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Introduction

1.1 OBJECTIVE AND CONTENTS OF THE BOOK

Hysteresis is a nonlinear phenomenon exhibited by systems stemming from various science and engineering areas: under a low-frequency periodic excitation, the relationship between the system's input and output is not the same for loading and unloading. More precisely, consider a single-input single-output (SISO) system excited by a periodic signal that has a loading–unloading shape. Then, hysteretic systems often present a periodic response that has the same frequency of the input. When this frequency goes to zero, the quasi-static response of the system has an output versus input plot that is a cycle (not a line as would be the case for linear systems).

A fundamental theory allowing a general mathematical framework for modelling hysteresis has not been developed up to now. For specific problems, models describing hysteretic systems can be derived from an understanding of physical laws. Usually this is an arduous task and the resulting models are too complex to be used in practical applications. In general, engineering practice seeks for alternative more simple models which, although not giving the 'best' description of the physical behaviour of the system, do keep relevant input–output features and are useful for characterization, design and control purposes. These models are referred to as phenomenological or semi-physical models.

Systems with Hysteresis: Analysis, Identification and Control using the Bouc–Wen Model
F. Ikhouane and J. Rodellar

In this context, several mathematical models have been proposed to describe the behaviour of hysteretic processes [1]. The Duhem model [2] uses the property that a hysteretic system's output changes its character when the input changes direction; the Ishlinskii hysteresis operator has been proposed as a model for plasticity–elasticity [3] and the Preisach model has been used for modelling electromagnetic hysteresis [4]. A survey of mathematical models for hysteresis may be found in [5]. In the areas of smart structures and civil engineering, another model has been used extensively to describe the hysteresis phenomenon: the so-called Bouc–Wen model [6,7]. It consists of a first-order nonlinear differential equation that relates the input displacement to the output restoring force in a rate-independent hysteretic way. The parameters that appear in the differential equation can be tuned to match the hysteresis loop of the system under study.

The current literature devoted to the Bouc–Wen model is extensive and focuses mainly on:

1. Tuning the model parameters to obtain a reasonable matching of the physical hysteretic system under consideration.
2. Use of the obtained tuned model for simulation and control purposes.

It is known that most works on this model have been practically oriented. In general, rigorous mathematical justifications of the techniques associated with the use of the model have been missing. To give an example, while many papers have been devoted to tuning the Bouc–Wen model parameters (that is the identification problem), rigorous proofs on the convergence of the identified model parameters to their true counterparts are still lacking. Most works rely mainly on numerical simulations to show this convergence.

The objective of this book is to contribute to fill this gap by providing the reader with a rigorous treatment of this model. This book is based on original works by the authors that have been published in scientific journals within the last three years. It includes a mathematical treatment of the subject along with several numerical simulation examples. The book covers basically four topics:

1. Analysis of the compatibility of the model with some laws of physics.

2. Relationship between the model parameters and the hysteresis loop.
3. Identification of the model parameters.
4. Control of systems that include a hysteretic part described by the Bouc–Wen model.

The first topic is about checking whether the semi-physical Bouc–Wen model is consistent with some general laws of physics. In particular, the conditions are given under which the model is input–output stable and passive. These conditions translate into inequalities that have to be satisfied by the Bouc–Wen model parameters in order to comply with the stability and the passivity properties. Also cited is a parallel work by other authors that checks the thermodynamical admissibility of the Bouc–Wen model [8]. The techniques used in this part of the book include Lyapunov techniques for checking the stability of the model and passivity methods for the analysis of energy dissipation. The result of this analysis is a set of inequalities to be held by the Bouc–Wen model parameters. These inequalities will prove to be fundamental in deriving a new form of the Bouc–Wen model that can be called the normalized one. This new form will be used extensively in the rest of the book. This first topic is the subject of Chapter 2.

The second topic is the subject of Chapters 3 and 4. Chapter 3 is devoted to the analytical description of the hysteresis loop. Indeed, it is well known that, under loading and unloading, physical systems with hysteresis do not follow the same path, which results in a hysteresis loop. Due to the nonlinearity of the Bouc–Wen model, the hysteresis loop has never been described analytically in an explicit way. This lack of knowledge has impeded analytical studies on the relationship between the model parameters and the shape and size of the hysteresis loop. Chapter 3 presents a novel result of the authors where, using a simple but rigorous mathematical framework, the hysteresis loop is described analytically using some explicit functions that can be computed numerically in an easy way. This analytical description is illustrated and commented upon by means of a numerical simulation example.

