

# Chapter 1

## Pre-Pre-Calculus

### *In This Chapter*

- ▶ Refreshing your memory on numbers and variables
- ▶ Accepting the importance of graphing
- ▶ Preparing for pre-calculus by grabbing a graphing calculator

**P**re-calculus is the bridge (or purgatory?) between Algebra II and calculus. In its scope, you'll review concepts you've seen before in math but then quickly build on them. You'll see some brand-new ideas, but even those build on the material you've seen before; the main difference is that the problems get much harder (take going from systems to nonlinear systems, for instance). You keep on building until the end of the course, which doubles as the beginning of calculus. Have no fear! We're here to help you cross the bridge (toll-free).

Because you've probably already taken algebra, Algebra II, and geometry, we assume throughout this book that there are certain things you already know how to do. (We go over them briefly in the Introduction to this book.) Just to make sure, though, we go over each of them in this chapter in a little more detail before we move on to the pre-calculus that is pre-calculus.



If we cover any topic in this chapter that you're not familiar with, don't remember how to do, or don't feel comfortable doing, we suggest that you pick up another *For Dummies* math book and start there. Don't feel like a failure in math if you need to do that. Even the pros have to look up things from time to time. These books can be like encyclopedias or the Internet — if you don't know the material, you look it up and get going from there.

## *Pre-Calculus: An Overview*

Don't you just love movie previews and trailers? Some people show up early to movies just to see what's coming out in the future. Well, consider this section a trailer that you see a couple months before the *Pre-Calculus For Dummies* movie comes out! (We wonder who will play us in the movie.) In the following

list, we present some material you've learned before in math, and then we give you some examples of where pre-calculus will take you next:

- ✓ **Algebra I and II:** Dealing with real numbers and solving equations and inequalities.

**Pre-calculus:** Expressing inequalities in a new way called *interval notation*.

Before, your solutions to inequalities were given as set notation. For example, one solution may look like  $x > 4$ . In pre-calc, you express this solution as an interval:  $(4, \infty)$ . (For more, see Chapter 2.)

- ✓ **Geometry:** Solving right triangles, where all sides are positive.

**Pre-calculus:** Solving non-right triangles, where the sides aren't necessarily always positive.

You've learned that a length can never be negative. Well, in pre-calc you use negative numbers for sides of triangles to show where these triangles lie in the coordinate plane (they can be in any of the four quadrants).

- ✓ **Geometry/trigonometry:** Using the Pythagorean Theorem to find the length of a triangle's sides.

**Pre-calculus:** Organizing information into one nice, neat package known as the unit circle (see Part II).

In this book, we give you a handy shortcut to finding the sides of triangles, which is an even handier shortcut to finding the trig values for the angles in those triangles.

- ✓ **Algebra I and II:** Graphing equations on a coordinate plane.

**Pre-calculus:** Graphing in a brand-new way, with the polar coordinate system (see Chapter 11).

Say goodbye to the good old days of graphing on the Cartesian coordinate plane. You have a new way to graph, and it involves goin' round in circles. We're not trying to make you dizzy; actually, polar coordinates can make you some pretty pictures.

- ✓ **Algebra II:** Dealing with imaginary numbers.

**Pre-calculus:** Adding, subtracting, multiplying, and dividing complex numbers gets boring when the complex numbers are in rectangular form  $(A + Bi)$ . In pre-calc, you'll become familiar with something new called *polar form* and use that to find solutions to equations that you didn't even know existed.

## All the Number Basics (No, Not How to Count Them!)

When entering pre-calculus, you should be comfy with sets of numbers (natural, integer, rational, and so on). By this point in your math career, you should also know how to perform operations with numbers. We quickly review these concepts in this section. Also, certain properties hold true for all sets of numbers; some math teachers may want you to know them by name, so we review these in this section, too.

### The multitude of number types: Terms to know

Dorky mathematicians love to name things simply because they can; it makes them feel special. In this spirit, mathematicians attached names to many sets of numbers to set them apart and cement their places in math students' heads for all of time:

- ✓ **The set of *natural* or *counting* numbers:**  $\{1, 2, 3 \dots\}$ . Notice that the set of natural numbers doesn't include 0.
- ✓ **The set of *whole* numbers:**  $\{0, 1, 2, 3 \dots\}$ . The set of whole numbers does include the number 0, however.
- ✓ **The set of *integers*:**  $\{\dots -3, -2, -1, 0, 1, 2, 3 \dots\}$ . The set of integers includes positives, negatives, and 0.



