# PART One

# Risk



Risk is about events that we can't foresee. Is there nevertheless some underlying connection between the frequency of past events and the frequency of future events? Between the magnitude of past risks and the magnitude of future risks? Can connections between past and future risks be quantified in some useful way that is not itself risky?

Paradoxically, the risks that are hardest to quantify are the risks of least concern to the institutional investor. The key is the tendency for certain kinds of risks to occur together—i.e., the degree of correlation between the risks. Although uncorrelated risks are the easiest for an institutional investor to diversify, so-called "market" risks, which can't be diversified away, are the easiest to quantify.

J.L.T.

CHAPTER 1

# Using Portfolio Composition to Estimate Risk

In recent years a number of financial scholars have commented on the marked degree of co-movement in the prices of securities. Statistical techniques have been applied to measuring the character and degree of co-movement by Donald Farrar, Hester and Feeney, and Benjamin King. Perhaps the best known model of stock prices that recognizes and incorporates the co-movement phenomenon is that of William Sharpe. In Sharpe's model fluctuations in the price of a particular common stock have two causes: (1) fluctuations in the general market level and (2) fluctuations unique to the stock in question. More complicated models than Sharpe's have been proposed and the Sharpe model has occasionally been criticized as being too simple to fit reality (see for example Benjamin King's discussion[1]). Nevertheless, its simplicity gives it great appeal.[2]

We are not the first to apply simple financial models to practical problems involving risk measurement. Marshall Blume tested the applicability of the Sharpe model to the problems of predicting the risk character of simulated rather than actual portfolios.[3] James Fanning, now of Rockefeller Brothers, and Marc Steglitz of Bankers Trust have measured risk in actual common stocks defined in terms of a related, but different, model and applied the results to estimating the risk character of actual portfolios containing these stocks. Although the present paper has benefited substantially from the work of Fanning and Steglitz, in terms of model and approach, we are much closer to Blume than Fanning and Steglitz.

Sometimes it is possible to identify stock price changes with particular news events. Even though the events that cause price changes sometimes seem to be unique, it is nevertheless useful to think of the events that affect prices as drawn at random from a large population, some of which can cause large price changes and some small, many of which have a high degree of uniqueness or individuality, but that, taken as an entire population, have a character that demonstrates some continuity



This chapter was coauthored with William W. Priest, Jr., Lawrence Fisher, and Catherine A. Higgins.

We are grateful to Marvin Lipson for programming the computer runs, the results of which are reported here. The data for the study were taken from "Price Relative" tapes supplied to us by the Center for Research in Security Prices, the Graduate School of Business, the University of Chicago.

over time. Labor unions will continue to strike; countries will continue to declare war or to make undeclared war; the Fed will continue by turns to tighten up and loosen the money supply; and soforth. Some of these events are felt throughout the economy and have their impact to a greater or lesser degree on the prices of most common stocks. The impact of other events is specific to at most a few companies or industries.

The Sharpe model specifies that price fluctuations in a particular common stock will be the sum of fluctuations due to fluctuations in the market index and fluctuations unique to the stock in question.[4] The risk character of the stock is completely specified under the assumptions of the Sharpe model by specifying two parameters: The first is sensitivity of the stock to market fluctuations. It is common knowledge, however, that price fluctuations in individual common stocks are not completely explained by a market index. We call the portion of price changes left unexplained by a market index the *residual* price changes. The second risk parameter in the Sharpe model is a number that expresses the average magnitude of the residual fluctuations. In Sharpe's model, residual fluctuations are assumed to be independent from one security to another.

Some companies are more sensitive to the impact of events affecting the market index than others. Rapidly growing companies, companies that manufacture capital goods, companies with high fixed costs, and highly levered companies all tend to be more sensitive than companies for which these factors are absent. Companies for which several of these factors are present simultaneously are likely to be particularly sensitive.

The second parameter in the model—the measure of the magnitude of residual fluctuations—tends to be larger for companies in which technological changes in products or processes are taking place very rapidly. It also tends to be larger for one-product companies, companies for which style is an important factor and for companies whose fortunes depend on a single executive. Widely diversified companies and companies with a balanced management team will tend to demonstrate less residual variability than others. A high level of fixed costs or a highly levered capital structure will, of course, amplify specific risk in the same way that it amplifies market risk.

Exhibit 1.1 demonstrates the meaning of the risk parameters for individual stocks in graphical terms. The horizontal axis measures the change in a market index (Fisher's Combination Investment Performance Index[5]). The vertical axis measures the change in the value of the security in question. Both are measured as the ratio of value at the end of a month (including intervening dividends) to value at the beginning. A straight line has been fitted to the data points in Exhibit 1.1. The slope of the line is a measure of the sensitivity of the value of the security to fluctuations in the market index. The spread of data points around the line of best fit is a measure of residual variability.

