Preface

This book was written in response to the growing demand for a text that provides a unified treatment of complex valued adaptive filters, both linear and nonlinear, and methods for the processing of both complex circular and complex noncircular signals. We believe that this is the first attempt to bring together established adaptive filtering algorithms in $\mathbb{C}$ and the recent developments in the statistics of complex variable under the umbrella of powerful mathematical frameworks of $\mathbb{CR}$ (Wirtinger) calculus and augmented complex statistics. Combining the results from the authors' original research and current established methods, this books serves as a rigorous account of existing and novel complex signal processing methods, and provides next generation solutions for adaptive filtering of the generality of complex valued signals. The introductory chapters can be used as a text for a course on adaptive filtering. It is our hope that people as excited as we are by the possibilities opened by the more advanced work in this book will further develop these ideas into new and useful applications.

The title reflects our ambition to write a book which addresses several major problems in modern complex adaptive filtering. Real world data are non-Gaussian, nonstationary and generated by nonlinear systems with possibly long impulse responses. For the processing of such signals we therefore need *nonlinear* architectures to deal with nonlinearity and non-Gaussianity, *feedback* to deal with long responses, and *adaptive* mode of operation to deal with the nonstationary nature of the data. These have all been brought together in this book, hence the title "*Complex Valued Nonlinear Adaptive Filters*". The subtitle reflects some more intricate aspects of the processing of complex random variables, and that the class of nonlinear filters addressed in this work can be viewed as temporal neural networks. This material can also be used to supplement courses on neural networks, as the algorithms developed can be used to train neural networks for pattern processing and classification.

Complex valued signals play a pivotal role in communications, array signal processing, power, environmental, and biomedical signal processing and related fields. These signals are either complex by design, such as symbols used in data communications (e.g. quadrature phase shift keying), or are made complex by convenience of representation. The latter class includes analytic signals and signals coming from many important modern applications in magnetic source imaging, interferometric radar, direction of arrival estimation and smart antennas, mathematical biosciences, mobile communications, optics and seismics. Existing books do not take into account the effects on performance of a unique property of complex statistics – complex noncircularity, and employ several convenient mathematical shortcuts in the treatment of complex random variables.

Adaptive filters based on widely linear models introduced in this work are derived rigorously, and are suited for the processing of a much wider class of *complex noncircular signals* (directional processes, vector fields), and offer a number of theoretical performance gains.

Perhaps the first time we became involved in practical applications of complex adaptive filtering was when trying to perform short term wind forecasting by treating wind speed and direction, which are routinely processed separately, as a unique complex valued quantity. Our results outperformed the standard approaches. This opened a can of worms, as it became apparent that the standard techniques were not adequate, and that mathematical foundations and practical tools for the applications of complex valued adaptive filters to the generality of complex signals are scattered throughout the literature. For instance, an often confusing aspect of complex adaptive filtering is that the cost (objective) function to be minimised is a real function (measure of error power) of complex variables, and is not analytic. Thus, standard complex differentiability (Cauchy-Riemann conditions) does not apply, and we need to resort to pseudoderivatives. We identified the need for a rigorous, concise, and unified treatment of the statistics of complex variables, methods for dealing with nonlinearity and noncircularity, and enhanced solutions for adaptive signal processing in $\mathbb{C}$, and were encouraged by our series editor Simon Haykin and the staff from Wiley Chichester to produce this text.

The first two chapters give the introduction to the field and illustrate the benefits of the processing in the complex domain. Chapter 1 provides a personal view of the history of complex numbers. They are truly fascinating and, unlike other number systems which were introduced as solutions to practical problems, they arose as a product of intellectual exercise. Complex numbers were formalised in the mid-19th century by Gauss and Euler in order to provide solutions for the fundamental theorem of algebra; within 50 years (and without the Internet) they became a linchpin of electromagnetic field and relativity theory. Chapter 2 offers theoretical and practical justification for converting many apparently real valued signal processing problems into the complex domain, where they can benefit from the convenience of representation and the power and beauty of complex calculus. It illustrates the duality between the processing in $\mathbb{R}^2$ and $\mathbb{C}$, and the benefits of complex valued processing – unlike $\mathbb{R}^2$ the field of complex numbers forms a division algebra and provides a rigorous mathematics framework for the treatment of phase, nonlinearity and coupling between signal components.

