Chapter 1
Dyscalculia, Dyslexia and Mathematics

Introduction

In 1981, when we moved from working in mainstream schools and began teaching in schools for dyslexic learners, our initial expectation was that teaching mathematics would be much the same as before. At that time, we could not find any source of guidance to confirm or contradict this expectation. We thought dyslexia meant difficulties with languages, not mathematics. Experience would change this impression!

Over the last 25 years, we have accumulated experience, tried out new (and old) ideas, researched, read what little appropriate material was available (there is still far less published on learning difficulties in mathematics than on languages), learned from our learners and have become convinced that difficulties in mathematics go hand in hand with difficulties in language and that a different teaching attitude and approach is needed.

The first four chapters of this book look at some of the background that influenced the evolution of these teaching methods and continues to underpin their ongoing evolution. This necessitates a look at the learner, the subject (mathematics) and the teacher. The main mathematical focus of this book is number, primarily because this is the first area of mathematics studied by children and thus provides the first opportunity to fail. Our experience suggests that number remains the main source of difficulty for most learners, even in secondary education. We also know that the foundations for all studies leading to General Certificate of Secondary Education (GCSE), and beyond, are based in these early learning experiences. The evaluations and expectations of a child's mathematical potential are often based, not always correctly, on performance in early work on number. The remaining chapters describe some of the methods we use to teach our dyslexic learners, with the ever-present caveat that no one method will work for all learners.

One of the main reasons for the first four chapters is that the methods described in the subsequent chapters will not meet the needs of every single child. As Watson (2005) states, 'There is no standard recipe for mathematical success'. The joyous range of characteristics that makes each child an individual ensures that this is true, so teachers need an understanding of the child and the subject to be able to adjust methods and improvise, from secure foundations and principles, to meet those individual needs.

We also believe that a greater understanding of the ways dyslexic and dyscalculic students learn and fail mathematics will enhance our understanding of how all children learn and fail mathematics. The extrapolation from this is that many, if not all, of the methods advocated in this book will also help many non-dyslexic students learn mathematics.

Definitions of Dyslexia

The year 1996 marked the 100th anniversary of the publication of the first paper describing a dyslexic learner (Pringle-Morgan, 1896, reproduced in the BDA Handbook 1996), yet it was only in the 1980s that definitions of dyslexia began to include difficulties in learning mathematics (often as numeracy) alongside difficulties in learning languages. Our hypothesis is that a profile similar to the one created by the factors that can create difficulties in learning mathematics (Chapter 2) could affect learning languages. A brief survey of definitions of dyslexia (and learning disabilities, an American term) shows how difficulties with learning mathematics were introduced alongside difficulties with learning languages. Now in the new millennium, it seems that the definitions of dyslexia are moving back to focus solely on language. Perhaps this is due to the current interest in dyscalculia and the trend in the United Kingdom to see specific learning difficulties used as an umbrella term, to cover dyslexia, dyscalculia, dyspraxia and dysgraphia, rather than as a label that was interchangeable with dyslexia.

In 1968 the World Federation of Neurology defined dyslexia as

> A disorder manifested by a difficulty in learning to read, despite conventional instruction, adequate intelligence and socio-cultural opportunity. It is dependent upon fundamental cognitive difficulties that are frequently of a constitutional character.

By 1972 the Department of Education and Science for England and Wales included number abilities in its definition of specific reading (sic) difficulties. In the United States, the Interagency Conference's (Kavanagh and Truss, 1988) definition of learning disabilities included 'significant difficulties in the acquisition of mathematical abilities' and, in the United Kingdom, Chasty (1989) defined specific learning difficulties as follows:

> Organising or learning difficulties, which restrict the students competence in information processing, in fine motor skills and working memory, so causing limitations in some or all of the skills of speech, reading, spelling, writing, essay writing, numeracy and behaviour.

In 1992 Miles and Miles, in their book Dyslexia and Mathematics, wrote:

> The central theme of this book is that the difficulties experienced by dyslexics in mathematics are manifestations of the same limitations which also affect their reading and spelling.