The important distinction between market variability and residual variability in the individual security is that they affect portfolio returns in different ways. Sensitivity of a portfolio to variations in the market index is the average of the sensitivities of the individual securities held, weighted by the amounts. Residual variability in individual securities, on the other hand, tends to combine in such a way that it looms relatively less important in a portfolio (in comparison with market variability) than in the securities which comprise it. The spread is measured in terms of a number statisticians call the *residual variance*, which in terms of the present application is

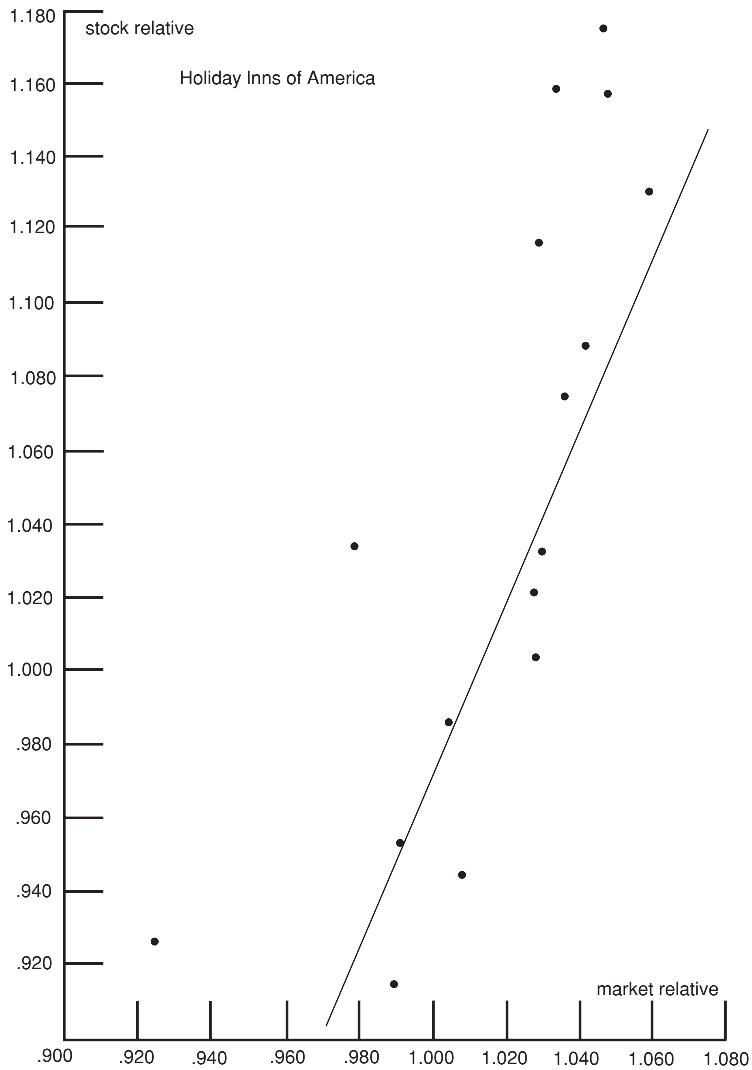


**EXHIBIT 1.1** Price Relatives for a Common Stock Versus Price Relatives for "The Market"

an average of the squares of the (residual) fluctuations over the time period covered by the sample. Under the assumptions of the Sharpe model, there is a very simple rule for determining how specific risk in individual securities combines to determine specific risk in the portfolio: The residual variance for the portfolio can be expressed in terms of residual variance $\sigma_i^2$ for the individual stocks. Letting $x_i$ equal the number of shares of the ith stock held in the portfolio and assuming that the relevant measures are expressed on a per-share basis, we have

$$\text{Residual variance} = \Sigma x \sigma_i^2 \tag{1.1}$$

In like fashion we can define *market variance* as the average over time of (squared) fluctuations in portfolio value due to market fluctuations. Letting regression-slope coefficients $B_i$ measure sensitivity of prices of individual stocks to fluctuations in the general market, and a variance $\sigma_m^2$ describe the variability of the general market, we have

$$\text{Market variance} = [\Sigma x_i B_i]^2 {\sigma_m}^2 \tag{1.2}$$

Finally, expressions (1) and (2) can be combined to estimate total portfolio variance $\sigma^2$. We have

$$\sigma^2 = [\Sigma x_i B_i]^2 {\sigma_m}^2 + \Sigma {x_i}^2 {\sigma_i}^2 \tag{1.3}$$

Equation 1.3 will hold only approximately since the residual variances for individual stocks are not strictly independent. As previously noted, a number of writers have challenged the Sharpe model on the assumptions underlying the way specific risks in individual securities combine in a portfolio. Nevertheless Equation 1.3 is probably

