## Section 1

# NUMERATION AND NUMBER PROPERTIES



**Objective 1**
Order three or more whole numbers up to ten thousands.

**Discussion**
To be able to compare three or more whole numbers in the thousands or ten thousands and to order them from least to greatest or from greatest to least, students must be able to apply the place value concept. The following activities provide them with experience in making such comparisons.

**Activity 1: Manipulative Stage**

**Materials**

Building Mat 1-1a for each pair of students
100 small counters for each pair of students (same color or same style)
Worksheet 1-1a
Regular pencil

**Procedure**

1. Give each pair of students a copy of Building Mat 1-1a—a base 10 mat that contains columns for thousands, hundreds, tens, and ones (left to right). The columns should be subdivided into three rows under the column headings (see mat illustrations).

2. Each pair also needs a set of 100 small counters. The counters may be the same color and same size.

3. For practice, partners should take turns placing counters in the different spaces within a row on the mat to represent three-digit or four-digit numbers, specifically numbers from 100 to 3,000. These practice numbers should be written on the board for students to build on their mats.

4. After students are comfortable with showing individual numbers on the mat, give each student a copy of Worksheet 1-1a, which contains sets of three numbers listed in mixed order.

5. Students will show each set of three numbers on their mat at the same time, building one number per row. The numbers should be randomly ordered on the mat at first, that is, not placed in any particular row.

6. Students will then compare the quantities of counters in the three rows within the same column to determine which quantity is greatest (or least). The process is repeated for each column until the desired order is found. The final sequence is then recorded in the appropriate box on the worksheet.

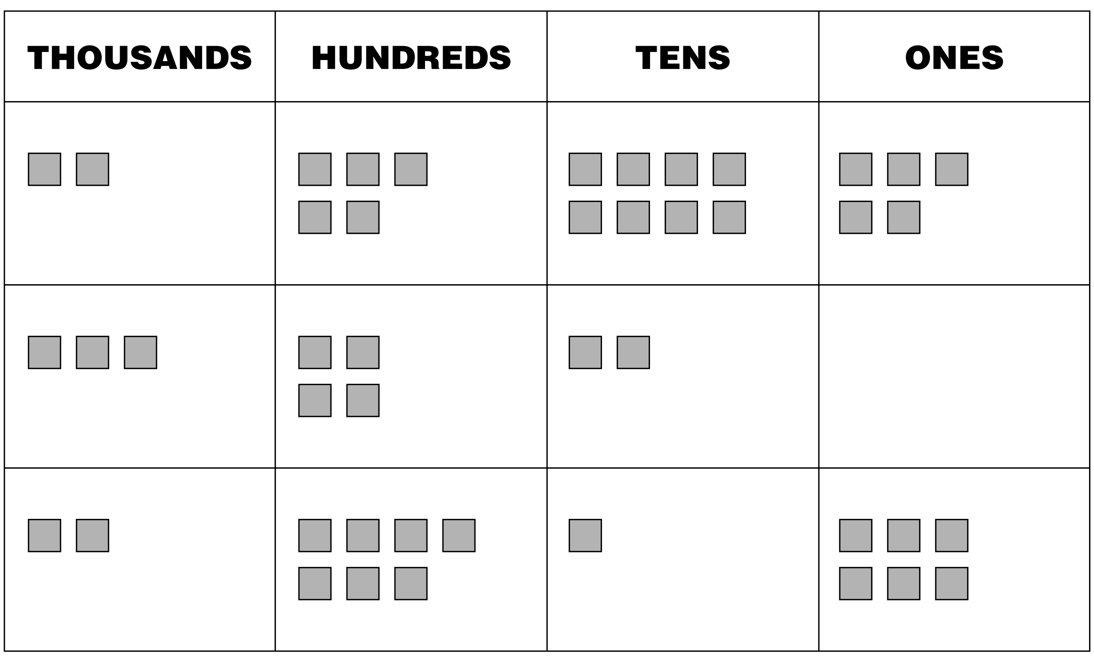
7. If numbers are to be ordered from greatest to least, the counters for the greatest number will be placed on the top row, followed by the other two numbers in correct order. If numbers are to be ordered from least to greatest, the counters for the least number will be placed on the top row, followed by the other two numbers appropriately.

8. Guide students through the first exercise before they proceed to the others.

For the first exercise on Worksheet 1-1a, consider the set that contains these three numbers: 2,716; 3,420; 2,585. The numbers are to be ordered from *greatest* to *least.*

Students should place counters in the three rows of Building Mat 1-1a to show the three numbers in some random order.

Here is a possible initial arrangement of the counters for the three numbers as they might appear on the building mat.



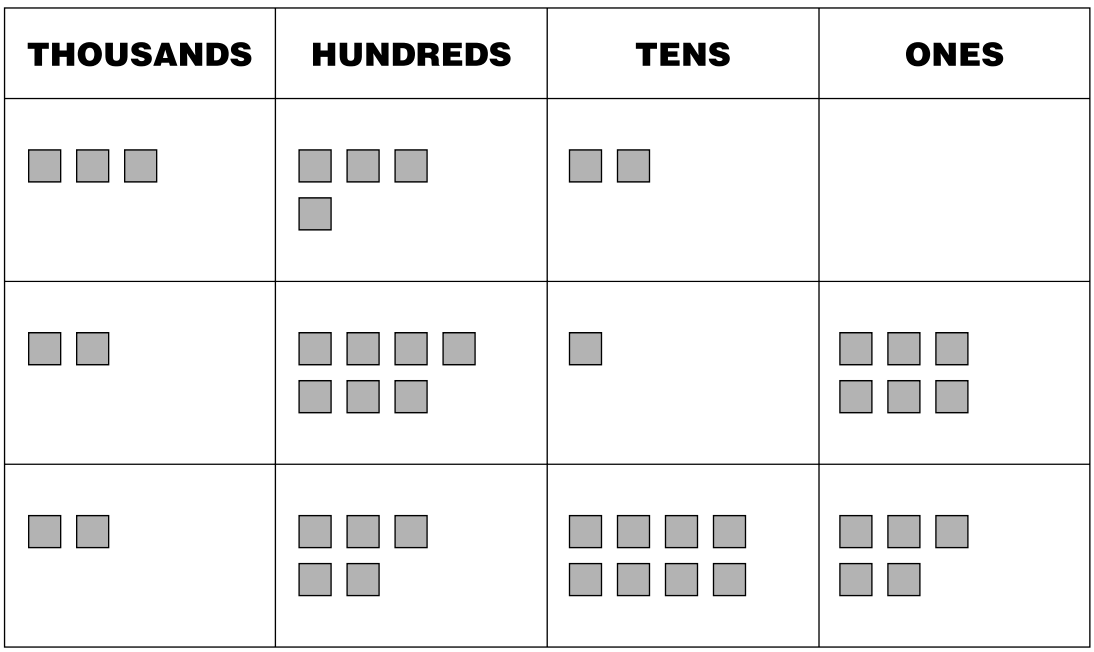

First appearance of counters on mat.

After each number has been built on the mat, ask students to look at the counters to find which number has the *most thousands.* Since 3,420 has 3 thousands and the other two numbers both have 2 thousands, have students move the counters for 3,420 to the top row of the mat, exchanging counters with whatever number was built there initially. (Note: If *two* numbers in the set had each had 3 thousands, students would have moved those two numbers to the top two rows of the mat and the third number down to the bottom row. Then the numbers in the top two rows would have been compared in their hundreds columns, and the one having more hundreds counters would be moved to the top row.) In this example, the lower two numbers have the same quantity of counters in their thousands columns, so they must now be compared in their hundreds columns.

The number 2,716 has 7 counters in the hundreds column, which is more than the other number—2,585—has. The counters for 2,716 should then be moved to the second row of the mat (if not already there) and the counters for 2,585 moved to the bottom row.

Have students record their results in the box shown under the given set of numbers on Worksheet 1-1a by writing the number name for the counters in the top row of the mat in the leftmost box, the middle row number in the middle box, and the bottom row number in the rightmost box. Since the top row of the mat was chosen for having the *greatest* number, the recording box shows the numbers sequenced in decreasing or descending order, or *from greatest to least.*

Here is how the final arrangement should appear on the mat, along with the numbers recorded in the box on the worksheet:




Final appearance of counters on mat.

| | | |
|---|---|---|
| 3,420 | 2,716 | 2,585 |

Recording box on student's worksheet.

Repeat this procedure with other sets of numbers on Worksheet 1-1a that must be arranged from *greatest* to *least.* If an exercise requires that a set of numbers be ordered from *least* to *greatest*, students should begin by finding the *least* quantity of counters in the thousands column and moving the corresponding number to the *top row* of the mat. Continue the comparing of the other two numbers to find which one has *fewer* counters in the hundreds column; the counters for this second number identified should then be moved to the middle row of the mat. The third number's counters will end up on the bottom row of the mat. Since the number in the top row is always recorded in the leftmost box, the final recording of the three numbers will list the numbers *from least to greatest.* (Note: If *two* numbers in the set had each had the least thousands, students would have moved those two numbers to the top two rows of the mat and the third number down to the bottom row. Then the numbers in the top two rows would have been compared in their hundreds columns, and the one having fewer hundreds counters would be moved to the top row.)

**Answer Key for Worksheet 1-1a**

1. 3,420; 2,716; 2,585
2. 3,172; 3,086; 1,843
3. 3,006; 3,530; 4,125
4. 3,618; 4,237; 5,207
5. 3,758; 3,728; 3,258

**BUILDING MAT 1-1a**

| THOUSANDS | HUNDREDS | TENS | ONES |
| --- | --- | --- | --- |
| | | | |
| | | | |
| | | | |

WORKSHEET 1-1a

## Ordering Whole Numbers by Building

Name ______________________

Date ______________________

Use counters on Building Mat 1-1a to order each given set of whole numbers. Then record the final arrangement of the three numbers in the box below the given set, recording from left to right in the required order.

1. Order from greatest to least: 2,716; 3,420; 2,585.

| | | |
|---|---|---|
| | | |

2. Order from greatest to least: 1,843; 3,172; 3,086.

| | | |
|---|---|---|
| | | |

3. Order from least to greatest: 3,006; 4,125; 3,530.

| | | |
|---|---|---|
| | | |

4. Order from least to greatest: 5,207; 3,618; 4,237.

| | | |
|---|---|---|
| | | |

5. Order from greatest to least: 3,258; 3,758; 3,728.

| | | |
|---|---|---|
| | | |

### Activity 2: Pictorial Stage

## Materials

Worksheet 1-1b
Regular pencil

## Procedure

1. Give each student a copy of Worksheet 1-1b. Have students work with partners, but each student will complete her or his own worksheet.

2. For each set of four whole numbers given on Worksheet 1-1b, students will represent the numbers with small circles on the base 10 frame on the worksheet. After they have determined the correct order of the four numbers, they will record the numbers in that order below the base 10 frame.

3. Some sets will be ordered from least to greatest, while others will be ordered from greatest to least.

4. Guide students through the first exercise on Worksheet 1-1b, then allow them to continue with the other exercises.

For the first exercise, consider this set of numbers: 3,425; 5,401; 3,213; 1,140. The first number listed in the set (3,425) should be drawn in the top row of the base 10 frame, the second number in the second row, and so on. To show a number, students should draw small circles in each column of the chosen row to represent each digit of the number.

In this exercise, the numbers are to be ordered *from least to greatest*. The greatest *place value* involved in the four numbers is thousands. Ask students to find the number with the *least* quantity of circles in the thousands column. Have them write #1 to the left of that number's row on the frame. The number is 1,140. Then have students find the number with the *greatest* quantity of circles in the thousands column and write #4 to the left of that number's row. The number is 5,401.

For the two remaining numbers, if their thousands columns differed, students would label the lower amount as #2 and the greater amount as #3. In this example, however, the thousands columns are equal in 3,425 and 3,213. Students must now compare the hundreds columns of the two numbers (they would continue to tens and ones, if necessary). The number with the lower quantity of hundreds (3,213) becomes #2. The remaining number (3,425) becomes #3.

After all four numbers are labeled, students should write the numbers in sequence below the frame, following the labeled ordering. This exercise's final frame and recorded sequence *from least to greatest* are shown here:

| | TEN THOUSANDS | THOUSANDS | HUNDREDS | TENS | ONES |
|---|---|---|---|---|---|
| **#3** | | ○○○ | ○○○○ | ○○ | ○○○○○ |
| **#4** | | ○○○○○ | ○○○○ | | ○ |
| **#2** | | ○○○ | ○○ | ○ | ○○○ |
| **#1** | | ○ | ○ | ○○○○ | |

Final Order: 1,140; 3,213; 3,425; 5,401

**Answer Key for Worksheet 1-1b**

1. 1,140; 3,213; 3,425; 5,401
2. 6,119; 5,026; 4,840; 4,385
3. 8,705; 9,235; 12,483; 12,630
4. 32,550; 32,450; 30,859; 30,824
5. 24,800; 24,760; 24,318; 24,035
6. 5,875; 8,860; 10,520; 14,423

WORKSHEET 1-1b

Name ____________________

Ordering Whole Numbers by Drawing

Date ____________________

Order each set of four whole numbers by drawing and comparing small circles on the frame provided. Then record the final arrangement of the four numbers below the frame, recording from left to right in the required order.

1. Order from least to greatest: 3,425; 5,401; 3,213; 1,140.

| TEN THOUSANDS | THOUSANDS | HUNDREDS | TENS | ONES |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

Final Order: ____________________________________________

2. Order from greatest to least: 4,385; 6,119; 4,840; 5,026.

| TEN THOUSANDS | THOUSANDS | HUNDREDS | TENS | ONES |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

Final Order: ____________________________________________

WORKSHEET 1-1b Continued

Name ____________________

Date ____________________

3. Order from least to greatest: 12,483; 8,705; 12,630; 9,235.

| TEN THOUSANDS | THOUSANDS | HUNDREDS | TENS | ONES |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

Final Order: ____________________________________________

4. Order from greatest to least: 32,450; 30,824; 30,859; 32,550.

| TEN THOUSANDS | THOUSANDS | HUNDREDS | TENS | ONES |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

Final Order: ____________________________________________

WORKSHEET 1-1b Continued

Name ____________________

Date ____________________

5. Order from greatest to least: 24,318; 24,760; 24,035; 24,800.

| TEN THOUSANDS | THOUSANDS | HUNDREDS | TENS | ONES |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

Final Order: ____________________

6. Order from least to greatest: 14,423; 8,860; 10,520; 5,875.

| TEN THOUSANDS | THOUSANDS | HUNDREDS | TENS | ONES |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

Final Order: ____________________

### Activity 3: Independent Practice

## Materials

Worksheet 1-1c
Regular pencils

## Procedure

Students work independently to complete Worksheet 1-1c. When all are finished, discuss the results.

