Section 1

NUMBER, OPERATION, AND QUANTITATIVE REASONING

Objective 1

Compare and order fractions, decimals (including tenths and hundredths), and percents, and find their approximate locations on a number line.

Students must be able to compare different real numbers. The comparing and ordering of several decimal numbers requires a strong understanding of place value. The skill to order signed decimal numbers, both positive and negative numbers within the same set, is also expected at the middle school level. Thus, students need experience with ordering a variety of real numbers. The following activities provide such experience with decimals, fractions, and percents. The relationship of positive numbers to negative numbers is emphasized, and mastery of basic equivalent decimals, fractions, and percents will be assumed.

Activity 1: Manipulative Stage

Materials

Pattern 1–1a and Pattern 1–1b for number cards for each pair of students
Worksheet 1–1a
Scissors
Regular pencil

Procedure

1. Give each pair of students a copy of Pattern 1–1a, Pattern 1–1b, and scissors. Each student should also receive a copy of Worksheet 1–1a.

2. Each pair should cut apart the number cards shown on the two pattern sheets. These cards will be used to determine the order of each set of numbers on the worksheet.

3. For each exercise, partners should select a tentatively lesser number in the set and build it with the appropriate number cards. Then they should add on other cards having positive values in order to change from the selected number's value

to a total card value equal to another number in the set. If this can be done, the selected number is less than the new number found. If this cannot be done, the selected number is greater than any of the other numbers in the set.

4. Other numbers from the set may need to be tested in this same manner before the final order of all numbers in the set can be determined. Once the order is found, students should record the numbers in the correct order below the exercise, using the appropriate < or > sign.

5. Guide students through Exercise 1 on Worksheet 1–1a before they proceed to the other exercises.

For the first exercise on Worksheet 1–1a, the set of numbers, +2.5, –3, –1.5, +0.75, must be ordered *from least to greatest*. Since the numbers in this exercise are all decimals, only the integer and decimal cards need to be used.

As an example, have students select –1.5 as the first number to build with the number cards. Students should place one (–1) card and one (–0.5) card on the desktop; these two cards have the total value of –1.5. Ask the students to place more cards beside these two cards but to use only positive cards. The process of adding only positive amounts to some given real number will produce new numbers greater than the given number. When appropriate, encourage them to use positive cards to form 0-pairs with any given negative cards. For example, if a (–0.5) card has been used, a (+0.5) card might be added to form a 0-pair with it.

Students should find the total value of the cards after each new card is added. If one (+1) card is added, the total value will be –0.5; if two (+0.5) cards are also added, the total value will be +0.5. Then if two more (+1) cards are added, the value will be +2.5, where +2.5 is a number in the original set. This result indicates that +2.5 is greater than –1.5 in the set. Here is a possible card arrangement for this process. Notice that negative amounts are placed to the left and positive amounts are added to the right, reflecting a general sense of direction. Also, positive amounts are placed below negative amounts in order to form 0-pairs easily. If no negative amount were present, there would just be a row of positive amounts being joined to the right.

–1	–0.5			
+1	+0.5	+0.5	+1	+1

Notice that when increasing from –1.5 to +2.5, the number –3 was not found as a total value. So –3 needs to be tested. After three (–1) cards are placed on the desktop, one (+1) card might be added to produce a value of –2. Then one (–1) card might be traded for two (–0.5) cards. One (+0.5) card may now be added, yielding a total card value of –1.5. The total card value has increased from –3 to –1.5, a member of the given set, so –1.5 is greater than –3. Here is a possible arrangement of the final cards used:

–1	–1	–0.5	–0.5
+1		+0.5	

Similarly, +0.75 can be built by adding one (+1) card, one (+0.5) card, and one (+0.75) card to the cards for –1.5; by adding to the (+0.75) card one (+0.25) card, one (+1) card, and one (+0.5) card, +0.75 can be increased to +2.5. These two tests confirm that $-1.5 < +0.75$ and $+0.75 < +2.5$. Applying some logical reasoning, students should now be able to write the four numbers in the required increasing order below Exercise 1 on Worksheet 1–1a:

$$-3 < -1.5 < +0.75 < +2.5$$

Answer Key for Worksheet 1–1a

1. $-3 < -1.5 < +0.75 < +2.5$
2. $+3.5 > 0 > 0.5 > -4$
3. $-3.5 < -2.5 < -1 < +3.0$
4. $+1.25 > +0.50 > -0.5 > -2.75$
5. $+\frac{3}{4} > 0 > -1 > -1\frac{1}{2}$
6. $-3\frac{1}{2} < -2 < +2\frac{1}{4} < +4$
7. $+1\frac{3}{4} > -\frac{1}{2} > -1\frac{3}{4} > -2$
8. $-3 < -\frac{3}{4} < +1\frac{1}{4} < +2$

WORKSHEET 1–1a Name ____________

Ordering Real Numbers by Building Date ____________

Find the required order of each set of real numbers by using the number cards from Pattern 1–1a or Pattern 1–1b. Record the correct order below each set, using < or >.

1. Order from least to greatest: +2.5, –3, –1.5, +0.75

2. Order from greatest to least: –4, 0, +3.5, –0.5

3. Order from least to greatest: –2.5, –3.5, +3.0, –1

4. Order from greatest to least: +1.25, –0.5, –2.75, +0.50

5. Order from greatest to least: $-1\frac{1}{2}$, 0, $+\frac{3}{4}$, –1

6. Order from least to greatest: +4, –2, $+2\frac{1}{4}$, $-3\frac{1}{2}$

7. Order from greatest to least: –2, $+1\frac{3}{4}$, $-\frac{1}{2}$, $-1\frac{3}{4}$

8. Order from least to greatest: +2, $+1\frac{1}{4}$, –3, $-\frac{3}{4}$

Instructions: Cut the cards apart.

Pattern 1–1a. Decimal Cards

–1	–1	–1	–1
+1	+1	+1	+1
+1	+1	+1	+1
+0.5	+0.5	+0.25	+0.25
+0.75	+0.75	–0.5	–0.5
–0.25	–0.25	–0.75	–0.75

Instructions: Cut the cards apart.

Pattern 1–1b. Fraction Cards

–1	–1	–1	–1
+1	+1	+1	+1
+1	+1	+1	+1
$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{4}$	$+\frac{1}{4}$
$+\frac{3}{4}$	$+\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{3}{4}$	$-\frac{3}{4}$

Activity 2: Pictorial Stage

Materials

Worksheet 1–1b
Regular pencil

Procedure

1. Give each student a copy of Worksheet 1–1b. Have students work in pairs.

2. Review the methods for finding equivalent percents, decimals, and fractions. For example, students should recognize 25% as "25 per hundred" or "25-hundredths" by definition and then write it in decimal form as 0.25. They should also be able to write "25-hundredths" in fraction form as 25/100 and then reduce it to 1/4. This renaming process is a prerequisite for this activity.

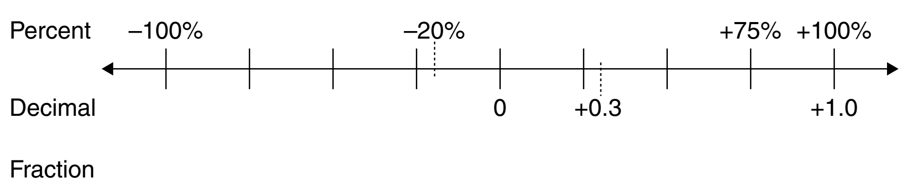
3. Using the unit length from 0 to +1 marked off on the number line on the worksheet as the whole for comparison purposes, have students locate points on the number line whose distances from 0 represent specific portions of the unit bar length. These distances also order the numbers they represent. Portions in the positive direction will indicate increases, and portions in the negative direction will indicate decreases.

4. Guide students through Exercise 1 on Worksheet 1–1b before they proceed to the other exercise.

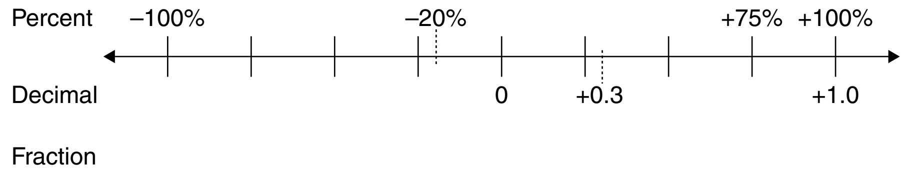
In Exercise 1, a situation concerning increases and decreases in profit is given. Students are asked to rank the three levels of performance and then represent their order on the number line. The total annual sales used to determine a store's profit may differ among the stores, but it is only their levels of performance (rates) that are being compared. Hence, students must think of the 75% increase as +0.75, the 0.3 increase as +0.3, and the 20% decrease as –0.20.

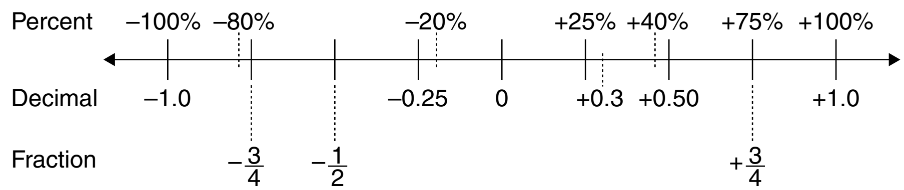
These three decimal numbers may now be easily compared and ordered, from least to greatest or from greatest to least. If ordered from least to greatest, the order will be –0.20, +0.3, +0.75. Then students should use the original numbers and record the following below Exercise 1: –20% < +0.3 < +75%. If they decide to order from greatest to least, they will record: +75% > +0.3 > –20%.

The ordered amounts should finally be marked on the number line. Percents, fractions, and decimals should be written in different horizontal lines for later ease of reading and comparing. Since markings are not shown on the number line for –20% or +0.3, students must estimate where their markings should be relative to the markings already shown. Here is the number line containing the numbers for Exercise 1:



Answer Key to Worksheet 1–1b

1. 

2. 

WORKSHEET 1–1b

Ordering Real Numbers on a Number Line

Name ______________________

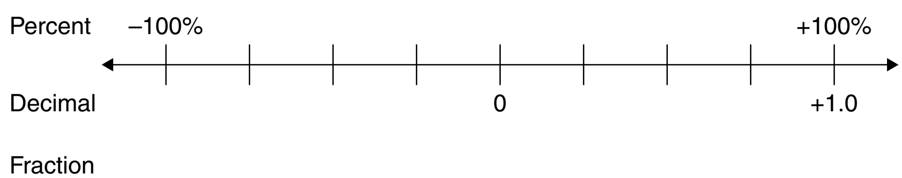
Date ______________________

Order the real numbers in the list by marking and labeling a point on the number line for each number. Each fraction, decimal, or percent represents a certain portion of the indicated unit length from 0 to +1 or from 0 to –1 on the number line. The positive unit length represents a "100% increase," and the negative unit length represents a "100% decrease." A point marked on a unit length will represent a portion of either the increase or the decrease. Answer any additional questions given in the exercises.

1. This year, store 1 showed a 75% increase over last year's profits, store 2 showed a 0.3 increase, and store 3 showed a 20% decrease in profits. Use signed numbers with $<$ or $>$ to rank these three levels of performance. Estimate and label positions on the number line to represent the ranking.

2. Order the following numbers by estimating and labeling their positions on the number line:

$$+25\%,\ +0.50,\ -\frac{3}{4},\ -0.25,\ -1.0,\ +40\%,\ +\frac{3}{4},\ -80\%,\ -\frac{1}{2}$$



Activity 3: Independent Practice

Materials

Worksheet 1–1c
Regular pencil

Procedure

Give each student a copy of Worksheet 1–1c. Have students work independently. When all are finished, discuss their results.

Answer Key for Worksheet 1–1c

1. B
2. C
3. A
4. 0.2

Possible Testing Errors That May Occur for This Objective

- The numbers are sequenced by size but in reverse order; for example, they are arranged in decreasing order, but the test item requires them to be in increasing order. Students clearly understand how to order the numbers but did not read the test item carefully.
- The first and last numbers listed in the sequence are correct, but the other numbers are randomly ordered between those two numbers. Students focus on the "least" and the "greatest" numbers, but disregard any others given in the list.
- The positive and negative signs are ignored, and the numbers are ordered only by their absolute values. Hence, +2, –3.5, and –1.8 are ordered from least to greatest as 1.8, 2, and 3.5.
- Decimal points are ignored, so decimal numbers like 4.25, 10.8, and 0.586 are ordered from greatest to least as 586, 425, and 108. This results in the incorrect answer choice being selected (0.586, 4.25, 10.8).

WORKSHEET 1–1c

Name ______________________

Ordering Real Numbers Involving Percents, Decimals, and Fractions

Date ______________________

Solve the problems provided. Draw a number line, and use it to order numbers if helpful.

1. The Triangle Mall manager reviewed a report that showed the percent increase in sales at 4 stores.

Store 1	**Store 2**	**Store 3**	**Store 4**
3.5%	0.78%	2.75%	2.06%

Which lists the percent increase in sales from greatest to least for all 4 stores?

A. 3.5%, 0.78%, 2.75%, 2.06%

B. 3.5%, 2.75%, 2.06%, 0.78%

C. 0.78%, 2.06%, 2.75%, 3.5%

D. 2.75%, 2.06%, 0.78%, 3.5%

2. The high temperatures in Anchorage, Alaska, for 5 consecutive days were –5 degrees, 0.6 degrees, –3.8 degrees, –1.9 degrees, and 4.2 degrees Fahrenheit. Which shows the temperatures in order from coldest to warmest?

A. 4.2°F, 0.6°F, –1.9°F, –3.8°F, –5°F

B. –5°F, 0.6°F, –1.9°F, –3.8°F, 4.2°F

C. –5°F, –3.8°F, –1.9°F, 0.6°F, 4.2°F

D. 4.2°F, –3.8°F, –1.9°F, 0.6°F, –5°F

3. A librarian arranged some books on the shelf using the Dewey decimal system. Which group of book numbers is listed in order from least to greatest?

A. 724, 724.29, 724.3, 724.39

B. 105.4, 105.04, 108.21, 110.0

C. 391.5, 397.53, 399.62, 399.05

D. 549.01, 549.10, 549.02, 549.4

4. Joe is offered sales commissions of 18%, 0.2, and 3/20 of every dollar sold by three different stores, respectively. Which rate is the highest?

Objective 2

Multiply decimals to solve word problems.

The algorithm for the multiplication of decimal numbers needs to be developed well before students are required to independently work story problems requiring decimal multiplication. The algorithm depends heavily on an understanding of multiplication as the repetition of all or part of a set. It is also very complex in terms of place value. Many students have difficulty with it because they do not understand how place value changes will cause partial products to be recorded in different columns.

Activities for the development of the algorithm are described next. It is assumed for this objective that the multiplication facts have already been fully developed with manipulative materials.

Activity 1: Manipulative Stage

Materials

Building Mat 1–2a
Set of base 10 blocks per pair of students (6 flats, 30 rods, 30 small cubes)
Worksheet 1–2a
Regular pencil

Procedure

1. Give each pair of students a set of base 10 blocks (6 flats, 30 rods, 30 small cubes) and a copy of Building Mat 1–2a. For the multiplicand or the set to be repeated, a flat will represent a one or a whole unit, a rod will represent a tenth of the one, and a small cube a hundredth of the one.

2. Give each student a copy of Worksheet 1–2a. Worksheet exercises will involve ones and tenths in one- or two-digit decimal numbers.

3. Have students model each exercise with their blocks. Then on Worksheet 1–2a below the exercise, have them record a word sentence that shows the results found on the building mat. They should count up their total blocks in the product region (that is, in the interior of the angle of the L-shape) of the mat in order to find their answers. Do not trade any blocks found in the product region. For example, if 10 hundredths are there, do not exchange them for 1 tenth. Always have students use proper place value language when verbally describing their steps to the class.

4. First discuss Exercise 1 on Worksheet 1–2a in detail with the students. Then allow them to work the other exercises with their partners.

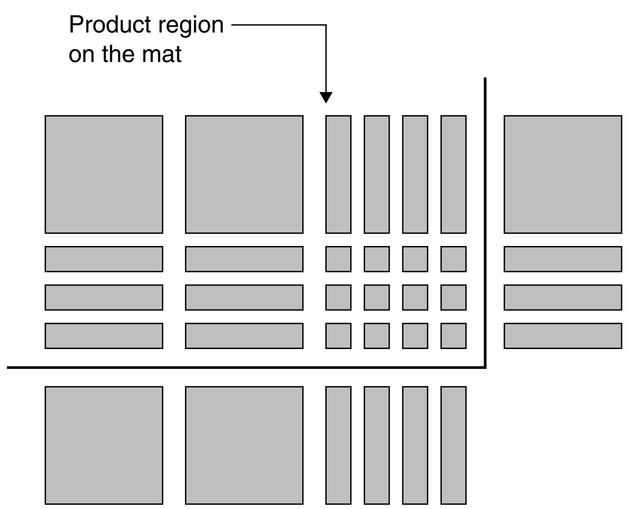
Here is the story problem for Exercise 1: “Marian has several bags of candy in the store display case. Each bag holds 2.4 ounces of candy. A customer needs 1.3 bags for a cookie recipe. How many ounces of candy does the customer need?”

Have students place 1 flat and 3 rods along the vertical bar of the L-frame for the multiplier, and place 2 flats and 4 rods below the horizontal bar of the frame for the multiplicand (see the illustration for block placement). For both factor sets, the rods should be placed closest to the corner of the L-frame and the flats placed toward the ends.

Discuss the idea that the flat or one along the vertical bar means to build one complete copy of each block below the horizontal bar. Each rod or tenth along the vertical bar means to build one part of ten parts of each block below the horizontal bar. So 1-tenth of a flat (or 1-tenth of one whole) will be a rod or tenth block in the product region, and 1-tenth of a rod (or 1-tenth of a tenth of a whole) will be a small cube or hundredth block in the product region.

Guide students to apply the blocks in the multiplier one block at a time. This process will build blocks in the product region one row at a time. For Exercise 1, when applied to the blocks in the horizontal multiplicand, each tenth block in the vertical multiplier will produce a product row that contains two tenths (rods) and four hundredths (cubes). The ones block or flat will produce a product row of two ones (flats) and four tenths (rods). Each row in the product represents an application of the distributive property of multiplication over addition.

Here is the final block arrangement on Building Mat 1–2a that represents the product of 1.3 bags of 2.4 ounces each:



The product is found by counting the blocks in the product region according to their place value, which yields 2 ones, 10 tenths, and 12 hundredths. Students should simplify the product mentally without removing any blocks physically from the mat. Allow students with weak place value skills to trade the 10 tenths for a one and trade 10 hundredths for a tenth in order to find the final numerical answer—3 ones, 1 tenth, and 2 hundredths, or 3.12 ounces—but have them return the original blocks to the product region of the mat once the answer is found.

Students should write below Exercise 1 on Worksheet 1–2a the following sentence about their results: “1.3 bags of 2.4 ounces each of candy will be 3.12 ounces in all.”

Answer Key for Worksheet 1–2a

Here are some possible sentences to write:

1. 1.3 bags of 2.4 ounces each of candy will be 3.12 ounces in all.
2. 2 cartons weighing 1.4 pounds each will weigh 2.8 pounds together. [Only 2 rows of blocks will be built in the product region.]
3. 0.4 of 1.2 yards equals 0.48 yard used for the tie. [Only 4 rows of blocks will be built in the product region.]
4. 1.2 of 1.5 liters of cream makes 1.8 liters of cream for the recipe.
5. 0.5 of 3 gallons makes 1.5 gallons of ice cream eaten at the party.

Building Mat 1–2a

WORKSHEET 1–2a

Building Decimal Products with Base 10 Blocks

Name ______________________

Date ______________________

Build with base 10 blocks on Building Mat 1–2a to solve the word problems provided. Below each exercise, write a word sentence about the result found for that word problem.

1. Marian has several bags of candy in the store display case. Each bag holds 2.4 ounces of candy. A customer needs 1.3 bags for a cookie recipe. How many ounces of candy does the customer need?

2. There are 2 cartons. Each carton weighs 1.4 pounds. What is the total weight in pounds of the 2 cartons?

3. George, a tailor, has 1.2 yards of silk fabric. He will use 0.4 of that amount to make a silk tie for a customer. How many yards of the fabric will he use?

4. The chef has 1.5 liters of cream. A recipe requires 1.2 times that amount of cream. How much cream will he need for the recipe?

5. Lin bought 3 gallons of ice cream for her party. Her friends ate only 0.5 of the ice cream. How many gallons of ice cream were eaten at the party?

Activity 2: Pictorial Stage

Materials

Worksheet 1–2b
Red pencil and regular pencil

Procedure

1. Give each student a copy of Worksheet 1–2b and a red pencil. Students will draw short and long rectangles on L-frames on the worksheet to represent base 10 blocks on a building mat and to find products for the exercises.

2. For each exercise, after drawing the product on the L-frame students will write a number sentence about the product below the L-frame.

3. Students will then label different parts of the product region and transfer the labels and their descriptions to the vertical numerical format for the multiplication algorithm.

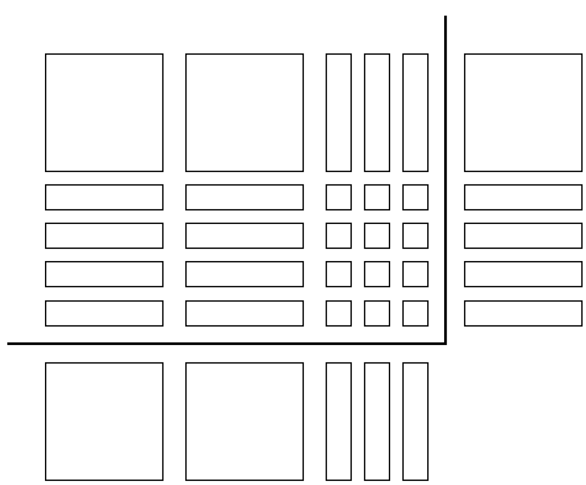
4. For each exercise, guide students to find a relationship between the digits of the two factors used and the arrangement of the rectangles in the L-frame.

5. Discuss Exercise 1 on Worksheet 1–2b in detail with students before having them work the other exercises on the worksheet. There will be three stages to apply to each exercise.

Here is a discussion for Exercise 1: "Luigi has 2.3 gallons of spaghetti sauce. He will need 1.4 times that amount to serve with spaghetti at a dinner. How many gallons of sauce will he need for the dinner?"

Drawing the Product Region

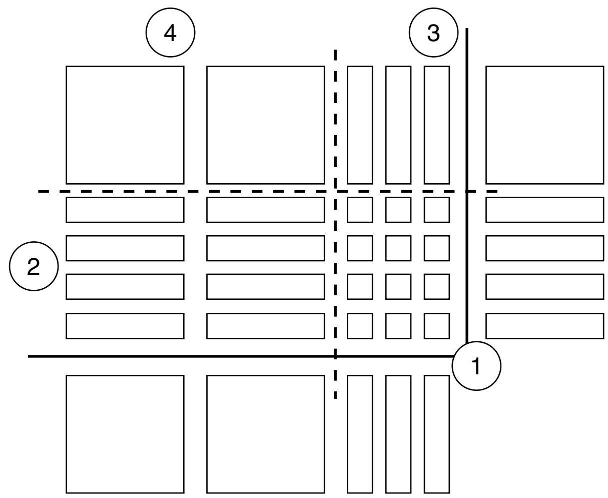
Following the repeated row method used in Activity 1 and drawing short and long rectangles to represent the ones, tenths, and hundredths, here is the final frame for Exercise 1 as drawn on Worksheet 1–2b:



After all students have finished the drawing, have them count the shapes by their value to find the product; 2 ones, 11 tenths, and 12 hundredths make 3 ones, 2 tenths, and 2 hundredths, or 3.22. Do not show any trades on the drawing. Write an equation or number sentence below the frame to show the product found: "$1.4 \times 2.3 = 3.22$ gallons of spaghetti sauce needed."

Labeling Parts of the Product Region

Next, discuss the four parts or smaller regions of shapes within the product region in Exercise 1. Separate the ones, tenths, and hundredths in the product region by drawing in red pencil a vertical bar between the left and right groups of shapes and a horizontal bar between the upper and lower levels of shapes. Also, label each of the four regions as shown (to stay in keeping with the order used in the traditional algorithm):



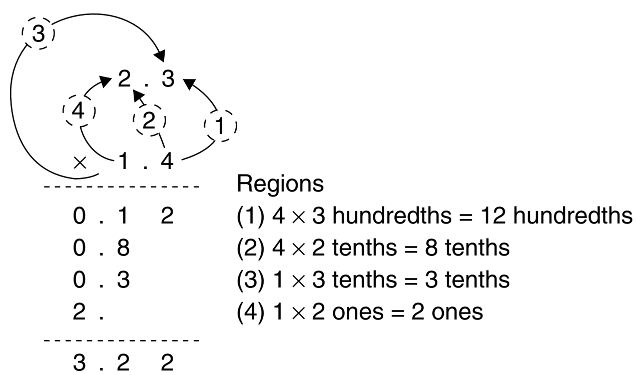
The arrangement of the shapes within each region of the example for Exercise 1 should be described in the following way: "Region 1 (lower right region) has four rows of 3 hundredths, or 12 hundredths; region 2 (lower left region) has four rows of 2 tenths, or 8 tenths; region 3 (upper right region) has one row of 3 tenths, or 3 tenths; and region 4 (upper left region) has one row of 2 ones, or 2 ones."

