# CHAPTER 1

# Discounted Cash Flow and the Gordon Model: *The Very Basics of Value*



## INTRODUCTION

We begin by focusing on "The Very Basics of Value." This subtitle is intentional because our purpose here is to explore the foundation of both the discounted cash flow model and the Gordon Model to enhance our understanding of these basic tools of valuation and finance. As will be shown, the discounted cash flow model and the Gordon Model can be used to develop the Integrated Theory of Business Valuation.

## COMMON QUESTIONS

In order to move the reader from theory to practice, we begin each chapter with a series of often vexing questions. We have structured the content of each chapter to provide answers to these questions. Keep in mind that it will not be uncommon to see certain questions repeated in other chapters.

- What are the necessary conditions for use of the Gordon Model?
- Where does the generalized valuation model, Value = Earnings × Multiple, come from?
- What are the conditions that define $g$, the long-term growth rate of core earnings used in the Gordon Model?
- What is the relationship between the net income and the net cash flow of business enterprises?
- When applying the DCF method, is the appropriate measure of benefits for discounting net income or net cash flow?
- What is the difference between the expected growth rate in the core *earnings* of a business and its expected growth rate in *value*?

- Are the DCF and single-period income capitalization methods intrinsically different?
- How fast can the earnings of an enterprise reasonably be expected to grow?
- When capitalizing net income rather than net cash flow, should adjustment factors to $r$, the discount rate, be applied?

Keep these questions in mind as we begin with a discussion of the discounted cash flow model.

## THE BASIC TOOLS OF VALUATION

### The Discounted Cash Flow Model

The value of a business enterprise can be described as:

- The value today (i.e., in *cash-equivalent terms*)
- of all expected future cash flows (or benefits) of the business
- forecasted or estimated over an indefinite time period (i.e., *into perpetuity*)
- that have been *discounted to the present* (expressed in terms of *present value* dollars) at an appropriate *discount rate* (which takes into consideration the riskiness of the projected cash flows of the business relative to alternative investments).

The valuation and finance literature consistently confirm this conceptual definition of the value of a business enterprise. In order to value a business, therefore, we need the following:

- A forecast of all expected future cash flows or benefits to be derived from ownership of the business; and,
- An appropriate discount rate with which to discount the cash flows to the present.

This conceptual definition of business value can be defined symbolically in Equation 1.1:

$$\textbf{Value} = V_0 = \left(\frac{\mathrm{CF}_1}{(1+r)^1} + \frac{\mathrm{CF}_2}{(1+r)^2} + \frac{\mathrm{CF}_3}{(1+r)^3} + \frac{\mathrm{CF}_4}{(1+r)^4} + \cdots + \frac{\mathrm{CF}_n}{(1+r)^n}\right) \tag{1.1}$$

Where:

$V_0$ is the value of the equity of a business today.
$\mathbf{CF_1}$ to $\mathbf{CF}_n$ represent the expected cash flows (or benefits) to be derived for periods 1 to $n$.[1]
$r$ is the discount rate that converts future dollars of CF into present dollars of value.

Equation 1.1 is the basic discounted cash flow (DCF) model. To employ the model in this form, however, the analyst must make a forecast of *all* the relevant cash flows into the indefinite future. For clarity, the cash flows or earnings discussed in this chapter are the net earnings and net cash flows of the enterprise or the business as a whole. $V_0$ is the value of the equity of the enterprise, or the present value of the expected cash flows to the owners of the equity of the enterprise.[2] Expanding the analysis to correspond to the total capital (equity plus debt) of an enterprise is beyond the scope of this chapter.

### The Gordon Model

In his 1962 finance text, Myron J. Gordon showed that under the appropriate assumptions, Equation 1.1 is equivalent to the simplified equation represented by Equation 1.2:[3]

$$V_0 = \frac{CF_1}{r - g} \tag{1.2}$$

The Gordon Model initially dealt with dividends, hence it has been called the Gordon Dividend Model, or the Gordon Growth Model.[4]

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[1]The discounted cash flow model is based on time periods of equal length. Because forecasts are often made on an annual basis in practice, we use the terms "periods" and "years" almost interchangeably for purposes of this theoretical discussion.
[2]For purposes of this book, we are discussing enterprises where there is little risk of imminent bankruptcy.
[3]Myron J. Gordon, *The Investment, Financing, and Valuation of the Corporation* (Homewood, IL: Richard D. Irwin, 1962).
[4]Equation 1.2 has become so generalized that it reflects what can be called the generalized valuation model. In practice, $CF_1$ often represents the estimate of earnings for the next period so we can generalize and refer to the cash flow measure as *Earnings*. The expression $(r - g)$ is known as the capitalization rate (see "Glossary," *ASA Business Valuation Standards* [Washington, DC: American Society of Appraisers, 2005], p. 21). And the expression $(1 / (r - g))$ is a multiple of earnings. So the Gordon Model is consistent with the general valuation model:

$$\boxed{\textbf{Value} = \textbf{Earnings} \times \textbf{Multiple}}$$

For Equations 1.1 and 1.2 to be equivalent, the following conditions must hold:

- **$CF_1$** is the measure of *expected cash flow* for the next period (sometimes derived as ($CF_0\, x\, 1 + g$) or otherwise derived specifically).
- Cash flows must grow at the constant rate of $\boldsymbol{g}$ into perpetuity.
- All cash flows must be: 1) distributed to owners; or, 2) reinvested in the enterprise at the discount rate, $\boldsymbol{r}$.
- The discount rate, $\boldsymbol{r}$, must be the appropriate discount rate for the selected measure of cash flow, **CF**.[5]

By comparing Equations 1.1 and 1.2, we see two ways to estimate the value of an enterprise. Equation 1.3 restates Equation 1.1 to reflect constant growth and relates it to Equation 1.2.

- The left portion of Equation 1.3 illustrates a forecast of cash flows growing at a constant rate into perpetuity, discounted to the present.
- With appropriate algebraic manipulation, the left portion of Equation 1.3 reduces to the Gordon Model.

$$\boldsymbol{V}_0 = \left( \frac{CF_0(1+g)}{(1+r)^1} + \frac{CF_0(1+g)^2}{(1+r)^2} + \cdots + \frac{CF_0(1+g)^n}{(1+r)^n} \right) = \frac{CF_1}{r-g} \tag{1.3}$$

### Two-Stage DCF Model

Recall the conditions that must hold for Equations 1.1 and 1.2 to be equivalent expressions. In practice, these conditions may limit the strict application of either expression.

- Application of Equation 1.1 requires a discrete forecast to time period $n$, or effectively into perpetuity. Few forecasts extend reliably beyond five or ten years in practice.

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These factors are so familiar that appraisers sometimes forget their source. Earnings in the generalized valuation model must be clearly defined and the "multiple" must be appropriate for the defined measure of earnings. These comments could be based on common sense, and they are. However, as will be shown, they are also theoretically sound.

[5]In the real world, businesses make reinvestments and accept the returns of these investments, some of which will exceed $r$ and some of which may be less than $r$. This model assumes that all reinvestments will achieve a return of $r$.

- Application of Equation 1.2 requires that the estimate of next year's cash flow grow into perpetuity at a constant rate of $g$. This condition may not be consistent with an analyst's expectations regarding near-term cash flow growth, which may be significantly different from longer-term expectations for growth.

In practice, these two limitations are overcome by use of a "two-stage" DCF model that combines elements of Equations 1.1 and 1.2. The two-stage DCF model is presented in Equation 1.4, and consists of the following two sets of forecast cash flows:

- *Interim Cash Flows (for finite period ending in Year f).* While accurate predictions regarding the future are certainly elusive, diligent analysts can often prepare reasonable forecasts of near-term financial results for many businesses. The left side of Equation 1.4 depicts the Present Value of Interim Cash Flows (PVICF).
- *Terminal Value (all remaining cash flows after Year f).* Following the discrete forecast period, the two-stage DCF model reverts to the Gordon Model, as the accuracy of the analyst's discrete financial forecast wanes, and violation of the constant-growth condition becomes less significant. When discounted to the present from the end of Year $f$, the Present Value of the Terminal Value (PVTV) is obtained.

$$V_0 = \left(\frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \cdots + \frac{CF_f}{(1+r)^f}\right) + \left(\frac{CF_{f+1}/(r-g)}{(1+r)^f}\right) \quad (1.4)$$

**Present Value of Interim Cash Flows (PVICF)** Using this portion of the basic DCF model, the analyst is not constrained by the requirement of constantly growing cash flows during the finite forecast period ending with Year $f$. This part of the equation is the present value of interim cash flows through the finite forecast period ending with Year $f$, or PVICF.

