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# A Mathematician's Waterloo

*All the measurements in the world are not worth one theorem by which the science of eternal truths is genuinely advanced.*

—Carl Friedrich Gauss



Napoleon Bonaparte was basking in the height of his glory in 1800, and so was another towering figure of the day—the great Italian-French mathematician Joseph-Louis Lagrange. Whether by military or mathematical might, France dominated Europe, and Napoleon and Lagrange were proof of it. In 1800, both were poised to further their mastery. Napoleon seemed set to knock over the rest of the continent, and Lagrange was ready to conquer the entire mathematical world.

In those days, France was flush with great mathematicians. The French Revolution had arrived squarely in the middle of some of the greatest mathematical progress in history. Prior to the revolution, Paris was the center of the mathematics world, and afterward there was an even greater exchange of mathematical ideas. The French capital attracted and educated some of the greatest minds of the day, including Pierre-Simon Laplace, Adrien-Marie Legendre, Siméon-Denis Poisson, Joseph Fourier, Augustin Cauchy, Lazare Carnot, and the young Sophie Germain. Lagrange was the elder statesman among them and the greatest of all.

Lagrange's reputation was hard-won and well deserved. Decades before, as a self-taught teenager, he had worked out a solution to a problem in calculus that had dogged thinkers for half a century despite attempts by some of the greatest minds of his day to solve it.

This solution launched Lagrange's fame, and he never looked back. Elected a member of the Berlin Academy of Sciences, he soon started solving some of the most profound scientific questions of his day.

Lagrange's prowess won him recognition in France almost instantly. He took several prizes offered by the French Academy in the 1760s for his work on the orbits of Earth's moon and the moons of Jupiter. One of the most famous of these writings was his deduction of the so-called problem of libration: why the same side of the moon always faces Earth. Lagrange showed that this was due to the mutual gravitational attractions of Earth, the moon, and the sun and that it could be deduced from Newton's law of gravitation.

By the age of twenty-five, Lagrange had been proclaimed the greatest mathematician alive. That was then. In 1800, he was still great and venerable, but he was an old man, looking back through the window of the French Revolution onto his glorious youth.

Time was ripe for another revolution in mathematics in 1800, and revolution was one thing Lagrange knew quite well. He had been a firsthand witness to the brutality of the French Revolution. Some of his closest acquaintances were put to the guillotine. As far as revolutions went, the French Revolution was premised on a logical, even mathematical, approach to government. Not all of its numbers were pretty, however. The turmoil began after a weak harvest in 1788 brought widespread food shortages in 1789. The panic over food created an untenable political situation that came to a head during the summer, when crowds stormed the Bastille, which, in addition to being a prison, had become a repository for gunpowder. This led to the collapse of the monarchy and the rise of a constitutional assembly, which within a few years declared a republic, passed a slew of new laws, and tried King Louis XVI for treason, chopping off his head in 1793.

In the year or so that followed, known as the Reign of Terror, more numbers became apparent. Some 2,639 people were decapitated in Paris. Thousands more lost their lives as mass executions played out across France. The horrors of the guillotine are well known, but this was not the only method of slaughter. In the city of Nantes, the victims of the Terror were killed by mass drowning.

During the Reign of Terror, Lagrange was in a precarious situation. He was a foreigner without any real home. He is said to have been an Italian by birth, a German by adoption, and a Frenchman by choice. His roots were in France—his father had been a French cavalry captain who entered the service of the Italian king of Sardinia, settled in Turin, and married into a wealthy family. Lagrange's father was much more adept at enjoying the proceeds of his rich lifestyle than at maintaining his wealth, however. A lousy money manager, he wound up losing both his own fortune and his wife's before his son would see any of it.

Lagrange left home early to seek his fortune, and he found it in the complex, abstract, and imaginatively free world of numbers and math. He rose to become a famous mathematician—the most famous. Frederick the Great appointed young Lagrange to be the director of the Berlin Academy in 1766, and he spent many of his best years there. After Frederick died in 1786, Lagrange had to leave Berlin because of anti-foreign sentiment, so he accepted an invitation by Louis XVI to come to Paris and join the French Academy of Sciences. He took up residence in the Louvre and became close friends with Marie Antoinette and the chemist Antoine Lavoisier.

Lagrange was a favorite of Marie Antoinette's. On the surface, his was perhaps as enviable a position as a mathematician of his day could hope for. But he fell into depression and decadence and became convinced that mathematics, too, was shrinking into decadence. Then the French Revolution arrived. Lagrange could have left at the outset, but where would he go? Not back to Berlin—and not back to Italy, a country he had left as a young man and to which he was no longer connected. For better or worse, France was now his home.

Lagrange lived to regret his decision and almost lost his life when the Reign of Terror started. When he was facing the guillotine, he was asked what he would do to make himself useful in the new revolutionary world. He insisted that it would be more worthwhile to keep him alive. To avoid being put to death, he replied, "I will teach arithmetic."

