# 1 Light, Vision and Photometry



## 1.1 LIGHT

Light is radiation in the form of electromagnetic waves that make vision possible to the human eye. Electromagnetic radiation can be classified by its wavelength or frequency, as shown in Figure 1.1. The wavelength of light is confined to a very narrow range limited by a short-wavelength edge between 360 and 400 nm (1 nm = $10^{-6}$ mm; see Appendix I) and a long-wavelength edge between 760 and 830 nm. Infrared radiation and ultraviolet radiation, which are not visible to human eye, are sometimes included in the category of light and referred to as infrared light and ultraviolet light. However, it is better to call these categories infrared radiation and ultraviolet radiation. When it is necessary to distinguish light from radiation not visible to human eye, it is referred to as visible light or visible radiation.

Newton (Figure 1.2) showed experimentally that white light, such as sunlight, is composed of various types of colored light. More specifically, he demonstrated the following facts by introducing sunlight into a prism (Figure 1.3).

1. White sunlight incident on a prism is separated into seven components differing in color, as observed in a rainbow. The seven colors are red, orange, yellow, green, blue, indigo, and violet (see Color Plate 1).

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2. The spectrum (i.e., the seven components of light differing in color) can be reunited to give the original white light by focusing the components back through a reversed prism.
3. If one color component alone is incident on a prism, it cannot be further separated into the seven colors.

We now know that, when observed in detail, the spectrum includes an infinite range of components of different wavelengths that cannot all be given different color names. The classification into seven named components is based on a simple set of basic

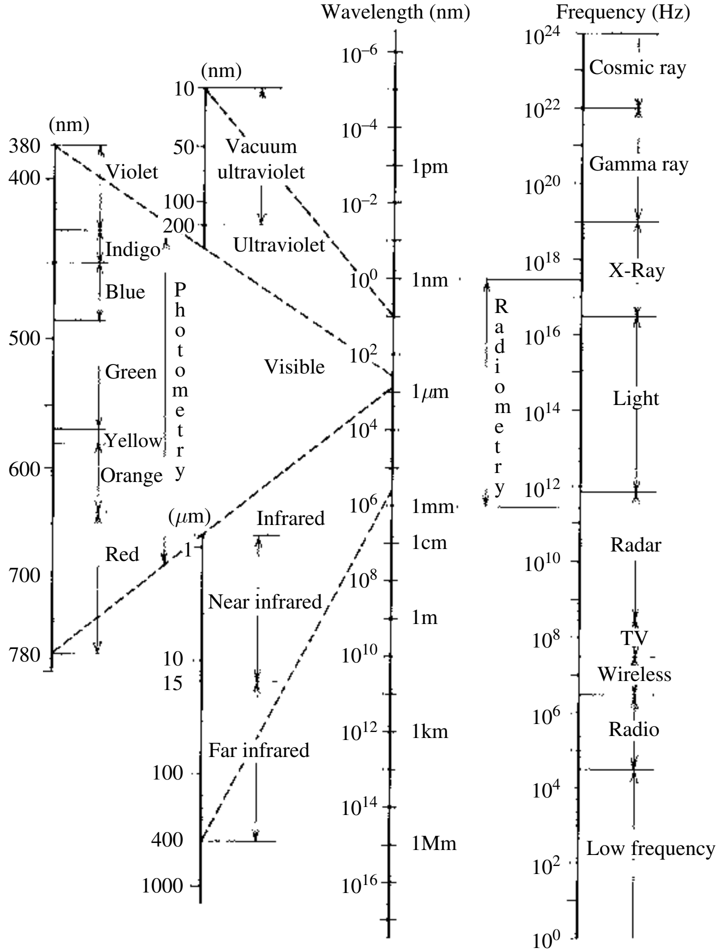


**Figure 1.1** Wavelengths of electromagnetic radiation and light

**Figure 1.2** Isaac Newton (1643–1727)

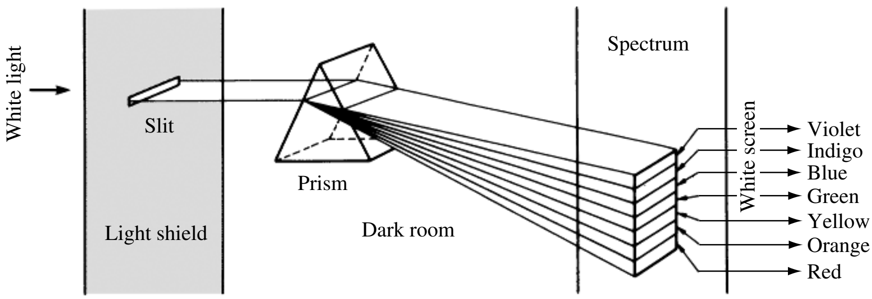


**Figure 1.3** Prismatic dispersion of white light

color names. On detailed examination, other colors can be named. For example, reddish orange colors appear between the pure red and pure orange parts of the spectrum.

Monochromatic light is light that cannot be separated into components. White light such as sunlight is polychromatic, i.e., a mixture of monochromatic lights. A spectrum is a band of color observed when a beam of white light is separated into components of light that are arranged in the order of their wavelengths. The approximate correspondence between wavelengths and colors is shown in Figure 1.1.

If one or more components is decreased in intensity and the components are recombined, colored light is obtained instead of the original white light. Thus, if an object illuminated with white light reflects the components with differing reflectance depending on the wavelengths, the human eye sees the object as colored. For example, a red object does not reflect much in the range from violet to yellow, but reflects the red component strongly. Thus, it is perceived to have a red color. In general, color is generated whenever white light is modified by reflection or transmission by an object.

## 1.2 MECHANISM OF THE HUMAN EYE

The visual system is very similar to a photographic system in that both respond to light and, in particular, to images. The human eyeball is a sphere about 24 mm in diameter, and its mechanism resembles that of a camera and photographic film. Figure 1.4 shows schematically an eye and a camera. The corresponding components are as follows:

| Camera | Eye |
|---|---|
| Black box | Sclera and choroid |
| Lens | Cornea and lens |
| Shutter | Eyelid |
| Diaphragm | Iris |
| Film | Retina |

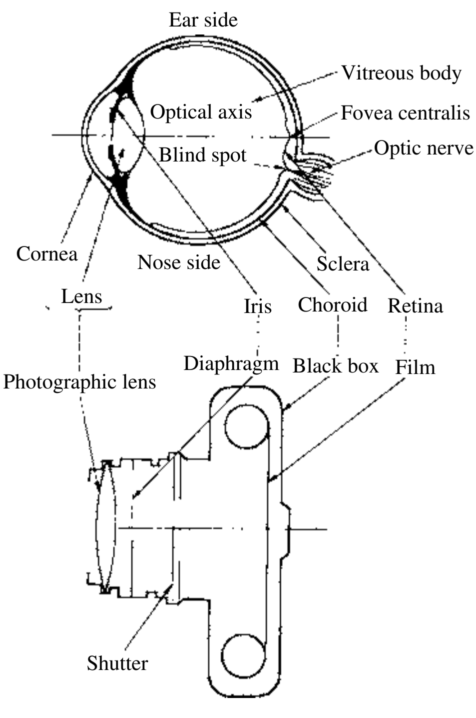


**Figure 1.4** Structure of eye and camera

Light incident on the eye induces a photochemical reaction in the retina, which corresponds to the photographic film The nerve impulse generated by the reaction is transmitted to the brain to give a visual sensation. The retina covers about two-thirds of the internal surface of the eyeball, and is a transparent film about 0.3 mm in thickness, with a complicated structure comprising several types of cell (Dowling and Boycott 1966). This is illustrated in Figure 1.5. The incident light enters the retina in the direction indicated by the arrows, and reaches the photosensitive neuroepithelial layer. The optic nerve layer, which is located in front of the neuroepithelial layer, performs various types of signal processing. It should be noted that the incident light reaches the neuroepithelial layer after it passes through the transparent optic nerve layer.

The photosensitive neuroepithelial layer, which corresponds to the fine photosensitive silver halide (e.g., AgCl, AgBr, or AgI) grains incorporated in a photographic film, consists of two types of cell. These are rods, which perceive brightness or darkness in relatively dark environments, and cones which perceive color in relatively bright environments. The names ‘rods’ and ‘cones’ are derived from the shapes of the cells. There are three types of cone cell, present in the ratio of about 32:16:1, which respond to long-, medium- and short-wavelength light, respectively. Thus, the eye can be thought

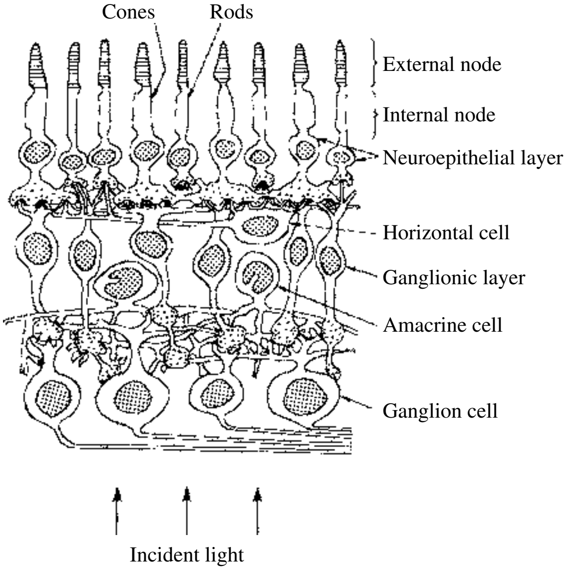


**Figure 1.5** Structure of retina (Dowling and Boycott 1966). Reproduced by permission of The Royal Society

of as being constructed of a high-speed black-and-white film (the rods) and a medium-speed color film (the cones).

There are about a hundred million rods and about seven million cones in a human retina. The end of each neuroepithelial cell (the hatched portion in Figure 1.5) is called the external node and contains a photosensitive pigment. The diameter of the external node is between 1 and 2 μm for a rod and between 1 and 5 μm for a cone. It can be seen therefore that the diameter of the external node is about the same as that of a photographic silver halide grain, which is between 0.05 and 3 μm. The human eye has about 60 000 elements per $mm^2$ at the center of the retina, an electronic camera has about 20 000, and a color photograph about 30 000.

