

Fundamental Parameters and Definitions for Antennas

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1.1 INTRODUCTION

To describe the performance of an antenna, definitions of various parameters are necessary. Some of the parameters are interrelated and not all of them need be specified for complete description of the antenna performance. Parameter definitions are given in this chapter. Many of those in quotation marks are from the *IEEE Standard Definitions of Terms for Antennas* (IEEE Std 145-1983).[†] This is a revision of the IEEE Std 145-1973. A more detailed discussion can be found in Ref. 1.

1.2 RADIATION PATTERN

An antenna *radiation pattern* or *antenna pattern* is defined as “a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates. In most cases, the radiation pattern is determined in the far-field region and is represented as a function of the directional coordinates. Radiation properties include power flux density, radiation intensity, field strength, directivity, phase, or polarization.” The radiation property of most concern is the two- or three-dimensional spatial distribution of radiated energy as a function of the observer’s position along a path or surface of constant radius. A convenient set of coordinates is shown in Figure 1.1. A trace of the received electric (magnetic) field at a constant radius is called the *amplitude field pattern*. On the other hand, a graph of the spatial variation of the power density along a constant radius is called an *amplitude power pattern*.

Often the *field* and *power* patterns are normalized with respect to their maximum value, yielding *normalized field* and *power patterns*. Also, the power pattern is usually plotted on a logarithmic scale or more commonly in decibels (dB). This scale is usually desirable because a logarithmic scale can accentuate in more detail those parts of the

[†]*IEEE Transactions on Antennas and Propagation*, Vol. AP-17, No. 3, May 1969; Vol. AP-22, No. 1, January 1974; and Vol. AP-31, No. 6, Part II, November 1983.

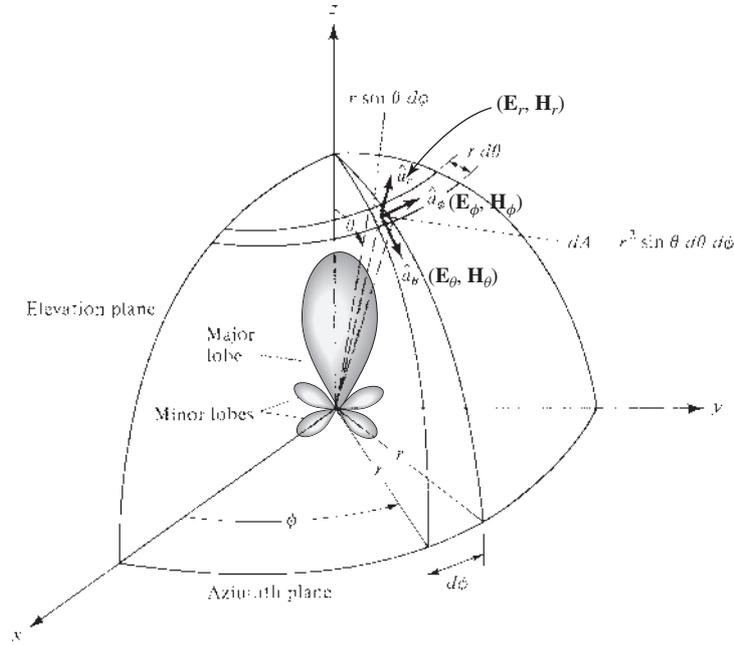


Figure 1.1 Coordinate system for antenna analysis.

pattern that have very low values, which later we refer to as minor lobes. For an antenna, (1) the *field* pattern in (*linear scale*) typically represents a plot of the magnitude of the electric or magnetic field as a function of the angular space; (2) the *power* pattern in (*linear scale*) typically represents a plot of the square of the magnitude of the electric or magnetic field as a function of the angular space; and (3) the *power* pattern in (*dB*) represents the magnitude of the electric or magnetic field, in decibels, as a function of the angular space.

To demonstrate this, the two-dimensional normalized field pattern (*plotted in linear scale*), power pattern (*plotted in linear scale*), and power pattern (*plotted on a logarithmic dB scale*) of a 10-element linear antenna array of isotropic sources, with a spacing of $d = 0.25\lambda$ between the elements, are shown in Figure 1.2. In this and subsequent patterns, the plus (+) and minus (-) signs in the lobes indicate the relative polarization of the amplitude between the various lobes, which changes (alternates) as the nulls are crossed. To find the points where the pattern achieves its half-power (-3 dB points), relative to the maximum value of the pattern, you set the value of (1) the *field* pattern at 0.707 value of its maximum, as shown in Figure 1.2a; (2) the *power* pattern (in a linear scale) at its 0.5 value of its maximum, as shown in Figure 1.2b; and (3) the *power* pattern (in dB) at -3 dB value of its maximum, as shown in Figure 1.2c. All three patterns yield the same angular separation between the two half-power points, 38.64° , on their respective patterns, referred to as *HPBW* and illustrated in Figure 1.2. This is discussed in detail in Section 1.5.

In practice, the three-dimensional pattern is measured and recorded in a series of two-dimensional patterns. However, for most practical applications, a few plots of the pattern as a function of θ for some particular values of ϕ , plus a few plots as a function of ϕ for some particular values of θ , give most of the useful and needed information.

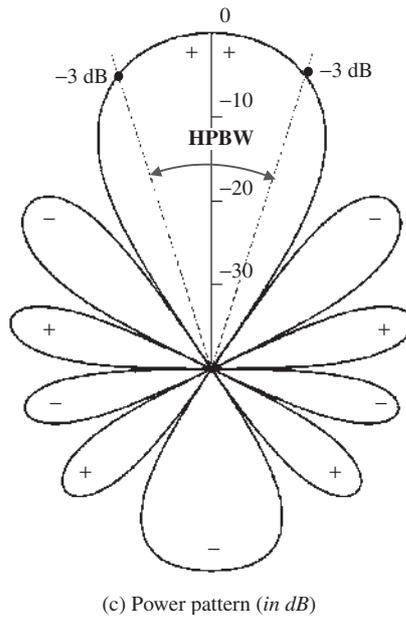
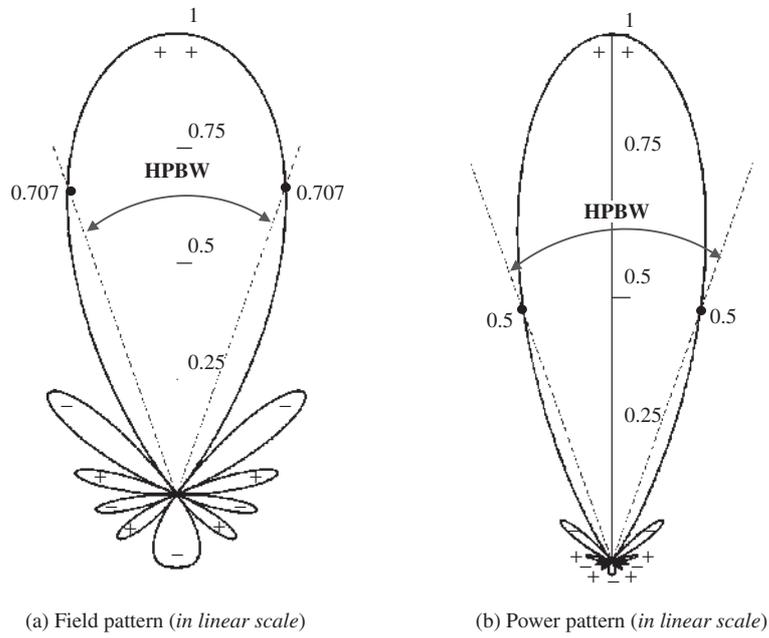


Figure 1.2 Two-dimensional normalized *field* pattern (linear scale), *power* pattern (linear scale), and *power* pattern (in dB) of a 10-element linear array with a spacing of $d = 0.25\lambda$.

1.2.1 Radiation Pattern Lobes

Various parts of a radiation pattern are referred to as *lobes*, which may be subclassified into *major* or *main*, *minor*, *side*, and *back* lobes.

A *radiation lobe* is a “portion of the radiation pattern bounded by regions of relatively weak radiation intensity.” Figure 1.3a demonstrates a symmetrical three-dimensional polar pattern with a number of radiation lobes. Some are of greater radiation intensity than others, but all are classified as lobes. Figure 1.3b illustrates a linear two-dimensional pattern (one plane of Figure 1.3a) where the same pattern characteristics are indicated.

MATLAB-based computer programs, designated as *polar* and *spherical*, have been developed and are included in the CD of [1]. These programs can be used to plot the two-dimensional patterns, both polar and semipolar (in *linear* and *dB scales*), in polar form and spherical three-dimensional patterns (in *linear* and *dB scales*). A description

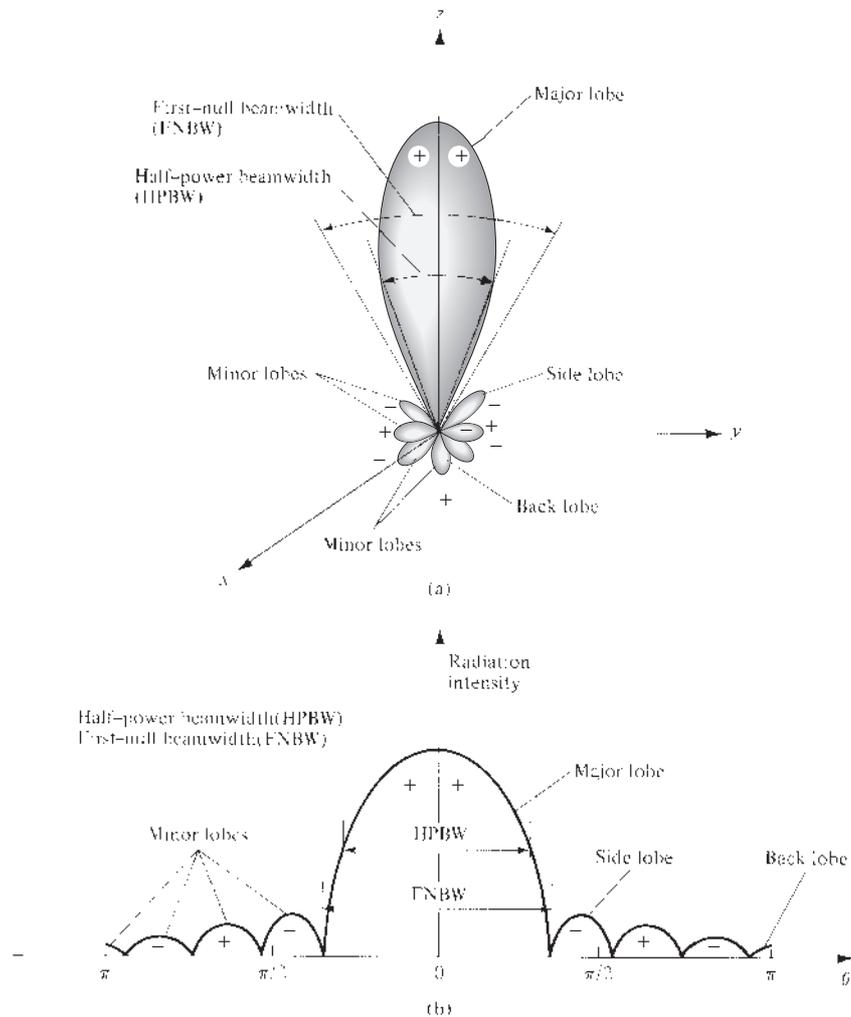


Figure 1.3 (a) Radiation lobes and beamwidths of an antenna pattern. (b) Linear plot of power pattern and its associated lobes and beamwidths.

of these programs is found in the CD attached to Ref. 1. Other programs that have been developed for plotting rectangular and polar plots are those of Refs. 1–5.

A *major lobe* (also called main beam) is defined as “the radiation lobe containing the direction of maximum radiation.” In Figure 1.3 the major lobe is pointing in the $\theta = 0$ direction. In some antennas, such as split-beam antennas, there may exist more than one major lobe. A *minor lobe* is any lobe except a major lobe. In Figures 1.3a and 1.3b all the lobes with the exception of the major can be classified as minor lobes. A *side lobe* is “a radiation lobe in any direction other than the intended lobe.” (Usually a side lobe is adjacent to the main lobe and occupies the hemisphere in the direction of the main beam.) A *back lobe* is “a radiation lobe whose axis makes an angle of approximately 180° with respect to the beam of an antenna.” Usually it refers to a minor lobe that occupies the hemisphere in a direction opposite to that of the major (main) lobe.

Minor lobes usually represent radiation in undesired directions, and they should be minimized. Side lobes are normally the largest of the minor lobes. The level of minor lobes is usually expressed as a ratio of the power density in the lobe in question to that of the major lobe. This ratio is often termed the side lobe ratio or side lobe level. Side lobe levels of -20 dB or smaller are usually not desirable in many applications. Attainment of a side lobe level smaller than -30 dB usually requires very careful design and construction. In most radar systems, low side lobe ratios are very important to minimize false target indications through the side lobes.

A normalized three-dimensional far-field amplitude pattern, plotted on a linear scale, of a 10-element linear antenna array of isotropic sources with a spacing of $d = 0.25\lambda$ and progressive phase shift $\beta = -0.6\pi$ between the elements is shown in Figure 1.4. It

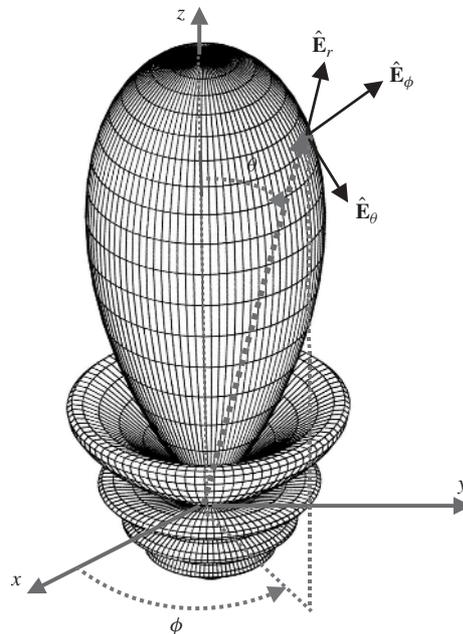


Figure 1.4 Normalized three-dimensional amplitude *field* pattern (in *linear scale*) of a 10-element linear array antenna with a uniform spacing of $d = 0.25\lambda$ and progressive phase shift $\beta = -0.6\pi$ between the elements.

is evident that this pattern has one major lobe, five minor lobes, and one back lobe. The level of the side lobe is about -9 dB relative to the maximum. A detailed presentation of arrays is found in Chapter 6 of Ref. 1. For an amplitude pattern of an antenna, there would be, in general, three electric-field components (E_r , E_θ , E_ϕ) at each observation point on the surface of a sphere of constant radius $r = r_c$, as shown in Figure 1.1. In the far field, the radial E_r component for all antennas is zero or, vanishingly small compared to either one, or both, of the other two components (see Section 3.6 of Chapter 3 of Ref. 1). Some antennas, depending on their geometry and also observation distance, may have only one, two, or all three components. In general, the magnitude of the total electric field would be $|\mathbf{E}| = \sqrt{|E_r|^2 + |E_\theta|^2 + |E_\phi|^2}$. The radial distance in Figure 1.4, and similar ones, represents the magnitude of $|\mathbf{E}|$.

1.2.2 Isotropic, Directional, and Omnidirectional Patterns

An *isotropic* radiator is defined as “a hypothetical lossless antenna having equal radiation in all directions.” Although it is ideal and not physically realizable, it is often taken as a reference for expressing the directive properties of actual antennas. A *directional* antenna is one “having the property of radiating or receiving electromagnetic waves more effectively in some directions than in others. This term is usually applied to an antenna whose maximum directivity is significantly greater than that of a half-wave dipole.” Examples of antennas with directional radiation patterns are shown in Figures 1.5 and 1.6. It is seen that the pattern in Figure 1.6 is nondirectional in the azimuth plane ($f(\phi)$, $\theta = \pi/2$) and directional in the elevation plane ($g(\theta)$, $\phi = \text{constant}$). This type of a pattern is designated as *omnidirectional*, and it is defined as one “having an essentially nondirectional pattern in a given plane (in this case in azimuth) and a directional pattern in any orthogonal plane (in this case in elevation).” An *omnidirectional* pattern is then a special type of a *directional* pattern.

1.2.3 Principal Patterns

For a linearly polarized antenna, performance is often described in terms of its principal *E*- and *H*-plane patterns. The *E*-plane is defined as “the plane containing the electric-field vector and the direction of maximum radiation,” and the *H*-plane as “the plane containing the magnetic-field vector and the direction of maximum radiation.” Although it is very difficult to illustrate the principal patterns without considering a specific example, it is the usual practice to orient most antennas so that at least one of the principal plane patterns coincides with one of the geometrical principal planes. An illustration is shown in Figure 1.5. For this example, the *x*-*z* plane (elevation plane; $\phi = 0$) is the principal *E*-plane and the *x*-*y* plane (azimuthal plane; $\theta = \pi/2$) is the principal *H*-plane. Other coordinate orientations can be selected.