Napoleon was already a rising star in France when he seized power and began setting up a new state of his own making with himself at the top. He had a keen interest in the educational system, which meant that he soon began to take a keen interest in Lagrange. He selected Lagrange to play a leading role in perfecting the metric system of weights and measures. By the dawning of the new century, Napoleon had come to refer to Lagrange as his ''high pyramid of the mathematical sciences.''

The year 1800 was a new day, both for France and Lagrange. The French had defeated the Dutch, crushed the Prussians, and annexed Belgium. There would soon be a pause followed by an even more explosive period of warfare. Mathematics was a changing field as well. It was quickly becoming more international, and nobody embodied this more than Lagrange. Napoleon made him a senator, a count of the empire, and a ''Grand officer of the legion of Honor.'' He rose higher and higher. Napoleon often consulted with Lagrange between campaigns—not for military advice, but for his perspective on matters of state as they related to philosophy and mathematics.

Lagrange began to lead France's two great academies, the École Polytechnique and the École Normale Supérieure, and was a professor of mathematics at both institutions. For the next century all the great French mathematicians either trained there or taught there or did both. This was where Lagrange reigned supreme. In 1800, nearly sixty-five years old, he was the premier mathematician in France, not through his acquaintance with Napoleon but by his dominance of the field through the previous two generations. He was positioned to profoundly influence legions of young mathematicians through his teaching and his original and groundbreaking work developing methods for dealing with rigid bodies, moving objects, fluids, and planetary systems. Lagrange's greatest discoveries were behind him in 1800, but he was perfectly positioned to reclaim his past glory. He was ready to make history again—and this time more dramatically than ever.

Such was the mood one day in 1800, when Lagrange stood up in front of an august body of his French peers, cleared his throat, and prepared to read what he must have thought would be one of his greatest breakthroughs. He was about to prove Euclid's fifth postulate—the mystery of mysteries. The oldest conundrum in mathematics, it dealt with the nature of parallel lines. Proving it had dogged mathematicians for thousands of years. Lagrange's own life was a microcosm of this history. One of his earliest encounters with mathematics was the work of Euclid, the ancient writer who had first proposed the fifth postulate thousands of years before, in 300 BC, in his treatise *Elements*. Lagrange stumbled upon this problem as a boy. He was aware of it for his entire career.

Euclid could not solve the fifth postulate, nor could the ancient Greek and Roman thinkers who followed him, nor the Arabian scholars who translated Euclid's work into their own language, nor the Renaissance intellectuals who translated the work into Latin and the European languages and studied it at their universities, nor the visionaries of the scientific revolution who developed mathematics as never before, nor finally any one of the many mathematicians who surrounded Lagrange. Some of the best minds of the previous twenty centuries took a stab at proving it, but they all missed the mark.

The audience surely crackled with anticipation as Lagrange took the podium to read his proof. The beauty of his work was well known. A dozen years before, he had published a book called the *Méchanique analytique*, which became the foundation of all later work on the science of mechanics. It was so beautifully written that Alexander Hamilton called it a scientific poem. A century later, it was still considered one of the ten most important mathematical books of all time.

Who in the audience could question that proving the fifth postulate was to have been Lagrange's greatest discovery yet? No mathematician in the previous two thousand years had been able to do it—a period that included Archimedes, Isaac Newton, and everyone in between. Some of the greatest thinkers in the history of mathematics had tried and failed. There was nearly a continuous chain of failed proofs stretching all the way back to Euclid, and it probably went back even further. Nobody knew for sure when mathematicians had started trying to prove the fifth postulate, and nobody in the audience that day could possibly account for how many had tried through the years. The only thing that was clear on the day that Lagrange stood ready to prove the fifth postulate in 1800 was that all before him had failed.

A postulate is a statement without justification—not in the sense that it is absurd but in the sense that it is not or cannot be proven. The fifth postulate basically says that two lines in a plane that are not parallel will eventually cross. Nothing seemed more obvious. Nevertheless, nobody had proved this to be true in all cases.

In Lagrange's day, the fifth postulate was called the scandal of elementary geometry, and working on it was the height of fashion for the cream of mathematicians in Europe. There had been a flood of papers on it in previous years; all aimed to prove it and all failed. There was nothing murkier in math.

How excited must Lagrange have been as he stood up to speak? Perhaps he saw this as a defining moment in history, like Napoleon marching into Germany, Poland, and Spain, his heart thumping like the constant drumbeat of marching armies. But to Lagrange's great embarrassment, he suffered a mathematical Waterloo instead. He made a simple mistake in his proof, and many in the audience recognized it immediately. Then he saw it himself, and abruptly ended his presentation, declaring, ''Il faut que j'y songe encore'' (''I shall have to think it over again''). He put his manuscript in his pocket and left. The meeting went on with further business.

History vindicates Lagrange somewhat. He was the last of a string of frustrated mathematicians who for thousands of years had been trying to prove the fifth postulate directly. They were like mountain climbers trying to scale the highest peak. But they would only ever get so high before they came to a chasm. Proving the fifth postulate was like trying to find a way across this chasm.