**EXHIBIT 1.2** Exponentially-Smoothed Estimates of Market Volatility ($B$) and Residual Variance ($\sigma^2$)

| Holiday Inns of America, Inc. | | |
|---|---|---|
| **Month** | ***B*** | $\sigma^2$ |
| 3/64 | 0.663 | 0.00643 |
| 4/64 | 4.604 | 0.01051 |
| 5/64 | 4.307 | 0.00880 |
| 6/64 | 4.300 | 0.00754 |
| 7/64 | 4.314 | 0.00656 |
| 8/64 | 3.797 | 0.00604 |
| 9/64 | 3.132 | 0.00571 |
| 10/64 | 2.725 | 0.00528 |
| 11/64 | 3.025 | 0.00670 |
| 12/64 | 2.812 | 0.00635 |
| 1/65 | 2.923 | 0.00589 |
| 2/65 | 3.073 | 0.00548 |
| 3/65 | 2.961 | 0.00520 |
| 4/65 | 2.938 | 0.00489 |
| 5/65 | 3.006 | 0.00461 |
| 6/65 | 2.943 | 0.00436 |
| 7/65 | 1.742 | 0.00505 |
| 8/65 | 1.746 | 0.00479 |
| 9/65 | 2.036 | 0.00535 |
| 10/65 | 2.194 | 0.00581 |
| 11/65 | 2.356 | 0.00579 |
| 12/65 | 2.423 | 0.00572 |
| 1/66 | 2.391 | 0.00551 |
| 2/66 | 2.410 | 0.00529 |
| 3/66 | 2.427 | 0.00527 |

**EXHIBIT 1.3** How Market Risk Varies: Examples from the U.S. Market

| Ranked by $B_i$ | | |
|---|---|---|
| 1. | Holiday Inns | 2.427 |
| 2. | Warner & Swasey | 2.088 |
| 3. | Admiral | 2.025 |
| 4. | Collins Radio | 1.954 |
| 5. | General Instru. | 1.898 |
| 6. | Vornado | 1.817 |
| 7. | Piper Aircraft | 1.798 |
| 8. | Beckman Instru. | 1.763 |
| 9. | Fairchild Camera | 1.717 |
| 10. | Northwest Airlines | 1.687 |
| 11. | Commonwealth Oil | 1.679 |
| 12. | Max Factor | 1.624 |
| 13. | Phila & Reading | 1.609 |
| 14. | Raytheon | 1.595 |
| 15. | Bell & Howell | 1.595 |
| 16. | Avon | 1.589 |
| 17. | Texas Instru. | 1.575 |
| 18. | TWA | 1.524 |
| 19. | Pan Am World Airways | 1.517 |
| 20. | Financial Federation | 1.506 |
| 21. | Ampex | 1.505 |
| 22. | First Charter Financial | 1.504 |
| 23. | Control Data | 1.500 |
| 24. | Crowell Collier | 1.498 |
| 25. | Foxboro | 1.496 |
| 26. | Universal Oil Prod. | 1.479 |
| 27. | EJ Korvette | 1.472 |
| 28. | William H. Rorer | 1.446 |
| 29. | Magnavox | 1.438 |
| 30. | Polaroid | 1.425 |
| 31. | Eastern Airlines | 1.404 |
| 32. | Cerro | 1.398 |
| 33. | Reynolds Metal | 1.394 |
| 34. | National Airlines | 1.378 |
| 35. | Perkin Elmer | 1.378 |
| 36. | Zenith Radio | 1.358 |
| 37. | Cons. Electronics Inds. | 1.375 |
| 38. | Rayonier | 1.449 |
| 39. | Celanese | 1.357 |
| 40. | Xerox | 1.352 |
| 41. | Revere Copper & Brass | 1.350 |
| 42. | Motorola | 1.344 |
| 43. | Litton Inds. | 1.337 |
| 44. | Penn RR | 1.332 |
| 45. | Ginn & Co. | 1.320 |
| 46. | Douglas Aircraft | 1.308 |
| 47. | Indian Head Mills | 1.307 |
| 48. | Mallory | 1.306 |
| 49. | Sunstrand | 1.303 |
| 50. | Delta Airlines | 1.302 |
| 51. | I T & T | 1.296 |
| 52. | Carter Products | 1.291 |
| 53. | General Precision | 1.290 |
| 54. | Sperry-Rand | 1.290 |
| 55. | Western Airlines | 1.277 |
| 56. | Hewlett Packard | 1.262 |
| 57. | ACF Inds. | 1.242 |
| 58. | Great Northern Paper | 1.233 |
| 59. | Owens Corning | 1.207 |
| 60. | Kayser Roth | 1.205 |
| 61. | Grace | 1.194 |
| 62. | Kaiser Alum. | 1.191 |
| 63. | Frito Lay | 1.17 |
| 64. | Air Prd. & Chem. | 1.157 |
| 65. | Allis Chalmers | 1.157 |
| 66. | Whirlpool | 1.156 |
| 67. | Pfizer Chas. | 1.133 |
| 68. | Corning Glass Works | 1.123 |
| 69. | General Dynamics | 1.120 |
| 70. | Bigelow Sanford | 1.115 |
| 71. | Texas Oil & Gas | 1.114 |
| 72. | Wetson & Co. | 1.105 |
| 73. | Pennzoil | 1.098 |
| 74. | National Can | 1.093 |
| 75. | Schering | 1.092 |
| 76. | Union Bag Camp Paper | 1.078 |
| 77. | Burroughs | 1.072 |
| 78. | Mueller Brass | 1.059 |
| 79. | Allied Supermarkets | 1.054 |
| 80. | MGM | 1.048 |
| 81. | Bethlehem Steel | 1.045 |
| 82. | Fibreboard Paper Prds. | 1.039 |
| 83. | Olin Mathieson | 1.035 |
| 84. | Harbison Walker | 1.030 |
| 85. | Chesebrough Ponds | 1.022 |
| 86. | Southern Co. | 1.019 |
| 87. | Aluminum Co Amer. | 1.011 |
| 88. | W. Virginia Pulp & Paper | 1.000 |
| 89. | Caterpillar Tractor | 0.985 |
| 90. | Beaunit | 0.982 |
| 91. | Crown Cork & Seal | 0.978 |
| 92. | Tidewater | 0.978 |
| 93. | Chrysler | 0.966 |
| 94. | Montgomery Ward | 0.953 |
| 95. | Texas Gulf Sulphur | 0.945 |
| 96. | Cons. Cigar | 0.936 |
| 97. | Ex-Cell-O | 0.936 |
| 98. | Westinghouse Electric | 0.932 |
| 99. | Upjohn | 0.920 |