**Answer Key for Worksheet 1-1c**

1. 7,351; 5,860; 5,249; 3,085
2. 11,845; 11,860; 12,400; 12,567
3. 9,114; 8,607; 8,549; 7,235; 7,102
4. 8,945; 9,845; 14,000; 20,873; 25,078
5. D
6. C

**Possible Testing Errors That May Occur for This Objective**

- The numbers are sequenced by size but in reverse order; for example, they are arranged in decreasing order, but the test item requires them to be in increasing order. Students clearly understand how to compare and order numbers, but they may not understand that "from least to greatest" means increasing and that "from greatest to least" means decreasing.
- The first and last numbers listed in the sequence are correct, but the other numbers are randomly ordered between those two numbers. Students focus on the "least" and the "greatest" numbers but disregard any others given in the list.
- The numbers are randomly sequenced without regard for value. Students do not understand the ordering process.

WORKSHEET 1-1c

Name ____________________

Date ____________________

In exercises 1 to 4, list each set of whole numbers in the given order:

1. Order from greatest to least: 5,249; 3,085; 7,351; 5,860.

2. Order from least to greatest: 12,400; 11,845; 11,860; 12,567.

3. Order from greatest to least: 8,549; 7,235; 8,607; 9,114; 7,102.

4. Order from least to greatest: 25,078; 9,845; 8,945; 14,000; 20,873.

In exercises 5 and 6, circle the letter of the correct response:

5. Which group of numbers is in order from *least* to *greatest*?

   A. 2,027 2,426 2,409 2,512
   B. 2,409 2,512 2,426 2,027
   C. 2,512 2,426 2,409 2,027
   D. 2,027 2,409 2,426 2,512

6. Which group of numbers is in order from *greatest* to *least*?

   A. 36,943 45,188 37,912 45,395
   B. 37,912 45,395 36,943 45,188
   C. 45,395 45,188 37,912 36,943
   D. 36,943 37,912 45,188 45,395

## Objective 2

Identify odd and even whole numbers.

## Discussion

Students have difficulty keeping *even* and *odd* numbers separated, mainly because they have never developed the two concepts and do not see the two types as complements of each other. Basically, *even* quantities are quantities that can be separated into two equal sets or amounts without extras being left over.

Hence, we have the idea that things come out *even*, meaning with no leftovers. Using counters arranged in two rows, students can discover for themselves what numbers are considered *even*. Students need to see how even and odd numbers relate to each other in order to be able to apply their general forms (2N + 1 and 2N) later on in algebra and other higher mathematics courses. The following activities are designed to help students differentiate between even and odd numbers and better understand the labels used for them.

### Activity 1: Manipulative Stage

## Materials

40 small counters per pair of students (1-inch paper squares, small disks, or square tiles)
Worksheet 1-2a
Paper and regular pencil

## Procedure

1. Give each pair of students a set of 40 counters (for example, 1-inch paper squares, small disks, or square tiles) and two copies of Worksheet 1-2a.

2. The worksheet contains several sets of four different numbers per set (two even and two odd, two large and two small) from 1 to 40. Assign each set to several different pairs of students.

3. For each number assigned to a pair of students, the students should try to arrange that quantity of counters or tiles in exactly *two* equal rows, if possible. Consider a row as going from left to right for this activity.

4. After all numbers have been built with the tiles, ask students to report which of their numbers made two equal rows and which made two unequal rows. Record their numbers in the appropriate column on the classroom board under the headings: "Equal Rows" and "Unequal Rows." Students should also record all reported numbers in the table on Worksheet 1-2a. Do not mention the ideas of *even* and *odd* at this time.

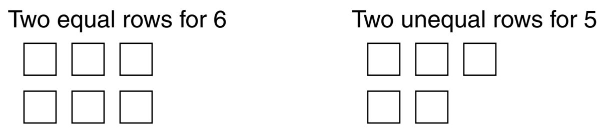
5. Because one tile cannot be arranged in *two* equal rows, the number 1 must be recorded in the "Unequal Rows" column, even though it makes only one row.

6. After all numbers have been recorded, ask students for their observations about the numbers in the different columns. Accept whatever reasonable ideas they might have at this time. Do not rush them to notice the digits in the ones place value position at this time. (Possible observation: In the "Unequal Rows" column for these numbers, one row of tiles had one more tile than the other row had.)

7. Guide students through building arrangements for two examples (5 and 6) before allowing them to build their own assigned set of numbers.

Here are examples for the numbers 5 and 6. Have students draw pictures of the arrangements on Worksheet 1-2a, then record the number 6 in their table under "Equal Rows" and record the number 5 under "Unequal Rows." Remind students that a "row" is considered as going from left to right in this activity.

Here are samples of built, then drawn, tile arrangements:




**Answer Key for Worksheet 1-2a Table**

Equal Rows: 6, 4, 20, 2, 26, 8, 30, 10, 24, 12, 36, 14, 28, 32, 16, 34

Unequal Rows: 5, 7, 31, 11, 35, 17, 23, 1, 29, 3, 27, 15, 21, 9, 19, 13, 37

WORKSHEET 1-2a
Building Equal Rows

Name ______________________
Date ______________________

Use small counters or tiles to build a two-row arrangement for each number in one of the given sets. The teacher will assign the set for you to build. Try to build two *equal* rows of tiles each time. On the back of the worksheet, draw a picture of each tile arrangement you build, then record the numbers you used in the appropriate columns of the table.

| Examples: | 5 | 6 |
|---|---|---|
| Row 1 | ________ | ________ |
| Row 2 | ________ | ________ |

Possible Sets to Build:

a. 4, 20, 7, 31
b. 2, 26, 11, 35
c. 8, 30, 17, 23
d. 10, 24, 1, 29
e. 12, 36, 3, 27
f. 14, 28, 15, 21
g. 2, 32, 9, 19
h. 16, 34, 13, 37

| Equal Rows | Unequal Rows |
|---|---|
| | |

**Activity 2: Pictorial Stage**

## Materials

Inch grid paper (8.5 inches by 11 inches)
Colored pencils, regular pencils
Scissors
Tape
Worksheet 1-2a (completed in Activity 1)
Worksheet 1-2b

## Procedure

1. Give each pair of students a sheet of inch grid paper, a colored pencil, scissors, and tape. Also give each student a copy of Worksheet 1-2b.

2. Assign each pair of students two different numbers from 10 to 50 (one even and one odd but not consecutive numbers). If possible, give each pair one small number and one large number to make the task easier. Here are suggested number pairs to use: (20, 49), (14, 43), (12, 41), (30, 17), (22, 47), (38, 19), (42, 21), (44, 11), (46, 15), (50, 25), (48, 23), (16, 39).

3. Each assigned number should be colored on the grid paper as two equal rows of squares, if possible. Grid pieces may have to be taped together in order to make some of the larger numbers. The represented number should be written on the colored grid spaces.

4. The arrangement of grid spaces should then be cut out and taped on the board under the appropriate headings: "Equal Rows" and "Unequal Rows." Do not try to order the cutouts by size; allow random placement.

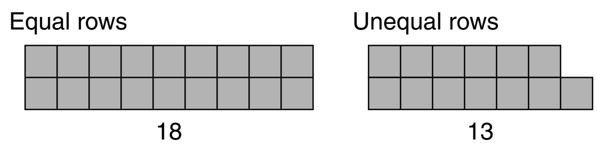
5. Once all the paper cutouts are taped in their proper columns, ask for observations again. Students should notice that all those in the "Unequal Rows" column have one extra square on the end. Tell students that since the cutouts in the "Equal Rows" column do not have the extra square, they will represent *even* numbers (their rows come out "even"). Thus, the numbers represented in the other column (unequal) will *not* be *even* numbers; therefore they will be called *odd* numbers (the extra square makes an "odd-sized" row).

6. Now write the number name beside each paper cutout on the board. Students should also record each of these numbers in the proper column of the table on Worksheet 1-2b. Record in the order shown on the board.

7. Have students take out their completed copies of Worksheet 1-2a. Ask students what they notice about the numbers in the tables of Worksheet 1-2a and Worksheet 1-2b. If necessary, call attention to the different digits in the tens and ones places. Ideally, students will notice that, for numbers in both columns of the two tables combined, the digit in the tens place can be 0 through 5 (since they only went to 50 with their numbers in the two activities), but the newly named *even* numbers have 0, 2, 4, 6, or 8 in the ones place, whereas the *odd* numbers have 1, 3, 5, 7, or 9 in the ones place. Thus, the ones place is really the indicator for even and odd numbers. Have students record their ideas at the bottom of Worksheet 1-2b.

8. Guide students through two examples of coloring grid spaces, cutting the shapes out, taping them on the board, and writing the appropriate numbers beside the cutout shapes before allowing the students to color their assigned amounts.

The following are examples for the numbers 13 and 18 and the column headings where their cutouts will be placed and the numbers recorded:




**Answer Key for Worksheet 1-2b**

Even Numbers: (unordered in table) 18, 20, 14, 12, 30, 22, 38, 42, 44, 46, 50, 48, 16

Odd Numbers: (unordered in table) 13, 49, 43, 41, 17, 47, 19, 21, 11, 15, 25, 23, 39

WORKSHEET 1-2b
Sorting Even and Odd Numbers

Name ______________________
Date ______________________

After cutting out grid shapes for different whole numbers assigned by the teacher, then sorting them on the board, record each number in its correct column in the table below.

| Equal Rows (Even Numbers) | Unequal Rows (Odd Numbers) |
|---|---|
| | |

Describe in your own words how *even* numbers differ from *odd* numbers.

______________________________________________________________

______________________________________________________________

______________________________________________________________

______________________________________________________________

______________________________________________________________

______________________________________________________________

______________________________________________________________

______________________________________________________________

______________________________________________________________

### Activity 3: Independent Practice

## Materials

Worksheet 1-2c
Regular pencil
Calculators (optional)

## Procedure

Students work independently to investigate numbers between 50 and 999 to see which of those amounts can be separated into two equal groups. This will allow students to see that in both even and odd numbers the tens and hundreds places can have 0 through 9. Discuss the idea that separating a quantity of counters into two equal rows can be represented by applying the written algorithm for division to the given number, using 2 as the divisor. If the two rows are equal, the remainder for the algorithm will be 0. If the two rows are unequal, the remainder will be 1. Since this is a discovery lesson, allow students to use the calculator if their division skills are weak. Any needed algorithmic review should occur at another time.

### Answer Key for Worksheet 1-2c

1. Even: 98, 86, 574, 128, 432, 700, 980; Odd: 65, 73, 751, 607, 831, 319, 245
2. Answers will vary.
3. Answers will vary.

### Possible Testing Errors That May Occur for This Objective

- A set of *consecutive* numbers such as 12, 13, 14, 15 is selected because students do not know the meaning of *even* and *odd* numbers.
- Students know that the two sets of numbers (0, 2, 4, 6, . . . and 1, 3, 5, 7, . . .) are different from each other, but they are confused as to the appropriate label for each set. For example, when the test item asks for a set of *odd* numbers to be identified, students select a set of *even* numbers such as 8, 10, 12, 14. Vocabulary needs to be emphasized.
- When students are asked to select an individual number that represents an *odd* number, they incorrectly select 0 as the *odd* number.

WORKSHEET 1-2c
Larger Even and Odd Numbers

Name ____________________
Date ____________________

1. Use division to decide whether each number listed below is even or odd. Write the correct name (even or odd) beside each number.

| | |
|---|---|
| 98 | 432 |
| 65 | 607 |
| 73 | 700 |
| 86 | 831 |
| 574 | 319 |
| 751 | 245 |
| 128 | 980 |

2. Now look at the digits in the hundreds, tens, and ones place value positions of the listed numbers. What do you notice? Describe in your own words a "rule" for deciding when a number is even or odd.

3. Practice: (a) List three *even* numbers between 165 and 483.

(b) List three *odd* numbers between 298 and 352.

## Objective 3

Form a generalization of the pattern found in a given ordered set (or sequence) of whole numbers, then generate more members of that set using that generalization.

## Discussion

Students need much practice with finding and extending patterns in sequences, particularly as the sequences occur within tables of values. Working with tables of values will provide excellent readiness for the study of functions in algebra in later years. The following activities offer such needed practice. Section 3, Objective 1 will provide additional experience in applying tables to solve word problems.

### Activity 1: Manipulative Stage

## Materials

100 square tiles or 1-inch paper squares for every four students
Worksheet 1-3a (two-column table)
Regular pencil

## Procedure

1. Give each group of four students a set of 100 square tiles, and give each individual student a copy of Worksheet 1-3a.

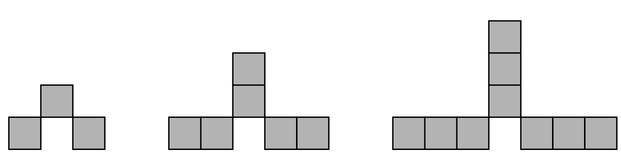
2. Have each group build a simple, flat design several times, increasing the tiles according to the same pattern or method each time to gradually enlarge the design. Students should complete the table on Worksheet 1-3a as they build, each time recording which design it is (or position in the sequence) and how many tiles are in the design.

3. After they have built the first four designs, ask them if they can predict how the sixth design will look. They should then continue to build their designs up to the sixth design in order to confirm their prediction.

4. Ask them how the numbers are changing in the left column (increase by 1 each time) and in the right column (increase by 3 each time for the three-wing design and by 4 each time for the tower design shown below).

5. Guide students through the building of the first sequence presented next and the recording of the amounts in the first table on Worksheet 1-3a. Then have them build the second sequence, using the tower design.

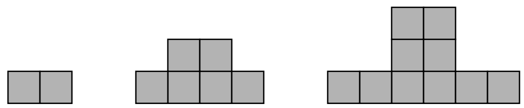
For the first sequence, have students build the following three shapes (three-wing designs) in order with their tiles:



Students should build the fourth design (four tiles per wing), then predict the sixth design. They should then confirm their prediction by building the fifth and the sixth designs. The finished table should show 1, 2, 3, 4, 5, and 6 in the left column and 3, 6, 9, 12, 15, and 18 in the right column.