It *had* to be true, and it *had* to be proven. The fifth postulate was not some obscure mathematical concept. Euclidean geometry was no mere mathematical treatment of abstract ideas. It cut to the very nature of space itself. Euclid's *Elements* was irreproachable. It was seen as the guidebook to truth in elementary geometry, and geometry was a treatment of reality—the space around us. To read Euclid was to know geometry, and to know geometry was to know reality. As the book was studied and translated through the years, numerous notes and remarks were compiled by various scholars who found ways to explain certain parts of the book. More than a thousand of these accumulated, but none of them could ever successfully address the one remaining unsolved problem.

Who could possibly question the reality of physical space? Certainly not Lagrange or anyone before him. The problem was not with his ideas. The problem was with geometry itself. Lagrange failed for the same reason that all mathematicians for centuries had failed: the fifth postulate could not be proven. But where Lagrange failed, three mathematicians would soon succeed.

In 1800, the world knew nothing of Carl Friedrich Gauss, Nikolai Lobachevsky, and János Bolyai, and the three knew nothing of one another. They would never meet, but they all shared at least one obsession throughout their lives: solving the fifth postulate. If ever there was a mystery calling out for a fresh approach, this was it. Working independently, Gauss, Lobachevsky, and Bolyai would each climb the same mountain and stare down the same chasm. Any effort to prove the fifth postulate was a bottomless pit, and even if they poured a lifetime of effort into it, they never would have reached its floor. Realizing this, many mathematicians had given up trying to find proof. But Gauss, Bolyai, and Lobachevsky all took a new approach. They would solve the mystery of the fifth postulate by asking a completely different question: what if the postulate was not true at all? They would first begin determining what space might be like in their alternative geometry. This would give them insight into the nature of three-dimensional space that few could imagine—certainly not Lagrange in 1800. The reason Lagrange and all the other mathematicians in history could not find this solution was that it required a leap of faith that none of them was ready to make. The solution to the fifth postulate lay in rejecting it entirely and creating a whole new world of geometry.

This new world was given many names in the nineteenth century—astral geometry, imaginary geometry, absolute geometry, hyperbolic geometry—and finally became known as non-Euclidean geometry. It is one of the great achievements of the human mind. It was as if for two thousand years mathematics was an orchestra composed entirely of drums. Mathematicians were like composers seeking to write the arrangement, but they were limited by the instrumentation. Then these three mathematicians came along and examined what music would be like if they were not constrained to the drum. They then invented the piano!

Instead of solving the oldest problem in Euclidean geometry, these three mathematicians invented non-Euclidean geometry. In doing so, they opened up the mathematical orchestra to millions of new arrangements, new problems, and new ways of looking at space. Non-Euclidean geometry was not a correction but a whole new geometry that introduced a strange space in which straight lines are curved and geometric objects become more distorted the larger they are. The oddest thing of all was that the strange new world turned out to be correct.

As Lagrange retreated, embarrassed, from the podium, how could he have known that the real answer to the mystery he had just failed to prove was already in the head of the young Gauss, who was just out of university? How could he have guessed that years later it would be discovered by Lobachevsky, who was then only a boy growing up in a remote part of Russia? How could he have even imagined that Bolyai, who was not even born in 1800, would also come up with non-Euclidean geometry on his own?

After these men formulated their theories, mathematics would never be the same.

Carl Friedrich Gauss coined the term *non-Euclidean,* but he never published anything on the subject in his lifetime. He was without a doubt the greatest mathematician of his day, perhaps one of the three greatest mathematicians who ever lived, so it is strange that one of his greatest discoveries went unmentioned until after his death.

Gauss was also an odd creature, so obsessed with numbers and numerical relationships that he could recall numerical solutions to complicated arithmetic problems he had solved years before. He used to keep many little notebooks filled with numbers, including days of historical note, biblical references, observations, and calculations. He even had one with the dates when his children's teeth came in. For amusement he would record the ages of all of his children and many of his friends in days. A few nights before he died, as if responding to a premonition of his own imminent demise, he did the same with his own age.

Gauss's obsession with numbers was not part of some grand delusional scheme trying to find meaning in a swarm of numerical nonsense. It was merely for enjoyment. Though to an unappreciative observer Gauss may have seemed like a madman, he really wasn't crazy at all. He was just firmly entrenched in numbers and math.

He was a mathematical super-genius—to call him a mathematical genius is to sell him short—possessed of an all-encompassing and deeply penetrating vision that mere genius never approached. Gauss claimed that he learned to do arithmetic before he could walk. He is said to have been able to do cube roots in his head by the age of eight. And he once told one of his students, ''You have no idea how much poetry is contained in the calculation of a logarithm table.'' In fact, he enjoyed using mathematical tables that were cheap and inaccurate so that he could correct them as he went.