Dealing with integers is like dealing with money: Think of positives as having it and negatives as owing it. This becomes important when operating on numbers (see the next section).

- ✓ **The set of *rational* numbers, which are the numbers that can be expressed as a fraction where the numerator and the denominator are both integers.** The word *rational* comes from the idea of a ratio (fraction or division) of two integers.

Examples of rational numbers include (but in no way are limited to)  $\frac{1}{2}$ ,  $-\frac{3}{4}$ , and 0.23. If you look at any rational number in decimal form, you notice that the decimal either stops or repeats.

Adding and subtracting fractions is all about finding a common denominator. And roots must be like terms in order to add and subtract them.

- ✓ **The set of *irrational numbers*, which are all numbers that can't be expressed as fractions.** Examples of irrational numbers include  $\sqrt{2}$ ,  $\sqrt{21}$ , and  $\pi$ .
- ✓ **The set of *all real numbers*, which comprise all the sets of numbers previously discussed.** For examples of a real number, think of one number . . . any number. Whatever it is, it's real. Any number from the previous lists would work as an example. The numbers that aren't real numbers are imaginary.

Like telemarketers and pop-up ads on the Net, real numbers are everywhere; you can't get away from them — not even in pre-calculus. Why? Because they include *all* numbers except the following:

- **A fraction with a zero as the denominator:** Such numbers don't exist.
- **The square root of a negative number:** These numbers are called *complex numbers* (see Chapter 11).
- **Infinity:** Infinity is a concept, not an actual number.
- ✓ **The set of *imaginary numbers*, which are square roots of negative numbers.** Imaginary numbers have an imaginary unit, like  $i$ ,  $4i$ , and  $-2i$ . Imaginary numbers were once made-up numbers, but mathematicians soon realized that these numbers pop up in the real world. We still call them imaginary because they're square roots of negative numbers, but they do exist. The imaginary unit is defined as  $i = \sqrt{-1}$ . (For more on these numbers, head to Chapter 11.)
- ✓ **The set of *complex numbers*, which are the sum and difference of a real number and an imaginary number.** Complex numbers appear like these examples:  $3 + 2i$ ,  $2 - \sqrt{2}i$ , and  $4 - \frac{3}{4}i$ . However, they also cover all the previous lists, including the real numbers (3 is the same thing as  $3 + 0i$ ) and imaginary numbers ( $2i$  is the same thing as  $0 + 2i$ ).



The set of complex numbers is the most complete set of numbers in the math vocabulary, because it includes real numbers (any number you can possibly think of), imaginary numbers ( $i$ ), or any combination of the two.

## *The fundamental operations you can perform on numbers*

From positives and negatives to fractions, decimals, and square roots, you should know how to perform all the basic operations on all real numbers. This means adding, subtracting, multiplying, dividing, taking powers of, and taking roots of numbers. The *order of operations* is the way in which you perform these operations.



The mnemonic device used most frequently to remember the order is PEMDAS, which stands for

1. Parentheses (and other grouping devices)
2. Exponents
3. Multiplication and Division, whichever is first from left to right
4. Addition and Subtraction, whichever is first from left to right



One type of operation most of our students overlook or forget to include on the previous list: the absolute value. *Absolute value* is the distance from 0 on the number line. Absolute value should be included with the parentheses step, because you have to consider what's inside the absolute value bars first (because the bars are a grouping device). Don't forget that absolute value is always positive. Hey, even if you're walking backward, you're still walking!

## *The properties of numbers: Truths to remember*

It's important to remember the properties of numbers because you'll use them consistently in pre-calc. However, you often won't see them used by name in pre-calc, but it's assumed that you know when you need to utilize them. The following list presents the properties of numbers:

- ✓ **Reflexive property:**  $a = a$ . For example,  $10 = 10$ .
- ✓ **Symmetric property:** If  $a = b$ , then  $b = a$ . For example, if  $5 + 3 = 8$ , then  $8 = 5 + 3$ .
- ✓ **Transitive property:** If  $a = b$  and  $b = c$ , then  $a = c$ . For example, if  $5 + 3 = 8$  and  $8 = 4 \cdot 2$ , then  $5 + 3 = 4 \cdot 2$ .
- ✓ **Commutative property of addition:**  $a + b = b + a$ . For example  $2 + 3 = 3 + 2$ .
- ✓ **Commutative property of multiplication:**  $a \cdot b = b \cdot a$ . For example,  $2 \cdot 3 = 3 \cdot 2$ .
- ✓ **Associative property of addition:**  $(a + b) + c = a + (b + c)$ . For example,  $(2 + 3) + 4 = 2 + (3 + 4)$ .
- ✓ **Associative property of multiplication:**  $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ . For example,  $(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$ .

