if $h = 4$ _____
10. _____ $(c + 4)^2$ if $c = 5$ _____
11. _____ $c + 4^2$ if $c = 5$ _____
12. _____ $c^2 + 4$ if $c = 5$ _____

NAME _____

DATE _____

SECTION

Evaluating Expressions Using Exponents

13. _____ $(n + 1)^2 + n$ if $n = 3$ _____

14. _____ a^6 if $a = 2$ _____

15. _____ $2 + m^3$ if $m = 6$ _____

16. _____ $3m + m^0$ if $m = 6$ _____

17. _____ $(2m)^3$ if $m = 6$ _____

18. _____ $2m^3$ if $m = 6$ _____

19. _____ $3^2 - a^3$ if $a = 2$ _____

20. _____ $5a^2 - a + 1$ if $a = 2$ _____

21. _____ $(x - 3)^2$ if $x = 5$ _____

22. _____ $x^2 - 3$ if $x = 5$ _____

23. _____ $4n^2$ if $n = 5$ _____

24. _____ v^2 if $v = 9$ _____

25. _____ $(8v)^0$ if $v = 2$ _____

26. _____ $8v^2$ if $v = 2$ _____

27. _____ $(2n + 3)^2$ if $n = 2$ _____

28. _____ $2n^2 + 11$ if $n = 2$ _____

29. _____ $n^2 - n - 1$ if $n = 4$ _____

Answer Bank

E. 19

B. 100

R. 432

K. 512

I.1

L. 81

0.64

N. 9

U. 22

S. 32

D. 11

A. 21

T. 1,728

M. 218

G. 29

P. 4

V. 256

H. 49

René Descartes (1596–1650), a French philosopher and mathematician, first used the modern notation for exponents _____

_____?

_____ in 1637.

NAME _____

DATE _____

SECTION _____



Writing Equations

An *equation* is a mathematical sentence that states that two quantities are equal. An equal sign (=) separates an equation into two parts, which are called sides of the equation.

Directions: Write an equation for each situation described below. Use the given variable. There may be more than one equation for each problem.

1. The number of days in September is one less than the number of days in October. Let s represent the number of days in September. There are thirty-one days in October.

2. The time in California is three hours earlier than the time in New York. It is 7 P.M. in New York. Let c represent the time in California.

3. The length of an NBA (National Basketball Association) court is forty-four feet longer than the width, which is fifty feet. Let l stand for the length of the court.

4. Each player begins a chess game with a total of sixteen chess pieces. This is twice the number of pawns. Let p stand for the number of pawns.

5. Forty-eight states of the United States are located in four time zones. This is $\frac{1}{6}$ of the number of time zones in the world. Let w represent the number of time zones in the world.

6. The total area of the United States is the combination of the area of its land and water. The total area of the U.S. is 3,787,319 square miles, of which 251,041 square miles is water. Let l stand for the area of the land.

7. A rabbit moves half as fast as a cheetah at full speed. A cheetah can run seventy miles per hour. Let r stand for the speed of the rabbit.

NAME _____

DATE _____

SECTION _____

Writing Equations

8. The average life span of a lion is one year more than twice the life span of a fox, which is seven years. Let l represent the life span of a lion.

9. The Pacific Ocean and the Atlantic Ocean are the two largest oceans in the world. The approximate area of the Atlantic is 86,560,000 square kilometers. The Pacific is a little more than 1.9 times as large. Let p represent the area of the Pacific Ocean.

10. It takes about seventeen muscles to smile and about nine more than twice that number to frown. Let f represent the number of muscles it takes to frown.

11. A newborn baby has more bones than an adult, because bones grow together as a person matures. The typical baby has 350 bones and an average adult has 206. Let b represent the difference in the number of bones.

12. The number of calories recommended for boys, ages 11 to 14, is 2,800 calories per day. The number recommended for girls, ages 11 to 14, is 400 calories less per day. Let g represent the number of calories recommended for girls each day.

13. The label on a bag of candy says there are about nine servings per bag, and that the serving size is eight pieces. Let p represent the number of candies in the bag.

14. The word processor was invented in 1965. The ballpoint pen was invented seventy-seven years earlier, and the typewriter was invented twenty-one years before the ballpoint pen. Let t stand for the year the typewriter was invented.