Note that in regions 3 and 4, there is only one row, which is determined by the 1 one to the right of the vertical bar of the L-frame (which is the 1 one in the multiplier). Some students will think there are three rows in region 3, but these are columns (vertical) and not rows (horizontal), as used in arrays in multiplication. Also notice that region 1 contains *hundredths* (as the product of tenths with tenths) and that region 3 (or the region above region 1) contains the next higher place value, *tenths*. In addition, each region on the left always contains the next-higher place value than that value found in the region immediately to the right. Students should now transfer the descriptions for the regions onto the vertical format to the right of the completed L-frame as shown. Notice where the decimal is placed in each partial product. The partial product for region 1 aligns at the right with the initial factors of the exercise, but the decimal for that first partial product is determined by that product's digits independent of the decimal positions in the factors above. The decimal position for region 1 then determines the place value alignment for the other partial products.

2.3	
× 1.4	Regions:
0.1 2	(1) 4 × 3 hundredths = 12 hundredths
0.8	(2) 4 × 2 tenths = 8 tenths
0.3	(3) 1 × 3 tenths = 3 tenths
2.	(4) 1 × 2 ones = 2 ones
3.2 2	

Pattern Search

After the regions of the finished frame for Exercise 1 have been transferred to the vertical format, ask students to compare the factors that were used to describe the shape arrangements in the four parts or smaller regions of the product region with the digits in the original two numbers (1.4 and 2.3) in Exercise 1. All digits in these two numbers must be unique so that the comparison will be easy for the students. They should notice that the factors in 4 × 3, 4 × 2, 1 × 3, and 1 × 2 use the same digits as those used in the original numbers, 1.4 and 2.3. Now have students draw red arrows on the original numbers to show the connection between these groups of numbers. The arrows should be drawn so that they do not cross each other and then labeled for the regions they represent.

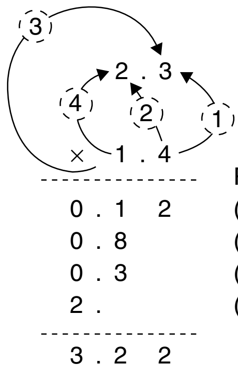


Have students repeat these three stages with the other exercises on Worksheet 1–2b. Exercise 3 will need only two regions of blocks; otherwise, the procedure remains the same. Emphasize the pattern of the arrows.

It is recommended that the four rows of partial products not be collapsed into two rows at this time. (Regions 1 and 2 are typically written as one row, and regions 3 and 4 are written as a second row in the traditional algorithm.) There is no need to do so mathematically, because students are no longer expected to multiply extremely large numbers by hand. It will also postpone the regroupings students have to do until they add at the end. The traditional multiplication algorithm is quite complex. Students need to go through the stages described here slowly so they will understand the process clearly. This entire development normally requires several class periods in order for most students to comprehend all the steps well. Once students see the pattern in the arrows for multiplying two-digit numbers by two-digit numbers, they will be able to extend the pattern to three-digit numbers as well. The arrow-region procedure reflects the place value changes that occur during multiplication and accounts for the directions concerning "column shifting" that have been used by teachers in the past.

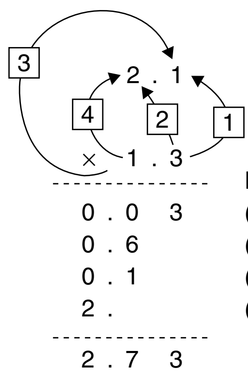
Answer Key for Worksheet 1–2b

1. 1.4 × 2.3 = 3.22 gallons of spaghetti sauce needed



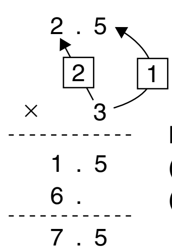
Regions
(1) 4 × 3 hundredths = 12 hundredths
(2) 4 × 2 tenths = 8 tenths
(3) 1 × 3 tenths = 3 tenths
(4) 1 × 2 ones = 2 ones

2. 1.3 × 2.1 = 2.73 gallons of gas used



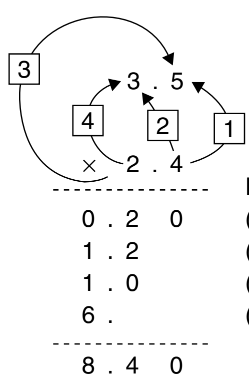
Regions
(1) 3 × 1 hundredth = 3 hundredths
(2) 3 × 2 tenths = 6 tenths
(3) 1 × 1 tenth = 1 tenth
(4) 1 × 2 ones = 2 ones

3. 3 × 2.5 = 7.5 liters of water in 3 buckets [only 2 regions of blocks needed]



Regions
(1) 3 × 5 tenths = 15 tenths
(2) 3 × 2 ones = 6 ones

4. 2.4 × 3.5 = 8.4 meters of fabric used in all



Regions
(1) 4 × 5 hundredths = 20 hundredths
(2) 4 × 3 tenths = 12 tenths
(3) 2 × 5 tenths = 10 tenths
(4) 2 × 3 ones = 6 ones

WORKSHEET 1–2b

Name ______________________

Drawing Decimal Products for Word Problems

Date ______________________

Draw rectangles on each L-frame to find the product for each exercise. Below the frame, write a number sentence that shows the product. Follow your teacher's instructions to finish each exercise.

1. Luigi has 2.3 gallons of spaghetti sauce. He will need 1.4 times that amount to serve with spaghetti at a dinner. How many gallons of sauce will he need for the dinner?

2. Marsha's motorcycle tank holds 2.1 gallons of gas. If her weekend trip will use 1.3 tanks of the gas, how many gallons of gas will be used for the trip?

WORKSHEET 1–2b Continued

Name ____________________

Date ____________________

3. A bucket holds 2.5 liters of water. How many liters of water will 3 buckets hold?

4. There are 3.5 meters of fabric on 1 bolt. Lily needs 2.4 bolts of fabric to make some curtains. How many meters of fabric will be used in all?

Activity 3: Independent Practice

Materials

Worksheet 1–2c
Red pencil and regular pencil

Procedure

Give each student a copy of Worksheet 1–2c and a red pencil. On the worksheet, students will work several exercises without drawing on L-frames. To check for understanding initially, have them reverse the labeling process used in Activity 2: they should draw the arrows first in red pencil, then use the arrow information to identify and record the region equations needed to find the partial products of the vertical algorithm.

For the example of a one-digit multiplier in Exercise 1, to work 6×3.4, arrow 1 will connect the 6 to the 4, which indicates the partial product 6×4 tenths = 24 tenths found in region 1. Arrow 1 represents ones multiplied by tenths, or complete copies of tenths. Arrow 2 will connect the 6 to the 3, indicating the partial product 6×3 ones = 18 ones. Applying the place value pattern found in Activity 2, ones are used for region 2 since ones are the next higher place value to the left of the tenths in region 1. When the 24 tenths and 18 ones are recorded and combined, the final product will be 20.4.

For the example of a two-digit multiplier in Exercise 2, when working 0.17×3.2, arrow 1 will connect the 7 to the 2, which indicates the partial product 7×2 thousandths = 14 thousandths. The blocks in region 1 will be thousandths because we are finding a hundredth of a tenth, which will be a thousandth of a whole. Arrow 2 will connect the 7 to the 3, indicating the partial product 7×3 hundredths = 21 hundredths, the next higher place value from thousandths. Arrow 3 will connect the 1 to the 2 for 1×2 hundredths = 2 hundredths. In this example, arrow 3 represents the group *above* the thousandths in region 1 of the L-frame, so region 3 must be hundredths. Finally arrow 4 will connect the 1 to the 3 for 1×3 tenths = 3 tenths. Region 4 contains tenths since region 3 to the right contains hundredths. When the partial products (14 thousandths, 21 hundredths, 2 hundredths, and 3 tenths) are recorded and combined, the final product will be 0.544.

The remaining exercises are word problems to be worked using the vertical notation of the algorithm that was developed in Activity 2. Below each word problem, students should record an equation that states the product found. Encourage students to find the partial products mentally, but allow them to continue drawing arrows or writing equations for the partial products to the right of the vertical notation if they find that helpful. After they become proficient in finding the partial products, they will no longer need to draw the arrows or record the region equations for the partial products.

Answer Key for Worksheet 1–2c

1. 20.4 (arrows are discussed in the text)

2. 0.544 (arrows are discussed in the text)

Be sure to keep the multiplier as the first factor in each equation below.

3. 6×2.8 pounds = 16.8 pounds of cheese

4. 13×2.5 kilograms = 32.5 kilograms of birdseed

5. $9 \times \$1.45 = \$13.05$ total for gas

6. $2.8 \times 2.8 = 7.84$ square meters, closest to 8 square meters

Possible Testing Errors That May Occur for This Objective

- The correct algorithm for multiplication is applied, but an incorrect multiplication fact is used when finding one of the partial products. For example, in 6×3.4, 6×4 tenths is recorded as the partial product 27 tenths instead of 24 tenths.
- All partial products are right-justified. For example, in 2.3×4.5, when finding the partial product for 2×5, students record the amount as 10 hundredths instead of 10 tenths.
- The two original factors in the problem are added instead of multiplied. For example, 1.3×2.4 is incorrectly computed as $1.3 + 2.4$.

WORKSHEET 1–2c — Name ____________

Finding Products with Decimals — Date ____________

To work Exercises 1 and 2, draw arrows with a red pencil. Then write equations to the right for the partial products in order to find the partial products and their sum for each exercise.

1. $\begin{array}{r} 3\,.\,4 \\ \times\quad 6 \\ \hline \end{array}$

2. $\begin{array}{r} 3\,.\,2 \\ \times 0\,.\,1\,7 \\ \hline \end{array}$

Solve Exercises 3 through 6 by using the multiplication algorithm. Show your steps on the back of the worksheet. Below each word problem, write a number sentence that shows the answer.

3. There are 6 display trays at the deli counter. Each tray holds 2.8 pounds of cheese. How many pounds of cheese total are on display at the deli counter?

4. Luis has 13 bags of birdseed for his bird feeder. Each bag weighs 2.5 kilograms. What is the weight of all 13 bags together?

5. Kandi bought 9 gallons of gasoline for her car at $1.45 per gallon. How much did she pay for the gasoline?

6. The area of a square is 8 square meters. Which best represents the length of a side of the square: 2.8 meters or 3.1 meters?

Objective 3

Divide decimals to solve word problems.

The division algorithm for decimal numbers needs to be developed carefully for students. The estimation of quotient digits and the backward trading or regrouping between place values are quite difficult for them. The activities described next present a method for introducing two- and three-digit dividends (ones, tenths, hundredths) with one- and two-digit divisors (ones, tenths). The divisor will represent the set being removed or copied from the dividend several times. It will be assumed here that the multiplication algorithm of decimals has already been mastered.

Activity 1: Manipulative Stage

Materials

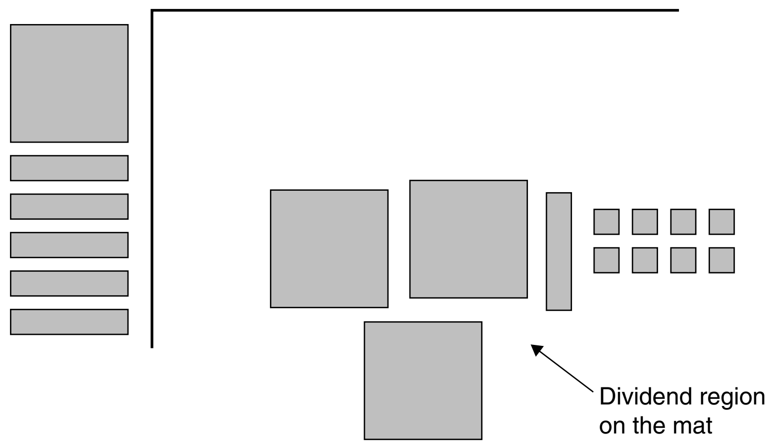
Set of base 10 blocks per pair of students (6 flats, 40 rods, 40 small cubes)
Building Mat 1–3a
Worksheet 1–3a
Regular pencil

Procedure

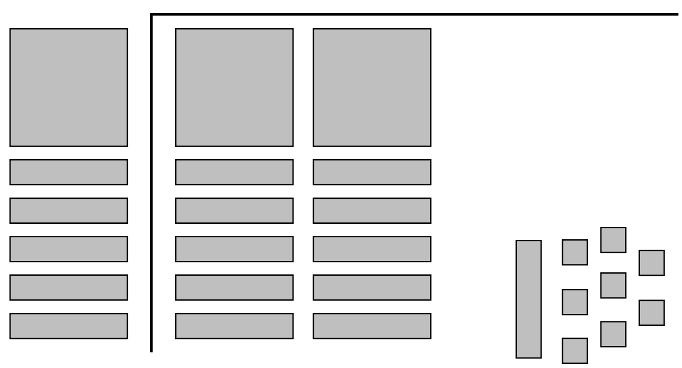
1. Give each pair of students a set of base 10 blocks (6 flats, 40 rods, 40 small cubes) and Building Mat 1–3a. The building mat should be positioned so that the short bar is vertical and at the left side of the mat and the long bar is horizontal and at the top of the mat.
2. Give each student a copy of Worksheet 1–3a.
3. Have students model each worksheet exercise with their blocks. Then they should record below the exercise a word sentence that shows the results found on the building mat.
4. Students should look at the blocks in each row of the dividend region of the building mat in order to determine the quotient. Always have students use proper place value language when verbally describing their steps to the class.
5. Discuss Exercise 1 on Worksheet 1–3a in detail with the class before allowing partners to work the other exercises on their own.

Consider Exercise 1 on Worksheet 1–3a: "Marge has 3.18 ounces of fudge in the store display case. A customer wants several bags of the fudge with 1.5 ounces in each bag. How many complete bags will Marge be able to prepare for the customer?"

Have students place 1 flat and 5 rods along the left side of the vertical bar of the L-frame for the divisor 1.5. The flat should be placed closest to the corner of the L-frame and the rods placed toward the end of the vertical bar. Then place 3 flats, 1 rod, and 8 small cubes below the horizontal bar of the frame near the bottom edge of the mat for the dividend 3.18 (see the illustration for initial block placement). The interior of the L-frame will contain the dividend each time. Eventually the quotient will be placed above the horizontal bar of the L-frame.

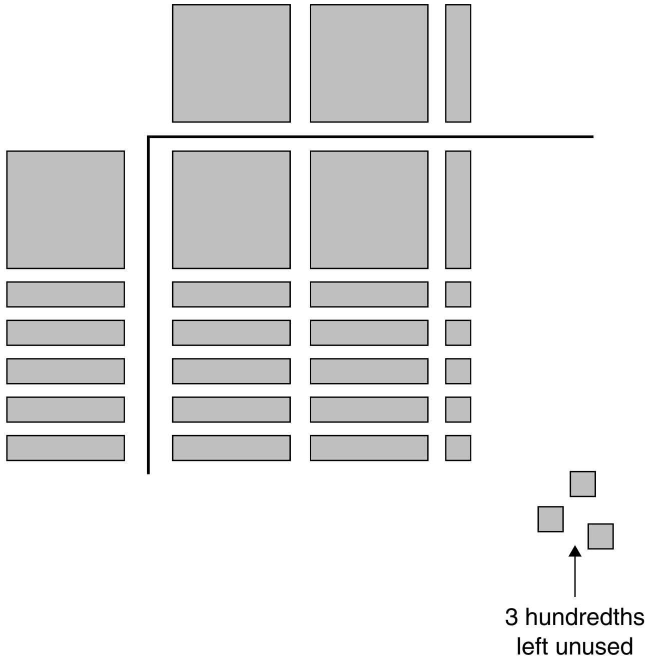


The divisor represents the set of blocks to be made or copied, using the blocks from the dividend. Students should try to make complete copies of the entire divisor set first. Two complete copies of the one (flat) and the five tenths (rods) can be made inside the L-frame, but a one will need to be traded for 10 tenths first in order to have enough tenths for the copies. After the two complete copies are made, 1 tenth and the 8 hundredths will remain in the dividend but not yet used in the new arrangement.



Students should now try to make a tenth of a copy of each block in the divisor. A tenth (rod) represents 1-tenth of the one (flat) in the divisor, and a hundredth (cube) represents 1-tenth of each tenth (rod) in the divisor. Hence, the remaining tenth should be placed beside the ones in the top row of the arrangement in the dividend, and each hundredth should be placed in a row of tenths in the dividend. So 1 tenth and 5 hundredths are 1-tenth of the divisor, which is 1 one and 5 tenths. Three hundredths will remain unused from the total dividend.

Now that all possible complete and partial (tenth) copies have been made, the quotient may be shown above the top bar of the L-frame. A flat or one should be placed above each column of blocks in the dividend that makes a complete copy of the divisor, and a rod or tenth should be placed above the column of blocks that shows 1-tenth of a copy of the divisor. Thus, the final quotient should have 2 ones and 1 tenth, or 2.1. Here is the final block arrangement on Building Mat 1–3a that represents 2.1 bags of 1.5 ounces each made from a total of 3.18 ounces with 0.03 ounce left unused:



Below Exercise 1 on Worksheet 1–3a, students should record a word sentence about their result, such as the following: "3.18 ounces of fudge will make 2.1 bags of 1.5 ounces per bag with 0.03 ounce left unused. So 2 complete bags can actually be made for the customer."

Answer Key for Worksheet 1–3a

1. 3.18 ounces of fudge will make 2.1 bags of 1.5 ounces per bag with 0.03 ounce left unused. So 2 complete bags can actually be made for the customer.
2. 3.0 pounds of salmon will make 1.5 packages at 2 pounds per package.
3. $1.15 will buy 2.3 pounds of carrots at $0.50 per pound.
4. 1.2 yards of fabric will make 3 ties, using 0.4 yard per tie.
5. 3.0 ounces of cream will make 1.2 servings of the dessert, using 2.5 ounces per dessert.

Building Mat 1–3a

WORKSHEET 1–3a

Building Decimal Quotients with Base 10 Blocks

Name ____________________

Date ____________________

Build with base 10 blocks on Building Mat 1–3a to solve the word problems provided. Below each exercise, write a word sentence about the result found for that word problem.

1. Marge has 3.18 ounces of fudge in the store display case. A customer wants several bags of the fudge with 1.5 ounces in each bag. How many complete bags will Marge be able to prepare for the customer?

2. The fishmonger has 3.0 pounds of salmon to put in individual packages. He wants to cut and wrap 2 pounds of salmon in each package. How many packages will he be able to make, including a partial package?

3. Sally has $1.15 and wants to buy carrots that sell for $0.50 per pound. How many pounds will Sally be able to buy if she spends all her money?

4. George, a tailor, has 1.2 yards of silk fabric. He needs 0.4 yard to make a silk tie. How many ties can he make with the fabric?

5. The chef has 3.0 ounces of cream. She needs 2.5 ounces of cream to make a single serving of a dessert. How many servings, including tenths of a serving, can be made?

Activity 2: Pictorial Stage

Materials

Worksheet 1–3b
Red pencil and regular pencil

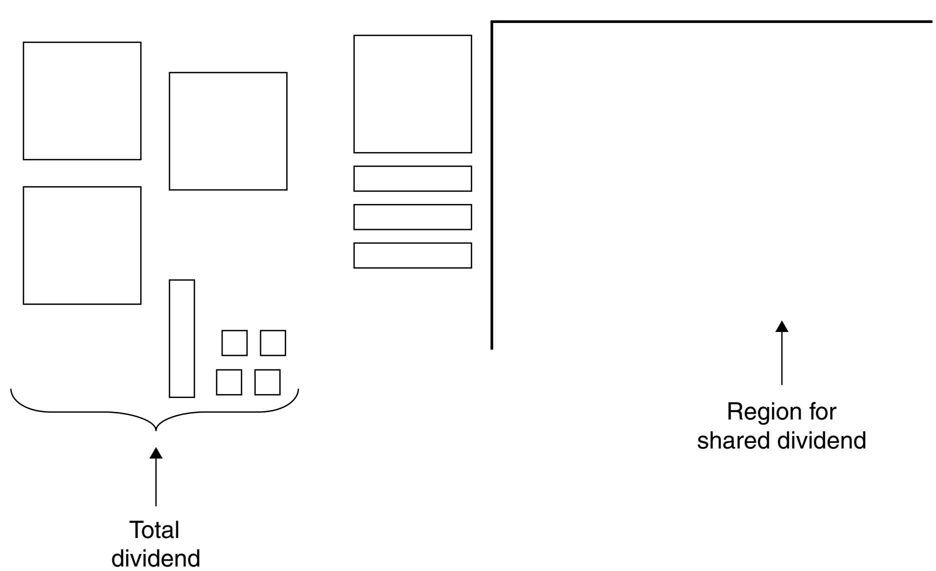
Procedure

1. Give each student a red pencil and a copy of Worksheet 1–3b.
2. Students will draw diagrams to find quotients for word problems. The drawings should look just like the finished mat in the example discussed for Activity 1.
3. After drawing the diagram for each exercise on Worksheet 1–3b, students will write a number sentence or equation below the diagram to record the results found.
4. Students will label the parts of the shared dividend and transfer the labels to the box format for the division algorithm. For each exercise, the box format will be recorded to the right of the diagram.
5. Students will look for patterns among the digits of the divisor and quotient that will help them identify the partial products involved in the division process.
6. Guide students through Exercise 1 before allowing them to proceed to others independently.

Exercise 1 on Worksheet 1–3b provides this example: "Pete has 3.14 pounds of bananas. He wants to bundle them into bags of 1.3 pounds each for his fresh produce display. How many bags, including tenths of a bag, will he be able to make?"

Drawing the Dividend Region

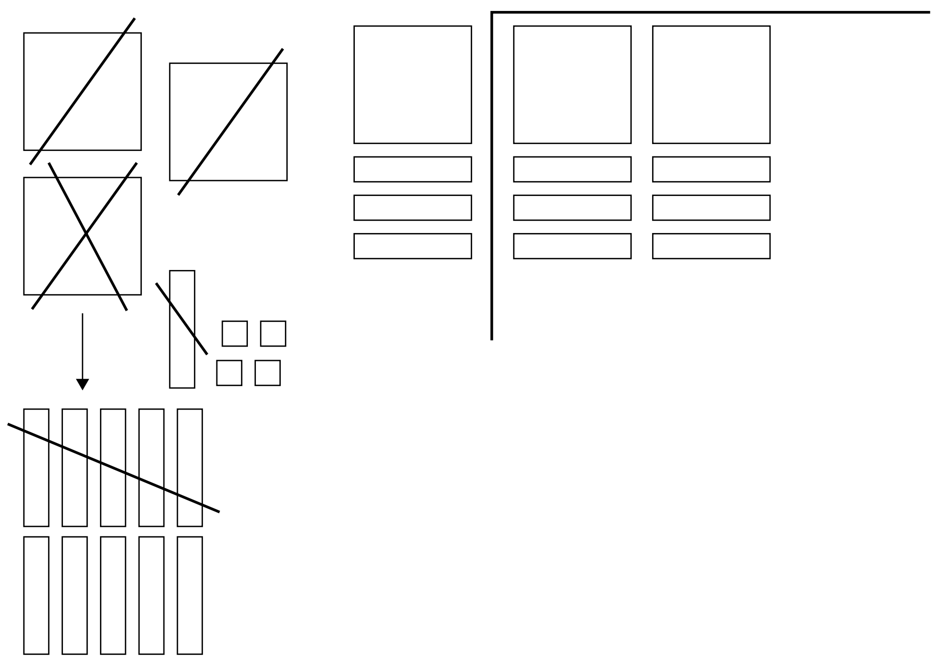
Following the copying method used in Activity 1 and drawing large squares for ones and long and short rectangles to represent the tenths and hundredths, respectively, here is the initial frame for the problem in Exercise 1:



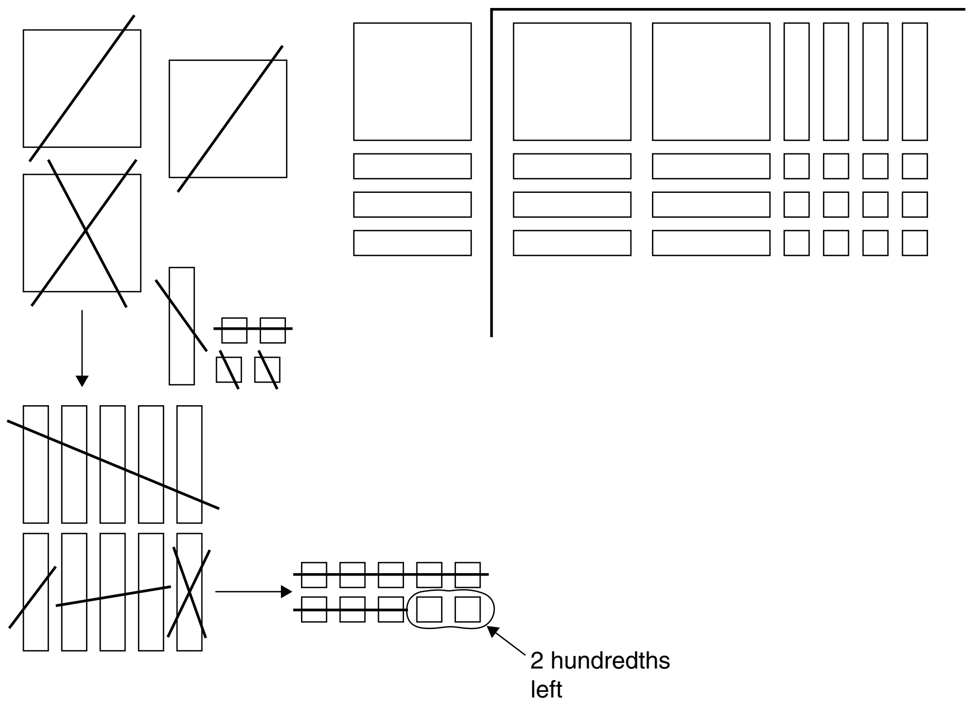
Note that the dividend shapes are drawn to the left of the L-frame this time instead of below the L-frame, as was done in Activity 1. This minimizes the need to erase. The large square and three long rectangles drawn immediately to the left of the vertical bar of the L-frame represent the divisor, 1.3, or the set to be copied.