Ask students how the numbers are changing as they go down the left column. (The numbers are increasing by 1 each time.) Then ask how the numbers are changing as they go down the right column. (The numbers are increasing by 3 each time.) So the next design will need three more tiles to build it than the previous design needed.

Now have students build the second sequence, using a tower design that increases by 4 tiles each time. Here are the first three designs for this second sequence:



**Answer Key for Worksheet 1-3a**

1. 3-wing: left column—1, 2, 3, 4, 5, 6

   right column—3, 6, 9, 12, 15, 18

2. Tower: left column—1, 2, 3, 4, 5, 6

   right column—2, 6, 10, 14, 18, 22

WORKSHEET 1-3a
Building Sequences of Shapes

Name ______________________
Date ______________________

Follow your teacher's instructions to complete each table below as you build different shapes according to a pattern.

1.

| NUMBER OF DESIGN | NUMBER OF TILES |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |

2.

| NUMBER OF DESIGN | NUMBER OF TILES |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |

### Activity 2: Pictorial Stage

## Materials

Worksheet 1-3b (tables)
Regular paper and pencils

## Procedure

1. After students have built several sequences of designs with the tiles, have students work with partners to draw several new sequences on their own paper. Give each student a copy of Worksheet 1-3b.

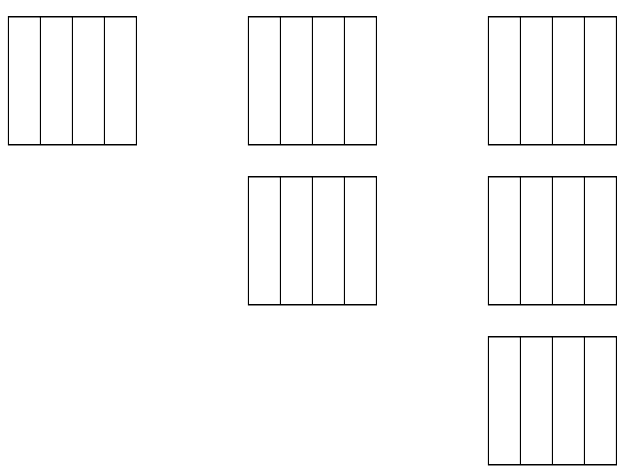
2. Follow the "predict-confirm" procedure described in the Manipulative Stage. Draw three designs of a sequence on the board for students to copy on their own papers, then extend to the sixth design.

3. For each completed sequence, students should complete a table on Worksheet 1-3b.

4. When students are finished with each sequence, ask about the changes in each column (or each row) of the table.

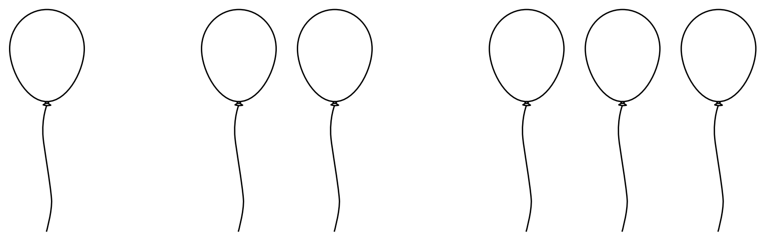
5. Guide students through drawing the first sequence and completing the table before having them continue with the other sequences.

For the first sequence, consider a real-world situation involving shelves and books in a bookcase. Have students draw simple diagrams to represent the objects. In this example, the first "set," or 1 shelf, consists of 4 books. Each new set must increase by 4 books over the previous set. The top row of the table will show "Number of Shelves," so 1 will be the first entry and 2 the second entry. The bottom row will show "Total Books," so 4 will be the first entry there and 8 the second entry. The numbers in the top row of the table will increase by 1 each time, and the numbers in the bottom row will increase by 4 each time. Here are the first three "sets" shown as diagrams ordered from left to right:

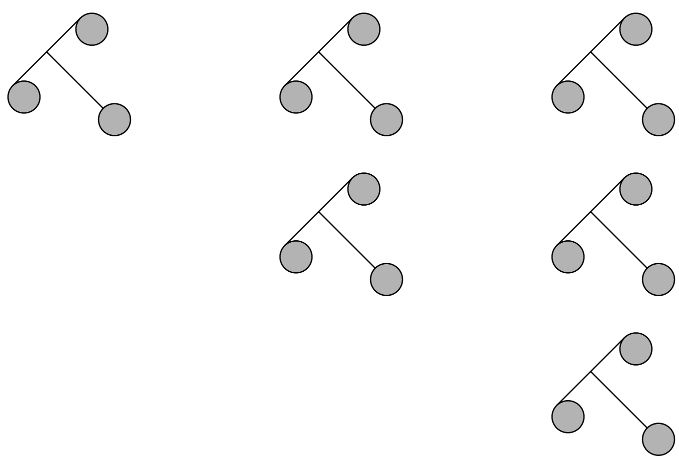


Here are three more sequences (with their first three possible designs) to use for exercises 2, 3, and 4 on Worksheet 1-3b:

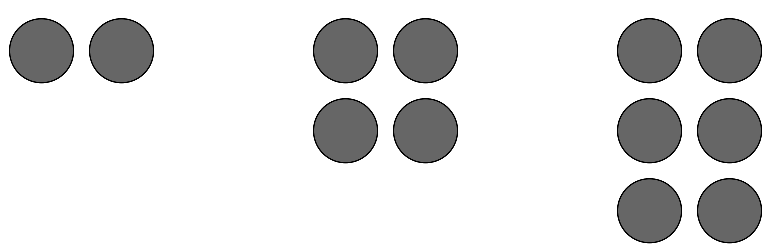
2. Number of Clowns vs. Total Balloons (1 balloon per clown)



3. Number of Vehicles vs. Number of Wheels (3 wheels per vehicle)



4. Number of Children vs. Number of Cookies (2 cookies per child)



**Answer Key for Worksheet 1-3b**

1. Top: 1, 2, 3, 4, 5, 6; Bottom: 4, 8, 12, 16, 20, 24
2. Top: 1, 2, 3, 4, 5, 6; Bottom: 1, 2, 3, 4, 5, 6
3. Left: 1, 2, 3, 4, 5, 6; Right: 3, 6, 9, 12, 15, 18
4. Left: 1, 2, 3, 4, 5, 6; Right: 2, 4, 6, 8, 10, 12

WORKSHEET 1-3b — Name ____________________

Drawing Sequences — Date ____________________

Follow your teacher's instructions to complete each table below as you draw different shapes or diagrams according to a pattern.

1.

| **Number of Shelves** | | | | | | |
|---|---|---|---|---|---|---|
| **Total Books** | | | | | | |

2.

| **Number of Clowns** | | | | | | |
|---|---|---|---|---|---|---|
| **Total Balloons** | | | | | | |

3.

| **Number of Vehicles** | **Number of Wheels** |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |

4.

| **Number of Children** | **Number of Cookies** |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |

**Activity 3: Independent Practice**

## Materials

Worksheet 1-3c (tables)
Regular pencils

## Procedure

Students work independently to complete the number sequences and tables on Worksheet 1-3c. Encourage them to look for how the numbers are changing in each row (or column of a table), then use that information to find the missing values.

### Answer Key for Worksheet 1-3c

1. A. 2, 4, 6, 8, 10, 12; B. 3, 6, 9, 12, 15, 18
2. A. 1, 5, 9, 13, 17, 21; B. 4, 11, 18, 25, 32, 39
3. Left: 1, 2, 3, 4, 5, 6; Right: 4, 8, 12, 16, 20, 24
4. Left: 1, 2, 3, 4, 5, 6; Right: 5, 10, 15, 20, 25, 30

### Possible Testing Errors That May Occur for This Objective

- The rate at which the first few numbers in the sequence have changed is not held constant to generate new terms of the sequence. For example, in the given sequence 3, 8, 13, . . ., the rate of change is 5 for the first three terms, but the student uses a rate of 7 to find 20 as the fourth term instead of 18.
- Only the first new entry for a given sequence is found when the test item asks for the second new entry instead. For example, in the sequence 3, 5, 7, . . ., the student selects the fourth term (9) as the answer instead of the required fifth term (11).
- The next new entry for a sequence is found by adding 1 to the value of the previous term, even though the rate of change for the first few terms is greater than 1. For example, the rate of change for 5, 8, 11, 14, . . . is 3, but the student selects 15 as the next term after 14.

WORKSHEET 1-3c Name ____________

Finding Changes in Sequences Date ____________

Look for patterns and complete the number sequences in items 1 and 2.

1. A. 2, 4, 6,____,____,____

   B. 3, 6, 9,____,____,____

2. A. 1, 5, 9,____,____,____

   B. 4, 11, 18,____,____,____

In items 3 and 4, find how the numbers change in each column of the table, then complete the blanks with the correct numbers.



3.

| Number of Cars | Number of People |
|---|---|
| 1 | 4 |
| 2 | 8 |
| 3 | 12 |
| | |
| 5 | |
| | 24 |

4.

| Number of Barrels | Number of Gallons |
|---|---|
| 1 | 5 |
| 2 | 10 |
| | 15 |
| | 20 |
| 5 | |
| | |

## Objective 4

Round whole numbers to the nearest ten, hundred, or thousand.

## Discussion

Students have great difficulty remembering the rule that is commonly given in the classroom for rounding whole numbers to a given place value. The skill of rounding is important when students need to estimate the answer to a particular computation; they must first round, then apply mental arithmetic to the rounded numbers to obtain the desired estimate. The following activities will help students develop their own rule for the rounding process.

### Activity 1: Manipulative Stage

## Materials

80 small counters in one color (color A)
30 small counters in another color (color B) for each pair of students
Building Mat 1-4a
Worksheet 1-4a
Regular pencil

## Procedure

1. Have students work in pairs to round numbers to the nearest ten, hundred, or thousand. Give each pair a copy of Building Mat 1-4a and two sets of small counters: 80 in color A and 30 in color B. All the counters should be the same size.

2. For practice, partners should take turns placing counters (color A) in the different spaces within a row on the mat to represent 3-digit or 4-digit numbers, specifically, numbers from 100 to 7,000. These practice numbers should be written on the board for students to build on their mats. For example, to show 3,275 in the first row of the building mat, students should place 3 counters in the thousands column, 2 counters in the hundreds column, 7 counters in the tens column, and 5 ones in the ones column.

3. After students are comfortable with showing individual numbers on the mat, have them build each number listed on Worksheet 1-4a on their mats, using color A counters, and round that number to the stated place value. In each case, they must decide how many *extra* ones, tens, or hundreds counters (color B) are needed to increase the given number to the next higher required place value amount. They must also decide how many *original* ones, tens, or hundreds (color A) must be removed from the building mat to decrease the given number to the nearest lower required place value amount. If the value of the extra counters (color B) needed is less than the value of the counters to be removed, the given number will be rounded up. If the value to be removed is less, the given number will be rounded down. If the two values are equal, we will agree to round up.

4. Guide students through each exercise on Worksheet 1-4a. Once students decide whether to round a number up or down, they should record the rounded result in the appropriate blank on the worksheet.

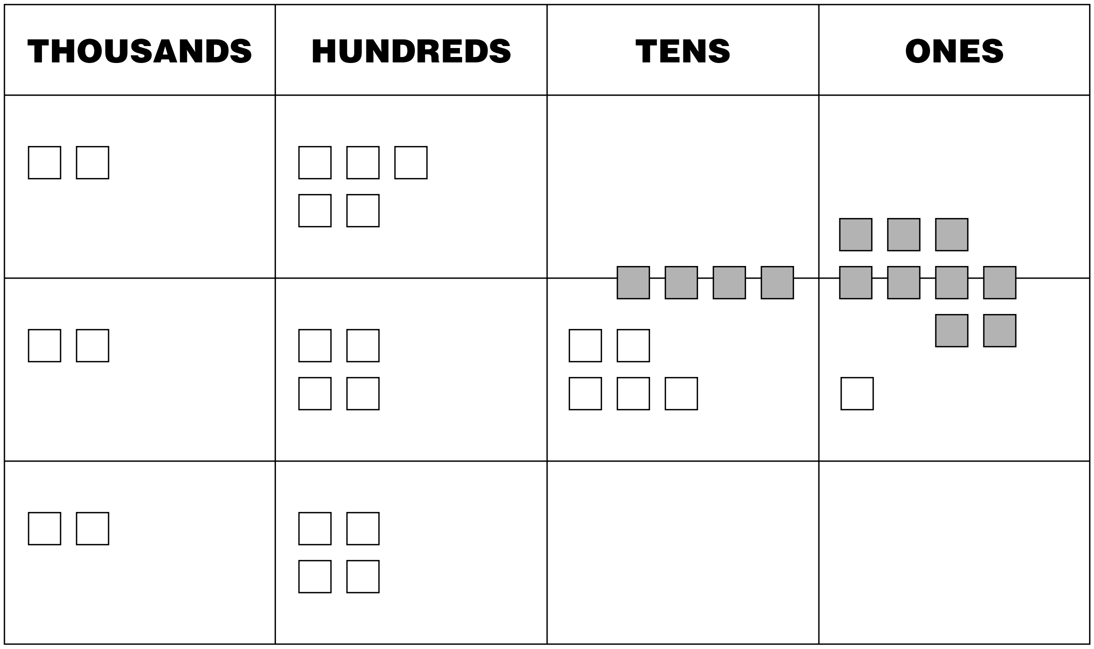
Here is the procedure to follow for rounding 2,451—the first number listed on Worksheet 1-4a—to the nearest hundred. Ask students to show this number on their mat in the *middle* row with their color A counters. After the number has been built on the mat, ask how many hundreds are really contained in this number. Ideally, students will recognize that there are actually 24 hundreds, but 20 of the hundreds have been traded for the 2 thousands. Discuss the idea that their number on the mat is really more than 24 hundreds but still less than 25 hundreds.

Have students place color A counters for 2 thousands and 5 hundreds in the *top* row of the mat to show the 25 hundreds simplified, then place color A counters for 2 thousands and 4 hundreds in the *bottom* row of the mat to show the 24 hundreds simplified. Students now need to find which amount of hundreds 2,451 is nearest—the 24 hundreds or the 25 hundreds.