Chapter 4 uses the analytical description of Chapter 3 to study the behaviour of the hysteresis loop when the Bouc–Wen model parameters change. This chapter is basically divided into two parts. The first part is focused on the variation of a given point of the hysteresis loop along the axes of abscissas and ordinates when the parameters of the normalized Bouc–Wen model vary. The results of this part are summarized in tables to facilitate their use. In the second part of Chapter 4, the hysteresis loop of the Bouc–Wen model is divided into four regions: the linear region, the plastic region and two regions of transition. The points that define each region are defined rigorously, which allows an analysis of the behaviour of the different regions with respect to the normalized Bouc–Wen model parameters. These regions are illustrated by means of several figures.

The third topic is the subject of Chapter 5. Identification of the parameters of the Bouc–Wen model is a crucial issue and a technical challenge for its practical use. This issue has been treated in the literature using numerical simulations, and, to the best of the authors' knowledge, no currently available method ensures analytically that the identified parameters converge to their true counterparts. In this chapter, a new identification technique is presented that uses the results of Chapter 3 to identify in an exact way the parameters of the normalized Bouc–Wen model. The technique consists of imposing two specific input displacement functions that are wave-periodic; this means that the displacements have a loading–unloading shape, and are periodic in time. Then the two obtained limit cycles are used to identify the Bouc–Wen model parameters. Chapter 5 is divided into two parts:

1. The first part presents the identification methodology and analyses its robustness with respect to external disturbances.
2. The second part of the chapter consists in applying this methodology to a magnetorheological (MR) damper, which is described by a model that includes a Bouc–Wen hysteresis. The values of the parameters of the model are taken from the literature and are unknown to the identification algorithm. Numerical simulations are carried out to illustrate the applicability of the identification method.

The fourth topic is the subject of Chapter 6. It consists of the control of a mechanical/structural system containing a hysteresis described by the Bouc–Wen model, and represents a base-isolated structure. The system parameters are not known exactly but they lie in known intervals. The control objective is to regulate the system around zero while maintaining the boundedness of the closed-loop signals. The control law is a simple proportional-integral-derivative

(PID) whose parameters are to be tuned in a specific way to guarantee the boundedness of all the closed-loop signals. Furthermore, the controller ensures the asymptotic convergence to zero of the mass displacement and velocity. The interest of this chapter is to show that a linear controller may ensure the control objective in the presence of a Bouc–Wen hysteresis.

1.2 THE BOUC–WEN MODEL: ORIGIN AND LITERATURE REVIEW

The starting point of the so-called Bouc–Wen model is the early paper by Bouc [6], where a functional that describes the hysteresis phenomenon was proposed. Consider Figure 1.1, where $\mathcal{F}$ is a force and x a displacement. Four values of $\mathcal{F}$ correspond to the single point $x = x_0$, which means that $\mathcal{F}$ is not a function. If it is considered that x is a function of time, then the value of the force at the instant time t will depend not only on the value of the displacement x at the time t, but also on the past values of x. The following simplifying assumption is made in Reference [6].

Assumption 1. *The graph of Figure 1.1 remains the same for all increasing functions $x(\cdot)$ between* 0 *and x_1, for all decreasing functions $x(\cdot)$ between the values x_1 and x_2, etc.*

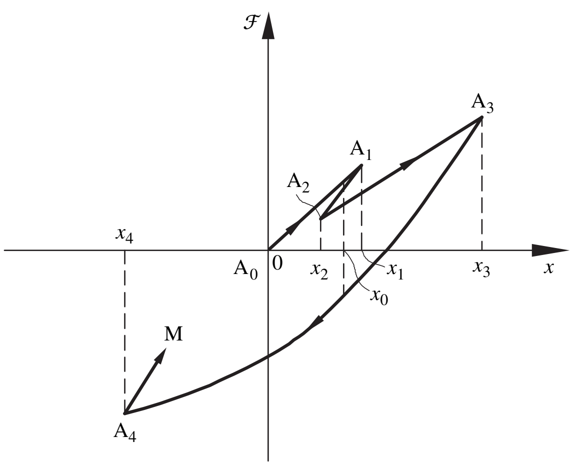

Figure 1.1 Graph force versus displacement for a hysteresis functional.