The distribution of neuroepithelial cells in the retina is shown in Figure 1.6 (Pirenne 1948). The cones are concentrated in the vicinity of the optical axis in the fovea centralis. The fovea centralis is a narrow portion of the retina, about 1.5 mm in diameter, in which approximately 100 000–150 000 cones are concentrated. Maximum resolution is therefore achieved in this narrow portion. In contrast to the cones, rods are rarely found in the vicinity of the fovea centralis, and are distributed over a wide region of the retina. Because the rods, and not the cones, function in dark environments, stars in the sky at night are seen more easily obliquely, i.e., with squinting eyes.

The signals generated by the photosensitive pigments in the neuroepithelial cells are processed by various cells, shown in Figure 1.5, and the processed signals are then transmitted to the brain through about one million optic nerves. Because no

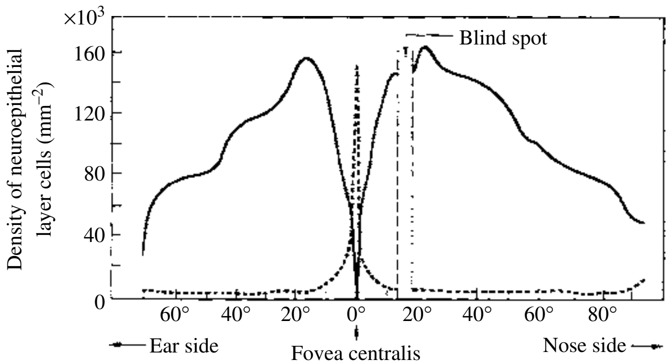


**Figure 1.6** Distribution of rods (solid line) and cones (broken line) (Pirenne 1948)

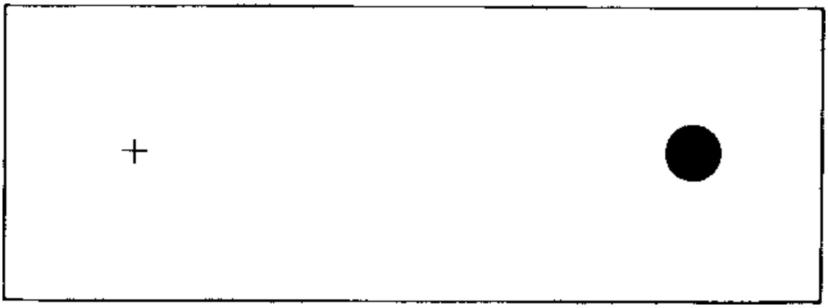

**Figure 1.7** Detection of blind spot

neuroepithelial cell is present in the portion of the retina where the optic nerve penetrates, this portion cannot sense light and is called the blind spot. The blind spot is located at an angle of 15° from the line of sight (optical axis) and is about 5° wide. This can be confirmed readily by a visual experiment using Figure 1.7. If the observer fixates his/her right eye on the cross while closing his/her left eye and adjusting the distance between the eye and the cross to about 20 cm, the solid circle disappears from sight. This occurs because the solid circle is imaged on the blind spot.

## 1.3 ADAPTATION AND RESPONSIVITY OF THE HUMAN EYE

The brightness (more correctly the illuminance) provided by natural and artificial light sources used in daily life ranges widely, as shown in Figure 1.8 (Shoumei Gakkai 1967). The human eye can see an object in direct sunlight where the illuminance is about 100 000 lx, or at night without moonlight at an illuminance of about 0.0003 lx (as described in Section 1.6, the unit of illuminance is the lux, abbreviated lx). To adapt the eye over such a wide range of illuminance, the pupil adjusts the quantity of light reaching the retina by changing its size. Thus, the pupil functions like the diaphragm of a camera. Because the pupil changes its diameter in a range from 2 to 7 mm, the quantity of light adjustable in this way covers a range of only a factor of 12.

Thus the change in pupil diameter is insufficient for full control of the quantity of light. Accordingly, the rods and cones share the function by changing the responsivity of the retina. In a relatively bright environment, the cones alone function to give what is

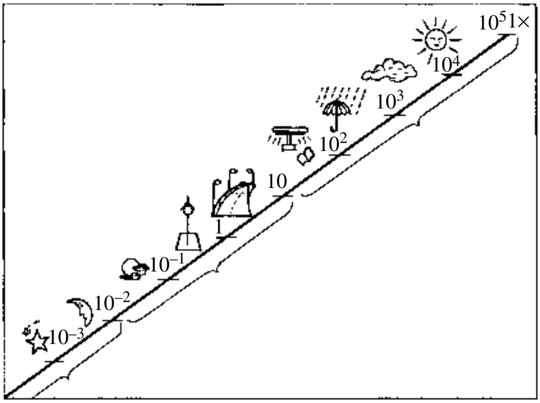


**Figure 1.8** Approximate values of brightness (illuminance) (Shoumei Gakkai 1967). Reproduced by permission of Shoumei Gakkai

called photopic vision. In a relatively dark environment, the rods alone function to realize what is called scotopic vision. In environments having an intermediate brightness between photopic vision and scotopic vision, both the cones and the rods function to provide what is called mesopic vision. Photopic vision is distinguished from scotopic vision by the luminance range in which it operates. Photopic vision occurs for luminances of about $3\,cd/m^2$ or higher (see Sections 1.5 and 1.6 for the definitions of photometric quantities and units), and scotopic vision occurs for luminances of about $0.003\,cd/m^2$ or lower. These numbers depend somewhat on other conditions, such as the color of the stimuli.

When one enters a bright environment from a dark one, one's vision changes from scotopic to photopic via mesopic. This change is completed in about 1 min, and the eye readily adapts to the bright environment. On the contrary, when one enters a dark environment from a bright one, vision changes from photopic to scotopic much more slowly. As shown in Figure 1.9, it takes about 30 min to completely accomplish the adaptation (Chapanis 1947).

In photopic vision, the photochemical reaction in the rods saturates, and they become inert to light and the cones alone are left active. The photochemical reaction in the cones continues to an upper limit of about $10^6\,cd/m^2$. If this limit is exceeded, the result is a blinding and uncomfortable sensation that can damage the eyes. Referring to Figure 1.9, the detectivity of the eye (the minimum luminance sensed as light) for white light changes from curve A to curve B as the rods take over from the cones upon transfer from light adaptation to dark adaptation.

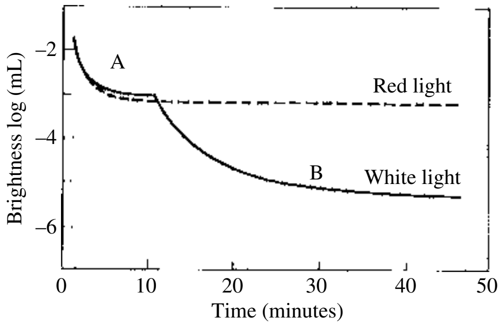


**Figure 1.9** Progress of dark adaptation

The ordinate of Figure 1.9 is luminance in units of millilambert (mL). These can be converted to $cd/m^2$ by using a conversion factor of $1\,mL = 3.183 (= 10/\pi) cd/m^2$. In the initial stage of dark adaptation (about 10 min), cones function to give curve A but, in the later stages, rods with their higher responsivity take over to yield curve B. However, under red light, the portion of curve B in which the rods function does not appear. This is because the rods do not respond to high (red) wavelengths.

In scotopic vision, rods are active and exhibit a relatively high response to light. With decreasing luminance, however, even the rods finally become insensitive. Depending on the experimental conditions, the luminance limit at which the rods lose sensitivity is about $10^{-6}$ $cd/m^2$. Taking into account absorption and scattering of light inside the eye and the absorption efficiency of the retina, this limit corresponds to about 5–14 photons incident on the rods. Cones on the other hand require about 100–1000 as many photons before they respond. By comparison, four or more photons are necessary to induce a reaction in the fine silver halide grains of high-speed photographic film. It can be seen that rods have a detectivity that compares well to that of photographic film.

## 1.4 SPECTRAL RESPONSIVITY AND THE STANDARD PHOTOMETRIC OBSERVER

The output of a photodetector divided by the radiant energy input is called its responsivity. The term can be applied to the human eye as well as to physical detectors. For the eye, the output is a brightness response. In the past, the term sensitivity was used, but responsivity is now preferred. The higher the responsivity, the higher is the output for a given input. When the responsivity is expressed as a function of wavelength, the curve is called the spectral responsivity. Equal amounts of radiant energy become less visible to the human eye with decreasing or increasing wavelength on either side of a maximum. Outside the visible region, which extends from about 380 to 780 nm, radiation becomes invisible. Thus the spectral responsivity of the eye is a function of wavelength, decreasing gradually to zero in the ultraviolet and infrared regions. Furthermore, since the manner of transfer from light adaptation to dark adaptation in white light differs from that in red light, it is clear that the spectral responsivity of rods differs from that of cones.

In general, the spectral responsivity of a photoreceptor is determined for each wavelength by introducing a monochromatic light of known radiant energy and then measuring the response in the form of a photocurrent, for example. The response of the eye, however, is not determined by a physical measurement, but rather in terms of a brightness sensation. Thus, to obtain the spectral responsivity of the eye, a means such as matching is employed. More specifically, in the matching method, a predetermined reference light having a certain wavelength is used, so that the brightness $\Phi_v$ of a test light having an arbitrary wavelength may be matched with that of the reference light. By measuring the radiant energy $\Phi_e$ of the test light in the match, $\Phi_v$ can be expressed as

$$\Phi_v = K\,\Phi_e \tag{1.1}$$

where $K$ is a measure of the brightness per unit radiant energy. Thus, $K$ is the responsivity of the eye. (At this stage, the quantities and units of brightness and radiant energy have not been defined so the units of $K$ are arbitrary. These matters are discussed later in this chapter.)

The brightness of two different lights can be matched by any of the following methods:

1. Direct comparison method. This method comprises directly comparing a test light of wavelength $\lambda_2$ with a reference light of wavelength $\lambda_1$. In principle it is the simplest method, but it is very difficult experimentally to match, for example, the brightness of a red light with that of a blue light. Consequently, the results fluctuate and the method suffers from poor precision.
2. Step-by-step method. Although it is difficult to compare the brightnesses of lights differing greatly in color, those having similar colors can be readily compared. Thus, by using a test light having a wavelength $\lambda_2$ near the wavelength $\lambda_1$ of the reference light,

the responsivity for a wavelength $\lambda_2$ can be determined. Then the responsivity for a light having a wavelength $\lambda_3$ near $\lambda_2$ can be obtained by using the previous test light as the reference light. By repeating this process sequentially, the spectral responsivity for the whole spectrum can be obtained, step-by-step.