Ask them how many tens and ones are needed to increase 2,451 to 2,500—the next hundreds number. Remind them that a trade from ones to tens might be involved. Students should place 9 counters (in color B) in the ones column on or near the line between the top and middle rows. The 9 ones and the single one already in the middle row could be traded for a new ten, but do not remove the ones counters; students must remember the new ten but not show it on the mat. They must be able to see the *extra* ones later. Looking at the 5 tens in the middle row and remembering the new ten, students will need 4 more tens to produce a trade from tens to hundreds. Have students place 4 new counters (in color B) in the tens column on the line between the top and middle rows of the mat. Now students should count the extra counters (in color B) added to the mat: 4 tens and 9 ones. This means that the original number is 49 away from the next higher hundreds number—2,500.

At this point, ask students what color A counters would need to be removed from the middle row to change the original number to the lower hundreds number—2,400. Only the 5 tens and the single one would need to be removed; that is, 2,451 is 51 away from the lower hundreds number. Thus 2,451 is *closer* to the next higher hundreds number, 2,500. Have students complete the statement on Worksheet 1-4a: "2,451 rounds to the hundreds number, *2,500.*"

Repeat this procedure with other numbers on the worksheet, some that will round up and others that will round down. Here is an example of the completed building mat for rounding 2,451 to the nearest hundreds number.




Appearance of counters on mat after extra counters added.

After students are comfortable with rounding *to* the nearest hundreds number, reverse the question. Have them show 3,200 on the top row of the building mat, then ask them to show some number in the middle row on the mat that would round up to 3,200. Have them give reasons for their choices. Repeat this process by having them show 2,500 on the bottom row of the mat. Ask students to show some number on the middle row that would round down to 2,500. Practice rounding to tens or to thousands in a similar manner.

**Answer Key for Worksheet 1-4a**

| | | | | |
|---|---|---|---|---|
| 1. 2,500 | 2. 1,700 | 3. 380 | 4. 7,000 | 5. 750 |
| 6. 4,000 | 7. 600 | 8. 4,360 | 9. 100 | 10. 5,070 |

**BUILDING MAT 1-4a**

| THOUSANDS | HUNDREDS | TENS | ONES |
|---|---|---|---|
| | | | |
| | | | |
| | | | |

WORKSHEET 1-4a

Name ______________________

## Rounding Whole Numbers on a Building Mat

Date ______________________

Round each number by building it with counters on the building mat and following your teacher's directions. Complete each sentence by recording the correct answer in the blank.

1. 2,451 rounds to the hundreds number, ____________________.
2. 1,736 rounds to the hundreds number, ____________________.
3. 382 rounds to the tens number, ____________________.
4. 6,500 rounds to the thousands number, ____________________.
5. 749 rounds to the tens number, ____________________.
6. 4,380 rounds to the thousands number, ____________________.
7. 593 rounds to the hundreds number, ____________________.
8. 4,358 rounds to the tens number, ____________________.
9. 146 rounds to the hundreds number, ____________________.
10. 5,073 rounds to the tens number, ____________________.

### Activity 2: Pictorial Stage

## Materials

Worksheet 1-4b
Regular pencil and red pencil

## Procedure

1. Give each student two copies of Worksheet 1-4b and a red pencil. Have students work with partners, but each student will complete her or his own copy of Worksheet 1-4b. The leftmost blank above each frame should be used to number each exercise in order as it is worked.

2. Have students write the following in the exercise blanks on their worksheets (you may fill these in before class):

| | |
|---|---|
| 1. 3,547, hundred | 5. 2,615, thousand |
| 2. 638, hundred | 6. 464, ten |
| 3. 1,459, ten | 7. 5,954, hundred |
| 4. 750, hundred | 8. 4,050, thousand |

3. Students should represent each number on the worksheet by drawing small circles in regular pencil in the middle row of the base 10 frame shown below that number.

4. Students should follow the same procedure used in the Manipulative Stage; instead of adding on extra counters, they will draw extra circles on the frame with red pencil. They will record the added value at the right of the top row of the frame. They will then record the value to be removed at the right of the bottom row of the frame.

5. After comparing the amount to be added with the amount to be removed, students will decide which way to round the original number and draw a red path around the small circles, representing the selected rounded amount on the frame. Remind students that if the two amounts are equal, they should round up. The new number will then be recorded in the rightmost blank above the frame.

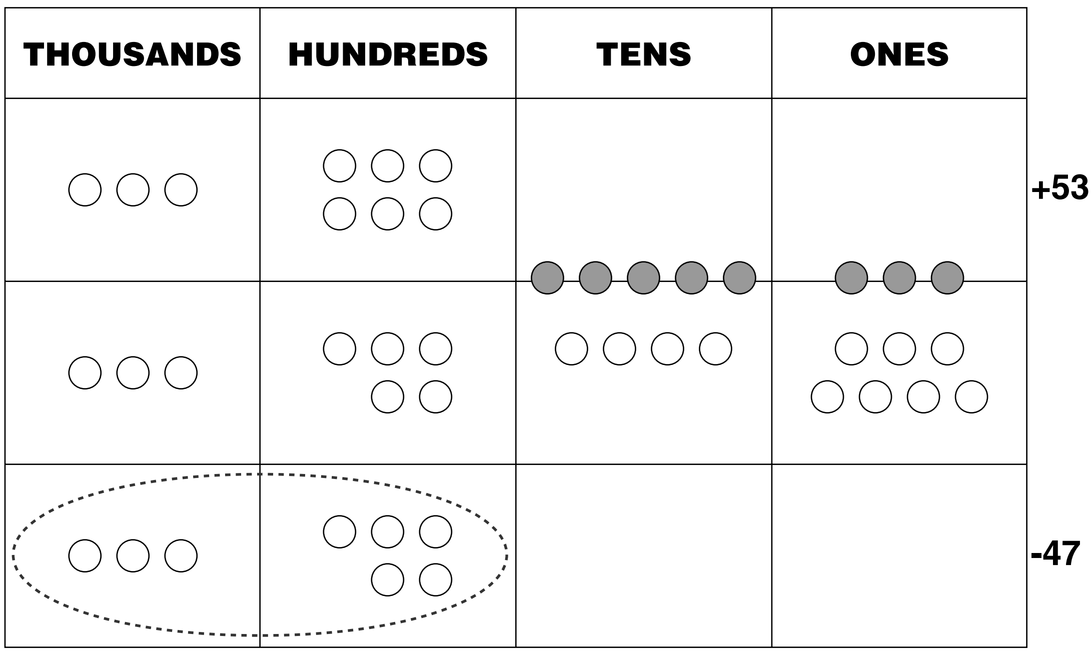
6. Guide students through the drawing procedure for the first exercise on the worksheet, then have them continue with their partners to complete the other exercises.

For the first exercise, consider the number 3,547, to round to the nearest hundreds number. Students should draw small circles in regular pencil on the middle row of the first frame on Worksheet 1-4b to represent 3 thousands, 5 hundreds, 4 tens, and 7 ones. Small circles for the next higher hundreds number (3,600) should be drawn on the top row of the frame, and small circles for the nearest lower hundreds number (3,500) should be drawn on the bottom row.

To increase 3,547 to the next higher hundreds number, small circles for 5 tens and 3 ones should be drawn in red pencil between the top and the middle rows of the frame. "+53" should be recorded at the right of the top row of the frame. To change 3,547 to the lower hundreds number (3,500), only 4 tens and 7 ones would need to be removed. "–47" should be recorded at the right of the bottom row of the frame.

Since 47 is less than 53, the original number (3,547) is *closer* to 3,500 than to 3,600. Therefore, it needs to round down to 3,500. Students should draw a red path around the small circles in the bottom row of the frame and record the new number above the frame. The completed frame and blanks are shown here:

1. 3,547 rounded to the nearest hundred is 3,500.




After students are comfortable with rounding *to* the nearest hundreds number pictorially, reverse the question. Have them draw 3,000 in the top row of a blank drawing frame, then ask them to draw circles to show some number in the middle row on the mat that would round *up* to the 3,000. Have them give reasons for their choices. Repeat this process by having them draw 1,900 in the bottom row of another blank drawing frame. Ask students to show some number in the middle row that would round *down* to 1,900. Practice rounding to tens or to thousands in a similar manner.

**Answer Key for Worksheet 1-4b**

| | | | |
|---|---|---|---|
| 1. 3,500 | 2. 600 | 3. 1,460 | 4. 800 |
| 5. 3,000 | 6. 460 | 7. 6,000 | 8. 4,000 |

WORKSHEET 1-4b

## Drawing Diagrams to Round Whole Numbers

Name ____________________

Date ____________________

Follow your teacher's directions to round numbers, using the frames provided below. Fill in the blanks with the results of each exercise.

_____ ________ rounded to the nearest ________ is ________.

| Thousands | Hundreds | Tens | Ones |
|---|---|---|---|
| | | | |
| | | | |
| | | | |

_____ ________ rounded to the nearest ________ is ________.

| Thousands | Hundreds | Tens | Ones |
|---|---|---|---|
| | | | |
| | | | |
| | | | |

WORKSHEET 1-4b Continued

Name ____________________

Date ____________________

_____ ________ rounded to the nearest ________ is ________.

| Thousands | Hundreds | Tens | Ones |
|---|---|---|---|
| | | | |
| | | | |
| | | | |

_____ ________ rounded to the nearest ________ is ________.

| Thousands | Hundreds | Tens | Ones |
|---|---|---|---|
| | | | |
| | | | |
| | | | |

## Activity 3: Independent Practice

### Materials

Worksheet 1-4c
Regular pencil

### Procedure

Students work with partners on Worksheet 1-4c to explore the rounding of whole numbers further. Ideally, they will discover the rule for rounding that is used in the elementary school curriculum: when rounding to a certain place value, only the digit in the adjacent lesser place value position needs to be considered. Other place value positions to the right do not matter. In the previous two activities, this idea was not presented. After students have completed the worksheet, have them share their conclusions.

#### Answer Key for Worksheet 1-4c

1. First four numbers round to 3,000; last five round to 4,000; least number to round up is 3,500.
2. 500
3. Hundreds; 5 through 9
4. Tens
5. Ones

#### Possible Testing Errors That May Occur for This Objective

- If a number is to be rounded to a certain place value, some students will simply change the given digits in the *lesser* place value positions to zeros, regardless of their values. For example, if rounding to the nearest hundred, they would incorrectly change the ones and tens digits in 1,377 to zeros, rounding the number down to 1,300 instead of up to 1,400.
- Students may just change the ones digit in the original number to zero but not actually do the rounding required. For example, 2,485 might be changed to 2,480 instead of being rounded to the nearest thousand.
- The original number may be correctly rounded to a different place value than the one required. Instead of rounding 876 to the nearest ten, for example, students might round it to the nearest hundred—900.

WORKSHEET 1-4c

Name ______________________

## Applying Rounding to Whole Numbers

Date ______________________

Work with a partner to complete this worksheet. You will need a red pencil.

1. In the following list of numbers, round each number to the nearest thousand by finding which thousands number is *closer* to the given number. In each given number, underline the hundreds, tens, and ones digits with a red pencil; the first number is already underlined. In the rightmost column, write whether the given number was rounded up or down.

| Number | | Rounded to Nearest Thousand | Rounded Up or Down? |
|---|---|---|---|
| 3,<u>340</u> | → | ____________ | ____________ |
| 3,375 | → | ____________ | ____________ |
| 3,484 | → | ____________ | ____________ |
| 3,499 | → | ____________ | ____________ |
| 3,500 | → | ____________ | ____________ |
| 3,501 | → | ____________ | ____________ |
| 3,575 | → | ____________ | ____________ |
| 3,684 | → | ____________ | ____________ |
| 3,699 | → | ____________ | ____________ |

In the list, what is the *least* number that rounds *up*? ____________

2. Fill in the blanks with the *least* 3-digit whole number that will cause the completed number to round up to 9,000.

8, ____ ____ ____

3. When rounding to the nearest thousand, which place value position seems to be the best one to test for whether to round up or round down? What digit values are needed in that position to cause a number to round up?

______________________________________________________________

______________________________________________________________

WORKSHEET 1-4c Continued

Name ______________________

Date ______________________

4. Which place value position is best to test when rounding to hundreds?

_______________________________________________

_______________________________________________

5. Which place value position is best to test when rounding to tens?

_______________________________________________

_______________________________________________

## Objective 5

Represent a proper fraction with various models (physical, pictorial).

## Discussion

The concept of relating part to whole is quite difficult for young students. In particular, the ratio format commonly used to name fractional amounts is often confusing to them. Many hands-on experiences are needed, especially in the modeling of word problems.

### Activity 1: Manipulative Stage

## Materials

Set of 40 colored square tiles or disks per pair of students (two different colors, 20 per color)
Worksheet 1-5a
Regular pencil

## Procedure

1. Give each pair of students a set of 40 colored square tiles or disks (two different colors, 20 per color). Tiles and disks are available commercially, or you may use cutout 1-inch paper squares. Also give each student a copy of Worksheet 1-5a, which contains several word problems involving fractions.

2. Each story problem on the worksheet will provide the description of a whole and some fractional part of that whole. Discuss each problem, and have students model the situation with their tiles.

3. As each result is found, students should record their findings as word sentences on their own worksheets. At the Manipulative Stage, use *word* names to describe fractional parts, for example, 3-*fourths* instead of $\frac{3}{4}$ of the whole.

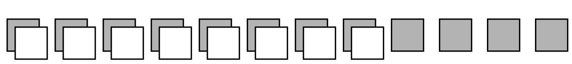
Here is the first exercise on Worksheet 1-5a to consider: “Maury made 12 hamburgers for her party. Eight of the hamburgers were eaten by her guests. What fraction name describes the portion of hamburgers eaten?”

Ask how many hamburgers were in the original set (12). Students should place 12 tiles in one color (for example, red) on the desktop. Ask how many of the hamburgers were eaten (8). Have the students cover 8 of the 12 red tiles with tiles of the second color (for example, blue) to show the amount eaten.

Then, because 8 out of 12 red tiles are covered, the fraction 8-twelfths describes the fractional part of the set that was eaten. Students should write the following sentence on their own worksheets:

“8 hamburgers equal 8-twelfths of 12 hamburgers.”