A row of shapes must eventually be drawn under the L-frame to correspond to each of the shapes in the divisor. Shapes in the dividend at the left are marked out (/) in regular pencil as they are redrawn under the L-frame. Trades are shown by marking out (×) shapes in red pencil and then drawing new, smaller shapes in regular pencil.

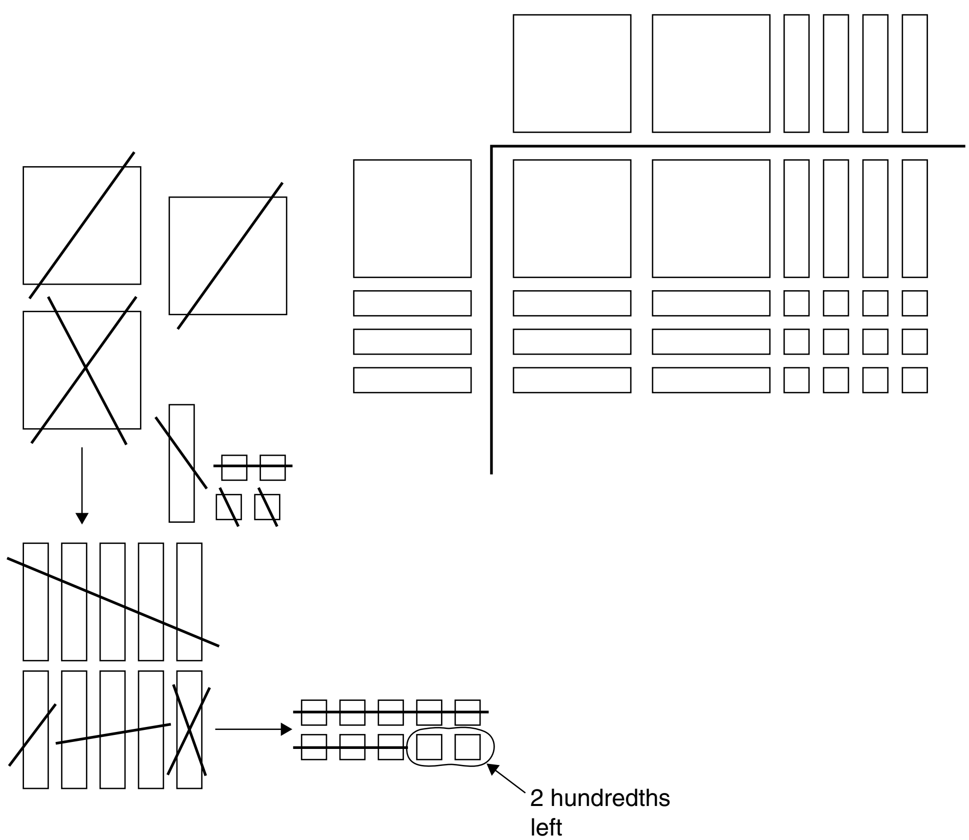
Students must copy the divisor under the L-frame. A complete copy cannot be made at first because there are not enough tenths available. So a one (large square) should be marked out in red pencil and traded for 10 new tenths (long rectangles) in the dividend. A complete copy (as a column) of 1 one and 3 tenths can then be drawn twice, so 2 ones and 6 tenths should be marked out in the dividend shapes at the left to show the transfer to the L-frame.



Since another complete copy of the divisor cannot be made, students should try to make partial copies. To do this, they need to compare the largest shape (long rectangle) remaining in the dividend to the largest shape (large square) in the divisor. Since the long rectangle represents 1 tenth of the large square, this indicates that a tenth of a copy of the divisor can probably be made. A tenth of a copy requires 1 tenth (long rectangle) and 3 hundredths (small squares), which equal a tenth of 1 one and 3 tenths, respectively. One partial copy can be drawn immediately, but a tenth in the dividend must be traded for 10 hundredths first before another three partial copies can be drawn under the L-frame. Two hundredths will remain in the dividend unused. These should be circled in the dividend and labeled.



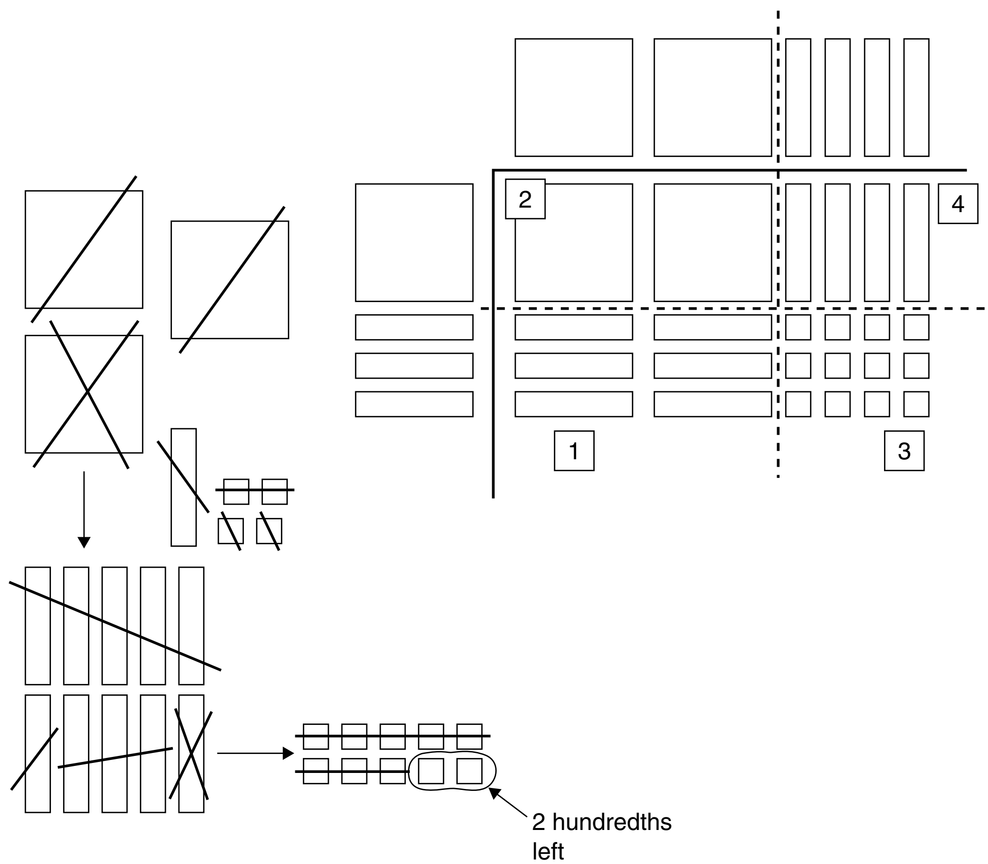
Now that the complete and partial copies of the divisor have been drawn under the L-frame, the quotient needs to be shown. A large square (one) should be drawn above the top bar of the L-frame over each complete copy, and a long rectangle (tenth) should be drawn over each tenth of a copy. The final quotient will contain 2 ones and 4 tenths, representing 2.4 copies of the divisor. This indicates that 2 complete bags and 0.4 of another bag of bananas could be made, excluding 0.02 of a pound of the original bananas. Here is the finished diagram for Exercise 1 on Worksheet 1–3b:



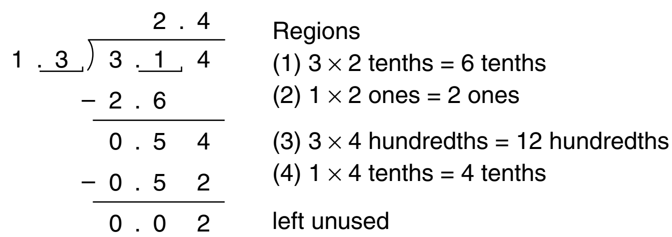
After all students have finished the drawing, they should write an equation below the frame to show the results: "3.14 ÷ 1.3 = 2.4 bags of bananas at 1.3 pounds each with 0.02 of a pound left unbagged." Draw frames for the other exercises on Worksheet 1–3b before continuing to the next step of this activity.

Labeling Parts of the Dividend Region

After drawing frames for all exercises on Worksheet 1–3b, return to the frame for Exercise 1. Label and discuss the four regions of shapes below the L-frame. Have students draw a vertical red bar to separate the complete copies from the partial copies. They should also draw a horizontal red bar between the upper level and lower level of shapes below the L-frame. Label the lower left region of tenths as region 1, the upper left region of ones as region 2, the lower right region of hundredths as region 3, and the upper right region of tenths as region 4. (There may not be four regions produced in every exercise, but the numbering always begins with the left or lower left region.)



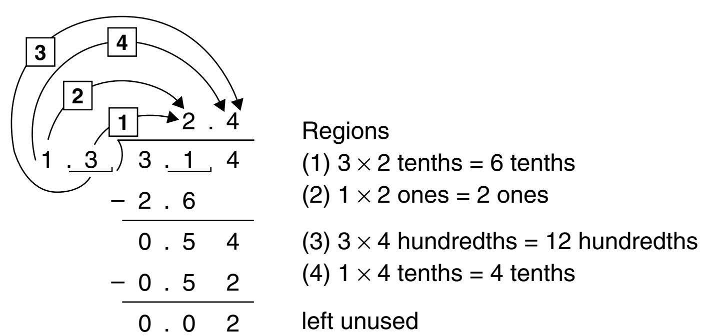
The arrangement of the shapes within each part under the L-frame of the example should be described in the following way: "Region 1 has 3 rows of 2 tenths, or 6 tenths; region 2 has 1 row of 2 ones, or 2 ones; region 3 has 3 rows of 4 hundredths, or 12 hundredths; and region 4 has 1 row of 4 tenths, or 4 tenths." Students should transfer these descriptions onto the box format to the right of the completed diagram of Exercise 1 as shown:



Because the 3 tenths in the divisor, along with the 1 one, must be copied in order to have a complete copy, the tenths in the divisor and the tenths in the dividend should be underlined in red pencil. This will indicate where the complete copies begin in the quotient and where the decimal point should be placed. Notice that regions directly above each other under the L-frame are recorded as one combined subtraction step in the box format. This follows the practice of our standard algorithm for division. If necessary for some students, each region might be subtracted from the dividend separately. Also note that the decimal point in the dividend continues down through the partial products being subtracted. Now repeat the labeling and transfer process for the other exercises on Worksheet 1–3b. Exercises 2 and 4 will involve only partial copies, and Exercise 4 will require a hundredth of a copy, as well as a tenth of a copy.

Pattern Search

After all finished frames have been transferred to the box format, return to Exercise 1. Ask students to compare the factors that were used to describe the shape arrangements in the four regions with the digits in the divisor and quotient (1.3 and 2.4) in the example. All digits in these two numbers are unique, so the comparison should be easy for the students. They should notice that the factors in 3 × 2, 1 × 2, 3 × 4, and 1 × 4 use the same digits as those used in the 1.3 and 2.4. Now have students draw arrows on the divisor and quotient to show the connection between these groups of numbers. The arrows should be drawn and labeled for the regions they represent.



Have students examine all the other exercises on Worksheet 1–3b that they have worked at this stage of development, in order to see if their digits have the same relationship as shown in the example of Exercise 1. The pattern should hold for all the problems. Thus, students should draw and label arrows on the rest of the exercises as they have done for Exercise 1.

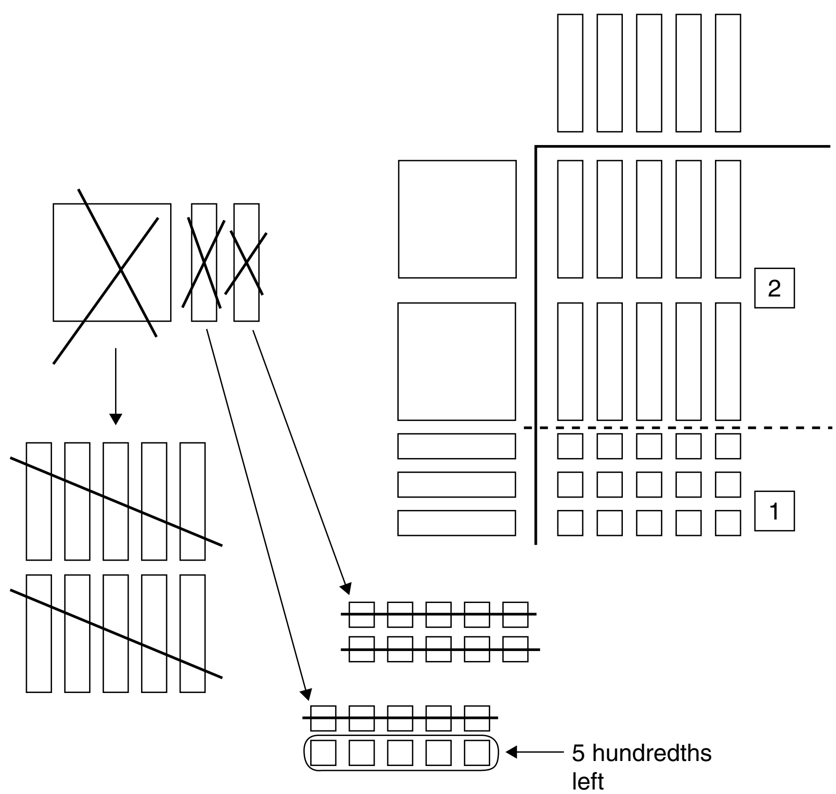
At this point, discuss how division is different from the other three operations in that students must *estimate* the first digit or left-most digit of the quotient, then compare the product of that digit and the divisor against the first left-most digits within the dividend. As in the case of Exercise 1, students have to look at the 3 ones and 1 tenth in the dividend to decide what factor is needed in the quotient to pair with the 1 one and 3 tenths in the divisor. The choice, 3, for the quotient digit would be too high since the arrow pattern would yield the product, 3 ones and 9 tenths, which is more than the 3 ones and 1 tenth found in the dividend. So the better estimate for the quotient's first digit would be 2.

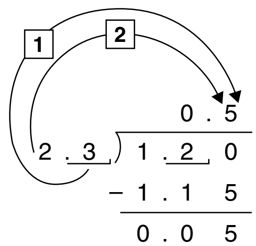
The traditional division algorithm is quite complex. Students need to go slowly through the stages described here so they will understand the process clearly. When the division algorithm is taught for the first time, this entire development usually requires several class periods in order for most students to comprehend all the steps well.

Answer Key for Worksheet 1–3b

1. 3.14 ÷ 1.3 = 2.4 bags of bananas at 1.3 pounds each with 0.02 of a pound left unbagged. [The diagram and box format are shown in the text.]

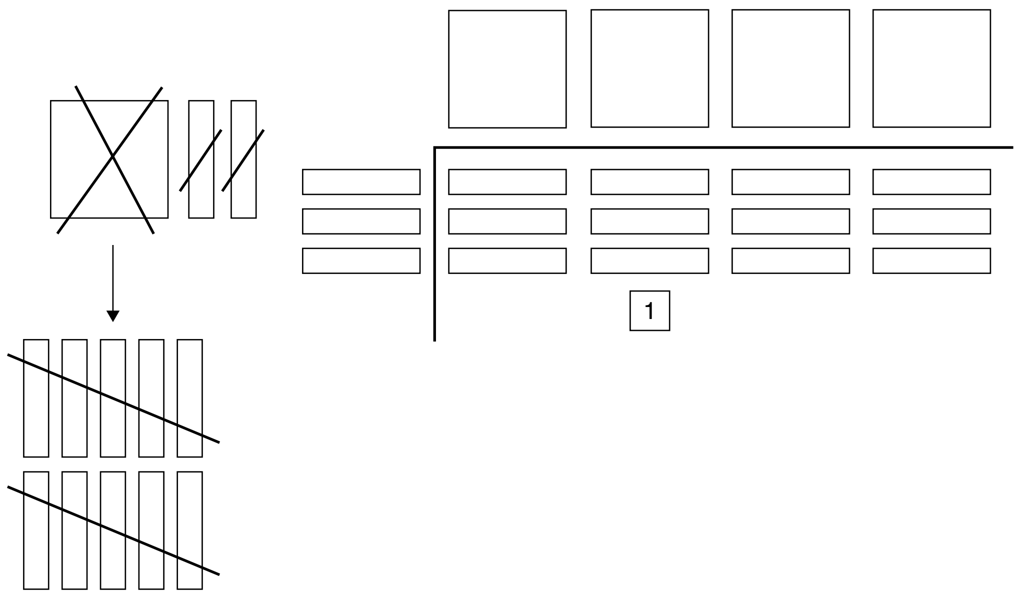
2. 1.2 ÷ 2.3 = 0.5 bag of birdseed at 2.3 pounds per bag with 0.05 of a pound not included. [There were 0 complete copies made.]

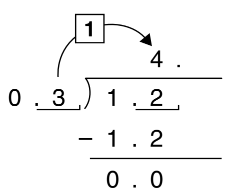




Regions
(1) 3 × 5 hundredths = 15 hundredths
(2) 2 × 5 tenths = 10 tenths
left unused

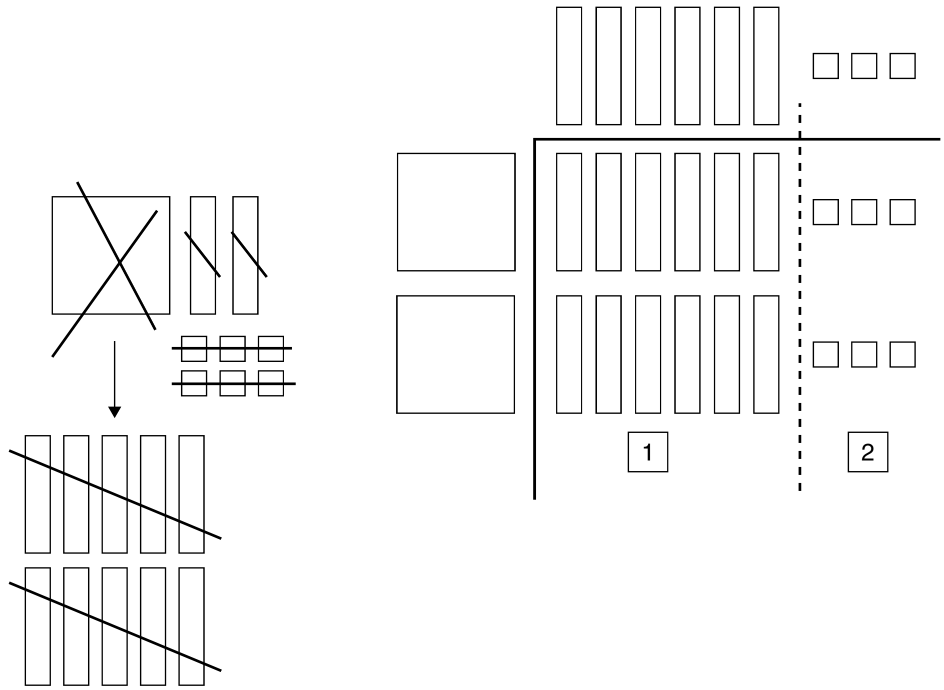
3. 1.2 ÷ 0.3 = 4 pencils bought at $0.30 per pencil. [Since no pennies were involved, no hundredths were drawn in the diagram.]

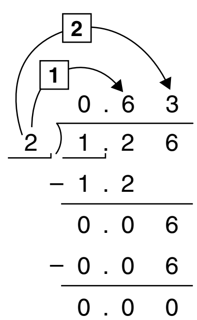




Region
(1) 3 × 4 tenths = 12 tenths

4. 1.26 ÷ 2 = 0.63 of a package of hamburger at 2 pounds per package.





Regions
(1) 2 × 6 tenths = 12 tenths

(2) 2 × 3 hundredths = 6 hundredths
left unused

WORKSHEET 1–3b
Drawing Decimal Quotients

Name ______________________
Date ______________________

Draw rectangles on each L-frame to find the quotient for each exercise. Below the frame, write a number sentence that shows the results. Follow your teacher's instructions to finish each exercise.

1. Pete has 3.14 pounds of bananas. He wants to bundle them into bags of 1.3 pounds each for his fresh produce display. How many bags, including tenths of a bag, will he be able to make?

2. Juan has only 1.2 pounds of birdseed left. The original bag contained 2.3 pounds. What fractional part of the original bag does he have left?

WORKSHEET 1–3b Continued

Name ____________________

Date ____________________

3. Lucy has \$1.20 and wants to buy pencils that sell for \$0.30 per pencil (including tax). How many pencils will Lucy be able to buy if she spends all her money?

4. The butcher wants to make a 2-pound package of hamburger. She only has 1.26 pounds of hamburger. How much of a package will she be able to make?

Activity 3: Independent Practice

Materials

Worksheet 1–3c
Red pencil and regular pencil

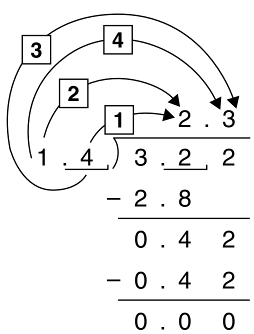
Procedure

Give each student a red pencil and a copy of Worksheet 1–3c. Students will work several problems without drawing on frames. Exercise 1 will show the quotient already written over the dividend but no partial products recorded below the dividend. Students should first draw and number the arrows. Then, using those arrows, they should write the region factor equations to the right of the problem and write the region products under the dividend to subtract. Exercises 2 through 4 are word problems to be worked with the box format. Encourage students to find the region factor equations mentally, but if necessary, allow them to continue drawing arrows and writing the equations beside the division box.

In all exercises, have students underline in red pencil the right-most digit in the divisor, as well as the left-most digits in the dividend that represent the first amount to be used for copies of the divisor. For example, in 2.19 ÷ 0.7, 2.1 and 0.7 should be underlined in red to show that 2 ones and 1 tenth in the dividend must first be used (through trading) to make copies of the 7 tenths in the divisor. Since three complete copies can be made, the left-most digit in the quotient should be marked as the ones place.

Answer Key for Worksheet 1–3c

1.



Regions
(1) 4 × 2 tenths = 8 tenths
(2) 1 × 2 ones = 2 ones

(3) 4 × 3 hundredths = 12 hundredths
(4) 1 × 3 tenths = 3 tenths

left unused

2. 5.7 ÷ 1.5 = 3.8 packages of cheese at 1.5 pounds per package

3. 3.2 ÷ 8 = 0.4 of a carton of yogurt left

4. \$32.10 ÷ \$5.35 = 6 CDs at \$5.35 each

Possible Testing Errors That May Occur for This Objective

- Students do not apply the division or separation/copying process described in the story situation. Rather, they incorrectly choose multiplication to solve the problem.
- The dividend and divisor numbers are reversed in the box format. For example, instead of finding 4.5 ÷ 6.1, students will compute 6.1 ÷ 4.5, using the lesser number as the divisor.

- An incorrect multiplication fact is used to find a partial product during the division process.
- After subtracting the first partial product found and getting a remainder, students drop that initial remainder when finding the second partial product. For example, in 2.29 ÷ 7, 1 tenth will remain; this tenth is ignored, and only the 9 hundredths are divided by 7 instead of the 19 hundredths.

WORKSHEET 1–3c Name ______________

Dividing with Decimal Numbers Date ______________

In Exercise 1, draw arrows with a red pencil. Then write factor equations to the right of the box in order to find the partial products to subtract during division. Be sure to number the arrows and the equations correctly.

1. $$1.4\overline{)3.22}\quad (2.3)$$

Solve Exercises 2 through 4 by using the box method of the division algorithm. Show your steps on the back of the worksheet. Below each word problem, write a number sentence that shows the answer.

2. A block of cheese weighs 5.7 pounds. How many packages of sliced cheese can be made from this block if each package is to weigh 1.5 pounds?

3. If Rachel has 3.2 ounces left from an 8-ounce carton of yogurt, what fractional part of the original carton does she have left?

4. Bert bought some CDs for a total cost of $32.10. If each CD sells for $5.35, how many CDs did Bert buy?

Objective 4

Estimate solutions to multistepped word problems by rounding with decimals.

