Chapter 1

Getting Comfortable with Math Speak

In This Chapter

- Introducing terminology and mathematical conventions
- Comparing sentence and equation structure for more clarity
- Using pictures for understanding
- Looking to tables and charts for organization of information

Mathematicians decided long ago to conserve on words and explanations and replace them with symbols and single letters. The only problem is that a completely different language was created, and you need to know how to translate from the cryptic language of symbols into the language of words. The operations have designations such as +, –, ×, and ÷. Algebraic equations use letters and arrangements of those letters and numbers to express relationships between different symbols.

In this chapter, you get a refresher of the math speak you've seen in the past. I review the vocabulary of algebra and geometry and give examples using the appropriate symbols and operations.

Latching onto the Lingo

Words used in mathematics are very precise. The words have the same meaning no matter who's doing the reading of a problem or when it's being done. These precise designations may seem restrictive, but being strict is necessary — you want to be able to count on a mathematical equation or expression meaning the same thing each time you use it.

For example, in mathematics, the word *rational* refers to a type of number or function. A person is *rational* if he acts in a controlled, logical way. A number is *rational* if it acts in a controlled, structured way. If you use the word *rational* to describe a number, and if the person you're talking to also knows what a rational number is, then you don't have to go into a long, drawn-out explanation about what you mean. You're both talking in the same language, so to speak.

Defining types of numbers

Numbers are classified by their characteristics. One number can have more than one classification. For example, the number 2 is a *whole* number, an *even* number, and a *prime* number. Knowing which numbers belong in which classification will help you when you're trying to solve problems in which the answer has to be of a certain type of number.

Naming numbers

Numbers have names that you speak. For example, when you write down a phone number that someone is reciting, you hear *two, one, six, nine, three, two, seven,* and you write down 216-9327. Some other names associated with numbers refer to how the numbers are classified.

- **Natural (counting):** The numbers starting with 1 and going up by ones forever: 1, 2, 3, 4, 5, . . .
- **Whole:** The numbers starting with 0 and going up by ones forever. Whole numbers are different from the natural numbers by just the number 0: 0, 1, 2, 3, 4, . . .
- **Integer:** The positive and negative whole numbers and 0: . . . ,–3, –2, –1, 0, 1, 2, 3, 4, . . .
- **Rational:** Numbers that can be written as $\frac{p}{q}$ where both p and q are integers, but q is never 0: $\frac{3}{4}, \frac{19}{8}, -\frac{5}{21}, \frac{24}{6}$, and so on
- **Even:** Numbers evenly divisible by 2: . . . ,–4, –2, 0, 2, 4, 6, . . .
- **Odd:** Numbers not evenly divisible by 2: . . . ,–3, –1, 1, 3, 5, 7, . . .
- **Prime** Numbers divisible evenly only by 1 and themselves: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, . . .
- **Composite:** Numbers that are not prime; numbers that are evenly divisible by some number other than just 1 and themselves: 4, 6, 8, 9, 10, 12, 14, 15, . . .

Relating numbers

Numbers of the same or even different classifications are often related in another way that makes them usable in problems. For example, if you want only *multiples of five,* you draw from evens, odds, and integers — several types to create a new relationship.

- **Consecutive:** A listing of numbers, in order, from smallest to largest, that have the same difference between them: 22, 33, 44, 55, . . . are consecutive multiples of 11 starting with 22.
- **Multiples:** Numbers that all have a common multiplier: 21, 28, and 63 are multiples of 7.

Gauging the geometric

Geometric figures appear frequently in mathematical applications — and in life. Geometric figures have names, classifications, and characteristics. The figures are also measured in two or more ways. Flat figures have the lengths of their sides, their whole perimeter, or their area measured. Solid figures have their surface area and volume measured. You can find all the formulas you need on the Cheat Sheet and in Chapters 18, 19, and 20. What you find here is a description of what the measures mean.

Plying perimeter

The *perimeter* is a linear measure: inches, feet, centimeters, miles, kilometers, and so on. Perimeter is a measure of distance — the distance around the outside of a flat figure. The perimeter of a figure made up of line segments is equal to the sum of the length of all the segments. The perimeter of a circle is also called its *circumference* and is always slightly more than three times the circle's diameter. In Figure 1-1, you see several sketches and their respective perimeters.