- ✓ **Additive identity:**  $a + 0 = a$ . For example,  $0 + -3 = -3$ .
- ✓ **Multiplicative identity:**  $a \cdot 1 = a$ . For example,  $4 \cdot 1 = 4$ .
- ✓ **Additive inverse property:**  $a + (-a) = 0$ . For example,  $2 + -2 = 0$ .
- ✓ **Multiplicative inverse property:**  $a \cdot (\frac{1}{a}) = 1$ . For example,  $2 \cdot \frac{1}{2} = 1$ .
- ✓ **Distributive property:**  $a(b + c) = a \cdot b + a \cdot c$ . For example  $10(2 + 3) = 10 \cdot 2 + 10 \cdot 3 = 50$ .
- ✓ **Multiplicative property of zero:**  $a \cdot 0 = 0$ . For example,  $5 \cdot 0 = 0$ .
- ✓ **Zero product property:** If  $a \cdot b = 0$ ,  $a = 0$  or  $b = 0$ . For example, if  $x(x + 2) = 0$ , then  $x = 0$  or  $x + 2 = 0$ .



If you're trying to perform an operation that isn't on the previous list, then the operation probably isn't correct. After all, algebra has been around since 1600 BC, and if a property exists, someone has probably already discovered it. For example, it may look inviting to say that  $10(2 + 3) = 10 \cdot 2 + 3 = 23$ , but that's incorrect. The correct answer is  $10 \cdot 2 + 10 \cdot 3 = 20 + 30 = 50$ . Knowing what you *can't* do is just as important as knowing what you *can* do.

## Putting Mathematical Statements in Visual Form: Fun with Graphs

Graphs are great visual tools. They're used to display what's going on in math problems, in companies, and in scientific experiments. For instance, graphs can be used to show how something (like real estate prices) changes over time. Surveys can be taken to get facts or opinions, the results of which can be displayed in a graph. Open up the newspaper on any given day and you can find a graph in there somewhere.

Hopefully that answers the question of why you need to understand how to construct graphs. Even though in real life you don't walk around with graph paper and a pencil to make the decisions you face, graphing is vital in math and in other walks of life. Regardless of the absence of graph paper, graphs indeed are everywhere.

For example, when a scientist goes out and collects data or measures things, he or she arranges the data as  $x$  and  $y$  values. Typically, the scientist is looking for some kind of general relationship between these two values to support his or her hypothesis. These values can then be graphed on a coordinate plane to show trends in data. A good scientist may show that the more you read this book, the more you understand pre-calculus! (Another scientist may show that people with longer arms have bigger feet. Boring!)

## Digesting basic terms and concepts

Graphing equations is such a huge part of pre-calc, and eventually calc, so we want to review the basics of graphing before we get into the more complicated and unfamiliar graphs you see later in the book.

Although some of the graphs in pre-calc will look very familiar, some will be new — and possibly intimidating. We're here to get you familiar with these graphs so that you can study them in detail in calculus. However, the information in this chapter is mostly information that your pre-calc teacher or book will assume that you remember from Algebra II. You did pay attention then, right?

Each point on the coordinate plane on which you construct graphs — made up of the horizontal, or  $x$ -axis, and the vertical, or  $y$ -axis, creating a plane of four quadrants — is called a *coordinate pair*  $(x, y)$ , which is often referred to as a *Cartesian coordinate pair*.



The name *Cartesian coordinates* comes from the French mathematician and philosopher who invented all this graphing stuff, René Descartes. Descartes worked to merge algebra and Euclidean geometry (flat geometry), and his work was influential in the development of analytic geometry, calculus, and cartography.

A *relation* is a set (meaning one or more) of ordered pairs that can be graphed on a coordinate plane. Each relation is kind of like a computer that expresses  $x$  as input and  $y$  as output. You know you're dealing with a relation when it's set in those curly-brackets (like these:  $\{ \}$ ) and has one or more points inside. For example,  $R = \{(2, -1), (3, 0), (-4, 5)\}$  is a relation with three ordered pairs. Think of each point as (input, output) just like from a computer.

The *domain* of a relation is the set of all the input values from least to greatest. The domain of set  $R$  is  $\{-4, 2, 3\}$ . The *range* is the set of all the output values, also from least to greatest. The range of  $R$  is  $\{-1, 0, 5\}$ . If any value in the domain or range is repeated, you don't have to list it twice. Usually, the domain is the  $x$  variable and the range is  $y$ .