Of course, not everything he did was so trivial. He also tackled some of the most important questions of his time and stretched his work across every field of mathematics, from number theory to geometry to probability to analysis. And he made significant contributions to related fields, such as astronomy and physics. He was so successful that many of his contemporaries considered him the greatest mathematician in the world even when he was a young man.

Among other innovations Gauss brought into the world, he introduced the ''normal'' distribution—the Gaussian, or so-called bell curve that represents how observed data are distributed. The bell curve was a way of representing statistically distributed data, and this distribution represented the important concept of probability. Gauss was the first person to use the letter *i* as a notation to indicate imaginary numbers, a standard still in use today, and he came up with the term *complex numbers* to describe those numbers that have both a real and an imaginary component.

Most of all, Gauss is remembered for his stunningly complete and sometimes perfect works of mathematical art. Around the same time that Lagrange was embarrassed in his attempt to prove the fifth postulate, he read Gauss's first book. The old mathematician was profoundly impressed with his much younger contemporary. ''Your *Disquisitiones* have with one stroke elevated you to the rank of the foremost mathematicians,'' Lagrange gushed in a letter to Gauss, also referring to one of the discoveries in the book as ''one of the most beautiful which has been made in a long time.''

Gauss spent his entire life chasing mathematical masterpieces. He considered everything else to be just scaffolding—and often unworthy of publication. He regarded himself in such poetic terms as well. He had Shakespeare's words from *King Lear* inscribed underneath a famous portrait of him by the artist Christian Albrecht Jensen: ''Thou, Nature, art my goddess; to thy laws my services are bound.''

Gauss's devotion to perfection may have stalled mathematics in his lifetime, however. He never published several of his discoveries, and more than one great mathematician of the nineteenth century discovered that some of their greatest work had been previously discovered by Gauss but remained unpublished in his notes. This was also the case with non-Euclidean geometry. According to his own recollections later in life, Gauss was a teenager when he started to figure it out. Throughout his lifetime, though, he rarely revealed even a hint about his thoughts.

Politically, this was a time of change in Europe. Revolution and reaction and their consequences were touching the continent. This would have been an interesting time for Gauss to shake up the mathematical world by exploring and revealing his strange new world of geometry. All he had to do was to publish something—anything. Instead, he said nothing.

When Gauss was a boy, he nearly drowned when he fell in a canal near his house. He was rescued, and nobody could have guessed how close the world came to suffering the cruel fate of losing one of its greatest minds.

Nobody thought much of the boy when he was a child. Born Johann Friedrich Carl Gauss, he went by Carl Friedrich Gauss his whole life. He was the only son of his impoverished parents, Gebhard Dietrich Gauss and Dorothea Benze, and was his father's second son. He had an older half-brother from his father's previous marriage. Gebhard's first wife died two years before Gauss was born.

Gauss's mother, Dorothea, was loving and devoted, but Gauss enjoyed few other advantages in childhood. To say his family was of simple means would be to put it mildly. He was born into severe poverty, at a time when the harsh misery of such an existence often bore down on those in his station. His grandfather was a stonemason and died at the age of thirty, having ruined his body by breathing in dust from the sandstone he worked with. His father lived slightly longer than that, and his mother lived to a ripe old age, but their lives were not much easier.

His paternal grandparents were subsistence farmers who sought to improve their fortunes by moving to the German city of Brunswick as ''half citizens''—part of an overall influx of immigrants to the city at the end of the eighteenth century. Gauss's father worked a series of dead-end jobs. He was a street butcher for a time and a gardener for another. He was a canal tender, then became a part-time bricklayer, his occupation when Gauss was a child.

When Gauss was born, his family was so poor that nobody bothered to make an official record of his birth. He was a commoner and had his childhood documented as befitting one of his station—which is to say nearly not at all. He didn't know when his birthday was until he was an adult—and then only because he figured it out himself. Even his mother could not remember his exact birthday, but she did recall it was a Wednesday and that it was a certain number of days removed from Easter. So when Gauss was a young man, he sat down one day and figured out a simple calculation to determine on which weekend Easter would fall in any given year between 1700 and 1899. Then he worked backward and determined his birthday.

The technique involves dividing the year by a series of numbers, doing another set of calculations with the answers, and then adding the remainders together with the number 22 to give the date of Easter. That Gauss had the patience and discipline to come up with a simple general calculation to solve this problem is incredible. Most people then, as now, would probably hasten to a library or some used bookstore and simply look up in an old record to see on which date Easter fell in 1777, and then figure out their date of birth from there. Gauss used his calculation to determine that he was born on April 30, 1777.

But this was a characteristic of Gauss. This simple boy of humble means would go on to accomplish great things. He would use mathematics to erase the brutality of his impoverished background in his escape from the misery his parents knew. He seems to have completely succeeded in doing so. In fact, as an older man, he was given to recalling fondly the simpler times of his childhood.