When using estimation, the general principle is to have students round numbers to levels that will allow them to mentally apply the various facts of the four operations and not have to regroup. Rounding each number to its highest place value is usually the best approach. Any rounding that causes a number to round to zero should be avoided unless the context of the situation allows it. In the following activities, students will be required to set up the series of equations needed to solve each multistepped word problem or to find a single equation that combines all the steps together. The original numbers in the equations will be rounded so that students may mentally estimate the answer to each computation.

Activity 1: Manipulative Stage

Materials

Building Mat 1–4a for each pair of students
50 small counters for each pair of students (same color or same style)
Worksheet 1–4a
Regular pencil

Procedure

1. Provide each pair of students with a building mat, counters, and two copies of Worksheet 1–4a, which involves one- to four-digit decimal numbers or whole numbers in two-step problems.

2. The numbers needed for each problem should be shown on the mat and then rounded. The rows on the mat are only for rounding, not for computation. For convenience, if two numbers need to be added or subtracted, they should be shown on the top two rows of the mat. If a number will serve as a multiplier or a divisor, it should be shown on the bottom row.

3. Once the numbers are rounded, students will perform the required computations mentally, if possible, or with the appropriate written algorithms, if preferred.

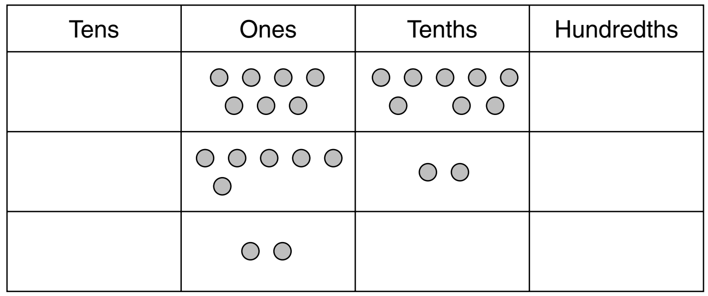
4. Remind students that they may need to use "backward thinking" here; that is, they may have to think of the second or final step first before they can decide what the first step should be.

5. The two number sentences, which contain the rounded numbers and were used to solve a word problem, should be recorded below that exercise on Worksheet 1–4a to show the estimated work. Also have students describe a numerical interval that contains the estimate they have found. (Intervals will vary.)

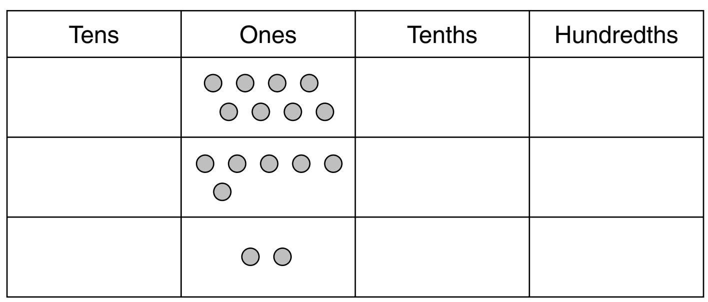
6. Guide students through the first exercise before they proceed to the others.

Consider Exercise 1: "Kelly rode her bicycle for 7.8 kilometers on Friday and for 6.2 kilometers on Saturday. Estimate how many kilometers she averaged per day for the two days."

Have students place counters on the top two rows of Building Mat 1–4a to represent 7.8 kilometers and 6.2 kilometers. They should also place counters on the bottom row to show the two days involved.



Discuss the idea that the top two numbers need to be added together, then divided by the bottom number in order to find the average kilometers per day. Since the top two numbers have ones as their higher place value, both numbers should be rounded to the nearest one: 7.8 will round up to 8, and 6.2 will round down to 6. This should be shown on the building mat by removing the 8 tenths from the top row but bringing in a new ones counter on that same row. On the second row, only the 2 tenths need to be removed from the mat. Since the 2 in the bottom row will serve as a divisor and is already a 1-digit number in the ones place, it does not need to be rounded. Here is the final mat arrangement:



The rounded numbers are now ready to be computed mentally. It is possible to use facts for each step. The first step will be "8 + 6 = 14 kilometers total for 2 days," and the second step will be "14 ÷ 2 = 7 kilometers estimated per day." Students should record these two equations below Exercise 1 on Worksheet 1–4a. An interval for the estimated solution also needs to be written below the exercise. These will vary. One possibility might be, "Estimate is between 5 and 10 kilometers." Intervals should be reasonable. "Between 0 and 50 kilometers" would not be considered reasonable.

Answer Key for Worksheet 1–4a

1. 8 + 6 = 14 kilometers total for 2 days; 14 ÷ 2 = 7 kilometers estimated per day. Possible interval: Estimate is between 5 and 10 kilometers.

2. $10 + $10 = $20; $30 – $20 = $10 left in savings. Possible interval: Estimate is between $5 and $20.

3. 3 × $2 = $6; $6 + $3 = $9, estimated cost for both cheeses. Possible interval: Estimate is between $8 and $10.

4. 1 + 3 = 4 pounds; $4 ÷ 4 = $1, estimated cost per pound of apples. Possible interval: Estimate is between $0.50 and $1.50.

5. 80 ÷ 4 = 20 miles estimated per day; 20 ÷ 5 = 4 miles estimated per hour. Possible interval: Estimate is between 3 and 6 miles per hour.

Building Mat 1-4a

Tens	Ones	Tenths	Hundredths

WORKSHEET 1–4a

Name ______________________

Estimating with Multistepped Problems

Date ______________________

Use Building Mat 1–4a and counters to round the numbers in each exercise. Compute mentally with the rounded numbers to estimate a solution for the problem, and write the final equations below the exercise. Write an interval that contains the estimated solution.

1. Kelly rode her bicycle for 7.8 kilometers on Friday and for 6.2 kilometers on Saturday. Estimate how many kilometers she averaged per day for the two days.

2. Lanny has $32.75 saved up. He wants to buy a new CD for $12.50 and a cassette tape for $9.95, including tax. Estimate how much money he will have left after the two purchases.

3. Mario bought 2.6 pounds of cheddar cheese that cost $1.80 per pound. He also bought some swiss cheese for $2.59. About how much did he pay for all the cheese?

4. Susan bought 1.4 pounds of gala apples, then another 2.7 pounds of apples. The total cost of the apples was $3.65. About how much per pound did the apples cost?

5. The Johnsons hiked 79.5 miles over 4 days. If they hiked approximately 6 hours per day, estimate their hiking rate in miles per hour.

Activity 2: Pictorial Stage

Materials

Worksheet 1–4b
Worksheet 1–4c
Red pencil and regular pencil

Procedure

1. Give each student a copy of Worksheet 1–4b and a red pencil. The worksheet contains small frames that look like the earlier building mat. Above each frame is a blank writing space for writing the initial numbers needed to solve the problem.

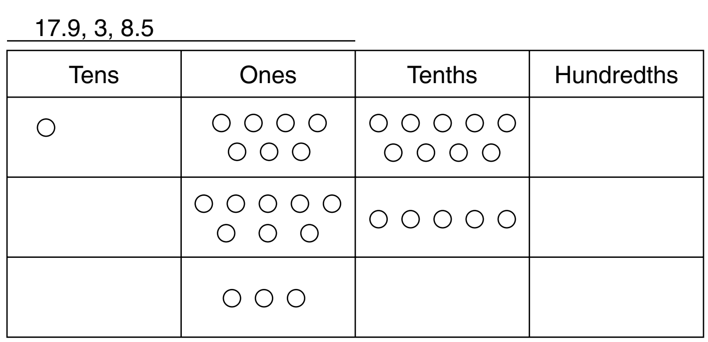
2. Each student will solve problems by completing the frames on her or his own worksheet but might share results with a partner. Instead of placing counters on the mat to show numbers, students will now draw circles in the columns of the frames. If a number must round up to the next place value, students should mark out the extra circles in the columns to the right of the new place value column, and draw a new circle in the selected place value column. If a number must round down to a certain place value, then the extra circles in the columns to the right of that place value will just be marked out in red pencil.

3. Also give each student a copy of Worksheet 1–4c, which contains several word problems involving up to four-digit numbers. Below each word problem, students will record the two equations used to estimate the solution and then write a final equation that combines the two equations.

4. Guide students through Exercise 1 before allowing them to proceed to others independently.

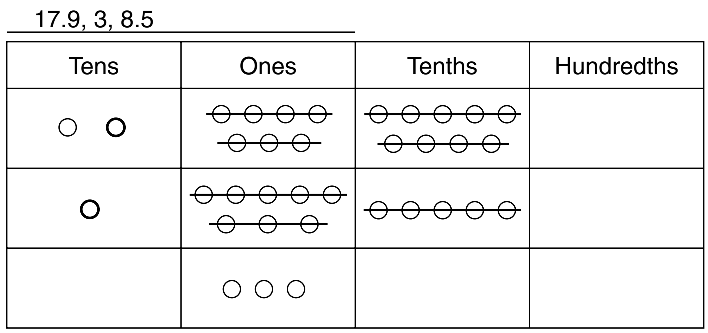
Here is Exercise 1 from Worksheet 1–4c to discuss with students as an example: “George has 17.9 ounces of sunflower seeds in a bag. If he triples this amount, then adds 8.5 more ounces to the bag, about how many ounces will he have in all?”

Have students write the numbers 17.9, 3, and 8.5 in the blank above the first frame on Worksheet 1–4b. These are the numbers needed to solve the problem. Students should draw circles in the appropriate columns to represent the three numbers. Since 3 serves as a multiplier, draw it on the bottom row of the frame. The top two rows may be used for 17.9 and 8.5. Here is how the frame will look initially:



Students should now decide how to round each number so that mental facts might be used to compute an estimate. They should round 17.9 to the nearest ten; this will be 2 tens, or 20, since there are 7 ones. With a red pencil, students should draw a new circle in the tens column and mark out all the circles in the ones and tenths columns.

They might round 8.5 to the nearest one or to the nearest ten. Since it will need to combine with the tens number for 17.9, it is better to round to the nearest ten, which will be 1 ten, or 10, since there are 8 ones. In red pencil, students should draw a new circle in the tens column and mark out all the circles in the ones and tenths columns. Since the number 3 is already a one-digit whole number, it will not be rounded. The final frame appears as follows:



Once students have rounded their numbers, they should write the equations needed to solve the problem. These equations should be recorded below Exercise 1 on Worksheet 1–4c. The equations are as follows: “$3 \times 20 = 60$” and “$60 + 10 = 70$ ounces total of seed estimated in the bag.” Write the factors in the first equation in correct order; 3×20 means 3 of 20. Discuss the facts used: $3 \times 2 = 6$ and $6 + 1 = 7$. Students should also combine the two equations into one equation and record it as well: “$(3 \times 20) + 10 = 70$ ounces estimated.” This is excellent preparation for future studies in algebra.

Answer Key for Worksheet 1–4c

1. $3 \times 20 = 60$; $60 + 10 = 70$ ounces estimated for seed in bag; $(3 \times 20) + 10 =$ 70 ounces estimated

2. \$40 – \$10 = \$30; \$30 + \$20 = \$50 estimated in savings; (\$40 – \$10) + \$20 = \$50 estimated

3. 4 × \$30 = \$120; \$120 + \$60 = \$180, estimate for two jobs; (4 × \$30) + \$60 = \$180 estimated

4. 10 ÷ 2 = 5 packages; 5 × \$3 = \$15, estimated earnings; (10 ÷ 2) × \$3 = \$15 estimated

WORKSHEET 1–4b

Drawing to Estimate with Multistepped Problems

Name ______________________

Date ______________________

Base 10 Drawing Frames

1. ______________________

Tens	Ones	Tenths	Hundredths

2. ______________________

Tens	Ones	Tenths	Hundredths

WORKSHEET 1–4b Continued

Name ______________________

Date ______________________

3. ______________________

Tens	Ones	Tenths	Hundredths

4. ______________________

Tens	Ones	Tenths	Hundredths

WORKSHEET 1–4c

Name ______________________

Date ______________________

Drawing to Estimate with Multistepped Problems

To solve each word problem, draw circles on a frame on Worksheet 1–4b to show each number. Then round as needed in order to use facts when solving. Below the word problem on this worksheet, record the final estimating equations, as well as a combination equation.

1. George has 17.9 ounces of sunflower seeds in a bag. If he triples this amount and then adds 8.5 more ounces to the bag, about how many ounces will he have in all?

2. Chui had $38.25 in her savings account. She took out $11.50 to go to a movie. A week later, she received $15.00 for a birthday gift and put it into her savings account. Estimate the new balance in the savings account.

3. Lana earned $27.50 each week for 4 weeks of baby-sitting. She also earned $63.95 total working at her aunt's candy store that same month. About how much did Lana earn from both jobs?

4. The grocer has 9.8 pounds of hamburger to put into 2-pound packages. If each package sells for $3.45, estimate how much the grocer will earn.

Activity 3: Independent Practice

Materials

Worksheet 1–4d
Regular pencil

Procedure

Students work independently to complete Worksheet 1–4d. When all are finished, discuss the results.

Answer Key for Worksheet 1–4d

1. B
2. C
3. B
4. A
5. D

Possible Testing Errors That May Occur for This Objective

- One or more numbers are rounded incorrectly.
- A fact error is made for one of the operations involved in the problem.
- In a two-step problem, students use only two of the three given numbers; that is, they perform only one of the required steps.
- The wrong operations are applied. For example, if multiplication and subtraction are needed, students might use multiplication and addition. This is generally caused by a misunderstanding of the meanings of the operations.

WORKSHEET 1–4d Name ______________________

Estimating with Multistepped Problems Date ______________________

Show your work on another sheet of paper. Circle the best answer choice for each problem on this worksheet. Be ready to discuss how you solved each problem with your classmates. Remember to try to round numbers so that facts may be used mentally.

1. A bakery had 12.6 pounds of fudge to sell. It sold 5 boxes containing 1.8 pounds each on Monday. Find the best estimate of the number of pounds of fudge remaining to be sold on Tuesday.

   A. 2 B. 3 C. 5 D. 7

2. Mr. Jeffers bought a digital camera on sale plus a flash attachment for $454.64 total. The discount off the original camera price was $194.30, and the flash attachment cost $89.99. Which is the best estimate of the original price of the camera?

   A. $400 B. $500 C. $600 D. $700

3. Amy sold $32.10 in candy, $14.80 in notebook paper, and $8.50 in pencils at the school store. What is the best interval for an estimate of her total sales?

   A. Less than $45

   B. Between $45 and $55

   C. Between $55 and $65

   D. More than $65

4. Eli has 8.25 yards of fabric for making some vests. Each vest requires 1.75 yards. He wants to sell each vest for $28.95. About how much should he expect to earn from selling the vests?

   A. $120 B. $180 C. $240 D. $300

5. Bill earned $18.50 each week for 2.5 weeks at one store, and then he earned $14.65 for one week at another store. Which expression is a reasonable estimate of his total earnings from the two stores?

   A. (2 × $20) + $10

   B. 3 × ($20 + $10)

   C. ($20 ÷ 2) + $10

   D. (3 × $20) + $10

Objective 5

Add fractions or mixed numbers to solve word problems.

Students have great difficulty transferring from their comfortable whole number world to the less familiar fraction world. They need many experiences involving fractional measurements. The use of manipulatives assists students in the adding or combining of different fractional part sizes by showing the need for a common denominator.

Activity 1: Manipulative Stage

Materials

Set of fraction bars per pair of students (Pattern 1–5a)
Building Mat 1–5a per pair of students
Worksheet 1–5a
Regular pencil

Procedure

1. For each set of fraction bars, prepare and cut out five copies of Pattern 1–5a. The whole bar on Building Mat 1–5a should be congruent to the whole bar on Pattern 1–5a. If preferred, each type of fraction might be colored to match the corresponding type in the teacher's set of fraction bars.

2. Give each pair of students a set of fraction bars, one copy of Building Mat 1–5a, and two copies of Worksheet 1–5a.

3. It is assumed for this activity that students have had experience with equivalent fractions. The trading action for finding common denominators will reinforce their understanding of the concept.

4. Students should build with the fraction bars on Building Mat 1–5a to solve each word problem on Worksheet 1–5a. Then they should write a word sentence below the exercise to describe the results.

5. Discuss Exercise 1 on Worksheet 1–5a before allowing students to solve the other exercises independently.

Consider the word problem in Exercise 1 on Worksheet 1–5a: "Carl jogged 1 and 1-half miles before he stopped to rest. Then he jogged another 3-fourths of a mile. How many miles in all did he jog?"

Have students place one whole fraction bar on the top bar of Building Mat 1–5a and then place a half bar on the second bar of the mat. To show the 3-fourths of a mile, they should place 3 of the fourth bars on the third bar of the mat. Here are the initial fraction bars on the building mat:

Ask students if all the fraction bars on the building mat are the same size. Since they are not, students must trade the bars for smaller fraction bars until they find a common bar size. Allow students to explore to find what size is needed. It is not necessary for them to find the largest common bar size (the "least common denominator"). Smaller bar sizes will also work as long as the size is common to all numbers shown on the mat.

A possible trade is to trade or replace the whole bar on the mat with 4 fourths and to trade or replace the half bar with 2 fourths. The third bar on the mat already contains 3 fourths. Once the trading is complete and all fraction bars on the mat are the same size, students should recount all the parts or fraction bars to find their total, which will be 9 fourths of a whole.

Discuss the idea that the total of 9 fourths of a whole can be grouped as 4 fourths, 4 fourths, and 1 fourth. So another way of thinking about the total might be 1 whole, 1 whole, and 1 fourth of a whole, which can be named as 2 and 1-fourth of a whole. Do not exchange each 4 fourths for 1 whole on the mat; only make the trade mentally at this time. The phrase "of a whole" needs to be used regularly so that students realize the significance of the whole to any of its fractional parts. In Exercise 1, the whole corresponds to the mile so that 1-fourth of a whole is equivalent to 1-fourth of a mile.

Because of the algorithm being developed in this objective, the improper fraction name needs to be recorded along with the mixed number name. Thus, students should write the following word sentence below Exercise 1: "Carl jogged 9-fourths of a mile or 2 and 1-fourth miles in all."

Answer Key for Worksheet 1–5a

1. Carl jogged 9-fourths of a mile or 2 and 1-fourth miles in all.
2. Kate ate 17-sixths of an ounce or 2 and 5-sixths ounces of candy.
3. Maria did homework for 19-twelfths of an hour or 1 and 7-twelfths hours.
4. 5-sixths of the whole class ate during lunch A or lunch B.
5. Sam cut lawns for 23-eighths of an hour or 2 and 7-eighths hours.

Building Mat 1-5a. Fraction Addition

WORKSHEET 1–5a

Adding with Fractions and Mixed Numbers

Name ______________________

Date ______________________

Use fraction bars on Building Mat 1–5a to solve the word problems provided. Write a word sentence below each exercise to record the result found.

1. Carl jogged 1 and 1-half miles before he stopped to rest. Then he jogged another 3-fourths of a mile. How many miles in all did he jog?

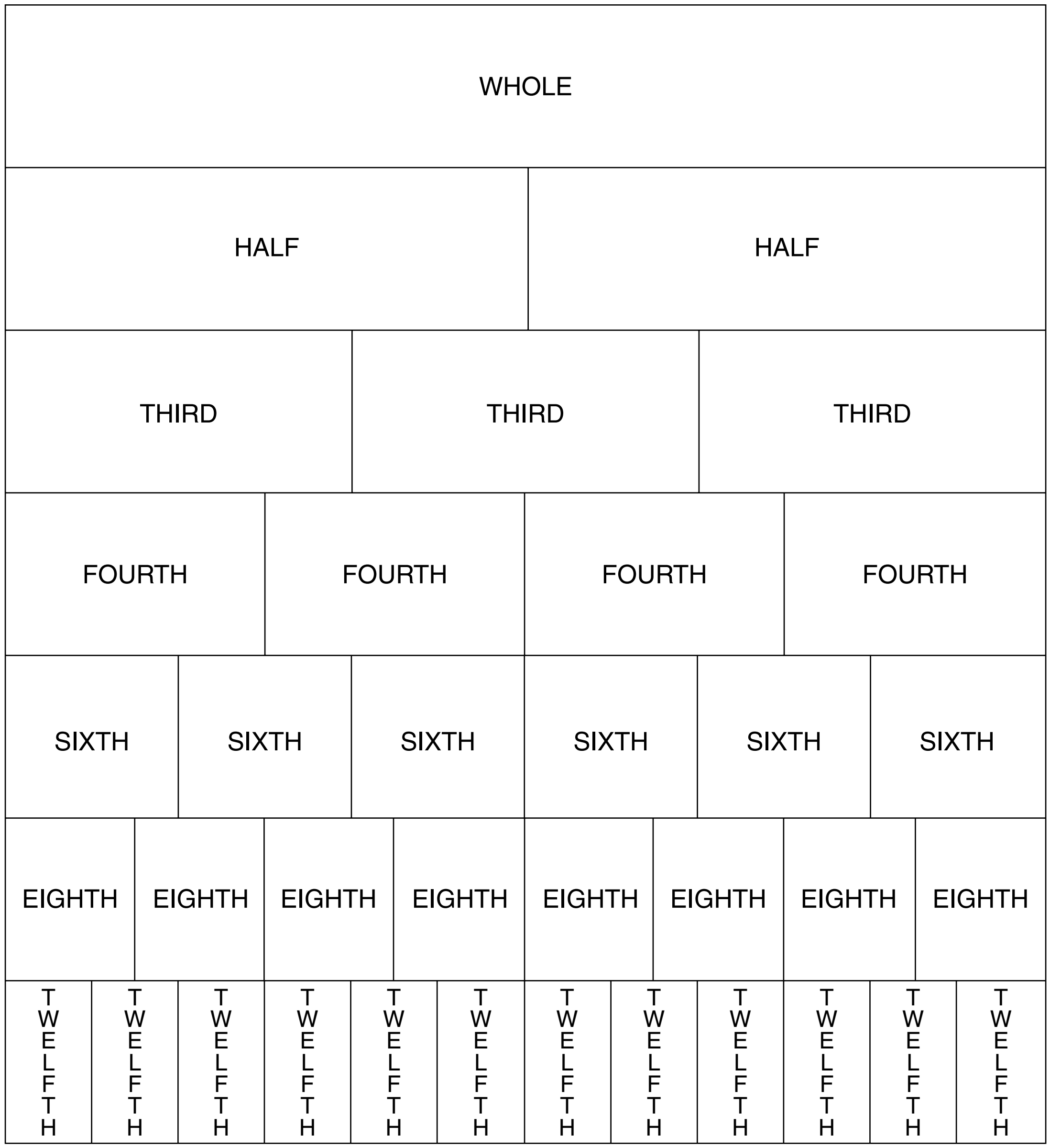
2. Kate ate 1 and 2-thirds ounces of rocky road fudge and 1 and 1-sixth ounces of peanut brittle. How many ounces total did Kate eat of both candies?

3. Maria worked on her math homework for 3-fourths of an hour and on her science homework for 5-sixths of an hour. How much time did she spend on homework?

4. One-third of the class ate during lunch A and 1-half of the class ate during lunch B. The rest ate during lunch C. What fractional part of the whole class ate during lunch A or lunch B?

5. Sam took 1 and 2-fourths hours to cut his lawn and another 1 and 3-eighths hours to cut a neighbor's lawn. How many hours did Sam take to cut both lawns?

Pattern 1–5a. Fraction Bars



Activity 2: Pictorial Stage

Materials

Worksheet 1–5b
Worksheet 1–5c
Red pencil and regular pencil

Procedure

1. Give each student a copy of Worksheet 1–5b, a copy of Worksheet 1–5c, and a red pencil. Now that students are comfortable adding with fraction bars on the building mat, they will work similar word problems with diagrams.

2. Each student should complete the frames on her or his own Worksheet 1–5b, but might share results with a partner.

3. For each exercise, students will subdivide whole bars on a frame to show different fractional amounts of a whole. They will shade the initial amounts, using diagonal stripes for easy recognition.

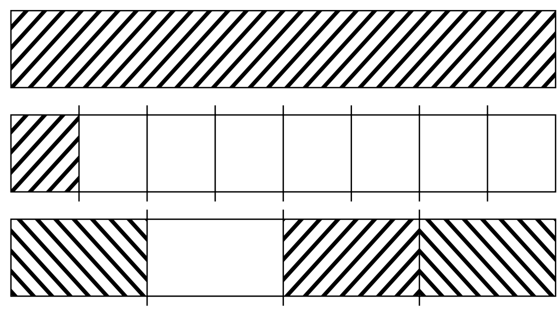
4. Students will then determine how each initial part must be traded in order to have only one common part size appearing on the frame. Trades will be shown by marking new subdivisions with red pencil.

5. The total shaded parts will be counted to find the answer to the word problem. An equation for this sum should be written below the exercise on Worksheet 1–5c. The equation should indicate the trades that were made in order to find the sum.

6. Exercise 1 on Worksheet 1–5c should be discussed with the class in detail before students are allowed to work the additional problems on the worksheet on their own.

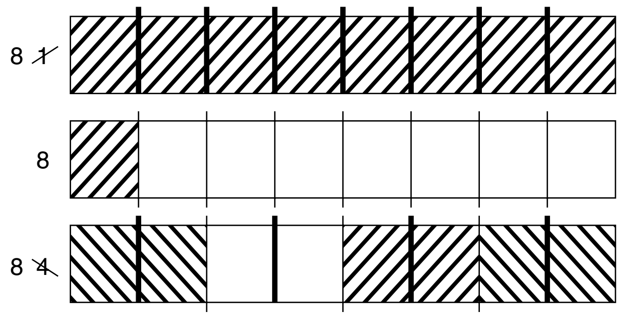
Here is Exercise 1 from Worksheet 1–5c to consider: “Dave bought 1 and 1-eighth pounds of taffy at the fair, and Robin bought 3-fourths of a pound of taffy. How much taffy did they buy in all?”