The foundations of standard complex adaptive filtering are established in Chapters 3–7. Chapter 3 provides an overview of adaptive filtering architectures, and introduces the background for their state space representations and links with polynomial filters and neural networks. Chapter 4 deals with the choice of complex nonlinear activation function and addresses the trade off between their boundedness and analyticity. The only continuously differentiable function in $\mathbb{C}$ that satisfies the Cauchy-Riemann conditions is a constant; to preserve boundedness some ad hoc approaches (also called split-complex) employ real valued nonlinearities on the real and imaginary parts. Our main interest is in complex functions of complex variables (also called fully complex) which are not bounded on the whole complex plane, but are complex differentiable and provide solutions which are generic extensions of the corresponding solutions in $\mathbb{R}$. Chapter 5 addresses the duality between gradient calculation in $\mathbb{R}^2$ and $\mathbb{C}$ and introduces the so called $\mathbb{CR}$ calculus which is suitable for general functions of complex variables, both holomorphic and non-holomorphic. This provides a unified framework for computing the Jacobians, Hessians, and gradients of cost functions, and serves as a basis for the derivation of learning algorithms throughout this book. Chapters 6 and 7 introduce standard complex valued adaptive filters, both linear and nonlinear; they are supported by rigorous proofs of convergence, and can be used to teach a course on adaptive filtering. The complex least mean square (CLMS) in Chapter 6 is derived step by step, whereas the learning algorithms for feedback structures in Chapter 7 are derived in a compact way, based on $\mathbb{CR}$

calculus. Furthermore, learning algorithms for both linear and nonlinear feedback architectures are introduced, starting from linear IIR filters to temporal recurrent neural networks.

Chapters 8–11 address several practical aspects of adaptive filtering, such as adaptive step-sizes, dynamical range extension, and a posteriori mode of operation. Chapter 8 provides a thorough review of adaptive step size algorithms and introduces the general normalised gradient descent (GNGD) algorithm for enhanced stability. Chapter 9 gives solutions for dynamical range extension of nonlinear neural adaptive filters, whereas Chapter 10 explains a posteriori algorithms and analyses them in the framework of fixed point theory. Chapter 11 rounds up the first part of the book and introduces fractional delay filters together with links between complex nonlinear functions and number theory.

Chapters 12–15 introduce linear and nonlinear adaptive filters based on widely linear models, which are suited to deal with complex noncircularity, thus providing theoretical and practical adaptive filtering solutions for the generality of complex signals. Chapter 12 provides a comprehensive overview of the latest results (2008) in the statistics of complex random signals, with a particular emphasis on complex noncircularity. It is shown that the standard complex Gaussian model is inadequate and the concepts of noise, stationarity, multicorrelation, and multispectra are re-introduced based on the augmented statistics. This has served as a basis for the development of the class of 'augmented' adaptive filtering algorithms, starting from the complex least square (ACLMS) algorithm through to augmented learning algorithms for IIR filters, recurrent neural networks, and augmented Kalman filters. Chapter 13 introduces the augmented least mean square algorithm, a quantum step in the adaptive signal processing of complex noncircular signals. It is shown that this approach is as good as standard approaches for circular data, whereas it outperforms standard filters for noncircular data. Chapter 14 provides an insight into the duality between complex valued linear adaptive filters and dual channel real adaptive filters. A correspondence is established between the ACLMS and the dual channel real LMS algorithms. Chapter 15 extends widely linear modelling in $\mathbb{C}$ to feedback and nonlinear architectures. The derivations are based on $\mathbb{CR}$ calculus and are provided for both the gradient descent and state space (Kalman filtering) models.

Chapter 16 addresses collaborative adaptive filtering in $\mathbb{C}$. It is shown that by employing collaborative filtering architectures we can gain insight into the nature of a signal in hand, and a simple test for complex noncircularity is proposed. Chapter 17 introduces complex empirical mode decomposition (EMD), a data driven time-frequency technique. This technique, when used for preprocessing complex valued data, provides a framework for "data fusion via fission", with a number of applications, especially in biomedical engineering and neuroscience. Chapter 18 provides a rigorous statistical testing framework for the validity of complex representation.

The material is supported by a number of Appendices (some of them based on [190]), ranging from the theory of complex variable through to fixed point theory. We believe this makes the book self-sufficient for a reader who has basic knowledge of adaptive signal processing. Simulations were performed for both circular and noncircular data sources, from benchmark linear and nonlinear models to real world wind and radar signals. The applications are set in a prediction setting, as prediction is at the core of adaptive filtering. The complex valued wind signal is our most frequently used test signal, due to its intermittent, non-Gaussian and noncircular nature. Gill Instruments provided ultrasonic anemometers used for our wind recordings.