15. The first modern Olympic Games were held in Athens, Greece, in 1896. Thirteen countries were represented and twelve more than twenty-three times that number of athletes participated. Let a stand for the number of athletes.

I-12

NAME _____

DATE _____

SECTION _____

Writing Equations and Inequalities I

An *equation* is a mathematical sentence in which the = symbol connects two expressions of the same value. An *inequality* is a mathematical sentence in which the symbols < (is less than) or > (is greater than) are used. In an inequality, the expressions on either side of the equation do not have the same value.

Directions: Each pair of phrases below and on the next page describes a number. Write =, <, or > on the blank between each pair of phrases to show the relationship between the numbers that the phrases represent. Hint: The number of equations < the number of inequalities.

- | | | |
|--|-------|--|
| 1. the number of singers in a trio | _____ | the number of singers in a quartet |
| 2. the number of digits in a ZIP Code | _____ | the number of digits in an area code |
| 3. items in a dozen | _____ | items in a baker's dozen |
| 4. perimeter of a 4×4 square | _____ | perimeter of a 3×5 rectangle |
| 5. the number of U.S. senators | _____ | the number of states |
| 6. total number of dots on a pair of dice | _____ | the number of squares on a checkerboard |
| 7. minimum number of players on a baseball field | _____ | minimum number of players on a football field |
| 8. length of a professional basketball game (in minutes) | _____ | length of a college basketball game (in minutes) |
| 9. length of a radius of a 10-inch circle | _____ | length of a diameter of a 10-inch circle |
| 10. the average of 90, 96, and 81 | _____ | the average of 90, 86, and 91 |
| 11. the number of quarters in a dollar | _____ | the number of quarts in a gallon |
| 12. the number of inches in a foot | _____ | the number of inches in a yard |
| 13. the number of sides on a stop sign | _____ | the number of tentacles of an octopus |

I-12
(continued)

NAME

DATE

SECTION

Writing Equations and Inequalities I

14. the number of oceans

15. 16 ounces

16. 2^5

17. the number of aces in a deck of cards

18. 100° C

19. the number of players required for backgammon

20. one square yard

21. the denomination of a bill with Andrew Jackson's portrait

22. the number of centimeters in a meter

23. the number of degrees in a circle

24. the number of legs of an ant

25. the number of stars in the Big Dipper
- the number of continents

1 pound

5^2

the number of red kings in a deck of cards

212° F

the number of players required for chess

three square feet

the denomination of a bill with Benjamin Franklin's portrait

the number of cents in a dollar

the number of degrees in a triangle

the number of legs of a spider

the number of stars in the Little Dipper

I-13

NAME _____

DATE _____

SECTION _____

Writing Equations and Inequalities II

Common situations can be expressed as equations or inequalities. While the $=$ sign is used to show expressions of equal value, several symbols can be used to express inequalities, including:

- $<$, which means is less than
- \leq , which means is less than or equal to
- $>$, which means is greater than
- \geq , which means is greater than or equal to

Directions: Write an equation or inequality for each problem. Use the appropriate variable.

1. The sign on a clearance rack in a store says \$10 and up. Let p represent the price of one item.

2. A person must be 18 years old to vote in the United States. Let a represent the age of a person eligible to vote.

3. In order to go on some rides in amusement parks, children must be over four feet tall. Let h represent the height required for admittance to these rides.

4. Children under twelve years of age can order from a child's menu in many restaurants. Let a represent the age at which kids can order a kid's meal.

5. A minimum charge at a restaurant is \$5 per person. Let t represent the total bill for five people.

6. When ordering from a particular catalog, the shipping and handling charges are \$7 for orders less than \$20. Let c represent the shipping and handling charge of a \$15 order.

7. The occupancy limit in the school cafeteria is 350 people. Let p represent the number of people that may safely occupy the room.

NAME _____

DATE _____

SECTION _____

Writing Equations and Inequalities II

8. The admissions fee to a nearby amusement park is as follows:

Children 2 and under — Free

Children between the ages of 2 and 12 — \$20

Children over 12 and adults — \$40

Let t represent the total admission fee for a family of two adults, a 3-year-old child, and a 10-year-old child.

9. The bus company charges \$120 per bus to transport up to forty-three passengers to and from the museum for the class trip. Let c represent the charge for transportation for eighty passengers.