To show the 1 and 1-eighth pounds, students should shade (use diagonal stripes) with regular pencil the top whole bar on the first frame of Worksheet 1–5b, then subdivide the second whole bar into 8 equal parts and shade one of the 8 parts. To show the 3-fourths of a pound, students should subdivide the third whole bar into four equal parts, then shade three of the four parts. Reverse the direction of the diagonal shading in adjacent parts for easy viewing. The initial shading should appear as follows:



Since a total is needed for all the shaded parts together, students must find a part size common to all three whole bars being used. They no longer have the advantage of actual fraction bars to test, so they must begin to count the total parts marked off on each whole bar in order to know the part size involved. For example, if a whole bar is divided into eight equal parts, the part size will be an eighth. The total number of parts on a bar might be written at the left or right end of the bar for easy reference. Thus, the top bar might have the number 1 written at the left end, the second bar might have 8 written at the left, and the third bar might have 4 written at the left.

Encourage students to think in terms of possible trades. In other words, can each fourth (the largest part size except for the whole bar on top) on the third bar be traded for two new parts, three new parts, and so forth? If each fourth trades for two new parts, there will be eight new parts forming the whole bar, yielding a new part size of an eighth. Since the second bar also has eight equal parts and the top whole bar can easily be changed to eighths, students should now use the red pencil to subdivide the top whole bar into eight equal parts and to subdivide each original part of the third bar into two new equal parts for a total of eight parts on the third bar. The numbers at the ends of the bars may also be changed to show the new total parts per bar. Here is the final diagram:



Since all part sizes are now the same (in this case, eighths), students should count to find the total shaded parts, which is 15 eighths from the three bars. Discuss how this total might be viewed as 8-eighths and 7-eighths of a whole, or as 1 whole and 7-eighths of another whole.

At this stage, students need to record their result with a notation that reflects the trades that were used. The top 1 whole bar was traded for 8 new parts, and the 4 fourths on the third whole bar were each traded for 2 new parts. The second bar remained unchanged. The notation that shows the trades over the entire bar, as well as for the shaded parts, is given below and should also be recorded below Exercise 1 on Worksheet 1–5c:

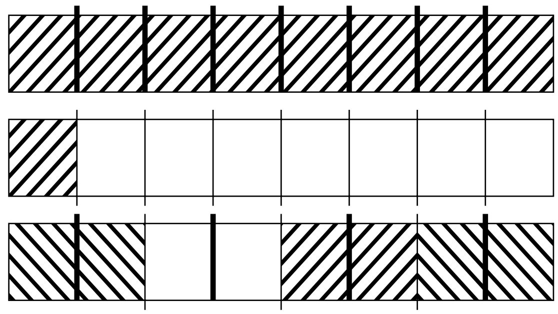
$$1+\frac{1}{8}+\frac{3}{4}=\frac{8}{8}+\frac{1}{8}+\frac{3\times 2}{4\times 2}=\frac{8}{8}+\frac{1}{8}+\frac{6}{8}=\frac{15}{8}\text{ pounds}$$

$$\text{or 1 and }\frac{7}{8}\text{ pounds of taffy}$$

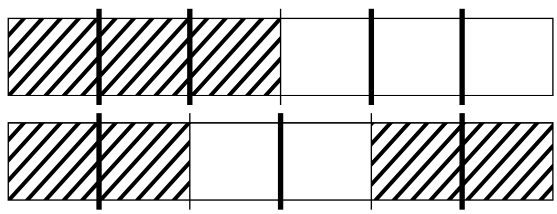
If the first amount had been 2 and $\frac{1}{8}$, the 2 would require 2 whole bars and 2 $\frac{1}{8}$ would be recorded as $1+1+\frac{1}{8}=\frac{8}{8}+\frac{8}{8}+\frac{1}{8}$.

Answer Key for Worksheet 1–5c

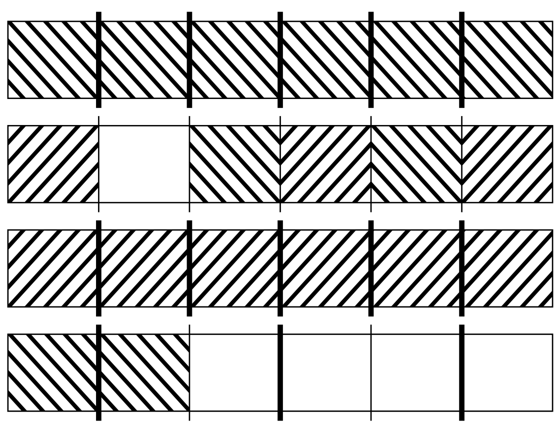
1. $1+\frac{1}{8}+\frac{3}{4}=\frac{8}{8}+\frac{1}{8}+\frac{3\times2}{4\times2}=\frac{8}{8}+\frac{1}{8}+\frac{6}{8}=\frac{15}{8}$ pounds,
or 1 and $\frac{7}{8}$ pounds of taffy



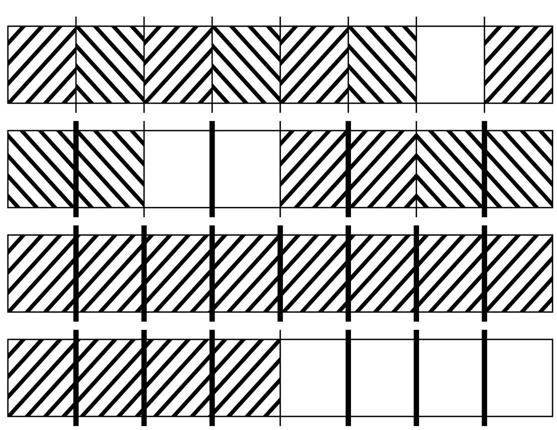
2. $\frac{1}{2}+\frac{2}{3}=\frac{1\times3}{2\times3}+\frac{2\times2}{3\times2}=\frac{3}{6}+\frac{4}{6}=\frac{7}{6}$ hrs., or 1 and $\frac{1}{6}$ hrs.



3. $1+\frac{5}{6}+1+\frac{1}{3}=\frac{6}{6}+\frac{5}{6}+\frac{6}{6}+\frac{1\times2}{3\times2}=\frac{6}{6}+\frac{5}{6}+\frac{6}{6}+\frac{2}{6}=\frac{19}{6}$ lbs., or 3 and $\frac{1}{6}$ lbs.



4. $\frac{7}{8}+\frac{3}{4}+1+\frac{1}{2}=\frac{7}{8}+\frac{3\times2}{4\times2}+\frac{8}{8}+\frac{1\times4}{2\times4}=\frac{7}{8}+\frac{6}{8}+\frac{8}{8}+\frac{4}{8}=\frac{25}{8}$ miles, or 3 and $\frac{1}{8}$ miles



WORKSHEET 1–5b
Drawing Frames

Name ______________________
Date ______________________

1.

2.

WORKSHEET 1–5b Continued

Name ______________________

Date ______________________

3.

4.

WORKSHEET 1–5c

Drawing to Add Fractions and Mixed Numbers

Name ______________________

Date ______________________

Use a red pencil and Worksheet 1–5b (the frames) to solve the word problems provided. Write an equation below each exercise to record the result found. Record any trades made in the equation.

1. Dave bought 1 and 1-eighth pounds of taffy at the fair, and Robin bought 3-fourths of a pound of taffy. How much taffy did they buy in all?

2. Lynda's kite flew in the air for 1-half of an hour, then later flew for another 2-thirds of an hour. What was the total time the kite flew in the air?

3. Juan bought 1 and 5-sixths pounds of birdseed in July. In August, he had to buy another 1 and 1-third pounds of seed. How much birdseed did he buy total for both months?

4. Caron walked 7-eighths of a mile on Monday, 3-fourths of a mile on Tuesday, 1 mile on Wednesday, and 1-half mile on Thursday. How many miles did she walk over the four days?

Activity 3: Independent Practice

Materials

Worksheet 1–5d
Regular pencil

Procedure

Give each student a copy of Worksheet 1–5d. Have students solve each exercise by using the addition algorithm, which was developed as the final notation in Activity 2. Discuss the alternative method that allows whole amounts, especially larger numbers, to be added separately from fractional amounts. For example, the addend 15 and $\frac{1}{2}$ may be viewed several ways: $1 + 1 + \ldots + 1 + \frac{1}{2} = \frac{2}{2} + \frac{2}{2} + \ldots + \frac{2}{2} + \frac{1}{2}$; $\frac{30}{2} + \frac{1}{2}$; or just $15 + \frac{1}{2}$. So in order to add 15 and $\frac{1}{2}$ to 24 and $\frac{3}{4}$, students might choose to add 15 to 24 to get 39 wholes, then add $\frac{1}{2}$ to $\frac{3}{4}$ to get $\frac{5}{4}$, or 1 whole and $\frac{1}{4}$ of a whole. The final answer would be 40 wholes and $\frac{1}{4}$ of a whole.

Encourage students to use the trading language for finding equivalent fractions as needed. The cup-quart conversion will be needed for Exercise 3. When all students have finished the worksheet, have several students share their answers with the rest of the class.

Answer Key for Worksheet 1–5d

1. A
2. C
3. B
4. C
5. D

Possible Testing Errors That May Occur for This Objective

- To add fractions with unlike denominators, students do not find a common denominator. Instead, they find the new numerator by adding the original numerators, and they find the new denominator by adding the original denominators.
- If the sum of several fractions produces an improper fraction (for example, 14-tenths), students will change the improper fraction to an incorrect mixed number. A subtraction fact error may be the cause.
- The computation yields a fractional sum that is not in lowest terms, and students do not recognize the reduced fraction in the response choices.

WORKSHEET 1–5d

Adding Fractions and Mixed Numbers to Solve Word Problems

Name ______________________

Date ______________________

Solve the word problems provided. Write an equation on the back of the worksheet to show the steps used for each exercise.

1. A jeweler used $\frac{3}{5}$ meter of silver chain to make a bracelet and $\frac{7}{10}$ meter to make a necklace. How many meters of silver chain were used in all to make the 2 pieces of jewelry?

   A. $1\frac{3}{10}$ m B. $\frac{10}{15}$ m C. $\frac{1}{10}$ m D. $1\frac{1}{5}$ m

2. Over a 3-month period, Dave's Market donated $12\frac{3}{4}$ pounds, $25\frac{3}{8}$ pounds, and $15\frac{1}{2}$ pounds of food, respectively, to the local shelter for abused children. How many pounds of food total were donated over the 3 months?

   A. $52\frac{3}{4}$ lbs. B. $53\frac{1}{2}$ lbs. C. $53\frac{5}{8}$ lbs. D. $53\frac{3}{4}$ lbs.

3. A recipe for apple sparkle punch requires the following ingredients: $4\frac{1}{2}$ cups of apple juice, $\frac{1}{4}$ cup of lime juice, 3 quarts of water, $3\frac{1}{2}$ cups of pineapple juice, and $2\frac{3}{4}$ cups of lime soda. What is the smallest-sized pitcher that will hold all the ingredients?

   A. 7-qt. B. 6-qt. C. 5-qt. D. 4-qt.

4. Julio hiked $16\frac{3}{5}$ kilometers on Monday, $11\frac{1}{2}$ kilometers on Tuesday, and 10 kilometers on Wednesday. What is the total number of kilometers Julio hiked during these 3 days?

   A. $37\frac{4}{7}$ km B. 38 km C. $38\frac{1}{10}$ km D. Not shown here

5. Marjorie has a casserole recipe that serves 4 people and requires $2\frac{7}{8}$ pounds of chicken. If she wants to serve 12 people, how many pounds of chicken will she need?

   A. $5\frac{3}{4}$ lbs. B. 6 lbs. C. $6\frac{7}{8}$ lbs. D. $8\frac{5}{8}$ lbs.

Objective 6

Subtract fractions or mixed numbers to solve word problems.

Students need many experiences involving fractional measurements. They have great difficulty transferring the four operations from their comfortable whole number world to the less familiar fraction world. Manipulatives assist students in applying the subtraction or removal process to different fractional part sizes by showing the need for a common denominator.

Activity 1: Manipulative Stage

Materials

Set of fraction bars per pair of students (use Pattern 1–5a)
Building Mat 1–6a per pair of students
Worksheet 1–6a
Regular pencil

Procedure

1. Use the sets of fraction bars prepared for addition in Objective 5. Each set of fraction bars should contain cut-out bars from five copies of Pattern 1–5a. The whole bar on Building Mat 1–6a should be congruent to the whole bar on Pattern 1–5a. An alternative is to have each type of fraction colored to match the corresponding type in the teacher's set of fraction bars.

2. Give each pair of students a set of fraction bars, one copy of Building Mat 1–6a, and two copies of Worksheet 1–6a.

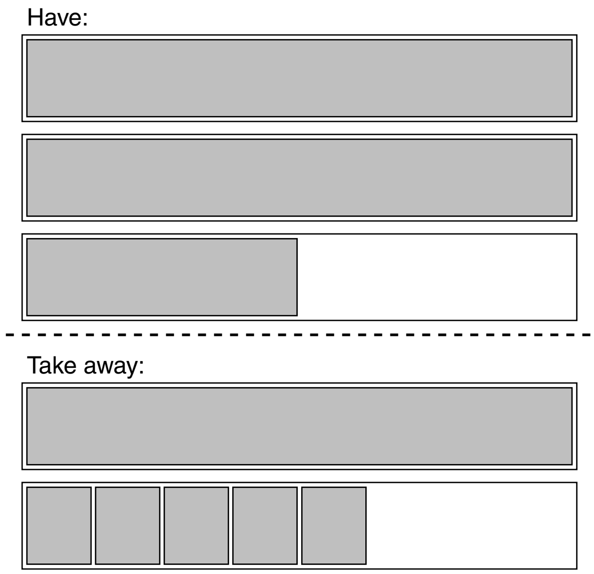
3. It is assumed for this activity that students have had experience with equivalent fractions. The trading action for finding common denominators, however, will reinforce their understanding of the concept.

4. Students should build with the fraction bars on Building Mat 1–6a to solve each word problem on Worksheet 1–6a. Then they should write a word sentence below the exercise to describe the results.

5. Discuss Exercise 1 on Worksheet 1–6a before allowing students to solve the other exercises independently.

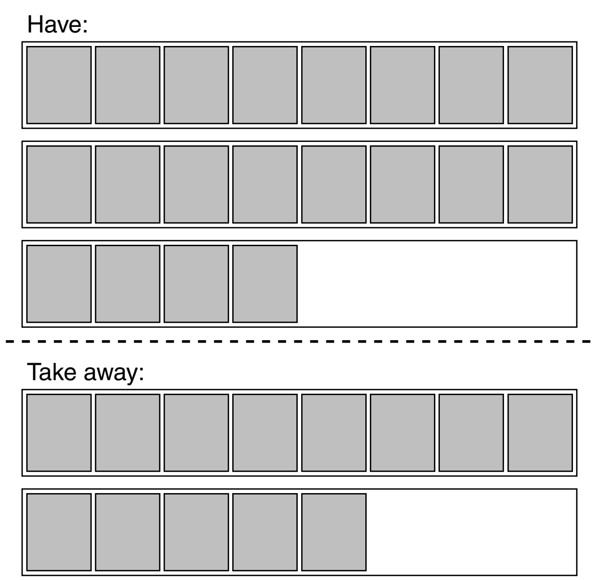
Consider the word problem in Exercise 1 on Worksheet 1–6a: "Sandy jogged 2 and 1-half miles in all on Saturday. She jogged 1 and 5-eighths miles of that distance before stopping to rest. How many more miles did she jog after she rested?"

Have students place one whole bar on each of the top two bars of Building Mat 1–6a and place a half bar on the third bar of the mat. This represents 2 and 1-half miles, the total distance in the problem. Students should then place 1 whole bar on the fourth bar of the mat below the phrase "Take away," and place five of the eighth bars on the bottom bar of the mat. This will show 1 and 5-eighths miles, the first distance Sandy jogged. The fraction bars placed above the broken line segment on the mat represent the *minuend*, and the bars placed below the broken line segment represent the *subtrahend*. Here are the fraction bars on the building mat as they appear initially:



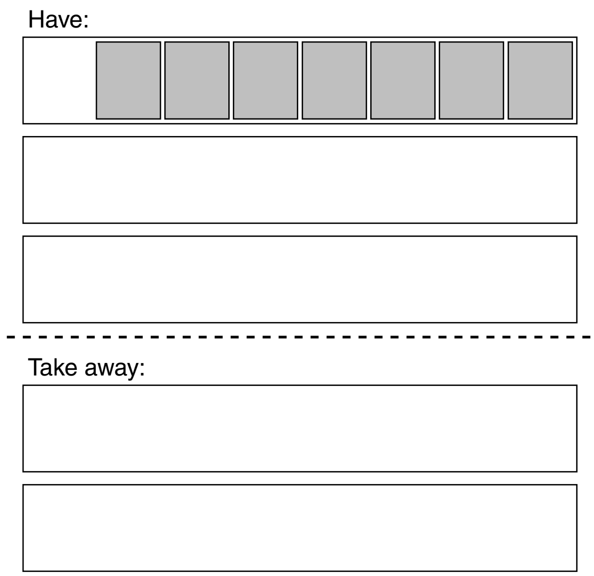
Ask students if all the fraction bars on the building mat are the same size. Since they are not, students must trade the bars for smaller fraction bars until they find a common bar size. Allow students to explore to find what size is needed. Any smaller bar size will work as long as the size is common to all numbers shown on the mat.

A possible trade is to trade or replace each whole bar on the mat with 8 eighths and to trade or replace the half bar with 4 eighths. The bottom bar on the mat already contains 5 eighths. Once the trading is complete and all fraction bars on the mat are the same size, there will be 20 eighths in the upper minuend region of the mat and 13 eighths in the lower subtrahend region of the mat. Students should begin to match each fraction bar in the lower region of the mat with a fraction bar in the upper region. Each matched pair should be removed from the mat as it is made. This action shows the removal process for subtraction. Here is the appearance of the mat before the matching begins:



The 13 eighths in the lower region will match to 13 eighths in the upper region, so 13 pairs of eighths will be removed from the building mat. As a result, 13 eighths have been taken away from 20 eighths, leaving 7 eighths as the difference in the upper region.

These 7 eighths may now be moved to the same bar on the mat if they are not already. They may be placed on either of the top three bars. Below Exercise 1 on Worksheet 1–6a, students should record their answer as a word sentence: “Sandy jogged 7-eighths of a mile after resting.” The final fraction bars remaining on the building mat after the removal will appear as follows:



Discuss the idea that if there had been more than 8 eighths of a whole left after the removal, the fraction bars could have been grouped in sets of 8. For example, if the difference had been 17 eighths of a whole, each 8 eighths could be exchanged for 1 whole, yielding 2 wholes and 1 eighth of another whole for the answer. Students should not exchange each 8 eighths for 1 whole on the mat; they should make the trade mentally. The phrase “of a whole” needs to be used regularly so that students realize the significance of the *whole* to any of its fractional *parts*. In Exercise 1, the whole corresponds to the mile, so that 1-eighth of a whole is equivalent to 1-eighth of a mile.

Because of the algorithm being developed in this objective, when the difference occurs as an improper fraction, the improper fraction name needs to be recorded along with the mixed number name in the final word sentence. As an example, students might include the following phrase in the word sentence for an exercise: “17-eighths of a mile, or 2 and 1-eighth miles left.”

Answer Key for Worksheet 1–6a

1. Sandy jogged 7-eighths of a mile after resting.
2. Allen had 1 and 1-sixth ounces of fudge left.
3. Sonja spent 5-sixths of an hour on science homework.
4. One-half of the eighth-grade class ate hot dogs on Monday.
5. Sam took 1 and 5-eighths hours to cut his own lawn.

Building Mat 1–6a. Fraction Subtraction

Have:

Take away:

WORKSHEET 1–6a

Subtracting with Fractions and Mixed Numbers

Name ______________________

Date ______________________

Use fraction bars on Building Mat 1–6a to solve the word problems provided. Write a word sentence below each exercise to record the result found.

1. Sandy jogged 2 and 1-half miles in all on Saturday. She jogged 1 and 5-eighths miles of that distance before stopping to rest. How many more miles did she jog after she rested?

2. Allen bought 2 and 5-sixths ounces of marshmallow fudge. He ate 1 and 2-thirds ounces of the fudge. How many ounces of the fudge did he have left?

3. Sonja spent 1 and 7-twelfths hours total on her math and science homework. If she studied for 3-fourths of an hour on her math homework, how much time did she spend on science homework?

4. Five-sixths of the eighth-grade class ate in the school cafeteria on Monday. Only pizzas and hot dogs were served that day. If 1-third of the class ate pizzas, what fractional part of the class ate hot dogs?

5. Sam took 3 hours to cut his lawn and a neighbor's lawn. If he took 1 and 3-eighths hours to cut the neighbor's lawn, how many hours did Sam take to cut his own lawn?

Activity 2: Pictorial Stage

Materials

Worksheet 1–6b
Worksheet 1–6c
Red pencil and regular pencil

Procedure

1. Give each student a copy of Worksheet 1–6b, a copy of Worksheet 1–6c, and a red pencil. Now that students are comfortable subtracting with fraction bars on the building mat, they will work similar word problems with diagrams.

2. Each student should complete the frames on her or his own Worksheet 1–6b, but might share results with a partner.

3. For each exercise, students will subdivide whole bars on a frame to show different fractional amounts of a whole. The initial amounts will be shaded, using diagonal stripes for easy recognition.

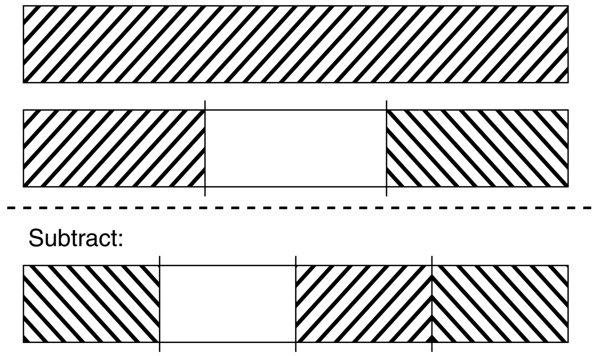
4. Students will determine how each initial part must be traded in order to have only one common part size appearing on the frame. Trades will be shown by marking new subdivisions with red pencil.

5. Each part in the lower region will be matched to a unique part in the upper region, and both parts will be marked out. This process continues until all parts in the lower region of the frame are marked out. The shaded parts remaining in the upper region will then be counted to find the answer to the word problem. An equation for this difference should be written below the exercise on Worksheet 1–6c. The equation should indicate the trades that were made in order to find the difference.

6. Exercise 1 on Worksheet 1–6c should be discussed with the class in detail before students are allowed to work the additional problems on the worksheet on their own.

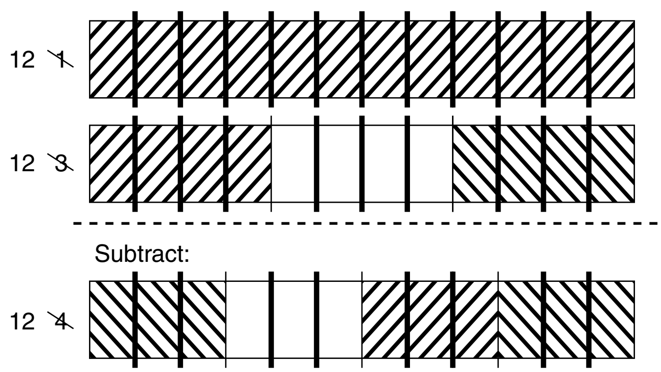
Here is Exercise 1 from Worksheet 1–6c to consider: "Marion bought 1 and 2-thirds pounds of taffy at the state fair. He ate 3-fourths of a pound of the taffy while at the fair. How many pounds of taffy did he have left?"

On the first frame of Worksheet 1–6b, have students shade the top bar completely (using diagonal stripes) to show 1 whole, and then subdivide the second bar into 3 equal parts and shade 2 of the parts to show 2-thirds of another whole. In the lower region below the broken line segment, have them subdivide the first bar into 4 equal parts and shade 3 of the parts; the shaded parts will represent the subtrahend, 3-fourths of a whole. The initial appearance of the diagram is shown here:

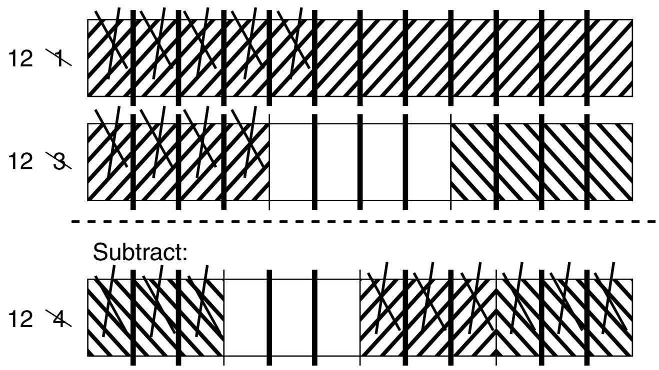


Have students count the total parts in each whole bar and write the number at the left end of the bar. The top bar contains only 1 part, the second bar has 3 parts, and the third bar has 4 parts. Students must decide how to trade or subdivide the parts of each whole bar so that all the whole bars contain the same total number of equal parts; hence, all parts will be the same size. Guide students through the thinking process.