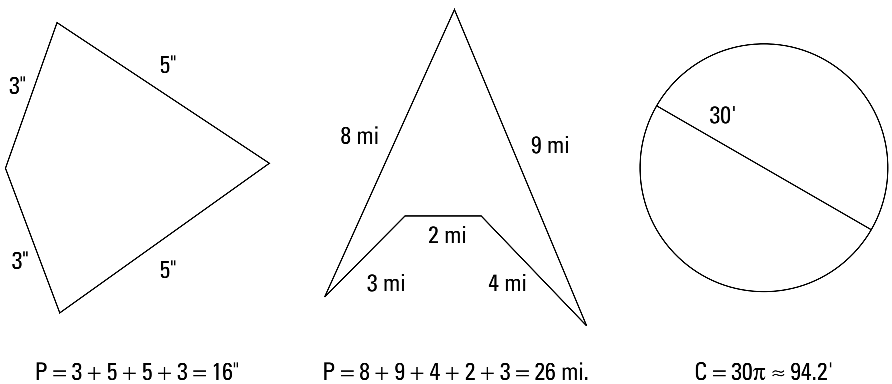

Figure 1-1: Add up the lengths of the segments to get the perimeter.

Assembling the area

The *area* of a figure is a two-dimensional measure. The area is a measure of how many squares you can fit into the figure. If the figure doesn't have 90-degree or squared-off angles, then you have to count up pieces of squares — break them up and put them back together — to get the whole area. Think about putting square tiles in a room — you have to cut some of them to go around cabinets or fit along a wall. The formulas that you use to compute areas help you with the piecing-together of squares.

In Figure 1-2, you see a triangle with an area of exactly 12 square units. See if you can figure out how the pieces go together to form a total of 12 squares. If that doesn't work, you can compute the area by just looking up the formula for the area of a triangle.

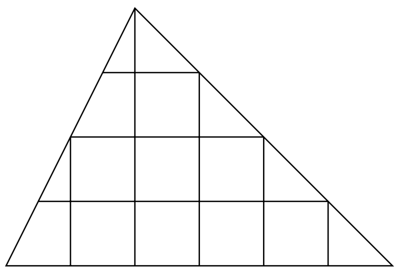

Figure 1-2: How many squares are in the triangle?

Coming to the surface with surface area

The *surface area* of a solid figure is the sum of the areas of all the sides. A four-sided figure has a triangle on each side, so you add up the areas of each of the triangles to get the total surface area. How do you get the area of each triangle? You go back to the formula for finding the area of triangles of that particular size — or just count how many squares! Figure 1-3 shows three of the six sides of a right rectangular prism and how each side has its area determined by all the squares it can fit on that side.

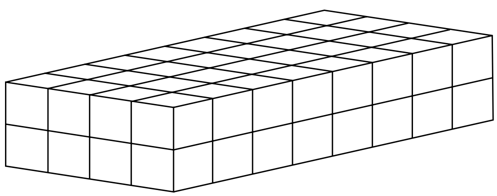

Figure 1-3: How much paper will you need to wrap the package?

The prism in Figure 1-3 has a surface area of 112 square units. That's how many squares cover the six surfaces of this solid figure. Formulas are much easier to use than actually trying to count squares.

Vanquishing volume

The *volume* of a solid figure is a three-dimensional type of unit. When you compute the volume of something, you're determining how many cubes (like sugar cubes or dice) will fit inside the figure. When the sides slant, of course, you have to slice, trim, and fit to make all the cubes go inside — or you can use a handy-dandy formula. Figure 1-4 shows how you can set cubes next to one another and then stack them to determine the volume of a solid.

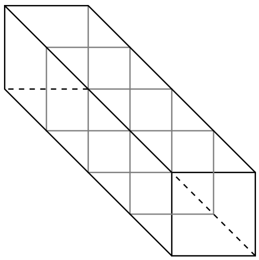

Figure 1-4: Cubes all in a row.

Formulating financials

Most people are interested in money, in one way or another. Money is the way people keep count of whether they can trade for what they want or need. Financial formulas aid with the computation of money-type situations.

The financial formulas here are divided into two different types: interest formulas and revenue formulas. The interest formulas both involve a percentage that needs to be changed into a decimal before being inserted into the formula. To change a percent into a decimal, you move the decimal point two places to the left. So 3.4 percent becomes 0.034 and 67 percent becomes 0.67.

The interest formulas are of two types: simple interest and compound interest. The simple-interest formula is $I = Prt$. The I indicates how much interest your money has earned — or how much interest you owe. The P is the principal — how much money you invested or are borrowing. The r represents the interest rate — the percentage that gets changed to a decimal. And the t stands for time, which is usually a number of years.

BRAINTEASER

"Tomorrow, tomorrow, I love ya, tomorrow . . ."