Light and Defries (1995) highlighted the comorbidity of language and mathematical difficulties in dyslexic twins. Comorbidity, the co-occurrence of two or more disorders in the same individual, has since become more of a mainstream term with the acknowledgement of specific learning difficulties other than dyslexia. For example, there has recently been a surge in the interest shown in dyscalculia (Shalev et al., 2001; DfES, 2001; Ramaa and Gowramma, 2002; Butterworth, 2003; Butterworth and Yeo, 2004; Henderson et al., 2003; Chinn, 2004; Hannell, 2005).

So definitions of dyslexia have now dropped any reference to mathematics and have focused on language, for example, as per the British Psychological Society (1999),

> Dyslexia is evident when accurate and fluent word reading and/or spelling develops very incompletely or with great difficulty. This focuses on literacy learning at 'word level' and implies that the problem is severe and persistent despite appropriate learning opportunities. It provides the basis for a staged process of assessment through reading.

This definition was adopted by the International Dyslexia Association (IDA) in 2002.

> Dyslexia is a specific learning difficulty that is neurobiological in origin. It is characterised by difficulties with accurate and/or fluent word recognition and by poor spelling and decoding abilities. These difficulties typically result from a deficit in the phonological component of language that is often unexpected in relation to other cognitive abilities and the provision of effective classroom instruction. Secondary consequences may include problems in reading comprehension and reduced reading experience that can impede growth of vocabulary and background knowledge.

If dyslexia and dyscalculia are to be defined as separate, distinct specific learning difficulties, then the concept of comorbidity becomes very relevant. An important question for researchers is to decide whether the comorbidity is causal, independent or different outcomes resulting from the same neurological basis. Our experience is that most of the dyslexics we have taught have had difficulties in at least some areas of mathematics. The outcomes, in terms of grades achieved in GCSE (the national exam for 16-year-old students in England) can be from A* to F and with one ex-student, who was severely dyslexic, a degree in mathematics. The theme of this book is of positive individual prognosis.

Recently, Yeo (2003) has looked at the issues surrounding dyspraxia, dyslexia and mathematics difficulties. The specific learning difficulty, dyspraxia, brings another set of issues in learning mathematics.

Resources and research

There is still a paucity of research in this field, particularly in comparison to research into language, as noted by Austin (1982), Sharma (1986), Miles and Miles (1992, 2004), Jordan and Montani (1997), Geary (2004)and Gersten et al. (2005). There are many examples of minimal mathematical content in publications on dyxlexia, including the Annals of the Orton Dyslexia Society (now the IDA), which had just three papers on mathematics in the 10 years from 1995 to 2004. At the last International Conference of the British Dyslexia Association (BDA) in 2004 the programme only contained 5 talks on mathematical difficulties out of some 200 talks. Similarly, in Belgium Desoete et al. (2004) from 1974 to 1997 only 28 articles on mathematical learning difficulties were cited in Psyc-Info, whereas there were 747 articles on reading disabilities.

There is, however, some reference material. Magne (1996) has compiled what must be the most extensive bibliography of publications on mathematical low achievement to that date, but he cast his net wide. Dowker (2005), Westwood (2004), and Miles and Miles (2004) have also produced thorough lists of references to relevant research. Geary remains a leading researcher in the field (for example, Geary, 1993, 1994, 2000).

One of the key factors for interventions for dyslexia is that the teaching and learning are multisensory. One of the earliest papers to suggest a multisensory approach to the teaching of mathematics to dyslexics was written by Steeves (1979), a pioneer in this field. Steeves advocated the same teaching principles for mathematics as Orton had suggested for language. Joffe (1980a, b, 1983), another pioneer in investigating dyslexia and mathematics, provided an excellent overview of the relationship between dyslexia and mathematics. Within these three relatively short papers, Joffe provided many observations that add to a clearer understanding of difficulties in learning mathematics. Most notably, Joffe drew attention to a deficit in the essential skill of generalising.

Equally, the interventions need mathematical structure and credibility. Sharma (see Berkshire Mathematics, Appendix 1) was a pioneer of this philosophy.

Sometimes the advice given by experts is contradictory, which may in part be due to the complexity of the interactions between learners and the various manifestations of mathematics. Ashlock et al. (1983), in an otherwise very useful book, state that all children learn and come to understand an idea in basically the same way, whereas Bley and Thornton (2005) begin their book with the sentence, 'Learning disabled children are unable to learn the way most children do'. (We consider the statement of Bley and Thornton to be the correct one, and hence this book!)