For example, if each of the three parts in the second bar is traded for 2 new parts, there will be 6 new parts for the whole bar. Similarly, if each of the 4 parts in the third bar is traded for 2 new parts, there will be 8 new parts for the whole bar. Now consider the second bar again with 3 new parts for each original part; there will be 9 new parts total for the whole bar. Continue this process until 12 parts per whole bar are discovered as the best choice. Then a 4-for-1 trade will be needed for the thirds on the second whole bar and a 3-for-1 trade will be needed for the fourths on the third whole bar. The number 12 should be recorded at the left end of each whole bar. With a red pencil, students should subdivide each whole bar in order to show 12 equal parts on each. The changed diagram will appear as follows:



The diagram is now ready for students to begin the subtraction or removal process. This is done by marking out 1 part in the subtrahend (the third bar in this case) and matching it to 1 part in the upper region (minuend) and marking out that part as well. This marking-out process continues one pair at a time until all parts in the lower region have been marked out. The parts that remain unmarked in the upper region will be the *difference* sought. Since there are 12-twelfths + 8-twelfths, or 20-twelfths, in the upper region and 9-twelfths must be removed, the difference will be 11-twelfths of a whole. Here is the completed diagram for this exercise:



Students should record equations or number sentences below Exercise 1 on Worksheet 1–6c to show their results. Because the upper region initially contained a mixed number, one number sentence will be needed to show the change of the mixed number to an improper fraction. Then another number sentence will be needed to show the subtraction. The number sentences should also reflect the trades that were used on each whole bar. Here is an example of a recording for Exercise 1:

$$1 + \frac{2}{3} = \frac{12}{12} + \frac{2 \times 4}{3 \times 4} = \frac{12}{12} + \frac{8}{12} = \frac{20}{12}$$

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12} \qquad \text{So } \frac{20}{12} - \frac{9}{12} = \frac{11}{12} \text{ lbs. of taffy left}$$

Answer Key for Worksheet 1–6c

The answer key provides possible number sentences to use.

1. The number sentences are given in the text (see above).
2. $\frac{7}{8} - \frac{2}{4} = \frac{7}{8} - \frac{2 \times 2}{4 \times 2} = \frac{7}{8} - \frac{4}{8} = \frac{3}{8}$ of a yard of fabric left
3. $1 + \frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$ and $1 + \frac{1}{3} = \frac{3}{3} + \frac{1}{3} = \frac{4}{3}$

   $\frac{3}{2} - \frac{4}{3} = \frac{3 \times 3}{2 \times 3} - \frac{4 \times 2}{3 \times 2} = \frac{9}{6} - \frac{8}{6} = \frac{1}{6}$ of a pound of seed left

   Alternative with whole numbers:

   wholes: 1 – 1 = 0 wholes left

   parts: $\frac{1}{2} - \frac{1}{3} = \frac{1 \times 3}{2 \times 3} - \frac{1 \times 2}{3 \times 2} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$ of a pound left
4. $2 = 1 + 1 = \frac{5}{5} + \frac{5}{5} = \frac{10}{5}$ and $1 + \frac{3}{5} = \frac{5}{5} + \frac{3}{5} = \frac{8}{5}$

   So $\frac{10}{5} - \frac{8}{5} = \frac{2}{5}$ of a mile more on Monday

   Alternative with whole numbers:

   wholes: 2 – 1 = 1 whole left

   parts: $1 - \frac{3}{5} = \frac{5}{5} - \frac{3}{5} = \frac{2}{5}$ of a mile more on Monday

WORKSHEET 1–6b
Drawing Frames

Name ____________________
Date ____________________

1.

Subtract:

2.

Subtract:

WORKSHEET 1–6b Continued

Name ______________________

Date ______________________

3.

Subtract:

4.

Subtract:

WORKSHEET 1–6c

Name ______________________

Drawing to Subtract Fractions and Mixed Numbers

Date ______________________

Use a red pencil and Worksheet 1–6b (frames) to solve the word problems provided. Write one or more equations below each exercise to record the result found. Record any trades made in the equations.

1. Marion bought 1 and 2-thirds pounds of taffy at the state fair. He ate 3-fourths of a pound of the taffy while at the fair. How many pounds of taffy did he have left?

2. Harriet had 7-eighths of a yard of fabric. She used 2-fourths of a yard of the fabric to make a chair cushion. How much fabric did Harriet have left?

3. Juan had 1 and 1-half pounds of birdseed. He gave 1 and 1-third pounds of the seed to his sister. How much birdseed did he have left?

4. Leslie walked 2 miles on Monday, then walked 1 and 3-fifths miles on Tuesday. How many more miles did she walk on Monday than on Tuesday?

Activity 3: Independent Practice

Materials

Worksheet 1–6d
Regular pencil

Procedure

Give each student a copy of Worksheet 1–6d. Have students solve each exercise by using the subtraction algorithm for fractions, which was developed as the final notation in Activity 2. Mastery of fraction addition is assumed. Discuss the alternative method that allows whole amounts, especially larger numbers, to be subtracted separately before fractional amounts are subtracted. For example, the subtrahend 18 and 1/2 may be viewed several ways: $1 + 1 + \ldots + 1 + \frac{1}{2} = \frac{2}{2} + \frac{2}{2} + \ldots + \frac{2}{2} + \frac{1}{2}$; $\frac{36}{2} + \frac{1}{2}$; or just $18 + \frac{1}{2}$. So in order to subtract 18 and $\frac{1}{2}$ from 23 and $\frac{1}{3}$, students might choose to subtract 18 from 23 to get 5 wholes left, then subtract $\frac{1}{2}$ from the remaining 5 wholes and the $\frac{1}{3}$. Then this becomes $\frac{32}{6}$ take away $\frac{3}{6}$, which equals $\frac{29}{6}$ or 4 wholes and $\frac{5}{6}$ of another whole as the final answer. Encourage students to use the trading language for finding equivalent fractions as needed. When all students have finished the worksheet, have several students share their answers with the rest of the class.

Answer Key for Worksheet 1–6d

1. C
2. A
3. D
4. A
5. B

Possible Testing Errors That May Occur for This Objective

- Students apply subtraction correctly but use incorrect equivalent fractions.
- Students make errors when changing a mixed number to an improper fraction; otherwise, the subtraction process is correct.
- Students subtract the numerators and subtract the denominators separately as though there are two different problems. No common denominator is found.
- Students add the fractions instead of subtracting them.

WORKSHEET 1–6d

Subtraction with Fractions and Mixed Numbers

Name ______________________

Date ______________________

Solve the word problems provided. Write an equation on the back of the worksheet to show the steps used for each exercise.

1. In a trip across a lake, Boat A had an average speed of $41\frac{1}{5}$ miles per hour, and Boat B had an average speed of $27\frac{1}{2}$ miles per hour. What was the difference between the two speeds?

   A. $14\frac{3}{10}$ mph B. $14\frac{1}{5}$ mph C. $13\frac{7}{10}$ mph D. $12\frac{1}{2}$ mph

2. Luis bought 5 gallons of gasoline in a large container. He poured $1\frac{3}{8}$ gallons of the gasoline into his lawn mower. How many gallons remain in the container?

   A. $3\frac{5}{8}$ gal. B. 4 gal. C. $4\frac{3}{8}$ gal. D. $6\frac{3}{8}$ gal.

3. A box contained $\frac{3}{4}$ of a pound of peanut brittle candy. George ate $\frac{3}{5}$ of a pound of the candy. How many pounds of candy were left in the box?

   A. $\frac{6}{9}$ lb. B. $\frac{1}{4}$ lb. C. $\frac{2}{5}$ lb. D. $\frac{3}{20}$ lb.

4. Katy spent $\frac{1}{6}$ of Saturday at the Heard Museum. Then she spent $\frac{1}{4}$ of the day baby-sitting. What fraction of Saturday was Katy not baby-sitting or visiting the museum?

   A. $\frac{7}{12}$ B. $\frac{5}{12}$ C. $\frac{2}{10}$ D. $\frac{1}{12}$

5. During spring break, Ann traveled $185\frac{3}{4}$ miles, and Carl traveled $256\frac{2}{3}$ miles. How many miles longer was Carl's trip than Ann's trip?

   A. $71\frac{2}{3}$ mi. B. $70\frac{11}{12}$ mi. C. $70\frac{1}{2}$ mi. D. $69\frac{3}{4}$ mi.

Objective 7

Divide fractions or mixed numbers to solve word problems.

The "common denominator" model will be used to develop the division algorithm for fractions. This approach seems to be easier for students to understand than the more traditional approach that depends on multiplication by the reciprocal of the divisor. In the exercises, the divisor will represent the set to be reproduced or copied, and the quotient will count the number of copies made. It is assumed for this activity that students have had experience with equivalent fractions. The trading action for finding common denominators, however, will reinforce their understanding of the concept.

Activity 1: Manipulative Stage

Materials

Set of fraction bars per pair of students (use Pattern 1–5a)
Building Mat 1–7a per pair of students
Worksheet 1–7a
Regular pencil

Procedure

1. Use the sets of fraction bars prepared for addition in Objective 5. Each set of fraction bars should contain cut-out bars from five copies of Pattern 1–5a. If preferred, each type of fraction might be colored to match the corresponding type in the teacher's set of fraction bars.

2. Give each pair of students a set of fraction bars, one copy of Building Mat 1–7a, and two copies of Worksheet 1–7a.

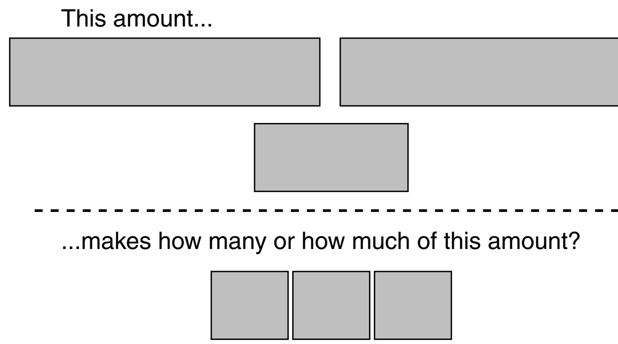
3. Students should build with the fraction bars on Building Mat 1–7a to solve each word problem on Worksheet 1–7a. Then they should write a word sentence below the exercise to describe the results.

4. With the common denominator method, students will trade all fraction bars on the building mat to a common bar size. Then they will compare the new fraction bars in the upper region of the building mat (dividend) to those in the lower region (divisor) to find how many copies the dividend amount will make of the divisor amount. The quotient will be the number of complete or partial copies made.

5. Discuss Exercise 1 on Worksheet 1–7a before allowing students to solve the other exercises independently.

Consider the word problem in Exercise 1 on Worksheet 1–7a: "Carrie has 2 and 1-half kilograms of fudge in the store display case. She wants to package the fudge in bags with 3-fourths of a kilogram per bag. How many bags, including a partial bag, will Carrie be able to prepare?"

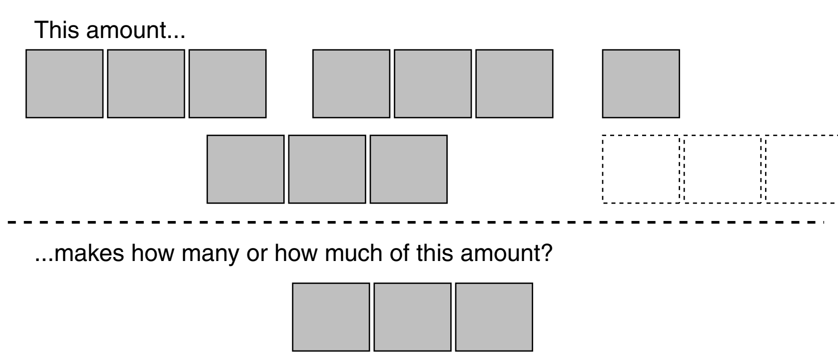
Have students place 2 whole bars and 1 half bar in the upper region of Building Mat 1–7a. This represents the dividend. They should then place 3 fourths bars in the lower region of the mat to represent the divisor. The 3 fourths bars should be touching end to end and will be viewed as a connected group. Here is a possible initial arrangement of the fraction bars on the building mat:



Students should determine a common fraction bar size for all bars on the mat. Since wholes, halves, and fourths are involved, all bars should be traded for fourths. The upper region will contain 4 fourths, 4 fourths, and 2 fourths, or 10 fourths total. The lower region will contain the original 3 fourths. Students should now slide the *group* of 3 fourths (the divisor) from the lower region to the upper region and try to match the *group* with similar groups from the 10 fourths (the dividend). Each new group of 3 fourths formed should be moved away from the other bars so that the groups are easily seen. Three such groups of 3 fourths each should be formed, leaving 1 fourth bar isolated in the upper region.

The divisor *group* should now be slid over and aligned with this single fourth. Discuss the idea that a complete group contains 3 fourths, so this single fourth represents one out of three parts needed for a complete group. Therefore, the single fourth is only a partial group and will be called *1-third of a complete group.* Once the partial group has been named, the original divisor group should be moved back down into the lower region so it will not be counted as part of the answer.

The dividend has been separated into three complete groups and 1 third of another complete group. That means 3 and 1-third bags of fudge can be prepared, using 3-fourths of a kilogram per bag. Students should write the following word sentence below Exercise 1 on Worksheet 1–7a: "Carrie can prepare 3 and 1-third bags of fudge." Here is the final appearance of the fraction bars on the mat. The dashed shapes indicate where the divisor group had been placed earlier in order to determine the partial group's name:



Answer Key for Worksheet 1–7a

1. Carrie can prepare 3 and 1-third bags of fudge. [Trade all bars to fourths.]

2. The tailor can make 5 scarves. [Trade all bars to thirds.]

3. The fishmonger can make 6 and 2-thirds packages of salmon. [Trade all bars to twelfths.]

4. Rita can fill 2-thirds of a container. [Do not trade; use three whole bars as group to form.]

5. The chef can make 4 and 4-fifths servings. [Trade all bars to eighths.]

Building Mat 1–7a. Fraction Division

This amount...

...makes how many or how much of this amount?

WORKSHEET 1–7a

Dividing with Fractions and Mixed Numbers

Name ______________________

Date ______________________

Use fraction bars on Building Mat 1–7a to solve the word problems provided. Write a word sentence below each exercise to record the result found.

1. Carrie has 2 and 1-half kilograms of fudge in the store display case. She wants to package the fudge in bags with 3-fourths of a kilogram per bag. How many bags, including a partial bag, will Carrie be able to prepare?

2. The tailor has 1 and 2-thirds yards of silk fabric. He needs 1-third of a yard to make a silk scarf. How many scarves can he make with the fabric?

3. The fishmonger has 1 and 2-thirds pounds of salmon to put in individual packages. He wants to cut and wrap 1-fourth of a pound of salmon in each package. How many packages will he be able to make, including a partial package?

4. Rita's recipe makes 2 liters of punch. How many 3-liter containers can be filled using all the punch from one recipe?

5. The chef has 3 cups of whipped cream and needs 5-eighths of a cup of cream for a single serving of a dessert. How many servings, including a partial serving, can be made?

Activity 2: Pictorial Stage

Materials

Worksheet 1–7b
Worksheet 1–7c
Red pencil and regular pencil

Procedure

1. Give each student a copy of Worksheet 1–7b, a copy of Worksheet 1–7c, and a red pencil. Now that students are comfortable dividing with fraction bars on the building mat, they will work similar word problems with diagrams.

2. Each student should complete the frames on her or his own Worksheet 1–7b, but might share results with a partner.

3. For each exercise, students will subdivide whole bars on a frame to show different fractional amounts of a whole. The initial amounts will be shaded, using diagonal stripes for easy recognition. The upper region of the frame will be for the dividend, and the lower region will be for the divisor. For ease of grouping, any fractional parts on the same whole bar in the divisor should be drawn adjacent to each other.

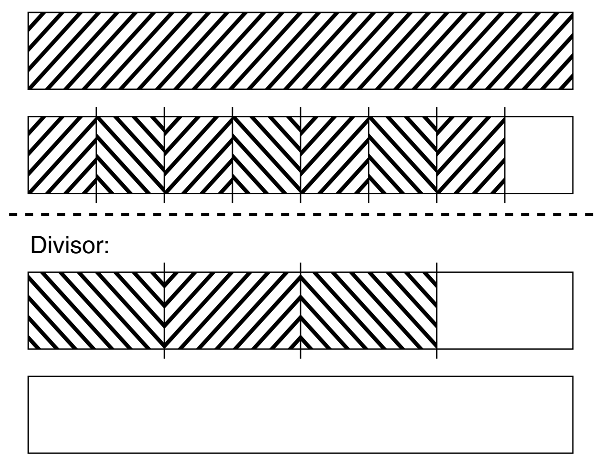
4. Students will determine how each initial part must be traded in order to have only one common part size appearing on the frame; that is, a common denominator is needed. Trades will be shown by marking new subdivisions with red pencil.

5. A ring will be drawn in red pencil around all the parts in the lower region (divisor) to signify the group to be formed. Then a matching group will be located in the upper region (dividend) and a red ring drawn around that new group. This process continues until all parts in the upper region of the frame have been ringed as a complete group or a partial group. The ringed groups in the upper region will then be labeled and counted to find the quotient or answer to the word problem. An equation for this process should be written below the exercise on Worksheet 1–7c. The equation should indicate the trades that were made in order to find the quotient.

6. Exercise 1 on Worksheet 1–7c should be discussed with the class in detail before students are allowed to work the additional problems on the worksheet independently.

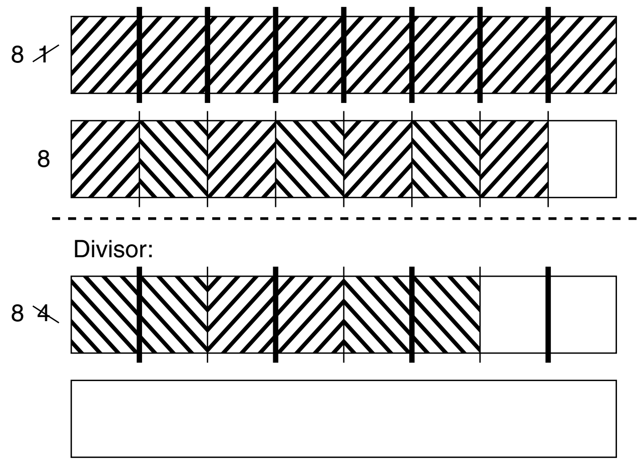
Here is Exercise 1 from Worksheet 1–7c to consider: "Jerry has 1 and 7-eighths kilograms of bananas. He wants to bundle them into bags weighing 3-fourths of a kilogram each. How many bags, including a partial bag, will he be able to make?"

On the first frame of Worksheet 1–7b, students should shade (with diagonal stripes) the top whole bar, then subdivide the second whole bar into 8 equal parts and shade 7 of those parts. Together, these two bars will represent the dividend, 1 and 7-eighths kilograms. On the third whole bar of the frame, which is the first whole bar in the lower region, students should subdivide the bar into 4 equal parts and shade 3 of those parts. This will represent the divisor, 3-fourths of a kilogram. Here is the appearance of the frame with the initial shadings:



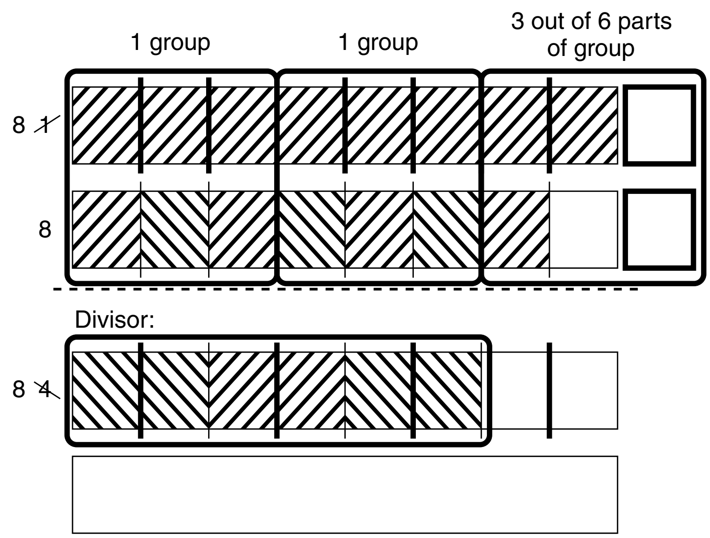
Have students count the total parts in each whole bar, and write the number at the left end of the bar. The top bar contains only 1 part, the second bar has 8 parts, and the third bar has 4 parts. Students must decide how to trade or subdivide the parts of each whole bar so that all the whole bars contain the same total number of equal parts; hence, all parts will be the same size. Guide students through the thinking process.

For example, if each of the 4 parts in the third bar is traded for 2 new parts, there will be 8 new parts for the whole bar. The second bar already has 8 original parts. Therefore, the top bar should also be traded for 8 new parts. So an 8-for-1 trade will be needed for the top whole bar and a 2-for-1 trade will be needed for the fourths on the third whole bar. The number 8 should be recorded at the left end of each whole bar that is changed. With a red pencil, students should subdivide each whole bar in order to show 8 equal parts on each. The changed diagram will appear as follows:



The diagram is now ready for students to begin the separation or grouping process. This is done by drawing a red ring around all the shaded parts in the divisor (the third bar in this case); this indicates how many parts will be in a *group*. Students should find and ring similar complete groups (similar by having the same number of parts in each group) in the upper region (dividend). This ringing process continues one group at a time until all parts in the upper region have been ringed. Finally, if some parts are left in the upper region, which are too few to form a complete group, additional parts should be drawn in red pencil adjacent to those parts to complete a group. Then a red ring should be drawn around that final partial group, including both the shaded parts and the newly drawn red parts. The shaded parts in the partial group compared to the total parts that should be in a complete group will identify the fractional name for the partial group formed.

In Exercise 1, the partial group will contain 3 shaded parts out of 6 total parts needed, so the partial group will represent 3-sixths of a complete group. Since after trading there are 8-eighths + 7-eighths, or 15-eighths, in the upper region and 6-eighths in the lower region (as the *group* to form), 2 complete groups and 3-sixths of another complete group should be ringed in the upper region. Here is the completed diagram for this exercise:



Students should record an equation or a number sentence below Exercise 1 on Worksheet 1–7c to show their results. It will be assumed that students are able to change the mixed number in the dividend directly to an improper fraction. Discuss the idea that the top bar showing 1 whole might be temporarily traded to eighths since its included common fraction is already eighths. Then both bars of eighths can be tested for possible trading, along with the third bar that contains fourths. The number sentence should also reflect the trades that were used on each whole bar of the diagram. Here is an example of a recording for Exercise 1:

$$1\frac{7}{8} \div \frac{3}{4} = \frac{15}{8} \div \frac{3\times 2}{4\times 2} = \frac{15}{8} \div \frac{6}{8} = 2 \text{ and } \frac{3}{6} \text{ bags of bananas,}$$

$$\text{or } 2 \text{ and } \frac{1}{2} \text{ bags of bananas}$$

Observe that after a common denominator or part size is found, only the new numerators contribute to the final answer or the quotient. In Exercise 1, it is 15 divided by 6 (the group size), which yields 2 complete groups and 3-sixths of another complete group. Encourage students to find other names for their answers when possible, but the initial quotient should be recorded first before a reduced form is given.

Answer Key for Worksheet 1–7c

Here are possible number sentences to use.

1. $1\frac{7}{8} \div \frac{3}{4} = \frac{15}{8} \div \frac{3 \times 2}{4 \times 2} = \frac{15}{8} \div \frac{6}{8} = 2$ and $\frac{3}{6}$ bags of bananas,

   or 2 and $\frac{1}{2}$ bags of bananas
2. $\frac{2}{3} \div 1\frac{1}{2} = \frac{2}{3} \div \frac{3}{2} = \frac{2 \times 2}{3 \times 2} \div \frac{3 \times 3}{2 \times 3} = \frac{4}{6} \div \frac{9}{6} = \frac{4}{9}$ of the original bag left
3. $2 \div \frac{1}{2} = \frac{4}{2} \div \frac{1}{2} = 4$ half-dollars in 2 whole dollars
4. $1\frac{1}{3} \div \frac{2}{5} = \frac{4}{3} \div \frac{2}{5} = \frac{4 \times 5}{3 \times 5} \div \frac{2 \times 3}{5 \times 3} = \frac{20}{15} \div \frac{6}{15} = 3$ and $\frac{2}{6}$ sections of cable, or 3 and $\frac{1}{3}$ sections of cable

WORKSHEET 1–7b
Drawing Frames

Name ____________________
Date ____________________

1.

Divisor:

2.

Divisor:

WORKSHEET 1–7b Continued

Name ____________________

Date ____________________

3.

Divisor:

4.