Even as a boy, Gauss's mathematical genius appeared. One Saturday when he was three, for instance, his father was dividing up the pay among the other bricklayers. With his son listening intently, he calculated the divided amounts. Gauss supposedly interjected and told his father that the calculation was incorrect. His father was often strict, harsh, and brutal. But he must have softened when, to the astonishment of all, the boy was right.

Gauss was the type of boy who must have seemed a little weird, even to the people who loved him. He was driven to learn and would spend hours doing nothing but reading books. His father, who was thrifty, made him and his brother go to bed as soon as it got dark so as to conserve candles and lamp oil and to save money by not having to heat the house. This did not prevent Gauss from studying his mathematics, though. The boy would sit for hours in the dark reading by the light of animal grease rendered from some carcass, which he had burning in a hollowed-out potato or turnip that he filled with the fat and lit with a homemade wick. He would read for hours by this putrid light until he fell asleep. These flickering flames were enough to illuminate page after page of mathematical text that the boy would pound into his skull every night.

This was a remarkable difference between Gauss and his parents. They were simple, semiliterate folk—if that. Their meager education gave them the barest tools for dealing with numbers, while Gauss received one of the finest educations that could be had in his day and grew to have an incredible facility for manipulating both words and numbers.

In some ways, this transformation from one generation to the next was amazing. Gauss was able to advance himself completely and escape the world of his parents entirely. His brother was a weaver, his father a bricklayer. Gauss's rise from son of an illiterate laborer to one of the top academics of his time was an unusual path for a boy of his station, almost miraculous. Certainly his parents had no expectation that his schooling would amount to anything spectacular. Perhaps they might have hoped for him to become a schoolteacher, maybe even a merchant. They never would have expected him to go to college and university for the next decade and a half and become the greatest mathematician of his day, because such an opportunity didn't exist for people like his parents.

Gauss's own brother thought of him as good for nothing because he always had his head buried in a book. And the boy might very well have been pulled early and permanently away from mathematics to live out his life as a meager peasant, breaking his back at his humble trade for pennies and dying completely forgotten.

However narrow his parent's view of the doors his education could open, they sought to encourage the boy because he definitely had a gift for numbers. At the age of seven, Gauss entered the nearby St. Katherine's school in Brunswick. Although only a small percentage of Germans kids went to school in those days, schools were becoming more accessible to the general populace, more than doubling the literacy rate in Germany over the eighteenth century. These gains were not always extended beyond the basic levels, however, and few continued after the first few years of elementary studies. Gauss might have stopped early too, except that he had the good fortune to impress the right people at the right time. He had that extremely rare combination of a unique mind and rare luck—much as he had been lucky when he was by chance seen falling into the canal by his home and plucked from a watery death. Gauss lived, and he drowned himself in numbers instead.

Two hundred students were crammed in a creaky, musty room at St. Katherine's school as their master, Mr. Büttner, stalked the aisles with a whip in his hand dispensing the cruel lessons of his curriculum. Incorrect answers would be met with lashes. Many of Büttner's students probably had whatever fleeting interest they might have had in mathematics and learning in general whipped out of them by those lashes—but not Gauss. He saved himself from the whip by showing flashes of genius, and his experience ignited a lifelong love of mathematics.

The day Mr. Büttner realized young Gauss had a real gift was probably one like any other. He assigned the children a bit of busy work: adding up every integer between 1 and 100. The students got to work, and Mr. Büttner monitored. Most of the students probably solved this problem in the straightforward manner: sequentially adding up all of the numbers in the set by adding each number to the growing sum:

$1 + 2 = 3;$
$3 + 3 = 6;$
$6 + 4 = 10;$
$10 + 5 = 15;$
$15 + 6 = 21;$
and so on.

How likely are you to make a mistake in any one of these operations? Perhaps minimally, if you know how to add your numbers well—unless you factor in the stress of doing all the additions while at the same time listening to Mr. Büttner as he crept up the aisles. Then you have to consider the number of additions you are making. Adding a hundred numbers in the straightforward manner like this requires ninety-nine operations, and long runs of addition are problematic because ninety-nine operations means ninety-nine chances to get the sum wrong. The chances of making at least one mistake in a hundred are understandably much larger. With the number of additions and the pressure of Mr. Büttner's whip and the cacophony of two hundred children furiously slapping chalk to two hundred slates, it is a wonder that any of the children could get the problem done, let alone correctly. All of the students made mistakes—except for one.

Gauss simply wrote a single number on his slate and immediately put it down. While many of the other boys in the class took a much longer time carefully adding up all the numbers, only to get the sum wrong in the end, Gauss had the answer in mere seconds.

While Mr. Büttner was waiting for the rest of the children to finish, he noticed Gauss sitting in his seat not working. What doubt and scorn he must have felt for the boy! Was little Gauss mocking him? He continued to pace the aisles as the other boys slowly finished their work on the problem. But when he checked the answers, he was surprised to see a single number written on Gauss's slate—shocked, moreover, to find that it was the correct answer of 5,050. How on earth could the boy have guessed the correct answer like that?