Dyscalculia

The concept of a specific mathematics difficulty, now named as *dyscalculia* in the United Kingdom, has slipped (Poustie, 2000) into common usage in our official documents (in sharp contrast to the acceptance of the word and concept of 'dyslexia'). The term is, however, not yet well defined. For some researchers it suggests learning difficulties that are solely related to mathematics, that is, there is an absence of a language difficulty. For some it seems to suggest a dire prognosis, that of a failure to do any mathematics or an inability to do mathematics. The little research that exists (when David Geary spoke at the 2002 IDA conference, he compared our knowledge of dyslexia to being close to adulthood and our knowledge of mathematical learning difficulties to being in its early infancy) suggests that the proportion of children in this category of a specific mathematical learning difficulty, without any comorbid condition, is small. As one would expect, the prevalence of dyscalculia will be dependent on how it is defined. It should also be noted that Geary (2004) describes dyscalculia as numerical and arithmetical deficits following overt brain injury, using instead the term 'mathematics learning difficulties' to describe the 5–8% of school-age children who have some form of memory or cognitive deficit that interferes with their ability to learn concepts or procedures in one or more mathematical domains.

The work of Kosc, a pioneer in the field of dyscalculia, and a review of the early literature on dyscalculia can be found in *Focus on Learning Difficulties in Mathematics* (Kosc, 1986). Butterworth and Yeo's new book (2004) 'Dyscalculia Guidance' provides a more recently compiled comprehensive reference list.

There are many parallels at many levels between dyslexia and dyscalculia and all that surrounds these specific learning difficulties, for example, prevalence, definition, teaching methods, etiology, perseveration, attitude of academics and so forth.

The definition of dyscalculia from the Department for Education and Skills (U.K.) booklet (2001) on supporting learners with dyslexia and dyscalculia in the National Numeracy Strategy is as follows:

> Dyscalculia is a condition that affects the ability to acquire mathematical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers, and have problems learning number facts and

> procedures. Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence.
>
> Very little is known about the prevalence of dyscalculia, its causes, or treatment. Purely dyscalculic learners who have difficulties only with numbers will have cognitive and language abilities in the normal range, and may excel in non-mathematical subjects. It is more likely that difficulties with numeracy accompany the language difficulties of dyslexia.

Perhaps it is not surprising, given that we do not have a clear agreed definition of the problem, that there is a range of figures given for the prevalence of dyscalculia and/or specific mathematics difficulties. For example, in a study by Lewis et al. (1994) of 1200 children aged 9 to 12, only 18 were identified as having specific mathematics difficulties in the absence of language difficulties. Lewis et al. did not find any one pattern or reason for this, but the study did focus on a difficulty only in mathematics, not a comorbid condition with language difficulties. The same distinction is made by Ramaa and Gowramma (2002) in a fascinating study of children in India. Ramaa and Gowramma used both inclusionary and exclusionary criteria to determine the presence of dyscalculia in primary schoolchildren. Both experiments suggest that the percentage of children identified as potentially dyscalculic was between 5.5 and 6%. Ramaa and Gowramma also list 13 observations from other researchers about the nature and factors associated with dyscalculia, including persistent reliance on counting procedures and extra stress, anxiety and depression. Sutherland (1988) states that, on the basis of his study, few children have specific problems with number alone. Rather, Miles (Miles and Miles, 1992) suggests that mathematical difficulties and language difficulties are likely to occur concurrently, and we come to the same conclusion in the last part of this chapter. More recently, Badian (1999) has produced figures for the prevalence of persistent arithmetic, reading, or arithmetic and reading disabilities, from a sample of over 1000 children, suggesting that for grades 1 to 8, 6.9% qualified as low in arithmetic, which included 3.9% low only in arithmetic.

Shalev et al. (2001) working in Israel, have suggested that developmental dyscalculia, taking a discrepancy model, has a significant familial aggregation. They estimate the prevalence of developmental dyscalculia to be between 3 and 6.5% of children in the general school population and conclude that there is a role for genetics in the evolution of this disorder. Inevitably this will raise a mathematical version of the nature/nurture debate.