**EXHIBIT 1.3** (*Continued*)

| | | | | | |
|---|---|---|---|---|---|
| 100. | Holt Rinehart & Winston | 0.914 | 117. | Marathon | 0.752 |
| 101. | Wesco Financial | 0.911 | 118. | Allied Chemical | 0.738 |
| 102. | Grumman | 0.904 | 119. | Gulf Oil | 0.727 |
| 103. | Halliburton | 0.893 | 120. | McDonnell Aircraft | 0.721 |
| 104. | Florida Pwr. & Light. | 0.888 | 121. | Socony Mobil Oil | 0.665 |
| 105. | Cone Mills | 0.887 | 122. | Texaco | 0.661 |
| 106. | Colgate Palmolive | 0.851 | 123. | Monsanto | 0.649 |
| 107. | Union Oil Cal. | 0.850 | 124. | Sunbeam | 0.639 |
| 108. | Merck | 0.850 | 125. | S. Carolina Elec. & Gas | 0.610 |
| 109. | Columbia Brdestg. | 0.837 | 126. | Central Southwest | 0.607 |
| 110. | Bobbie Brooks | 0.825 | 127. | Abbott Labs | 0.593 |
| 111. | Gillette | 0.819 | 128. | Standard Oil Cal. | 0.587 |
| 112. | Lockheed | 0.805 | 129. | Petrolane Gas Service | 0.576 |
| 113. | United Fruit | 0.782 | 130. | Beneficial Finance | 0.565 |
| 114. | Coastal States Gas Prod. | 0.774 | 131. | Gulf States Utilities | 0.549 |
| 115. | United Carr | 0.764 | 132. | Southwestern Public Sve. | 0.451 |
| 116. | IBM | 0.757 | 133. | AT&T | 0.403 |

the simplest model which has any reasonable hope of predicting the risk character of a diversified portfolio.

The values for the regression coefficients and residual variances are obtained by regressing price change histories for individual common stocks against a suitable market average. From these values and composition data, a model of the risk character of the portfolio is constructed. Predictions of a change in the value of the portfolio (given the change in market level) are compared with the actual change in order to test this model.

Our basic idea (in which we were anticipated by the work of Fanning and Steglitz, and also by that of Marshall Blume) is that if we knew the risk parameters for individual common stocks then we could estimate the risk character of a portfolio instantaneously—even though the composition was continuously changing. In order to test this idea we have studied all the common stocks held in an actual mutual-fund portfolio during a period of more than two years. The price history of each common stock was traced back as far as conveniently possible—in some cases, up to 40 years. From the price and dividend histories for the common stocks, we made running estimates of the risk parameters for each common stock held. Then, once a month for each month during the test period, we made an instantaneous estimate of the risk character of the mutual fund portfolio, based on its composition at the end of that month. Our estimate of *market* risk for the portfolio enabled us to predict how rapidly the value of the fund would change as the market level fluctuated. Our estimate of *residual* risk for the fund gives an estimate of the amount by which the true market value of the fund will differ from our predictions. For each month of the test period we estimated both risk parameters for the fund and observed the actual change in market level and the actual change in the value of the fund. How well we succeeded in predicting the observed changes in the value of the fund, given the actual changes in market level, is discussed at the end of this paper.