The omnidirectional pattern of Figure 1.6 has an infinite number of principal *E*-planes (elevation planes; $\phi = \phi_c$) and one principal *H*-plane (azimuthal plane; $\theta = 90^\circ$).

1.2.4 Field Regions

The space surrounding an antenna is usually subdivided into three regions: (1) *reactive near-field*, (2) *radiating near-field (Fresnel)*, and (3) *far-field (Fraunhofer)* regions as shown in Figure 1.7. These regions are so designated to identify the field structure in

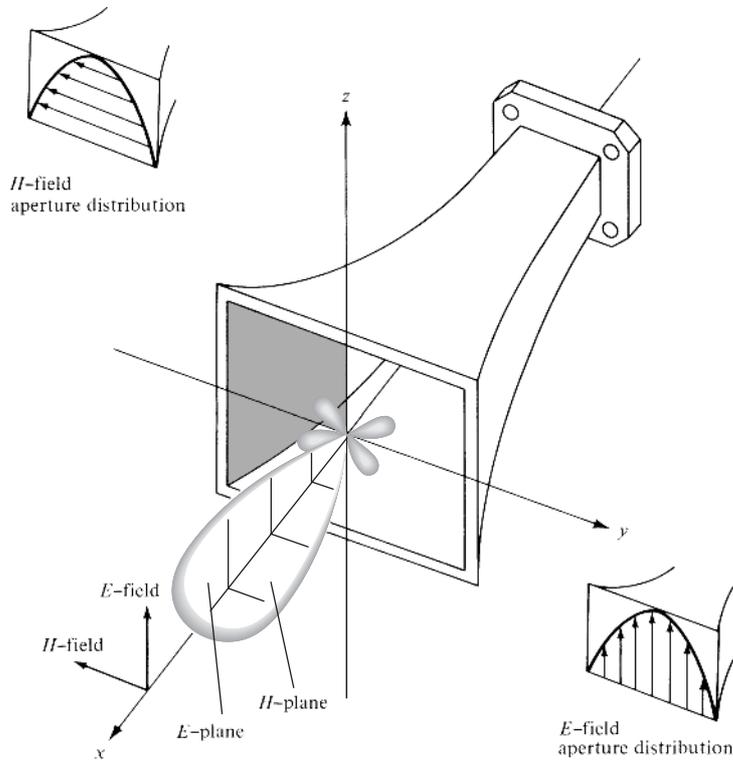


Figure 1.5 Principal E - and H -plane patterns for a pyramidal horn antenna.

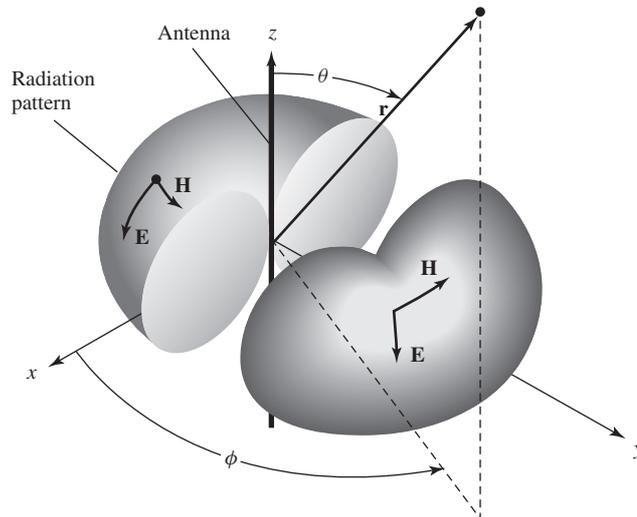


Figure 1.6 Omnidirectional antenna pattern.

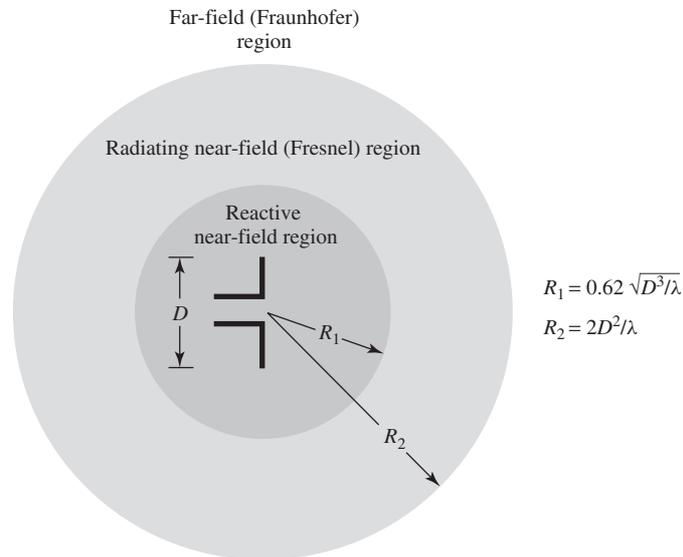


Figure 1.7 Field regions of an antenna.

each. Although no abrupt changes in the field configurations are noted as the boundaries are crossed, there are distinct differences among them. The boundaries separating these regions are not unique, although various criteria have been established and are commonly used to identify the regions.

Reactive near-field region is defined as “that portion of the near-field region immediately surrounding the antenna wherein the reactive field predominates.” For most antennas, the outer boundary of this region is commonly taken to exist at a distance $R < 0.62\sqrt{D^3/\lambda}$ from the antenna surface, where λ is the wavelength and D is the largest dimension of the antenna. “For a very short dipole, or equivalent radiator, the outer boundary is commonly taken to exist at a distance $\lambda/2\pi$ from the antenna surface.”

Radiating near-field (Fresnel) region is defined as “that region of the field of an antenna between the reactive near-field region and the far-field region wherein radiation fields predominate and wherein the angular field distribution is dependent upon the distance from the antenna. If the antenna has a maximum dimension that is not large compared to the wavelength, this region may not exist. For an antenna focused at infinity, the radiating near-field region is sometimes referred to as the Fresnel region on the basis of analogy to optical terminology. If the antenna has a maximum overall dimension which is very small compared to the wavelength, this field region may not exist.” The inner boundary is taken to be the distance $R \geq 0.62\sqrt{D^3/\lambda}$ and the outer boundary the distance $R < 2D^2/\lambda$, where D is the largest[†] dimension of the antenna. This criterion is based on a maximum phase error of $\pi/8$. In this region the field pattern is, in general, a function of the radial distance and the radial field component may be appreciable.

Far-field (Fraunhofer) region is defined as “that region of the field of an antenna where the angular field distribution is essentially independent of the distance from the

[†]To be valid, D must also be large compared to the wavelength ($D > \lambda$).

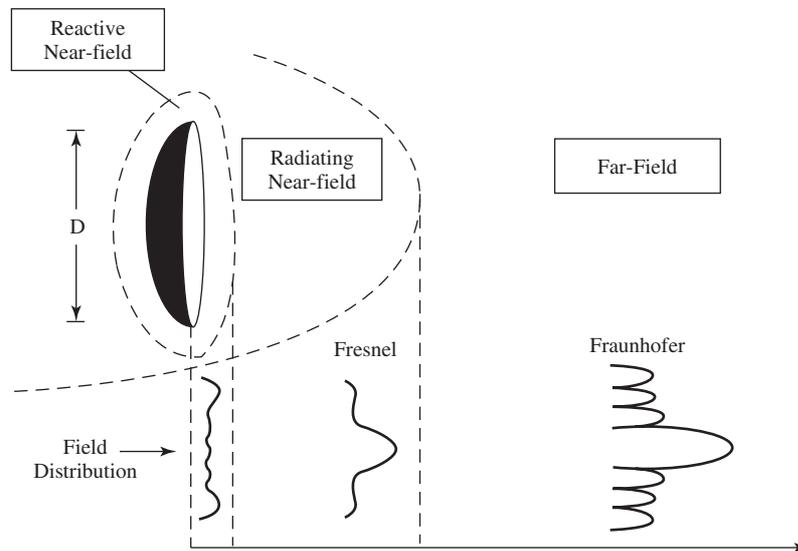


Figure 1.8 Typical changes of antenna amplitude pattern shape from reactive near field toward the far field. (From: Y. Rahmat-Samii, L. I. Williams, and R. G. Yoccarino, The UCLA bi-polar planar-near-field antenna measurement and diagnostics range, *IEEE Antennas Propag. Mag.*, Vol. 37, No. 6, December 1995. Copyright © 1995 IEEE.)

antenna. If the antenna has a maximum[†] overall dimension D , the far-field region is commonly taken to exist at distances greater than $2D^2/\lambda$ from the antenna, λ being the wavelength. The far-field patterns of certain antennas, such as multibeam reflector antennas, are sensitive to variations in phase over their apertures. For these antennas $2D^2/\lambda$ may be inadequate. In physical media, if the antenna has a maximum overall dimension, D , which is large compared to $\pi/|\gamma|$, the far-field region can be taken to begin approximately at a distance equal to $|\gamma|D^2/\pi$ from the antenna, γ being the propagation constant in the medium. For an antenna focused at infinity, the far-field region is sometimes referred to as the Fraunhofer region on the basis of analogy to optical terminology.” In this region, the field components are essentially transverse and the angular distribution is independent of the radial distance where the measurements are made. The inner boundary is taken to be the radial distance $R = 2D^2/\lambda$ and the outer one at infinity.

The amplitude pattern of an antenna, as the observation distance is varied from the reactive near field to the far field, changes in shape because of variations of the fields, both magnitude and phase. A typical progression of the shape of an antenna, with the largest dimension D , is shown in Figure 1.8. It is apparent that in the reactive near-field region the pattern is more spread out and nearly uniform, with slight variations. As the observation is moved to the radiating near-field region (Fresnel), the pattern begins to smooth and form lobes. In the far-field region (Fraunhofer), the pattern is well formed, usually consisting of few minor lobes and one, or more, major lobes.

[†]To be valid, D must also be large compared to the wavelength ($D > \lambda$).

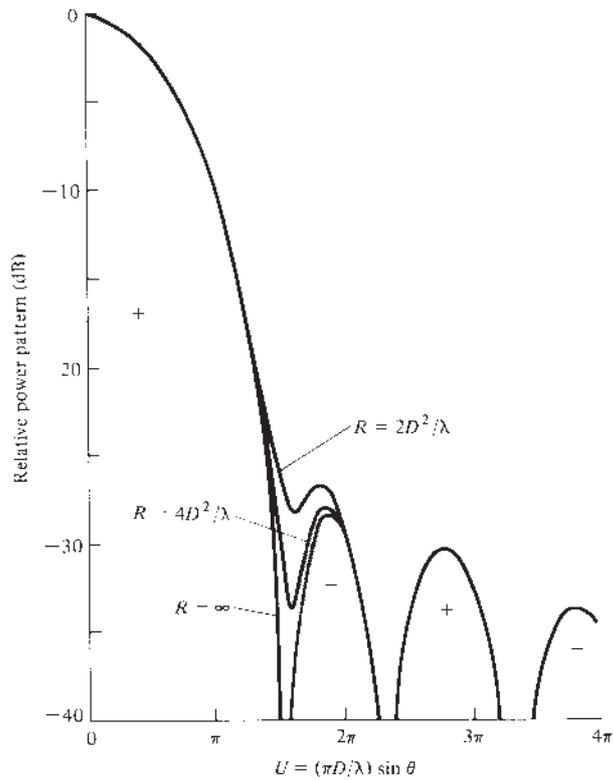


Figure 1.9 Calculated radiation patterns of a paraboloid antenna for different distances from the antenna. (From Ref. 6.)

To illustrate the pattern variation as a function of radial distance beyond the minimum $2D^2/\lambda$ far-field distance, in Figure 1.9 we have included three patterns of a parabolic reflector calculated at distances of $R = 2D^2/\lambda$, $4D^2/\lambda$, and infinity [6]. It is observed that the patterns are almost identical, except for some differences in the pattern structure around the first null and at a level below 25 dB. Because infinite distances are not realizable in practice, the most commonly used criterion for minimum distance of far-field observations is $2D^2/\lambda$.

1.2.5 Radian and Steradian

The measure of a plane angle is a radian. One *radian* is defined as the plane angle with its vertex at the center of a circle of radius r that is subtended by an arc whose length is the radius r . A graphical illustration is shown in Figure 1.10a. Since the circumference of a circle of radius r is $C = 2\pi r$, there are 2π rads ($2\pi r/r$) in a full circle.

The measure of a solid angle is a steradian. One *steradian* is defined as the solid angle with its vertex at the center of a sphere of radius r that is subtended by a spherical surface area equal to that of a square with each side of length r . A graphical illustration is shown in Figure 1.10b. Since the area of a sphere of radius r is $A = 4\pi r^2$, there are 4π sr ($4\pi r^2/r^2$) in a closed sphere.

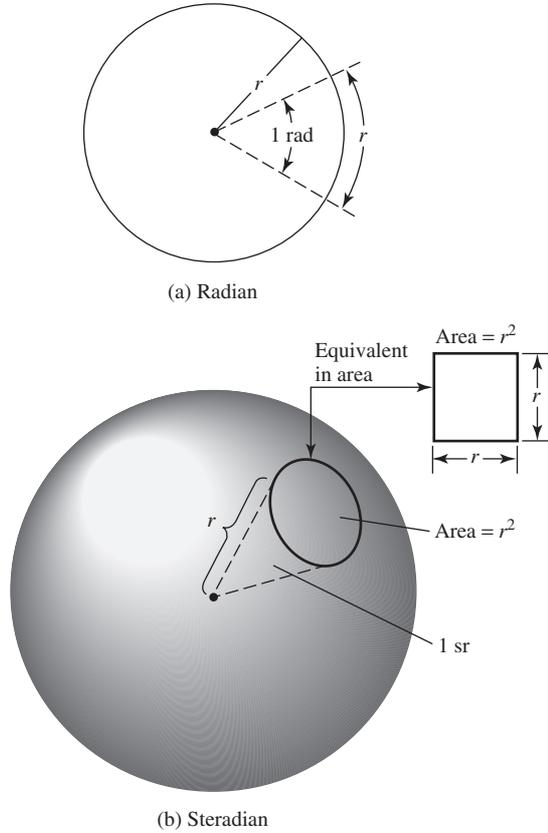


Figure 1.10 Geometrical arrangements for defining a radian and a steradian.

The infinitesimal area dA on the surface of a sphere of radius r , shown in Figure 1.1, is given by

$$dA = r^2 \sin \theta \, d\theta \, d\phi \quad (\text{m}^2) \quad (1.1)$$

Therefore the element of solid angle $d\Omega$ of a sphere can be written

$$d\Omega = \frac{dA}{r^2} = \sin \theta \, d\theta \, d\phi \quad (\text{sr}) \quad (1.2)$$

1.3 RADIATION POWER DENSITY

Electromagnetic waves are used to transport information through a wireless medium or a guiding structure, from one point to the other. It is then natural to assume that power and energy are associated with electromagnetic fields. The quantity used to describe the power associated with an electromagnetic wave is the instantaneous Poynting vector defined as

$$\mathcal{W} = \mathcal{E} \times \mathcal{H} \quad (1.3)$$

where

- \mathcal{W} = instantaneous Poynting vector (W/m²)
- \mathcal{E} = instantaneous electric-field intensity (V/m)
- \mathcal{H} = instantaneous magnetic-field intensity (A/m)

Note that script letters are used to denote instantaneous fields and quantities, while roman letters are used to represent their complex counterparts.

Since the Poynting vector is a power density, the total power crossing a closed surface can be obtained by integrating the normal component of the Poynting vector over the entire surface. In equation form

$$\mathcal{P} = \oint_s \mathcal{W} \cdot d\mathbf{s} = \oint_s \mathcal{W} \cdot \hat{\mathbf{n}} da \quad (1.4)$$

where

- \mathcal{P} = instantaneous total power (W)
- $\hat{\mathbf{n}}$ = unit vector normal to the surface
- da = infinitesimal area of the closed surface (m²)

The time-average Poynting vector (average power density) can be written

$$\mathbf{W}_{av}(x, y, z) = [\mathcal{W}'(x, y, z; t)]_{av} = \frac{1}{2} \text{Re}(\mathbf{E} \times \mathbf{H}^*) \quad (\text{W/m}^2) \quad (1.5)$$

The $\frac{1}{2}$ factor appears in Eq. (1.5) because the \mathbf{E} and \mathbf{H} fields represent peak values, and it should be omitted for RMS values. Based on the definition of Eq. (1.5), the average power radiated by an antenna (radiated power) can be written

$$\begin{aligned} P_{rad} = P_{av} &= \oint_s \mathbf{W}_{rad} \cdot d\mathbf{s} = \oint_s \mathbf{W}_{av} \cdot \hat{\mathbf{n}} da \\ &= \frac{1}{2} \oint_s \text{Re}(\mathbf{E} \times \mathbf{H}^*) \cdot d\mathbf{s} \end{aligned} \quad (1.6)$$

1.4 RADIATION INTENSITY

Radiation intensity in a given direction is defined as “the power radiated from an antenna per unit solid angle.” The radiation intensity is a far-field parameter, and it can be obtained by simply multiplying the radiation density by the square of the distance. In mathematical form it is expressed as

$$U = r^2 W_{rad} \quad (1.7)$$

where

$$\begin{aligned} U &= \text{radiation intensity} \quad (\text{W/unit solid angle}) \\ W_{\text{rad}} &= \text{radiation density} \quad (\text{W/m}^2) \end{aligned}$$

The radiation intensity is also related to the far-zone electric field of an antenna, referring to Figure 1.4, by

$$\begin{aligned} U(\theta, \phi) &= \frac{r^2}{2\eta} |\mathbf{E}(r, \theta, \phi)|^2 \simeq \frac{r^2}{2\eta} [|E_\theta(r, \theta, \phi)|^2 + |E_\phi(r, \theta, \phi)|^2] \quad (1.7a) \\ &\simeq \frac{1}{2\eta} [|E_\theta^0(\theta, \phi)|^2 + |E_\phi^0(\theta, \phi)|^2] \end{aligned}$$

where

$$\begin{aligned} \mathbf{E}(r, \theta, \phi) &= \text{far-zone electric-field intensity of the antenna} = \mathbf{E}^0(\theta, \phi) \frac{e^{-jkr}}{r} \\ E_\theta, E_\phi &= \text{far-zone electric-field components of the antenna} \\ \eta &= \text{intrinsic impedance of the medium} \end{aligned}$$

The radial electric-field component (E_r) is assumed, if present, to be small in the far zone. Thus the power pattern is also a measure of the radiation intensity.