The publication of Brian Butterworth's Screening Test for Dyscalculia (2003) and the inclusion of dyscalculia as a specific learning difficulty on a Department for Education and Skills (DfES) web site are helping to push dyscalculia into the educational spotlight in the United Kingdom. We contend that dyscalculia is going to be a complex concept, not least because there is unlikely to be a single reason behind the problem of the many, many people who fail to master mathematics, not all of whom will be dyscalculic.

Kosc (1974, 1986) a pioneer in the study of dyscalculia defined it in terms of brain abnormalities:

> Developmental dyscalculia is a structural disorder of mathematical abilities which has its origin in a genetic or congenital disorder of those parts of the brain that are the direct anatomico-physiological substrate of the maturation of the mathematical abilities adequate to age, without a simultaneous disorder of general mental functions.

More recently, O'Hare et al (1991) found right-hemisphere dysfunction in one case of childhood dyscalculia, with the difficulties manifesting as problems in understanding the abstract values of numbers; another child showing a poor understanding of number symbols and inability to write numbers from dictation was found to have left-hemisphere dysfunction.

Sharma (1990) comments that 'although there are significant differences between dyscalculia and acalculia, some authors have used the terms interchangeably . . . the descriptions of these terms are quite diverse to say the least'. He explains dyscalculia and acalculia as follows:

> Dyscalculia refers to a disorder in the ability to do or to learn mathematics, ie, difficulty in number conceptualisation, understanding number relationships and difficulty in learning algorithms and applying them. (An irregular impairment of ability.)
>
> Acalculia is the loss of fundamental processes of quantity and magnitude estimation. (A complete loss of the ability to count.)

It seems that some researchers are confusing acalculia with dyscalculia, tending to take the pessimistic line, which is basically viewing the problem as acalculia, whereas, if one views dyslexia and dyscalculia as similar in nature, then it would follow that many of the problems of learning mathematics can be circumvented, but will still persist into adulthood, with the danger of regression if hard-won skills are not regularly practised. This more optimistic view would not preclude great success in mathematics for some 'dyscalculics' in the same way that dyslexia has not held back some great writers and actors.

So, in some perspectives, dyscalculia infers lack of success in mathematics, which in turn suggests the questions, 'What does it mean to be successful at mathematics?' and 'What skills and strengths does a learner need to be successful at mathematics?' and 'Is it important to be successful at mathematics?'

In terms of comorbidity, Joffe's much quoted, pioneering paper (1980a) on mathematics and dyslexia included a statistic that has been applied over-enthusiastically and without careful consideration of how it was obtained, that is, '61% of dyslexics are retarded in arithmetic' (and thus, many have since assumed, 39% are not). The sample for this statistic was quite small, some 50 dyslexic learners. The mathematics test on which the statistic was largely based was the British Abilities Scales Basic Arithmetic Test, which is primarily a test of arithmetic skills. Although the test was not timed, Joffe noted that the high attainment group would have done less well if speed was a consideration. She also stated that the extrapolations from this paper would have to be cautious. Other writers seem to have overlooked Joffe's cautions and detailed observations, for example, she states, 'Computation was a slow and laborious process for a large proportion of the dyslexic sample.' The results from mathematics tests can depend on many factors and speed of working will be one of the most influential of these factors for a population that is often slow at processing written information.

At Mark College, a DfES-approved independent school for boys who have been diagnosed as dyslexic (often at the severe end of the spectrum), the results for GCSE mathematics are significantly above the national average. Usually, at least 75% of grades are at C and above compared to the national average of around 50%. Obviously we believe that if the teaching is appropriate, then a learning difficulty does not necessarily mean lack of achievement.

Later (Chapter 2), we look at the factors, such as short-term memory, working memory and long-term memory that contribute to success and failure in mathematics. These are likely to contribute to mathematics difficulties in general, and it is likely that a combination of many of these factors, within the learner and within the way he is taught, will create problems that could well be identified as dyscalculia. Butterworth's hypothesis in his recent paper (2005) is that developmental dyscalculia appears to be a specific problem with understanding, and accessing quickly (Landerl et al., 2004), basic numerical concepts and facts. He also notes that 'there are several major gaps in our knowledge'.