**EXHIBIT 1.4** How Specific Risk Varies: Examples from the U.S. Market

| Ranked by Residual Variance | | |
|---|---|---|
| 1. | Control Data Corp. | 0.02116 |
| 2. | Fairchild Camera & Instru. | 0.02071 |
| 3. | Texas Gulf Sulphur | 0.01576 |
| 4. | Texas Instru. | 0.01453 |
| 5. | Admiral Corp. | 0.01431 |
| 6. | EJ Korvette | 0.01344 |
| 7. | Collins Radio | 0.01326 |
| 8. | Wesco Financial Corp. | 0.01244 |
| 9. | Xerox Corp. | 0.01217 |
| 10. | Financial Federation | 0.01200 |
| 11. | General Instru. Corp. | 0.01145 |
| 12. | First Charter Finan. Corp. | 0.01045 |
| 13. | Ampex Corp. | 0.01011 |
| 14. | Cons. Electronics Inds. | 0.00946 |
| 15. | Beckman Instru. | 0.00939 |
| 16. | Commonwealth Oil Refining | 0.00900 |
| 17. | Raytheon Co. | 0.00893 |
| 18. | Vornado Inc. | 0.00892 |
| 19. | Max Factor & Co. | 0.00878 |
| 20. | Polaroid Corp. | 0.00874 |
| 21. | Magnavox Co. | 0.00861 |
| 22. | Crowell-Collier | 0.00820 |
| 23. | William H. Rorer | 0.00812 |
| 24. | Douglas Aircraft Co. | 0.00780 |
| 25. | Zenith Radio Corp. | 0.00765 |
| 26. | Texas Oil & Gas | 0.00762 |
| 27. | General Dynamics | 0.00752 |
| 28. | McDonnell Aircraft Corp. | 0.00730 |
| 29. | TWA Inc. | 0.00725 |
| 30. | Bell & Howell Co. | 0.00717 |
| 31. | Hewlett Packard Co. | 0.00711 |
| 32. | Crown Cork & Seal | 0.00705 |
| 33. | Piper Aircraft Corp. | 0.00697 |
| 34. | Universal Oil Prds. | 0.00694 |
| 35. | Northwest Airlines | 0.00692 |
| 36. | Perkin Elmer Corp. | 0.00691 |
| 37. | Carter Prod. Inc. | 0.00688 |
| 38. | Mueller Brass Co. | 0.00686 |
| 39. | Eastern Airlines | 0.00685 |
| 40. | Reynolds Metals | 0.00679 |
| 41. | National Can Corp. | 0.00668 |
| 42. | Foxboro Co. | 0.00666 |
| 43. | Pan Am Wld Airways | 0.00666 |
| 44. | Phil. & Reading Corp. | 0.00660 |
| 45. | National Airlines | 0.00659 |
| 46. | Allied Supermarkets | 0.00648 |
| 47. | Delta Air Lines Inc. | 0.00645 |
| 48. | General Precision | 0.00636 |
| 49. | Ginn & Co. | 0.00628 |
| 50. | Kaiser Alum. & Chem. | 0.00622 |
| 51. | Pennzoil Co. | 0.00619 |
| 52. | Beaunit Corp. | 0.00613 |
| 53. | Kayser Roth Corp. | 0.00603 |
| 54. | Lockheed Aircraft | 0.00603 |
| 55. | Litton Inds. | 0.00590 |
| 56. | Grumman Aircraft | 0.00582 |
| 57. | Western Airlines | 0.00578 |
| 58. | Bobbie Brooks | 0.00576 |
| 59. | Sunstrand Corp. | 0.00573 |
| 60. | Motorola Inc. | 0.00571 |
| 61. | United Fruit | 0.00565 |
| 62. | Great Northern Paper Co. | 0.00564 |
| 63. | Schering Corp. | 0.00558 |
| 64. | Sperry-Rand Corp. | 0.00540 |
| 65. | Wetson & Co. | 0.00540 |
| 66. | Cerro Corp. | 0.00539 |
| 67. | Southern Co. | 0.00537 |
| 68. | Chrysler Corp. | 0.00532 |
| 69. | Cons. Cigar Corp. | 0.00529 |
| 70. | Holiday Inns of Amer. | 0.00527 |
| 71. | Tidewater Oil Co. | 0.00515 |
| 72. | Bigelow-Sanford Inc. | 0.00504 |
| 73. | Air Prod. & Chem. | 0.00501 |
| 74. | Indian Head Mills | 0.00498 |
| 75. | Burroughs Corp. | 0.00493 |
| 76. | Warner & Swasey Co. | 0.00489 |
| 77. | Holt Rinehart & Winston | 0.00488 |
| 78. | Revere Copper & Brass | 0.00482 |
| 79. | MGM | 0.00480 |
| 80. | Whirlpool Corp. | 0.00476 |
| 81. | Celanese Corp Amer. | 0.00475 |
| 82. | Owens Corning Fiberglass | 0.00465 |
| 83. | Corning Glass Works | 0.00449 |
| 84. | Sunbeam Corp. | 0.00447 |
| 85. | Rayonier Inc. | 0.00439 |
| 86. | Alum. Co. Amer. | 0.00433 |
| 87. | Petrolane Gas Srv. Inc. | 0.00429 |
| 88. | Intern. T & T | 0.00428 |
| 89. | Gillette Co. | 0.00406 |
| 90. | ACF Inds. | 0.00404 |
| 91. | Penn. RR | 0.00404 |
| 92. | Columbia Brdcstg. | 0.00402 |
| 93. | W. Virginia Pulp & Paper | 0.00402 |
| 94. | Avon Prod. | 0.00401 |
| 95. | Frito Lay | 0.00396 |
| 96. | Fibreboard Paper Prds. | 0.00394 |
| 97. | Merck & Co. | 0.00387 |
| 98. | Mallory Pr. & Co | 0.00373 |