Assumption 1 is what, in the current literature, is called the *rate-independent property* [1]. To define the form of the functional $\mathcal{F}$, Reference [6] elaborates on previous works to propose the following form:

$$\frac{\mathrm{d}\mathcal{F}}{\mathrm{d}t} = g\left(x, \mathcal{F}, \operatorname{sign}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)\right)\frac{\mathrm{d}x}{\mathrm{d}t} \tag{1.1}$$

Consider the equation

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \mathcal{F}(t) = p(t) \tag{1.2}$$

for some given input $p(t)$ and initial conditions

$$\frac{\mathrm{d}x}{\mathrm{d}t}(t_0), \qquad x(t_0) \qquad \text{and} \qquad \mathcal{F}(t_0)$$

at the initial time instant t_0. Equations (1.1) and (1.2) describe completely a hysteretic oscillator.

Paper [6] notes that it is difficult to give explicitly the solution of Equation (1.1) due to the nonlinearity of the function g. For this reason, the author proposes the use of a variant of the Stieltjes integral to define the functional $\mathcal{F}$:

$$\mathcal{F}(t) = \mu^2 x(t) + \int_{\beta}^{t} F\left(V_s^t\right) \mathrm{d}x(s) \tag{1.3}$$

where $\beta \in [-\infty, +\infty)$ is the time instant after which the displacement and force are defined. The term V_s^t is the total variation of x in the time interval $[s, t]$. The function F is chosen in such a way that it satisfies some mathematical properties compatible with the hysteresis property. The following is an example of this choice given in Reference [6] so that these mathematical properties are satisfied:

$$F(u) = \sum_{i=1}^{N} A_i \mathrm{e}^{-\alpha_i u} \qquad \text{with } \alpha_i > 0 \tag{1.4}$$

Equations (1.2) to (1.4) can then be written in the form

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \mu^2 x + \sum_{i=1}^{N} Z_i = p(t) \tag{1.5}$$

$$\frac{\mathrm{d}Z_i}{\mathrm{d}t} + \alpha_i \left| \frac{\mathrm{d}x}{\mathrm{d}t} \right| Z_i - A_i \frac{\mathrm{d}x}{\mathrm{d}t} = 0, \qquad i = 1, \ldots, N \tag{1.6}$$

Equations (1.5) and (1.6) are what is now known as the Bouc model. The derivation of these equations is detailed in Reference [6]. The objective here is not to enter in these details, but only to give a short idea of the origin of the model. Equation (1.6) has been extended in Reference [7] to describe restoring forces with hysteresis in the following form:

$$\dot{z} = -\alpha|\dot{x}|z^n - \beta\dot{x}|z^n| + A\dot{x} \qquad \text{for } n \text{ odd} \tag{1.7}$$

$$\dot{z} = -\alpha|\dot{x}|z^{n-1}|z| - \beta\dot{x}z^n + A\dot{x} \qquad \text{for } n \text{ even} \tag{1.8}$$

Equations (1.7) and (1.8) constitute the earliest version of what is now called the Bouc–Wen model. The shape of the hysteresis loop is given in Reference [7] for different values of the model parameters. Some subsequent works have proposed different modifications of the model to take into account some physical properties observed experimentally in some hysteretic systems. In Reference [9], the authors consider the modelling of degradation in civil engineering structures. A multidegree of freedom shear beam structure is modelled in the form

$$m_i \left(\sum_{j=1}^{i} \ddot{u}_j + \ddot{\xi}_B \right) + q_i - q_{i+1} = 0 \qquad \text{for } i = 1, \ldots, n \tag{1.9}$$

in which m_i is the mass of the ith floor, $\ddot{\xi}_B$ is the ground acceleration and q_i is the ith restoring force, including viscous damping. The quantities u_i are the relative displacement of the ith and the $(i+1)$th stories, and q_i is given as

$$q_i = c_i\dot{u}_i + \alpha_i k_i u_i + (1-\alpha_i)k_i z_i \qquad \text{for } i = 1, \ldots, n \tag{1.10}$$

in which c_i is the viscous damping, k_i controls the initial tangent stiffness, α_i controls the ratio of post-yield to pre-yield stiffness and z_i is the ith hysteresis which obeys the equation

$$\dot{z}_i = \frac{A_i\dot{u}_i - \nu_i\left(\beta_i|\dot{u}_i||z_i|^{n_i-1}z_i + \gamma_i\dot{u}_i|z_i|^{n_i}\right)}{\eta_i} \qquad \text{for } i = 1, \ldots, n \tag{1.11}$$