> As for the importance of mathematics, there is the mathematics you need for everyday life, which rarely includes algebra, fractions (other than 1/4 and 1/2), coordinates or indeed much of what is taught in secondary schools. Mathematics for everyday does include money, measurement, time and percentage. As an example of a real life mathematics exercise, let us consider the question of paying for a family meal in a restaurant. It needs estimation skills, possibly accurate addition skills, subtraction skills if using cash, and percentage skills to calculate the tip. Some careers require mathematical knowledge and skills and mathematics has a tendency to be a part of many higher education courses, even if those courses seem a long way away from mathematics.

So there are a number of questions and issues that need better answers than current knowledge can provide. Some of these questions may look rhetorical, but they are framed within the context of seeking better awareness of the nature of dyscalculia.

What is mathematics?

Mathematics is not just arithmetic or manipulating numbers. It is possible that a person could be good at some topics in mathematics and a failure in other topics. Does dyscalculia imply an inability to succeed in any of the many topics that make up mathematics?

In terms of subject content, early mathematics mostly deals with numbers. Later it becomes more varied with new topics such as measure, algebra and spatial topics. Up to GCSE, despite the different headings, the major component remains as number. So the demands of mathematics can appear quite broad, and this can be very useful, but number can be a disproportionate part of early learning experiences. So it seems that poor number skills could be a key factor in dyscalculia, but it also suggests that we have to consider the match between the demands of the task and the skills of the learner.

In terms of approach, mathematics can be a written subject or a mental exercise. It can be formulaic or intuitive. It can be learnt and communicated in either way, or a combination of ways by the learner and it can be taught and communicated in either way or a combination of ways by the teacher. Mathematics can be concrete, but fairly quickly moves to the abstract and symbolic. It has many rules and a surprising number of inconsistencies. In terms of judgment, feedback and appraisal, mathematics is quite unique as a school subject. Work is usually a blunt 'right' or 'wrong' and that judgement is a consequence of mathematics itself, not of how the teacher appraises work. And mathematics has to be done quickly. Even on this brief overview it is obvious that the demands of mathematics are varied.

What is the role of memory?

We often pose the question in lectures 'What does the learner bring?' (to mathematics). We have already mentioned some factors such as anxiety. But what about memory? We know that Krutetskii (1976) lists mathematical memory as a requirement to be good at mathematics. We are sure that short-term memory and working memory are vital for mental arithmetic, particularly for those sequential, formula-based mathematics thinkers, but can a learner compensate for difficulties in some of these requirements and thus 'succeed' in mathematics?

In English schools, we have the excellent National Numeracy Strategy. This truly is, in our opinion, an excellent programme, but however excellent be the programme, it is virtually impossible for any one programme to meet the needs of every learner. An essential part of the National Numeracy Strategy (NNS) in the early years of education is mental arithmetic, which is an activity that needs effective memories, long, short and working. So a learner with a poor short-term memory could fail when it involves mental mathematics, even though he may have the potential to become an effective mathematician. If failure is internalised as a negative attributional style by the learner, then that potential may never be realised.

It is possible that Krutetskii's *mathematical memory* draws a parallel with Gardner's multiple intelligences. Perhaps there are multiple memories. That would explain some of the discrepancies we see in children's memory performances. Like any subject, there is a body of factual information in mathematics, and if a learner can remember and recall this information, then he will be greatly advantaged, and if he cannot, then failure is likely.

So a good memory may be required for doing mathematics in general. Short-term and working memories may be essential for mental mathematics and mathematical long-term memory will be essential for the number facts and formulae you need when you are doing mental arithmetic. Geary considers memory a key factor in mathematics learning difficulties (Geary, 2004).

Counting

The first number test on the Butterworth Dyscalculia Screener is a test for subitizing. This refers to the ability to look at a random cluster of dots and know how many are there, without counting. Most adults can subitize 5–7 items.

A person who has to rely entirely on counting for addition and subtraction is severely handicapped in terms of speed and accuracy. Such a person is even more handicapped when trying to use counting for multiplication and division. Often their page is covered with endless tally marks and often they are just lined up, not grouped as ~~1111~~ that is, in fives. Mathematics is done in counting steps of one. If you show them patterns of dots or groups, they prefer the lines.