3. Flicker method. By alternately introducing a reference light having a wavelength of $\lambda_1$ and a test light having a wavelength of $\lambda_2$ into the visual field, the color can be made to flicker, for example, between red and green. On increasing the frequency of repetition, the two colors merge into one at a frequency of about 30 Hz. Above this frequency, no color change is perceived. In the example given, the red and green lights merge to yield yellow. However, if the two lights differ in brightness from each other, the difference in brightness remains as a flicker even if the colors merge into one. On further increasing the frequency to a value higher than 50 Hz, the brightness as well as the color merges to yield a uniform visual field. By utilizing the frequency region in which the flicker attributed to brightness remains, but that for color disappears, the brightness of the test light can be matched with that of the reference light.

In the direct comparison method, results cannot be determined with high precision. However, a relatively stable result can be obtained by the step-by-step method or by the flicker method. By setting the reference light sufficiently dark and performing the experiment in scotopic vision, the spectral responsivity of the rods can be measured. On the other hand, if the reference light is set sufficiently bright, photopic vision operates and the responsivity curve of the cones can be measured. The spectral responsivity of the cones can also be obtained by narrowing the observation field to about 2°, because no rods are present in the fovea centralis.

As described above, the value of $K$ in Equation 1.1 corresponds to the responsivity of the eye. The value of $K$ is called the luminous efficacy of the radiation. The spectral luminous efficacy, $K(\lambda)$, can be determined by varying the wavelength, $\lambda$ and observing $K$ as a function of $\lambda$. The maximum value, $K_m$, of $K(\lambda)$ is called the maximum luminous efficacy, and the ratio of $K(\lambda)$ to $K_m$, is called the spectral luminous efficiency, $V(\lambda)$. The maximum luminous efficacies, $K_m$ and $K'_m$, are related to the spectral luminous efficacies, $K(\lambda)$ and $K'(\lambda)$, by the following equations

$$K(\lambda) = K_m V(\lambda) \tag{1.2a}$$

$$K'(\lambda) = K'_m V'(\lambda) \tag{1.2b}$$

where $V(\lambda)$ and $V'(\lambda)$ are the spectral luminous efficiencies in photopic and scotopic vision, respectively. Because $K(\lambda) \leq K_m$ and $K'(\lambda) \leq K'_m$, the maximum values for $V(\lambda)$ and $V'(\lambda)$ are 1.0.

Once the spectral responsivity for brightness is known, the brightness of lights differing in color can be treated quantitatively. However, to compare brightness on a worldwide basis, it is necessary for everyone to use the same spectral responsivity function. The Commission Internationale de l'Élairage (CIE) is an international organization that researches and recommends standards related to light. The CIE has established two spectral responsivity curves that are universally used.

In 1924, the CIE established the spectral luminous efficiency for photopic vision, $V(\lambda)$, based on the average observed values from 7 studies involving 251 people with normal color vision. Similarly, in 1951, it established the spectral luminous efficiency for scotopic vision, $V'(\lambda)$ (Figure 1.10 and Table 1.1). The responsivities thus established are the average values for a large number of observers. Although a real observer does not necessarily exist with exactly the spectral responsivities illustrated in Figure 1.10, virtual observers with these responsivities are known as the CIE Standard Photometric Observers.

The spectral luminous efficiency functions recommended by the CIE were determined by the step-by-step method and the flicker method described above. These experimental methods use special conditions for the evaluation of brightness whereas direct comparison is usually employed in practical situations. However, as is illustrated in Figure 1.11, the fluctuation of the values obtained by direct comparison is too large (Ikeda *et al.* 1982). Hence, results of the direct comparison method are not used as basic data for establishing spectral luminous efficiency. However, numerous observed

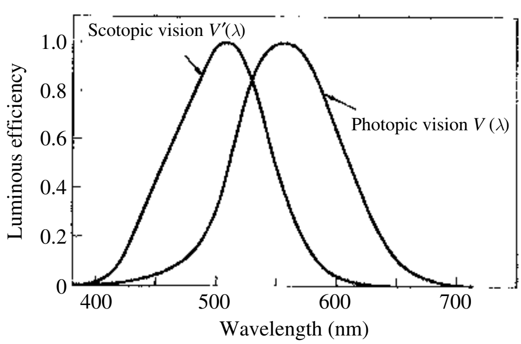


**Figure 1.10** Spectral luminous efficiency in photopic and scotopic vision

**Table 1.1** Spectral luminous efficiency in photopic and scotopic vision

| Wavelength (nm) | Photopic vision $V(\lambda)$ | Scotopic vision $V'(\lambda)$ |
|---|---|---|
| 380 | 0.0000 | 0.0006 |
| 390 | 0.0001 | 0.0022 |
| 400 | 0.0004 | 0.0093 |
| 410 | 0.0012 | 0.0348 |
| 420 | 0.0040 | 0.0966 |
| 430 | 0.0116 | 0.1988 |
| 440 | 0.0230 | 0.3281 |
| 450 | 0.0380 | 0.4550 |
| 460 | 0.0600 | 0.5670 |
| 470 | 0.0910 | 0.6760 |
| 480 | 0.1390 | 0.7930 |
| 490 | 0.2080 | 0.9040 |
| 500 | 0.3230 | 0.9820 |
| 510 | 0.5030 | 0.9970 |
| 520 | 0.7100 | 0.9350 |
| 530 | 0.8620 | 0.8110 |
| 540 | 0.9540 | 0.6500 |
| 550 | 0.9950 | 0.4810 |
| 560 | 0.9950 | 0.3288 |
| 570 | 0.9520 | 0.2076 |
| 580 | 0.8700 | 0.1212 |
| 590 | 0.7570 | 0.0655 |
| 600 | 0.6310 | 0.0332 |
| 610 | 0.5030 | 0.0159 |
| 620 | 0.3810 | 0.0074 |
| 630 | 0.2650 | 0.0033 |
| 640 | 0.1750 | 0.0015 |
| 650 | 0.1070 | 0.0007 |
| 660 | 0.0610 | 0.0003 |
| 670 | 0.0320 | 0.0001 |
| 680 | 0.0170 | 0.0001 |
| 690 | 0.0082 | 0.0000 |
| 700 | 0.0041 | 0.0000 |
| 710 | 0.0021 | 0.0000 |
| 720 | 0.0010 | 0.0000 |
| 730 | 0.0005 | 0.0000 |
| 740 | 0.0003 | 0.0000 |
| 750 | 0.0001 | 0.0000 |
| 760 | 0.0001 | 0.0000 |
| 770 | 0.0000 | 0.0000 |
| 780 | 0.0000 | 0.0000 |

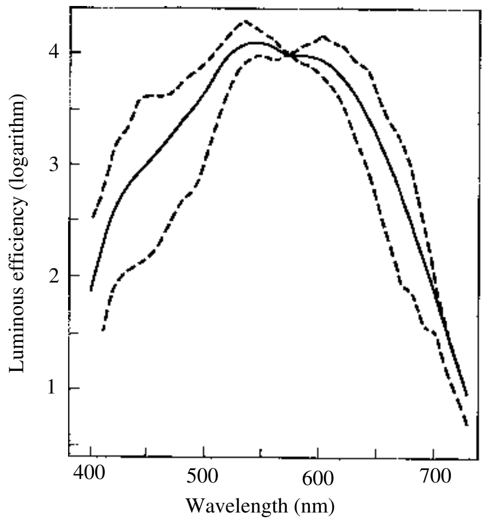


**Figure 1.11** Variations (broken lines) and $V_b(\lambda)$ (solid line) for spectral luminous efficiency obtained by the direct comparison method (Ikeda *et al.* 1982). Reproduced by permission of Optical Society of America

results have been accumulated by the direct comparison method, and the average values $V_b(\lambda)$ have been found to deviate significantly from the values $V(\lambda)$ established by the CIE (see Figure 1.12).

It might seem appropriate therefore to replace $V(\lambda)$ by $V_b(\lambda)$. However $V(\lambda)$ is the prevailing standard in the world, and it is not simple to change it. Furthermore, $V_b(\lambda)$ applies to the direct comparison of brightnesses and is not appropriate for predicting other psychophysical phenomena, such as visual acuity, for which $V(\lambda)$ works better. Thus, the CIE has recommended $V_b(\lambda)$ separately in addition to $V(\lambda)$ (CIE 1988). These issues are discussed in more detail on page 29 and in Note 1.2.

The spectral luminous efficiency function for mesopic vision $V^*(\lambda)$, is intermediate between $V(\lambda)$ and $V'(\lambda)$. However, a CIE standard photometric observer for mesopic vision is not yet established. Brightness in mesopic vision cannot be determined unless $V^*(\lambda)$ is established, but the problem is not simple because the relative contribution of $V(\lambda)$ and $V'(\lambda)$ is complicated. Several methods have been proposed for the determination of $V^*(\lambda)$ including one by Sagawa *et al.* (1992).

As the eye's vision moves from photopic to scotopic via mesopic, the wavelength of maximum luminous efficacy moves to shorter wavelengths because the spectral luminous efficiency changes from

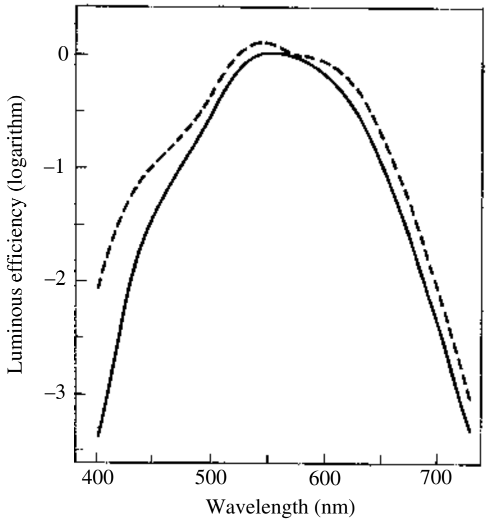


**Figure 1.12** Spectral luminous efficiency $V(\lambda)$ for the CIE Standard Observer for photopic vision (solid line) and spectral luminous efficiency $V_b(\lambda)$ determined by the direct comparison method (broken line) (CIE 1988)

$V(\lambda)$ to $V'(\lambda)$. Thus, a phenomenon occurs in which two colors having the same brightness in photopic vision, have different brightness in scotopic vision. For example, the brightness of a red color tends to decrease relative to a blue color with the transfer to scotopic vision, even if the red and blue colors are of the same brightness in photopic vision. This phenomenon is called the Purkinje phenomenon after its discoverer.