If yesterday had been Wednesday's tomorrow, and if tomorrow is Sunday's yesterday, then what day is it today?

Answer: It's Friday.

Compounding interest means that you split up the rate of interest into a designated number of subintervals (every three months, twice a year, daily, and so on), figure the interest earned during that subinterval, add the interest to the principal, and then figure the next interval's interest on the sum of the original principal plus the interest you've added. As you may expect, you'll have more money in the end if you deposit it where you can earn compound interest rather than just a flat amount. The formula for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$. The A represents the total amount of money — all the principal plus the interest earned. The r and t are the same as in simple interest. The n represents the number of times each year that the interest is compounded. Most banks compound quarterly, so the value of n is 4 in those cases.

Interpreting the Operations

What would mathematics be without its operations? The basic operations are addition, subtraction, multiplication, and division. You then add raising to powers and finding roots. Many more operations exist, but these six basic operations are the ones you'll find in this book. Also listed here are some of the special names for multiplying by two or three.

Naming the results

Each operation has a result, and just naming that result is sometimes more convenient than going into a big explanation as to what you want done. You can economize with words, space, time, and ink. The following are results of operations most commonly used.

- **Sum:** The result of adding
- **Difference:** The result of subtracting

- **Product:** The result of multiplying
- **Twice or double:** The result of multiplying by two
- **Thrice or triple:** The result of multiplying by three
- **Quotient:** The result of dividing
- **Half:** The result of dividing by two

Assigning the variables

A variable is something that changes. In mathematics, a variable is represented by a letter — usually one from the end of the alphabet — and it *always* represents a number (usually an unknown number). For example, if you're doing a problem involving Jake and Jim and their ages, you can let x represent Jake's *age,* but you can't let x represent Jake.

As you work on a problem, it's a good idea to make a notation as to what you're letting the variable or variables represent, so you don't forget or get confused when constructing an equation to solve the problem.

Aligning symbols and word forms

One of the things that people see as a challenge in word problems is that they're full of *words!* After you've changed the words to symbols and equations, it's smooth sailing. But you have to get from there to here. Table 1-1 lists some typical translations of words into symbols and an example of their use.

Table 1-1 Translating into Math Shorthand

Words	*Symbols*	*Examples*
Is, are	=	Jack **is** twice as old as Jill: $x = 2y$, where x represents Jack's age, and y represents Jill's age.
And, total	+	Six dimes **and** 5 quarters: $10x + 25q$, where x represents the number of dimes and q represents the number of quarters; each is multiplied by its monetary value.
Less, fewer	–	Jon worked 3 **fewer** hours than Jim: $x = y - 3$, where x represents the number of hours that Jon worked, and y represents the number of hours that Jim worked.

(continued)

Table 1-1 *(continued)*

Words	*Symbols*	*Examples*
Of, times	$\times$	One-half **of** Clare's money: $\frac{1}{2}x$, where x represents Clare's money.
Ratio	$\div$	The ratio of pennies to quarters: $\frac{x}{y}$, where x represents the number of pennies, and y represents the number of quarters
Approximately	$\approx$	The fraction $\frac{22}{7}$ is about 3.14: $\frac{22}{7} \approx 3.14$

Drawing a Picture

One of the most powerful tools you can use when working on word problems is drawing a picture. Most people are very visual — they understand relationships between things when they write something down and/or draw a picture illustrating the situation.

Visualizing relationships

The words in a math problem suggest how different parts of the situation are connected — or not connected. Drawing a picture helps to make the connections and, often, suggests how to proceed with a solution.

For example, consider a word problem starting out with: "A plane is flying east at 600 mph while another plane is flying north at 500 mph. . . ." You need more information than this to determine what the question and answer are, but a picture suggests what process to use. Look at Figure 1-5, where two possible scenarios for the statement are illustrated.

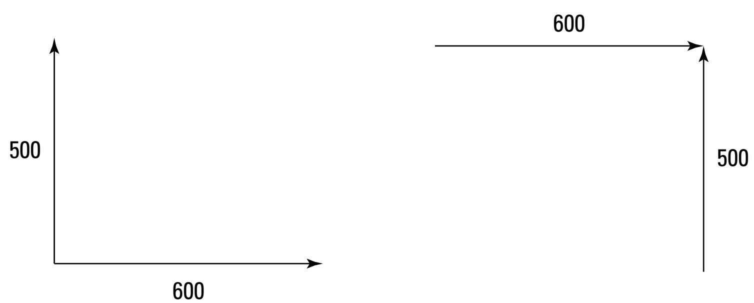

Figure 1-5: The planes are leaving or approaching one another.