It is not just the ability to 'see' and use 5. It is the ability to see 9 as one less than 10, to see $6 + 5$ as $5 + 5 + 1$, and to count on in twos, tens and fives, especially if the pattern is not the basic one of 10, 20, 30 . . . but 13, 23, 33, 43. . . .

Students need to progress beyond the counting strategy.

It is the ability to go beyond counting in ones by seeing the patterns and relationships in numbers (Chinn and Ashcroft, 2004).

What distinguishes the dyscalculic learner from the garden-variety poor mathematician?

Stanovich (1991) asked, 'How do we distinguish between a 'garden-variety' poor reader and a dyslexic?' A key question to ask is, 'How do we distinguish between a 'garden-variety' poor mathematician and a dyscalculic?' We would suggest that the answer to this latter question has a lot to do with perseveration of the difficulty in the face of skilled, varied and appropriate intervention.

This leads to further questions, such as, 'Can you be a good reader and still be a dyslexic? Can you be good at some areas of mathematics and still be dyscalculic?' Our hypothesis is that the answer to both questions is 'Yes', but that is partly because mathematics is made up of many topics, some of which make quite different demands (and for both these questions, good and appropriate teaching can make such a difference). It has also to do with this difficulty being a continuum and that the interaction of a learner's position on that spectrum and the way he is taught creates the potential to move forwards or backwards along that spectrum of achievement.

The temptation is to return to the thought that problems with numbers are at the core of dyscalculia. It is numbers that will prevail in real life, when algebra is just a distant memory. And it is likely that the main problem is in accessing these facts accurately and quickly, usually straight from memory, rather than via inefficient strategies such as counting. There is also the practice among some educators to hold learners at the number stage in the mistaken belief that mastery of number, often judged in terms of mechanical recall of facts and procedures, is an essential prerequisite for success in mathematics.

Not all factors involved in learning difficulties are solely within the cognitive domain. A difficulty may be exacerbated by a bureaucratic decision. For example, some bureaucrats specify a level of achievement that defines whether a child's learning difficulties may be addressed in school or even assessed, influenced in this decision, at least in part, by economic considerations. But, even then, is a child's dyslexia or dyscalculia defined solely by achievement scores? Is there room to consider the individual and what he brings to the situation?

In terms of diagnosing dyscalculia, one of the few papers (Macaruso et al., 1992) looked at the assessment of patients with acquired dyscalculia, exploring which mathematical tasks should be incorporated into a diagnostic protocol. These tasks included understanding of the symbols and words used for the four operations, oral and written arithmetic and transcoding numbers.

What is appropriate teaching?

For many teachers, the first reaction to hearing that a child is diagnosed as dyscalculic will be 'So he's dyscalculic, how can I teach him?' We are certain that use of the range of methods and strategies we have developed at Mark College for teaching our dyslexic learners will also be effective with dyscalculic learners. Indeed we have probably taught many learners who have the comorbid problems of dyslexia and dyscalculia. What we address as teachers is the way the learner presents, not a learner defined solely by some stereotypical attributes.

Our colleague, Julie Kay when faced with a learner who is struggling with learning mathematics asks herself the questions, 'Where do I begin? How far back in mathematics do I go to start the intervention?' This may be a difference, should we need one, between the dyscalculic and the dyslexic who is also bad at mathematics. It may be that the starting point for the intervention is further back in the curriculum for the dyscalculic than for the dyslexic. (This may be yet another topic needing research.) It may also be that the subsequent rates of progress are different. Kaufmann et al. (2003) advocate a numeracy intervention programme that involves both basic numerical knowledge and conceptual knowledge, and that there is a need for explicit teaching of numerical domains that often have been neglected in school mathematics. In other words, 'How far back do you start to explain mathematics?'

And for a final thought in this section, we ask, 'What is the influence of the style of curriculum?' We know, for example, from a European study in which one of the authors was involved (Chinn et al., 2001), that the design of the mathematics curriculum certainly affects the thinking style in mathematics for many pupils.

What are the other interactions and factors?