The total power is obtained by integrating the radiation intensity, as given by Eq. (1.7), over the entire solid angle of 4π . Thus

$$P_{\text{rad}} = \oint_{\Omega} U d\Omega = \int_0^{2\pi} \int_0^\pi U \sin \theta d\theta d\phi \quad (1.8)$$

where $d\Omega = \text{element of solid angle} = \sin \theta d\theta d\phi$.

1.5 BEAMWIDTH

Associated with the pattern of an antenna is a parameter designated as *beamwidth*. The *beamwidth* of a pattern is defined as the angular separation between two identical points on opposite sides of the pattern maximum. In an antenna pattern, there are a number of beamwidths. One of the most widely used beamwidths is the *half-power beamwidth (HPBW)*, which is defined by IEEE as: "In a plane containing the direction of the maximum of a beam, the angle between the two directions in which the radiation intensity is one-half value of the beam." This is demonstrated in Figure 1.2. Another important beamwidth is the angular separation between the first nulls of the pattern, and it is referred to as the *first-null beamwidth (FNBW)*. Both the *HPBW* and *FNBW* are demonstrated for the pattern in Figure 1.11. Other beamwidths are those where the pattern is -10 dB from the maximum, or any other value. However, in practice, the term *beamwidth*, with no other identification, usually refers to the *HPBW*.

The beamwidth of an antenna is a very important figure-of-merit and often is used as a trade-off between it and the side lobe level; that is, as the beamwidth decreases, the side lobe increases and vice versa. In addition, the beamwidth of the antenna is also used

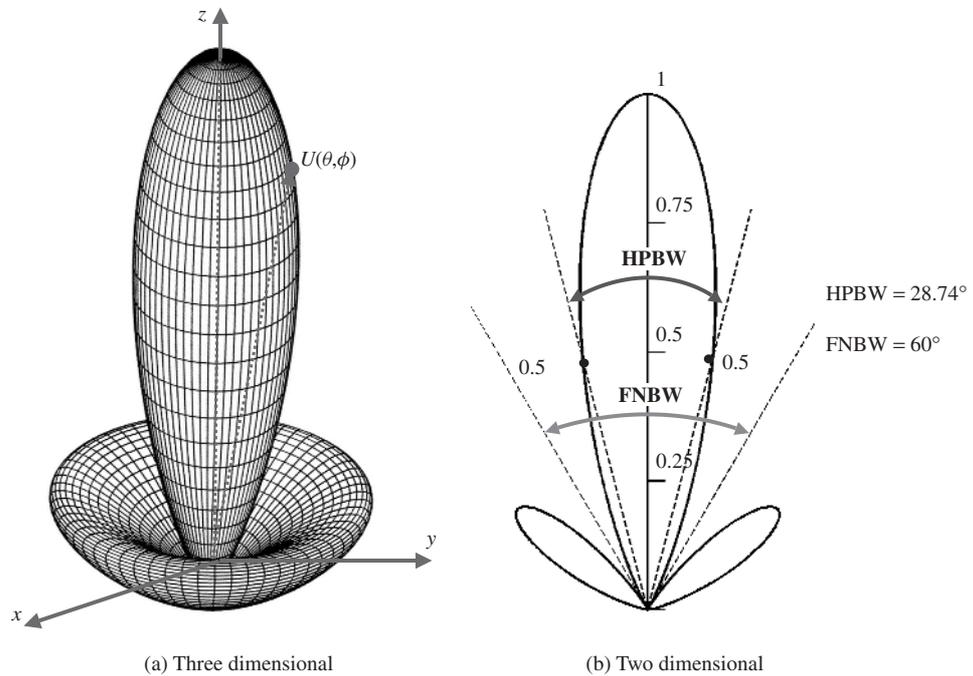


Figure 1.11 Three- and two-dimensional power patterns (in linear scale) of $U(\theta) = \cos^2(\theta) \cos^2(3\theta)$.

to describe the resolution capabilities of the antenna to distinguish between two adjacent radiating sources or radar targets. The most common resolution criterion states that *the resolution capability of an antenna to distinguish between two sources is equal to half the first-null beamwidth (FNBW/2), which is usually used to approximate the half-power beamwidth (HPBW)* [7, 8]. That is, two sources separated by angular distances equal to or greater than $\text{FNBW}/2 \approx \text{HPBW}$ of an antenna with a uniform distribution can be resolved. If the separation is smaller, then the antenna will tend to smooth the angular separation distance.

1.6 DIRECTIVITY

In the 1983 version of the *IEEE Standard Definitions of Terms for Antennas*, there was a substantive change in the definition of *directivity*, compared to the definition of the 1973 version. Basically the term *directivity* in the 1983 version has been used to replace the term *directive gain* of the 1973 version. In the 1983 version the term *directive gain* has been deprecated. According to the authors of the 1983 standards, “this change brings this standard in line with common usage among antenna engineers and with other international standards, notably those of the International Electrotechnical Commission (IEC).” Therefore *directivity of an antenna* is defined as “the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions. The average radiation intensity is equal to the total power radiated by the antenna divided

by 4π . If the direction is not specified, the direction of maximum radiation intensity is implied.” Stated more simply, the directivity of a nonisotropic source is equal to the ratio of its radiation intensity in a given direction over that of an isotropic source. In mathematical form, it can be written

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{\text{rad}}} \quad (1.9)$$

If the direction is not specified, it implies the direction of maximum radiation intensity (maximum directivity) expressed as

$$D_{\text{max}} = D_0 = \frac{U|_{\text{max}}}{U_0} = \frac{U_{\text{max}}}{U_0} = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} \quad (1.9a)$$

where

- D = directivity (dimensionless)
- D_0 = maximum directivity (dimensionless)
- U = radiation intensity (W/unit solid angle)
- U_{max} = maximum radiation intensity (W/unit solid angle)
- U_0 = radiation intensity of isotropic source (W/unit solid angle)
- P_{rad} = total radiated power (W)

For an isotropic source, it is very obvious from Eq. (1.9) or (1.9a) that the directivity is unity since U , U_{max} , and U_0 are all equal to each other.

For antennas with orthogonal polarization components, we define the *partial directivity of an antenna for a given polarization in a given direction* as “that part of the radiation intensity corresponding to a given polarization divided by the total radiation intensity averaged over all directions.” With this definition for the partial directivity, then in a given direction “the total directivity is the sum of the partial directivities for any two orthogonal polarizations.” For a spherical coordinate system, the total maximum directivity D_0 for the orthogonal θ and ϕ components of an antenna can be written

$$D_0 = D_\theta + D_\phi \quad (1.10)$$

while the partial directivities D_θ and D_ϕ are expressed as

$$D_\theta = \frac{4\pi U_\theta}{(P_{\text{rad}})_\theta + (P_{\text{rad}})_\phi} \quad (1.10a)$$

$$D_\phi = \frac{4\pi U_\phi}{(P_{\text{rad}})_\theta + (P_{\text{rad}})_\phi} \quad (1.10b)$$

where

- U_θ = radiation intensity in a given direction contained in θ field component
- U_ϕ = radiation intensity in a given direction contained in ϕ field component
- $(P_{\text{rad}})_\theta$ = radiated power in all directions contained in θ field component
- $(P_{\text{rad}})_\phi$ = radiated power in all directions contained in ϕ field component

The directivity of an isotropic source is unity since its power is radiated equally well in all directions. *For all other sources, the maximum directivity will always be greater than unity, and it is a relative “figure-of-merit” that gives an indication of the directional properties of the antenna as compared with those of an isotropic source.* In equation form, this is indicated in Eq. (1.9a). The directivity can be smaller than unity; in fact it can be equal to zero. *The values of directivity will be equal to or greater than zero and equal to or less than the maximum directivity ($0 \leq D \leq D_0$).*

A more general expression for the directivity can be developed to include sources with radiation patterns that may be functions of both spherical coordinate angles θ and ϕ . The radiation intensity of an antenna can be written

$$U = B_0 F(\theta, \phi) \simeq \frac{1}{2\eta} [|E_\theta^0(\theta, \phi)|^2 + |E_\phi^0(\theta, \phi)|^2] \quad (1.11)$$

where B_0 is a constant, and E_θ^0 and E_ϕ^0 are the antenna's far-zone electric-field components. The maximum value of Eq. (1.11) is given by

$$U_{\max} = B_0 F(\theta, \phi)|_{\max} = B_0 F_{\max}(\theta, \phi) \quad (1.11a)$$

The maximum directivity can be written

$$D_0 = \frac{4\pi}{\left(\int_0^{2\pi} \int_0^\pi F(\theta, \phi) \sin \theta \, d\theta \, d\phi \right) / F(\theta, \phi)|_{\max}} = \frac{4\pi}{\Omega_A} \quad (1.12)$$

where Ω_A is the *beam solid angle*, and it is given by

$$\Omega_A = \frac{1}{F(\theta, \phi)|_{\max}} \int_0^{2\pi} \int_0^\pi F(\theta, \phi) \sin \theta \, d\theta \, d\phi = \int_0^{2\pi} \int_0^\pi F_n(\theta, \phi) \sin \theta \, d\theta \, d\phi \quad (1.12a)$$

$$F_n(\theta, \phi) = \frac{F(\theta, \phi)}{F(\theta, \phi)|_{\max}} \quad (1.12b)$$

Dividing by $F(\theta, \phi)|_{\max}$ merely normalizes the radiation intensity $F(\theta, \phi)$, and it makes its maximum value unity.

The beam solid angle Ω_A is defined as the solid angle through which all the power of the antenna would flow if its radiation intensity is constant (and equal to the maximum value of U) for all angles within Ω_A .

1.6.1 Directional Patterns

Instead of using the exact expression of Eq. (1.12) to compute the directivity, it is often convenient to derive simpler expressions, even if they are approximate, to compute the directivity. These can also be used for design purposes. For antennas with one narrow major lobe and very negligible minor lobes, the beam solid angle is approximately equal to the product of the half-power beamwidths in two perpendicular planes [7] shown in Figure 1.12(a). For a rotationally symmetric pattern, the half-power beamwidths in any two perpendicular planes are the same, as illustrated in Figure 1.12(b).

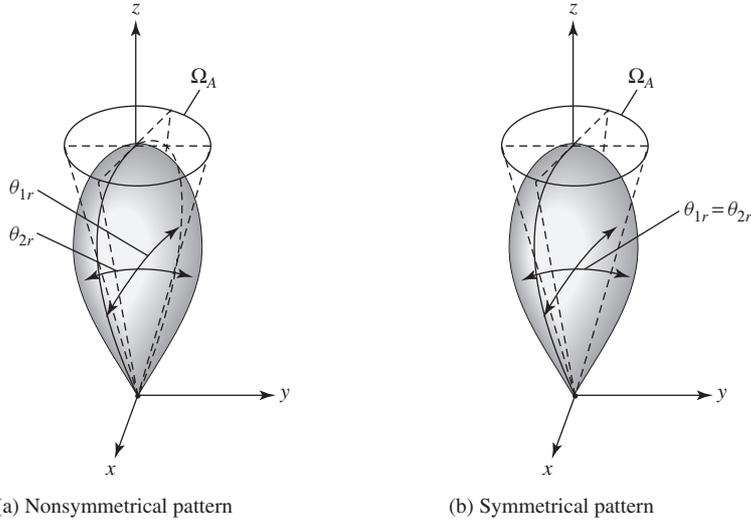


Figure 1.12 Beam solid angles for nonsymmetrical and symmetrical radiation patterns.

With this approximation, Eq. (1.12) can be approximated by

$$D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} \tag{1.13}$$

The beam solid angle Ω_A has been approximated by

$$\Omega_A \simeq \Theta_{1r}\Theta_{2r} \tag{1.13a}$$

where

Θ_{1r} = half-power beamwidth in one plane (rad)

Θ_{2r} = half-power beamwidth in a plane at a right angle to the other (rad)

If the beamwidths are known in degrees, Eq. (1.13) can be written

$$D_0 \simeq \frac{4\pi(180/\pi)^2}{\Theta_{1d}\Theta_{2d}} = \frac{41,253}{\Theta_{1d}\Theta_{2d}} \tag{1.14}$$

where

Θ_{1d} = half-power beamwidth in one plane (degrees)

Θ_{2d} = half-power beamwidth in a plane at a right angle to the other (degrees)

For planar arrays, a better approximation to Eq. (1.14) is [9]

$$D_0 \simeq \frac{32,400}{\Omega_A(\text{degrees})^2} = \frac{32,400}{\Theta_{1d}\Theta_{2d}} \tag{1.14a}$$

The validity of Eqs. (1.13) and (1.14) is based on a pattern that has only one major lobe and any minor lobes, if present, should be of very low intensity. For a pattern with two identical major lobes, the value of the maximum directivity using Eq. (1.13) or (1.14) will be twice its actual value. For patterns with significant minor lobes, the values of maximum directivity obtained using Eq. (1.13) or (1.14), which neglect any minor lobes, will usually be too high.

Many times it is desirable to express the directivity in decibels (dB) instead of dimensionless quantities. The expressions for converting the dimensionless quantities of directivity and maximum directivity to decibels (dB) are

$$D(\text{dB}) = 10 \log_{10}[D(\text{dimensionless})] \quad (1.15a)$$

$$D_0(\text{dB}) = 10 \log_{10}[D_0(\text{dimensionless})] \quad (1.15b)$$

It has also been proposed [10] that the maximum directivity of an antenna can also be obtained approximately by using the formula

$$\frac{1}{D_0} = \frac{1}{2} \left(\frac{1}{D_1} + \frac{1}{D_2} \right) \quad (1.16)$$

where

$$D_1 \simeq \frac{1}{\left(\frac{1}{2 \ln 2} \int_0^{\Theta_{1r}/2} \sin \theta d\theta \right)} \simeq \frac{16 \ln 2}{\Theta_{1r}^2} \quad (1.16a)$$

$$D_2 \simeq \frac{1}{\left(\frac{1}{2 \ln 2} \int_0^{\Theta_{2r}/2} \sin \theta d\theta \right)} \simeq \frac{16 \ln 2}{\Theta_{2r}^2} \quad (1.16b)$$

Θ_{1r} and Θ_{2r} are the half-power beamwidths (in radians) of the E and H planes, respectively. Formula (1.16) will be referred to as the arithmetic mean of the maximum directivity. Using Eqs. (1.16a) and (1.16b) we can write Eq. (1.16) as

$$\frac{1}{D_0} \simeq \frac{1}{2 \ln 2} \left(\frac{\Theta_{1r}^2}{16} + \frac{\Theta_{2r}^2}{16} \right) = \frac{\Theta_{1r}^2 + \Theta_{2r}^2}{32 \ln 2} \quad (1.17)$$

or

$$D_0 \simeq \frac{32 \ln 2}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{22.181}{\Theta_{1r}^2 + \Theta_{2r}^2} \quad (1.17a)$$

$$D_0 \simeq \frac{22.181(180/\pi)^2}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} \quad (1.17b)$$

where Θ_{1d} and Θ_{2d} are the half-power beamwidths in degrees.