The reason why a darkroom operator wears red eyeglasses or why the illumination in a submarine is often red can be explained by this phenomenon. As can be seen from Figure 1.10, only the cones function in red light, whereas the rods are not stimulated. Accordingly, once the operator returns to the darkroom or observes the dark surface of the sea through a periscope without wearing the red eyeglasses, the rods can be put to work immediately because they have remained dark-adapted.

Instead of using matching, the spectral responsivity of the human eye can be measured by a threshold technique. A threshold is the minimum radiant energy that makes light visible. Responsivity can be obtained as the inverse of the threshold. In other words, the higher the responsivity, the lower the threshold. Continuing the analogy with photography, responsivity (sensitivity) there is expressed by film speed as, for example, ISO (ASA) 1000, which is an inverse of a value similar to a threshold value. Figure 1.13 shows the threshold values for rods and cones as a function of wavelength. In theory, the spectral responsivity obtained from thresholds may not agree with the spectral luminous efficiency obtained by matching. However, on deriving a spectral responsivity curve from Figure 1.13 and comparing it with that in Figure 1.10, it is confirmed that the two are approximately the same.

In Figure 1.13 two curves are shown that define three regions I, II, and III. Region I corresponds to color vision in which the cones work to sense light and color. Region II is a colorless region in which only light is sensed but not color. In this region, only the rods function. Nothing is sensed in region III because it is below the threshold value for both cones and rods. A light having a wavelength of 650 nm or longer is red-colored if it can be seen at all. No colorless region is present for these wavelengths. This is one of the reasons why a red light is used in important cases such as traffic signals.

Although the spectral responsivity of cones differs somewhat from that of rods, they both have a maximum in the middle of the visible wavelength region. With recent progress in artificial illumination, activity at night can be carried out without any problem. However, considering the long history of the human race, activity under artificial illumination accounts for a very short period

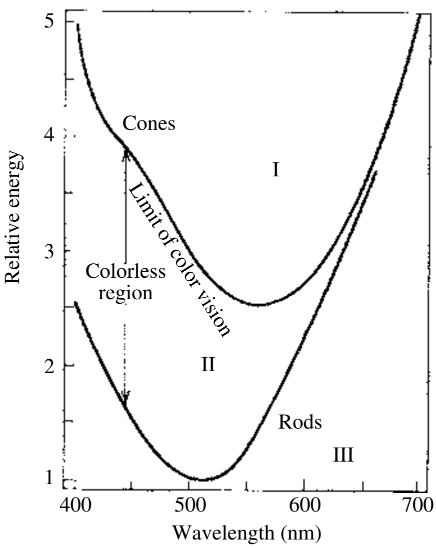


**Figure 1.13** Spectral responsivity of cones and rods obtained from threshold measurements (Wald 1945). Reprinted with permission from Human vision and the spectrum, *Science* **101**. Copyright (1945) AAAS

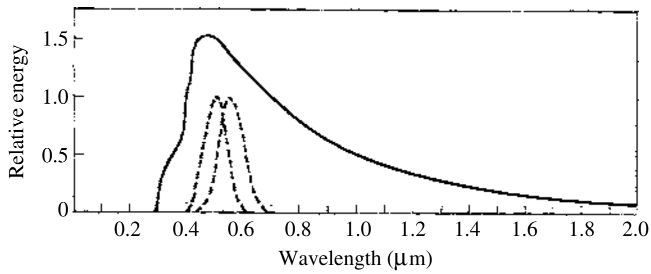


**Figure 1.14** Spectral distribution of daylight (solid line) and spectral sensitivity (broken lines) of cones (right) and rods (left) (Ikeda 1975b). Reproduced by permission of Asakura Shoten

and, historically, human activity has taken place principally under daylight. Presumably, the evolution of the spectral responsivity of the human eye occurred so as to fully and efficiently utilize daylight. Figure 1.14 shows the spectral distribution of daylight and the spectral responsivity of rods and cones, and it can be seen that the maximum responsivities occur at a wavelengths roughly corresponding to those at which the radiant energy of daylight is at its maximum (Ikeda 1975b).

## 1.5 DEFINITION OF PHOTOMETRIC QUANTITIES

In daily life, the term 'brightness' is used without thinking carefully about its meaning. For instance, laser radiation is considered 'bright', but its brightness is insufficient if it is to be used to illuminate a room. On the other hand, a few fluorescent lamps will provide sufficient light for room illumination although, considered as single lamps, they are not considered to be as 'bright' as lasers. To explain this apparent paradox, the brightness of light must be judged by a value normalized by solid angle or area, just as a substance is judged to be light or heavy by considering its mass per unit volume, i.e., its density. Photometry comprises measuring the 'brightness' of lights and normalizing the measurements in various ways to obtain photometric quantities.

Photometric quantities include quantity of light, luminous flux, luminous intensity, illuminance, luminous exitance, and luminance. Luminous flux $\Phi_v$ is a quantity obtained by evaluating radiant flux $\Phi_e$ according to the luminous efficacy of the human eye. Quantity of light $Q_v$ is obtained by integrating the luminous flux over time and the other quantities are obtained by various geometric normalizations. Thus, any photometric quantity can be obtained by evaluating the corresponding radiometric quantity according to the luminous efficacy. As noted above, the appropriate value of luminous efficacy depends on the wavelength of the radiation and on the type of vision involved (photopic, scotopic or mesopic). This is discussed in more detail in Section 1.7 after the geometric factors have been explained. A radiometric quantity is a physical quantity measured in units such as joules (J) or watts (W), whereas a photometric quantity is an operationally defined quantity designed to represent the way in which the human visual system evaluates the corresponding radiometric (physical) quantity. Accordingly, it is called a psychophysical quantity. The tristimulus values of color to be described later are also psychophysical quantities.

In geometric terms, the definitions of photometric quantities are the same as for the corresponding radiometric quantities. Accordingly, the explanations below are made in terms of photometric quantities, but they apply equally well to radiometric ones. For photometric quantities, a subscript v is normally added to the symbol for each quantity, whereas for radiometric quantities, a subscript e is added.

Figure 1.15 illustrates a luminous flux $d\Phi$ that passes through two plane elements, A and B. Because the luminous flux passing through the plane elements A and B remains the same (in the absence of any absorption or scattering), it can be expressed as

$$d\Phi = I\,dm = I'\,dm' \tag{1.3}$$

where, $dm$ and $dm'$ are the 'sizes' of A and B, respectively, and $I$ and $I'$ are the 'densities' of luminous flux, respectively.

If the light source is infinitesimally small, as shown in Figure 1.16, it becomes a point. Then, because light travels in straight lines, the plane elements are cross sections of the same cone, and the following relation can be obtained by defining the

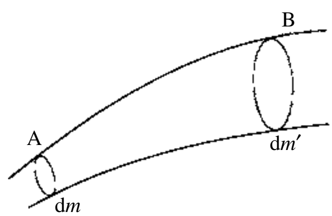


**Figure 1.15** Luminous flux passing through plane elements A and B

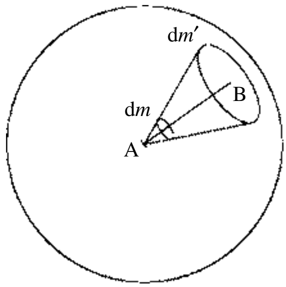


**Figure 1.16** Luminous flux and luminous intensity inside a cone

'sizes' of the plane elements by the solid angles d$\omega$ and d$\omega'$ that they subtend at the source. Since

$$\mathrm{d}\omega = \mathrm{d}\omega' \tag{1.4}$$

the value of $I$ for the point light source can be obtained as

$$I = I' = \mathrm{d}\Phi/\mathrm{d}\omega \tag{1.5}$$

where $I$ is called the luminous intensity, d$\omega$ is the solid angle and d$\Phi$ is the flux within d$\omega$.

If, as is shown in Figure 1.17, the light source is a planar surface, A and if the size of the light source is not negligible with respect to the distance of observation, luminous intensity cannot be defined so easily. However, because a planar light source can be regarded as a group of plane elements, each acting as a point light source, the luminous flux in the plane B increases as the area of the source increases. Thus it is useful to consider the luminous intensity of the light source per unit area. This quantity is called luminance, $L$ and is expressed as

$$L = \mathrm{d}I/(\mathrm{d}S\cos\theta) \tag{1.6}$$

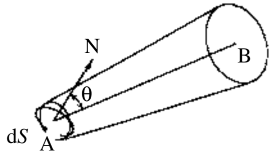


**Figure 1.17** Luminous flux and luminance from a planar light source

where d$I$ is the intensity emitted from a plane element of area d$S$, $\theta$ is the angle between the normal N of the plane A and the light beam AB. The projected area of the plane element, perpendicular to the light beam is d$S$ cos $\theta$. Combining Equations 1.5 and 1.6 gives

$$L = \mathrm{d}\Phi/(\mathrm{d}S\cos\theta\,\mathrm{d}\omega) \tag{1.7}$$

Although the form of Equation 1.7 is not strictly correct from a mathematical point of view because $\mathrm{d}\Phi/(\mathrm{d}S\ \mathrm{d}\omega)$ should be written as a double partial derivative $\partial^2\Phi/\partial S\partial\omega$, this form of the equation is usually used in photometry because it relates more clearly to the measurement process.

Referring to Figure 1.17, if the luminous flux, d$\Phi$ at point B changes with $\theta$ in proportion to cos$\theta$, the luminance, $L$ remains constant because the apparent area of the light source also changes in proportion to cos$\theta$. This type of light source is called a perfect diffusing planar light source.