There are many reasons why a child or an adult may fail to acquire mathematical skills and knowledge. For example, a child who finds symbols confusing may have been successful with mental arithmetic, but finds written arithmetic very challenging. There may be other examples of an onset of failure at different times that will most likely depend on the match between the demands of the curriculum and the skills and deficits of the learner, for example, a dyslexic will probably find word problems especially difficult, and a child who is not dyslexic but is learning at the concrete level may find the abstract nature of algebra difficult. A child who is a holistic learner may start to fail in mathematics if his new teacher uses a sequential and formula-based inchworm teaching style. A learner may have a poor mathematical memory and the demands on memory may suddenly exceed his capacity.

A difficulty will depend on the interaction between the demands of the task, the skills of the teacher and the skills and attitudes of the learner. For example, if one of the demands of mental arithmetic is that it be done quickly, then any learner who retrieves and processes facts slowly will present with learning difficulties. Learning difficulties are obviously dependent on the interaction between the learner and the learning task.

None of the underlying contributing factors discussed above are truly independent. Anxiety, for example is a consequence of many influences. Our hypothesis is that the factors mentioned are the key ones. There may well be others and the pattern and interactions will vary from individual to individual, but these are what we consider to be the difficulties at the core of dyscalculia.

Of the definitions quoted, the version of the National Numeracy Strategy (DfES, 2001) seems to be the most realistic. We have added some extra notes into the definition, which may then be better seen as a description (and thus not as a label).

Dyscalculia is a perseverant condition that affects the ability to acquire mathematical skills despite appropriate instruction. Dyscalculic learners may have difficulty understanding simple number concepts (such as place value and use of the four operations, $+$, $-$, $\times$ and $\div$), lack an intuitive grasp of numbers (including the value of numbers and understanding and using the interrelationship of numbers), and have problems learning, retrieving and using number facts quickly (for example, multiplication tables) and procedures (for example, long division). Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence (and have no way of knowing or checking that the answer is correct).

The NNS version focuses on number, which makes sense in the light of relevant research. It mentions memory and includes those who present as competent in some areas, but whose performance has no underlying understanding of number. An addendum could list some of the key contributors, such as the following:

A learner's difficulties with mathematics may be exacerbated by anxiety, poor short-term memory, inability to use and understand symbols, inflexible learning style and inappropriate teaching.

The Nature of Mathematics

In order to teach successfully, you need knowledge of the learner and knowledge of the subject. You may not need to be a degree level mathematician, but to teach mathematics effectively you must have a good understanding of the nature of mathematics and its progression beyond the immediate topics being taught. Mathematics is a subject that builds on previous knowledge as it extends knowledge. Of late we have become more convinced of the need for teachers to be flexible in ways of teaching and doing mathematics and to recognise and accept this flexibility in their pupils, too. To some extent, the new Wave 3 intervention materials (in England) that are designed to address the mathematical learning needs of those who are failing in the National Numeracy Strategy illustrate this. They are detailed, logically sequenced, heavily scripted but lacking overviews, analysis of 'what the learner brings' and 'where the learning is heading'.

Number and arithmetic are the first experience of mathematics for most children and the mathematics most people use in later life. Early experience of success or failure at this stage sets the scene for later stages, academically and emotionally. Some learners learn competence in limited areas of arithmetic, for example, they are comfortable with addition, but cannot carry out subtractions. What can create significant problems for learners are programmes that require mastery before progression (for example, Kumon mathematics) because mastery, especially of rote learning tasks, and even more especially under the pressure of working quickly, is a transient stage for many dyslexics. Consolidation and sustained mastery without frequent reviews, revision and careful interlinking of the developing strands of mathematics is a difficult task for most dyslexics. Finding the right balance between mastery and progress will be a consequence of knowing the child well and of the adaptation and structure of the teaching programme.

In terms of subject content, early mathematics consists of mostly numbers. Later it becomes more varied with new topics introduced, such as measure, algebra and spatial topics. Up to GCSE, despite the different headings, the major component remains as number. So the demands of mathematics can appear quite broad, and this can be very useful, but number can remain a disproportionate part of early learning experiences.