Here is an example of 8 out of 12 tiles being covered:



**Answer Key for Worksheet 1-5a**

1. 8 hamburgers equal 8-twelfths of 12 hamburgers.
2. 11 broken chairs equal 11-twentieths of 20 chairs.
3. 5 red sectors equal 5-eighths of 8 sectors.
4. 10 eggs equal 10-fifteenths of 15 eggs.
5. 5 uneaten cupcakes equal 5-ninths of 9 cupcakes.

WORKSHEET 1-5a
Building Parts of a Whole

Name ____________________
Date ____________________

Read and discuss each word problem that follows. Use tiles to build the fractional amount mentioned in the word problem, then write a word sentence to describe the amount.

1. Maury made 12 hamburgers for her party. Eight of the hamburgers were eaten by her guests. What fraction name describes the portion of hamburgers eaten?

2. There are 20 chairs in the classroom. Eleven of the chairs are broken. What fractional part of the total chairs is broken?

3. A circular game spinner contains eight equal sectors. Five of the sectors are red, and the other sectors are green. What fraction of the total spinner sectors is red?

4. There are 15 eggs in a bowl. Ten of the eggs will be used for baking cakes. What fraction of the bowl of eggs will be used for the cakes?

5. Nine cupcakes were on the tray. Luis and his friends ate four of the cupcakes. What fractional part of the original cupcakes was left on the tray?

### Activity 2: Pictorial Stage

## Materials

Worksheet 1-5b
Regular pencil

## Procedure

1. After students have practiced with the tiles to model fractions and to name those fractions, have them draw diagrams to show fractions instead. Give each student a copy of Worksheet 1-5b containing several word problems and with drawing space left between problems.

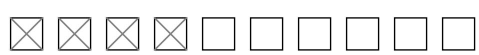
2. Repeat the procedure followed in the Manipulative Stage, but the recording format will be different. The drawings (squares) should look like the tiles used earlier, but instead of placing new tiles on top of the original tiles to show the parts needed, an X will be marked on those squares that are identified for some special reason. Encourage students to draw their squares to look as "equal" in size as possible.

3. The recording of the results should be written below or beside the drawing on the worksheet. The ratio format for fractions will now be used. The total mentioned in the word problem should always be included with the fraction name.

4. Guide students through the first exercise on Worksheet 1-5b before allowing them to draw models on their own for the remaining exercises on the worksheet. When all students are finished, have them share their results.

The first exercise on Worksheet 1-5b is as follows: "Marion plans to ride his bicycle 10 miles today. After riding 4 miles, he stops to rest. What fraction of the total trip still remains for him to do?"

Here is how the situation might be represented. Ten squares are drawn for the 10 miles, and 4 of those squares are marked with an X to show the 4 miles completed. The 6 remaining unmarked squares represent the miles of the trip Marion still must do. A sentence is written beside the drawing to express the result.



$\frac{6}{10}$ of the 10-mile trip needs to be done.

**Answer Key for Worksheet 1-5b (shapes other than squares might be used)**

1. ☒ ☒ ☒ ☒ ☐ ☐ ☐ ☐ ☐ ☐
   $\frac{6}{10}$ of the 10-mile trip needs to be done.

2. ☒ ☒ ☒ ☒ ☒ ☐ ☐
   $\frac{5}{7}$ of the 7 spoons are polished.

3. ☒ ☒ ☒ ☒ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐
   $\frac{4}{12}$ of the 12 runners are from Carter Elementary School.

4. ☒ ☒ ☒ ☒ ☒ ☒ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐
   Wakoto and her friends make $\frac{6}{14}$ of the 14 children.

WORKSHEET 1-5b
Drawing Models for Fractions

Name ______________________
Date ______________________

Draw pictures to represent fractional amounts in the following word problems. Write a word sentence about each fraction shown.

1. Marion plans to ride his bicycle 10 miles today. After riding 4 miles, he stops to rest. What fraction of the total trip still remains for him to do?

2. Esperanza has seven silver spoons she has collected while on different vacations. She wants to polish the spoons and has already polished five of them. What fraction of the seven spoons has she already polished?

3. Twelve students will run in the school race today. Four students are from Carter Elementary School. What fraction of the runners are from Carter Elementary School?

4. Fourteen children will be allowed to attend a special preview of a new movie. Wakoto and five of her friends will go to the preview. What fractional part of the total children at the preview will she and her friends represent?

### Activity 3: Independent Practice

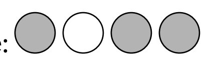
## Materials

Worksheet 1-5c
Regular pencil

## Procedure

Have students work independently to write or apply fraction names for amounts described in diagrams or word problems on Worksheet 1-5c. When all have finished, have them share their results.

**Answer Key for Worksheet 1-5c**

1. $\frac{4}{7}$ of 7 triangles are shaded.
2. $\frac{4}{6}$ of the whole bar is not shaded.
3. $\frac{2}{10}$ of the 10 pencils are red.
4. 6 cookies total were in the package.
5. Models will vary. Sample: 

**Possible Testing Errors That May Occur for This Objective**

- The number of *shaded* parts is compared to the total number of parts in a diagram when the test item asks for *unshaded* parts to be compared to the total instead.
- The number of *shaded* parts (as numerator) is compared to the number of *unshaded* parts (as denominator) instead of being compared to the total number of parts as the denominator.
- The positions of the numerator and the denominator are reversed. The total number of parts (as numerator) is compared to the number of shaded parts (as denominator).
- A pictorial model with unequal parts is matched to a fraction name.

WORKSHEET 1-5c

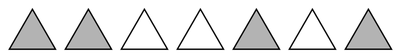
Naming Parts of a Whole or Total

Name

Date

Write a word sentence to answer each question that follows. Be sure to include the total or whole amount in any fraction name given.

1. What fraction of the triangles is shaded?



2. How much of the bar is not shaded?

3. Claire has ten pencils in her school box. Five pencils are yellow and two pencils are red. What fractional part of the ten pencils is red?

4. Sam has four cookies, which represent 4-sixths of the total cookies in the original package. How many cookies in all were in the package before it was opened?

5. Draw a diagram in which 3-fourths of the whole diagram is shaded. Remember that all the fourths must be equal in size.

## Objective 6

Find equivalent fractions that are less than one.

## Discussion

The concept of relating part to whole is quite difficult for young students. In addition, with equivalent fractions they must be able to trade or rename that part and its whole in another way. That is, they need to know how to change the original fraction name to an equivalent fraction name. Many hands-on experiences are needed, especially in the modeling of word problems.

### Activity 1: Manipulative Stage

## Materials

Set of 40 colored square tiles or disks per pair of students (two different colors, 20 per color)
Worksheet 1-6a
Regular pencil

## Procedure

1. Give each pair of students a set of 40 colored square tiles or disks (two different colors, 20 per color), and give each student a copy of Worksheet 1-6a. Tiles and disks are available commercially, or you may use cutout 1-inch paper squares.

2. The worksheet contains several word problems. Each story provides the description of a whole and some fractional part of that whole. Discuss each problem and have students model the situation with their tiles. For two to three problems, students should model only the original fraction in the story.

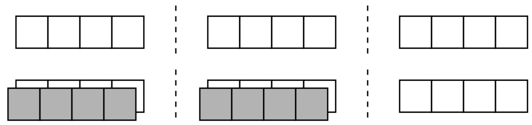
3. After some practice, have them begin to reduce that fraction to a simpler form (smaller numerator and denominator) or trade or group the given parts differently for a higher count (larger numerator and denominator), depending on the situation. An example of each type will be discussed below.

4. Students should record their findings as word sentences on their worksheets. At the Manipulative Stage, use *word* names to describe fractional parts; for example, 3-*fourths* instead of $\frac{3}{4}$ of the whole.

Here is the first exercise on Worksheet 1-6a to discuss with your students: "Tory made 12 hot dogs for her party. Two-thirds of the hot dogs were eaten by her guests. What other fraction name describes the portion of hot dogs eaten?"

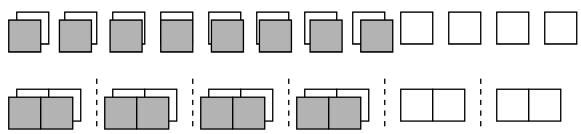
Ask how many hot dogs were in the original set (12). Students should place 12 tiles in one color (for example, red) on the desktop. Ask what portion of the hot dogs was eaten ($\frac{2}{3}$). Do not ask "how many?" because that implies the quantity—8 hot dogs.

Students should separate the 12 tiles into 3 equal groups of red tiles; as a result, 4 connected red tiles will be in each group. Have them cover 2 groups of these red tiles with tiles of the second color (for example, blue) to show the portion eaten. Then, since 8 out of 12 red tiles are covered, the fraction 8-twelfths also describes the fractional part of the set that was eaten. Students should write the following sentence below Exercise 1 on their worksheets: "2-thirds of 12 hot dogs is equal to 8-twelfths of 12 hot dogs."

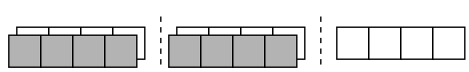


After modeling several stories in which the given simpler fraction is renamed with a larger numerator and a larger denominator, have students begin to reduce some fractions. Exercise 4 on Worksheet 1-6a is an example of the reduction process. Consider the situation: "Eight-twelfths of 12 pencils total have been sold at the school store. What is a simpler fraction name for the portion of pencils sold?"

Students should place 12 red tiles on the desktop, then cover 8 of them with blue tiles. This shows that 8-twelfths of the total pencils have been sold. Ask students if the covered tiles can be grouped 2 at a time and if the uncovered tiles can also be grouped the same way (yes, there will be 4 groups of 2 covered tiles each, blue on red, and 2 groups of 2 uncovered red tiles each). Ask if the blue-topped group and the red group could each be grouped 3 tiles at a time (neither set of tiles can be grouped 3 at a time).



Ask if both can be grouped 4 tiles at a time (yes, the 8 covered or blue-topped tiles can make 2 groups of 4, and the 4 uncovered red tiles can make 1 group of 4). Have students now move their tiles into the larger groups of 4 tiles each, making 2 groups with the covered tiles and 1 group with the uncovered red tiles. Notice that *all* 12 original red tiles are now in 3 groups of 4 red tiles each; 2 of the groups are topped with blue tiles. Discuss the idea that the pencils sold can now be compared to the total pencils by using groups of 4 each instead of counting *individual* pencils; that is, 2 *groups* of pencils out of a total of 3 *groups* of pencils were sold at the school store. A new fraction name for what was sold will be "2-thirds of the 12 pencils total."



Note that the earlier grouping by 2's also allows students to compare 4 groups of 2 covered tiles to a total of 6 groups of 2 tiles each, so "4-sixths of the 12 pencils" would also be a possibility for a new fraction name. Two-thirds and 4-sixths are both reduced forms of 8-twelfths; 2-thirds is just the simplest form. Tell students to look for the lowest possible numbers of groups (that is, the largest group size) when trying to form new groups. Always test for the group size in an orderly way: try groups of 2 tiles each, then groups of 3 tiles each, and so on.

Students should write a sentence below Exercise 4 on the worksheet to record both new fraction names: "8-twelfths of 12 pencils total equals 4-sixths or 2-thirds of 12 pencils."

**Answer Key for Worksheet 1-6a**

1. 2-thirds of 12 hot dogs is equal to 8-twelfths of 12 hot dogs.
2. 1-sixth of 18 stickers or 3-eighteenths of the stickers are sold.
3. 4-fifths of a 20-gallon container or 8-tenths of the container is filled with water.
4. 8-twelfths of 12 pencils total equals 4-sixths or 2-thirds of the 12 pencils.
5. 4-eighths of a pizza equals 2-fourths or 1-half of a pizza.
6. 5-sixths of a dozen eggs cannot be simplified to lower terms.

WORKSHEET 1-6a

Name ______________________

# Building Equivalent Fractions with Tiles

Date ______________________

Build with tiles to solve each exercise that follows. For each exercise, write a word sentence that describes the answer.

1. Tory made 12 hot dogs for her party. Two-thirds of the hot dogs were eaten by her guests. What other fraction name describes the portion of hot dogs eaten?

   ______________________________________________

   ______________________________________________

2. One-sixth of 18 school stickers were sold on the first day of school. What other fraction name tells the portion of stickers that were sold?

   ______________________________________________

   ______________________________________________

3. Only 4-fifths of a 20-gallon container is filled with water. How many tenths of the container is filled with water?

   ______________________________________________

   ______________________________________________

4. Eight-twelfths of 12 pencils total have been sold at the school store. What is a simpler fraction name for the portion of pencils sold?

   ______________________________________________

   ______________________________________________

5. Four-eighths of an 8-slice pizza remains on the buffet tray. What is a simpler fraction name for this uneaten portion of pizza?

   ______________________________________________

   ______________________________________________

6. Joel found 5-sixths of a dozen eggs in the refrigerator. Is there a simpler fraction name for this amount of eggs?

   ______________________________________________

   ______________________________________________

**Activity 2: Pictorial Stage**

## Materials

Worksheet 1-6b
Regular pencil and red pencil

## Procedure

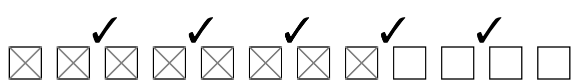
1. After students have practiced with the tiles to model fractions and to rename those fractions, have them draw diagrams to show fractions instead. Give each student a red pencil and a copy of Worksheet 1-6b containing several word problems and drawing space between problems.

2. Repeat the procedure followed in the Manipulative Stage, but the recording format will be different. The drawings should look like the tiles used earlier, but instead of placing new tiles on top of the original tiles to show the part needed, an X will be marked on those squares that are identified for some special reason. New groups formed will be ringed in red pencil.

3. Some exercises on the worksheet will involve the reduction of a given fraction; others will ask for equivalent fractions that have larger numerators and larger denominators than the original fraction. One example of each type will be discussed below.

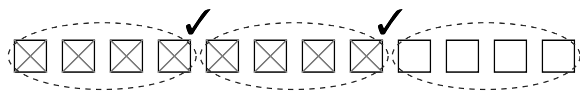
Here is the first exercise on Worksheet 1-6b: "Eight-twelfths of 12 cupcakes are eaten at a birthday party. What is the simplest fraction name for the part that is eaten?"