**Present Value of the Terminal Value (PVTV)** Using the Gordon Model, all cash flows are capitalized after Year $f$, assuming cash flows are growing from that point at the constant rate of g. This portion of the equation therefore represents the present value of $CF_{f+1} = CF_f \times (1+g)$.

Appraisers using the two-stage DCF model typically employ discrete forecast periods ranging from about three to ten years or so, followed by application of the Gordon Model as shown in Equation 1.4.[6]

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[6]Alternatively, in practice, many appraisers and market participants use a market-based method that applies current market multiples to the forecasted cash

We can use the two-stage DCF model to illustrate the equivalency between the DCF method and the Gordon Model under the conditions previously specified. In this case, the "proof" of equivalency will be practical rather than algebraic.

### Practical Proof: DCF = Gordon Model

Consider a business enterprise that is expected to generate earnings of $1.0 million next year, followed by growth of 10% per year into the indefinite future.[7] Further, assume that the appropriate discount rate is 20%. Given these assumptions, we can value the enterprise using the Gordon Model (Equation 1.2). We can also value the enterprise using the DCF methodology from Equation 1.4.

Exhibit 1.1 depicts the Gordon Model valuation.

The indicated value for the enterprise using the Gordon Model is $10.0 million. The capitalization rate, $(r - g)$, is 10% (20% – 10%), and the multiple of cash flow is 10.0x (1/10%). Recall the conditions for use of the Gordon Model: *Cash flows are growing at the constant rate of g, and all cash flows are either distributed or reinvested in the enterprise at the discount rate, r*. An additional condition is that the cash flows are distributed (or reinvested) at the end of each year of the forecast. This will be clear in the DCF method shown next.

**EXHIBIT 1.1** Application of the Gordon Model

| **Gordon Model Value Indication** | |
|---|---|
| Next Year's Expected Cash Flow ($CF_1$) ($000's) | **$1,000** |
| Constant Growth Rate of CF (g) | **10.0%** |
| Discount Rate *(r)* | **20.0%** |
| Capitalization Rate $(r - g)$ | **10.0%** |
| Multiple of CF $(1 / (r - g))$ | **10.0x** |
| **Value of Enterprise** | $10,000 |

flow for Year *f* or Year *f*-plus-1. This alternative practice, if employed with reasonable multiples from the public marketplace, should not be considered unusual or incorrect. For a further discussion on this point, see "Practical Observations" at the conclusion of this chapter.

[7] In the "Practical Observations" section at the end of this chapter, we suggest that a long-term *g* of 10% may be on the high side for many discounted cash flow applications. For purposes of this example we ask the reader's indulgence. A 10% growth rate is convenient for calculations and therefore facilitates this discussion.

**Simplified Discounted Cash Flow Valuation Model (Two-Stage Model)**

| | | Interim Cash Flows | | | | | |
|---|---|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** | **5** | **6** |
| Expected Cash Flows ($000's) | *g* = 10% | **$1,000** | $1,100 | $1,210 | $1,331 | $1,464 | **$1,611** |
| Present Value Factors | *r* = 20% | 0.8333 | 0.6944 | 0.5787 | 0.4823 | 0.4019 | |
| *Present Values of Interim Cash Flows* | | *$833* | *$764* | *$700* | *$642* | *$588* | |
| *Terminal Value* | | | | | | **$16,105** | |
| *Present Value of Terminal Value* | | | | | | $6,472 | |

| | | |
|---|---|---|
| Present Value of Interim Cash Flows (PVICF) | $3,528 | *35%* |
| Present Value of Terminal Value (PVTV) | $6,472 | *65%* |
| **Value of Enterprise** | **$10,000** | *100%* |

To estimate the terminal value (present value at the end of year 5 of all cash flows beyond year 5) we apply the Gordon Model to the next (6th) year's cash flow estimate: $CF_6 / (r - g)$

**EXHIBIT 1.2** Application of Two-Stage DCF Model

We can now develop a parallel valuation using the DCF methodology. In doing so, we employ Equation 1.4 in Exhibit 1.2. First, we calculate the present value of cash flows for the finite period (PVICF). At the end of the finite forecast period, we use the Gordon Model to derive the value of all remaining cash flows (from year 6 into perpetuity). We discount this Terminal Value to the present at the discount rate, *r*, to derive the Present Value of the Terminal Value (PVTV). Recall that in this example, it is assumed that cash flows are growing at the constant rate of *g*, or 10%, during the finite forecast period as well as in the perpetuity calculation.

The DCF valuation conclusion is $10.0 million, or precisely the same as the conclusion of the Gordon Model in Exhibit 1.1. In this example, the conditions for use of the Gordon Model are consistent with the explicit assumptions of the DCF model. Value is the sum of the present values of the five interim cash flows ($3.5 million), and the terminal value ($6.5 million). Note the following about this example:

- We assume receipt of each of the interim cash flows by the owners of the enterprise.
- The Present Value of Interim Cash Flows (PVICF) represents $3.5 million, or 35%, of the concluded value of $10 million.
- The Present Value of the Terminal Value (PVTV) represents $6.5 million, or 65% of the total value. This analysis should alert readers to the importance of the terminal value estimation in DCF valuations. For example, with 10% compound growth in cash flow for five years, the

terminal value accounts for almost two-thirds of the total value. If cash flow growth were faster or there were losses during the finite forecast period, the influence of the terminal value on the conclusion would be amplified.

- The starting point for the model is the valuation date (denoted as year 0, or the day prior to the start of year 1). The cash flows are received at the end of each year of the forecast, such that the present value factors for years 1 and 2 are calculated as follows:

  Year 1: $(1/(1+20\%)^1 = 0.8333)$
  Year 2: $(1/(1+20\%)^2 = 0.6944)$

These calculations illustrate discounting in Exhibit 1.2 for the whole periods, i.e., one full year, two full years, and so on. Assessing the merit of this assumption is beyond "the very basics of value."[8] The purpose at this point is simply to focus on the assumptions of the model.

## DIVIDENDS, REINVESTMENT, & GROWTH

Owners of the example enterprise expect to receive a total return equal to the discount rate of 20%. How does this happen? There are two components of the expected return: the current return from expected distributions and the expected growth in the value of the enterprise. The first is the expected return from interim cash flows, which can be described as the yield on current value. For the first period in Exhibit 1.2, cash flow is $1.0 million, which reflects a 10% yield on the current value of $10.0 million. We can also calculate the expected value at the end of each period and see that the yield on current value for each subsequent year is also 10%.[9]

The expected growth rate in value is also 10%, as can be confirmed by the growth of value from $10.0 million today to $16.1 million at the end of

[8] Sensitivity to changes in assumptions is a fact of life in valuation. For example, changing the assumption to reflect receipt of cash flows at mid-year into perpetuity would raise value in this example from $10 million to $10.95 million, or increase it by 9.5%. The sensitivity of the Gordon Model and the DCF model to changes in assumptions is beyond the scope of this chapter.

[9] For example, projected value at the end of year 2 is equal to $12.1 million (year 3 cash flow of $1,210 capitalized by $r - g$ of 10%). Expected cash flow for Year 3 divided by value at the end of year 2 is 10% ($1.210 million / $12.1 million). Under the assumptions of Exhibit 1.2, this expected current return, or current yield, will be 10% for every year.

Year 5 (($\$10.0 \times (1 + 10\%)^5) = \$16.1$). Therefore, the total expected return for the owners of the enterprise in Exhibit 1.2 is 20%, or the discount rate. This is comprised of the yield on current value of 10%, plus the expected growth in value of 10%. The total return of 20% is achieved with full distribution of all interim cash flows.

### Intuitive Impact of Reinvesting Cash Flows

Each period, the owners of a business make one of three decisions:

- Distribute (through dividend or share repurchase) all cash flows; or,
- Retain all cash flows in the business for reinvestment; or,
- Distribute a portion of the cash flows and retain the remainder for reinvestment.