Numbers can be exciting, challenging tools (McLeish, 1991) or the cause of great anxiety (Buxton, 1981; Cope, 1988). Mathematics is a sequential subject, building on early skills and knowledge to take the student on to new skills and knowledge. It is a subject involving organisation and patterns (Ashcroft and Chinn, 2004) and abstract ideas and concepts. Gaps in the early stages of understanding can only handicap the learner in later stages, in the speed of processing number problems if not anything else.

Mathematics has an interrelating/sequential/reflective structure. It is a subject in which one learns the parts; the parts build on each other to make a whole; knowing the whole enables one to reflect with more understanding on the parts, which in turn strengthens the whole. Knowing the whole also enables one to understand the sequences and interactions of the parts and the way they support each other so that the getting there clarifies the stages of the journey. Teachers are (usually) in the fortunate position of being conversant with the subject and can bring to the work knowledge and experience beyond the topic they are teaching. The learner is rarely in this position and is thus vulnerable to assumptions about his levels of knowledge and experience, which are often made unconsciously by the teacher.

It is important that the learner develops a clear, broad and flexible understanding of number and processes at each stage, and that he begins to see the interrelationships, patterns, generalisations and concepts clearly and without anxiety. To teach a child to attain this understanding of mathematics requires that you also need to understand mathematics and numbers. This is not to say that every teacher who teaches arithmetic needs a degree in mathematics, but it is to say that they need to understand where mathematics is going beyond the level at which they teach and where it has come from, so that what they teach is of benefit to the child at the time and helps, not hinders, him later on as his mathematics develops. Teachers need to be mindful of what are the concepts that follow what they have taught, because the development of a concept starts long before it is addressed directly.

To illustrate this point, consider the strategy advocated in this book for teaching the 9 times table (see Chapter 6). The method uses previous information (the 10 times table), estimation, refinement of estimation and patterns. Although a child may not need to realise that he is doing all these things when he learns how to work out 6 × 9, the processes are being used, concepts are being introduced and foundations are being laid. We agree with Madsen et al. (1995) that instruction should be conceptually oriented.

A second illustration of the influence of early ideas involves a subtraction such as

$$\begin{array}{r} 93 \\ \underline{-47} \end{array}$$

A likely error is the answer 54, which occurs when the child subtracts 3 from 7. This is an easier process than the correct one, but can also be the consequence of earlier subtraction experience where the child is told to 'Take the smaller number from the larger number.' Dyslexics have a tendency to take instructions literally and feel safer in the consistency of procedures. There is also the problem that a first learning experience is often a dominant learning experience (Buswell and Judd, 1925), which means that the consequences of that experience being incorrect are very detrimental.

Margaret Rawson said of teaching English to dyslexics, 'Teach the language as it is to the child as he is'. Harry Chasty says, 'If the child does not learn the way you teach, then you must teach the way he learns'. This advice is apposite for teaching mathematics. One of the attributes of an effective teacher is clear communication. This is usually a consequence of knowing the child, usually enhanced by listening to the child, and presenting work in a way that preempts as many of the potential difficulties as possible. Thus the teacher needs to understand the way each child learns and fails to learn, though individual learning can be frustrating in that a lesson that works superbly with one child may not work at all with another (see Chapter 2). This combined understanding of the child and all his strengths, weaknesses and potentials together with a knowledge of the nature, structure and skills of mathematics will help pre-empt many of the potential learning problems. In modern UK terminology, it can keep the child at the earliest stage of intervention, Wave 1.

Finally, it should be remembered that an insecure learner values consistency. This characteristic must be linked to automaticity, in that automaticity allows the brain to devote more capacity to what is different or an extension of a known procedure. Consistency will also reduce anxiety.

Although we will refer to it again later, the culture of mathematics is that calculations should be done quickly. This, of course, dramatically handicaps any child who is a slow processor and heightens any sense of anxiety.

We believe that there are only a few key concepts in mathematics as taught to most children up to the age of 16 and that these concepts therefore reappear regularly throughout a child's progression through his school years. The benefit of this is that the child may strengthen that concept as each new manifestation appears. The drawback is that the child may never develop the concept if he has not generalised all or even some of the preceding experiences. It is a vital part of the teacher's role to ensure that as many children as possible develop a sound understanding of these concepts, rather than produce a rote-learned regurgitation of a mass of unconnected memories.