Divisor:

WORKSHEET 1–7c

Name ______________________

Drawing to Divide with Fractions and Mixed Numbers

Date ______________________

Draw on a frame on Worksheet 1–7b to find the quotient for each exercise on this worksheet. Below each exercise, write a number sentence or equation that shows the results. Show the trades used by recording them within the number sentences.

1. Jerry has 1 and 7-eighths kilograms of bananas. He wants to bundle them into bags weighing 3-fourths of a kilogram each. How many bags, including a partial bag, will he be able to make?

2. Cindy has only 2-thirds of a pound of birdseed left. The original bag contained 1 and 1-half pounds. What fractional part of the original bag does she have left?

3. How many half-dollars are in 2 whole dollars?

4. The city utility company has 1 and 1-third kilometers of cable on a large spool. The company needs sections cut from this cable so that each section is 2-fifths of a kilometer long. How many sections, including a partial section, can be cut from the cable?

Activity 3: Independent Practice

Materials

Worksheet 1–7d
Regular pencil

Procedure

Give each student a copy of Worksheet 1–7d. Have students solve each exercise by using the common denominator algorithm for dividing fractions, which was developed as the final notation in Activity 2. Encourage students to use the trading language for finding equivalent fractions as needed. It is assumed that students have already mastered changing mixed numbers to improper fractions. Be careful to avoid shortcuts when explaining these changes. For example, students should understand that for 12 and $\frac{1}{3}$, each of the 12 wholes trades for 3 thirds to agree with the given $\frac{1}{3}$. After trading, there will be 12 of the $\frac{3}{3}$, which combine with the given $\frac{1}{3}$. So we have $\frac{36}{3} + \frac{1}{3}$, or $\frac{37}{3}$ as the improper fraction equivalent to the mixed number, 12 and $\frac{1}{3}$. When all students have finished the worksheet, have several students share their answers with the rest of the class.

Answer Key for Worksheet 1–7d

1. C
2. D
3. B
4. B
5. D [The correct answer is $1\frac{5}{6}$ packages.]

Possible Testing Errors That May Occur for This Objective

- Students apply division correctly but use incorrect equivalent fractions. An incorrect multiplication fact may have been used when finding an equivalent fraction.
- Students make errors when changing a mixed number to an improper fraction; otherwise, the division process is correct.
- Students divide the original numerators and divide the original denominators separately as though there are two different division problems.
- Students multiply the fractions instead of dividing them. They do not apply the division or separation/copying process described in the story situation.
- The order of the dividend and the divisor is reversed before the division algorithm is applied.

WORKSHEET 1–7d

Dividing with Fractions and Mixed Numbers

Name ______________________

Date ______________________

Solve the word problems provided. Write an equation on the back of the worksheet to show the steps used for each exercise. Reduce fractions in answers to the lowest terms.

1. Sheryl rode her bike for $27\frac{1}{2}$ miles total over several days. If she averaged $5\frac{1}{2}$ miles per day, how many days did she ride her bike?

   A. 7 days B. 6 days C. 5 days D. 4 days

2. A block of cheese weighs $5\frac{7}{8}$ pounds. How many packages of sliced cheese can be made from this block if each package is to weigh $1\frac{1}{2}$ pounds?

   A. $7\frac{3}{8}$ pkg. B. $5\frac{1}{2}$ pkg. C. $4\frac{3}{8}$ pkg. D. $3\frac{11}{12}$ pkg.

3. If José has $3\frac{1}{5}$ ounces left from an 8-ounce carton of yogurt, what fractional part of the original carton does he have left?

   A. $\frac{1}{5}$ carton B. $\frac{2}{5}$ carton C. $\frac{3}{4}$ carton D. Not shown here

4. The farmer has $15\frac{3}{4}$ kilograms of carrots. He plans to sell them in bunches at $2\frac{1}{4}$ kilograms per bunch. How many bunches will he be able to make?

   A. $7\frac{1}{2}$ bunches B. 7 bunches C. $6\frac{3}{4}$ bunches D. $5\frac{1}{2}$ bunches

5. The fishmonger has $3\frac{2}{3}$ pounds of tuna to put in individual packages. He wants to cut and wrap 2 pounds of tuna in each package. How many packages will he be able to make, including a partial package?

   A. $1\frac{1}{6}$ pkg. B. $1\frac{2}{3}$ pkg. C. 2 pkg. D. Not shown here

Objective 8

Multiply fractions or mixed numbers to solve word problems.

Often multiplication of fractions is viewed as a simple concept to teach. This is because students seem to quickly recognize the pattern of "numerator times numerator" and "denominator times denominator." Unfortunately, many do not truly understand the process. When interviewed, students have revealed that they see fraction multiplication as two separate problems: a multiplication problem with whole numbers above the bars, and another similar problem below the bars. To avoid the phenomenon of getting the right answer for the wrong reason, students need to experience whole number multipliers with fraction multiplicands before they are introduced to fraction multipliers. The lessons that follow use such a sequence. It is assumed for this activity that students have had experience with equivalent fractions. They will often be trading given fraction bars for several copies of smaller fraction bars. They should also be comfortable with changing improper fractions to mixed numbers or mixed numbers to improper fractions.

Activity 1: Manipulative Stage

Materials

Set of fraction bars per pair of students (use Pattern 1–5a)
Building Mat 1–8a per pair of students
Worksheet 1–8a
Regular pencil

Procedure

1. Use the sets of fraction bars prepared for addition in Objective 5. Each set should contain cut-out bars from five copies of Pattern 1–5a. The whole bars in the student sets must be congruent to the whole bars drawn on Building Mat 1–8a. If preferred, each type of fraction might be colored to match the corresponding type in the teacher's set of fraction bars.

2. Give each pair of students a set of fraction bars, one copy of Building Mat 1–8a, and two copies of Worksheet 1–8a.

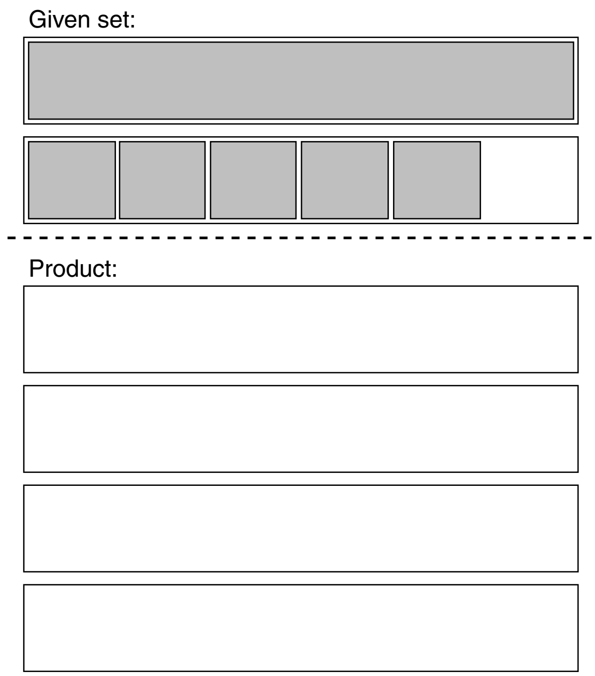
3. Students should build with the fraction bars on Building Mat 1–8a to solve each word problem on Worksheet 1–8a. Then they should write a word sentence below the exercise to describe the results.

4. Students will first work with exercises that use a whole number as the multiplier (Exercises 1 and 2). Then they will be introduced to fractions as multipliers in Exercises 3 through 5. Do not require students to write the whole number 1 as the *missing denominator* for a whole number factor. This step is unnecessary, and a denominator of 1 seems to have little meaning to students. One pattern will be identified when a whole number factor is present, and another pattern will be identified when both factors are fractions. These patterns are discussed in Activity 3.

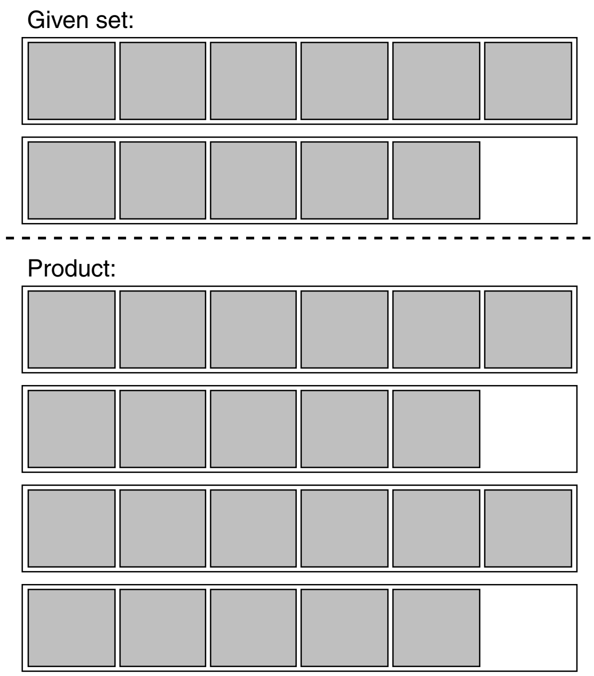
5. Discuss Exercises 1 and 3 on Worksheet 1–8a before allowing students to solve the other exercises independently.

Here is Exercise 1 on Worksheet 1–8a to discuss with the students: “Kerry sells candy in bags of 1 and 5-sixths ounces each. A customer wants 2 of the bags. How many ounces will Kerry sell to the customer?”

Students should place 1 whole bar on the top bar of Building Mat 1–8a and 5 sixths bars on the second bar of the mat. These fraction bars are in the Given Set region of the mat and represent the multiplicand, 1 and 5-sixths ounces. To help with the development of the algorithm, have students trade the 1 whole bar for 6 sixths bars on the mat. The Given Set region will then contain 11 sixths bars total. The initial fraction bars are shown here before the whole bar is traded for sixths:



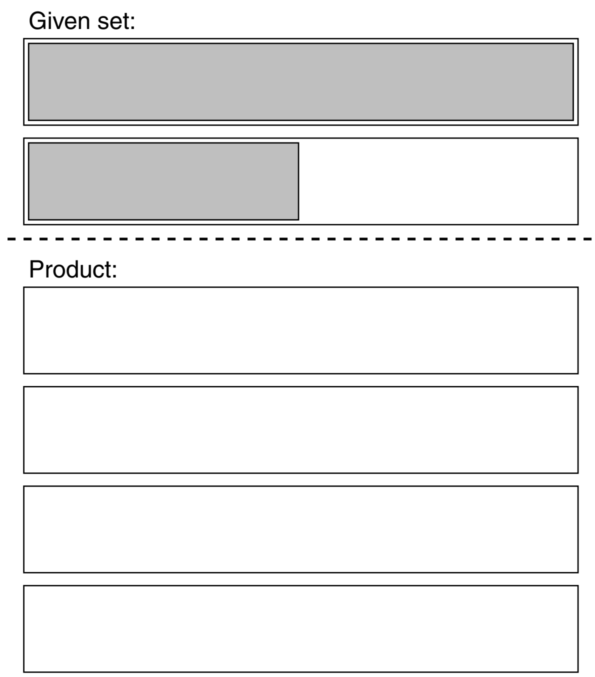
The multiplier tells how many *complete* or *partial* copies of the multiplicand are needed. Since Exercise 1 involves a whole number 2 as the multiplier, this indicates that complete copies are needed. Thus, students should build 2 complete copies of the multiplicand, 11-sixths, on the whole bars in the Product region of the building mat. The sixths bars placed on the whole bars in the lower region should be grouped like those in the upper region, so that they reflect the repetition involved. When all bars have been placed on the building mat, the building mat will appear as follows:



Now 6-sixths + 5-sixths + 6-sixths + 5-sixths, or 22-sixths total, appear in the Product region as the final product. On Worksheet 1–8a below Exercise 1, students should record a word sentence about their result. A possible sentence might be the following: "Kerry will sell 22-sixths ounces of candy, which is also 3 and 4-sixths ounces or 3 and 2-thirds ounces." Be sure to record 22-sixths as the first answer in the sentence. The numerator-denominator format will be needed to show the numerical pattern in the algorithm.

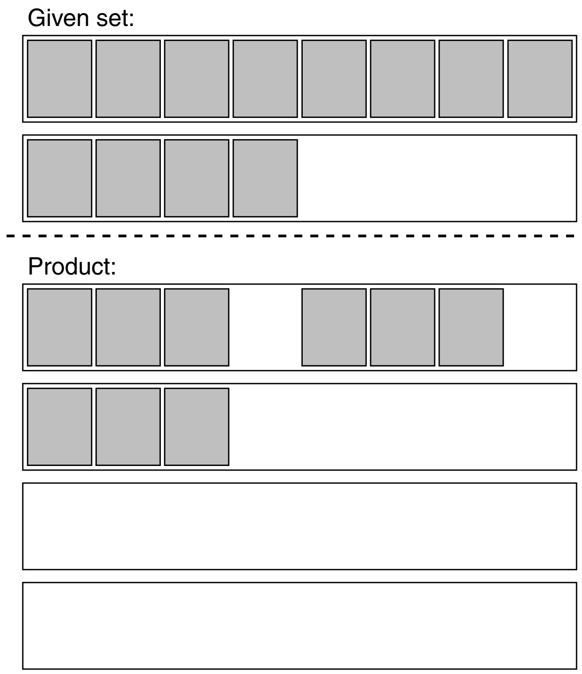
Now discuss Exercise 3 of Worksheet 1–8a with students. This exercise introduces a common fraction as the multiplier: "Jana has 1 and 1-half yards of felt. She will use 3-fourths of that amount to make a small table cover. How many yards of the felt will she use?"

Since the multiplier is 3-fourths, guide students to use the language "Copy 3 out of every 4 equal parts of 1 and 1-half" rather than "3-fourths of 1 and 1-half." Three out of every 4 equal parts indicates the action to be taken on the multiplicand. In other words, a partial copy of the multiplicand must be built, not a complete copy. Students should first place the multiplicand, 1 whole bar and 1 half bar, on the top two bars of Building Mat 1–8a as shown:



All the fraction bars in the Given Set region should now be traded to the same bar size. This procedure preserves the traditional algorithm being developed in this activity. Since a half bar is already present, the whole bar should be traded for 2 halves, making 3 halves total on the top 2 bars of the mat.

The multiplier 3-fourths now requires that students trade *each* half bar on the upper region of the mat for 4 equal parts, which will be eighths in this case, and then copy 3 of those 4 parts on a whole bar in the Product region of the mat. This trading-copying process will be done three times because there are 3 halves in the Given Set region of the mat. Each half bar should be traded for 4 new parts (eighths), and then 3 of those parts copied and placed in the Product region, before the next half bar is traded. This helps students to focus on the process indicated by the multiplier. Here is the final mat arrangement after all trading and copying have been completed:



The Product region of the mat now contains 9 eighths bars, so the answer is 9-eighths yards. Notice how the 9 eighths have been grouped into 3 sets in the lower region of the mat. This is to show how each 3-eighths corresponds to the 1-half in the upper region from which it was copied. Students should record a word sentence below Exercise 3 on Worksheet 1–8a that is similar to the following: “Jana will use 9-eighths yards of the felt, which is 1 and 1-eighth yards.” Be sure to record “9-eighths” first in the answer.

Answer Key for Worksheet 1–8a

The answer key provides possible sentences to use.

1. Kerry will sell 22-sixths ounces of candy, which is also 3 and 4-sixths ounces or 3 and 2-thirds ounces.
2. Three cartons weigh 21-eighths kilograms, or 2 and 5-eighths kilograms.
3. Jana will use 9-eighths yards of the felt, which is 1 and 1-eighth yards.
4. A one-way trip will be 3-eighths of a kilometer.
5. Josh will serve 10-twelfths of a gallon of ice cream, which is also 5-sixths of a gallon.

Building Mat 1–8a. Fraction Multiplication

Given set:

Product:

WORKSHEET 1–8a Name ____________

Building Products with Fraction Bars Date ____________

Build with fraction bars on Building Mat 1–8a to solve the word problems provided. Below each exercise, write a word sentence about the result found for that word problem. When appropriate, state both the improper fraction and the equivalent mixed number.

1. Kerry sells candy in bags of 1 and 5-sixths ounces each. A customer wants 2 of the bags. How many ounces will Kerry sell to the customer?

2. There are 3 cartons. Each carton weighs 7-eighths of a kilogram. What is the total weight in kilograms of the 3 cartons?

3. Jana has 1 and 1-half yards of felt. She will use 3-fourths of that amount to make a small table cover. How many yards of the felt will she use?

4. The round trip between home and school is 3-fourths of a kilometer. How many kilometers equal 1-half of the round trip, or a 1-way trip?

5. Josh has 1 and 1-fourth gallons of ice cream. He plans to serve 2-thirds of the ice cream for dinner. How many gallons of ice cream will be served for dinner?

Activity 2: Pictorial Stage

Materials

Worksheet 1–8b
Worksheet 1–8c
Red pencil and regular pencil

Procedure

1. Give each student a copy of Worksheet 1–8b, a copy of Worksheet 1–8c, and a red pencil. Now that students are comfortable multiplying with fraction bars on the building mat, they will work similar word problems with diagrams.

2. Each student should complete the frames on her or his own Worksheet 1–8b, but might share results with a partner.

3. For each exercise, students will subdivide whole bars in the upper region of a frame to show different fractional amounts of a whole for the multiplicand. The initial amounts will be shaded, using diagonal stripes for easy recognition.

4. If the multiplicand is a mixed number, the shaded whole bars must be traded to the smaller part size in order to have only one common part size appearing on the frame. These initial trades will be shown by marking new subdivisions with regular pencil.

5. The multiplier will then be applied to each individual part in the upper region of the frame. The denominator of the multiplier indicates how to subdivide each part of the multiplicand into new parts, and the numerator tells how many of the new parts to copy in the lower region of the frame. The subdividing into new parts should be shown with red pencil.

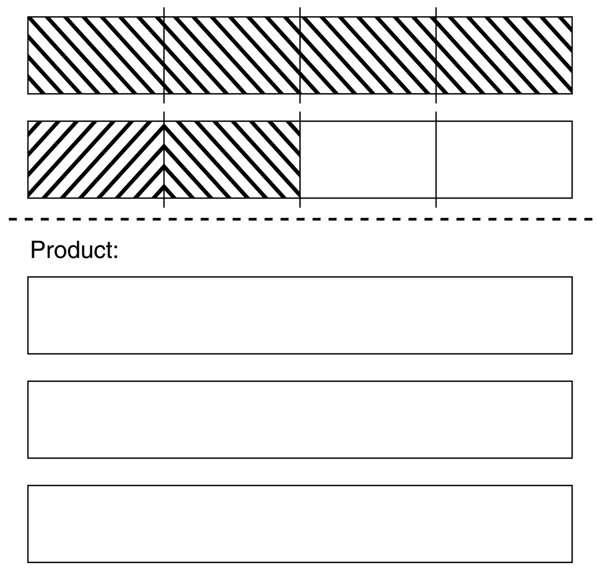
6. In order to copy parts of the correct size on the whole bars in the lower region of the frame, students must first subdivide those whole bars into the same number of parts shown on the whole bars in the upper region after subdivisions have been made with red pencil. Thus, the same final part size will be used on all whole bars of the frame. This allows a fractional name to be given to the copies drawn for the product.

7. Students must count the new parts drawn in the lower region to find the answer to the word problem. An equation for this product should be written below the exercise on Worksheet 1–8c.

8. Exercise 1 on Worksheet 1–8c should be discussed with the class in detail before students are allowed to work the additional problems on the worksheet on their own.

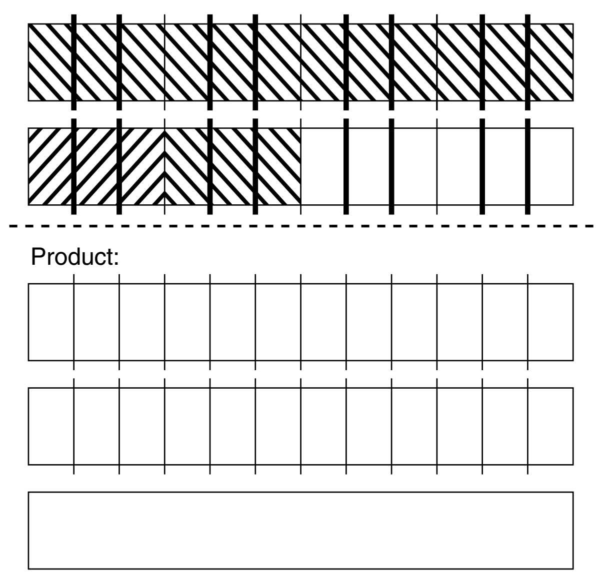
Here is Exercise 1 from Worksheet 1–8c to consider: "Harold has 1 and 2-fourths gallons of spaghetti sauce. He will need 2-thirds of that amount to serve with spaghetti at a dinner. How many gallons of sauce will he need for the dinner?"

On the first frame of Worksheet 1–8b, have students shade (with diagonal stripes) the top whole bar and 2-fourths of the second whole bar to represent the multiplicand, 1 and 2-fourths. With their regular pencil, they should subdivide the top whole bar into 4 equal parts to show fourths, so that both whole bars in the upper region of the frame contain fourths. There will be 4-fourths and 2-fourths, or 6-fourths total, in the upper region. Here is the frame after this first trade on the top bar:

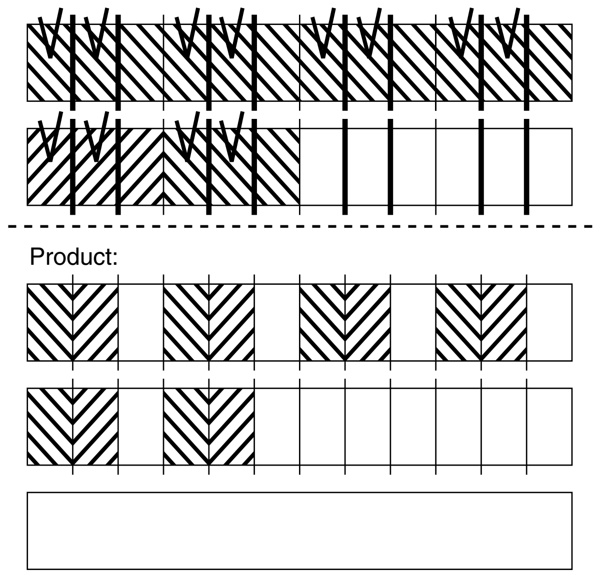


Since the multiplier is 2-thirds, remind students that this means that *each* fourth in the upper region must be subdivided into 3 equal parts. Then 2 of those 3 parts should be copied on a whole bar in the lower region of the frame. Discuss the idea that if each fourth of a whole bar in the upper region is subdivided into 3 new parts, the whole bar will be subdivided into 12 new parts total. Thus, the new parts being formed will be called twelfths of the whole bar.

Have students use regular pencil to divide the first two whole bars in the Product region of the frame into 12 equal parts per bar. They should then draw vertical segments in red pencil to subdivide each fourth in the upper region into 3 equal parts. Here is the frame after the subdividing is completed:



For every 3 shaded parts in the upper region students should mark 2 of the new parts with a red arrow. These 2 parts should be copied on one of the whole bars in the lower region by shading with regular pencil 2 of the twelfths already shown there. This process should continue until all shaded fourths have been considered. Encourage students to copy these parts in positions on the Product whole bars that are similar to their original locations in the upper region. This spacing will help students focus on the process being used. Here is how the final frame should appear:



The total shaded parts in the Product region show the final answer to be 12-twelfths of a whole. Students should write a multiplication equation below Exercise 1 on Worksheet 1–8c that represents the change of the original mixed number to an improper fraction, as well as the final product. The other subdividing of the shaded parts will be reflected only in the final answer. Do not show any other changes, such as canceling, within the equation. This is necessary in order to see the numerical patterns for the algorithms, which will be discussed in Activity 3. Be sure to record the multiplier as the first factor and the multiplicand as the second factor. Here is a possible equation to use for Exercise 1:

$$\frac{2}{3} \times 1\frac{2}{4} = \frac{2}{3} \times \frac{6}{4} = \frac{12}{12} \text{ gallons, or 1 gallon of sauce used}$$

Answer Key for Worksheet 1–8c
This answer key provides possible equations to use.

1. $\frac{2}{3} \times 1\frac{2}{4} = \frac{2}{3} \times \frac{6}{4} = \frac{12}{12}$ gallons, or 1 gallon of sauce used
2. $\frac{1}{4} \times \frac{2}{3} = \frac{2}{12}$ of a gallon of gas, or $\frac{1}{6}$ of a gallon used
3. $3 \times \frac{5}{8} = \frac{15}{8}$ liters of water, or $1\frac{7}{8}$ liters

   [Do not write a 1 below the 3; it is not necessary, as will be discussed in Activity 3.]
4. $\frac{5}{6} \times 1\frac{1}{2} = \frac{5}{6} \times \frac{3}{2} = \frac{15}{12}$ meters of fabric, or $1\frac{3}{12}$ m or $1\frac{1}{4}$ m

WORKSHEET 1–8b

Drawing Products on Fraction Bar Frames

Name ______________________

Date ______________________

1.

Product:

2.

Product:

WORKSHEET 1–8b Continued

Name ______________________

Date ______________________

3.

Product:

4.

Product:

WORKSHEET 1–8c

Name ______________________

Drawing Products Using Fractions as Factors

Date ______________________

Draw on a frame of Worksheet 1–8b to find the product for each exercise provided. Below the exercise on this worksheet, write number sentences that show the product.

