CHAPTER 1

INTRODUCTION

1-1. Communications Systems and Statistics

A simple communications system is the cascade of an information source, a communication link, and an information user where, as shown in Fig. 1-1, the communication link consists of a transmitter, a channel,

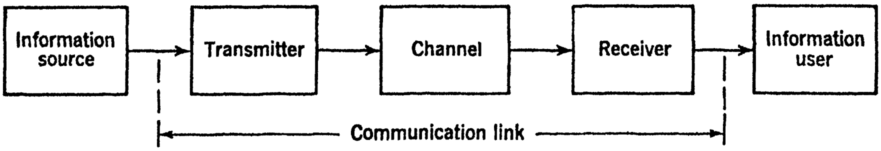

FIG. 1-1. A communications system.

and a receiver. The transmitter modulates or encodes information supplied by the source into a signal form suitable to the channel, the channel conveys the signal over the space intervening between the transmitter and receiver, and the receiver demodulates or decodes the signal into a form suitable to the user. Typical communication links are (1) the frequency-modulated, very-high-frequency radio link that brings entertainment to the listener, (2) the time-division multiplex radio link that transmits various data to some controlled device, e.g., a ship or an airplane, (3) the Teletype machine, connecting wire or cable, and associated terminal apparatus that convey telegrams from one point to another, and (4) the eye and pertinent portions of the nervous system that transmit a visual image to the brain.

Randomness or unpredictability can enter a communications system in three ways: the information generated by the source may not be completely predictable, the communication link may be randomly disturbed, and the user may misinterpret the information presented to it. It is in fact a fundamental tenet of information theory that the output of a source *must* be unpredictable to some degree in order to impart information at all; if the source output were completely predictable, the user could, without communication from the source, state at any instant the entire future output of the source. Disturbances in the communication link can occur in many ways. Both the transmitter and the receiver can add noise. If the channel is a radio channel, it can, for example, add atmospheric noise, galactic noise, or man-made interference. In addition, it might be subject to a randomly variable multipath propagation mode which causes a single transmitted signal to appear as a multiplicity of interfering signals at the receiver.

Even though the signals and noises in a communications system are random, the behavior of that system can be determined to a considerable degree if various average properties of the signals and noises are known. Such properties might include the average intensity or power, the distribution of power with frequency, and the distribution of the instantaneous amplitude. The determination of the relations among the various average properties falls into the domain of probability theory and statistics. The purpose of this book is to introduce to the reader the application of statistical techniques to the study of communications systems.† In general we shall assume the statistical properties of the signals and noises to be given; the study of optimum statistical properties for signals may be found in works on information theory‡ and will not be discussed here.

1-2. The Book

Approximately the first half of this book is devoted to a development of those elements of probability theory and statistics which are particularly pertinent to a study of random signals and noises in communications systems. The remainder is devoted to applications.

Survey of the Text. In Chap. 2, the *probability* of an *event*, i.e., the outcome of an experiment, is introduced in terms of the relative frequency of occurrence of that event and a set of axioms of probability theory is presented. The probabilities of multiple events are then discussed and the concept of *statistical independence* is mentioned. The representation of an event by a point in a *sample space* is made in Chap. 3, and a *random variable* is defined there as a function over a sample space. Probabilities are then introduced on the sample space, and probability distribution functions and probability density functions of random variables are defined. The probability concepts are next extended to random functions of time by the definition of a *random process* as a family of random variables indexed by the parameter t.

The notion of *statistical average* is introduced in Chap. 4 in terms of the common arithmetic average, and certain statistical averages are considered. In particular, the *characteristic function* of the probability distribution function of the random variable x is defined to be the statistical

† An excellent short introduction to the subject matter of this book may be found in Bennett (II). (See Bibliography.)

‡ For example Shannon (I) and (II).

average of $\exp(jvx)$ and is shown to be the Fourier transform of the probability density function of x. The *correlation coefficient* of the two random variables x and y is next stated to be the statistical average of a normalized product of x and y and is related to the problem of obtaining the best mean-square prediction of y given x. The random variables x and y are then said to be *linearly independent* if their correlation coefficient is zero. Finally, in Chap. 4 the relation between time averages and statistical averages is investigated.

In Chap. 5, sampling is introduced and the *sample mean* in particular is discussed at some length. A simple form of the central limit theorem is derived, and the relation between the relative frequency of occurrence of an event and the probability of that event is further studied.

The *spectral density*, i.e., the distribution of power with frequency, of a function of time is considered in Chap. 6 and is shown to be the Fourier transform of the autocorrelation function of the time function. The concept of spectral density is then extended to random processes, and the problem of representing a random process by a series of orthogonal time functions with linearly independent random coefficients is discussed.

The determination of the statistical properties of a physical process is illustrated in Chap. 7 by a study of the shot noise generated in thermionic vacuum tubes. First, the properties of shot noise in temperature-limited diodes are found in a manner similar to that used by Rice (I), and then the results so obtained are extended to space-charge-limited tubes.

One of the most commonly occurring and thoroughly studied classes of random processes is that of the gaussian processes. The statistical properties of these processes are reviewed in Chap. 8. In particular, the properties of a narrow-band gaussian random process are considered in some detail, as are the joint statistics of a sine wave and a narrow-band gaussian process.

The techniques developed in Chaps. 2 through 8 are applied to a study of the passage of random signals and noises through linear systems in Chaps. 9, 10, and 11. The analysis of the response of a linear system to an ordinary function of time is reviewed in Chap. 9 and extended there to random time functions. The correlation functions and spectral densities at the output of a linear system in response to random inputs are then determined; and the problem of obtaining the probability density function of the output of a linear system is treated. These results are applied in Chap. 10 to a study of noise in amplifiers. *Noise figure* is defined, and some of its properties are discussed. The synthesis of *optimum* linear systems is discussed in Chap. 11. In particular, the theory of least-mean-square error smoothing and predicting, using either the infinite past of the input or only a finite portion of the past, is investigated.

The passage of random processes through a class of nonlinear devices which have no memory is considered in Chaps. 12 and 13. In Chap. 12, this problem is treated directly as a transformation of variables using the nonlinear transfer characteristic of the device in question, and specific results are obtained for the full-wave square-law detector and the half-wave linear detector. In Chap. 13, the *transfer function* of a nonlinear device is defined to be the Fourier transform of the transfer characteristic of the device. The transfer function is then used to determine the autocorrelation function and spectral density of the output of a nonlinear device in response to an input consisting of the sum of a sine wave and a gaussian random process. Particular results are then obtained for the class of νth-law nonlinear devices.

Chap. 14 presents an introduction to the application of statistical-hypothesis testing and parameter estimation to problems of signal detection and extraction. The statistical principles needed are developed, including the Neyman-Pearson hypothesis test and other tests involving the likelihood ratio and the maximum-likelihood method of parameter estimation. Applications are made to radar and to radio-communications systems using a binary alphabet.

The Bibliography. The various sources referred to in this book have been collected in the Bibliography at the end of the book. Only those sources were included which seemed to us to be particularly pertinent either to the text itself or to the problems at the ends of the chapters; no attempt was made to be all-inclusive. Extensive lists of references may be found in the bibliographies of Chessin, Green, and Stumpers and in the books of Blanc-Lapierre and Fortet, Bunimovich, Cramér, Doob, Gnedenko and Kolmogorov, and Solodovnikov (see Bibliography).