Intuitively, the value of a business whose cash flows are reinvested should grow more rapidly than an otherwise similar but fully distributing business. This makes sense because retained cash flows increase the asset base on which the company generates a return. Said another way, the business that retains a greater portion of its earnings can experience more rapid growth in expected future earnings (upon which expected future value is based).[10]

### Reinvestment and the Gordon Model

As presented in Equation 1.2, the Gordon Model calculates the present value of a cash flow stream growing at a constant rate into perpetuity. The $g$ in Equation 1.2 reflects the expected growth rate in the cash flows (or earnings) of the enterprise. Assuming equality of cash flow and earnings, Equation 1.2 can be rewritten in generalized form as Equation 1.5 to show this relationship specifically:

$$V_0 = \frac{\text{Earnings}}{r - g_e} \tag{1.5}$$

In this case, $g_e$ is the expected constant growth rate in earnings (consistent with the distribution of all earnings to shareholders).

[10]However, retention of earnings does not necessarily imply optimal returns to shareholders. This will become clear when we focus on the importance of the expected reinvestment rate for nondistributing or partially distributing enterprises.

We stated earlier that the Gordon Model expresses the value of a security today as the present value of its expected dividends growing at a constant rate into perpetuity ($g_d$).

$$P_0 = \frac{D_1}{r - g_d} \tag{1.6}$$

Where:
$P_0$ is the expected price of the security
$D_1$ is the expected dividend for the security at the end of period 1
$g_d$ is the expected growth rate of the dividend, $D_1$
$D_1$ represents the portion of earnings to be distributed. To relate Equations 1.5 and 1.6, we can express $D_1$ as follows:

$$D_1 = \text{Earnings} \times \text{DPO}$$
$$\text{DPO} = \text{Dividend Payout Ratio ((dividends as a percentage of earnings))}$$

Equation 1.6 can be rewritten as Equation 1.7:

$$P_0 = \frac{\text{Earnings} * \text{DPO}}{r - g_d} \tag{1.7}$$

If all earnings are distributed (DPO = 100%), Equation 1.7 is equal to Equation 1.6, and the expected growth rate of the dividend ($g_d$) is equal to the expected growth rate of earnings ($g_e$). Further, if we hold constant the discount rate ($r$), the price of the security ($P_0$), and the expected earnings, the expected growth rate in the dividend ($g_d$) must vary inversely with the dividend payout ratio.[11]

In Exhibit 1.3, the expected growth in dividends ($g_d$) is shown to equal the expected growth in the value of the enterprise, which we denote as $g_v$.

**EXHIBIT 1.3** Relationship between Growth in Value and Dividends

$$\frac{D_1}{r - g_d} \times (1 + g_v) = \frac{D_2}{r - g_d}$$
$$1 + g_v = \frac{D_2}{r - g_d} \times \frac{r - g_d}{D_1}$$
$$1 + g_v = D_2/D_1 = 1 + g_d$$
$$g_v = g_d$$

[11] This insight is not particularly new; however, its implications for business valuation are not yet generally recognized. We will explore these implications in the remainder of this chapter.

Substituting $g_v$ for $g_d$ in Equation 1.6 yields the following:

$$P_0 = \frac{D_1}{r - g_v}$$
$$g_v = r - D_1/P_0 \qquad (1.8)$$

In other words, the expected growth in value is equal to the discount rate less the expected dividend yield (Equation 1.8). If the dividend payout percentage is 100%, the expected growth in value is equal to the discount rate less the earnings yield. If the dividend payout percentage is 0% (and all earnings are retained), the expected growth in value is equal to the discount rate. This analysis confirms the intuitive logic that reinvestment accelerates the expected growth in value over the base level of earnings growth without reinvestment. With reinvestment at $r$, the expected $g_v$ increases to offset the diminution in dividend yield such that the expected reinvestment at $r$ will generate the required return of $r$ for the enterprise.

## The Core Business vs. Reinvestment Decisions

We have demonstrated that, under the conditions of the Gordon Model, the value of a business enterprise is unaffected by the level of reinvestment, although the level of reinvestment does affect the components—dividend yield and capital appreciation—of total return.

In order to understand the effect of reinvestment decisions, it is helpful to think conceptually (and somewhat artificially in terms of the way we look at businesses) of all business enterprises as having two components—a core business and a series of incremental investments:

- *The core business*. The core business is the existing enterprise. The core level of earnings is normally expected to grow at a rate consistent with the company's market position and management capabilities (in the context of the relevant economy). When business appraisers discuss the expected (long-term) growth rate of earnings, they should be referring to the growth of this core level of earnings, or $g_e$.

  What is the expected growth in core earnings? This very important concept needs explanation. We define $g_e$ as the level of (constant) long-term growth available to a business assuming that all the net earnings of the business are distributed (i.e., DPO = 100%). This assumption has several important implications, including:
  - Inflationary price increases are achieved (to the extent reasonably available over time).
  - Productivity enhancements are also captured (to the extent reasonably available over time).

- Positive net present value capital investments may be available.[12]

  In other words, the core business operates under the constraint of no earnings retention.[13] Under this constraint, value can be estimated using Equation 1.7 as follows:

$$V_0 = \frac{E_1 \times \text{DPO}}{r - g_d} = \frac{E_1 \times 100\%}{r - g_d} = \frac{\text{Earnings}}{r - g_d} \tag{1.9}$$

  The long-term level of expected core earnings growth for private companies will seldom exceed 10%. In fact, the *long-term* level of expected core earnings growth for larger public companies seldom exceeds 10%, in spite of the fact that earnings for the next one, three, or five years might be expected to grow at rates of 15%, 25%, or more.[14]

- *Incremental investments*. Healthy business enterprises are earnings (cash flow) machines. They are designed to engage in economic activities and to generate earnings and cash flow. When earnings are retained in a business, such earnings should be viewed as being *reinvested* in the business. Over time, the bulk of all value growth in a business tends to result from reinvestment decisions, rather than to the growth in core earnings.

While the distinction between the core business and incremental investments, or the cumulative impact of reinvestment decisions, may seem artificial, it is essential to understanding the nature of value creation. The DCF model can be used to examine both the core business and reinvestment decisions to facilitate this understanding. To do so, we will now focus on *future values*, rather than the *present values* that are the result of the DCF model as presented in Exhibit 1.2. This inversion does not pose any conceptual problems. After all, without the expectation of future value, there is no present value. We use the same valuation example as that in Exhibit 1.2.

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[12]The prospect for positive NPV capital investment (i.e., that which earns a return in excess of $r$) frees companies from the straitjacket imposed by some analysts suggesting that $g_e$ can never exceed the level of inflation. Such an artificial constraint ignores expectations for future value creation and is inconsistent with observed capitalization ratios in the public and guideline transaction markets.

[13]Note that, in the short term, the conditions of no earnings retention can also be satisfied if "excess" capital expenditures and working capital investments are funded with borrowings. The long-term ramifications of such a decision are beyond the scope of this chapter.

[14]Bear in mind that typical public company EPS growth estimates of 10%–20% almost always include the effect of substantial near term reinvestment of earnings.

| | | Today | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|
| **Enterprise Discount Rate** | **20%** | | | | | | | |
| **Expected Growth in Earnings ($G_e$)** | **10%** | | **Future Cash Flows and Values** | | | | | |
| | | **Today** | **1** | **2** | **3** | **4** | **5** | **6** |
| Expected Growth in Value ($G_v$) of Enterprise | | $10,000 | $11,000 | $12,100 | $13,310 | $14,641 | **$16,105** | |
| Projected Cash Flows (Core Earnings = $G_e$) | | $909 | $1,000 | $1,100 | $1,210 | $1,331 | $1,464 | **$1,611** |
| Earnings on Reinvested Cash Flows @ *R* | **20%** | | | $200 | $460 | $794 | $1,219 | *$1,756* |
| Accumulated Reinvested Cash Flows | | | $1,000 | $2,300 | $3,970 | $6,095 | **$8,778** | |
| Present Value of Reinvested Cash Flows | 35.3% | $3,528 | | | | | | |
| Present Value of Terminal Value | 64.7% | $6,472 | | | | | **$16,105** | |
| Value Indication Today | 100.0% | $10,000 | | | | | | |
| | | | | | | | **FV** | **% of FV** |
| Expected Future Value of Core Business | | | $11,000 | $12,100 | $13,310 | $14,641 | $16,105 | 64.7% |
| Expected Future Value of Reinvestments | | | $1,000 | $2,300 | $3,970 | $6,095 | $8,778 | 35.3% |
| | | | $12,000 | $14,400 | $17,280 | $20,736 | $24,883 | 100.0% |
| Expected Appreciation in Value | | | 10.0% | 10.0% | 10.0% | 10.0% | 10.0% | |
| Earnings "Yield" | | | 10.0% | 10.0% | 10.0% | 10.0% | 10.0% | |
| Total Expected Return (by Year) | | | 20.0% | 20.0% | 20.0% | 20.0% | 20.0% | |

**EXHIBIT 1.4** Illustration of Core Business and Cumulative Reinvestments

Exhibit 1.4 adapts the DCF model of Exhibit 1.2 to focus on expected future values, consistent with the two components of the enterprise. Note the primary difference between Exhibit 1.4 and Exhibit 1.2. In Exhibit 1.2, all cash flows were *distributed* and investors achieved a return equal to the discount rate. In Exhibit 1.4, the cash flows are *reinvested* in the business at the discount rate of 20%. The future value of the core business is determined as of the end of each year, given the next year's cash flow expectations, the discount rate of 20%, and expected growth rate in (core) earnings of 10%, or $g_e$.

