Chapter 1

Introduction

1.1 Twenty years of development at EDF

Since 1987, the year in which the Telemac project was launched, the Research and Development Department of Electricité de France (EDF) embarked upon a sustained effort to develop a hydro-informatics system dedicated to free surface flows. The subject is of considerable strategic interest. Dams, thermal and nuclear power stations on sea shores or river banks are all structures which interact with an aquatic environment that has to be protected and from which we need to protect ourselves. The quality of water, the calculation of tides and swells during storms, dam break flood waves, floods, the impact and stability of development, and sedimentology, are all crucial questions for EDF, and for all these disciplines a good command of the mechanics of free surface flows is indispensable.

Historically, physical models first ensured this command but these have progressively been replaced by numerical modelling. Today, the computer has become an unavoidable investigative tool and physical models, although still indispensable in certain key domains like spillways and dyke stability, have tended to become validation tools in other spheres.

In the last decade, numerical simulation has stood out as a vital tool for environmental hydraulics, through the concerted action of two important trends:

- Technical progress, with, on the one hand, exponential progress in the speed and memory capacity of computers and, on the other hand, the advent of new algorithms and numerical methods.
- The rise in requirements brought about, on the one hand, by a reinforcement of legislation (stocking of hydrocarbons, emergency plans, decrees on intakes and discharge, compensation water, etc.) and, on the other hand, by a series of so-called natural catastrophes. However, their anthropogenic origin (greenhouse effect, regional development) cannot be overlooked, and requires re-examination of the hypotheses on the sizing of structures.

Thus every technological advance, whether it arises from a new machine architecture or a new modelling technique, has been used immediately and advantageously to

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solve more and more complex problems. This situation remains unchanged and specific theoretical challenges like turbulence, or the complex physical conditions arising in sedimentology, pose problems which leave us far from our goal. From simple software solving a single type of equation, we have gradually progressed to the notion of hydro-informatics and sets of codes dealing with computerized data and capable of communicating for the processing of complex problems.

The stakes are high. The investment for a hydro-informatics system is of the order of 500000 to 1 million euros a year, but the advantages that can be obtained from it are much higher. Thus, numerical simulation of the impact of civil engineering work during a flood recently prevented an expense of 12 million euros. Another one, by helping to predict in a convincing manner the evolution of a thermal patch, helped to reduce the expense of a measurement collection campaign. However, this concerns only money matters.

To develop its hydro-informatics Telemac system, the National Laboratory of Hydraulics and Environment of EDF relied on the finite element method in which it had acquired experience in pressure flow, in particular with the N3S code. Originally, EDF had at disposal a background in finite differences on curvilinear meshing which largely predetermined the exotic architecture of the first Telemac-2D code: a theoretical framework and formalism in finite elements but with regular curvilinear meshing. The immediate enthusiasm of the first users for real non-structured meshing was quickly followed by a confirmation of their interest, and the modelling approach has therefore been maintained. However it should be noted that, in 2006, the three main numerical techniques relying on meshing, namely: finite differences, finite elements and finite volumes, are all still very widely used. Finite differences do indeed offer, thanks to their simplicity, the possibility of developing high-order numerical schemes at a low cost, while finite elements offer at the same time a strict theoretical framework and a flexibility to discretize the domain of computation, while finite volumes help to solve simply all the equations written in conservative form. Each of these three main families of techniques is subdivided into a large number of variants and, today, it is quite difficult to differentiate between the current and the outdated ones. The trend emerging currently would appear to be a gradual rejection of finite differences and a convergence on finite elements and finite volumes involving non-structured meshing. New techniques like distributive schemes are perhaps precursors of this evolution. Other techniques known as “Lagrangian”, like the SPH (Smoothed Particle Hydrodynamics) method, which have already produced promising results in classically difficult domains like mixed flows, could play an important role in the future. Without predicting the form that future methods take, we shall just state that the finite element method is currently the most widely used in France by research departments and organizations in environmental hydraulics. In this work, while restricting ourselves to hydrodynamics, we intend to put forward certain technical advances achieved while building the Telemac system. We shall achieve this in a coherent framework from the establishment of the basic equations to their numerical implementation. In Chapter 2, we establish several variants of the Navier–Stokes and the Saint-Venant equations, the tracer equation and a few outlines of turbulence equations. After a short Chapter 3 on the finite element method, Chapter 4 elaborates the numerical resolution of the Saint-Venant equations. Also proposed in this chapter is a numerical scheme for the Boussinesq equations. Chapter 5 deals with the resolution of the Navier–Stokes equations, first with and then without a hydrostatic hypothesis. Chapter 6 gives a glimpse of some methods adapted to account for the hyperbolic nature of the transport equations. Chapter 7 gives in detail the computerized implementation of the finite element method utilizing modern forms of matrix storage: element-by-element storage and edge-based storage. The optimization of algorithms is dealt with mainly from the point of view of computers with vector architecture. Parallelism with distributed memory computers is dealt with later in Chapter 8. In Chapter 9 details are given of the techniques of parameter estimation in the Saint-Venant equations using an adjoint method. Finally, Chapter 10 presents some applications which include the calculation of the dam-break wave of the Malpasset dam.