1.6.2 Omnidirectional Patterns

Some antennas (such as dipoles, loops, and broadside arrays) exhibit omnidirectional patterns, as illustrated by the three-dimensional patterns in Figure 1.13. Approximate directivity formulas have been derived [11, 12] for antennas with omnidirectional patterns similar to the ones shown in Figure 1.13. The approximate directivity formula for an omnidirectional pattern as a function of the pattern half-power beamwidth (in degrees), which is reported by McDonald in [11], was derived based on the array factor of a broadside collinear array, and it is given by

$$D_0 \simeq \frac{101}{\text{HPBW (degrees)} - 0.0027[\text{HPBW (degrees)}]^2} \quad (1.18a)$$

However, that reported by Pozar [12] is given by

$$D_0 \simeq -172.4 + 191\sqrt{0.818 + 1/\text{HPBW (degrees)}} \quad (1.18b)$$

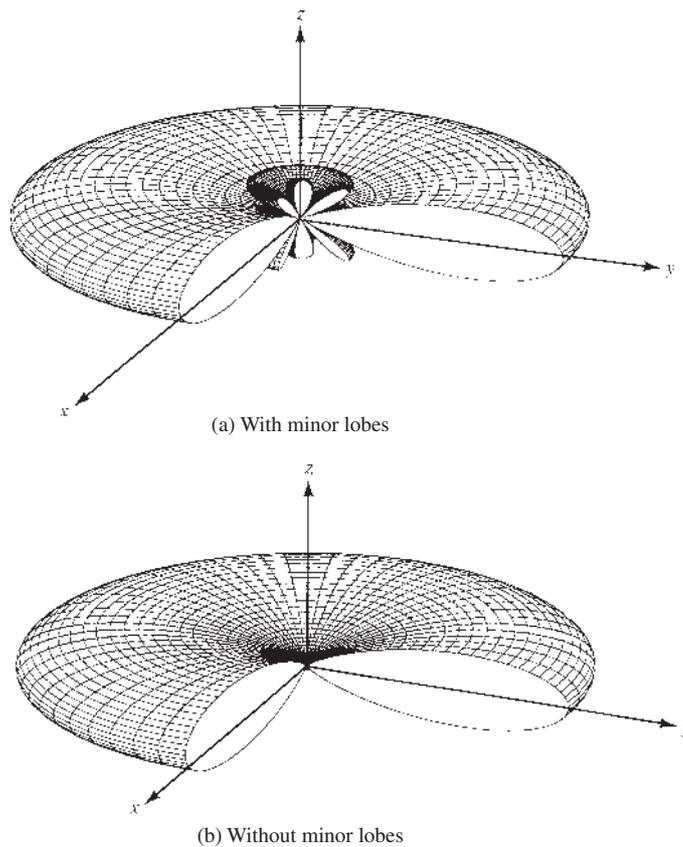


Figure 1.13 Omnidirectional patterns with and without minor lobes.

1.7 NUMERICAL TECHNIQUES

For most practical antennas, their radiation patterns are so complex that closed-form mathematical expressions are not available. Even in those cases where expressions are available, their form is so complex that integration to find the radiated power, required to compute the maximum directivity, cannot be performed. Instead of using the approximate expressions of Kraus, Tai and Pereira, McDonald, or Pozar, alternate and more accurate techniques may be desirable. With the high speed computer systems now available, the answer may be to apply numerical methods.

Let us assume that the radiation intensity of a given antenna is given by

$$U = B_0 F(\theta, \phi) \quad (1.19)$$

where B_0 is a constant. The directivity for such a system is given, in general, by

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} \quad (1.20)$$

where

$$P_{\text{rad}} = B_0 \int_0^{2\pi} \left(\int_0^\pi F(\theta, \phi) \sin \theta d\theta \right) d\phi \quad (1.20a)$$

For N uniform divisions over the π interval of θ and M uniform divisions over the 2π interval of ϕ , the digital form of the radiated power (Eq. (1.20a)) can be written as

$$P_{\text{rad}} = B_0 \left(\frac{\pi}{N} \right) \left(\frac{2\pi}{M} \right) \sum_{j=1}^M \left(\sum_{i=1}^N F(\theta_i, \phi_j) \sin \theta_i \right) \quad (1.21)$$

where θ_i and ϕ_j represent the discrete values of θ and ϕ .

A MATLAB and FORTRAN computer program called *Directivity* has been developed to compute the maximum directivity of any antenna whose radiation intensity is $U = F(\theta, \phi)$ based on the formulation of Eq. (1.21). The intensity function F does not have to be a function of both θ and ϕ . The program is included in the CD attached to [1]. It contains a *subroutine* for which the intensity factor $U = F(\theta, \phi)$ for the required application must be specified by the user. As an illustration, the antenna intensity $U = \sin \theta \sin^2 \phi$ has been inserted in the subroutine. In addition, the upper and lower limits of θ and ϕ must be specified for each application of the same pattern.

1.8 ANTENNA EFFICIENCY

Associated with an antenna are a number of efficiencies that can be defined using Figure 1.14. The total antenna efficiency e_0 is used to take into account losses at the input terminals and within the structure of the antenna. Such losses may be due, referring to Figure 1.14(b), to (1) reflections because of the mismatch between the transmission line and the antenna and (2) I^2R losses (conduction and dielectric).

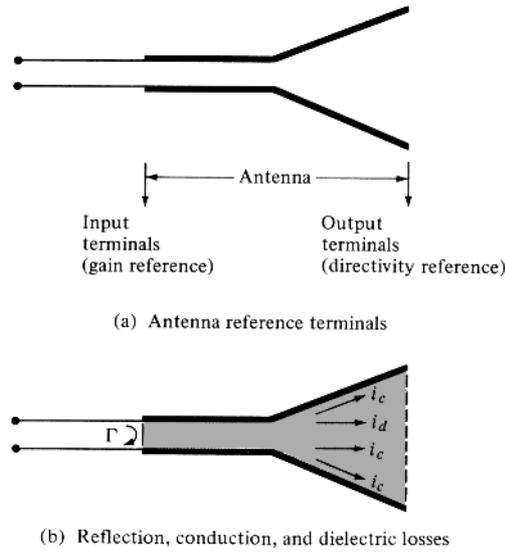


Figure 1.14 Reference terminals and losses of an antenna.

In general, the overall efficiency can be written

$$e_0 = e_r e_c e_d \tag{1.22}$$

where

- e_0 = total efficiency (dimensionless)
- e_r = reflection (mismatch) efficiency = $(1 - |\Gamma|^2)$ (dimensionless)
- e_c = conduction efficiency (dimensionless)
- e_d = dielectric efficiency (dimensionless)

Γ = voltage reflection coefficient at the input terminals of the antenna
 $[\Gamma = (Z_{in} - Z_0)/(Z_{in} + Z_0)$ where Z_{in} = antenna input impedance and Z_0 = characteristic impedance of the transmission line]

VSWR = voltage standing wave ratio = $(1 + |\Gamma|)/(1 - |\Gamma|)$

Usually e_c and e_d are very difficult to compute, but they can be determined experimentally. Even by measurements they cannot be separated, and it is usually more convenient to write Eq. (1.22) as

$$e_0 = e_r e_{cd} = e_{cd} (1 - |\Gamma|^2) \tag{1.23}$$

where $e_{cd} = e_c e_d$ = antenna radiation efficiency, which is used to relate the gain and directivity.

1.9 GAIN

Another useful measure describing the performance of an antenna is the *gain*. Although the gain of the antenna is closely related to the directivity, it is a measure that takes into account the efficiency of the antenna as well as its directional capabilities.

Gain of an antenna (in a given direction) is defined as “the ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically. The radiation intensity corresponding to the isotropically radiated power is equal to the power accepted (input) by the antenna divided by 4π .”

In most cases we deal with *relative gain*, which is defined as “the ratio of the power gain in a given direction to the power gain of a reference antenna in its referenced direction.” The power input must be the same for both antennas. The reference antenna is usually a dipole, horn, or any other antenna whose gain can be calculated or it is known. In most cases, however, the reference antenna is a *lossless isotropic source*. Thus

$$G = \frac{4\pi U(\theta, \phi)}{P_{\text{in}}(\text{lossless isotropic source})} \quad (\text{dimensionless}) \quad (1.24)$$

When the direction is not stated, the power gain is usually taken in the direction of maximum radiation.

Referring to Figure 1.14(a), we can write that the total radiated power (P_{rad}) is related to the total input power (P_{in}) by

$$P_{\text{rad}} = e_{cd} P_{\text{in}} \quad (1.25)$$

where e_{cd} is the antenna radiation efficiency (dimensionless), which is defined in Eqs. (1.22) and (1.23). According to the IEEE Standards, “gain does not include losses arising from impedance mismatches (reflection losses) and polarization mismatches (losses).”

Here we define two gains: one, referred to as *gain* (G), and the other, referred to as *absolute gain* (G_{abs}), that also takes into account the reflection/mismatch losses represented in both Eqs. (1.22) and (1.23).

Using Eq. (1.25) reduces Eq. (1.24) to

$$G(\theta, \phi) = e_{cd} \left(4\pi \frac{U(\theta, \phi)}{P_{\text{rad}}} \right) \quad (1.26)$$

which is related to the directivity of Eq. (1.9) by

$$\boxed{G(\theta, \phi) = e_{cd} D(\theta, \phi)} \quad (1.27)$$

In a similar manner, the maximum value of the gain is related to the maximum directivity of Eq. (1.9a) and (1.12) by

$$\boxed{G_0 = G(\theta, \phi)|_{\text{max}} = e_{cd} D(\theta, \phi)|_{\text{max}} = e_{cd} D_0} \quad (1.27a)$$

While Eq. (1.25) does take into account the losses of the antenna element itself, it does not take into account the losses when the antenna element is connected to a transmission line, as shown in Figure 1.14. These connection losses are usually referred to as *reflections (mismatch) losses*, and they are taken into account by introducing a reflection (mismatch) efficiency e_r , which is related to the reflection coefficient as shown

in Eq. (1.23) or $e_r = (1 - |\Gamma|^2)$. Thus we can introduce an *absolute gain* G_{abs} that takes into account the reflection/mismatch losses (due to the connection of the antenna element to the transmission line), and it can be written

$$\boxed{G_{\text{abs}}(\theta, \phi) = e_r G(\theta, \phi) = (1 - |\Gamma|^2) G(\theta, \phi) = e_r e_{cd} D(\theta, \phi) = e_o D(\theta, \phi)} \quad (1.28)$$

where e_o is the overall efficiency as defined in Eqs. (1.22) and (1.23). Similarly, the *maximum absolute gain* $G_{0\text{abs}}$ of Eq. (1.28) is related to the maximum directivity D_0 by

$$\boxed{G_{0\text{abs}} = G_{\text{abs}}(\theta, \phi)|_{\text{max}} = e_r G(\theta, \phi)|_{\text{max}} = (1 - |\Gamma|^2) G(\theta, \phi)|_{\text{max}} = e_r e_{cd} D(\theta, \phi)|_{\text{max}} = e_o D_0} \quad (1.28a)$$

If the antenna is matched to the transmission line, that is, the antenna input impedance Z_{in} is equal to the characteristic impedance Z_0 of the line ($|\Gamma| = 0$), then the two gains are equal ($G_{\text{abs}} = G$).

As was done with the directivity, we can define the *partial gain of an antenna for a given polarization in a given direction* as “that part of the radiation intensity corresponding to a given polarization divided by the total radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically.” With this definition for the partial gain, then, in a given direction, “the total gain is the sum of the partial gains for any two orthogonal polarizations.” For a spherical coordinate system, the total maximum gain G_0 for the orthogonal θ and ϕ components of an antenna can be written, in a similar form as was the maximum directivity in Eqs. (1.10), (1.10a) and (1.10b), as

$$G_0 = G_\theta + G_\phi \quad (1.29)$$

while the partial gains G_θ and G_ϕ are expressed as

$$G_\theta = \frac{4\pi U_\theta}{P_{\text{in}}} \quad (1.29a)$$

$$G_\phi = \frac{4\pi U_\phi}{P_{\text{in}}} \quad (1.29b)$$

where

U_θ = radiation intensity in a given direction contained in E_θ field component

U_ϕ = radiation intensity in a given direction contained in E_ϕ field component

P_{in} = total input (accepted) power

For many practical antennas an approximate formula for the gain, corresponding to Eq. (1.14) or (1.14a) for the directivity, is

$$\boxed{G_0 \simeq \frac{30,000}{\Theta_{1d}\Theta_{2d}}} \quad (1.30)$$

In practice, whenever the term “gain” is used, it usually refers to the *maximum gain* as defined by Eq. (1.27a) or (1.28a).

Usually the gain is given in terms of decibels instead of the dimensionless quantity of Eq. (1.27a). The conversion formula is given by

$$G_0(\text{dB}) = 10 \log_{10}[e_{cd} D_0 \text{ (dimensionless)}] \quad (1.31)$$

1.10 BEAM EFFICIENCY

Another parameter that is frequently used to judge the quality of transmitting and receiving antennas is the *beam efficiency*. For an antenna with its major lobe directed along the z -axis ($\theta = 0$), as shown in Figure 1.1, the beam efficiency (BE) is defined by

$$\text{BE} = \frac{\text{power transmitted (received) within cone angle } \theta_1}{\text{power transmitted (received) by the antenna}} \quad (\text{dimensionless}) \quad (1.32)$$

where θ_1 is the half-angle of the cone within which the percentage of the total power is to be found. Equation (1.32) can be written

$$\text{BE} = \frac{\int_0^{2\pi} \int_0^{\theta_1} U(\theta, \phi) \sin \theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin \theta \, d\theta \, d\phi} \quad (1.33)$$

If θ_1 is chosen as the angle where the first null or minimum occurs (see Figure 1.1), then the beam efficiency will indicate the amount of power in the major lobe compared to the total power. A very high beam efficiency (between the nulls or minimums), usually in the high 90s, is necessary for antennas used in radiometry, astronomy, radar, and other applications where received signals through the minor lobes must be minimized.

1.11 BANDWIDTH

The *bandwidth* of an antenna is defined as “the range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard.” The bandwidth can be considered to be the range of frequencies, on either side of a center frequency (usually the resonance frequency for a dipole), where the antenna characteristics (such as input impedance, pattern, beamwidth, polarization, side lobe level, gain, beam direction, radiation efficiency) are within an acceptable value of those at the center frequency. For broadband antennas, the bandwidth is usually expressed as the ratio of the upper-to-lower frequencies of acceptable operation. For example, a 10:1 bandwidth indicates that the upper frequency is 10 times greater than the lower. For narrowband antennas, the bandwidth is expressed as a percentage of the frequency difference (upper minus lower) over the center frequency of the bandwidth. For example, a 5% bandwidth indicates that the frequency difference of acceptable operation is 5% of the center frequency of the bandwidth.

Because the characteristics (input impedance, pattern, gain, polarization, etc.) of an antenna do not necessarily vary in the same manner or are not even critically affected by the frequency, there is no unique characterization of the bandwidth. The specifications are set in each case to meet the needs of the particular application. Usually there is a distinction made between pattern and input impedance variations. Accordingly *pattern bandwidth* and *impedance bandwidth* are used to emphasize this distinction. Associated with pattern bandwidth are gain, side lobe level, beamwidth, polarization, and beam direction while input impedance and radiation efficiency are related to impedance bandwidth. For example, the pattern of a linear dipole with overall length less than a half-wavelength ($l < \lambda/2$) is insensitive to frequency. The limiting factor for this antenna is its impedance, and its bandwidth can be formulated in terms of the Q . The Q of antennas or arrays with dimensions large compared to the wavelength, excluding superdirective designs, is near unity. Therefore the bandwidth is usually formulated in terms of beamwidth, side lobe level, and pattern characteristics. For intermediate length antennas, the bandwidth may be limited by either pattern or impedance variations, depending on the particular application. For these antennas, a 2:1 bandwidth indicates a good design. For others, large bandwidths are needed. Antennas with very large bandwidths (like 40:1 or greater) have been designed in recent years. These are known as *frequency-independent* antennas.

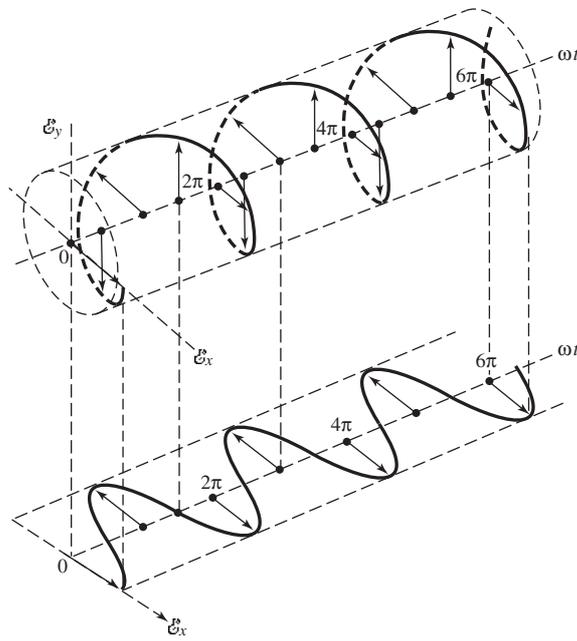
The above discussion presumes that the coupling networks (transformers, baluns, etc.) and/or the dimensions of the antenna are not altered in any manner as the frequency is changed. It is possible to increase the acceptable frequency range of a narrowband antenna if proper adjustments can be made on the critical dimensions of the antenna and/or on the coupling networks as the frequency is changed. Although not an easy or possible task in general, there are applications where this can be accomplished. The most common examples are the antenna of a car radio and the “rabbit ears” of a television. Both usually have adjustable lengths that can be used to tune the antenna for better reception.