**EXHIBIT 1.4** (*Continued*)

| | | | | | |
|---|---|---|---|---|---|
| 99. | Caterpillar Tractor | 0.00363 | 117. | Olin Mathieson Chem. | 0.00274 |
| 100. | United Carr Inc. | 0.00359 | 118. | Gulf Oil | 0.00267 |
| 101. | Bethlehem Steel | 0.00356 | 119. | Westinghouse Electric | 0.00267 |
| 102. | Abbott Lab | 0.00354 | 120. | Grace WR & Co. | 0.00255 |
| 103. | Upjohn Co. | 0.00353 | 121. | Union Oil of Cal. | 0.00245 |
| 104. | Coastal States Gas Prd. Co. | 0.00352 | 122. | IBM | 0.00242 |
| 105. | Chesebrough Ponds Inc. | 0.00341 | 123. | Florida Pwr. & Light | 0.00234 |
| 106. | Ex-Cell-O | 0.00340 | 124. | Beneficial Finance | 0.00232 |
| 107. | Cone Mills | 0.00336 | 125. | Socony Mobil Oil | 0.00225 |
| 108. | Colgate Palmolive | 0.00330 | 126. | Texaco | 0.00217 |
| 109. | Halliburton Co. | 0.00331 | 127. | Gulf States Utilities | 0.00216 |
| 110. | Monsanto Co. | 0.00329 | 128. | S. Carolina Electric & Gas | 0.00203 |
| 111. | Allis Chalmers Mfg. | 0.00327 | | | |
| 112. | Marathon Oil Co. | 0.00323 | 129. | Central S. West | 0.00200 |
| 113. | Union Bag Camp Paper | 0.00318 | 130. | Allied Chem. Corp. | 0.00185 |
| 114. | Montgomery Ward | 0.00309 | 131. | Southwestern Public Svc. | 0.00181 |
| 115. | Pfizer Chas | 0.00294 | | | |
| 116. | Harbison Walker Refractories | 0.00291 | 132. | Standard Oil of Cal. | 0.00174 |
| | | | 133. | AT&T | 0.00091 |

Although the risk character of a fund may change quickly if the composition of the fund changes, our scheme assumes that risk parameters for individual common stocks change relatively slowly. In most cases the assumption seems valid to us, since they change through the gradual evolution of products, manufacturing processes, and markets. The risk character of a company's common stock may change quickly, however, if the company enters into a wide-ranging diversification program or undergoes a profound change in capital structure. The essence of the measurement problem is that they are measured subject to random fluctuations in the data and that reliable estimates can be obtained only with samples large enough to "average out" these random fluctuations to some degree. Unfortunately, over time, the underlying parameters that we are attempting to measure are themselves changing. Thus we are confronted with an inescapable dilemma: If we confine our samples to very recent data, possible error due to random fluctuations in sample data may be excessively large. If, on the other hand, we include in our sample a longer time span, we may be including data that are no longer relevant because of changes over time in the risk character of the common stock in question. In principle, there are ways of weighting more and less recent data that are optimal in the sense of minimizing the combined effects of both problems. We are currently experimenting with techniques that select optimum weights automatically. For this study, however, we used exponential smoothing techniques with arbitrary weights (see Appendix 1.1).

Using exponential smoothing we obtained running (that is, continually changing) estimates of the risk character of a large group of common stocks covering as much as 40 years. Exhibit 1.2 shows how estimates of the risk parameters for a single common stock have behaved over time.

Perhaps the most striking thing about our estimates of risk parameters for individual common stocks is the range of values encountered in our modest sample. Exhibit