Several observations about the future value analysis of Exhibit 1.4 help our understanding of the value creation process:

- The expected future value of the core business is $16.1 million, which is identical to the terminal value calculation in the DCF model in Exhibit 1.2. The terminal value comprises 65% of expected future value, just as the present value of the terminal value provided 65% of present value.
- The expected future value of cumulative reinvestments of cash flow is $8.8 million, or 35% of expected future value at the end of five years. The present value of the expected future value of reinvestments is the present value of expected interim cash flows, or $3.5 million (from Exhibit 1.2).
- All reinvestments are assumed to provide a return equal to the discount rate of 20%. If this assumption is violated, the present value of the cash flows to be received by the shareholders will differ from the value of the business enterprise calculated using the Gordon Model. For example, if this company could grow core earnings at 10% and reinvested all cash flows at a net rate of 5% in cash and liquid securities for the first five years, rather than 20%, the present value of the expected cash

flows would fall to $9.2 million from $10 million. This result holds true even if the terminal value is calculated based on the assumption that reinvestments after the terminal year earn the discount rate. We will investigate the impact of this issue on the value of enterprises and minority interests in those enterprises in later chapters.

- The expected return from an investment in this company is 20% per year over the five-year forecast period. The expected return has two sources, the expected growth in value of the core business (10% per year based on $g_e$) and the incremental capital appreciation attributable to reinvestment, which is equivalent to 10% in this case (or $r$ of 20% minus $g_e$ of 10%). Note in Exhibit 1.4 that the forecasted cash flows for Year 6 are $1.611 million and that the earnings on reinvested cash flows are $1.756 million.

In Chart 1.1, we can see the increasing importance of reinvestment in terms of expected future value for a ten-year forecast. Chart 1.1 continues to use the base example valuation but carries the discrete forecast period to ten years:

Chart 1.1 illustrates the magic of compound interest in the form of expected future values of a business. The expected growth in value of the core business, the bottom area of the chart, is based on the expected growth of core earnings, or 10%. As a result, this base value grows from $10 million

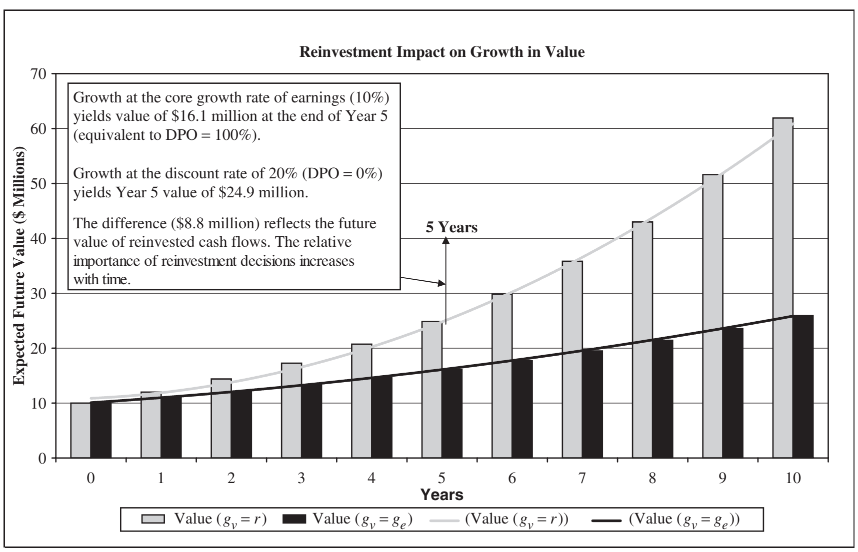


**CHART 1.1** Reinvestment Impact on Growth in Value

today to $16.1 million in five years. The compounding effect of reinvestment decisions is shown in the upper area of the chart. The upper boundary of the chart provides the cumulative effect of the growth of the core business and reinvestment decisions. Expected future value grows to $24.9 million after five years, aided by $8.8 million of future value of reinvested cash flows. The relative importance of reinvestment decisions is magnified with the passage of time, as can be seen as the forecast is extended to ten years in Chart 1.1.

It should be clear from the preceding discussion that the expected growth rate of core earnings is one driver in the determination of expected future value (and therefore, present value). In the present case, $g_e$ is 10%. If all reinvested earnings are invested to yield $r$, the discount rate of 20%, then the total realized return in the example is 20%. The cumulative impact of reinvestment of cash flows raises the total return from 10% (based on $g_e$) to 20%, or $r$.[15]

The (present) value of the business in the example is $10 million. The (future) value of the business at the end of five years will be $24.9 million, which is the sum of the value of the core business growing at $g_e$ and the accumulated value of all reinvestments, which have been made at $r$. Accordingly, the expected growth in value ($g_v$) is equal to $r$, the discount rate of 20%.

At this point, it should be clear that $g_e$ and $g_v$ are different concepts. The inherent growth potential of the core business ($g_e$) is unaffected by the level of reinvestment.[16] The impact of reinvestment decisions is, however, manifest in the expected growth in value. Exhibit 1.5 summarizes the relationship between $g_e$, $g_v$, and the dividend payout ratio.

**EXHIBIT 1.5** Range of Potential Reinvestment Decisions

| | |
|---|---|
| No cash flows are retained<br>DPO = 100%<br>No reinvestment | $g_v = g_e$ |

| | |
|---|---|
| All cash flows are retained<br>DPO = 0%<br>All earning reinvested at $r$ | $g_v = r$ |

[15]Of course, the same return would be earned by shareholders if all earnings were distributed to them and the business did not grow beyond its core earnings.

[16]The potential for positive NPV projects does suggest, however, that $g_e$ can be affected by the quality of available investments.

In other words, $g_v$ will equal or exceed $g_e$ and be less than or equal to $r$, depending upon the expectations regarding the dividend payout ratio.

Two important observations have been made thus far:

- The Gordon Model is equivalent to a discounted cash flow model with certain restrictive conditions, namely, (a) earnings grow at a constant rate into perpetuity and (b) all earnings are either distributed to shareholders or reinvested by the company at the discount rate.
- The expected growth in core earnings of an enterprise ($g_e$ from Equation 1.5) is a distinct concept from the expected growth in value of an enterprise, or $g_v$. The expected growth in core earnings is a function of the markets in which a company operates, the quality of its management, the strength of the economy, inflation, long-term productivity enhancements, and other variables. The expected growth in value is a function of the expected dividend and reinvestment policy of the enterprise and the risk of the enterprise (as manifest in the required return), in addition to the expected growth in core earnings.

We can view the Gordon Model as a summary formulation for the valuation of public and private securities. It is a shorthand way of expressing key relationships between expected earnings (or cash flow), expected growth of those earnings, and risk. Reinvested earnings, if successfully deployed at the discount rate, accelerate the growth in value, $g_v$, toward the discount rate, $r$. If all earnings are retained, and successfully reinvested at the discount rate, then the expected growth rate in value will equal the discount rate.

### Core Earnings Growth ($g_e$) vs. Analysts' Expected Earnings Growth ($g^*$)

In this section, we focus specifically on the relationship between expected growth in core earnings and the expected growth in reported earnings in the public securities markets that we call *analysts*' $g$, or $g^*$.