1.12 POLARIZATION

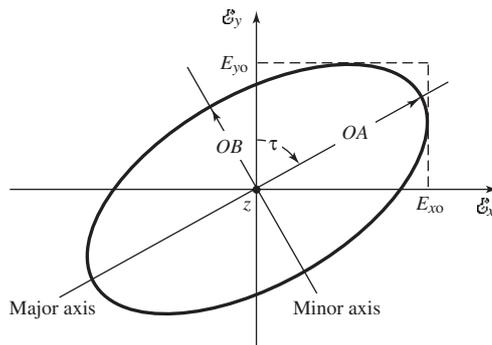
Polarization of an antenna in a given direction is defined as “the polarization of the wave transmitted (radiated) by the antenna. *Note:* When the direction is not stated, the polarization is taken to be the polarization in the direction of maximum gain.” In practice, polarization of the radiated energy varies with the direction from the center of the antenna, so that different parts of the pattern may have different polarizations.

Polarization of a radiated wave is defined as “that property of an electromagnetic wave describing the time-varying direction and relative magnitude of the electric-field vector; specifically, the figure traced as a function of time by the extremity of the vector at a fixed location in space, and the sense in which it is traced, *as observed along the direction of propagation.*” Polarization then is the curve traced by the end point of the arrow (vector) representing the instantaneous electric field. The field must be observed along the direction of propagation. A typical trace as a function of time is shown in Figure 1.15.

The polarization of a wave can be defined in terms of a wave *radiated (transmitted)* or *received* by an antenna in a given direction. The polarization of a wave *radiated* by an antenna in a specified direction at a point in the far field is defined as “the polarization of the (locally) plane wave which is used to represent the radiated wave at that point. At



(a) Rotation of wave



(b) Polarization ellipse

Figure 1.15 Rotation of a plane electromagnetic wave and its polarization ellipse at $z = 0$ as a function of time.

any point in the far field of an antenna the radiated wave can be represented by a plane wave whose electric-field strength is the same as that of the wave and whose direction of propagation is in the radial direction from the antenna. As the radial distance approaches infinity, the radius of curvature of the radiated wave's phase front also approaches infinity and thus in any specified direction the wave appears locally as a plane wave." This is a far-field characteristic of waves radiated by all practical antennas. The polarization of a wave *received* by an antenna is defined as the "polarization of a plane wave, incident from a given direction and having a given power flux density, which results in maximum available power at the antenna terminals."

Polarization may be classified as *linear*, *circular*, or *elliptical*. If the vector that describes the electric field at a point in space as a function of time is always directed along a line, the field is said to be *linearly* polarized. In general, however, the figure that the electric field traces is an ellipse, and the field is said to be elliptically polarized. Linear and circular polarizations are special cases of elliptical, and they can be obtained when the ellipse becomes a straight line or a circle, respectively. The figure of the electric field is traced in a *clockwise* (CW) or *counterclockwise* (CCW) sense. *Clockwise* rotation of the electric-field vector is also designated as *right-hand polarization* and *counterclockwise* as *left-hand polarization*.

In general, the polarization characteristics of an antenna can be represented by its *polarization pattern* whose one definition is “the spatial distribution of the polarizations of a field vector excited (radiated) by an antenna taken over its radiation sphere. When describing the polarizations over the radiation sphere, or portion of it, reference lines shall be specified over the sphere, in order to measure the tilt angles (see tilt angle) of the polarization ellipses and the direction of polarization for linear polarizations. An obvious choice, though by no means the only one, is a family of lines tangent at each point on the sphere to either the θ or ϕ coordinate line associated with a spherical coordinate system of the radiation sphere. At each point on the radiation sphere the polarization is usually resolved into a pair of orthogonal polarizations, the *co-polarization* and *cross polarization*. To accomplish this, the co-polarization must be specified at each point on the radiation sphere. . . . *Co-polarization* represents the polarization the antenna is intended to radiate (receive) while *Cross polarization* represents the polarization orthogonal to a specified polarization which is usually the co-polarization.

“For certain linearly polarized antennas, it is common practice to define the co-polarization in the following manner: First specify the orientation of the co-polar electric-field vector at a pole of the radiation sphere. Then, for all other directions of interest (points on the radiation sphere), require that the angle that the co-polar electric-field vector makes with each great circle line through the pole remain constant over that circle, the angle being that at the pole.

“In practice, the axis of the antenna’s main beam should be directed along the polar axis of the radiation sphere. The antenna is then appropriately oriented about this axis to align the direction of its polarization with that of the defined co-polarization at the pole. . . . This manner of defining co-polarization can be extended to the case of elliptical polarization by defining the constant angles using the major axes of the polarization ellipses rather than the co-polar electric-field vector. The sense of polarization (rotation) must also be specified.”

The polarization of the wave radiated by the antenna can also be represented on the Poincaré sphere [13–16]. Each point on the Poincaré sphere represents a unique polarization. The north pole represents left circular polarization, the south pole represents right circular, and points along the equator represent linear polarization of different tilt angles. All other points on the Poincaré sphere represent elliptical polarization. For details, see Figure 17.24 of Chapter 17 [1].

1.12.1 Linear, Circular, and Elliptical Polarizations

We summarize the discussion on polarization by stating the general characteristics and the *necessary and sufficient* conditions that the wave must have in order to possess *linear*, *circular*, or *elliptical* polarization.

Linear Polarization A time-harmonic wave is linearly polarized at a given point in space if the electric-field (or magnetic-field) vector at that point is always oriented along the same straight line at every instant of time. This is accomplished if the field vector (electric or magnetic) possesses the following:

1. Only one component, or
2. Two orthogonal linear components that are in time phase or 180° (or multiples of 180°) out-of-phase.

Circular Polarization A time-harmonic wave is circularly polarized at a given point in space if the electric (or magnetic) field vector at that point traces a circle as a function of time.

The *necessary and sufficient* conditions to accomplish this are if the field vector (electric or magnetic) possesses *all* of the following:

1. The field must have two orthogonal linear components, and
2. The two components must have the same magnitude, and
3. The two components must have a time-phase difference of odd multiples of 90° .

The sense of rotation is always determined by rotating the phase-leading component toward the phase-lagging component and observing the field rotation as the wave is viewed as it travels away from the observer. If the rotation is clockwise, the wave is right-hand (or clockwise) circularly polarized; if the rotation is counterclockwise, the wave is left-hand (or counterclockwise) circularly polarized. The rotation of the phase-leading component toward the phase-lagging component should be done along the angular separation between the two components that is less than 180° . Phases equal to or greater than 0° and less than 180° should be considered leading whereas those equal to or greater than 180° and less than 360° should be considered lagging.

Elliptical Polarization A time-harmonic wave is elliptically polarized if the tip of the field vector (electric or magnetic) traces an elliptical locus in space. At various instants of time the field vector changes continuously with time in such a manner as to describe an elliptical locus. It is right-hand (clockwise) elliptically polarized if the field vector rotates clockwise, and it is left-hand (counterclockwise) elliptically polarized if the field vector of the ellipse rotates counterclockwise [13]. The sense of rotation is determined using the same rules as for the circular polarization. In addition to the sense of rotation, elliptically polarized waves are also specified by their axial ratio whose magnitude is the ratio of the major to the minor axis.

A wave is elliptically polarized if it is not linearly or circularly polarized. Although linear and circular polarizations are special cases of elliptical, usually in practice elliptical polarization refers to other than linear or circular. The *necessary and sufficient* conditions to accomplish this are if the field vector (electric or magnetic) possesses *all* of the following:

1. The field must have two orthogonal linear components, and
2. The two components can be of the same or different magnitude.

3. (a) If the two components are not of the same magnitude, the time-phase difference between the two components must not be 0° or multiples of 180° (because it will then be linear). (b) If the two components are of the same magnitude, the time-phase difference between the two components must not be odd multiples of 90° (because it will then be circular).

If the wave is elliptically polarized with two components not of the same magnitude but with odd multiples of 90° time-phase difference, the polarization ellipse will not be tilted but it will be aligned with the principal axes of the field components. The major axis of the ellipse will align with the axis of the field component that is the larger of the two, while the minor axis of the ellipse will align with the axis of the field component that is the smaller of the two.

For elliptical polarization of a wave traveling along the negative z axis, the curve traced at a given z position as a function of time is, in general, a tilted ellipse, as shown in Figure 1.15(b). The ratio of the major axis to the minor axis is referred to as the *axial ratio* (AR), and it is equal to

$$\text{AR} = \frac{\text{major axis}}{\text{minor axis}} = \frac{OA}{OB}, \quad 1 \leq \text{AR} \leq \infty \quad (1.34)$$

where

$$OA = \left[\frac{1}{2} \{ E_{x_0}^2 + E_{y_0}^2 + [E_{x_0}^4 + E_{y_0}^4 + 2E_{x_0}^2 E_{y_0}^2 \cos(2\Delta\phi)]^{1/2} \} \right]^{1/2} \quad (1.34a)$$

$$OB = \left[\frac{1}{2} \{ E_{x_0}^2 + E_{y_0}^2 - [E_{x_0}^4 + E_{y_0}^4 + 2E_{x_0}^2 E_{y_0}^2 \cos(2\Delta\phi)]^{1/2} \} \right]^{1/2} \quad (1.34b)$$

where E_{x_0} and E_{y_0} represent, respectively, the maximum magnitudes of the two electric field components while $\Delta\phi$ is the time-phase difference between them. The tilt of the ellipse, relative to the y axis, is represented by the angle τ given by

$$\tau = \frac{\pi}{2} - \frac{1}{2} \tan^{-1} \left[\frac{2E_{x_0}E_{y_0}}{E_{x_0}^2 - E_{y_0}^2} \cos(\Delta\phi) \right] \quad (1.35)$$

When the ellipse is aligned with the principal axes [$\tau = n\pi/2$, $n = 0, 1, 2, \dots$], the major (minor) axis is equal to $E_{x_0}(E_{y_0})$ or $E_{y_0}(E_{x_0})$ and the axial ratio is equal to E_{x_0}/E_{y_0} or E_{y_0}/E_{x_0} .

1.12.2 Polarization Loss Factor and Efficiency

In general, the polarization of the receiving antenna will not be the same as the polarization of the incoming (incident) wave. This is commonly stated as “polarization mismatch.” The amount of power extracted by the antenna from the incoming signal will not be maximum because of the polarization loss. Assuming that the electric field of the incoming wave can be written

$$\mathbf{E}_i = \hat{\rho}_w E_i \quad (1.36)$$

where $\hat{\rho}_w$ is the unit vector of the wave, and the polarization of the electric field of the receiving antenna can be expressed as

$$\mathbf{E}_a = \hat{\rho}_a E_a \quad (1.37)$$

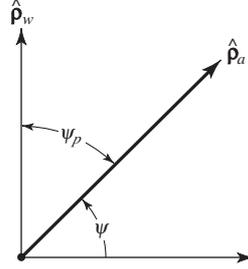


Figure 1.16 Polarization unit vectors of incident wave ($\hat{\rho}_w$) and antenna ($\hat{\rho}_a$), and polarization loss factor (PLF).

where $\hat{\rho}_a$ is its unit vector (polarization vector), the polarization loss can be taken into account by introducing a *polarization loss factor* (PLF). It is defined, based on the polarization of the antenna in its transmitting mode, as

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |\cos \psi_p|^2 \quad (\text{dimensionless}) \quad (1.38)$$

where ψ_p is the angle between the two unit vectors. The relative alignment of the polarization of the incoming wave and of the antenna is shown in Figure 1.16. If the antenna is polarization matched, its PLF will be unity and the antenna will extract maximum power from the incoming wave.

Another figure of merit that is used to describe the polarization characteristics of a wave and that of an antenna is the *polarization efficiency* (*polarization mismatch* or *loss factor*), which is defined as “the ratio of the power received by an antenna from a given plane wave of arbitrary polarization to the power that would be received by the same antenna from a plane wave of the same power flux density and direction of propagation, whose state of polarization has been adjusted for a maximum received power.” This is similar to the PLF and it is expressed as

$$p_e = \frac{|\ell_e \cdot \mathbf{E}^{\text{inc}}|^2}{|\ell_e|^2 |\mathbf{E}^{\text{inc}}|^2} \quad (1.39)$$

where

- ℓ_e = vector effective length of the antenna
- \mathbf{E}^{inc} = incident electric field

The vector effective length ℓ_e of the antenna has not yet been defined, and it is introduced in Section 1.15. It is a vector that describes the polarization characteristics of the antenna. Both the PLF and p_e lead to the same answers.

The conjugate (*) is not used in Eq. (1.38) or (1.39) so that a right-hand circularly polarized incident wave (when viewed in its direction of propagation) is matched to a

right-hand circularly polarized receiving antenna (when its polarization is determined in the transmitting mode). Similarly, a left-hand circularly polarized wave will be matched to a left-hand circularly polarized antenna.

Based on the definitions of the wave transmitted and received by an antenna, the polarization of an antenna in the *receiving* mode is related to that in the *transmitting* mode as follows:

1. "In the same plane of polarization, the polarization ellipses have the same axial ratio, the same sense of polarization (rotation) and the same spatial orientation.
2. "Since their senses of polarization and spatial orientation are specified by viewing their polarization ellipses in the respective directions in which they are propagating, one should note that:
 - (a) Although their senses of polarization are the same, they would appear to be opposite if both waves were viewed in the same direction.
 - (b) Their tilt angles are such that they are the negative of one another with respect to a common reference."

Since the polarization of an antenna will almost always be defined in its transmitting mode, according to the IEEE Std 145-1983, "the receiving polarization may be used to specify the polarization characteristic of a nonreciprocal antenna which may transmit and receive arbitrarily different polarizations."

The polarization loss must always be taken into account in the link calculations design of a communication system because in some cases it may be a very critical factor. Link calculations of communication systems for outer space explorations are very stringent because of limitations in spacecraft weight. In such cases, power is a limiting consideration. The design must properly take into account all loss factors to ensure a successful operation of the system.

An antenna that is elliptically polarized is that composed of two crossed dipoles, as shown in Figure 1.17. The two crossed dipoles provide the two orthogonal field components that are not necessarily of the same field intensity toward all observation angles. If the two dipoles are identical, the field intensity of each along zenith (perpendicular to the plane of the two dipoles) would be of the same intensity. Also, if the two dipoles were fed with a 90° time-phase difference (phase quadrature), the polarization along zenith would be circular and elliptical toward other directions. One way to obtain the 90° time-phase difference $\Delta\phi$ between the two orthogonal field components, radiated respectively by the two dipoles, is by feeding one of the two dipoles with a transmission line that is $\lambda/4$ longer or shorter than that of the other ($\Delta\phi = k\Delta\ell = (2\pi/\lambda)(\lambda/4) = \pi/2$). One of the lengths (longer or shorter) will provide right-hand (CW) rotation while the other will provide left-hand (CCW) rotation.

1.13 INPUT IMPEDANCE

Input impedance is defined as "the impedance presented by an antenna at its terminals or the ratio of the voltage to current at a pair of terminals or the ratio of the appropriate components of the electric to magnetic fields at a point." In this section we are primarily interested in the input impedance at a pair of terminals that are the input terminals of the antenna. In Figure 1.18a these terminals are designated as $a-b$. The ratio of the

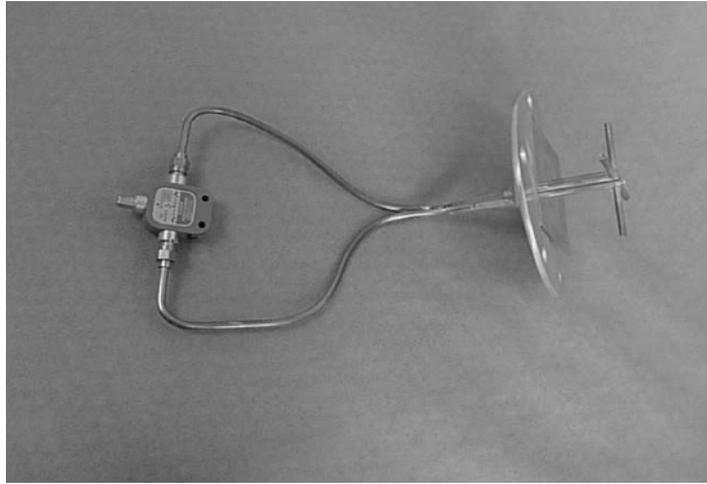


Figure 1.17 Geometry of elliptically polarized cross-dipole antenna.

voltage to current at these terminals, with no load attached, defines the impedance of the antenna as

$$\boxed{Z_A = R_A + jX_A} \quad (1.40)$$

where

Z_A = antenna impedance at terminals $a-b$ (ohms)

R_A = antenna resistance at terminals $a-b$ (ohms)

X_A = antenna reactance at terminals $a-b$ (ohms)

In general, the resistive part of Eq. (1.40) consists of two components; that is,

$$\boxed{R_A = R_r + R_L} \quad (1.41)$$

where

R_r = radiation resistance of the antenna

R_L = loss resistance of the antenna

The radiation resistance is used to represent the power delivered to the antenna for radiation.