1.2 Some smoother pebbles...

For a knowledgeable reader familiar with the classical techniques and the main numerical schemes used in free surface flow mechanics, and who simply wants more information about the new numerical schemes and the optimization techniques proposed in this book, here is a short résumé.

1.2.1 Saint-Venant equations

The algorithm leading to a linear system coupling water depth and velocity dates from 1988 and has since been continuously employed by hundreds of users. It has successfully replaced the Uzawa algorithm used earlier. Its strength goes beyond the theoretical limits of the equations, including its extension for solution of the Boussinesq equations. Among the essential points of this algorithm we may cite the choice of a non-conservative form, the weakly formulated impermeability conditions at solid boundaries and preconditioning called "c–u", based on a celerity–velocity form of the Saint-Venant equations, where the depth is replaced by the celerity of shallow water waves.

The special processing of exposed beds and dry areas, which does not rely on any notion of minimum water depth and assures an exact conservation of mass, is undoubtedly the advance which has contributed most to the success of the Telemac-2D software, on account of the range of application domains that it allows and the user satisfaction it achieves.

For the resolution of equations in Mercator projection, the use of local coordinates for each element is also a novel approach, which avoids changing the equations without giving rise to a topological problem.

Simultaneous use of porosity and drag force to simulate the presence of vertical obstacles like bridge piers or trees was presented for the first time in the year 2000.

1.2.2 Navier–Stokes equations

The proposed hydrostatic option, with a fractional step method and a sigma transform, is relatively classical. The solution presented for the continuity equation is new and we owe it to Jean-Marc Janin, the person in charge and trustworthy guardian for several years of the Telemac-3D software. From a staggered vertical velocity, he draws, thanks to the distributive schemes which he applies to prisms, an astonishingly conservative and monotonic model for the advection–diffusion equation of a tracer.

For non-hydrostatic Navier–Stokes equations, we attribute the pioneering research work to Jacek Jankowski who, in 1998, implemented the first version of this option in Telemac-3D. In 2001, the use of the Saint-Venant equations for the "hydrostatic" step increased the robustness tremendously, to the extent of obtaining the calculation of the solitary wave with only one layer of prisms in the three-dimensional meshing, a calculation which is undoubtedly very new.

In the same year, the algorithm for processing exposed beds, initially developed in two dimensions, was extended to three dimensions. This progress led to the calculation of an exposed beach with a non-hydrostatic option, a computation including both dry areas and torrential flow.

In 2004, the idea of the wave equation, which was already used in the Saint-Venant equations, was extended to three dimensions.

1.2.3 Finite element techniques and optimization

The discovery of the "non-assembled matrices" method in 1987 at EDF gave an impetus to the Telemac project. At the same time, the emergence of symbolic computation software enabled the rejection of Gauss quadrature. It was only a little later that the connection was made with the method known as the American EBE (Element By Element), which was older by four years. Reading the publications on this technique then served to us "on a plate" the Crout preconditioning in EBE storage. Our vectorization algorithm for assembling vectors, founded on a specific numbering of elements, was certainly original, as is also the processing of Dirichlet boundary conditions (i.e. imposed values), realized with a matrix–vector product. Interpreting the main required operations in finite elements as matrix–vector products, e.g. diagonal preconditioning, has then greatly facilitated the passage to parallelism.

Jean-Marc Janin conceived the idea of assembling matrices and sharing the terms between the elements by an exchange of elementary terms through segments. This way of storing, which already enabled a frontal matrix–vector product, was completely supplanted by a very similar edge-based storage, this idea being picked up during a congress. In the absence of precise references, all algorithms built on edge-based storage, and about which an idea is given in Section 7.3.4, have been recreated, and adapted to parallelism.