The Gordon Model calculates the present value of a growing perpetuity. In other words, it is a mathematical relationship akin to the formula for determining the present value of an annuity. In the context of a publicly traded stock, we can specify the Gordon Model as follows:

$$P_0 = \frac{D_1}{r - g_d} \tag{1.10}$$

The price of a publicly traded stock today reflects the present value of all expected future dividends. Ignoring for a moment the possibility of share repurchases by the company, the receipt of dividends represents the only return to shareholders from ownership of the stock—other than a sale of stock in the public market, where all expected future dividends are continuously capitalized in the market price. We derive the price/earnings multiple by dividing both sides of the equation by earnings for the coming year ($E_1$).

$$P_0/E_1 = \frac{D_1/E_1}{r - g_d} \tag{1.11}$$

Recognize that the expression ($D_1/E_1$) is the dividend payout ratio, or DPO.

$$P_0/E_1 = \frac{\text{DPO}}{r - g_d} \tag{1.12}$$

Now, assume that DPO equals 100%. Therefore, the *P/E* of Equation 1.12 is ($1/(r - g)$). This should clarify that valuation analysts, who typically derive earnings multiples as ($1/(r - g)$), are making an implied assumption that all earnings of the company will be distributed, i.e., that the DPO = 100%.

We know it is a rare public company that distributes all of its earnings to shareholders. Therefore, it is important to understand the relationship between the expected earnings growth rates discussed by public securities analysts, the dividend payout ratios of public companies, and the expected earnings growth rates that analysts apply in the derivation of valuation multiples for closely held companies.

Assume the hypothetical company described in Exhibit 1.1 is publicly traded. As shown in Exhibit 1.1, an earnings multiple of 10.0x is appropriate, given the discount rate and core earnings growth assumptions. Assume further that the consensus estimate of analysts is that the company's reported earnings will grow at an annual rate of 17.5%. Does this imply that the company is undervalued with an earnings multiple of 10.0x? Not necessarily.

Why? Assume the company is expected to distribute approximately 25% of earnings as dividends. As shown in Exhibit 1.3, the retention (and subsequent reinvestment) of earnings fuels incremental earnings growth beyond that of the core earnings stream. The public securities analyst is concerned with growth in reported earnings, which includes both core earnings and those attributable to prior reinvestment. According to the dividend discount model described in Equations 1.10 through 1.12, the estimated 17.5% growth in reported earnings is consistent with the earnings multiple of 10.0x and the dividend payout ratio of 25%. Note that the $g_d$ in Equation 1.12 is $g^*$, or 17.5%. In Exhibit 1.6, these values are substituted into Equation 1.12.

**EXHIBIT 1.6** Illustration of the Price/Earnings Ratio

$$10.0 = \frac{25\%}{20\% - 17.5\%}$$

**EXHIBIT 1.7** Overstatement of the Price/Earnings Ratio

$$40.0 = \frac{1}{20\% - 17.5\%}$$

Note that if there is a constant DPO, then $g_d = g^*$. If the valuation analyst had relied upon the $(1/(r - g))$ framework for determining the earnings multiple, consideration of the growth in reported earnings rather than core earnings would result in a price/earnings multiple of 40.0x and a material overvaluation of the company (Exhibit 1.7).

We can see then that an important and predictable relationship exists among the growth in core earnings, the dividend payout ratio, and the expected growth in reported earnings. This analysis assumes a constant dividend payout ratio (or its complement, a constant earnings retention ratio), so the growth in reported earnings will be equal to the growth rate of the dividend.

We can now work with the Gordon Model equation to develop the following relationship between the expected growth rate of core earnings and the expected growth rate in dividends, or assuming a constant payout ratio, earnings in the series of equations labeled Equation 1.13.

$$P_0 = \frac{D_1}{r - g_d} = \frac{E_1}{r - g_e}$$

$$D_1(r - g_e) = E_1(r - g_d)$$

$$g_d = \frac{E_1 - D_1}{E_1} + \frac{D_1}{E_1}(g_e)$$

$$g_d = \text{RR} \times (r) + \text{DPO} \times (g_e) \tag{1.13}$$

The derived relationship is intuitively appealing. RR signifies the earnings retention rate. Reinvested earnings contribute to growth at the discount rate. The portion distributed contributes only the core earnings growth rate. The overall reported earnings growth rate is the weighted average of the two components.

The table in Exhibit 1.8 illustrates these relationships.

**EXHIBIT 1.8** Relationship between Reinvestment and Reported Earnings Growth

| (A) Retention Ratio | (B) Discount Rate | (C) Product | (D) Payout Ratio | (E) $g_e$ Core Earnings Growth | (F) $(D-E)$ Product | (G) $(C+F)$ $g^*$ Reported Earnings Growth |
|---|---|---|---|---|---|---|
| 0% | 20% | 0% | 100% | 10% | 10% | 10% |
| 20% | 20% | 4% | 80% | 10% | 8% | 12% |
| 40% | 20% | 8% | 60% | 10% | 6% | 14% |
| 60% | 20% | 12% | 40% | 10% | 4% | 16% |
| 80% | 20% | 16% | 20% | 10% | 2% | 18% |
| 100% | 20% | 20% | 0% | 10% | 0% | 20% |

As Exhibit 1.8 illustrates, the expected growth in core earnings ($g_e$) is equal to that of reported earnings ($g^*$) only when the dividend payout ratio is 100%, or when there are no expected earnings from reinvested cash flows. Exhibit 1.8 also indicates that, for a given level of core earnings growth, reported earnings growth is inversely related to the dividend payout ratio.

We have seen that the growth in reported earnings estimated by public securities analysts is conceptually distinct from the core earnings growth rate. As our example has illustrated, failure to understand the relationship between these growth rates can result in significant overvaluation of a business. Put more simply, investors do not pay for earnings both as they are created (core earnings) and as the earnings subsequently generate returns after being reinvested by the company (earnings on reinvestment). Investors will only pay for a given dollar of earnings once. If an analyst relies on an estimate of growth in reported earnings, the valuation analysis should be based on cash flows actually received by the investor (a dividend discount model, rather than a single-period income capitalization model based on earnings).[17]

## NET INCOME VS. NET CASH FLOW

In the preceding section, we made what might appear to be an artificial distinction between the core growth in core earnings and the growth in

[17] If a single-period income capitalization model is used, it should be appropriately adjusted for the dividend payout ratio (which would be complicated if the DPO is not expected to be constant over time).

reported earnings. However, the distinction is critical to properly using the Gordon Model and the discounted cash flow model.

### Multiple *g*'s and One *r* for the Gordon Model

Equation 1.14 illustrates four equalities using the algebraic framework of the Gordon Model. Three critical insights should be drawn from these equations.

$$V_0 = \frac{\text{Earnings}}{r - g_e} = \frac{D_1}{r - g_d} = \frac{\text{Earnings} * \text{DPO}}{r - g_d} = \frac{\text{CF}_1}{r - g_{cf}} \qquad (1.14)$$

Recall that Earnings are net of depreciation and taxes, with no reinvestment into the business. Earnings are derived from the core, or existing, business.

$V_0$ is constant. We show multiple expressions that indicate the same value for an enterprise. Now consider the following:

- *Insight 1*. Differences between Earnings and expected cash flow ($\text{CF}_1$) are the result of differences in dividend payout policies.
- *Insight 2*. The expected growth rate, *g*, *varies* with the earnings measure employed (i.e., with DPO changes). This should be apparent, because earnings paid out cannot be retained to finance future growth.
- *Insight 3*. *r*, the discount rate *remains unchanged* with the degree of earnings retention or distribution.

We have shown that there are multiple g's involved in single-period capitalization models:

- $g_e$ is the growth in core earnings. It is associated with the first identity, which capitalizes Earnings.
- $g_d$ is the expected growth rate associated with a particular dividend, $D_1$.
- And $g_{cf}$ is the expected growth rate associated with a particular dividend payout policy, which is to say, with a particular earnings retention or reinvestment policy.

In other words, as the portion of net earnings that is capitalized changes, *g* must change to retain the equality of $V_0$.

Now focus on the fact that *r* did not change in any of the equations. In other words, *r* is the discount rate applicable to expected Earnings, to the expected dividend next period, and to the expected net cash flow of the enterprise. We have a symbolic answer to the frequently asked question: "Does *r* relate to net income or to net cash flow?" Clearly the answer is yes. We now explore the implications of this observation.

### Focus Again on $g_e$—the Long-Term Expected Growth Rate in Earnings

Although they were just stated, the assumptions defining $g_e$ bear repeating. $g_e$ is the constant, long-term growth in earnings achievable by a business that distributes all reported earnings each year. In other words, this level of growth occurs within the following constraints:

- Inflationary price increases are achieved over time.
- Productivity enhancements are also captured over time.
- Incremental working capital requirements are negligible, with incremental assets being financed by incremental liabilities.
- There may be potential for positive NPV capital investments.