where A_i, ν_i, β_i, γ_i, η_i and n_i are parameters that control the hysteresis shape and the degradation of the system. System degradation is introduced into the model for z_i by allowing the parameters of the model to vary as a function of the response duration and severity. Pinching has been considered in Reference [10] by modifying the Bouc–Wen model in the form

$$\dot{z} = h(z)\,\frac{\dot{u} - \nu\left(\beta|\dot{u}||z|^{n-1}z + \gamma\dot{u}|z|^{n}\right)}{\eta} \tag{1.12}$$

where $h(z)$ is the function that describes pinching. A discussion on how to choose this function for wood systems is given in Reference [11]. Other modifications of the Bouc–Wen model include ones to describe a soft soil [12], an asymmetric response as observed in shape memory alloys [13], the response of steel buildings under earthquakes [14], the drift observed under a zero-mean, broad-band, stationary-random load [15] and the behaviour of low yield strength steel [16]. In a parallel research line, extensions of the Bouc or Bouc–Wen models to the multivariate case have been done in References [17] and [18].

Figure 1.2 illustrates that the literature on the Bouc–Wen model has increased rapidly during the last few years. It quantifies the number of papers published in journal papers, most of which are quoted in the references given at the end of the book.

One of the main issues in the literature devoted to the Bouc–Wen model is parameter identification. Several techniques have been

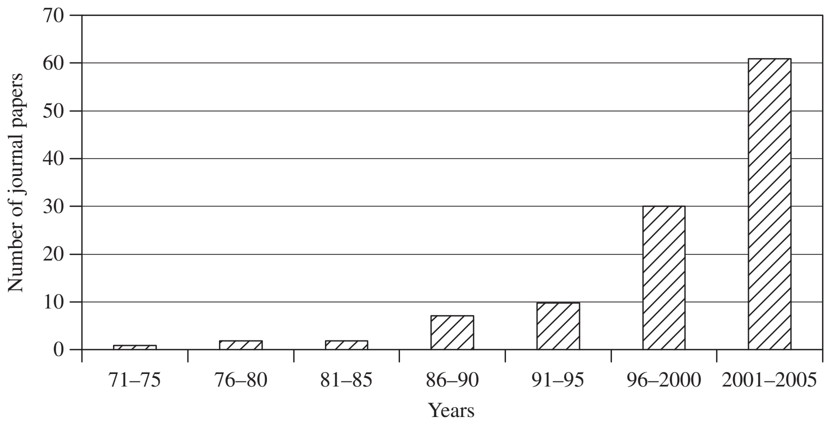

Figure 1.2 Evolution of the Bouc–Wen model literature.

used to deal with this problem. In Reference [19], a nonrecursive least error minimization algorithm is used. A recursive least-squares algorithm has been used in Reference [20], along with the Newton method and the extended Kalman filtering technique. More recent works that use some version of the least-squares algorithm include References [21] to [25]. For example, Reference [21] considers a second-order single-degree-of-freedom system which is a mass subject to a nonlinear restoring force and an external excitation. The restoring force is represented as a Bouc–Wen hysteresis whose input is the velocity of the mass. When the mass is exactly known, the restoring force can be calculated knowing the instantaneous external excitation and the acceleration of the mass. In this case, all the Bouc–Wen model parameters appear linearly except the exponent of the differential equation. This nonlinearity is coped with by assuming knowledge of an upper bound on the exponent and writing the Bouc–Wen differential equation as a sum of terms whose number is the upper bound. Then, a first-order filter is used to write the nonlinear system in a way that allows the use of the least-squares algorithm to identify the system parameters. The case of unknown mass is treated similarly by using an on-line estimation of the restoring force.

Genetic-type algorithms for the determination of the Bouc–Wen model parameters have been used in References [26] to [29]. For example, Reference [27] uses a differential evolution algorithm whose main difference with conventional genetic algorithms is in the way the mechanisms of mutations and crossover are performed using real floating point numbers instead of long strings of zeros and ones. This algorithm starts with an initial pool of 15 three-dimensional vectors drawn from uniform probability distributions. The differential evolution mutates a randomly selected number of the featured generation with vector differentials. Each differential is the difference between two randomly selected vectors, scaled with a parameter. This process generates a new mutated vector. Natural selection is implemented via a comparison process between the cost of the trial vector and the cost of the target vector. The differential evolution algorithm generates a new set of 15 three-dimensional vectors, which is a new generation with improved characteristics.