In the case of luminous intensity, the plane element is defined by the solid angle it subtends at the source. If, instead, the plane element is considered in terms of its area, another quantity, illuminance can be defined as shown in Figure 1.18. Illuminance is the luminous flux incident on a given plane (plane B in Figure 1.18) per unit area. If the luminous flux leaving the plane per unit area is considered, the term luminous exitance is used instead of illuminance but is expressed by the same equation. Thus, illuminance $E$ and luminous exitance $M$ can be expressed by

$$E = \mathrm{d}\Phi/\mathrm{d}S \tag{1.8a}$$

$$M = \mathrm{d}\Phi/\mathrm{d}S \tag{1.8b}$$

where, d$S$ is the area of a plane on which luminous flux d$\Phi$ is incident (Equation 1.8a) or from which luminous flux d$\Phi$ is leaving (Equation 1.8b).

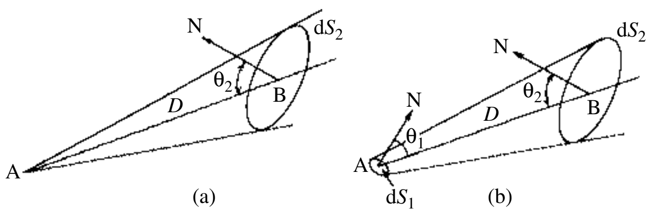


**Figure 1.18** Luminous flux and illuminance from a point light source (a) and from a planar light source (b)

Figure 1.18(a) illustrates the illuminance $E$ on plane B resulting from a point source, A. By taking the angle between the normal to the plane B and the light beam AB as $\theta_2$, the area of the plane B projected perpendicular to AB is expressed as $dS_2 \cos\theta_2$. This projected plane subtends a solid angle of $d\omega$ at point A expressed by

$$d\omega = dS_2 \cos\theta_2 / D^2 \tag{1.9}$$

where $D$ is the distance between A and B. The flux $d\Phi$ is obtained from Equation 1.5 as

$$d\Phi = I\, d\omega = I\, dS_2 \cdot cos\theta_2 / D^2 \tag{1.10}$$

Thus, the illuminance, $E$ incident on plane B is

$$E = d\Phi / dS_2 = I \cos\theta_2 / D^2 \tag{1.11}$$

In the case of a planar light source, A as is shown in Figure 1.18(b), $d\Phi$ can be expressed, from Equation 1.7 as

$$d\Phi = L\, dS_1 \cos\theta_1\, d\omega = L\, dS_1 \cos\theta_1 dS_2 \cos\theta_2 / D^2 \tag{1.12}$$

Thus, the illuminance $E$ can be expressed by

$$E = d\Phi / dS_2 = L\, dS_1 \cos\theta_1 \cos\theta_2 / D^2 \tag{1.13}$$

Table 1.2 shows the definition of photometric quantities together with the corresponding radiometric quantities. Figure 1.19 schematically illustrates the photometric quantities (Noguchi 1987). In Table 1.2, $t$ represents time in seconds (s), $\omega$ represents solid angle in steradians (sr), $S$ represents area in square meters ($m^2$), and $\theta$ represents the angle between the normal of the plane element and the direction of observation. The solid angle is expressed in units of steradian, which is defined as the solid angle that, having its vertex at the center of a sphere of radius $r$, cuts off an area $r^2$ in the surface of the sphere. Thus, the whole surface of a sphere yields a solid angle of $4\pi$ sr because the surface area of a sphere is $4\pi r^2$.

## 1.6 PHOTOMETRIC UNITS

In the SI (the International System of units), luminous intensity is a base unit. The SI evolved from the MKSA unit system. Three units, the kelvin (abbreviated K) for temperature, the mole (abbreviated

**Table 1.2** Definition of radiometric and photometric quantities

| radiometric quantities | | |
|---|---|---|
| Quantity | Definition | Unit |
| radiant energy | $Q_e$ | J |
| radiant flux | $\Phi_e = dQ_e/dt$ | W (J/s) |
| radiant intensity | $I_e = d\Phi_e/d\omega$ | W/sr |
| irradiance | $E_e = d\Phi_e/dS$ | $W/m^2$ |
| radiant exitance | $M_e = d\Phi_e/dS$ | $W/m^2$ |
| radiance | $L_e = d\Phi_e/(dS \cdot \cos\theta \cdot d\omega)$ | $W/(sr \cdot m^2)$ |
| **photometric quantities** | | |
| Quantity | Definition | Unit |
| quantity of light | $Q_v$ | lm · s |
| luminous flux | $\Phi_v = dQ_v/dt$ | lm |
| luminous intensity | $I_v = d\Phi_v/d\omega$ | lm/sr (cd) |
| illuminance | $E_v = d\Phi_v/dS$ | $lm/m^2$ (lx) |
| luminous exitance | $M_v = d\Phi_v/dS$ | $lm/m^2$ |
| luminance | $L_v = d\Phi_v/(dS \cdot \cos\theta \cdot d\omega)$ | $lm/(sr \cdot m^2)$ |

($t$: time, $\omega$: solid angle, $S$: area, and $\theta$: the angle between the normal of the plane element and the direction of observation)

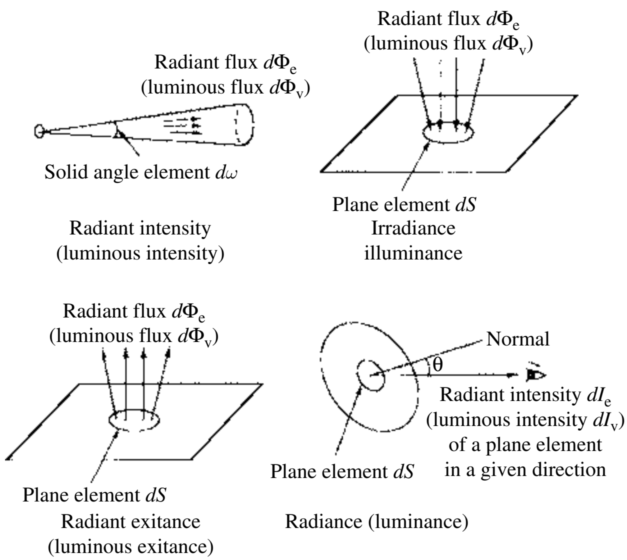


**Figure 1.19** Definition of radiometric and photometric quantities (Noguchi 1987). Reproduced by permission of Ohm Sha

mol) for amount of substance, and the candela (abbreviated cd) for luminous intensity are added to the four basic units of the MKSA system — meter (m), kilogram (kg), second (s), and ampere (A) — to provide the seven base units of the SI. In addition to the base units, the radian (rad) for planar angle and the steradian (sr) for solid angle are included as derived units.

For the radiometric quantities listed in Table 1.2, the joule (J) is the unit of radiant energy and the watt (W) is the unit of radiant flux. For the photometric quantities, the base unit is the candela (cd). Units derived from the candela include the lumen (lm) for luminous flux and the lux (lx) for illuminance. As is shown in Table 1.2, the lumen is defined as the luminous flux emitted into a solid angle of one steradian by a point source having a uniform luminous intensity of one candela. The lux is defined as the illuminance produced by a luminous flux of one lumen uniformly distributed over an area of one square meter.

Over time, the standard for the candela has evolved and increased in precision, as is the case with the other base units such as the meter. The unit of length was determined, in earlier times, in accordance with the length of a part of the human body such as one foot. Later a more universal unit, the 'meter' was introduced and was originally defined to be one ten-millionth of the quadrant from the equator to the North Pole through Dunkirk. However, developments in the precision of measurements of optical wavelengths led to the adoption of a definition in terms of the wavelength of a spectrum line of krypton. Nowadays, the meter is defined as the length of the path traveled by light in vacuum during a time interval of 1/299792458 s.

Similarly, in earlier times, the standard of luminous intensity was provided by burning a specified candle made of whale oil. This is the origin of the unit name 'candle' or 'candela'. There was also an era when a more stable pentane lamp was used in place of the whale-oil candle to provide the standard. Subsequently, further precise standards were established. In 1948, the candela was defined as the 'luminous intensity in the direction perpendicular to a surface of area $1/60\,\mathrm{cm}^2$ of a black body at the temperature of the freezing point of platinum (2042 K) under a pressure of 101325 Pa (one atmosphere)'. Figure 1.20 illustrates the mechanism of a standard which realized the candela according to the above definition (Wyszecki and Stiles 1982). In 1979, the standard was revised, and the candela was re-defined as the 'luminous intensity in a given direction of a source that emits monochromatic radiation of $540 \times 10^{12}$ Hz and that has a radiant intensity in that direction of

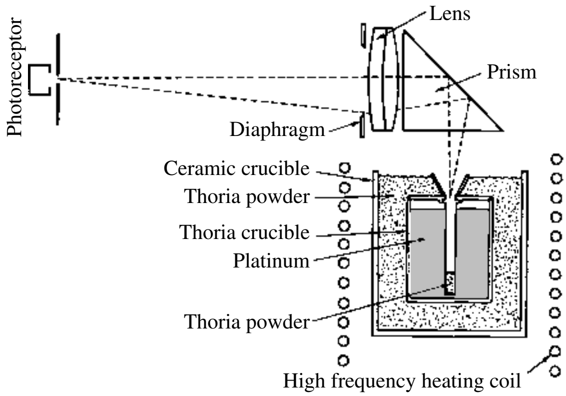


**Figure 1.20** Standard for luminous intensity (Wyszecki and Stiles 1982). Reproduced by permission of John Wiley & Sons, Ltd

1/683 watt (W) per steradian'. ($540 \times 10^{12}$ Hz is equivalent to a wavelength of about 555 nm, which is where $V(\lambda)$ is unity.)

As described above, the standard for photometric units started from a concrete concept such as a candle, but is now defined precisely in terms of the radiometric quantity of monochromatic light. However, it should be re-emphasized that photometric quantities are psychophysical. At frequencies other than $540 \times 10^{12}$ Hz, they must be linked to radiometric quantities by a convention (the $V(\lambda)$ curve) that represents the response of the human visual system. For practical implementation in the industrial field, a standard luminous intensity is provided by a standard photometric light source calibrated by the national metrology institute in each nation because of the difficulty in realizing the above definition directly.

Among the photometric quantities, illuminance and luminance are widely used in the industrial field. The simplest system of units for these and other photometric quantities is the SI. In this system, illuminance is measured in $lm/m^2$ (usually called lux, abbreviated lx), and luminance is measured in $cd/m^2$ (sometimes called nits, abbreviated nt). However, various other units are often used depending on needs and on history. The conversion factors for some of the most common of these units are listed in Table 1.3 (Ohyama 1969). However there are still other units that are not given in the table. The 'equivalent' units shown in Table 1.3 are introduced to relate illuminance with luminance. For example, the luminance of a perfect diffusing plane under an illuminance of 1 phot is 1 equivalent phot. In the measurement of sensitivity in photography, the illuminance unit meter-candle (m cd) is used conventionally. One meter-candle is equivalent to one lux.