$g_e$ is the long-term expected growth rate of the core earnings of a business. Otherwise, there would be some "automatic" level of reinvestment for which there would be no incremental return. Recall that the owners of businesses make one of three decisions each period:

1. Distribute all cash flows or earnings (through dividends or share repurchase) to the owners; or,
2. Retain all cash flows or earnings in the business and reinvest them; or,
3. Distribute a portion of cash flows and retain the rest for reinvestment.

There is no reason to retain earnings if there are no reinvestment opportunities. Reinvestment implies incremental return, or an acceleration of growth from the level of $g_e$ toward $r$, the discount rate. Investors always demand returns equal to the discount rate, $r$. That return can come in the form of current return (yield) or capital appreciation, which is fueled by reinvestment of net earnings.

### Focus Again on $g^*$—the Long-Term Growth Rate in Cash Flow

If $g_e$ is the long-term growth in the net earnings of an enterprise, what is $g^*$? In Equation 1.14, we note that the $g$ of the Gordon Model framework changes with dividend payout policy. This is to be expected, because funds that are distributed provide current returns and are not available to finance future growth. In Exhibit 1.8, we showed that $g^*$, which was characterized as the growth in reported earnings, was different than $g_e$ because of differences in the dividend payout ratio.

We use $g^*$ to represent the expected growth in both reported earnings and net cash flow (assuming a constant dividend payout ratio). Consider the typical definition of net cash flow, which is defined as:

**Net Income (after taxes)**
+ Noncash Charges (depreciation and amortization and, possibly, deferred taxes)
− Net Capital Expenditures (new purchases of fixed assets less disposals)
+/− Incremental Changes in Working Capital
+/− Net Changes in Interest-Bearing Debt
= Net Cash Flow

It is not necessarily obvious from examining this definition, but the reconciling factor between Net Income (*Earnings* from Equation 1.14) and Net Cash Flow ($CF_1$ from Equation 1.14) is the firm's dividend policy. For a firm with attractive growth prospects, net cash flow is usually less than net income as at least a portion of earnings are reinvested to exploit those growth opportunities. The net cash flow ($CF_1$) is distributed, while the difference between net income and net cash flow, the net reinvestment, is retained in the firm to finance growth.[18]

We now see that $g^*$, which was developed in the previous section "Core Earnings Growth vs. Analysts' Expected Earnings Growth," as the *analysts*' *g*, or the expected growth in reported earnings, is also the expected growth rate in net cash flow under the assumption of a constant dividend payout policy.

### The Relationship between Net Income and Net Cash Flow

Exhibit 1.8 presented one way of illustrating the relationship between net income and net cash flow in terms of expected growth rates. Exhibit 1.8 demonstrates that the expected growth rate in reported earnings (net cash flow) increases as the retention rate increases. However, a picture is often worth the proverbial thousand words.

Chart 1.2 shows the long-term relationship between net income (*Earnings*) and net cash flow ($CF_1$) in graphical form as two "strategies" are illustrated. The first strategy distributes 100% of earnings and the second distributes only 75%, retaining 25% to finance future growth. Investors are

[18] Astute readers may object that for private companies, net cash flow is not always distributed on a pro rata basis, and undistributed earnings are not always reinvested efficiently. These objections are valid, and are addressed in detail in Chapter 7.

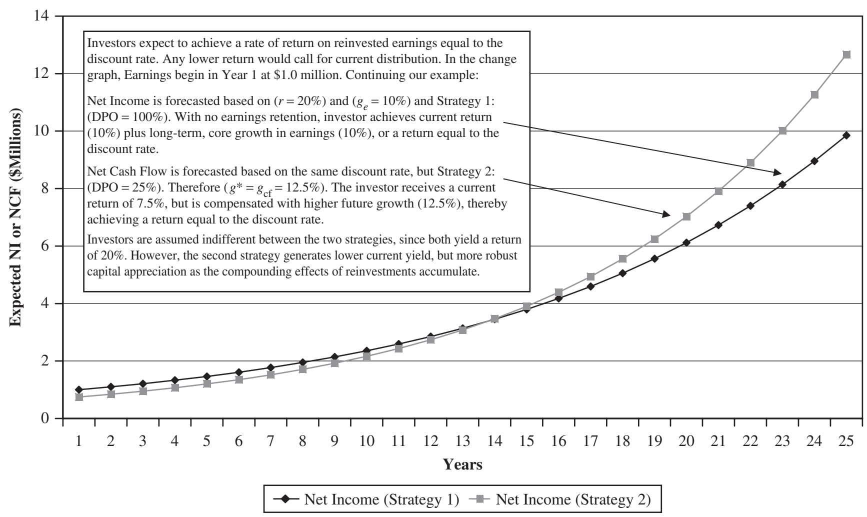


**CHART 1.2** Expected Net Income vs. Expected Net Cash Flow

assumed to be indifferent to the two strategies. The first provides a higher current return and lower expected growth. The second provides a lower current yield, but higher expected capital appreciation.

## Does *r* Relate to Net Income or to Net Cash Flow?

In 1989, Mercer wrote an article introducing the Adjusted Capital Asset Pricing Model (ACAPM), which presented a methodology for building up discount rates based on the Capital Asset Pricing Model(CAPM).[19] While some appraisers had been using similar techniques for some time, to the best of our knowledge, the 1989 article was the first published presentation of the build-up method using the CAPM. Appraisers were (and remain) somewhat divided regarding whether build-up method discount rates are properly applied to the net income or the net cash flow of enterprises.

From a practical viewpoint, analysts at Mercer Capital did (and still do) capitalize net income estimates, rather than net cash flow estimates, because

[19] Z. Christopher Mercer, "The Adjusted Capital Asset Pricing Model for Developing Capitalization Rates: An Extension of Previous 'Build-Up' Methodologies Based Upon the Capital Asset Pricing Model," *Business Valuation Review*, Vol. 8, No. 4 (1989): pp. 147–156. The concepts in this 1989 article form the foundation for the discussion of discount rates in Chapter 6.

we have consistently achieved reasonable results doing so. Other appraisers, in making the case that net cash flow is the appropriate measure for capitalization, have argued the following (with our comments in brackets):

- For companies with attractive growth prospects, net cash flow is generally less than net income. This point was made based on the definition of net cash flow that we examined earlier:

  **Net Income (after taxes)**
  \+ Noncash Charges (depreciation and amortization and, possibly, deferred taxes)
  − Net Capital Expenditures (new purchases of fixed assets less disposals)
  +/− Incremental Changes in Working Capital
  +/− Net Changes in Interest-Bearing Debt
  = Net Cash Flow

  [Recall that if Net Cash Flow is less than Net Income, a portion of earnings is being retained (i.e., DPO < 100%).]
- If the same discount rate and growth rate are developed and used to capitalize both net income and an estimate of net cash flow, capitalized net income will exceed capitalized net cash flow. [We have demonstrated that differences between net income and net cash flow are directly related to differences in growth rates. There is no conceptual or practical link to differences in discount rates.]
- Because the returns used in the Ibbotson data series are derived from *net cash flow to investors* (i.e., dividends plus capital appreciation), the appropriate income measure to capitalize (or to discount) is therefore *the net cash flow of enterprises*. [This conclusion is inconsistent with the analysis in this chapter.]

Given these premises, it would follow that if the ACAPM (build-up) discount rate is used to capitalize net income rather than net cash flow, an *adjustment factor* must be employed to convert the net cash flow discount rate to one applicable to net income. But no one could determine what it should be, except in a general range of 2% to 6% or so. In light of these comments, Mercer wrote an article in 1990 with the title "Adjusting Capitalization Rates for Differences Between Net Income and Net Free Cash Flow."[20] While the title of the article mentions adjusting *capitalization rates*, the article actually developed an adjustment factor to adjust *discount rates*.

[20]Z. Christopher Mercer, "Adjusting Capitalization Rates for Differences Between Net Income and Net Free Cash Flow," *Business Valuation Review*, Vol. 11, No. 4 (1992): p. 201.

The adjustment factor was then evaluated over a range of expected growth assumptions to determine the impact on capitalization rates.

The article showed that under relevant ranges of assumptions regarding earnings retention (dividend payout) policies, the factor would be fairly small. It further concluded that the magnitude of any adjustment factor applicable to $r$ was within the range of judgments routinely made by appraisers regarding discount rates and capitalization rates. These judgments include the choice of Treasury rates, the selection of arithmetically or geometrically derived equity risk premiums (or something in between), and the estimation of size premiums and other company-specific risk premiums.