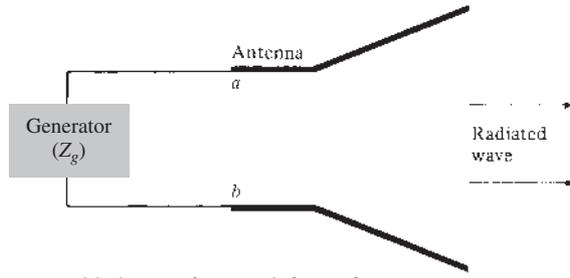
If we assume that the antenna is attached to a generator with internal impedance

$$Z_g = R_g + jX_g \quad (1.42)$$

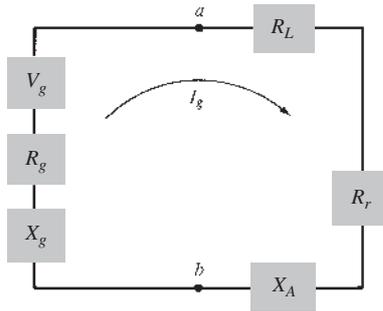
where

R_g = resistance of generator impedance (ohms)

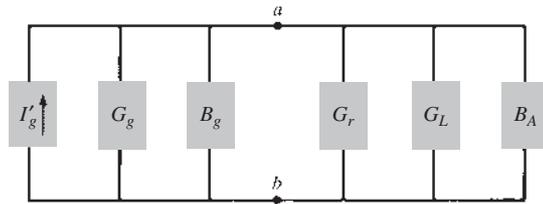
X_g = reactance of generator impedance (ohms)



(a) Antenna in transmitting mode



(b) Thevenin equivalent



(c) Norton equivalent

Figure 1.18 Transmitting antenna and its equivalent circuits.

and the antenna is used in the transmitting mode, we can represent the antenna and generator by an equivalent circuit[†] shown in Figure 1.18b.

The maximum power delivered to the antenna occurs when we have conjugate matching; that is, when

$$R_r + R_L = R_g \tag{1.43a}$$

$$X_A = -X_g \tag{1.43b}$$

Under conjugate matching, of the power that is provided by the generator, half is dissipated as heat in the internal resistance (R_g) of the generator and the other half is delivered

[†]This circuit can be used to represent small and simple antennas. It cannot be used for antennas with lossy dielectric or antennas over lossy ground because their loss resistance cannot be represented in series with the radiation resistance.

to the antenna. This only happens when we have *conjugate matching*. Of the power that is delivered to the antenna, part is radiated through the mechanism provided by the radiation resistance and the other is dissipated as heat, which influences part of the overall efficiency of the antenna. If the antenna is lossless and matched to the transmission line ($e_o = 1$), then half of the total power supplied by the generator is radiated by the antenna during conjugate matching, and the other half is dissipated as heat in the generator. Thus to radiate half of the available power through R_r you must dissipate the other half as heat in the generator through R_g . These two powers are, respectively, analogous to the power transferred to the load and the power scattered by the antenna in the receiving mode. In Figure 1.18 it is assumed that the generator is directly connected to the antenna. If there is a transmission line between the two, which is usually the case, then Z_g represents the equivalent impedance of the generator transferred to the input terminals of the antenna using the impedance transfer equation. If, in addition, the transmission line is lossy, then the available power to be radiated by the antenna will be reduced by the losses of the transmission line. Figure 1.18c illustrates the Norton equivalent of the antenna and its source in the transmitting mode.

The input impedance of an antenna is generally a function of frequency. Thus the antenna will be matched to the interconnecting transmission line and other associated equipment only within a bandwidth. In addition, the input impedance of the antenna depends on many factors including its geometry, its method of excitation, and its proximity to surrounding objects. Because of their complex geometries, only a limited number of practical antennas have been investigated analytically. For many others, the input impedance has been determined experimentally.

1.14 ANTENNA RADIATION EFFICIENCY

The antenna efficiency that takes into account the reflection, conduction, and dielectric losses was discussed in Section 1.8. The conduction and dielectric losses of an antenna are very difficult to compute and in most cases they are measured. Even with measurements, they are difficult to separate and they are usually lumped together to form the e_{cd} efficiency. The resistance R_L is used to represent the conduction–dielectric losses.

The *conduction–dielectric efficiency* e_{cd} is defined as *the ratio of the power delivered to the radiation resistance R_r to the power delivered to R_r and R_L* .

The radiation efficiency can be written

$$e_{cd} = \frac{R_r}{R_L + R_r} \quad (\text{dimensionless}) \quad (1.44)$$

For a metal rod of length l and uniform cross-sectional area A , the dc resistance is given by

$$R_{dc} = \frac{1}{\sigma} \frac{l}{A} \quad (\text{ohms}) \quad (1.45a)$$

If the skin depth δ ($\delta = \sqrt{2/\omega\mu_0\sigma}$) of the metal is very small compared to the smallest diagonal of the cross section of the rod, the current is confined to a thin layer near the

conductor surface. Therefore the high frequency resistance, based on a *uniform current distribution*, can be written

$$R_{\text{hf}} = \frac{l}{P} R_s = \frac{l}{P} \sqrt{\frac{\omega \mu_0}{2\sigma}} \quad (\text{ohms}) \quad (1.45b)$$

where P is the perimeter of the cross section of the rod ($P = C = 2\pi b$ for a circular wire of radius b), R_s is the conductor surface resistance, ω is the angular frequency, μ_0 is the permeability of free space, and σ is the conductivity of the metal. For a $\lambda/2$ dipole with a sinusoidal current distribution $R_L = \frac{1}{2} R_{\text{hf}}$, where R_{hf} is given by Eq. (1.45b).

1.15 ANTENNA VECTOR EFFECTIVE LENGTH AND EQUIVALENT AREAS

An antenna in the receiving mode, whether it is in the form of a wire, horn, aperture, array, or dielectric rod, is used to capture (collect) electromagnetic waves and to extract power from them, as shown in Figures 1.19. For each antenna, an equivalent length and a number of equivalent areas can then be defined.

These equivalent quantities are used to describe the receiving characteristics of an antenna, whether it be a linear or an aperture type, when a wave is incident on the antenna.

1.15.1 Vector Effective Length

The effective length of an antenna, whether it be a linear or an aperture antenna, is a quantity that is used to determine the voltage induced on the open-circuit terminals of the antenna when a wave impinges on it. The vector effective length $\boldsymbol{\ell}_e$ for an antenna is usually a complex vector quantity represented by

$$\boldsymbol{\ell}_e(\theta, \phi) = \hat{\mathbf{a}}_\theta l_\theta(\theta, \phi) + \hat{\mathbf{a}}_\phi l_\phi(\theta, \phi) \quad (1.46)$$

It should be noted that it is also referred to as the *effective height*. It is a far-field quantity and it is related to the *far-zone* field \mathbf{E}_a radiated by the antenna, with current I_{in} in its terminals, by [13–18]

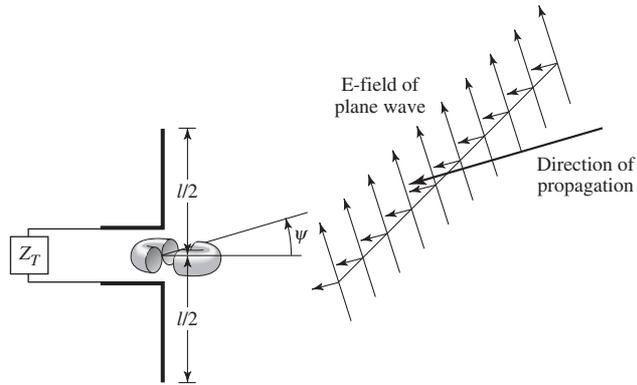
$$\mathbf{E}_a = \hat{\mathbf{a}}_\theta E_\theta + \hat{\mathbf{a}}_\phi E_\phi = -j\eta \frac{k I_{\text{in}}}{4\pi r} \boldsymbol{\ell}_e e^{-jkr} \quad (1.47)$$

The effective length represents the antenna in its transmitting and receiving modes, and it is particularly useful in relating the open-circuit voltage V_{oc} of receiving antennas. This relation can be expressed as

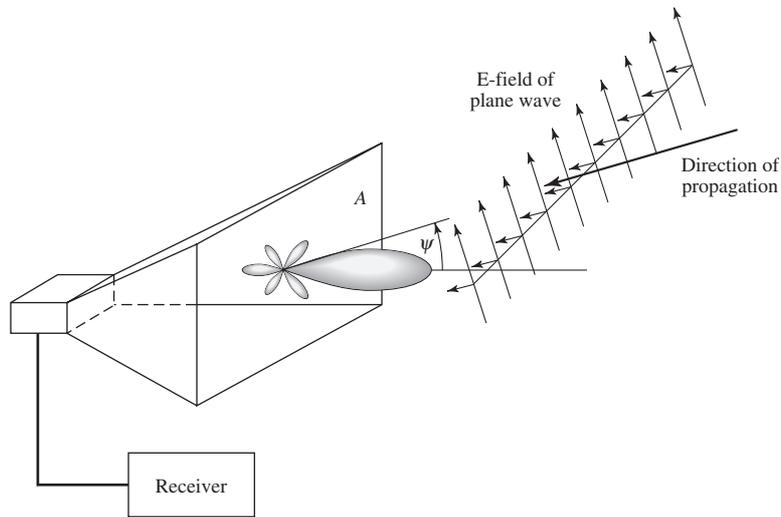
$$V_{\text{oc}} = \mathbf{E}^i \cdot \boldsymbol{\ell}_e \quad (1.48)$$

where

- V_{oc} = open-circuit voltage at antenna terminals
- \mathbf{E}^i = incident electric field
- $\boldsymbol{\ell}_e$ = vector effective length



(a) Dipole antenna in receiving mode



(b) Aperture antenna in receiving mode

Figure 1.19 Uniform plane wave incident on dipole and aperture antennas.

In Eq. (1.48) V_{oc} can be thought of as the voltage induced in a linear antenna of length l_e when l_e and \mathbf{E}^i are linearly polarized [19, 20]. From relation Eq. (1.48) the *effective length of a linearly polarized antenna receiving a plane wave in a given direction* is defined as “the ratio of the magnitude of the open-circuit voltage developed at the terminals of the antenna to the magnitude of the electric-field strength in the direction of the antenna polarization. Alternatively, the effective length is the length of a thin straight conductor oriented perpendicular to the given direction and parallel to the antenna polarization, having a uniform current equal to that at the antenna terminals and producing the same far-field strength as the antenna in that direction.”

In addition, as shown in Section 1.12.2, the antenna vector effective length is used to determine the polarization efficiency of the antenna.

1.15.2 Antenna Equivalent Areas

With each antenna, we can associate a number of equivalent areas. These are used to describe the power capturing characteristics of the antenna when a wave impinges on it. One of these equivalent areas is the *effective area (aperture)*, which in a given direction is defined as “the ratio of the available power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction, the wave being polarization-matched to the antenna. If the direction is not specified, the direction of maximum radiation intensity is implied.” In equation form it is written as

$$A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 R_T / 2}{W_i} \quad (1.49)$$

where

A_e = effective area (effective aperture) (m²)

P_T = power delivered to the load (W)

W_i = power density of incident wave (W/m²)

The effective aperture is the area that when multiplied by the incident power density gives the power delivered to the load. We can write Eq. (1.49) as

$$A_e = \frac{|V_T|^2}{2W_i} \left(\frac{R_T}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2} \right) \quad (1.50)$$

Under conditions of maximum power transfer (*conjugate matching*), $R_r + R_L = R_T$ and $X_A = -X_T$, the effective area of Eq. (1.50) reduces to the maximum effective aperture given by

$$A_{em} = \frac{|V_T|^2}{8W_i} \left(\frac{R_T}{(R_L + R_r)^2} \right) = \frac{|V_T|^2}{8W_i} \left(\frac{1}{R_r + R_L} \right) \quad (1.51)$$

1.16 MAXIMUM DIRECTIVITY AND MAXIMUM EFFECTIVE AREA

In general then, the *maximum effective area* (A_{em}) of any antenna is related to its *maximum directivity* (D_0) by [1]

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 \quad (1.52)$$

Thus when Eq. (1.52) is multiplied by the power density of the incident wave it leads to the maximum power that can be delivered to the load. This assumes that there are no conduction-dielectric losses (radiation efficiency e_{cd} is unity), the antenna is matched to the load (reflection efficiency e_r is unity), and the polarization of the impinging wave matches that of the antenna (polarization loss factor PLF and polarization efficiency p_e are unity). If there are losses associated with an antenna, its maximum effective aperture of Eq. (1.52) must be modified to account for conduction-dielectric losses (radiation efficiency). Thus

$$A_{em} = e_{cd} \left(\frac{\lambda^2}{4\pi} \right) D_0 \quad (1.53)$$

The maximum value of Eq. (1.53) assumes that the antenna is matched to the load and the incoming wave is polarization-matched to the antenna. If reflection and polarization losses are also included, then the maximum effective area of Eq. (1.53) is represented by

$$\begin{aligned}
 A_{em} &= e_0 \left(\frac{\lambda^2}{4\pi} \right) D_0 |\hat{\rho}_w \cdot \hat{\rho}_a|^2 \\
 &= e_{cd} (1 - |\Gamma|^2) \left(\frac{\lambda^2}{4\pi} \right) D_0 |\hat{\rho}_w \cdot \hat{\rho}_a|^2
 \end{aligned}
 \tag{1.54}$$

1.17 FRIIS TRANSMISSION EQUATION AND RADAR RANGE EQUATION

The analysis and design of radar and communications systems often require the use of the *Friis transmission equation* and the *radar range equation*. Because of the importance [21] of the two equations, a few pages will be devoted to them.

1.17.1 Friis Transmission Equation

The Friis transmission equation relates the power received P_r to the power transmitted P_t between two antennas separated by a distance $R > 2D^2/\lambda$, where D is the largest dimension of either antenna. Referring to Figure 1.20, we can write the ratio of received power P_r to transmitted power P_t as

$$\frac{P_r}{P_t} = e_t e_r \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{(4\pi R)^2}
 \tag{1.55}$$

where

- e_t = radiation efficiency of transmitting antenna
- e_r = radiation efficiency of receiving antenna
- D_t = directivity of transmitting antenna
- D_r = directivity of receiving antenna

The power received based on Eq. (1.55) assumes that the transmitting and receiving antennas are matched to their respective lines or loads (reflection efficiencies are unity)

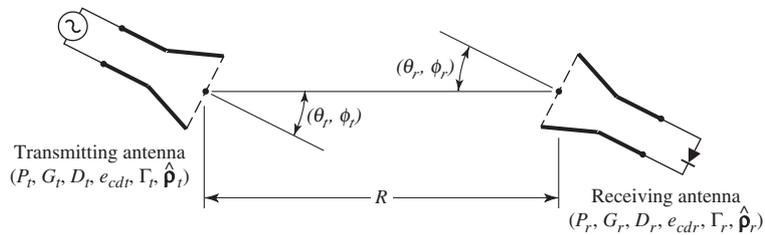


Figure 1.20 Geometrical orientation of transmitting and receiving antennas for Friis transmission equation.

and the polarization of the receiving antenna is polarization-matched to the impinging wave (polarization loss factor and polarization efficiency are unity). If these two factors are also included, then the ratio of the received to the input power of Eq. (1.55) is represented by

$$\frac{P_r}{P_t} = e_{cdt}e_{cdr}(1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) \left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \phi_t)D_r(\theta_r, \phi_r)|\hat{\rho}_t \cdot \hat{\rho}_r|^2 \quad (1.56)$$

For reflection and polarization-matched antennas aligned for maximum directional radiation and reception, Eq. (1.56) reduces to

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{0t}G_{0r} \quad (1.57)$$

Equation Eq. (1.55) or (1.56), or (1.57) is known as the *Friis transmission equation*, and it relates the power P_r (delivered to the receiver load) to the input power of the transmitting antenna P_t . The term $(\lambda/4\pi R)^2$ is called the *free-space loss factor*, and it takes into account the losses due to the spherical spreading of the energy by the antenna.

1.17.2 Radar Range Equation

Now let us assume that the transmitted power is incident on a target, as shown in Figure 1.21. We now introduce a quantity known as the *radar cross section* or *echo area* (σ) of a target, which is defined as *the area intercepting that amount of power which, when scattered isotropically, produces at the receiver a density that is equal to that scattered by the actual target* [1]. In equation form,

$$\lim_{R \rightarrow \infty} \left(\frac{\sigma W_i}{4\pi R^2}\right) = W_s \quad (1.58)$$

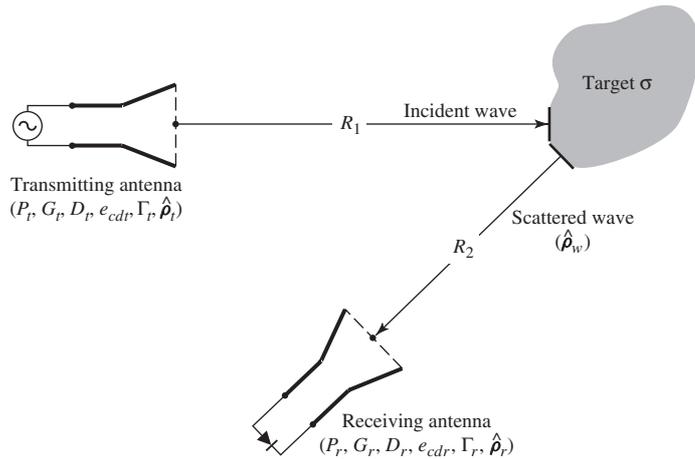


Figure 1.21 Geometrical arrangement of transmitter, target, and receiver for radar range equation.

or

$$\begin{aligned} \sigma &= \lim_{R \rightarrow \infty} \left(4\pi R^2 \frac{W_s}{W_i} \right) = \lim_{R \rightarrow \infty} \left(4\pi R^2 \frac{|\mathbf{E}^s|^2}{|\mathbf{E}^i|^2} \right) \\ &= \lim_{R \rightarrow \infty} \left(4\pi R^2 \frac{|\mathbf{H}^s|^2}{|\mathbf{H}^i|^2} \right) \end{aligned} \quad (1.59)$$

where

- σ = radar cross section or echo area (m²)
- R = observation distance from target (m)
- W_i = incident power density (W/m²)
- W_s = scattered power density (W/m²)
- \mathbf{E}^i (\mathbf{E}^s) = incident (scattered) electric field (V/m)
- \mathbf{H}^i (\mathbf{H}^s) = incident (scattered) magnetic field (A/m)

Any of the definitions in Eq. (1.59) can be used to derive the radar cross section of any antenna or target. For some polarization one of the definitions based either on the power density, electric field, or magnetic field may simplify the derivation, although all should give the same answers [13].