**Table 1.3** Units for illuminance and luminance (Ohyama 1969). Reproduced by permission of Seishin Shobo

(a) Illuminance

| | Lux (lx) | Phot (ph) | Milliphot (mph) | Footcandle (fcd) |
|---|---|---|---|---|
| 1 Lux ($lm/m^2$) | 1 | 0.0001 | 0.1 | 0.0929 |
| 1 Phot | 1000 | 1 | 1000 | 929 |
| 1 Milliphot | 10 | 0.001 | 1 | 0.929 |
| 1 Footcandle | 10.76 | 0.001076 | 1.076 | 1 |

(b) Luminance

| | Nit (nt) | Stilb (sb) | Apostilb (asb) | Lambert (L) | Millilambert (mL) | Footlambert (fL) |
|---|---|---|---|---|---|---|
| 1 Nit ($cd/m^2$) | 1 | 0.0001 | 3.142 | 0.0003142 | 0.3142 | 0.2919 |
| 1 Stilb | 10000 | 1 | 31420 | 3.142 | 3142 | 2919 |
| 1 Apostilb | 0.3183 | 0.00003183 | 1 | 0.0001 | 0.1 | 0.0929 |
| 1 Lambert | 3183 | 0.3183 | 10000 | 1 | 1000 | 929 |
| 1 Millilambert | 3.183 | 0.0003183 | 10 | 0.001 | 1 | 0.929 |
| 1 Footlambert | 3.426 | 0.0003426 | 10.76 | 0.001076 | 1.076 | 1 |
| 1 Candela/$ft^2$ | 10.76 | 0.001076 | 33.82 | 0.003382 | 3.382 | 3.142 |
| 1 Candela/$in^2$ | 1550 | 0.155 | 4869 | 0.4869 | 486.9 | 452.4 |

Notes:
(1) 1 foot (ft) equals 0.3048 m
(2) The coefficient 3.142 in the table is the value of $\pi$
(3) In luminance, the following equivalent units are sometimes used:

1 equivalent phot = 1 L
1 equivalent lux = 1 blondel = 1 asb
1 equivalent fcd = 1 fL

In another special case, retinal illuminance is used for expressing illuminance on the retinal surface by taking into consideration the diameter of the pupil of the eye. Retinal illuminance is used principally in the field of physiological optics and has the unit troland (td). When the eye views a surface of uniform luminance, the number of trolands is equal to the product of the area in $mm^2$ of the limiting pupil (natural or artificial) and the luminance of the surface in $cd/m^2$. Thus, the retinal illuminance $E_r$ when a surface having a luminance of $L$ $cd/m^2$ is viewed with a pupil of $r$ mm diameter can be written as

$$E_r = \pi r^2 L \tag{1.14}$$

Taking into consideration that 1 mL = 10 $/\pi$ $cd/m^2$, a surface having a luminance of 1 mL provides a retinal illuminance of 10 td through a pupil having a radius of 1 mm.

## 1.7 CALCULATION AND MEASUREMENT OF PHOTOMETRIC QUANTITIES

As described above, radiometric quantities can be converted into photometric quantities and vice versa by the equation (photometric quantity) = $K(\lambda)\times$ (radiometric quantity). One lumen corresponds by definition to 1/683 W of a monochromatic light of $\lambda_m$ = 555 nm, which is the wavelength where $V(\lambda)$ has its maximum value of unity. Thus, $K_m$, the maximum luminous efficacy of radiation for photopic vision is 683 lm/W. In the case of scotopic vision, on the other hand, $V'(\lambda)$ = 0.40175 at $\lambda$ = 555 nm as can be obtained by interpolating the values in Table 1.1. For monochromatic light having a radiant intensity of 1/683 W at $\lambda$ = 555 nm to yield a luminous flux of 1 lm in scotopic vision, $K'_m$, the maximum luminous efficacy for scotopic vision must be set to 683/0.40175 = 1700 lm/W. This maximum occurs at $\lambda'_m$ = 507 nm. Thus, Equations 1.2a and 1.2b can be written explicitly as

$$K(\lambda) = K_m V(\lambda) = 638 V(\lambda) \tag{1.15a}$$

$$K'(\lambda) = K'_m V'(\lambda) = 1700 V'(\lambda) \tag{1.15b}$$

Figure 1.21 shows the comparison between the spectral luminous efficacies $K(\lambda)$ and $K'(\lambda)$ obtained through Figure 1.10 and Equations 1.15a and 1.15b. In general, photometric quantities for photopic vision are used more frequently than those for scotopic

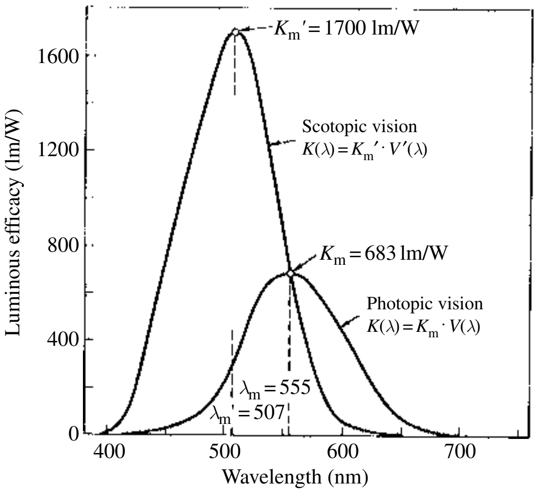


**Figure 1.21** Spectral luminous efficacy $K(\lambda)$ and $K'(\lambda)$ for photopic and scotopic vision respectively

vision. Accordingly, the following explanation is given using the symbols for photopic vision.

In the above description, only monochromatic light is treated. However, in daily life, monochromatic light is rare, and, in general, polychromatic light, such as sunlight or light emitted from an artificial lamp with a continuous or partly continuous polychromatic spectrum, is used. By finely dividing a polychromatic light as is shown in Figure 1.22, it can be considered to consist of a number of near-monochromatic components.

The quotient $X_\lambda$ of a radiometric or photometric quantity $\mathrm{d}X$ in an elementary range of wavelength $\mathrm{d}\lambda$ at a wavelength, $\lambda$ by that range, i.e., $\mathrm{d}X/\mathrm{d}\lambda$, is called the spectral concentration, and the spectral concentration expressed as a function of wavelength is called the spectral distribution and denoted by $X_\lambda(\lambda)$. In particular, the spectral concentration of radiant flux, $\Phi_{e,\lambda} = \mathrm{d}\Phi_e/\mathrm{d}\lambda$ is called spectral radiant flux, and the spectral distribution of radiant flux is denoted by $\Phi_{e,\lambda}(\lambda)$. By finely dividing the spectral distribution $\Phi_{e,\lambda}(\lambda)$ into $n$ sections of wavelength width $\Delta\lambda$, the $i$th radiant flux (with a wavelength of $\lambda_i$) can be written as $\Phi_{e,\lambda}(\lambda_i)\Delta\lambda$. Thus, the total radiant flux $\Phi_e$ for polychromatic light with a spectral distribution of $\Phi_{e,\lambda}(\lambda)$ can be found by summation as follows

$$\Phi_e = \sum_{i=1}^{n} \Phi_{e,\lambda}(\lambda_i)\Delta\lambda \tag{1.16}$$

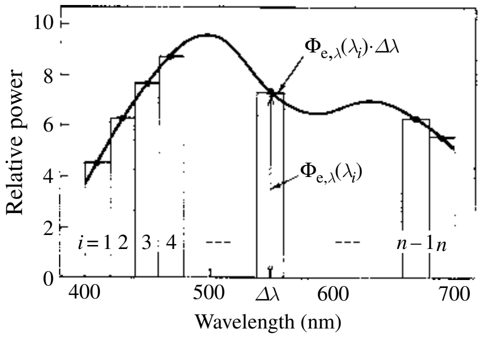


**Figure 1.22** Approximation of spectral distribution $\Phi_e(\lambda)$ by narrow-band near-monochromatic components

where the summation is over the entire wavelength range of the distribution $\Phi_{e,\lambda}(\lambda)$.

In the case of a physical quantity such as radiant flux, the additivity implied in Equation 1.16 holds true in a strict sense. However, in the case of luminous quantities, it is difficult to predetermine theoretically whether it is correct to apply additivity. Experimentally, however, it can be confirmed that additivity holds for luminous flux within a certain precision. That is, two radiant fluxes $\Phi_{e,\lambda}(\lambda_1)\Delta\lambda$ and $\Phi_{e,\lambda}(\lambda_2)\Delta\lambda$ in narrow bands of wavelength $\Delta\lambda$, at wavelengths $\lambda_1$ and $\lambda_2$, can be safely converted into luminous fluxes $\Phi_{v,\lambda}(\lambda_1)\Delta\lambda$ and $\Phi_{v,\lambda}(\lambda_2)\Delta\lambda$, and added together. According to Equations 1.1 and 1.2, the individual luminous fluxes are written as

$$\Phi_{v,\lambda}(\lambda_1)\Delta\lambda = K_m\Phi_{e,\lambda}(\lambda_1)V(\lambda_1)\Delta\lambda \tag{1.17a}$$

$$\Phi_{v,\lambda}(\lambda_2)\Delta\lambda = K_m\Phi_{e,\lambda}(\lambda_2)V(\lambda_2)\Delta\lambda \tag{1.17b}$$

The fact that additivity is valid means that, when the two radiant fluxes are viewed together, the total luminous flux is

$$\begin{aligned}\Phi_v &= \Phi_{v,\lambda}(\lambda_1)\Delta\lambda + \Phi_{v,\lambda}(\lambda_2)\Delta\lambda\\ &= K_m\{\Phi_{e,\lambda}(\lambda_1)V(\lambda_1) + \Phi_{e,\lambda}(\lambda_2)V(\lambda_2)\}\Delta\lambda\end{aligned} \tag{1.18}$$

This is known as Abney's Law. By extending Equation 1.18 to $n$ radiant fluxes, the luminous flux of polychromatic light can be obtained as

$$\Phi_v = K_m\{\Phi_{e,\lambda}(\lambda_1)V(\lambda_1) + \Phi_{e,\lambda}(\lambda_2)V(\lambda_2) + \cdots + \Phi_{e,\lambda}(\lambda_n)V(\lambda_n)\}\Delta\lambda \tag{1.19}$$

By taking limits $\Delta\lambda \rightarrow 0$ and $n \rightarrow \infty$, the following equation is obtained

$$\Phi_v = K_m \int_{vis} \Phi_{e,\lambda}(\lambda) V(\lambda) d\lambda \tag{1.20}$$

where the integral is taken over the entire visible wavelength region.