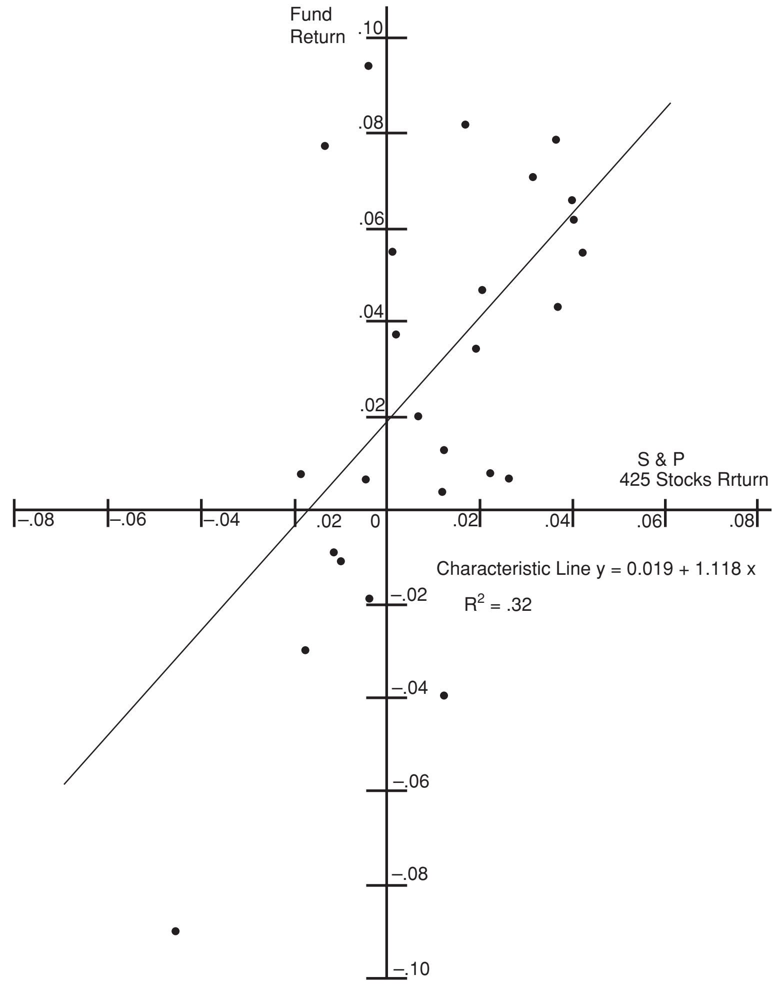


**EXHIBIT 1.5** Fund Return versus Market Return

1.3 shows current estimates for market risk of stocks in the sample. A regression-slope coefficient $B_i$ equal to 1 implies that the stock in question has an average degree of market risk, and that if the market rises or falls 10 percent other things equal the stock in question will rise or fall respectively 10 percent. Values $B_i$ in Exhibit 1.3 range from less than $\frac{1}{2}$ to more than $2\frac{1}{2}$. In other words some stocks in Exhibit 1.3 have more than 5 times as much market risk as some others. Clearly the degree of market risk in a portfolio is determined, not only by the proportions devoted to common stock and fixed income securities respectively, but also by the kind of common stocks held.

Exhibit 1.4 shows current estimates of $\sigma_i^2$, the spread in residual risk for the common stocks studied. Here too the range is impressive. As one might expect there is some tendency for stocks that rank high in Exhibit 1.3 to rank high in Exhibit 1.4.