The ratio of received power P_r to transmitted power P_t , that has been scattered by the target with a radar cross section of σ , can be written

$$\frac{P_r}{P_t} = e_{cdt} e_{cdr} \sigma \frac{D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2 \quad (1.60)$$

Expression (1.60) is used to relate the received power to the input power, and it takes into account only conduction–dielectric losses (radiation efficiency) of the transmitting and receiving antennas. It does not include reflection losses (reflection efficiency) and polarization losses (polarization loss factor or polarization efficiency). If these two losses are also included, then Eq. (1.60) must be expressed as

$$\begin{aligned} \frac{P_r}{P_t} &= e_{cdt} e_{cdr} (1 - |\Gamma_t|^2) (1 - |\Gamma_r|^2) \sigma \frac{D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \\ &\quad \times \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2 |\hat{\rho}_w \cdot \hat{\rho}_r|^2 \end{aligned} \quad (1.61)$$

where

- $\hat{\rho}_w$ = polarization unit vector of the scattered waves
- $\hat{\rho}_r$ = polarization unit vector of the receiving antenna

For polarization-matched antennas aligned for maximum directional radiation and reception, Eq. (1.61) reduces to

$$\frac{P_r}{P_t} = \sigma \frac{G_{0t} G_{0r}}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2 \quad (1.62)$$

Equation (1.60), or (1.61), or (1.62) is known as the *radar range equation*. It relates the power P_r (delivered to the receiver load) to the input power P_t transmitted by an antenna, after it has been scattered by a target with a radar cross section (echo area) of σ .

1.17.3 Antenna Radar Cross Section

The radar cross section, usually referred to as RCS, is a far-field parameter, which is used to characterize the scattering properties of a radar target. For a target, there is *monostatic* or *backscattering RCS* when the transmitter and receiver of Figure 1.21 are at the same location, and a *bistatic RCS* when the transmitter and receiver are not at the same location. In designing low-observable or low-profile (stealth) targets, it is the parameter that you attempt to minimize. For complex targets (such as aircraft, spacecraft, missiles, ships, tanks, or automobiles) it is a complex parameter to derive. In general, the RCS of a target is a function of the polarization of the incident wave, the angle of incidence, the angle of observation, the geometry of the target, the electrical properties of the target, and the frequency of operation. The units of RCS of three-dimensional targets are meters squared (m^2) or for normalized values decibels per squared meter (dBsm) or RCS per squared wavelength in decibels (RCS/λ^2 in dB). Representative values of some typical targets are shown in Table 1.1 [22]. Although the frequency was not stated [22], these numbers could be representative at X-band.

The RCS of a target can be controlled using primarily two basic methods: *shaping* and the use of *materials*. Shaping is used to attempt to direct the scattered energy toward directions other than the desired. However, for many targets shaping has to be compromised in order to meet other requirements, such as aerodynamic specifications for flying targets. Materials are used to trap the incident energy within the target and to dissipate part of the energy as heat or to direct it toward directions other than the desired. Usually both methods, shaping and materials, are used together in order to optimize the performance of a radar target. One of the “golden rules” to observe in order to achieve low RCS is to “*round corners, avoid flat and concave surfaces, and use material treatment in flare spots.*”

TABLE 1.1 RCS of Some Typical Targets

Object	Typical RCSs [22]	
	RCS (m^2)	RCS (dBsm)
Pickup truck	200	23
Automobile	100	20
Jumbo jet airliner	100	20
Large bomber or commercial jet	40	16
Cabin cruiser boat	10	10
Large fighter aircraft	6	7.78
Small fighter aircraft or four-passenger jet	2	3
Adult male	1	0
Conventional winged missile	0.5	-3
Bird	0.01	-20
Insect	0.00001	-50
Advanced tactical fighter	0.000001	-60

There are many methods of analysis to predict the RCS of a target [13, 22–33]. Some of them are full-wave methods, others are designated as asymptotic methods, either low frequency or high frequency, and some are considered as numerical methods. The methods of analysis are often contingent on the shape, size, and material composition of the target. Some targets, because of their geometrical complexity, are often simplified and are decomposed into a number of basic shapes (such as strips, plates, cylinders, cones, wedges), which when put together represent a very good replica of the actual target. This has been used extensively and proved to be a very good approach. The topic is very extensive to be treated here in any detail, and the reader is referred to the literature [13, 22–33]. There are a plethora of references but because of space limitations, only a limited number are included here to get the reader started on the subject. Chapter 21 in this and book is devoted to the antenna scattering and design considerations.

Antennas individually are radar targets that many exhibit large radar cross section. In many applications, antennas are mounted on the surface of other complex targets (such as aircraft, spacecraft, satellites, missiles, or automobiles) and become part of the overall radar target. In such configurations, many antennas, especially aperture types (such as waveguides and horns) become large contributors to the total RCS, monostatic or bistatic, of the target. Therefore in designing low-observable targets, the antenna type, location, and contributions become important considerations of the overall design.

The scattering and transmitting (radiation) characteristics of an antenna are related [34–36]. There are various methods that can be used to analyze the fields scattered by an antenna. The presentation here parallels that in Refs. 23 and 37–40. In general, the electric field scattered by an antenna with a load impedance Z_L can be expressed by

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(0) - \frac{I_s}{I_t} \frac{Z_L}{Z_L + Z_A} \mathbf{E}^t \quad (1.63)$$

where

$\mathbf{E}^s(Z_L)$ = electric field scattered by antenna with a load Z_L

$\mathbf{E}^s(0)$ = electric field scattered by short-circuited antenna ($Z_L = 0$)

I_s = short-circuited current induced by the incident field on the antenna with $Z_L = 0$

I_t = antenna current in transmitting mode

$Z_A = R_A + jX_A$ = antenna input impedance

\mathbf{E}^t = electric field radiated by the antenna in transmitting mode

By defining an antenna reflection coefficient of

$$\Gamma_A = \frac{Z_L - Z_A}{Z_L + Z_A} \quad (1.64)$$

the scattered field of Eq. (1.63) can be written

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(0) - \frac{I_s}{I_t} \frac{1}{2} (1 + \Gamma_A) \mathbf{E}^t \quad (1.65)$$

Therefore according to Eq. (1.65) the scattered field by an antenna with a load Z_L is equal to the scattered field when the antenna is short-circuited ($Z_L = 0$) minus a term related to the reflection coefficient and the field transmitted by the antenna.

Green has expressed the field scattered by an antenna terminated with a load Z_L in a more convenient form that allows it to be separated into the *structural* and *antenna mode* scattering terms [23, 37–40]. This is accomplished by assuming that the antenna is loaded with a conjugate-matched impedance ($Z_L = Z_A^*$). Doing this and using Eq. (1.63) generates another equation for the field scattered by the antenna with a load $Z_L = Z_A^*$. When this new equation is subtracted from Eq. (1.63) it eliminates the short-circuited scattered field, and we can write that the field scattered by the antenna with a load Z_L is

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(Z_A^*) - \frac{I_s}{I_t} \frac{\Gamma^* Z_A}{2R_A} \mathbf{E}^t \quad (1.66)$$

$$\Gamma^* = \frac{Z_L - Z_A^*}{Z_L + Z_A^*} \quad (1.66a)$$

where

$\mathbf{E}^s(Z_L)$ = electric field scattered by the antenna with load Z_L

$\mathbf{E}^s(Z_A^*)$ = electric field scattered by the antenna with a conjugate-matched load

$I(Z_A^*)$ = current induced by the incident wave at the antenna terminals matched with a conjugate impedance load

Γ^* = conjugate-matched reflection coefficient

Z_L = load impedance attached to antenna terminals

For the short-circuited case and the conjugate-matched transmitting (radiating) case, the product of their currents and antenna impedance are related by [34]

$$I_s Z_A = I_m^* (Z_A + Z_A^*) = 2R_A I_m^* \quad (1.67)$$

where I_m^* is the scattering current when the antenna is conjugate-matched ($Z_L = Z_A^*$). Substituting Eq. (1.67) into (1.66) for I_s reduces Eq. (1.66) to

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(Z_A^*) - \frac{I_m^*}{I_t} \Gamma^* \mathbf{E}^t \quad (1.68)$$

It can also be shown that if the antenna is matched with a load Z_A (instead of Z_A^*), then Eq. (1.68) can be written

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(Z_A) - \frac{I_m}{I_t} \Gamma_A \mathbf{E}^t \quad (1.69)$$

Therefore the field scattered by an antenna loaded with an impedance Z_L is related to the field radiated by the antenna in the transmitting mode in three different ways, as shown by Eqs. (1.65), (1.68), and (1.69). According to Eq. (1.65) the field scattered by an antenna when it is loaded with an impedance Z_L is equal to the field scattered by the antenna when it is short-circuited ($Z_L = 0$) minus a term related to the antenna reflection coefficient and the field transmitted by the antenna. In addition, according to Eq. (1.68), the field scattered by an antenna when it is terminated with an impedance Z_L is equal

to the field scattered by the antenna when it is conjugate-matched with an impedance Z_A^* minus the field transmitted (radiated) times the conjugate reflection coefficient. The second term is weighted by the two currents. Alternatively, according to Eq. (1.69), the field scattered by the antenna when it is terminated with an impedance Z_L is equal to the field scattered by the antenna when it is matched with an impedance Z_A minus the field transmitted (radiated) times the reflection coefficient weighted by the two currents.

In Eq. (1.68) the first term consists of the *structural* scattering term and the second of the *antenna mode* scattering term. The *structural* scattering term is introduced by the currents induced on the surface of the antenna by the incident field when the antenna is conjugate-matched, and it is independent of the load impedance. The *antenna mode* scattering term is only a function of the radiation characteristics of the antenna, and its scattering pattern is the square of the antenna radiation pattern. The antenna mode depends on the power absorbed in the load of a lossless antenna and the power that is radiated by the antenna due to a load mismatch. This term vanishes when the antenna is conjugate-matched.

From the scattered field expression of Eq. (1.65), it can be shown that the total radar cross section of the antenna terminated with a load Z_L can be written as [40]

$$\sigma = |\sqrt{\sigma^s} - (1 + \Gamma_A)\sqrt{\sigma^a}e^{j\phi_r}|^2 \quad (1.70)$$

where

- σ = total RCS with antenna terminated with Z_L
- σ^s = RCS due to structural term
- σ^a = RCS due to antenna mode term
- ϕ_r = relative phase between the structural and antenna mode terms

If the antenna is short-circuited ($\Gamma_A = -1$), then according to Eq. (1.70)

$$\sigma_{\text{short}} = \sigma^s \quad (1.71)$$

If the antenna is open-circuited ($\Gamma_A = +1$), then according to Eq. (1.70)

$$\sigma_{\text{open}} = |\sqrt{\sigma^s} - 2\sqrt{\sigma^a}e^{j\phi_r}|^2 = \sigma_{\text{residual}} \quad (1.72)$$

Lastly, if the antenna is matched $Z_L = Z_A$ ($\Gamma_A = 0$), then according to (1.70)

$$\sigma_{\text{match}} = |\sqrt{\sigma^s} - \sqrt{\sigma^a}e^{j\phi_r}|^2 \quad (1.73)$$

Therefore under matched conditions, according to Eq. (1.73), the range of values (minimum to maximum) of the radar cross section is

$$|\sigma^s - \sigma^a| \leq \sigma \leq |\sigma^s + \sigma^a| \quad (1.74)$$

The minimum value occurs when the two RCSs are in phase while the maximum occurs when they are out of phase.

To produce a zero RCS, Eq. (1.70) must vanish. This is accomplished if

$$\operatorname{Re}(\Gamma_A) = -1 + \cos \phi_r \sqrt{\sigma^s / \sigma^a} \quad (1.75a)$$

$$\operatorname{Im}(\Gamma_A) = -\sin \phi_r \sqrt{\sigma^s / \sigma^a} \quad (1.75b)$$

Assuming positive values of resistances, the real value of Γ_A cannot be greater than unity. Thus there are some cases where the RCS cannot be reduced to zero by choosing Z_L . Because Z_A can be complex, there is no limit on the imaginary part of Γ_A .

In general, the structural and antenna mode scattering terms are very difficult to predict and usually require that the antenna is solved as a boundary-value problem. However, these two terms have been obtained experimentally utilizing the Smith chart [37–39].

For a monostatic system the receiving and transmitting antennas are collocated. In addition, if the antennas are identical ($G_{or} = G_{ot} = G_0$) and are polarization-matched ($P_r = P_t = 1$), the total radar cross section of the antenna for backscattering can be written as

$$\sigma = \frac{\lambda_0^2}{4\pi} G_0^2 |A - \Gamma^*|^2 \quad (1.76)$$

where A is a complex parameter independent of the load.

If the antenna is a thin dipole, then $A \simeq 1$ and Eq. (1.76) reduces to

$$\begin{aligned} \sigma &\simeq \frac{\lambda_0^2}{4\pi} G_0^2 |1 - \Gamma^*|^2 = \frac{\lambda_0^2}{4\pi} G_0^2 \left| 1 - \frac{Z_L - Z_A^*}{Z_L + Z_A} \right|^2 \\ &= \frac{\lambda_0^2}{4\pi} G_0^2 \left| \frac{2R_A}{Z_L + Z_A} \right|^2 \end{aligned} \quad (1.77)$$

If in addition we assume that the dipole length is $l = \lambda_0/2$ and is short-circuited ($Z_L = 0$), then the normalized radar cross section of Eq. (1.77) is equal to

$$\frac{\sigma}{\lambda_0^2} \simeq \frac{G_0^2}{\pi} = \frac{(1.643)^2}{\pi} = 0.8593 \simeq 0.86 \quad (1.78)$$

which agrees with the experimental corresponding maximum monostatic value of Figure 1.22 and those reported in the literature [41, 42].

Shown in Figure 1.22 is the measured E -plane monostatic RCS of a half-wavelength dipole when it is matched to a load, short-circuited (straight wire), and open-circuited (gap at the feed). The aspect angle is measured from the normal to the wire. As expected, the RCS is a function of the observation (aspect) angle. Also it is apparent that there are appreciable differences between the three responses. For the short-circuited case, the maximum value is approximately -24 dBsm, which closely agrees with the computed value of -22.5 dBsm using Eq. (1.78). Similar responses for the monostatic RCS of a pyramidal horn are shown in Figure 1.23(a) for the E -plane and in Figure 1.23(b) for the H -plane. The antenna is a commercial X-band (8.2–12.4 GHz) 20-dB standard gain horn with aperture dimension of 9.2 cm by 12.4 cm. The length of the horn is 25.6 cm. As for the dipole, there are differences between the three responses for each plane. It is seen that the short-circuited response exhibits the largest return.

Antenna RCS from model measurements [43] and microstrip patches [44, 45] have been reported.

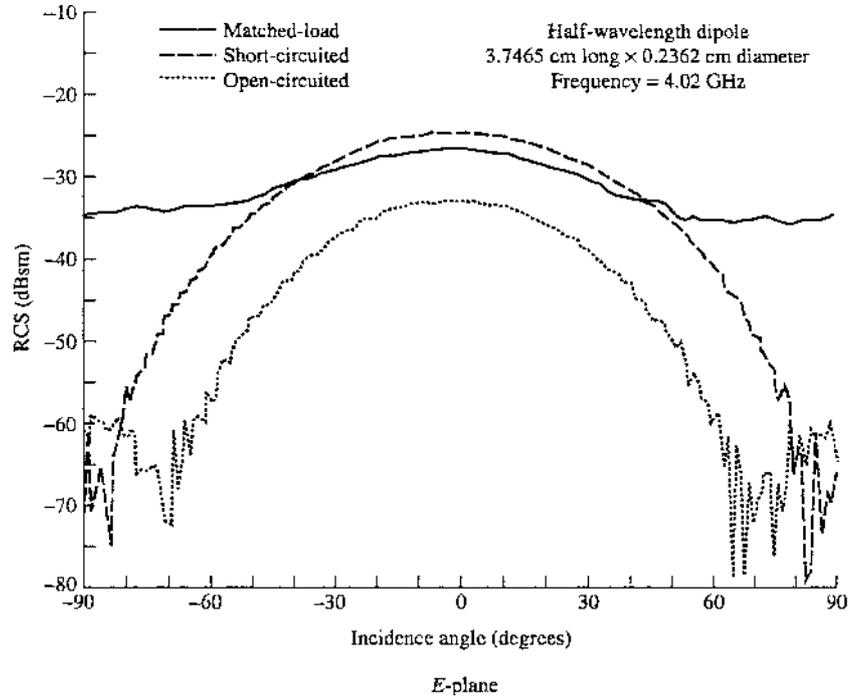


Figure 1.22 *E*-plane monostatic RCS ($\sigma_{\theta\theta}$) versus incidence angle for a half-wavelength dipole.