The analysis in this chapter, however, suggests that the appropriate focus is on different growth rates attributable to net income and net cash flow, rather than different discount rates. There is no adjustment factor for $r$, but rather to $g$, to reflect the effect of earnings retention and reinvestment. In Equation 1.13 (and repeated in Equation 1.15), we developed the means to convert an estimate of $g_e$ into $g^*$, or the expected growth rate in net cash flow given a particular $r$ and retention policy:

$$G^* = \text{RR} \times (r) + \text{DPO} \times (g_e) \tag{1.15}$$

### Further Analysis Regarding Net Income vs. Net Cash Flow

At least two of the most prominent business valuation texts suggest that discount rates (derived using a variety of methods) are applicable to net cash flow rather than net income. For example, consider the following treatment of this topic:

> 501.10 Both of the methods mentioned above [either guideline company or build-up] result in a discount rate for *net cash flow,* which is the benefit stream used in the discounted cash flow method. However, another common benefit stream that may be appropriate is *net earnings*. This benefit stream is used in the capitalized net earnings method. Whatever benefit stream is selected (net cash flow or net earnings), the corresponding discount rate or cap rate must be stated in that same manner. For example, a net cash flow discount rate should not be used to discount net earnings. Instead, a separate net earnings discount rate must be developed, or the benefit stream should be adjusted to net cash flow.[21] [emphasis in original]

[21] Jay E. Fishman, Shannon P. Pratt, and J. Clifford Griffith, *Guide to Business Valuations* 17th ed. (Fort Worth, TX: Practitioners Publishing Company, 2007), pp. 5–6. For convenience, we refer to this as the Fishman text.

The Fishman text then discusses two methods to convert net cash flow discount rates to net income discount rates (at pages 5–8 to 5–10). The first method is based on judgmental comparisons to a rule of thumb range of 3% to 6%. The second method is based on the procedures outlined in the 1990 article quoted earlier. The Fishman text outlines the procedure from the 1990 article for converting a net cash flow discount rate (using the build-up method or the ACAPM method) to one applicable to net income. The discussion in the Fishman text relates specifically to single-period income capitalization methods. The same methodology is discussed in the fourth edition of Pratt's *Valuing a Business*.[22]

It should be clear from the discussion in this chapter, however, that the market's discount rate does not change as a result of changes in dividend policy or with changes in earnings retention decisions. Market-derived discount rates apply to enterprise cash flows. This is true whether they are derived directly from guideline company analysis or indirectly using build-up methods (a more detailed treatment is presented in Chapter 6).

A practical example will illustrate. Exhibit 1.9 displays a two-stage valuation model to value the net cash flows of a hypothetical public

**Hypothetical Public Company Analysis**
**Two-Stage Model to Value Free Cash Flow to Equity**
**Based on Varying Dividend Payout Policies**
(*$Thousands*)

| **Enterprise Assumptions** | **Assume Same** | **Assume Change** | |
|---|---|---|---|
| **Discount Rate ($r$)** | **16.0%** | | |
| **Expected Near-Term Earnings Growth** | | **6.0%** | *Stage 1: For Years 1–10* |
| **Expected Long-Term Earnings Growth** | **6.0%** | **6.0%** | *Stage 2: After Year 10 to Perpetuity* |
| **Dividend Payout %** | | **100.0%** | |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Expected Earnings | Base | **$1,000.0** | $1,060.0 | $1,123.6 | $1,191.0 | $1,262.5 | $1,338.2 | $1,418.5 | $1,503.6 | $1,593.8 | $1,689.5 | **$1,790.8** |
| Less Distributions | | −$1,000.0 | −$1,060.0 | −$1,123.6 | −$1,191.0 | −$1,262.5 | −$1,338.2 | −$1,418.5 | −$1,503.6 | −$1,593.8 | −$1,689.5 | |
| Reinvested Earnings | | $0.0 | $0.0 | $0.0 | $0.0 | $0.0 | $0.0 | $0.0 | $0.0 | $0.0 | $0.0 | |
| Free Cash Flow to Equity | | **$1,000.0** | **$1,060.0** | **$1,123.6** | **$1,191.0** | **$1,262.5** | **$1,338.2** | **$1,418.5** | **$1,503.6** | **$1,593.8** | **$1,689.5** | |
| Terminal Value (CF11 / ($r - g$) | | | | | | | | | | | **$17,908.5** | |
| Total Cash Flows | | $1,000.0 | $1,060.0 | $1,123.6 | $1,191.0 | $1,262.5 | $1,338.2 | $1,418.5 | $1,503.6 | $1,593.8 | $19,598.0 | |
| Present Value Factors | | 0.8621 | 0.7432 | 0.6407 | 0.5523 | 0.4761 | 0.4104 | 0.3538 | 0.3050 | 0.2630 | 0.2267 | |
| Sum of Annual Shareholder CFs | | $5,940.4 | 59.4% | | | | | | | | | |
| Present Value of Terminal Value | | $4,059.6 | 40.6% | | | | | | | | | |
| **Sum of Present Values** | | **$10,000.0** | 100.0% | | | | | | | | | |

**EXHIBIT 1.9** Application of Two-Stage DCF Model to Public Company

[22]Shannon P. Pratt, Robert F. Reilly, and Robert P. Schweihs, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 4th ed. (New York, NY: McGraw-Hill, 2000), pp. 151–201.

**Summary of Free Cash Flow to Equity Model**
**for Four Different Assumptions re Dividend Payout Policy**
(*$Thousands*)

| Assumptions/Results | DPO #1 | DPO #2 | DPO #3 | DPO #4 | |
|---|---|---|---|---|---|
| Discount Rate | **16.0%** | **16.0%** | **16.0%** | **16.0%** | Only difference is DPO policy – otherwise companies exactly identical |
| Expected Near-Term Earnings Growth | **6.0%** | **11.0%** | **13.5%** | **16.0%** | Increasing reinvestment fuels near-term growth of earnings |
| Expected Long-Term Earnings Growth | 6.0% | 6.0% | 6.0% | 6.0% | The second stage calls for 6% long-term growth in all cases |
| Dividend Payout % | **100.0%** | **50.0%** | **25.0%** | **0.0%** | Assume four different levels of constant free cash flow to equity holders |
| Year 1 Expected Net Income | $1,000.0 | $1,000.0 | $1,000.0 | $1,000.0 | |
| Indicated Value | $10,000.0 | $10,000.0 | $10,000.0 | $10,000.0 | Dividend policy should not impact enterprise value |
| Year 11 Net Income | $1,790.8 | $2,839.4 | $3,547.8 | $4,411.4 | The "cost" of a current, higher payout is lower expected future |
| Terminal Value | $17,908.0 | $28,394.0 | $35,478.0 | $44,114.0 | earnings and lower expected terminal values |
| Present Value of Dividends | $5,940.4 | $3,563.5 | $1,957.7 | $0.0 | As the DPO % decreases, the shareholder returns are shifted to |
| Present Value of Terminal Value | $4,059.6 | $6,436.5 | $8,042.3 | $10,000.0 | the future |
| Portion of Expected Value as Terminal Value | 40.6% | 64.4% | 80.4% | 100.0% | |
| | | **Model Discounts/Capitalizes** | | | |
| | Net Income | FCF | FCF | Nothing | **Near-Term (Years 1–10)** |
| | Net Income | Net Income | Net Income | Net Income | **Terminal Value** |
| | The Gordon Model | As the DPO % varies (the ratio of Net Cash Flow to Net Income), the model discounts Free Cash Flow during the interim periods and Net Income for the Terminal Value. The discount rate does not change. | | | |

**EXHIBIT 1.10** Influence of Dividend Policy on Components of Two-Stage DCF Model

company. The discrete forecast period is ten years. The discount rate is 16.0%, and the expected growth rate in core earnings is 6.0%.

The value of the enterprise is $10.0 million. Note that application of the Gordon Model ($1,000/(6% − 6%)) yields the same conclusion. The purpose of this illustration is to show clearly that because all earnings are distributed, growth in reported earnings is equal to expected growth in core earnings at 6.0%.

In Exhibit 1.10, the model is run under three other assumptions regarding dividend payout: 50%, 25%, and 0%. This confirms our prior analysis as summarized in Exhibit 1.8.

In the two-stage model, there is a relationship between the dividend payout policy and the ability of the enterprise to grow during the discrete forecast period. If DPO = 100%, the enterprise can grow at its expected growth in core earnings ($g_e$) of 6.0%. At the other extreme, if DPO = 0%, and all earnings are reinvested, the business can grow earnings (and value) at 16.0%, or at $r$, the discount rate (assuming reinvestment at the discount rate). And with dividend payouts in between, near-term earnings growth is accelerated from the core rate of 6.0% toward the discount rate (again, assuming reinvestment at $r$).