1. Harold has 1 and 2-fourths gallons of spaghetti sauce. He will need 2-thirds of that amount to serve with spaghetti at a dinner. How many gallons of sauce will he need for the dinner?

2. Jody's motorcycle has 2-thirds of a gallon of gas. A trip to her aunt's house will use 1-fourth of the gas. How many gallons of gas will be used for the trip?

3. A bucket holds 5-eighths of a liter of water. How many liters of water will 3 buckets hold?

4. There are 1 and 1-half meters of fabric left on a bolt. Dana will use 5-sixths of the fabric to make some scarves. How many meters of fabric will be used in all for the scarves?

Activity 3: Independent Practice

Materials

Worksheet 1–8d
Regular pencil

Procedure

Give each student a copy of Worksheet 1–8d. Before students begin to work, discuss several exercises from Activities 1 and 2. On the board, write four or five equations from previous exercises where one factor was a whole number. For example, consider Exercise 1 from Activity 1, even though students did not record equations at that stage. The abstract equation would be $2 \times 1\frac{5}{6} = 2 \times \frac{11}{6} = \frac{22}{6}$. Do not change the answer to a mixed number at this point, and do not cancel among factors since a pattern is being sought. Have students compare the numbers used within each equation, beginning after a mixed number has been changed to an improper fraction. Guide students to discover that *the numerator of the answer equals the whole number factor multiplied by the numerator of the fraction factor, and the denominator of the answer just equals the denominator of the fraction factor.* This is a very easy pattern for students to see as long as it is developed *before* the pattern for fraction × fraction. No denominator of 1 needs to be created for the whole number factor. Students should write the italicized description in their notebooks.

Next, write four or five equations on the board that have fractions for both factors. For example, consider Exercise 5 from Activity 1: $\frac{2}{3} \times 1\frac{1}{4} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12}$. Do not reduce the final fraction, and do not cancel among the factors since a pattern is being sought. Again, have students compare the numbers used within each equation, beginning after a mixed number has been changed to an improper fraction. Guide students to discover that *the numerator of the answer equals the product of the numerators of the two fraction factors, and the denominator of the answer equals the product of the denominators of the two fraction factors.* Students should now write this second description in their notebooks.

Now have students solve each exercise on Worksheet 1–8d by writing an equation below the exercise similar to those equations recorded on Worksheet 1–8c. Encourage students to use the trading language for changing mixed numbers to improper fractions when needed and to apply the two multiplication algorithms they have discovered through their pattern searches. When all students have finished the worksheet, have several students share their answers with the rest of the class.

Answer Key for Worksheet 1–8d

1. $4 \times 2\frac{3}{10} = 4 \times \frac{23}{10} = \frac{92}{10}$ lbs., or $9\frac{2}{10}$ lbs. or $9\frac{1}{5}$ lbs.
2. $9 \times \frac{7}{12} = \frac{63}{12}$ kg, or $5\frac{3}{12}$ kg or $5\frac{1}{4}$ kg
3. $5 \times 3\frac{5}{8} = 5 \times \frac{29}{8} = \frac{145}{8}$ gal., or $18\frac{1}{8}$ gal.
4. $\frac{3}{4} \times 8\frac{4}{5} = \frac{3}{4} \times \frac{44}{5} = \frac{132}{20}$ sq m, or $6\frac{12}{20}$ sq m or $6\frac{3}{5}$ sq m
5. $\frac{1}{8} \times 35\frac{2}{3} = \frac{1}{8} \times \frac{107}{3} = \frac{107}{24}$ mi., or $4\frac{11}{24}$ mi.
6. $\frac{2}{3} \times \frac{15}{16} = \frac{30}{48}$ km, or $\frac{5}{8}$ km

Possible Testing Errors That May Occur for This Objective

- When one factor is a whole number, some students multiply the whole number by both the numerator and the denominator of the fraction factor. For example, $3 \times \frac{2}{5}$ incorrectly becomes $\frac{6}{15}$ instead of $\frac{6}{5}$.
- Students who have been taught to cancel among the factors may cancel incorrectly, producing incorrect new factors to use for the final product.
- If the product produces an improper fraction (for example, 14-tenths), students may change the improper fraction to an incorrect mixed number. A division or subtraction fact error may be the cause.
- The computation yields a fractional sum that is not in lowest terms, and students do not recognize the reduced fraction in the response choices.

WORKSHEET 1–8d

Name ______________________

Date ______________________

Multiplying Fractions and Mixed Numbers to Solve Word Problems

Solve the word problems provided. Write an equation below each exercise to show the multiplication steps used.

1. There are 4 display trays at the deli counter. Each tray holds $2\frac{3}{10}$ pounds of cheese. How many pounds of cheese total are on display at the deli counter?

2. Roy has 9 bags of birdseed for his bird feeder. Each bag weighs $\frac{7}{12}$ of a kilogram. What is the weight of all 9 bags together?

3. Candace filled each of 5 large containers with $3\frac{5}{8}$ gallons of gasoline. How many gallons total were in the 5 containers together?

4. The area of a garden is $8\frac{4}{5}$ square meters, and 3-fourths of the garden is planted with grass. How many square meters of grass are in the garden?

5. Juan drove $35\frac{2}{3}$ miles on business. He did 7-eighths of his driving during daylight. How many miles did he drive after dark?

6. Brooks jogged $\frac{15}{16}$ of a kilometer on Saturday, and his brother jogged 2-thirds of that distance. How far did his brother jog in kilometers?

Objective 9

Develop and apply scientific notation to solve word problems.

Students who are being introduced to scientific notation need activities that will help them focus on the meanings of the terms within the notation. The following lessons provide that experience. It is assumed that students have already mastered multiplication facts and the multiplication of more than two factors (for example, $3 \times 2 \times 2$). They should also understand the relationship between an exponent and its base. Thus, the focus here is on the grouping of certain factors of a number and the notation that shows such groupings.

Activity 1: Manipulative Stage

Materials

Packet of 15 paper squares, 2 inches by 2 inches, per pair of students (see packet details in step 1 of the procedure)
One flat coffee stirrer per pair of students
Worksheet 1–9a
Regular pencil

Procedure

1. Give each pair of students two copies of Worksheet 1–9a, a packet of 15 paper squares that are 2 inches by 2 inches, and one flat coffee stirrer. Five of the paper squares should have the numbers 1 through 5 written on them, with a different number on each square. Ten more squares should contain the number 10 on each square.

2. For each number on Worksheet 1–9a, write the number on the board. Some numbers will be in standard form, and others will be in scientific notation. Have students represent the number on their desktops with their paper squares.

3. Guide students to rearrange the paper squares and find a new name for the number in either scientific notation or standard form. The new name should be recorded beside the exercise on Worksheet 1–9a.

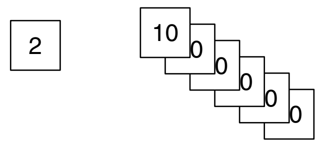
4. Discussion of Exercises 1–3 is provided. After the discussion, allow students to work the other exercises independently.

Consider Exercise 1: "Change 2,000,000 to scientific notation."

Write the number on the board: 2,000,000. Ask students to locate the left-most digit that is not zero, which will be 2, and to identify its place value position (1,000,000). Tell students to first think of the number as $2 \times 1{,}000{,}000$, then as the product of the 2 and several 10s. Have them show this last product by placing paper squares on their desktop to represent the *factors*. From left to right, the row of paper squares should show one 2 and six 10s:

[2] [10] [10] [10] [10] [10] [10]

Now have students place each set of similar factors in its own stack. The 2 will be alone, and the 10s will be in another stack. Tell them that the *exponent*, a number written higher on the writing line than the factor itself, will indicate how high the factor's stack is, and that the factors are still being multiplied, even though they are arranged in stacks. Since we are developing scientific notation, we will omit the exponent 1 when there is only one paper square in a stack:

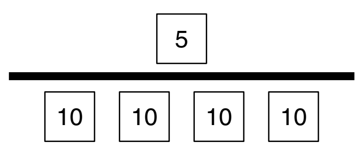


Students should now record the following beside Exercise 1 on Worksheet 1–9a: "$\mathit{2},000,000 = \mathit{2} \times (10 \times 10 \times 10 \times 10 \times 10 \times 10) = 2 \times 10^6$." Parentheses should be used to show each factor's own grouping or stack.

Now consider Exercise 2: "Change 0.0005 to standard form."

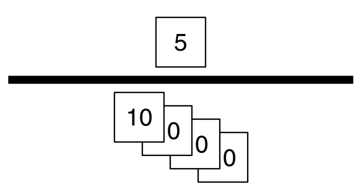
Write the number on the board: 0.0005. Have students state the word name for the number: 5 ten-thousandths. The 5 is the left-most digit that is not zero, and it is in the ten-thousandths position.

Since this is a fraction less than 1, tell students to think of 5 over 10,000. To show this fraction with paper squares, students should place the paper square for the number 5 on the desktop and place the flat coffee stirrer below the 5. Then below the coffee stirrer (the denominator position), they should place four paper squares containing 10s to show the product equal to 10,000:



The 10s should now be placed in a stack together. Since the stack of four 10s is below the 5, or in the denominator, the stack must be expressed with a negative exponent when shown as a factor in the numerator. Thus, students should record the following beside Exercise 2 on Worksheet 1–9a:

$$0.0005 = \frac{5}{10 \times 10 \times 10 \times 10} = \frac{5}{10^4} = 5 \times 10^{-4}$$



Now discuss Exercise 3: "Change 4×10^{-3} to standard form."

To work this exercise, students merely need to reverse the procedure of Exercise 2. That is, since a negative exponent is on the 10 factor, three 10s should be placed in a stack below the coffee stirrer, and one 4 should be placed above the coffee stirrer. Then the 10s should be unstacked to form a row of three 10s, representing $10 \times 10 \times 10$, or 1,000. The squares now show the fraction: 4-thousandths, whose number name is 0.004. Students should record the following beside Exercise 3:

$$4 \times 10^{-3} = \frac{4}{10^3} = \frac{4}{10 \times 10 \times 10} = 0.004$$

Answer Key for Worksheet 1–9a

1. $2{,}000{,}000 = 2 \times (10 \times 10 \times 10 \times 10 \times 10 \times 10) = 2 \times 10^6$
2. $0.0005 = \frac{5}{10 \times 10 \times 10 \times 10} = \frac{5}{10^4} = 5 \times 10^{-4}$
3. $4 \times 10^{-3} = \frac{4}{10^3} = \frac{4}{10 \times 10 \times 10} = 0.004$
4. $5{,}000{,}000{,}000 = 5 \times (10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10) = 5 \times 10^9$
5. $0.00002 = \frac{2}{10 \times 10 \times 10 \times 10 \times 10} = \frac{2}{10^5} = 2 \times 10^{-5}$
6. $3 \times 10^6 = 3 \times (10 \times 10 \times 10 \times 10 \times 10 \times 10) = 3{,}000{,}000$

WORKSHEET 1–9a
Building with Scientific Notation

Name ____________________
Date ____________________

Build with paper squares to work each exercise provided. Write an equation below each exercise to show how the given number is changed to its new form.

1. Change 2,000,000 to scientific notation.

2. Change 0.0005 to scientific notation.

3. Change 4×10^{-3} to standard form.

4. Change 5,000,000,000 to scientific notation.

5. Change 0.00002 to scientific notation.

6. Change 3×10^{6} to standard form.

Activity 2: Pictorial Stage

Materials

Worksheet 1–9b
Red pencil and regular pencil

Procedure

1. Give each student a copy of Worksheet 1–9b and a red pencil.
2. Have students change each number on Worksheet 1–9b according to the instructions. Have them express each number as the product of a decimal number less than 10 and several factors of 10. Then have students draw rings in red pencil around the groups of similar factors in the written notation. The red pencil will be used to identify factors to be represented by an exponent.
3. For each exercise, the resulting equation will reflect a notation similar to that used in Activity 1 and should be recorded below the exercise on the worksheet.
4. Discuss Exercises 1 and 2 with the class before allowing students to work the other exercises independently.

Consider Exercise 1 on Worksheet 1–9b: "Change 8,500 to scientific notation."

For the number 8,500, which is greater than 1, have students identify the place value of the left-most nonzero digit. The digit 8 is the left-most digit and is in the thousands place. The number 8,500 equals 8 thousands plus a little more; that is, $8,500 > (8 \times 1,000)$. So $\underline{8},500$ is equivalent to $\underline{8}.5 \times 1,000$. Students should then write the equation below Exercise 1 on Worksheet 1–9b as follows: "$\underline{8},500 = \underline{8}.5 \times 1,000 = \underline{8}.5 \times 10 \times 10 \times 10$," underlining the digit 8 in each expression.

Have students draw a ring in red pencil around the group of 10s (equivalent to forming a stack of paper squares at the manipulative stage). Tell them that the number of factors inside a red ring indicates the "exponent," while the factor itself is the "base." Also remind students that a single factor, like 8.5 in this example, will be written without the exponent, 1. Students should then record the final exponential form below Exercise 1. The completed equation is shown below:

$$\underline{8},500 = \underline{8}.5 \times 1,000 = \underline{8}.5 \times \boxed{10 \times 10 \times 10} = \underline{8}.5 \times 10^3$$

Now discuss Exercise 2: "Change 0.000034 to scientific notation."

The number 0.000034 is less than 1. Have students identify and underline the left-most digit that is not zero. They should also identify the digit's place value. The digit is 3, and it is in the hundred thousandths position. Since $0.000034 > 0.00003$, we also know $\frac{3.4}{100,000} > \frac{3}{100,000}$, which preserves the original place value (here shown by the denominator).

The 100,000 should be factored into a product of 10s, and a ring drawn with red pencil around the product. Since the group of 10s is in the denominator, it must be shown with a negative exponent in the numerator. Students should write the following equation below Exercise 2 on Worksheet 1–9b, with the digit 3 underlined in each expression:

$$0.0000\underline{3}4 = \frac{\underline{3}.4}{100{,}000} = \frac{\underline{3}.4}{(10 \times 10 \times 10 \times 10 \times 10)} = \underline{3}.4 \times 10^{-5}$$

Answer Key for Worksheet 1–9b

1. $\underline{8},500 = \underline{8}.5 \times 1{,}000 = \underline{8}.5 \times (10 \times 10 \times 10) = \underline{8}.5 \times 10^{3}$
2. $0.0000\underline{3}4 = \frac{\underline{3}.4}{100{,}000} = \frac{\underline{3}.4}{(10 \times 10 \times 10 \times 10 \times 10)} = \underline{3}.4 \times 10^{-5}$
3. $\underline{4}.6 \times 10^{-2} = \frac{4.6}{10 \times 10} = \frac{4.6}{100} = 0.0\underline{4}6$
4. $\underline{5}30{,}000{,}000 = \underline{5}.3 \times 100{,}000{,}000 = \underline{5}.3 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 =$ $\underline{5}.3 \times 10^{8}$
5. $0.000\underline{1}7 = \frac{1.7}{10{,}000} = \frac{1.7}{10 \times 10 \times 10 \times 10} = \underline{1}.7 \times 10^{-4}$
6. $\underline{6}.25 \times 10^{5} = \underline{6}.25 \times 10 \times 10 \times 10 \times 10 \times 10 = \underline{6}.25 \times 100{,}000 = \underline{6}25{,}000$

WORKSHEET 1–9b
Factoring with Scientific Notation

Name ____________________
Date ____________________

Ring in red pencil the factors of 10 needed to work each exercise provided. Write an equation below each exercise to show how the given number is changed to its new form.

1. Change 8,500 to scientific notation.

2. Change 0.000034 to scientific notation.

3. Change 4.6×10^{-2} to standard form.

4. Change 530,000,000 to scientific notation.

5. Change 0.00017 to scientific notation.

6. Change 6.25×10^{5} to standard form.

Activity 3: Independent Practice

Materials

Worksheet 1–9c
Regular pencil

Procedure

Give each student a copy of Worksheet 1–9c. After all have completed the worksheet, ask several students to share their answers with the rest of the class.

Answer Key for Worksheet 1–9c

1. C
2. A
3. B
4. A
5. D
6. D

Possible Testing Errors That May Occur for This Objective

- Students count the digits in the single factor along with the exponent on the 10 and ignore the decimal point. For example, in 3.5×10^4, they count the two digits in 3.5 and the exponent 4 for a total of 6 digits, which causes them to select 350,000 as the final number instead of 35,000.
- Students ignore the negative sign on the exponent and select a number greater than one. That is, the decimal is always moved to the right based on the absolute value of the exponent.
- Students use the exponent to determine the correct number of places to move the decimal, but they count in the wrong direction. For example, if they need to change 2.7 by 3 places to the left to get 0.0027, they will instead go 3 places to the right to get 2,700.

WORKSHEET 1–9c

Name ______________________

Date ______________________

Solving Word Problems with Scientific Notation

Use scientific notation to work each exercise provided.

1. The average nitrogen gas molecule has a diameter of 3.7×10^{-10} meters. Which length is equivalent to the diameter's measure?

   A. 370,000,000,000 m

   B. 3,700,000,000 m

   C. 0.00000000037 m

   D. 0.0000000037 m

2. A certain radio signal travels at 3.4×10^{8} meters per second. Which is another way to express this measure?

   A. 340,000,000 m/sec

   B. 34,000,000 m/sec

   C. 0.000000034 m/sec

   D. 0.00000034 m/sec

3. Three large cities have a total population of 9,000,000 people. This amount may also be expressed with an exponent as:

   A. 9×10^{7}

   B. 9×10^{6}

   C. 9×10^{-6}

   D. 9×10^{-7}

WORKSHEET 1–9c Continued

Name ______________________

Date ______________________

4. An average nitrogen gas molecule travels 0.000000066 meter before smashing into another gas molecule. Express this distance as a decimal number.

   A. 6.6×10^{-8} m

   B. 6.6×10^{-7} m

   C. 6.6×10^{9} m

   D. 6.6×10^{8} m

5. Hydrofluoric acid is considered a weak acid in water because it has a low K_a value of 6.8×10^{-4}. This is equivalent to which decimal number?

   A. 68,000

   B. 680,000

   C. 0.000068

   D. 0.00068

6. At certain points in its orbit, the planet Mars is about 248,000,000 miles from earth. What is this distance in scientific notation?

   A. 2.48×10^{-8} mi

   B. 2.48×10^{-9} mi

   C. 2.48×10^{7} mi

   D. 2.48×10^{8} mi

Section 1

Name ______________________

Date ______________________

NUMBER, OPERATION, AND QUANTITATIVE REASONING: PRACTICE TEST ANSWER SHEET

Directions: Use the Answer Sheet to darken the letter of the choice that best answers each question.

1.	○ A	○ B	○ C	○ D	10.	○ A	○ B	○ C	○ D
2.	○ A	○ B	○ C	○ D	11.	○ A	○ B	○ C	○ D
3.	○ A	○ B	○ C	○ D	12.	○ A	○ B	○ C	○ D
4.	○ A	○ B	○ C	○ D	13.	○ A	○ B	○ C	○ D
5.	○ A	○ B	○ C	○ D	14.	○ A	○ B	○ C	○ D
6.	○ A	○ B	○ C	○ D	15.	○ A	○ B	○ C	○ D
7.	○ A	○ B	○ C	○ D	16.	○ A	○ B	○ C	○ D
8.	○ A	○ B	○ C	○ D	17.	○ A	○ B	○ C	○ D
9.	○ A	○ B	○ C	○ D	18.	○ A	○ B	○ C	○ D

SECTION 1: NUMBER, OPERATION, AND QUANTITATIVE REASONING: PRACTICE TEST

1. A librarian arranged some books on the shelf using the Dewey decimal system. Which group of book numbers is listed in order from least to greatest?

   A. 715, 715.29, 715.3, 715.39

   B. 108.4, 108.04, 110.21, 112.0

   C. 390.5, 396.53, 398.62, 398.05

   D. 529.01, 529.10, 529.02, 529.4

2. Joe is offered sales commissions of 18%, 0.2, $\frac{1}{4}$, and $\frac{3}{20}$ of every dollar sold by four different stores, respectively. Which rate is the highest?

   A. 18% B. 0.2 C. $\frac{1}{4}$ D. $\frac{3}{20}$

3. Michelle has 12 bags of birdseed for her bird feeder. Each bag weighs 3.6 kilograms. What is the weight of all 12 bags together?

   A. 33.2 kg B. 41.2 kg C. 42.2 kg D. 43.2 kg

4. Larry bought 8 gallons of gasoline for his car at $1.39 per gallon. How much did he pay for the gasoline?

   A. $11.42 B. $11.12 C. $11.04 D. $11.03

5. A block of cheese weighs 4.8 pounds. How many full packages of sliced cheese can be made from this block if each package is to weigh 1.5 pounds?

   A. 5 pkg. B. 4 pkg. C. 3 pkg. D. 2 pkg.

6. Chui bought some CDs for a total cost of $43.75. If each CD sells for $6.25, how many CDs did Chui buy?

   A. 7 CDs B. 6 CDs C. 5 CDs D. 4 CDs

SECTION 1: NUMBER, OPERATION, AND QUANTITATIVE REASONING: PRACTICE TEST (Continued)

7. A tailor has 8.75 yards of fabric for making some vests. Each vest requires 2.6 yards. He wants to sell each vest for \$8.99. Which is a reasonable estimate of how much he should expect to earn from selling the vests?

   A. \$50 B. \$43 C. \$35 D. \$27

8. Jorge earned \$89.50 each week for 2.5 weeks at one store, then earned \$62.95 for one week at another store. Which expression is a reasonable estimate of his total earnings from the two stores?

   A. $(2 \times \$90) + \$60$ C. $(\$90 \div 3) + \$60$

   B. $(3 \times \$90) + \$60$ D. $2 \times (\$90 + \$60)$

9. Sharon hiked $11\frac{2}{5}$ miles on Monday, $10\frac{1}{2}$ miles on Tuesday, and 8 miles on Wednesday. What is the total number of miles Sharon hiked during these 3 days?

   A. 29 mi. B. $29\frac{3}{7}$ mi. C. $29\frac{9}{10}$ mi. D. Not shown here

10. A jeweler used $\frac{3}{4}$ of a meter of silver chain to make a bracelet and $\frac{7}{8}$ of a meter to make a necklace. How many meters of silver chain were used in all to make the 2 pieces of jewelry?

    A. $1\frac{5}{8}$ m B. $1\frac{1}{2}$ m C. $\frac{5}{6}$ m D. $\frac{13}{16}$ m

11. Bill bought 8 gallons of gasoline in a large container. He used $3\frac{5}{8}$ gallons of the gasoline for his boat motor. How many gallons remain in the container?

    A. $5\frac{5}{8}$ gal. B. $4\frac{5}{8}$ gal. C. $4\frac{1}{2}$ gal. D. $4\frac{3}{8}$ gal.

12. During spring break, Maria traveled $147\frac{2}{3}$ miles, and Carlos traveled $195\frac{3}{4}$ miles. How many miles shorter was Maria's trip than Carlos's trip?

    A. $52\frac{1}{12}$ mi. B. $48\frac{5}{7}$ mi. C. $48\frac{1}{2}$ mi. D. $47\frac{1}{2}$ mi.

13. If Dan has $2\frac{4}{5}$ ounces left from an 8-ounce carton of yogurt, what fractional part of the original carton does he have left?

    A. $\frac{1}{4}$ carton B. $\frac{7}{20}$ carton C. $2\frac{6}{7}$ carton D. Not shown here

SECTION 1: NUMBER, OPERATION, AND QUANTITATIVE REASONING: PRACTICE TEST (Continued)

14. Kristen rode her bike for $32\frac{1}{4}$ miles total over several days. If she averaged $5\frac{3}{8}$ miles per day, how many days did she ride her bike?

    A. 7 days B. 6 days C. 5 days D. 4 days

15. Landon jogged $2\frac{7}{10}$ kilometers on Saturday, but his brother Harry jogged only two-thirds of that distance. How far did Harry jog in kilometers?

    A. $1\frac{4}{5}$ km B. $1\frac{1}{3}$ km C. $\frac{14}{15}$ km D. Not shown here

16. Kendra filled each of 8 containers with $1\frac{5}{6}$ liters of punch for a party. How many liters of punch total were in the 8 containers together?

    A. $7\frac{2}{3}$ L B. $8\frac{5}{6}$ L C. $12\frac{1}{2}$ L D. $14\frac{2}{3}$ L

17. Four large cities have a total population of 13,000,000 people. This amount may also be expressed in scientific notation as:

    A. 1.3×10^{7} B. 1.3×10^{6} C. 1.3×10^{-6} D. 1.3×10^{-7}

18. A gas molecule has an average diameter of 2.5×10^{-10} meters. Which length is equivalent to the diameter's measure?

    A. 250,000,000,000 m C. 0.00000000025 m

    B. 2,500,000,000 m D. 0.0000000025 m

Section 1: Number, Operation, and Quantitative Reasoning: Answer Key for Practice Test

The objective being tested is shown in brackets beside the answer.

1. A [1]
2. C [1]
3. D [2]
4. B [2]
5. C [3]
6. A [3]
7. D [4]
8. B [4]
9. C [5]
10. A [5]
11. D [6]
12. C [6]
13. B [7]
14. B [7]
15. A [8]
16. D [8]
17. A [9]
18. C [9]