1.18 ANTENNA TEMPERATURE

Every object with a physical temperature above absolute zero ($0\text{ K} = -273^\circ\text{C}$) radiates energy [8]. The amount of energy radiated is usually represented by an equivalent temperature T_B , better known as brightness temperature, and it is defined as

$$T_B(\theta, \phi) = \epsilon(\theta, \phi)T_m = (1 - |\Gamma|^2)T_m \tag{1.79}$$

where

T_B = brightness temperature (equivalent temperature; K)

ϵ = emissivity (dimensionless)

T_m = molecular (physical) temperature (K)

$\Gamma(\theta, \phi)$ = reflection coefficient of the surface for the polarization of the wave

Since the values of emissivity are $0 \leq \epsilon \leq 1$, the maximum value the brightness temperature can achieve is equal to the molecular temperature. Usually the emissivity is a function of the frequency of operation, polarization of the emitted energy, and molecular structure of the object. Some of the better natural emitters of energy at microwave frequencies are (1) the ground with equivalent temperature of about 300 K and (2) the

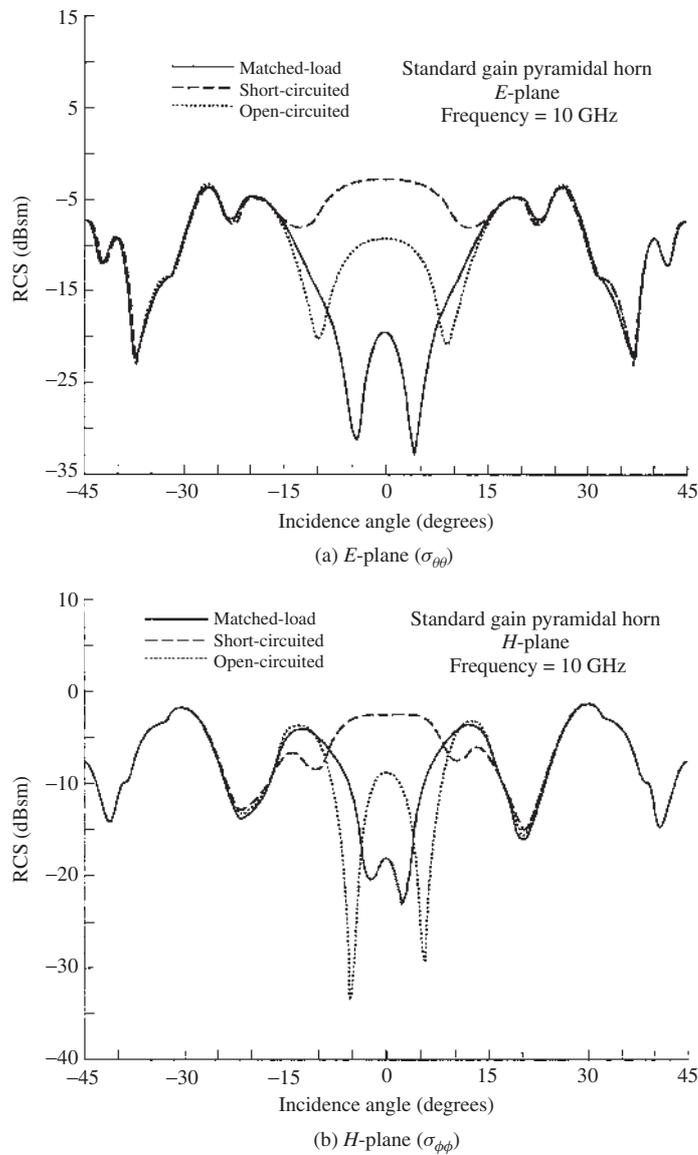


Figure 1.23 *E*- and *H*-plane monostatic RCS versus incidence angle for a pyramidal horn.

sky with equivalent temperature of about 5 K when looking toward zenith and about 100–150 K toward the horizon.

The brightness temperature emitted by the different sources is intercepted by antennas, and it appears at their terminals as an antenna temperature. The temperature appearing at the terminals of an antenna is that given by Eq. (1.79), after it is weighted by the gain pattern of the antenna. In equation form, this can be written

$$T_A = \frac{\int_0^{2\pi} \int_0^\pi T_B(\theta, \phi) G(\theta, \phi) \sin \theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^\pi G(\theta, \phi) \sin \theta \, d\theta \, d\phi} \quad (1.80)$$

where

T_A = antenna temperature (effective noise temperature of the antenna radiation resistance; K)

$G(\theta, \phi)$ = gain (power) pattern of the antenna

Assuming no losses or other contributions between the antenna and the receiver, the noise power transferred to the receiver is given by

$$P_r = k T_A \Delta f \quad (1.81)$$

where

P_r = antenna noise power (W)

k = Boltzmann's constant (1.38×10^{-23} J/K)

T_A = antenna temperature (K)

Δf = bandwidth (Hz)

If the antenna and transmission line are maintained at certain physical temperatures, and the transmission line between the antenna and receiver is lossy, the antenna temperature T_A as seen by the receiver through Eq. (1.81) must be modified to include the other contributions and the line losses. If the antenna itself is maintained at a certain physical temperature T_p and a transmission line of length l , constant physical temperature T_0 throughout its length, and uniform attenuation of α (Np/unit length) is used to connect an antenna to a receiver, as shown in Figure 1.24, the effective antenna temperature at the receiver terminals is given by

$$T_a = T_A e^{-2\alpha l} + T_{AP} e^{-2\alpha l} + T_0(1 - e^{-2\alpha l}) \quad (1.82)$$

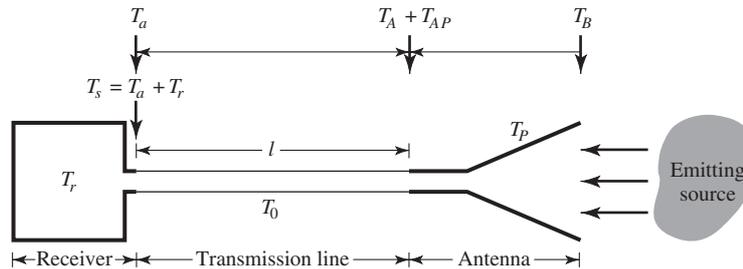


Figure 1.24 Antenna, transmission line, and receiver arrangement for system noise power calculation.

where

$$T_{AP} = \left(\frac{1}{e_A} - 1 \right) T_p \quad (1.82a)$$

T_a = antenna temperature at the receiver terminals (K)

T_A = antenna noise temperature at the antenna terminals (Eq. (1.80)) (K)

T_{AP} = antenna temperature at the antenna terminals due to physical temperature (Eq. (1.82a)) (K)

T_p = antenna physical temperature (K)

α = attenuation coefficient of transmission line (Np/m)

e_A = thermal efficiency of antenna (dimensionless)

l = length of transmission line (m)

T_0 = physical temperature of the transmission line (K)

The antenna noise power of Eq. (1.81) must also be modified and written as

$$P_r = kT_a \Delta f \quad (1.83)$$

where T_a is the antenna temperature at the receiver input as given by Eq. (1.82).

If the receiver itself has a certain noise temperature T_r (due to thermal noise in the receiver components), the *system noise power at the receiver terminals* is given by

$$P_s = k(T_a + T_r) \Delta f = kT_s \Delta f \quad (1.84)$$

where

P_s = system noise power (at receiver terminals)

T_a = antenna noise temperature (at receiver terminals)

T_r = receiver noise temperature (at receiver terminals)

$T_s = T_a + T_r$ = effective system noise temperature (at receiver terminals)

A graphical relation of all the parameters is shown in Figure 1.24. The effective system noise temperature T_s of radio astronomy antennas and receivers varies from very few degrees (typically $\simeq 10$ K) to thousands of Kelvins depending on the type of antenna, receiver, and frequency of operation. Antenna temperature changes at the antenna terminals, due to variations in the target emissions, may be as small as a fraction of one degree. To detect such changes, the receiver must be very sensitive and be able to differentiate changes of a fraction of a degree.

A summary of the pertinent parameters and associated formulas and equation numbers for this chapter are listed in Table 1.2.

TABLE 1.2 Summary of Important Parameters and Associated Formulas and Equation Numbers

Parameter	Formula	Equation Number
Infinitesimal area of sphere	$dA = r^2 \sin \theta d\theta d\phi$	(1.1)
Elemental solid angle of sphere	$d\Omega = \sin \theta d\theta d\phi$	(1.2)
Average power density	$\mathbf{W}_{\text{av}} = \frac{1}{2} \text{Re}(\mathbf{E} \times \mathbf{H}^*)$	(1.5)
Radiated power/average radiated power	$P_{\text{rad}} = P_{\text{av}} = \iint_S \mathbf{W}_{\text{av}} \cdot d\mathbf{s} = \frac{1}{2} \iint_S \text{Re}(\mathbf{E} \times \mathbf{H}^*) \cdot d\mathbf{s}$	(1.6)
Radiation density of isotropic radiator	$W_0 = \frac{P_{\text{rad}}}{4\pi r^2}$	
Radiation intensity (far field)	$U = r^2 W_{\text{rad}} = B_0 F(\theta, \phi)$ $\simeq \frac{r^2}{2\eta} [E_\theta(r, \theta, \phi) ^2 + E_\phi(r, \theta, \phi) ^2]$	(1.7), (1.7a)
Directivity $D(\theta, \phi)$	$D = \frac{U}{U_0} = \frac{4\pi U}{P_{\text{rad}}} = \frac{4\pi}{\Omega_A}$	(1.9), (1.12)
Beam solid angle Ω_A	$\Omega_A = \int_0^{2\pi} \int_0^\pi F_n(\theta, \phi) \sin \theta d\theta d\phi$ $F_n(\theta, \phi) = \frac{F(\theta, \phi)}{ F(\theta, \phi) _{\text{max}}}$	(1.12a) (1.12b)
Maximum directivity D_0	$D_{\text{max}} = D_0 = \frac{U_{\text{max}}}{U_0} = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}}$	(1.9a)
Partial directivities D_θ, D_ϕ	$D_0 = D_\theta + D_\phi$ $D_\theta = \frac{4\pi U_\theta}{P_{\text{rad}}} = \frac{4\pi U_\theta}{(P_{\text{rad}})_\theta + (P_{\text{rad}})_\phi}$ $D_\phi = \frac{4\pi U_\phi}{P_{\text{rad}}} = \frac{4\pi U_\phi}{(P_{\text{rad}})_\theta + (P_{\text{rad}})_\phi}$	(1.10) (1.10a) (1.10b)
Approximate maximum directivity (<i>one main lobe pattern</i>)	$D_0 \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{41,253}{\Theta_{1d}\Theta_{2d}}$ (Kraus)	(1.13), (1.14)
	$D_0 \simeq \frac{32 \ln 2}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{22,181}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2}$ (Tai-Pereira)	(1.17a), (1.17b)
Approximate maximum directivity (<i>omnidirectional pattern</i>)	$D_0 \simeq \frac{101}{\text{HPBW (degrees)} - 0.0027[\text{HPBW (degrees)}]^2}$ (McDonald)	(1.18a)
	$D_0 \simeq -172.4 + 191 \sqrt{0.818 + \frac{1}{\text{HPBW (degrees)}}}$ (Poazar)	(1.18b)

TABLE 1.2 (Continued)

Parameter	Formula	Equation Number
Gain $G(\theta, \phi)$	$G = \frac{4\pi U(\theta, \phi)}{P_{\text{in}}} = e_{cd} \left(\frac{4\pi U(\theta, \phi)}{P_{\text{rad}}} \right) = e_{cd} D(\theta, \phi)$ $P_{\text{rad}} = e_{cd} P_{\text{in}}$	(1.24), (1.27), (1.25)
Antenna radiation efficiency e_{cd}	$e_{cd} = \frac{R_r}{R_r + R_L}$	(1.44)
Loss resistance R_L (straight wire/uniform current)	$R_L = R_{\text{hf}} = \frac{l}{P} \sqrt{\frac{\omega\mu_0}{2\sigma}}$	(1.45b)
Loss resistance R_L (straight wire/ $\lambda/2$ dipole)	$R_L = \frac{l}{2P} \sqrt{\frac{\omega\mu_0}{2\sigma}}$	
Maximum gain G_0	$G_0 = e_{cd} D_{\text{max}} = e_{cd} D_0$	(1.27a)
Partial gains G_θ, G_ϕ	$G_0 = G_\theta + G_\phi$ $G_\theta = \frac{4\pi U_\theta}{P_{\text{in}}}, \quad G_\phi = \frac{4\pi U_\phi}{P_{\text{in}}}$	(1.29) (1.29a) (1.29b)
Absolute gain G_{abs}	$G_{\text{abs}} = e_r G(\theta, \phi) = e_r e_{cd} D(\theta, \phi)$ $= (1 - \Gamma ^2) e_{cd} D(\theta, \phi) = e_0 D(\theta, \phi)$	(1.28) (1.28b)
Total antenna efficiency e_0	$e_0 = e_r e_c e_d = e_r e_{cd} = (1 - \Gamma ^2) e_{cd}$	(1.22), (1.23)
Reflection efficiency e_r	$e_r = (1 - \Gamma ^2)$	(1.23)
Beam efficiency BE	$\text{BE} = \frac{\int_0^{2\pi} \int_0^{\theta_1} U(\theta, \phi) \sin \theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^\pi U(\theta, \phi) \sin \theta \, d\theta \, d\phi}$	(1.33)
Polarization loss factor (PLF)	$\text{PLF} = \hat{\rho}_w \cdot \hat{\rho}_a ^2$	(1.38)
Vector effective length $\ell_e(\theta, \phi)$	$\ell_e(\theta, \phi) = \hat{\mathbf{a}}_\theta l_\theta(\theta, \phi) + \hat{\mathbf{a}}_\phi l_\phi(\theta, \phi)$	(1.46)
Polarization efficiency p_e	$p_e = \frac{ \ell_e \cdot \mathbf{E}^{\text{inc}} ^2}{ \ell_e ^2 \mathbf{E}^{\text{inc}} ^2}$	(1.39)
Antenna impedance Z_A	$Z_A = R_A + jX_A = (R_r + R_L) + jX_A$	(1.40), (1.41)
Maximum effective area A_{em}	$A_{em} = \frac{ V_T ^2}{8W_i} \left(\frac{1}{R_r + R_L} \right) = e_{cd} \left(\frac{\lambda^2}{4\pi} \right) D_0 \hat{\rho}_w \cdot \hat{\rho}_a ^2$ $= \left(\frac{\lambda^2}{4\pi} \right) G_0 \hat{\rho}_w \cdot \hat{\rho}_a ^2$	(1.51), (1.53), (1.54)

(continued)

TABLE 1.2 (Continued)

Parameter	Formula	Equation Number
Aperture efficiency ε_{ap}	$\varepsilon_{ap} = \frac{A_{em}}{A_p} = \frac{\text{maximum effective area}}{\text{physical area}}$	
Friis transmission equation	$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{0r} G_{0t} \hat{\rho}_t \cdot \hat{\rho}_r ^2$	(1.56), (1.57)
Radar range equation	$\frac{P_r}{P_t} = \sigma \frac{G_{0r} G_{0t}}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2}\right)^2 \hat{\rho}_w \cdot \hat{\rho}_r ^2$	(1.61), (1.62)
Radar cross section (RCS)	$\begin{aligned} \sigma &= \lim_{R \rightarrow \infty} \left(4\pi R^2 \frac{W_s}{W_i}\right) = \lim_{R \rightarrow \infty} \left(4\pi R^2 \frac{ \mathbf{E}^s ^2}{ \mathbf{E}^i ^2}\right) \\ &= \lim_{R \rightarrow \infty} \left(4\pi R^2 \frac{ \mathbf{H}^s ^2}{ \mathbf{H}^i ^2}\right) \end{aligned}$	(1.59)
Brightness temperature $T_B(\theta, \phi)$	$T_B(\theta, \phi) = \varepsilon(\theta, \phi) T_m = (1 - \Gamma ^2) T_m$	(1.79)
Antenna temperature T_A	$T_A = \frac{\int_0^{2\pi} \int_0^\pi T_B(\theta, \phi) G(\theta, \phi) \sin \theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^\pi G(\theta, \phi) \sin \theta \, d\theta \, d\phi}$	(1.80)

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