10. Tables in a restaurant accommodate ten people. Let p represent the number of people able to sit at each table.

11. An aquarium provides one free ticket per eight students. Let t represent the number of free tickets for a group of 125 students.

12. It takes ten minutes to boil an egg. Let t represent the time it takes to boil a half-dozen eggs.

13. To qualify for the honor roll, a student must have at least three A's and no grade less than a B on his or her report card. Let n represent the number of A's required for a student to qualify for the honor roll.

14. A family of five traveled 390 miles to Florida. Let d represent the total distance they traveled.

15. A student who is absent from school more than three days in a row must bring in a medical excuse when he or she returns to school. Let d represent the number of consecutive absences that require a note.

NAME _____

DATE _____

SECTION _____

Identifying the Solution of an Equation

A *solution* to an equation is a number that makes the equation a true statement when the solution is substituted for the variable.

Directions: A group of equations is written below a set of numbers. All but one of the numbers is a solution to one of the equations. (One number in each set is not a solution to any of the equations.) Write the letter of the solution in the blank before each equation. Then write the letters in order, starting with the first equation, to complete the statement at the bottom of the sheet.

Set 1: L. 54 Y. 8 C. 20 B. 5 A. 52

_____ $3a - 2 = 13$

_____ $4x - 1 = 31$

_____ $\frac{n}{4} - 5 = 8$

_____ $15 = 6 + \frac{m}{6}$

Set 2: K. 12 R. 30 W. 36 A. 8 H. 4

_____ $3n + 29 = 65$

_____ $18 = 4t + 2$

_____ $\frac{x}{3} + 2 = 14$

_____ $3g - 4 = 20$

Set 3: Z. 30 M. 18 I. 9 R. 7 A. 110 I. 85

_____ $18 + 2b = 32$

_____ $2n + 5 = 23$

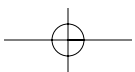
_____ $\frac{a}{10} + 3 = 6$

_____ $3x - 4 = 50$

_____ $\frac{2n}{5} - 23 = 11$

Algebra comes from the Arabic word “al-jabr,” which appeared in the title of a book written around 825 _____

_____, an Islamic mathematician.



NAME

DATE

SECTION

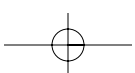
I-15

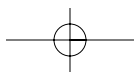
Determining the Solutions of Equations and Inequalities

A *solution* of an equation or an inequality is a number that makes the statement true.

Directions: Circle the letter(s) of the *solution or solutions* for each equation or inequality. If there is no solution to a problem, circle the letter for “none.” Then write the letters in order, starting with the first problem, to complete the statement at the end of the activity. A fact about inequalities will be revealed.

1. $a > 3$	T H E {1, 2, 3}	I none
2. $c \geq 10$	M T I {0, 10, 20}	N none
3. $w < 5$	S T H {0, 5, 10}	E none
4. $0 < x$	O I M {0, 1, 2}	F none
5. $3x \geq 36$	P O S {12, 14, 16}	T none
6. $x^2 = 2x$	S A T {2, 4, 8}	N none
7. $2x = 17$	T H E {8, 9, 34}	I none
8. $2x > 17$	N B L {8, 9, 34}	A none
9. $\frac{x}{x} = 1$	E T O {3, 6, 9}	F none
10. $2x^2 \geq 9$	A L I {2, 4, 8}	T none





NAME _____

DATE _____

SECTION _____

I-15

(continued)

Determining the Solutions of Equations and Inequalities

11. $x + 7 < 10$	S T I {1, 2, 3}	N none
12. $3r - 7 > 9$	N A L {5, 6, 7}	O none
13. $\frac{n}{4} - 1 > 0$	T L O {4, 8, 12}	D none
14. $\frac{h}{14} > 3$	I N A {14, 28, 42}	F none
15. $\frac{1}{x} \geq 0$	T H E {1, 2, 3}	I none
16. $15 < \frac{a}{5}$	O F A {15, 25, 75}	S none
17. $12 \leq 2n + 6$	O L U {3, 6, 9}	I none
18. $35 \leq 10x + 20$	N T I {1, 2, 3}	S none
19. $s - 9 \geq 15$	D O N {15, 24, 36}	F none
20. $\frac{24}{n} < 8$	I V E {1, 2, 3}	S none

_____.