In practice, Abney's law of additivity holds very well when the luminous quantities are used to predict psychophysical phenomena such as the perception of fine detail, reaction times and heterochromatic flicker photometry. However, it holds less well for predicting the relative perceived brightness of stimuli of different colors. (see Note 1.2)

Photometric methods can be classified into two types, visual and physical. Visual photometry is simple, but has the following disadvantages:

1. The spectral luminous efficiency $V(\lambda)$ for the CIE standard observer is an average value, and hence it does not agree exactly with the spectral luminous efficiency for most individual observers.
2. It is extremely difficult to compare quantitatively two radiations differing in brightness. The only viable method is the null method in which one of the two radiations is adjusted until the two are equal to each other.
3. Even with the null method, it is difficult to compare the brightness of two radiations that differ in color (e.g., red and green), and thus the results have a large uncertainty.
4. The results are influenced by the degree of fatigue of the observer and by the experimental environment.

Disadvantages 2 can be overcome to a certain degree by using an optical attenuator (for example, an optical bench). However, an optical bench requires an extremely large space (typically an area of about $3 \times 14$ m for 10 m bench). Disadvantage 3 can be overcome by using the flicker method but, although the flicker method enables results with higher precision than the direct comparison method, it requires special measuring conditions, and the results are still of low reliability compared with other physical measurements. Accordingly, in the derivation of the $V(\lambda)$ curve, an average value had to be determined by repeating the measurements a number of times.

In the light of these drawbacks of visual photometry, physical photometry is usually used in practice. Physical photometry can be performed in the following two ways.

1. Measurement of the corresponding spectral radiant quantity, usually by measuring a ratio at successive wavelengths with the value obtained with a standard photometric light source, followed by calculation of the photometric quantity using Equation 1.20.
2. Direct measurement of the photometric quantity by using a photoelectric detector such as a photoelectric cell, a photoelectric tube, or a photomultiplier in combination with a suitable optical filter to provide a photodetector having a relative spectral responsivity approximately equal to $V(\lambda)$. Such a detector is known as a $V(\lambda)$ photodetector.

The spectral method 1 comprises a comparative measurement for each wavelength. Accordingly, the spectral concentration of the radiometric quantity as well as the photometric quantity can be measured with high precision, and the relative spectral responsivity of the photoreceptor need not be the same as $V(\lambda)$. An instrument for measuring photometric quantities by means of a $V(\lambda)$ photoreceptor (method 2) is called a photometer. The key to this method is the accuracy of the approximation to $V(\lambda)$ by the relative spectral responsivity of the photometer. Photometers capable of approximating $V(\lambda)$ with a sufficiently high precision for most applications have been developed and are commercially available. An example is shown in Figure 1.23. Such photometers are useful for simple photometry.

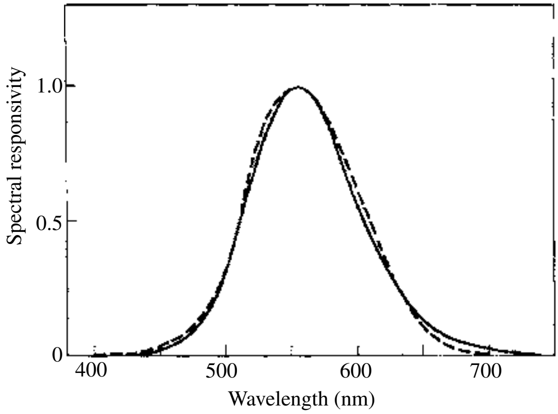


**Figure 1.23** Spectral responsivity (solid line) of a photometer and $V(\lambda)$ (broken line)

## 1.8 RELATIONS BETWEEN PHOTOMETRIC QUANTITIES

The photometric quantities described in Section 1.5 have different dimensions and hence cannot, in general, be converted one into another. However, if the quantities are obtained under a light source with certain well-defined characteristics, some quantities can be converted to others by simple relations. For example, the values of photometric quantities obtained under a perfect diffusing planar light source or a point light source can be converted in the following manner.

### 1.8.1 Perfect Diffusing Planar Light Source

From Equation 1.6, the luminance, $L_n$ normal to a plane element of area d$S$ on a surface, and the luminance, $L_\theta$ in a direction making an angle of θ with respect to the normal, are

$$L_n = dI_n/\mathrm{d}S \qquad \text{and} \qquad L_\theta = \mathrm{d}I_\theta/(\mathrm{d}S\cos\theta) \tag{1.21}$$

where d$I_n$ and d$I_\theta$ are the luminous intensities normal to the plane element and in a direction making an angle, θ with respect to the normal, respectively. In the case of a perfect diffusing planar light source, the luminance is the same for all directions so

$$L_\theta = L_n \tag{1.22a}$$

$$\mathrm{d}I_\theta = \mathrm{d}I_n\cos\theta \tag{1.22b}$$

This is known as Lambert's cosine law. If the plane surface satisfying Lambert's cosine law is a reflecting surface instead of a light source, the reflecting surface is called a uniform reflecting diffuser. In particular, a uniform reflecting diffuser is called a perfect reflecting diffuser if it has a reflectance $\rho = 1.0$. In daily life, a surface of gypsum or white chalk is close to a perfect reflecting diffuser. For a perfect reflecting diffuser, the luminous exitance $M$ (lm/m$^2$) and the illuminance $E$ (lm/m$^2$) can be related to luminance $L$ (cd/m$^2$) by the following equations (see Note 1.1)

$$E = \pi L \tag{1.23a}$$

$$M = \pi L \tag{1.23b}$$

Thus, if the illuminance $E$ on a perfect reflecting diffuser is known, the luminance $L$ of the light reflected is

$$L = E/\pi \tag{1.24}$$

In the case of a uniform reflecting diffuser with $\rho < 1.0$, the luminous exitance $M = \rho E$, where $E$ is the illuminance, and hence

$$L = \rho E/\pi \tag{1.25}$$

Table 1.3 shows that $1\,\text{mL} = 3.183\,\text{cd/m}^2$. Accordingly, a luminance of 1 mL for a perfect reflecting diffuser corresponds to an illuminance of 10 lx since, from Equation 1.23a, E = $\pi L = 3.142 \times 3.183 = 10$.

### 1.8.2 Uniform Point Light Source

The total luminous flux $\Phi$ of a point light source having the same luminous intensity $I$ in all directions can be obtained from Equation 1.5 as

$$\Phi = 4\pi I \tag{1.26}$$

This relation is based on the fact that the total solid angle with respect to a point light source is $4\pi$ sr.

### 1.8.3 Correlation with Brightness

Illuminance, luminance, and retinal illuminance can be roughly related with each other in a typical situation as shown in Figure 1.24. The luminance is obtained according to Equation 1.25 by taking $\rho = 0.5$. The conversion from luminance to retinal illuminance assumes a pupil size of $15\,\text{mm}^2$ at a luminance of $1\,\text{cd/m}^2$ and $44\,\text{mm}^2$ at a luminance of $0.001\,\text{cd/m}^2$. These values represent typical values of the eye's natural pupil at these luminances.

The question arises which photometric quantity correlates best with the sensation of 'brightness'. For the sake of simplicity, let us consider the case of a 100 W incandescent lamp. The efficiency of an incandescent lamp is about 15 lm/W. Accordingly, a luminous flux, $\Phi$ of $100 \times 15 = 1500$ lm is emitted from the lamp. Thus, from Equation 1.26, the luminous intensity $I$ is $I = \Phi/4\pi = 119$ lm/sr. The illuminance $E$ at a distance $D$ of 40 cm from the incandescent lamp

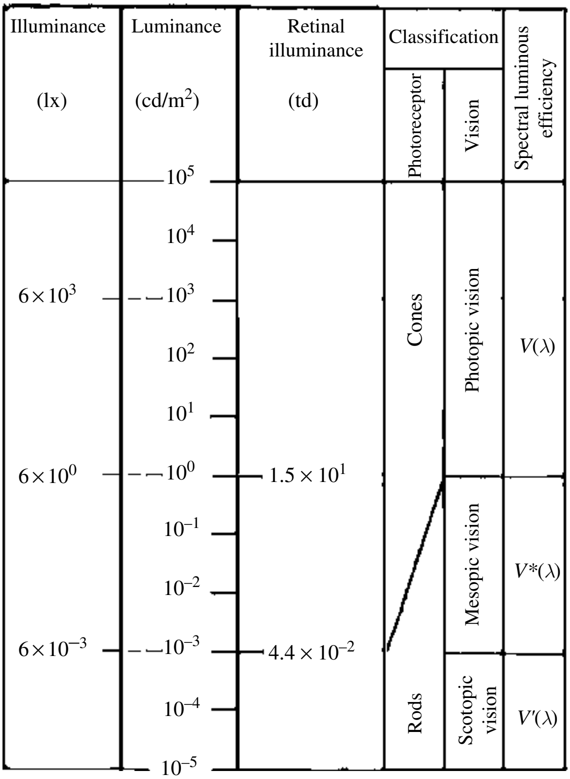


**Figure 1.24** Illuminance and corresponding luminance (for $\rho = 0.5$)

can be obtained from Equation 1.11 as $E = I/D^2 = 119/(0.4)^2 = 744$ lx when $\theta_2 = 0$. Thus, a 100 W lamp can supply an illuminance suitable for reading a book, but the question remains whether or not illuminance is suitable as a quantity for representing brightness as sensed by the human eye.