In fact, Gauss didn't guess at all. He calculated the answer. What accounted for his speed and accuracy was that he approached the problem differently. Although it appears to require ninety-nine additions of one hundred numbers, the problem is actually much simpler. It can be solved with a simple multiplication of two numbers. Gauss saw that you could add $1 + 100$, $2 + 99$, $50 + 51$, and similar pairs of ascending and descending numbers together. All pairs added up to a sum of 101, and there were exactly 50 such pairs. Thus adding together all the numbers between one and 100 was simply a matter of multiplying 50 and 101, a calculation Gauss found simple enough to do in his head.

To Mr. Büttner's credit, he recognized that this simple little son of a bricklayer had a genius that would have to be fostered. He ordered a more advanced arithmetic book for the boy, and his assistant, Johann Christian Martin Bartels, took a special interest in Gauss and began tutoring him. Bartels was eight years older than young Gauss. He grew up in the same area, lived close by, and was very interested in mathematics himself. Bartels was perhaps the last person Gauss met who knew more about mathematics than he—and only because as an untrained boy, Gauss knew so little at the time, not because Bartels was a particularly knowledgeable scholar. For the rest of Gauss's life, he would have contemporaries—collaborators. But he would always be the master and never again the pupil. Still, Bartels taught a lot to the boy in these sessions.

Working with Gauss on basic mathematics profoundly influenced Bartels as well. After this experience, he began a love affair with the subject that would last the rest of his life. Later in his life, he became a respectable mathematician who had a decent career as a teacher, and he published a number of essays before he died. Bartels profoundly influenced the development of non-Euclidean geometry because he taught two of its inventors how to do mathematics—both Gauss and later the Russian mathematician Lobachevsky, who invented non-Euclidean geometry independently.

Together, Büttner and Bartels also helped free up Gauss's evenings for private study. As a poor child of simple means, he had to do what is now known as piecework most evenings in order to supplement the family's meager income. These were the sort of time-consuming, mindless weaving or assembly tasks that could be farmed out to a boy. His father had a spinning wheel set up in the house, and every evening Carl had to spin a certain amount of flax yarn, which was used to make twine and fishing nets.

But Büttner and Bartels convinced Gauss's father to release the boy from this busywork and let him study instead. They were so convincing that not only did Gauss's father agree to release his son from further burdens and forgo the modest addition to his income, but as if to emphasize this point, he carried the spinning wheel out back, picked up an ax, and chopped it into firewood.

By far the most profound influence Bartels had on Gauss was to bring him into contact with the people who would really make a difference in the young boy's future—the local nobles who could afford to send the lad to better schools and pay for him to concentrate on mathematics. There was little hope of ever being able to succeed without the intercession of a rich, noble benefactor, and Bartels brought Gauss to the attention of just such a man: Eberhard August Wilhelm von Zimmermann.

Zimmermann was a professor and a close adviser to the local ruler, Duke Karl Wilhelm Ferdinand of Brunswick-Wolfenbüttel. Bartels told Zimmermann about Gauss, and Zimmermann brought the boy to the duke's attention. In those days it was fashionable for noble lords like the duke to foster genius within their realm to adorn their courts with mathematicians and other scholars. It was not unusual for a nobleman such as Duke Ferdinand to sponsor a brilliant peasant such as Gauss.

Duke Ferdinand's initial overall impression of Gauss, from the report he received from Zimmermann, was enhanced by the duke's wife, who happened upon the boy reading a book one day in the palace yard. She walked up to him and asked him about it and was immediately impressed that the thoughtful boy had such an in-depth understanding of the subject. She went back to the palace and urged the duke to summon the boy back.

The duke sent one of his men to fetch Gauss, but a misunderstanding caused the man to mistakenly invite Gauss's older brother instead. George Gauss, realizing this was a mistake, sent the messenger along to see his brother. He thought little of the invitation and had no desire to appear before the duke. Years later, however, George would be filled with regret over this when his little brother was a world-famous mathematician. ''If I had known,'' he said, ''I would be a professor now.'' Perhaps he would have become one—if he had been as gifted a thinker as his half-brother. We will never know. What is clear, though, is that young Carl Gauss was someone extraordinary indeed.

In the nineteenth century, there were plenty of parlor-trick mathematicians who became famous for their ability—usually as children—to multiply large numbers in their heads quickly. One boy named Zacharias Dase, who lived at the same time as Gauss, was famous for doing things like computing the square root of a hundred-digit number in less than an hour and multiplying two twenty-digit numbers in six minutes flat. Other mathematical whiz kids would travel as sideshow attractions and perform similar complicated arithmetic tricks for the amused crowds. Such prodigies were usually children, because they often lost their ability to do the tricks as they grew older. In the age before the Internet, before computers, and before electronic calculating machines of any sort, they were the original wunderkinds of the mathematical world.