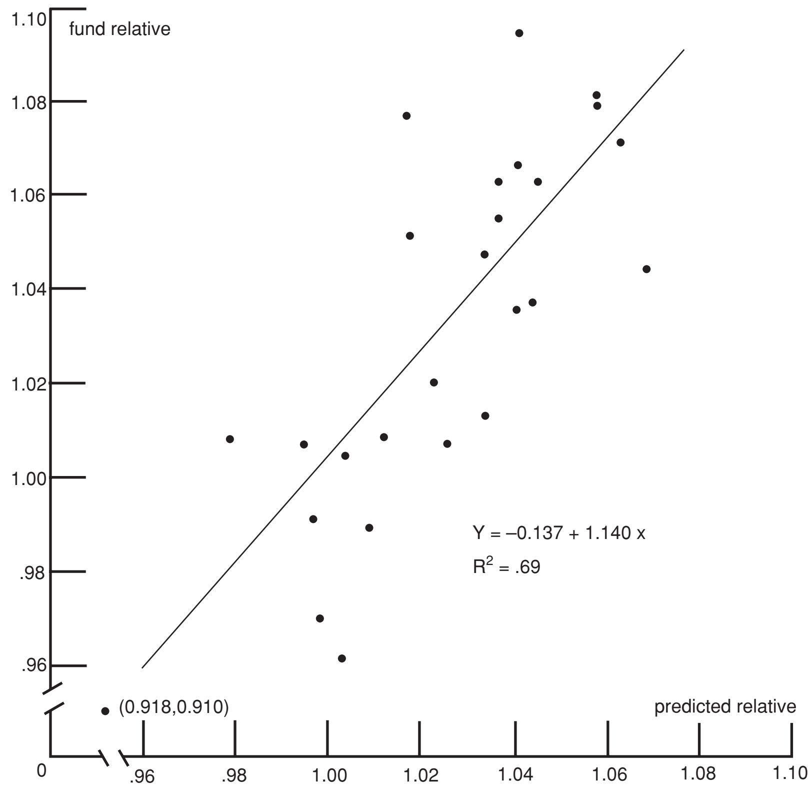


**EXHIBIT 1.6** Fund Return versus Predictions Based on Risk Character of Stocks Listed

Ultimately our interest in the risk character of common stocks derives entirely from their possible impact on the risk character of a portfolio. The risk character of the mutual fund portfolio considered in this study[6] is displayed in Exhibit 1.5. The rate of return for an appropriate market index (the same Fisher Index referred to above) is measured along the horizontal axis and a rate of return for the fund is measured on the vertical axis. (The scatter diagram in Exhibit 1.5 covers 27 consecutive months of investment results for the fund.) The slope of the regression line fitted to these points (the Characteristic Line) is a measure of the average level of market risk in the fund over this period. If the actual level of market risk in the fund had been maintained constant over the period, then the dispersion of month-to-month results around the line of best fit would be a measure of the degree of specific risk in the fund. If on the other hand, the actual degree of market risk in the fund was changing from month to month, then the dispersion of the data around the line of best fit overstates the degree of specific risk in the fund. It is obviously necessary to accumulate data over a substantial period of time in order to measure market risk in a fund using the Characteristic Line technique.

Exhibit 1.6 compares actual month-to-month results for the fund with results predicted, using the technique described in this paper. A comparison of Exhibit 1.5

and 1.6 shows that our forecast of investment results for the fund is improved by using risk estimates for the individual common stocks held. In fact, roughly half the variance left unexplained by the Characteristic Line is accounted for by allowing for changes in the composition of the fund (hence changes in the risk character of the fund) over the sample period. We conclude that our risk-measuring technique is producing numbers that are both meaningful and useful—not only for estimating portfolio risk after the fact but also for estimating the impact on fund risk of making contemplated changes in the composition of the fund.

## APPENDIX 1.1

The following formulas indicate schematically how we used Exponential Smoothing to get continuously-updated estimates of risk parameters for individual common stocks. Let $x_i(t)$ be the rate of return for the ith stock in period $t$ and define $\overline{x}_i(t)$ implicitly as our estimate of the current expected value around which $x(t)$ is fluctuating. Let $\mu(t)$ be the rate of return for an appropriate market index and define $\overline{\mu}(t)$ analogously to $\overline{x}(t)$. Then define the covariance matrix $\sigma_i^2$ by

$$\sigma_i^2 = \begin{pmatrix} \sigma_{ii}^2 & \sigma_{i\mu}^2 \\ \sigma_{i\mu}^2 & \sigma_{\mu\mu}^2 \end{pmatrix} = \begin{pmatrix} (x_i - \bar{x}_i)^2 & (x_i - \bar{x}_i)(\mu - \bar{\mu}_i) \\ (x_i - \bar{x}_i)(\mu - \bar{\mu}) & (\mu - \bar{\mu})^2 \end{pmatrix} \tag{1.4}$$

and define $\sigma_i^2(t)$ implicitly by the relation

$$\sigma^2(t) = \alpha\sigma^2(t) + (1 - \alpha)\bar{\sigma}^2(t - 1)$$

where $\alpha$ is the same smoothing constant as before. Then our estimates of the regression parameters $\beta_i(t)$ and $\sigma_i^2(t)$ are given by

$$\beta_i(t) = \frac{\sigma_{i\mu}^2(t)}{\sigma_{\mu\mu}^2(t)} \tag{1.5}$$

$$\sigma_i^2(t) = \sigma_{ii}^2(t) - \beta_i^2(t)\sigma_{\mu\mu}^2(t) \tag{1.6}$$

It can be seen from these formulas that, when the true values of $\beta_i$ and $\sigma_i^2$ are changing along a steady trend, our estimates will tend to lag somewhat behind the true values.

### Notes

1. "Market and Industry Factors in Stock Price Behavior," *Journal of Business*, Volume 39, Number 1, Part II ("Supplement," January, 1966), pp. 139–190.
2. It should be noted that in conversation with one of us (Fisher) in 1964 or 1965, Harry Markowitz expressed the opinion that the degree of clustering of fluctuations found by

King would not cause portfolios to show riskiness substantially different from that estimated using the Sharpe model.

3. Unpublished monograph *The Empirical Adequacy of Portfolio Theory*, submitted to *Journal of Business*, July, 1968.
4. Sharpe's model was nearly anticipated by M. F. M. Osborne in his celebrated paper "Brownian Motion in the Stock Market," (*Operations Research*, Vol. 7, March–April, 1959). Osborne considered an "ensemble consisting of 1,000 pennies and one gold piece." The outcome of the toss of the gold piece affected the prices of 1,000 stocks; the effect of the outcome of tossing each of the 1,000 pennies was unique to a single stock.
5. Described in "Some New Stock Market Indexes," *Journal of Business, loc. cit.*, pp. 191–225.
6. The "fund" studied was the portion of Diversified Growth Stock Fund that was invested in common stocks listed on the New York Stock Exchange. These stocks comprised over 90 per cent of the net assets of the fund throughout the period studied (December 31, 1964 to March 31, 1966).