This exercise involves reduction of the given fraction. Students should draw 12 small squares on the worksheet, then make a large X inside each of 8 of the squares to show that 8 out of 12 cupcakes were eaten. Ask students if they can group the X-squares in pairs and then group the plain squares in pairs. (Yes, they can make 4 groups of 2 with the X-squares and 2 groups of 2 with the plain squares.) Lightly drawn tick marks can be made with pencil to show various groupings being tested. Ask if they can group the X-squares to have 3 squares in each group and similarly group the plain squares to have 3 per group. (No, they cannot group by three's.)



Now ask students to try to group the X-squares with 4 in each group and similarly group the plain squares with 4 in a group. (Yes, they will have 2 groups of 4 X-squares each and 1 group of 4 plain squares each.) Because there are only 4 plain squares total, no grouping size larger than 4 will be possible for the plain squares and the X-squares simultaneously.

Have students draw a red path around each group of 4 squares, as shown next. This will show the least number of groups possible when 8-twelfths of 12 cupcakes are eaten. In other words, because 2 groups out of the 3 groups ringed in red pencil contain all X-squares, we can say that $\frac{2}{3}$ of the 12 cupcakes have been eaten.



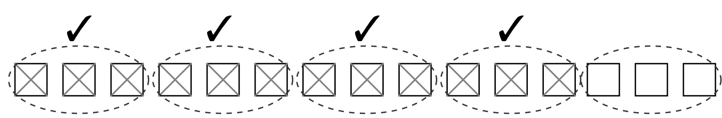
The recording of the results should be written below or beside the drawing on the worksheet. The ratio format for fractions will now be used, along with products that show the regrouping process used. Multiplication (to show the groupings) is recommended here, instead of the division often shown in elementary textbooks, because it is parallel to the format used in algebra later. Here is the recording for this particular diagram, where $2 \times 4$ represents 2 groups of 4 X-squares each and $3 \times 4$ represents 3 groups total of 4 squares per group.

$$\frac{8}{12} = \frac{2 \times 4}{3 \times 4} = \frac{2}{3} \text{ of the 12 cupcakes were eaten}$$

Exercise 2 on Worksheet 1-6b is an example of trading to a larger numerator and larger denominator:

"Tom gave 4-fifths of his 15 baseball cards to his friend, Rosie. What is another fraction name for the part he gave to Rosie?"

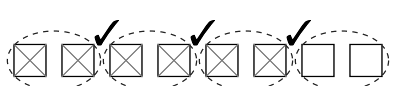
Have students draw 5 rings in red pencil on the worksheet, then draw equal amounts of small squares inside the rings, using a total of 15 squares. Three squares should be drawn inside each ring. Because 4 groups of baseball cards out of 5 groups total were given to Rosie, have students place a check mark next to 4 of the 5 red rings, then draw a large X inside each square found inside the checked red rings.

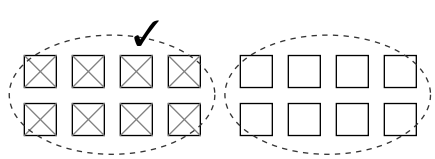


Twelve of the 15 squares are now marked with X, so we can say that $\frac{12}{15}$ of the total baseball cards were given to Rosie. Here is the final recording for this diagram, using 3 squares per group:

$$\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15} \text{ of the 15 baseball cards were given away}$$

**Answer Key for Worksheet 1-6b**

1. $\frac{8}{12} = \frac{2 \times 4}{3 \times 4} = \frac{2}{3}$ of the 12 cupcakes were eaten (diagram given in text).
2. $\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$ of the baseball cards were given away (diagram given in text).
3. 

   $\frac{6}{8} = \frac{3 \times 2}{4 \times 2} = \frac{3}{4}$ of a pizza
4. 

   $\frac{1}{2} = \frac{1 \times 8}{2 \times 8} = \frac{8}{16}$ of the class

WORKSHEET 1-6b

Finding Equivalent Fractions by Drawing

Name ______________________

Date ______________________

Draw to find the answers to the following exercises. Record your results according to the teacher's instructions.

1. Eight-twelfths of 12 cupcakes are eaten at a birthday party. What is the simplest fraction name for the part that is eaten?

2. Tom gave 4-fifths of his 15 baseball cards to his friend, Rosie. What is another fraction name for the part he gave to Rosie?

3. Sean ate 6-eighths of a pizza cut into 8 slices. Find the fraction in lowest terms that is equivalent to 6-eighths of the pizza.

4. There are 16 students in an art class. One-half of these students also study music. What is another fraction name for the part of the class studying music?

**Activity 3: Independent Practice**

## Materials

Worksheet 1-6c
Regular pencil

## Procedure

Students will work exercises on Worksheet 1-6c independently, then share their results with the class. Encourage them to draw diagrams whenever necessary.

**Answer Key for Worksheet 1-6c**

1. $\frac{10}{15}$ of a whole
2. $\frac{4}{7}$
3. 3 groups; 3 pencils per group
4. [4] in numerator and denominator; 24 as new denominator
5. $\frac{2}{5}$ of the class or $\frac{12}{30}$ of the class; other possible answers are $\frac{4}{10}$ or $\frac{6}{15}$ of the class.
6. $\frac{3}{5}$ of the fabric

**Possible Testing Errors That May Occur for This Objective**

- A response that is relatively close to the original fraction is given. For example, $\frac{6}{10}$ or $\frac{2}{5}$ is given as equivalent to $\frac{1}{2}$ because 6 is about half of 10 or 2 is about half of 5.
- The common factor is removed from the denominator but not from the numerator, which is simply changed to 1. For example, in $\frac{6}{8}$ the factor 2 is removed from 8 but not from 6, which is just changed to 1. This yields the fraction $\frac{1}{4}$ instead of $\frac{3}{4}$.
- The denominator is multiplied by a certain factor to produce the new denominator, but the original numerator is not multiplied by the same factor. For example, to change $\frac{2}{3}$ to twelfths, students multiply 3 by 4 for the new denominator, but keep 2 as the new numerator, producing $\frac{2}{12}$ as the new but incorrect equivalent fraction for $\frac{2}{3}$.

WORKSHEET 1-6c
Finding Equivalent Fractions

Name ______________________
Date ______________________

Work the following exercises. Be ready to share your answers with the entire class. Use multiplication to record your steps.

1. Change $\frac{2}{3}$ to fifteenths of a whole.

2. Express $\frac{8}{14}$ as a fraction in lowest terms.

3. Nine red pencils out of 21 pencils total is equivalent to how many *groups* of red pencils out of *7 groups* of pencils total? How many pencils must be in each group?

4. Complete: $\frac{5}{6} = \frac{5 \times [\quad]}{6 \times [\quad]} = \frac{20}{?}$

5. Three-fifths of a class of 30 students will attend a band concert. Find two equivalent fraction names for the portion of the class that is not going to the concert.

6. Out of 20 yards of fabric, $\frac{12}{20}$ of the fabric will be used to make a flag. Rename $\frac{12}{20}$ as an equivalent fraction in lowest terms.

## Objective 7

Compare and order two proper fractions, using models (physical, pictorial).

## Discussion

The concept of relating part to whole is quite difficult for young students. The trading or regrouping process that occurs in finding equivalent fractions is also hard for them to comprehend at this age. Many hands-on experiences are needed, especially in the modeling of word problems that involve the comparison of two fractions.

### Activity 1: Manipulative Stage

## Materials

Set of 60 colored square tiles or disks per pair of students (two different colors, 30 per color)
Worksheet 1-7a
Regular pencil

## Procedure

1. Give each pair of students a set of 60 colored square tiles or disks (two different colors, 30 per color). Tiles and disks are available commercially, or you may use cutout 1-inch paper squares. Also give each student a copy of Worksheet 1-7a, which contains several word problems that require two fractional amounts to be compared. Each story provides the description of a whole and some fractional parts of that whole.

2. Discuss each problem and have students model the situation with their tiles. If the whole is *a set of individual objects* (the "discrete" model), have students physically separate their tiles on the desktop. If the whole is *a single object* (the "continuous" model), have students place the tiles edge-to-edge to form a predetermined bar length. The touching tiles should then be slightly separated to form "sections" of the "whole" bar when new groupings are needed. For example, if students need to find $\frac{1}{2}$ of a whole bar and $\frac{2}{3}$ of another whole bar of the same size, tell students to take 6 tiles ($2 \times 3 = 6$, a common denominator) and make a whole bar that is 6 tiles long. The 6 touching tiles may easily be separated into 2 equal groups or into 3 equal groups when necessary.

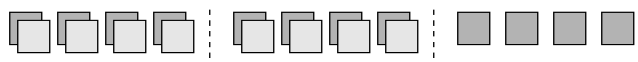
3. Students should record their findings as word sentences on their own papers. At the Manipulative Stage, use *word* names to describe fractional parts; for example, 3-*fourths* instead of $\frac{3}{4}$ of the whole.

Here is the first exercise on Worksheet 1-7a to consider: "Cary made 12 hamburgers for her party. Two-thirds of the hamburgers were made with mustard. One-sixth of the hamburgers were made with mayonnaise. Which portion of hamburgers was greater, the portion with mustard or the portion with mayonnaise?"

Ask how many hamburgers were in the original set (12). Students should place 12 tiles in one color (for example, red) on the desktop. (Note: This example uses the discrete model, so the 12 tiles are not touching initially.)

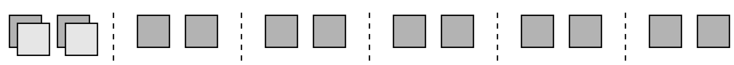
Ask what *portion* of the hamburgers had mustard ($\frac{2}{3}$). Do not ask "how many" at this time; that implies the quantity—8 hamburgers. Students should separate the 12 tiles into 3 equal groups of red tiles; as a result, 4 red tiles will be in each group. Have them cover 2 groups of these red tiles with tiles of the second color (for example, blue) to show the portion made with mustard. Then, since 8 out of 12 red tiles are covered, the fraction 8-twelfths also describes the fractional part of the set that was made with mustard. Students should write the following sentence on their own worksheets: "2-thirds of 12 hamburgers is equal to 8-twelfths of 12 hamburgers, so 8 hamburgers were made with mustard."

Here is a sample of tiles showing "2-thirds of 12 hamburgers" (discrete model):



Now repeat the process to show 1-sixth of 12 tiles. Have students put 12 new red tiles on the desktop, leaving the first set of red and blue tiles intact. Separate the new 12 tiles into 6 equal groups; 2 red tiles will be in each group. Because 1-sixth of the hamburgers were made with mayonnaise, have students cover one group of red tiles with blue tiles. Because 2 red tiles are now covered with blue tiles, the fraction 2-twelfths also describes the fractional part of the set that was made with mayonnaise. Students should now write the following sentence: "1-sixth of 12 hamburgers is equal to 2-twelfths of 12 hamburgers, so 2 hamburgers were made with mayonnaise."

Here is a sample of tiles showing "1-sixth of 12 hamburgers" (discrete model):



Discuss the idea that since 8 tiles are more than 2 tiles, we know that 8-twelfths of 12 must also be more than 2-twelfths of 12. Thinking backward to the original portions given, we conclude that 2-thirds of 12 is more than 1-sixth of 12. Students should finally record: "2-thirds of 12 is more than 1-sixth of 12, so more hamburgers were made with mustard than with mayonnaise."

For Exercise 3 on Worksheet 1-7a, the number of wieners in a package is not given. Students will need to build a "continuous" row of tiles (touching tiles) that can be separated into 4 equal groups to show fourths, as well as into 8 equal groups to show eighths. The least amount of tiles that will be able to be separated both ways is 8 tiles. Be sure to discuss with the students the types of separations required by the exercise so they will understand why 8 tiles are being used. A discussion of "least common denominator" is not necessary at this point.

**Answer Key for Worksheet 1-7a**

Here are suggested sentences to record for each exercise. Complete statements should be encouraged.

1. 2-thirds of 12 hamburgers is equal to 8-twelfths of 12 hamburgers, so 8 hamburgers were made with mustard. 1-sixth of 12 hamburgers is equal to 2-twelfths of 12 hamburgers, so 2 hamburgers were made with mayonnaise. 2-thirds of 12 is more than 1-sixth of 12, so more hamburgers were made with mustard than with mayonnaise.

2. 3-fifths of 15 boxes is equal to 9-fifteenths of 15 boxes, so Jena sold 9 boxes. Susan sold 8-fifteenths of 15 boxes, which is 8 boxes. So 8-fifteenths of 15 boxes is less than 3-fifths of 15 boxes.

3. The north concession stand used 3-fourths of a package of wieners, which is 6-eighths of a package. The south concession stand used 5-eighths of a package. So 3-fourths of a package is greater than 5-eighths of a package. The north concession stand sold more than the south concession stand did.

WORKSHEET 1-7a

Comparing Fractions with Tiles

Name ______________________

Date ______________________

For each exercise that follows, show each fraction with tiles and compare the two fractions. Write word sentences about what you discover.

1. Cary made 12 hamburgers for her party. Two-thirds of the hamburgers were made with mustard. One-sixth of the hamburgers were made with mayonnaise. Which portion of hamburgers was greater, the portion with mustard or the portion with mayonnaise?

2. On Saturday, Jena sold 3-fifths of her 15 boxes of cookies for her Girls' Club camp fund. On the same day, Susan sold 8-fifteenths of another 15 boxes. Which fractional amount was less, 3-fifths or 8-fifteenths of 15 boxes?

3. Before the football game began, the north concession stand used 3-fourths of a package of wieners to make hot dogs. The south concession stand used 5-eighths of a package. Which concession stand sold the greater part of a package of wieners? (Hint: For a "package of wieners," build a "whole" bar that is 8 tiles long. The tiles should be touching, because the number of wieners in a package is not known.)

**Activity 2: Pictorial Stage**

## Materials

Worksheet 1-7b
Regular notebook paper
Regular pencil and red pencil

## Procedure

1. After students have practiced with the tiles to model fractions and to rename and compare those fractions, have them draw diagrams to compare fractions instead. Give each student a red pencil and a copy of Worksheet 1-7b, which contains several word problems. Students will draw their diagrams on their own notebook paper.

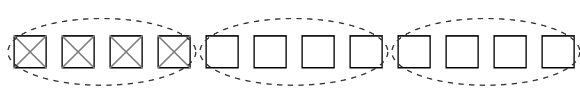
2. Repeat the procedure followed in the Manipulative Stage, but the recording format will be different. The ratio format for fractions will now be used, along with products that show the trading or grouping process used.