Gauss was no mere amusement, though. He certainly could perform outstanding feats of mathematics in his head, but his talent was much deeper and more creative. Throughout his life, he sought, and often succeeded, in tackling problems that required months of study or more. By the time he was eleven, he was already a great mathematician. He began to construct and use what are known as infinite series to solve problems. He also worked out a general form of the binomial theorem—something that had puzzled the best mathematicians in a previous century until Isaac Newton discovered it. Newton was older and better educated when he did his work. Gauss was still a boy.

Gauss, on the strength of his promising progress, was rewarded by the duke with a lavish education far beyond anything his parents could have afforded. Under the duke's patronage, Gauss enrolled in Brunswick's prestigious Collegium Carolinum in 1792, and then in the University of Göttingen in 1794. Duke Ferdinand paid for his tuition, provided him with books, and supported him with a modest stipend from his personal fortune after Gauss graduated. The duke supported Gauss even when his state treasury was on the brink of bankruptcy and, as one of his biographies reads, enabled Gauss to permanently exchange ''the humble pursuits of trade for those of science.''

Gauss was forever grateful to his benefactor. ''I owe to your kindness, which freed me from other cares and permitted me to devote myself to this work,'' he wrote to the duke. ''For if your grace had not opened up for me the access to the sciences, if your unremitting benefactions had not encouraged my studies up to this day . . . I would never have been able to dedicate myself completely to the mathematical sciences to which I am inclined by nature.''

The end of the eighteenth century was an exciting time for mathematics, and at the Collegium Carolinum, Gauss read the works of Euler, Lagrange, Newton, and all the great mathematicians of earlier generations. He also began a serious study of geometry and became acquainted with the fifth postulate.

More than fifty years later Gauss would tell a friend that in 1792, when he was fifteen and studying at the Collegium Carolinum, he first realized the basis for non-Euclidean geometry. The foundation of his idea later in life was that if the fifth postulate were not true, there would still be a consistent geometry. In other words, he thought he could reject the fifth postulate and still consider valid geometrical concepts. But it is hard to say concretely what his earliest concepts were because he never published them. Nor are there handwritten notes or any other indication of his views. Still, it was incredible that he was doing this at a time when the rest of the mathematicians in Europe, like Lagrange, were still trying to prove the postulate.

From the Collegium Carolinum, Gauss matriculated into the University of Göttingen, a school founded by King George II of England and built upon the models of Cambridge and Oxford. It had a nice endowment and an equally generous amount of academic freedom. It quickly became one of the most prestigious institutions in Germany and remained so for many years. When Gauss went there, the school was enjoying some of its greatest years and was attracting students from all over Europe. Even scholars from as far away as the United States came to visit the university. Benjamin Franklin, for instance, visited Göttingen when he was formulating plans for the University of Pennsylvania.

Despite his apparent interest and his obvious talent in mathematics, however, Gauss almost didn't pursue the subject when he arrived at Göttingen. He had a gift for words as well, and he might have easily followed something else like law that would put him on a safer path to a more secure income. Humanistic studies were exceptionally strong at Göttingen, and it would have been a natural fit. What turned him around was his first mathematical discovery—also one of his most curious. He discovered how to construct a seventeen-sided figure with only a ruler and a compass.

On March 29, 1796, the day Napoleon left Paris for Italy, where he would lead French troops on his Italian campaign, win the acclaim of the empire, and earn the moniker ''the little corporal,'' Gauss awoke in the morning thinking about regular polygons—shapes like triangles, hexagons, and pentagons where all the sides are the same length and all the angles formed by the adjoining sides are equal.

Since ancient times, mathematicians had been able to construct regular polygons of a certain number of sides. Shapes like the triangle (three sides), square (four), pentagon (five), hexagon (six), and octagon (eight) were all known from the time of the ancient Greeks and before. For millennia, mathematicians, even schoolchildren, had learned to construct such shapes. There seemed to be a limit, however, to how many sides a polygon constructed this way could have.

''It has generally been said since then that the field of elementary geometry extends no farther,'' Gauss wrote in a German journal in 1796. ''At least I know of no successful attempt to extend its limits.'' Extending the limits of geometry is exactly what he did.

In the history of the world, nobody had ever been able to make a septadecagon, a regular seventeen-sided polygon, with only a ruler and a compass. It was not that nobody had thought of constructing such a figure. In fact, mathematicians had attempted to do this since ancient times. Nineteen-year-old Gauss discovered how. As he later wrote, ''After intense consideration of the relation of all the roots to one another on arithmetical grounds, I succeeded on the morning of [that] day before I had got out of bed.''

Gauss did much more than construct a seventeen-sided figure, however. He also solved the problem generally and figured out a formula for determining larger polygons that could also be constructed with a ruler and a compass. In fact, he worked out that a polygon with a given number of sides could be constructed *if* the number of sides is a prime number equal to something known as the Fermat number $2^{2^n} + 1$. This meant that he could construct polygons with a number of sides equal to 3, 5, 17, 257, 65537, . . .

If a seventeen-sided polygon had been so elusive, it is unimaginable that anyone had thought of constructing a 257-sided polygon, let alone a polygon with 65,537 sides.