Let us consider the case further. Although the entire page of the book is exposed to light of the same illuminance, the white portion of the paper is brighter, and the printed black portion darker. As an example, let the reflectance, $\rho$ of the white portion be 0.9, and that of the black portion 0.03. Because the luminous exitance $M$ is found by $M = \rho E$, the luminous exitance of the white portion is $M = 0.9 \times 744 = 670\,\text{lm/m}^2$, and that of the black portion is $M = 0.03 \times 744 = 22\,\text{lm/m}^2$. These values are in good agreement with the brightness sensed by the eye. However, the light is reflected from the page in all directions, and only a small portion of it enters the eye. If the reflectance varies with the direction of viewing, as it does for a glossy ink for example, exitance would not be a good indicator of brightness because it measures the total reflected flux. The quantity that shows the variation with angle is luminance. Hence luminance is the photometric quantity corresponding best to the brightness perceived by the eye (see Note 1.2). In our example, the luminance $L$ is obtained as $L = 670/\pi = 213\,\text{cd/m}^2$ for the white portion, and as $L = 22/\pi = 7\,\text{cd/m}^2$ for the black portion.

## NOTE 1.1 LUMINOUS EXITANCE, ILLUMINANCE, AND LUMINANCE OF A PERFECT DIFFUSING PLANE LIGHT SOURCE

Referring to Figure 1.25, a plane element of area $dS_1$ is considered at point A on a perfect diffusing planar light source with a luminance $L$. The luminous exitance of $dS_1$ can be derived as follows. A cone with a height of $D_0$ and a base plane of radius $R$ is drawn in the direction normal to $dS_1$, and a plane element of area $dS_2$ is considered on this base plane. The position of $dS_2$ is defined by polar coordinates $r$ and $\varphi$ in the base plane. Then, the solid angle $d\omega$ subtended by $dS_2$ with respect to A is

$$d\omega = dS_2 \cos\theta / D^2 \tag{1.27}$$

In the case of a perfect diffusing planar light source, the luminance is the same for all directions. Accordingly, the following equation can be derived from Equation 1.7

$$d\Phi = L\, dS_1 \cos\theta d\omega = L\, dS_1 \cos^2\theta dS_2 / D^2 \tag{1.28}$$

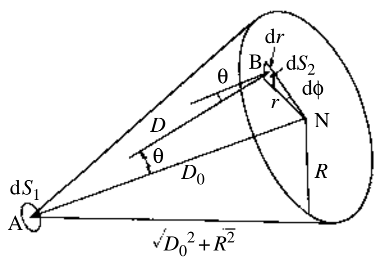


**Figure 1.25** Luminous exitance for a perfect diffusing planar light source

By simple geometry, it can be seen from Figure 1.25 that the following equations hold

$$D^2 = D_0^2 + r^2 \tag{1.29a}$$

$$\mathrm{d}S_2 = r\mathrm{d}r\mathrm{d}\varphi \tag{1.29b}$$

$$D_0 = D\cos\theta \tag{1.29c}$$

Thus, the following relation can be obtained from Equation 1.28 by substitution

$$\mathrm{d}\Phi = L\,\mathrm{d}\,S_1\,D_0^2\,r\,\mathrm{d}r\,\mathrm{d}\varphi/(D_0^2 + r^2)^2 \tag{1.30}$$

Thus, the luminous flux, Φ inside the cone is

$$\begin{aligned}\Phi &= \int_0^R \int_0^{2\pi} L\,\mathrm{d}S_1\,D_0^2\,r\,\mathrm{d}\varphi\,\mathrm{d}r/(D_0^2 + r^2)^2 \\ &= 2\pi L\,\mathrm{d}S_1 D_0 \int_0^R r\,\mathrm{d}r/(D_0^2 + r^2)^2 \\ &= \pi L \mathrm{d}S_1 R^2/(D_0^2 + R^2)\end{aligned} \tag{1.31}$$

By taking $R \rightarrow \infty$

$$\Phi = \pi L\,\mathrm{d}S_1 \tag{1.32}$$

The luminous exitance, $M$ can then be obtained as

$$M = \Phi/\mathrm{d}S_1 = \pi L \tag{1.33}$$

to yield the relation between luminous exitance and luminance for a perfect diffusing planar light source.

Next, referring to Figure 1.26, the illuminance is considered at point B at a distance $D_0$ from the center of a disk of radius $R$ on a perfect diffusing planar light source of luminance $L$. A plane

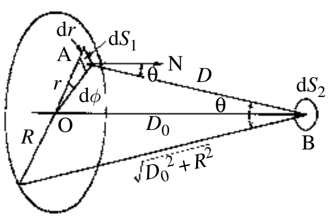


**Figure 1.26** Illuminance for a perfect diffusing planar light source

element of area $dS_1$ is considered at point A (defined by polar coordinates r and $\phi$) on the light source. Then, by substituting $\theta_1 = \theta_2 = \theta$ into Equation 1.13, the following expression can be obtained for the illuminance $dE$ produced by $dS_1$.

$$dE = L\,dS_1 \cos^2\theta / D^2 \tag{1.34}$$

As in the previous example, the following geometrical relationships exist

$$D^2 = D_0^2 + r^2 \tag{1.35a}$$

$$dS_1 = r\,dr\,d\varphi \tag{1.35b}$$

$$D_0 = D\cos\theta \tag{1.35c}$$

Thus, by substitution, the following relation can be obtained from Equation 1.34

$$dE = L\,D_0^2\,r\,dr\,d\varphi / (D_0^2 + r^2)^2 \tag{1.36}$$

Then, the luminance for the whole area of the plane light source is

$$\begin{aligned} E &= \int_0^R \int_0^{2\pi} L\,D_0^2\,r\,d\varphi\,dr / (D_0^2 + r^2)^2 \\ &= 2\pi L\,D_0^2 \int_0^R r\,dr / (D_0^2 + r^2)^2 \\ &= \pi L\,R^2 / (D_0^2 + R^2) \end{aligned} \tag{1.37}$$

By taking $R \to \infty$

$$E = \pi L \tag{1.38}$$

It can be seen from Equations 1.33 and 1.38 that the illuminance $E$ produced by an infinite planar light source is equal to the luminous exitance, $M$ of the source. This indicates that the maximum illuminance obtained by infinitely increasing the area of a perfect diffusing planar light source depends only on its luminous exitance.

## NOTE 1.2 LUMINANCE AND BRIGHTNESS

Among the photometric quantities, the one that corresponds best to the sensation of brightness is luminance. Thus, lights having the same luminance might be expected to have the same brightness, irrespective of color. If the luminances of a red and a blue light were the same, for example, the brightness of the red light might be expected to appear the same as that of the blue light. However, in fact, the brightnesses are not the same. More specifically, if a white light is taken as the reference light and a red light is taken as the test light, the red light appears brighter than the white light when the luminance is the same for both. That is, the ratio of brightness to luminance $B/L$, where $L$ is the luminance and $B$ is the brightness, is found to vary with the color. This phenomenon is known as the Helmholtz–Kohlrausch effect. Possible reasons for this variation of $B/L$ are:

1. The function $V(\lambda)$ is not correct.
2. Abney's law expressed by Equation 1.18 (the additivity of luminous fluxes) does not hold in a strict sense.

With respect to the function $V(\lambda)$, it has been shown by numerous experiments that the function used at present is too low in the short-wavelength region. Hence, Judd proposed to revise $V(\lambda)$ as shown in Figure 1.27. Judd's modified function was adopted by the CIE in 1988 as a supplemental spectral luminous efficiency $V_{\mathrm{M}}(\lambda)$ to be used in those conditions where luminance measurements

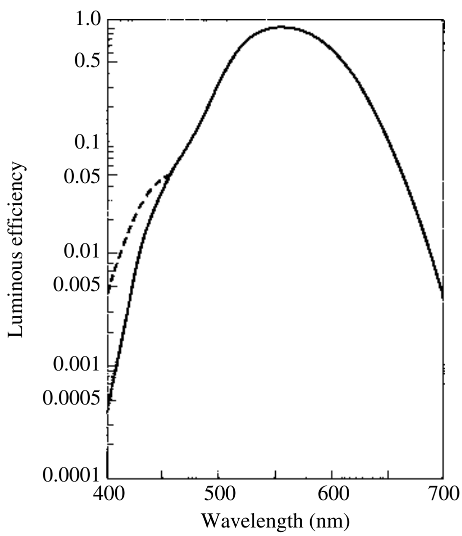


**Figure 1.27** $V(\lambda)$ of CIE 1924 (solid line) and $V_M(\lambda)$ of CIE 1988 (broken lines) (CIE 1990)

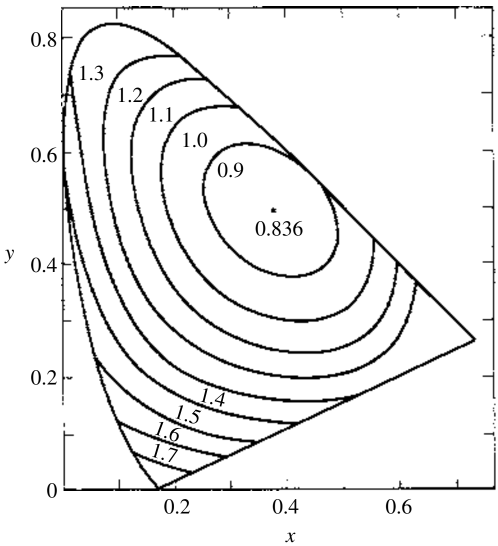


**Figure 1.28** Correction factor $10^f$ for brightness as a function of chromaticity

of short wavelengths consistent with the color vision of normal observers is desired (CIE 1990).

However, even if the modified $V_M(\lambda)$ or any of the other proposed $V(\lambda)$ functions are used, a constant $B/L$ still cannot be obtained. Moreover, a number of experimental results indicate that Abney's law does not hold in a strict sense for brightness judgments. Accordingly, the reason for the variation of $B/L$ is that both effects listed above occur at the same time.

The evaluation of brightness is a particularly important practical problem. Thus, until an analytical solution is found, the CIE has recommended that, to compare the brightness of two objects of different color, one with a luminance $L_1$ and another with a luminance $L_2$, the following experimentally deduced equation be used (Kaiser 1986):

$$\log(L_1) + f_1 = \log(L_2) + f_2 \tag{1.39}$$

where $f$ is a correction factor determined by

$$f = 0.256 - 0.184\,y - 2.527\,xy + 4.656\,x^3y + 4.657\,xy^4 \tag{1.40}$$

and $x$, $y$ are chromaticity coordinates (see Chapter 3). If the equation holds true, it can be concluded that $L_1$ and $L_2$ are of the same brightness. Figure 1.28 shows the correction factor $10^f$ in graphical form.