If different variables appear, such as  $m$  and  $n$ , input (domain) and output (range) usually go alphabetically, unless you're told otherwise. In this case,  $m$  would be your input/domain and  $n$  would be your output/range. But when written as a point, a relation is always (input, output).

## Graphing equalities versus inequalities

When you first figured out how to graph a line on the coordinate plane, you learned to pick domain values ( $x$ ) and plug them into the equation to solve for the range ( $y$ ). Then you went through the process multiple times, expressed each pair as a coordinate point, and connected the dots to make a line. Some mathematicians call this the ol' *plug and chug* method.

After a while of that tedious work, somebody said to you, "Hold on! There's a shortcut." That shortcut is called *slope-intercept form* —  $y = mx + b$ . The variable  $m$  stands for the *slope* of the line (see the next section), and  $b$  stands for the  $y$ -intercept (or where the line crosses the  $y$ -axis). You can change equations that aren't written in slope-intercept form by solving for  $y$ . For example, graphing  $2x - 3y = 12$  requires you to subtract  $2x$  from both sides first to get  $-3y = -2x + 12$ . Then you divide every term by  $-3$  to get  $y = \frac{2x}{3} - 4$ . This graph starts at  $-4$  on the  $y$ -axis; to find the next point, you move up two and right three (using the slope). Slope is always a fraction because it's rise over run — in this case  $\frac{2}{3}$ .

*Inequalities* are used for comparisons, which are a big part of pre-calc. They show a relationship between two expressions (we're talking greater than, less than, or equal to). Graphing inequalities starts exactly the same as graphing equalities, but at the end of the graphing process (you still put the equation in slope-intercept form and graph), you have two decisions to make:

- ✓ Is the line *dashed* —  $y <$  or  $y >$  — or is the line *solid* —  $y \leq$  or  $y \geq$ ?
- ✓ Do you shade under the line —  $y <$  or  $y \leq$  — or do you shade above the line —  $y >$  or  $y \geq$ ? Simple inequalities (like  $x < 3$ ) express all possible answers. For inequalities, you show all possible answers by shading the side of line that works in the original equation.

For example, when graphing  $y < 2x - 5$ , you follow these steps:

1. Start off at  $-5$  on the  $y$ -axis and mark a point.
2. Move up two and right one to find a second point.
3. When connecting the dots, you produce a straight line that will be dashed.
4. Shade on the bottom half of the graph to show all possible points in the solution.

## Gathering information from graphs

After getting you used to coordinate points and graphing equations of lines on the coordinate plane, typical math books and teachers will begin to ask



you questions about the points and lines that you've been graphing. The three main things you'll be asked to find are the distance between two points, the midpoint of the segment connecting two points, and the exact slope of a line that passes between two points. Away we go in the following sections!

### Calculating distance



Knowing how to calculate distance by using the information from a graph comes in handy in pre-calc in a big way, so allow us to review a few things first. *Distance* is how far apart two objects, or two points, are. To find the distance,  $d$ , between the two points  $(x_1, y_1)$  and  $(x_2, y_2)$  on a coordinate plane, for example, use the following formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

You can use this equation to find the length of the segment between two points on a coordinate plane whenever the need arises. For example, to find the distance between A(-6, 4) and B(2, 1), first identify the parts:  $x_1 = -6$  and  $y_1 = 4$ ;  $x_2 = 2$  and  $y_2 = 1$ . Plug these values into the distance formula:

$d = \sqrt{(2 - -6)^2 + (1 - 4)^2}$ . This simplifies to  $\sqrt{73}$ .

### Finding the midpoint



Finding the midpoint of a segment will pop up in pre-calc topics like conics (Chapter 12). To find the midpoint of the segment connecting two points, you just average their  $x$  values and  $y$  values and express the answer as an ordered pair:

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

You can use this formula to find the center of various graphs on a coordinate plane, but for now you're just finding the midpoint. You find the midpoint of the segment connecting the two points  $\overline{AB}$  (see the previous section) by using the previous formula. This would give you  $\left( \frac{-6 + 2}{2}, \frac{4 + 1}{2} \right)$ , or  $(-2, \frac{5}{2})$ .

### Figuring a line's slope



When you graph a linear equation, slope plays a role. The *slope* of a line tells how steep the line is on the coordinate plane. When you're given two points  $(x_1, y_1)$  and  $(x_2, y_2)$  and are asked to find the slope of the line between them, you use the following formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

If you use the same two points A and B from the previous sections and plug the values into the formula, the slope is  $-\frac{3}{4}$ .