The precise relationship between the planes has to be given, but both sketches suggest that a triangle can be formed by connecting the ends of the arrows. Right triangles suggest the Pythagorean theorem, and other triangles come with their respective perimeter and area formulas. In any case, the picture solidifies the situation and makes interpretations possible.

Another example where a picture is helpful involves a situation where you're cutting a piece of paper. The word problem starts out with: "A rectangular piece of paper has equal squares cut out of its corners. . . ." You draw a rectangle, and you show what it looks like to remove squares that are all the same size. Figure 1-6 illustrates one interpretation.

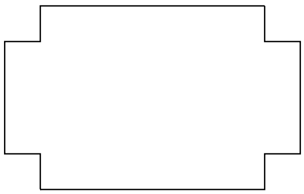

Figure 1-6: Cut squares from the corners.

With the figure in view, you see that the lengths of the outer edges are reduced by two times some unknown amount. The picture helps you write expressions about the relationships between the original piece of paper and the cut-up one.

Labeling accurately

Pictures are great for clarifying the words in a problem, but equally important are the labels that you put on the picture. By labeling the different parts — especially with their units in feet, miles per hour, and so on — you improve your chances of writing an expression or equation that represents the situation.

You're told "A trapezoidal piece of land has 300 feet between the two parallel sides, and the other two sides are 400 feet and 500 feet in length, while the two parallel bases are 600 feet and 1,200 feet." This statement has five different numbers in it, and you need to sort them out. Figure 1-7 shows how the different measures sort out from the statement.

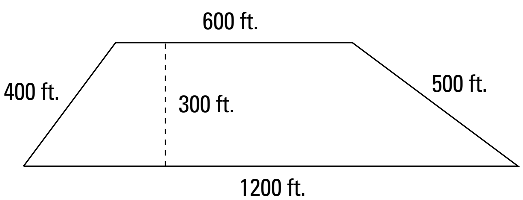

Figure 1-7: The trapezoid has a height of 300 feet.

BRAINTEASER

Watch out for the black cat

A completely black cat was ambling down the street during a total blackout — electricity was down throughout the entire town. Not a single streetlight had been on for hours. Just as the cat was crossing the middle of the street, a Jeep with two broken headlights came racing toward where the cat was. Just as the Jeep got to where the cat was crossing, it swerved out of the way to avoid hitting the cat. How could the driver of the Jeep have seen the cat in time to swerve and avoid hitting it?

Answer: This happened in the middle of the day. It didn't matter that no street lights were on — everyone could see just fine.

Constructing a Table or Chart

A really nice way to determine what's going on with a word problem is to make a list of different possibilities and see what fits in the list or what pattern forms. Patterns often suggest a formula or equation; the values in the listing sometimes even provide the exact answer. Just as with pictures, making a chart is a way of visualizing what's going on.

Finding the values

When creating a table or chart, designate a variable to represent a part of the problem, and see what the results are as you systematically change that variable. For example, if you're trying to find two numbers the product of which is 60 and the sum of which is as small as possible, let the first number be x. Then the other number is $\frac{60}{x}$. Add the two numbers together to see what you get. Table 1-2 shows the different values for the two numbers and the sum — if you stick to whole numbers.

Table 1-2 Finding the Smallest Possible Sum

x (The First Number)	$\frac{60}{x}$ *(The Second Number)*	*The Sum of the Numbers*
1	60	1 + 60 = 61
2	30	2 + 30 = 32
3	20	3 + 20 = 23

x (The First Number)	$\frac{60}{x}$ *(The Second Number)*	*The Sum of the Numbers*
4	15	4 + 15 = 19
5	12	5 + 12 = 17
6	10	6 + 10 = 16
10	6	10 + 6 = 16

I stop here, because I'm just going to get the same pairs of numbers in the opposite order. If the rule is that the numbers can't be fractions, then the two numbers with a product of 60 and with the smallest possible sum are 6 and 10.

Increasing in steps

When making a table or chart, you want to be as systematic as possible so you don't miss anything – especially if that *anything* is the correct answer. After you've determined a variable to represent a quantity in the problem, you need to go up in logical steps — by ones or twos or halves or whatever is appropriate. In Table 1-2, in the preceding section, you can see that I went up in steps of 1 until I got to the 6. One more than 6 is 7, but 7 doesn't divide into 60 evenly, so I skipped it. Even though the work isn't shown here, I mentally tried 7, 8, and 9 and discarded them, because they didn't work in the problem. When you're working with more complicated situations, you don't want to skip any steps — show them all.