This analysis illustrates again that the discount rate *relates both to the net income and the net cash flow* of business enterprises. Distribution policy does not change the discount rate, but rather, distribution policy (and the implied reinvestment policy) affects growth in reported earnings and cash flows, and capital appreciation.

In practical application, these observations suggest at least the following:

- When applying single-period income capitalization methods, it is entirely appropriate to estimate ongoing earning power based on the net income of an enterprise, and then to capitalize that earning power using a build-up discount rate and expected growth in core earnings ($g_e$).
- When using the discounted cash flow model, the appropriate measure to discount during a discrete forecast period is the net cash flow of the business. This net cash flow would be that level of cash flow distributed to shareholders after all capital expenditures, working capital requirements, debt service, and the like. When developing the terminal value, it is then appropriate to capitalize the expected net income of the enterprise, using the expected growth rate in core earnings at the end of the forecast period.

## PRACTICAL OBSERVATIONS

In this chapter, we have analyzed and reconciled the discounted cash flow model with the Gordon Model. Hopefully, readers will have gained fresh perspective into both models. At this point it is helpful to place our discussion of the DCF and the Gordon Model into the context of everyday valuation practice.

There are three basic approaches to valuation. Within the basic approaches, there are numerous valuation methods, and within various methods, appraisers apply appropriate valuation procedures. Definitions for each of the three valuation approaches can be found in the most current *Business Valuation Standards* of the American Society of Appraisers, but they can be described generally as follows:

- *Cost Approach, or Asset-Based Approach.* The cost approach considers the cost to reproduce or replace the service capability of assets. In business valuation, methods under the cost approach are usually asset-based methods.
- *Income Approach.* Under the income approach, measures of income are discounted to the present or capitalized.[23] The discounted cash flow method is a method under the income approach, as is the single-period

[23]As demonstrated in this chapter, valuation methods that capitalize a measure of current income are a subset of methods that discount projections of future income to the present.

income capitalization method represented by the Gordon Model. The two-stage DCF model represented by Equation 1.4 incorporates both the DCF method (for the PVICF) and the single-period income capitalization (for the PVTV).

- *Market Approach*. The market approach compares financial measures for a subject company with valuation metrics taken from the markets—either the public securities markets, the market for similar companies, or even the market for the securities of the subject (public or private) company. Typical valuation methods under the market approach are the guideline (public) company method and the guideline transactions method.

This book outlines the Integrated Theory of Business Valuation. However, theory must be applied in everyday practice. At this point, it will be helpful to make several observations about the discounted cash flow method. An examination of Equation 1.4 and experience lead to a number of important observations about the discounted cash flow method:

- *The projected earnings are important*. While this observation may seem obvious, the projected earnings must be reasonable for the subject company. What does this mean? It means that the projection for the interim period must make sense in the context of a company's past (if it has one), the market within which the company operates, the performance of similar companies, the capabilities of management, other logical benchmarks, and common sense. A spreadsheet will forecast anything, depending on the inputs. It is up to the appraiser to make logical and reasonable assumptions when forecasting earnings and cash flow.
- *The interim period of the forecast is a matter of appraiser judgment*. If income is stable and growing at a fairly constant rate, a single-period income capitalization method may be appropriate. The DCF method is most helpful when the expected cash flows over the next year or two (or three or four or more) are significantly different from those that may be expected after a finite period of growth, decline, recovery, or stabilization. Appraisers may forecast for any relevant period, although most forecasts are in the range of three to ten years, with five years being the most common.
- *The discount rate, r, should be appropriate to the measure of cash flow selected*. This chapter on the very basics of valuation is conceptual and does not address the practical development of the discount rate, which is treated in Chapter 6. But note that the measure of cash flow can vary from Net Earnings (DPO $= 100\%$) to Net Cash Flow (DPO $< 100\%$), while the discount rate does not change. All cash flows considered

thus far have been net (after-tax) cash flows. If pre-tax cash flows are considered, the discount rate should be adjusted appropriately.[24]

- *The terminal value estimation is critical.* In a typical five-year DCF forecast, the terminal value will account for 60% to 80% or more of the total present value for the method. Obviously, the development of the terminal value is important.
  - Other things being equal, the higher the discount rate, $r$, the lower will be the terminal value (and the indication of value for the method), and the terminal value will account for a lower portion of the total present value.
  - Other things being equal, the higher the expected growth rate, $g$ (i.e., the long-term expected growth in core earnings, or $g_e$), used in the terminal value calculation, the higher will be the terminal value, and it will account for a larger portion of the total present value.
  - In theory, the $g_e$ in the terminal value should not be very high—and most appraisers use long-term $g$'s in the range of 3% or 4% up to 8% or 10% on the high side. Double-digit long-term $g$'s are typically considered unusual, because the implied earnings become astounding over time. Further, higher $g$'s almost certainly include the effect of reinvestment on reported earnings rather than simply growth in core earnings.
  - While this book will scarcely address the r to be used in a DCF forecast based on the total capital of an enterprise, i.e., the weighted average cost of capital (WACC), the terminal value determination is even more sensitive to the selection of g in such scenarios than for forecasts of cash flows to equity holders.
- *Not all appraisers use the Gordon Model to develop the terminal value*. In practice, many appraisers (and market participants) use market-based methods to develop the terminal value multiple. Current market multiples (of net income, pre-tax income, EBITDA, debt-free net income, or others, as appropriate to the selected cash flow measure) are often applied to the forecasted cash flow in Year $f$, the last year of the discrete forecast, or to the year $f + 1$. Some appraisers have suggested that this method is "wrong" because it mixes an income

[24]In our opinion, it is generally preferable to adjust the cash flows rather than the discount rate. Alternatively, analysts need to be keenly aware of whether the projected cash flows are applicable to equity only, or to all capital providers. The Integrated Theory presented in this book deals with cash flows applicable to equity. If the projected cash flows are applicable to all capital providers, the appropriate discount rate is the weighted average cost of capital (WACC), rather than the cost of equity.

approach method (DCF for the finite forecast) and a market approach method (usually guideline company methods for the terminal value). It is unclear why such a mixing is necessarily wrong. Given this procedure's usefulness and widespread use in developing reasonable indications of value using the DCF method, it should not be considered unusual or incorrect—provided that reasonable multiples from the public marketplace (or the market for transactions) are selected.[25] But "reasonable multiples" from the public marketplace today may not be reasonable for application five to ten years from now, particularly if the industry is in a very rapid growth phase and growth is expected to slow in a few years.

- What projections should be used? This observation is the corollary to the statement that the cash flow forecast is important. In many cases, the management of a subject enterprise will provide a forecast (or forecasts) of expected future performance. Appraisers using such forecasts are obligated to test their reasonableness and to develop discount rates that reasonably reflect of the risks of achieving the forecasted results. In the absence of management forecasts, appraisers must take care to develop forecasts that make sense in the context of the relevant market and industry, the company's history, its outlook, and the capabilities of its management.

The discounted cash flow method is an excellent tool for appraisers. However, its use is neither appropriate nor necessary in every appraisal. Some appraisers seem to believe that the DCF method provides "ultimate valuation truth." It does not. It can be used directly to provide reasonable valuation indications and, like other valuation methods, it can be misused. The DCF method can also be used effectively to test the reasonableness of other valuation methods or conclusions.

[25]This point about the reliability of "mixing" approaches is further substantiated by common practice. If the Gordon Model is used to develop a terminal multiple, the very first test of the reasonableness of the derived multiple is to test it in the context of current public market multiples. For example, an appraiser used $1 / (r-g)$ to develop a terminal multiple of 20.0x debt-free net income in a subject company's DCF method. The credibility of that multiple will be supported if the median debt-free net income multiple for his guideline public group is in the range of 18x to 22x or so. However, its credibility might be questioned if the range of similar multiples in his guideline group was from 10x to 14x.

## CONCLUSION

The "very basics of value" are not so basic. This chapter has analyzed the discounted cash flow model and the Gordon Model in considerable detail. Hopefully, we have provided fresh insights and a growing understanding of these two tools that appraisers often use without fully appreciating their implications.

The "very basics of value" form the foundation for the Integrated Theory of Business Valuation introduced in Chapter 3. Before proceeding to the Integrated Theory, however, we examine certain fundamental principles of valuation that are important to applying the "very basics of value" to investing and financial decision-making in the real world, as well as in the hypothetical world of fair market value.