The seventeen-sided figure delighted Gauss. He always considered it one of his most important discoveries. As mentioned earlier, he kept a notebook of significant dates of importance in his lifetime, such as when someone spotted a new planet or when he had a breakthrough mathematical insight. The discovery of the seventeen-sided polygon was one of these dates. So highly did Gauss regard this figure that he told his friends that he wanted to decorate his tombstone with it. When he died more than half a century later, it did indeed become the thematic adornment on the back of the monument erected in his honor. (It was actually a seventeen-sided star, because the stonemason thought a seventeen-sided figure would look too much like a circle.)

Perhaps one of the reasons that the septadecagon figure was so gratifying to Gauss was that it turned him permanently toward mathematics. It was, in a sense, this discovery that launched his long and amazingly productive career. He never looked back, and by the time he died he was considered the ''prince of mathematics,'' the most famous mathematician of his day and one of the three greatest of all time.

Gauss may have been the last mathematician to contribute to every field of mathematics that existed in his day, including geometry. He made some of the most significant discoveries of his time in geometry—not the least of which was the solution to that burning question of the fifth postulate, which Lagrange and many others had failed to prove by the end of the eighteenth century. In those days, while Lagrange was preparing to read his failed proof in Paris, Gauss began to acquire an almost unique lack of faith in the fifth postulate. Instead of asking how to prove it, he began to question why it needed to be proven at all.

Almost nobody had asked questions like this before Gauss, because the fifth postulate was a central part of geometry, and geometry was an established subject communicated through the centuries as the wisdom of the ancients. The fifth postulate was part of this ancient and incontrovertible tradition handed down through the ages and studied for thousands of years in the form of Euclid's text on geometry, the *Elements*.

The *Elements* was one of the most successful textbooks of all time; it survived the rise and fall of the Roman Empire, persisted in the Middle East, and was translated into Latin, Arabic, English, and a half dozen other languages, becoming a standard source for centuries—even before the advent of the printing press. The next-longest-running scientific book, Ptolemy's *Almagest*, was only in use from the second century AD to the late Renaissance.

After the printing press arrived in Europe, the *Elements* was the first mathematical book to be pressed and bound, and on average about two editions were printed each year after that. By the time Gauss was born, more than a thousand editions had already been printed, and the book had been translated into all the modern European languages. The *Elements* is perhaps second only to the Bible in this regard, and like the Bible, it had become more than just lines on a page. It was the embodiment of a certain type of knowledge—a guide to analytical thinking and the scientific method through the ages.

Its author, Euclid, was also unassailably famous. He is perhaps the only mathematician ever who captured his entire field in one book. Because of the book's endurance, Euclid is sometimes considered the leading mathematics teacher of all time. He was so famous that some Arabian writers claimed that he was really an Arab, the native son of Tyre.

In fact, Euclid was a native son of Alexandria, Egypt. And geometry was not his unique invention, but something he invented in part and borrowed, stole, and largely cobbled together from other, much earlier sources. Centuries of Greek mathematicians who came before Euclid really deserve much of the credit for the *Elements*. Hundred of years of discoveries by people who have been partly or completely lost in time—men and women who witnessed the flower of Greek civilization and are now forgotten—contributed in some way. All of these discoveries found their way into the *Elements*.

Euclid's *Elements* also owes a lot to people who will be remembered forever—like Plato and Aristotle. Plato was not interested in experimentation and regarded it to be a base art. Mathematics, on the other hand, he appreciated fully. He was influenced by one of his teachers, Theodorus of Cyrene, who taught him mathematics. Plato's philosophy was a mathematical one, and he brought a love of mathematics to learned circles that persists to this day. His academy endured in one form or another for more than nine centuries. Moreover, the idea of the academy as a place of learning would greatly influence the course of learning in general and mathematics in particular.

Aristotle was even more influential. He seems to have been the first to consider organized research and the first to classify knowledge into the different disciplines that existed in his time. He was the first to attempt to formally organize science into a logical approach, and for nothing else, Aristotle would be remembered for his large contribution to the Western culture of improving the systematic presentation of mathematics and other subjects.

Aristotle is said to have derived his theory of science from the very fruitful system of mathematics that existed during his lifetime. He organized his approach to science the way that mathematics was organized—as a system of laws derived by logical approaches to proof based on a minimal number of assumptions. Science, like mathematics, took simple definitions and postulates and built upon them.

Plato and Aristotle were both alive slightly before Euclid, so they never saw his masterwork, but they were likely familiar with the same books that Euclid used to write much of the *Elements*. And Euclid was deeply influenced by them. He took the axiomatic system of Aristotle and furthered it, adapting it and applying it to mathematics in his book.

Euclid's book also owes something to the most famous of the ancient Greek mathematicians—Thales and Pythagoras and their followers. They lived hundreds of years before anyone had ever heard of the fifth postulate, and they imported mathematics from Egypt and Babylon in the seventh century BC. The mystery of the fifth postulate really began with them.