3. For the discrete model, the drawings should look like the tiles used earlier, but instead of placing new tiles on top of the original tiles to show the portion needed, an X will be marked on those squares that are identified for some special reason. New groups formed will be ringed in red pencil.

4. Guide students through each exercise. The first two exercises will be discussed next. The first exercise involves a discrete model, and the second exercise involves a continuous model. Students need experience with both types.

Consider Exercise 1 on Worksheet 1-7b: "Allen made 12 cupcakes for his party. One-third of the cupcakes were blueberry. Three-sixths of the cupcakes were banana. Which portion of the cupcakes was greater, blueberry or banana?"

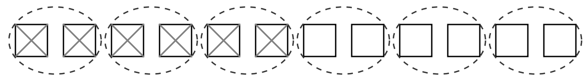
We start with "$\frac{1}{3}$ of 12 cupcakes," so students should first draw 12 squares on their own notebook paper. Then red rings will be drawn to form 3 equal groups with the 12 squares. One of the 3 groups is selected by marking an X in each square inside that group. As a result, $\frac{4}{12}$ of the total squares will be marked.



Because 4 squares are found in each ringed group, the change from counting ringed groups to counting individual squares will be recorded below Exercise 1 on Worksheet 1-7b as follows:

$$\frac{1}{3} = \frac{1 \times 4}{3 \times 4} = \frac{4}{12} \text{ of the 12 cupcakes are blueberry}$$

Similarly, "$\frac{3}{6}$ of 12 cupcakes" would be drawn on the notebook paper as follows:



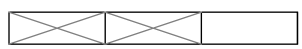
Because 2 squares are in each ringed group, we show the change from counting ringed groups to counting individual squares by also recording the following below Exercise 1 on Worksheet 1-7b:

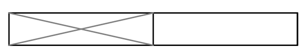
$$\frac{3}{6} = \frac{3 \times 2}{6 \times 2} = \frac{6}{12} \text{ of the 12 cupcakes are banana}$$

Because $\frac{6}{12}$ is more than $\frac{4}{12}$, we finally record on the worksheet:

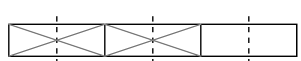
$$\frac{1}{3} < \frac{3}{6}, \text{ so there are more banana cupcakes.}$$

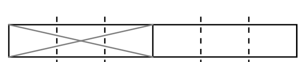
For the continuous model found in Exercise 2, students will first draw two "whole" bars of the same length (the whole candy bar is a continuous model) on their own notebook paper. At the Manipulative Stage, they were told how many tiles to connect together to form the "whole" bar needed. At this new stage, they will decide how many parts will finally be needed. To compare "$\frac{2}{3}$ of a candy bar" to "$\frac{1}{2}$ of a candy bar," students will subdivide each bar and mark its parts as follows:


$\frac{2}{3}$ of whole bar


$\frac{1}{2}$ of whole bar

Encourage students to carefully subdivide their bars each time so that equal parts are formed within each bar; the parts should at least pass the "eye" test for congruence. After the initial fractional amounts are represented, students must decide how to trade or subdivide each original part shown on each whole bar so that the two whole bars have the same total number of new parts at the end. The new parts should be marked off in red pencil. In the present exercise, if each third in the first bar is changed to *two* new equal parts and each half in the second bar is changed to *three* new equal parts, then each whole bar will contain six new equal parts total. The new markings should be drawn as shown here:


$\frac{2}{3}$ of whole bar


$\frac{1}{2}$ of whole bar

The recordings below Exercise 2 on Worksheet 1-7b for the changes to the two bars will be as follows:

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6} \text{ of the whole candy bar}$$

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6} \text{ of the whole candy bar}$$

Since we know that 4 parts are more than 3 parts, or $\frac{4}{6}$ of the candy bar is more than $\frac{3}{6}$ of the candy bar, the final comparison should now be recorded on the worksheet:

$$\frac{2}{3} > \frac{1}{2} \text{ of the candy bar}$$

**Answer Key for Worksheet 1-7b**

1. $\frac{1}{3} = \frac{1 \times 4}{3 \times 4} = \frac{4}{12}$ of the 12 cupcakes are blueberry

   $\frac{3}{6} = \frac{3 \times 2}{6 \times 2} = \frac{6}{12}$ of the 12 cupcakes are banana

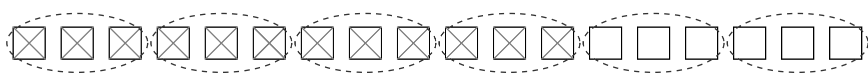
   $\frac{1}{3} < \frac{3}{6}$ , so there are more banana cupcakes (diagrams shown in text).

2. $\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$ of the whole candy bar

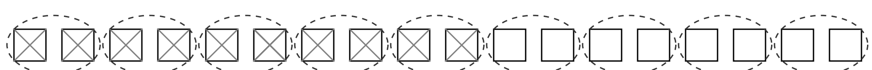
   $\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$ of the whole candy bar

   $\frac{2}{3} > \frac{1}{2}$ of the candy bar (diagrams shown in text)

3. Harry:

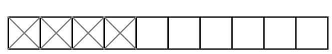
   Lynn:

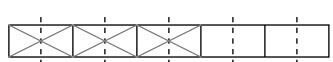
   

   Harry: $\frac{4}{6} = \frac{4 \times 3}{6 \times 3} = \frac{12}{18}$ of the 18 cans

   Lynn: $\frac{5}{9} = \frac{5 \times 2}{9 \times 2} = \frac{10}{18}$ of the 18 cans

   $\frac{4}{6} > \frac{5}{9}$, so $\frac{5}{9}$ of the 18 cans was less.

4.  $\frac{4}{10}$ of whole kg

    $\frac{3}{5}$ of whole kg

   $\frac{4}{10}$ is unchanged.

   $\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$ of a kg

   $\frac{4}{10} < \frac{3}{5}$, so $\frac{4}{10}$ of a kilogram weighs less.

WORKSHEET 1-7b

## Drawing Diagrams to Compare Fractions

Name ______________________

Date ______________________

For each exercise that follows, draw diagrams on your own notebook paper to compare the two fractions. Follow your teacher's directions to record your results on this worksheet.

1. Allen made 12 cupcakes for his party. One-third of the cupcakes were blueberry. Three-sixths of the cupcakes were banana. Which portion of the cupcakes was greater, blueberry or banana?

2. Joe ate $\frac{2}{3}$ of a candy bar. Phil had the same kind of candy bar but ate $\frac{1}{2}$ of his bar. Who ate more candy, Joe or Phil?

3. Each shipping carton holds 18 cans of vegetables. Harry placed 4-sixths of his carton's cans on the display shelf, and Lynn placed 5-ninths of her carton's cans on the shelf. Which fractional part of the cans was less?

4. Which weighs less: $\frac{4}{10}$ of a kilogram or $\frac{3}{5}$ of a kilogram?

**Activity 3: Independent Practice**

## Materials

Worksheet 1-7c
Regular pencil

## Procedure

Students will solve each problem on Worksheet 1-7c independently, using the fraction conversion format developed in Activity 2. When all are finished with the worksheet, have various students explain their results.

**Answer Key for Worksheet 1-7c**

1. $\frac{15}{16}$ of a whole
2. $\frac{2}{3}$ of a pound
3. $<$
4. $\frac{3}{8} < \frac{1}{2} < \frac{2}{3}$
5. Charlie

**Possible Testing Errors That May Occur for This Objective**

- Two fractions are incorrectly considered equal when the same constant is subtracted from both the numerator and the denominator of one fraction to obtain the numerator and the denominator of the second fraction. For example, in $\frac{9}{12}$, 6 is subtracted from 9 and from 12 to get the "fraction" $\frac{3}{6}$; therefore, the student considers $\frac{9}{12}$ equal to $\frac{3}{6}$.
- One shaded amount on a given whole bar is correctly named, but the shaded amount on the second whole bar is not correctly named. The final fraction names found are then compared with the correct inequality sign. For example, students are to compare diagrams that represent $\frac{9}{12}$ and $\frac{2}{3}$ but use $\frac{9}{12}$ and $\frac{5}{6}$ instead, correctly finding $\frac{9}{12} < \frac{5}{6}$ numerically. Students ignore the visual clues in the diagrams that show $\frac{9}{12}$ to be *more than* $\frac{2}{3}$. The test item requires $\frac{9}{12} > \frac{2}{3}$ as the response in order to be correct.
- The unshaded parts are compared to the shaded parts on each whole bar shown instead of the shaded parts being compared to the total parts on each bar. For example, for 9 shaded parts out of 12 parts total on the first whole bar, students incorrectly use $\frac{3}{9}$ as the fraction name. Then for 4 shaded parts out of 6 parts total on the second whole bar, they incorrectly use $\frac{2}{4}$ as the fraction name. The response, $\frac{3}{9} > \frac{2}{4}$, is selected because the first shaded amount (actually $\frac{9}{12}$) looks larger in size than the second shaded amount (actually $\frac{4}{6}$). So the "greater than" sign seems reasonable. The correct response for the test item, however, should be $\frac{9}{12} > \frac{4}{6}$ .

WORKSHEET 1-7c
Comparing Proper Fractions

Name ______________________
Date ______________________

Solve the following exercises. Be ready to share your results with the entire class. Record any steps used to change a fraction name to another name.

1. Which is greater: $\frac{7}{8}$ or $\frac{15}{16}$ of a whole?

2. Jose wants to buy either 2-thirds of a pound of fudge or 3-fourths of a pound of peanut brittle. Which portion of a pound weighs less?

3. Complete with <, >, or =: $\frac{3}{5}$ ______ $\frac{7}{10}$ for the same whole.

4. Order from least to greatest, based on the same whole: $\frac{1}{2}, \frac{2}{3}, \frac{3}{8}$.

5. Each student in Ms. Garza's class has 3 dozen donuts to sell at the school carnival. By noon, Charlie has sold 7-twelfths of his donuts and Maria has sold 10-eighteenths of her donuts. Who has sold more donuts?

## Objective 8

Match numerals or number names of decimals involving tenths and hundredths with their equivalent word names (including mixed numbers).

## Discussion

Understanding fractional language is difficult for students. It is easier for them to think of 10 ones as 1 ten than to think of a one as 1 out of 10 parts, or 1-tenth of 1 ten. This difficulty extends into the tenths and hundredths place value positions.

Students need practice with fractional language, both orally and in written form. Base 10 blocks are helpful in the development of decimal fractions and provide practice with the naming process.

### Activity 1: Manipulative Stage

## Materials

Demonstration set of base 10 blocks (1 large cube, 10 flats, 10 rods, and 10 small cubes) for the teacher
Worksheet 1-8a
Regular pencil
For each pair of students, a bag of approximately 60 small counters (centimeter cubes, buttons, and so on, all same color and size)
Building Mat 1-8a (includes ones, tenths, hundredths)

## Procedure

1. Use a set of base 10 blocks (1 large cube, 10 flats, 10 rods, and 10 small cubes) to demonstrate to the class the fractional relationship each block has with the next larger size; for example, a flat is 1-tenth of a large cube, a rod is 1-tenth of a flat, and a small cube is 1-tenth of a rod. Then define a flat as the "whole" or "ones block" and discuss what fractional part each smaller block is of this "whole"; for example, a rod will be 1-tenth of the "whole" (10 rods = 1 flat) and a small cube will be 1-hundredth of the "whole." Students should already know that 100 small cubes are equivalent in size to 1 flat, so 100 small cubes should not be necessary for the discussion.

2. Then define a large cube as the *new* "whole" or "ones block" and discuss what fractional part each smaller block is of this "whole." A flat will be 1-tenth of the "whole" (10 flats = 1 large cube), a rod will be 1-hundredth of the "whole" (100 rods = 1 large cube), and a small cube will be 1-thousandth of the "whole" (1,000 small cubes = 1 large cube). Again, the larger amounts of blocks should not be needed. Help students reason through how many total blocks are required. For example, show that 10 rods cover one flat, then reason that 10 flats must represent 10 of 10 rods, or 100 rods, when the flats form the large cube.

3. Discuss the idea that the fractional name of a block changes when the "whole" block changes; for example, the small cube is 1-thousandth *of the large cube*, but it is 1-hundredth *of the flat.* When using fraction language, students must always be aware of what the "whole" is.

4. Now give each student a copy of Worksheet 1-8a. Give each pair of students a bag of approximately 60 small counters (centimeter cubes, buttons, and so on) of the same size and color. Because of limits on the building mat space and the quantities of each kind of block, it is difficult to continue working with the base 10 blocks. Therefore, we must move to nonproportional materials, such as the small counters, in order to show a variety of decimal fractions. Students must now rely on their *understanding* that it takes 10 tenths to make a "whole," 100 hundredths to make a "whole," 10 hundredths to make a tenth of a "whole," and so on. Remind them that each counter placed in the ones column of the building mat now represents the "whole."

5. Students need to practice the different ways amounts of counters on the building mat can be described or named. Have them place one counter in the tenths column. Write its *decimal name* on the board ("0.1"), then read that decimal name as "one-tenth of the whole." Also write "one-tenth of the whole" on the board as the *word name*, and write the equivalent common fraction name ("$\frac{1}{10}$") beside the "0.1" to reinforce the name relationships. Note that both $\frac{1}{10}$ and 0.1 should be *read aloud* as "one-tenth of the whole." Avoid language such as "zero point one" for 0.1 or "one over ten" for $\frac{1}{10}$.

6. Ask students to trade the tenth currently on the building mat for 10 hundredths. To do this, they should remove the counter from the tenths column and put 10 counters in the hundredths column. Now 0.1 has become 0.10. Write the new decimal number on the board, read the number as "ten hundredths of the whole," then write that same phrase on the board as the word name. Also write the common fraction name ("$\frac{10}{100}$") beside the "0.10."

7. Call out different amounts of counters described on Worksheet 1-8a for students to place in the appropriate columns of Building Mat 1-8a. They should make any allowable trades from each column to the adjacent column to the left in order to simplify the amount of counters on the mat. For example, if there are 14 counters in the hundredths column, 10 of the counters should be removed from that column and a new counter placed in the tenths column instead; 4 counters will remain in the hundredths column.