Positive slopes always move up and to the right on the plane. Negative slopes either move down and right or up and left. (Note that if you moved the slope down and left, it would be  $\nearrow$ , which is really positive.) Horizontal lines have zero slope, and vertical lines have undefined slope.



If you ever get the different types of slopes confused, remember the skier on the ski-slope:

- ✓ When he's going uphill, he's doing a lot of work (+ slope).
- ✓ When he's going downhill, the hill is doing the work for him (– slope).
- ✓ When he's standing still on flat ground, he's not doing any work at all (0 slope).
- ✓ When he hits a wall (the vertical line), he's dead and he can't ski anymore (undefined slope)!

## *Getting a Grip on a Graphing Calculator*

We *highly* recommend that you purchase a graphing calculator for pre-calculus work. Since the invention of the graphing calculator, math classes have begun to change their scope. Some teachers feel that the majority of the work should be done using the calculator. More conservative math teachers, however, won't even let you use one. Your instructor should make his views quite apparent that first day of school. A graphing calculator does so many things for you, and even if a teacher won't allow you to use one on a test, you can almost always use one to check your work on homework problems.

There are many different types of graphing calculators, and their individual inner workings are all different. As far as which one to purchase, ask advice from someone who has already taken a pre-calc class, and then look around on the Internet for the best deal. (In our opinion, the TI-89 or TI-89 Titanium is the greatest calculator ever, as of press time — if you can figure out how to use it [we're still learning!].)



Just a hint: If you can find one with an exact/approximate mode, you'll thank us later because it will give you exact values (rather than decimal approximations), which is often what teachers are looking for.

We recommend that if, by chance, you're allowed to use a graphing calculator, you still do the work by hand. Then use your graphing calculator to check your work. This way, you won't become dependent on technology to do work for you; someday, you may not be allowed to use one (a college math placement test, for example).



Many of the more theoretical concepts in this book, and in pre-calc in general, are lost when you use your graphing calculator. All you're told is "plug in the numbers and get the answer." Sure, you get your answer, but do you really know what the calculator did to get that answer? Nope. For this purpose, this book goes back and forth between using the calculator and doing complicated problems long hand. But whether you're allowed to use the graphing calculator or not, be smart with its use. If you plan on moving on to calculus after this course, you need to know the theory and concepts behind each topic.

We can't even begin to teach you how to use your unique graphing calculator, but the good *For Dummies* folks at Wiley supply you with entire books on the use of them, depending on the type you own. We can, however, give you some general "heads up" on their use. Here's a list of hints that should help you use your graphing calculator:

- ✓ **Always double-check that the mode in your calculator is set according to the problem you're working on.** Look for a button somewhere on the calculator that says "mode." Depending on the brand of calculator, it will allow you to change things like degrees or radians, or  $f(x)$  or  $r(\theta)$ , which we discuss in Chapter 11. For example, if you're working in degrees, you must make sure that your calculator knows that before you ask it to solve a problem. The same goes for working in radians. Some calculators have over ten different modes to choose from. Be careful!
- ✓ **Make sure you can solve for  $y$  before you try to construct a graph.** You can graph anything in your graphing calculator as long as you can solve for  $y$ . The calculators are set up to accept only equations that have been solved for  $y$ .



Equations that you have to solve for  $x$  often aren't true functions and aren't studied in pre-calc — except conic sections, and students generally aren't allowed to use graphing calculators for this material because it's entirely based on graphing (see Chapter 12).

- ✓ **Be aware of all the shortcut menus available to you and use as many of the calculator's functions as you can.** Typically, under your calculator's graphing menu you can find shortcuts to other mathematical concepts (like changing a decimal to a fraction, finding roots of numbers, or entering matrices and then performing operations with them). Each brand of graphing calculator is unique, so read the manual. Shortcuts give you great ways to check your answers!
- ✓ **Type in an expression exactly the way it looks and the calc will do the work and simplify the expression.** All graphing calculators do order of operations for you, so you won't even have to worry about the order. Just be aware that some built-in math shortcuts automatically start with grouping parentheses.

For example, the calculator we use starts a square root off as  $\sqrt{\phantom{x}}$ , so all information we type after that is automatically inside the square root sign until we close the parentheses. For instance,  $\sqrt{4+5}$  and  $\sqrt{4}+5$  represent two different calculations and, therefore, two different values (3 and 7, respectively). Some smart calculators even solve the equation for you. In the near future, you probably won't even have to take a pre-calc class; the calculator will take it for you!

Okay, now you're ready to take flight into pre-calculus. Good luck to you and enjoy the ride!