Methods that use the frequency domain have been utilized in References [30] to [33]. For example, Reference [30] considers a second-order system coupled with a Bouc hysteresis. The nonlinear system is excited with a periodic input and the Bouc model parameters are determined by using a first harmonic approximation. A higher number of harmonics is considered in Reference [31].

Neural networks have been used in Reference [34]. In this work, an inverse model for a magnetorheological damper has been developed using a multilayer perception network and system identification-based ARX model.

Bayesian parameter estimation is used in References [35] to [38]. For example, Reference [35] uses a modified version of the extended Kalman filter and the particle filter to determine the parameters of a second-order Bouc–Wen hysteresis.

A nonparametric identification method has been proposed in Reference [39]. The nonlinear hysteresis part of the system is written as a linear combination of polynomial functions with unknown coefficients. These coefficients are determined using a least-squares algorithm.

Other proposed identification techniques are included in References [40] to [48].

Control of mechanical systems and structures with Bouc–Wen hysteretic behaviour has also spurred much effort in the current literature. In this sense, it may be useful to distinguish between active and semi-active control. A control law is said to be active when the control signal directly feeds an actuator that applies the desired feedback control force. With an active control scheme, energy is injected into the closed-loop system. A control law is semi-active when the corresponding actuator does not pour energy into the closed loop. Instead, the control signal is generated by the controller to modify the characteristics of an adaptive passive-like actuator. Examples of semi-active actuators are the devices based on smart materials, in particular the magnetorheological dampers.

Now a brief overview of the recent control literature related to the Bouc–Wen model is given. Active control is described in References [49] to [58]. In Reference [49] fuzzy control is used for a structure modelled as a second-order single-degree-of-freedom structural system that includes a Bouc–Wen hysteresis. In Reference [51], an H_∞ controller is proposed to cope with the presence of uncertainties. In the other references nonlinear controllers based on Lyapunov techniques are used to ensure stability and some degree of performance in spite of the uncertainties.

Semi-active control is often used in relation to MR dampers. Reference [59] gives a state-of-the-art review of semi-active control systems for the seismic protection of structures. Recent references include [53] and [60] to [72]. For example, Reference [60] considers several semi-active control strategies using MR dampers for the control of a six-storey building. These control algorithms include a Lyapunov controller, decentralized bang-bang controller, modulated homogeneous friction algorithm and a clipped optimal controller. Each algorithm uses measurements of the absolute acceleration and device displacements for determining the control action to ensure that the algorithms would be implementable on a physical structure. The performance of the algorithms is compared through a numerical example, and the advantages of each algorithm are discussed.

The Bouc–Wen model has been extensively used for modelling hysteresis in structural and mechanical systems [44, 62, 73–95]. For example, Reference [88] considers an MR damper for which a dynamic model is to be developed. The damper force is written as the sum of several terms:

1. The damper friction due to seals and measurement bias.
2. The product of the equivalent mass which represents the MR fluid stiction phenomenon and inertial effect, and the acceleration of the piston.
3. The product of the piston velocity and the post-yield plastic damping coefficient.
4. The product of the piston position and the factor that accounts for the accumulator stiffness and the MR fluid compressibility.
5. A hysteretic term.

The hysteresis part of the model is assumed to follow a Bouc–Wen equation. Experiments are carried out to verify the validity of the model.

There are other works that have used the Bouc–Wen model [8, 96–123]. These works are difficult to classify into a single homogeneous group as their research subjects are diverse. However, they mostly deal with the analysis of some properties of systems that include a Bouc–Wen hysteresis. For example, Reference [107] analyses the influence of hysteresis dissipation on chaotic responses, Reference [113] studies the nonlinear response of a Bouc–Wen hysteretic oscillator under evolutionary excitation and Reference [110] addresses strategies for finding the design point in nonlinear finite element reliability analysis.

This book treats the univariate basic Bouc–Wen model, that is the one that has one input and one output, and describes only the hysteresis phenomenon regardless of other types of nonlinear behaviours (like pinching and others). This choice is motivated by the fact that most references treat only this basic Bouc–Wen model. The extension of the results of this book to the multivariate model, which may include other types of nonlinearities, is still an open problem and a possible subject for future research.