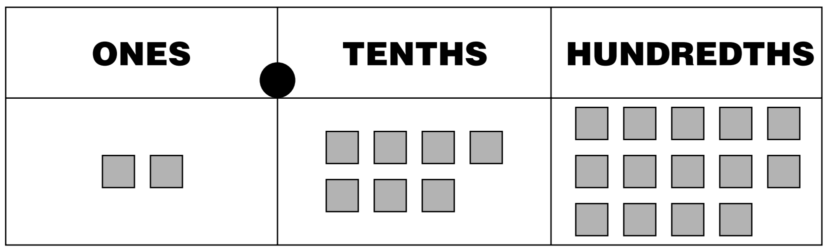
8. Once the initial counters have been simplified, have students record the decimal name, the word name, and the common fraction name (or mixed number name) under the appropriate description on Worksheet 1-8a. Common fraction forms are being included here, even though the objective does not mention them. It is a good time to reinforce equivalent decimal and fraction notations.

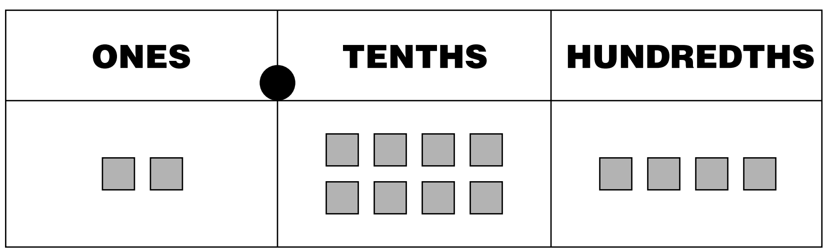
9. The first exercise on Worksheet 1-8a will be discussed next.

Consider Exercise 1 on Worksheet 1-8a: "14 hundredths, 7 tenths, 2 ones." The three amounts should be shown with counters on the mat. Ten of the hundredths will trade for 1 tenth. This will leave 4 counters in the hundredths column, 8 counters in the tenths column, and 2 counters in the ones column. Have students record the result on Worksheet 1-8a as the decimal number name (2.84), which directly reflects the amount of counters in each column.

Because the rightmost column of the mat containing counters is hundredths, the final word name will require hundredths as the denominator. Remind students that the 8 tenths can trade for 80 hundredths, so in terms of hundredths, we have 80+4, or 84 hundredths *in value* on the mat. (Do not show the actual trading with the counters because the quantity of counters needed is so large.) Thus, the final amount on the mat should be read as "two and eighty-four hundredths of the whole." This phrase should be written on the worksheet as the word name. The mixed number name, 2 and $\frac{84}{100}$, should also be recorded. Use "and" in the mixed number name at this stage to help students differentiate between whole and fractional notation.

Here is the building mat as it should appear with the initial set of counters and with the final set of counters:




**Answer Key for Worksheet 1-8a**

1. 2.84; two and eighty-four hundredths of the whole; 2 and $\frac{84}{100}$
2. 0.43; forty-three hundredths of the whole; $\frac{43}{100}$
3. 1.02; one and two hundredths of the whole; 1 and $\frac{2}{100}$
4. 3.5; three and five tenths of the whole; 3 and $\frac{5}{10}$ (alternatively, since the exercise had hundredths at first: 3.50; three and fifty hundredths of the whole; 3 and $\frac{50}{100}$)
5. 0.21; twenty-one hundredths of the whole; $\frac{21}{100}$
6. 4.2; four and two tenths of the whole; 4 and $\frac{2}{10}$

**BUILDING MAT 1-8a.**

| ONES | TENTHS | HUNDREDTHS |
|---|---|---|
| | | |

WORKSHEET 1-8a

Building Decimal Numbers

Name ______________________

Date ______________________

Show each set of counters listed next on your building mat and simplify the counters. Write the decimal name, word name, and fraction name for each set below its description.

1. 14 hundredths, 7 tenths, 2 ones

2. 3 tenths, 13 hundredths

3. 9 tenths, 12 hundredths

4. 14 tenths, 2 ones, 10 hundredths

5. 11 hundredths, 1 tenth

6. 3 ones, 12 tenths

**Activity 2: Pictorial Stage**

## Materials

Worksheet 1-8b
Regular pencil

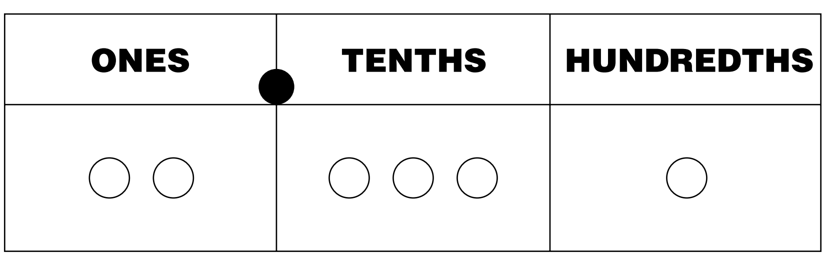
## Procedure

1. Give each student a copy of Worksheet 1-8b. Repeat the process used in the Manipulative Stage, except have students draw small circles in the columns of base 10 frames on the worksheet instead of using counters on the building mat. This time 0 to 9 circles will be used in each column. Focus on the naming technique without involving any trading.
2. For each exercise on the worksheet, have students draw circles in each column of a base 10 frame according to the amounts stated above the frame.
3. They should write the decimal name below the frame, read its correct name aloud, then write the corresponding word name and common fraction name beside or below the decimal name. Do *not* teach reduction of fractions at this time; it will interfere with the place value study that you are doing here.
4. An example from Worksheet 1-8b will be discussed next.

Consider Exercise 1 from the worksheet: "2 ones, 3 tenths, and 1 hundredth." Students should draw 2 circles in the ones column, 3 circles in the tenths column, and 1 circle in the hundredths column. Using the quantity in each column, they should record 2.31 below the base 10 frame.

Because the rightmost column containing circles is hundredths, the final word name will require hundredths as the denominator. Students should realize that 3 tenths could trade for 30 hundredths, so the tenths and hundredths combined equal to 30 + 1, or 31 hundredths. The word name to be recorded will then be "two and thirty-one hundredths of the whole." The fraction or mixed number name will be 2 and $\frac{31}{100}$.

Here is an example of the completed frame:




2.31; two and thirty-one hundredths of the whole; 2 and $\frac{31}{100}$

**Answer Key for Worksheet 1-8b**

1. 2.31; two and thirty-one hundredths of the whole; 2 and $\frac{31}{100}$
2. 4.07; four and seven hundredths of the whole; 4 and $\frac{7}{100}$
3. 5.6; five and six tenths of the whole; 5 and $\frac{6}{10}$
4. 0.83; eighty-three hundredths of the whole; $\frac{83}{100}$

WORKSHEET 1-8b Name ______________________

Drawing Decimal Numbers Date ______________________

For each exercise, show the given place value amounts by drawing small circles on the base 10 frame. Write the decimal name, word name, and fraction name for the total amount below the frame or on the back of this sheet.

1. 2 ones, 3 tenths, 1 hundredth

| ONES | TENTHS | HUNDREDTHS |
|---|---|---|
| | | |

2. 7 hundredths, 4 ones

| ONES | TENTHS | HUNDREDTHS |
|---|---|---|
| | | |

3. 5 ones, 6 tenths

| ONES | TENTHS | HUNDREDTHS |
|---|---|---|
| | | |

4. 3 hundredths, 8 tenths

| ONES | TENTHS | HUNDREDTHS |
|---|---|---|
| | | |

## Activity 3: Independent Practice

### Materials

Worksheet 1-8c
Regular pencil

### Procedure

Students work independently to complete Worksheet 1-8c. Remind them that the rightmost place value position shown in a decimal number name indicates the denominator when a fractional part is involved. Also encourage them to use the phrase "of the whole" whenever describing a fractional amount. When all are finished, have them share their answers with the class.

**Answer Key for Worksheet 1-8c**

1. 5.94
2. twenty-five hundredths of the whole
3. 0.60
4. eight and three hundredths of the whole
5. twelve and seven tenths of the whole
6. 1.75

**Possible Testing Errors That May Occur for This Objective**

- Students recognize the place value required for the denominator of the decimal fraction but ignore the zero in a numeral when naming the numerator. For example, students name 2.40 as "two and four hundredths."
- If a mixed number is changed to an improper fraction before being written in decimal form, the numerator is not correctly aligned with the place value positions. As an example, "one and five hundredths" is changed to $\frac{105}{100}$, but the entire numerator is written to the right of the decimal point as 0.105, instead of as 1.05.
- Students do not know the place value positions to the right of the decimal point. For example, "five and six hundredths" might be written as 5.6 instead of 5.06.

WORKSHEET 1-8c Name ____________

Naming Decimal Numbers Date ____________

For each exercise, write the required equivalent name.

1. Write the decimal number name for "five and ninety-four hundredths of the whole."

2. Write the word name for the decimal number 0.25.

3. Write the decimal number name for "sixty hundredths of the whole."

4. Write the word name for the decimal number 8.03.

5. Write the word name for 12.7.

6. What decimal number name is equal to the mixed number name 1 and $\frac{75}{100}$?

Section 1

Name ____________________

Date ____________________

# NUMERATION AND NUMBER PROPERTIES: PRACTICE TEST ANSWER SHEET



**Directions:** Use the Answer Sheet to darken the letter of the choice that best answers each question.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1. | ○ A | ○ B | ○ C | ○ D | 9. | ○ A | ○ B | ○ C | ○ D |
| 2. | ○ A | ○ B | ○ C | ○ D | 10. | ○ A | ○ B | ○ C | ○ D |
| 3. | ○ A | ○ B | ○ C | ○ D | 11. | ○ A | ○ B | ○ C | ○ D |
| 4. | ○ A | ○ B | ○ C | ○ D | 12. | ○ A | ○ B | ○ C | ○ D |
| 5. | ○ A | ○ B | ○ C | ○ D | 13. | ○ A | ○ B | ○ C | ○ D |
| 6. | ○ A | ○ B | ○ C | ○ D | 14. | ○ A | ○ B | ○ C | ○ D |
| 7. | ○ A | ○ B | ○ C | ○ D | 15. | ○ A | ○ B | ○ C | ○ D |
| 8. | ○ A | ○ B | ○ C | ○ D | 16. | ○ A | ○ B | ○ C | ○ D |

## SECTION 1: NUMERATION AND NUMBER PROPERTIES: PRACTICE TEST

1. Order from greatest to least: 12,500; 11,745; 12,567; 11,760.

   A. 11,745; 11,760; 12,500; 12,567

   B. 12,500; 11,745; 12,567; 11,760

   C. 12,567; 12,500; 11,760; 11,745

   D. 11,760; 11,745; 12,567; 12,500

2. Order from least to greatest: 7,549; 6,235; 7,607; 8,114; 6,102.

   A. 6,235; 6,102; 7,607; 7,549; 8,114

   B. 8,114; 7,607; 7,549; 6,235; 6,102

   C. 6,102; 8,114; 6,235; 7,549; 7,607

   D. 6,102; 6,235; 7,549; 7,607; 8,114

3. Which set contains only even numbers?

   A. 4, 10, 26 B. 5, 6, 7 C. 9, 15, 23 D. 5, 10, 15

4. Which number is an odd number?

   A. 18 B. 27 C. 2 D. 0

5. If the pattern shown in row A continues, what number will be the sixth term in the row?

| | | | | | | |
|---|---|---|---|---|---|---|
| **A** | 2 | 4 | 6 | | | |

   A. 8 B.10 C. 15 D. 12

6. If the patterns shown in row X and row Y continue, what number will be in row Y below the number 4 in row X?

| | | | | | | |
|---|---|---|---|---|---|---|
| **X** | 1 | 2 | | 4 | | 6 |
| **Y** | 1 | 4 | 7 | | | |

   A. 10 B. 4 C. 11 D. 8

## SECTION 1: NUMERATION AND NUMBER PROPERTIES: PRACTICE TEST

7. At the Convention Center, 1,560 parking spaces are being used by people attending a garden show. Estimate to the nearest thousand how many parking spaces are being used.

   A. 1,600 B. 2,000 C. 1,000 D. 1,500

8. What is 3,248 feet rounded to the nearest hundred feet?

   A. 3,000 ft B. 3,300 ft C. 3,250 ft D. 3,200 ft

9. What fraction of the triangles is shaded?

   A. $\frac{2}{5}$ B. $\frac{5}{7}$ C. $\frac{2}{7}$ D. $\frac{2}{3}$

10. George has 15 pencils in his school box. Five pencils are yellow and eight pencils are blue. What fractional part of the 15 pencils is blue?

    A. $\frac{8}{15}$ B. $\frac{5}{8}$ C. $\frac{5}{15}$ D. $\frac{13}{15}$

11. Express $\frac{8}{12}$ as a fraction in lowest terms.

    A. $\frac{4}{12}$ B. $\frac{2}{4}$ C. $\frac{2}{3}$ D. $\frac{4}{6}$

12. Two-fifths of a class of 30 students will attend a band concert. What is an equivalent fraction name for the portion of the class that is not going to the concert?

    A. $\frac{6}{10}$ B. $\frac{6}{15}$ C. $\frac{2}{5}$ D. $\frac{4}{10}$

13. Marge wants to buy either 2-thirds of a pound of hamburger meat or 3-fourths of a pound of fajita meat. Which portion of a pound weighs less?

    A. $\frac{2}{3}$ B. $\frac{5}{7}$ C. $\frac{3}{4}$ D. $\frac{1}{4}$

14. Which statement is correct when each fraction names a part of the same whole?

    A. $\frac{2}{5} = \frac{3}{10}$ B. $\frac{2}{5} < \frac{3}{10}$ C. $\frac{2}{5} > \frac{3}{10}$ D. $\frac{2}{10} = \frac{3}{5}$

15. Write the decimal number name for "five and sixty-three hundredths of the whole."

    A. 563 B. 5.63 C. 0.563 D. 56.3

16. Write the word name for the decimal number 0.09.

    A. Nine B. Nine thousandths C. Nine tenths D. Nine hundredths

## Section 1: Numeration and Number Properties: Answer Key for Practice Test

The objective being tested is shown in brackets beside the answer.

1. C [1]
2. D [1]
3. A [2]
4. B [2]
5. D [3]
6. A [3]
7. B [4]
8. D [4]
9. B [5]
10. A [5]
11. C [6]
12. A [6]
13. A [7]
14. C [7]
15. B [8]
16. D [8]