

Designing a Series of Suspension Footbridges

- ▶ *Basic definitions of statics: loads, forces, tension, compression, stress*
- ▶ *Free-body diagrams; vectors and scalars; static equilibrium of concurrent forces*
- ▶ *The force polygon and funicular polygon for funicular structures; Bow's notation*
- ▶ *Detailing steel rod elements in tension, and anchoring to rock*
- ▶ *Lateral stability; stiffening a tensile structure*
- ▶ *Construction detailing and planning*

We have been commissioned to design a series of footbridges for a new scenic trail that will wind through a deep, narrow canyon in a national park in the southwestern United States. The walls of the canyon are often vertical and sometimes overhang, so that the trail must move from one side of the canyon to the other at a number of locations to follow a route that will avoid the steepest walls and minimize excavation of the rock. The lengths (*spans*) of the bridges will vary between 40 and 100 ft. The *decks* of all the bridges (the walking surfaces) will be 4 ft wide.

DESIGN CONCEPT

We have already developed, in cooperation with the Park Service, a basic design concept and a simple system of components for making these bridges (Figures 1.1, 1.2). Because of the remoteness of the bridge sites and the difficulty of working in the narrow canyon while standing on

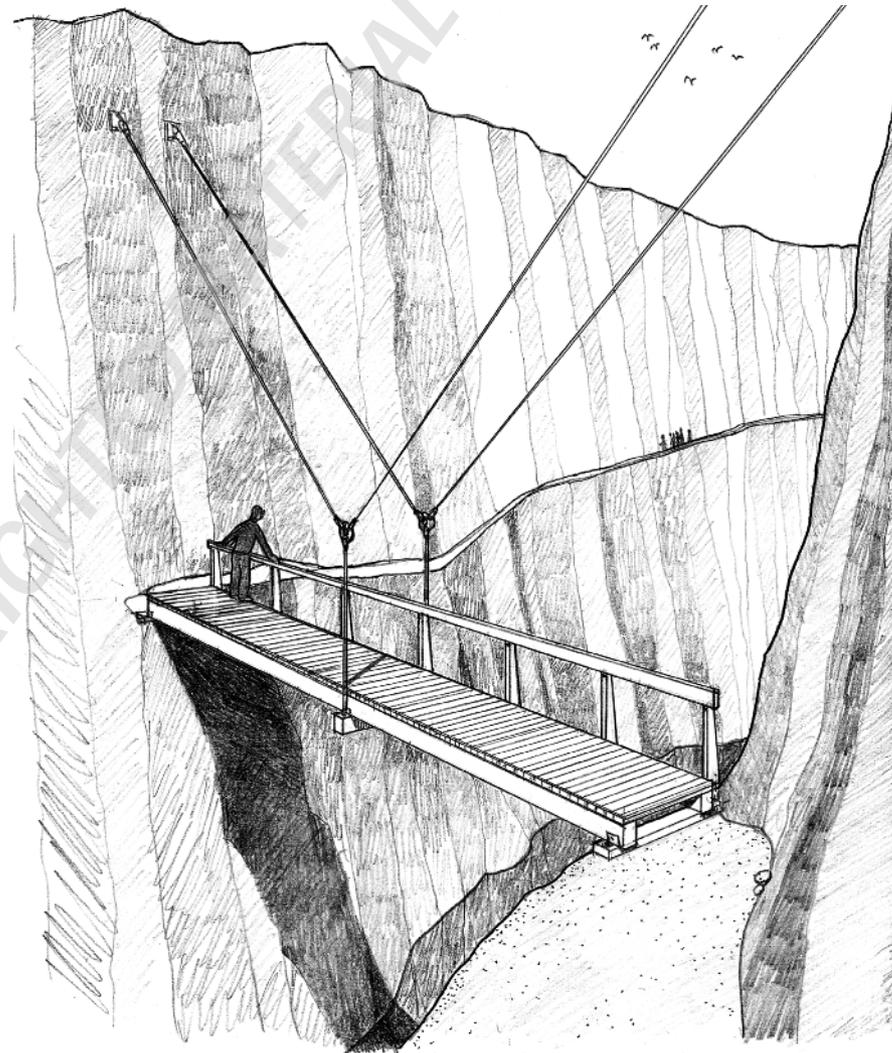


Figure 1.1 This suspension span of 40 ft is the first bridge we will consider in this chapter.

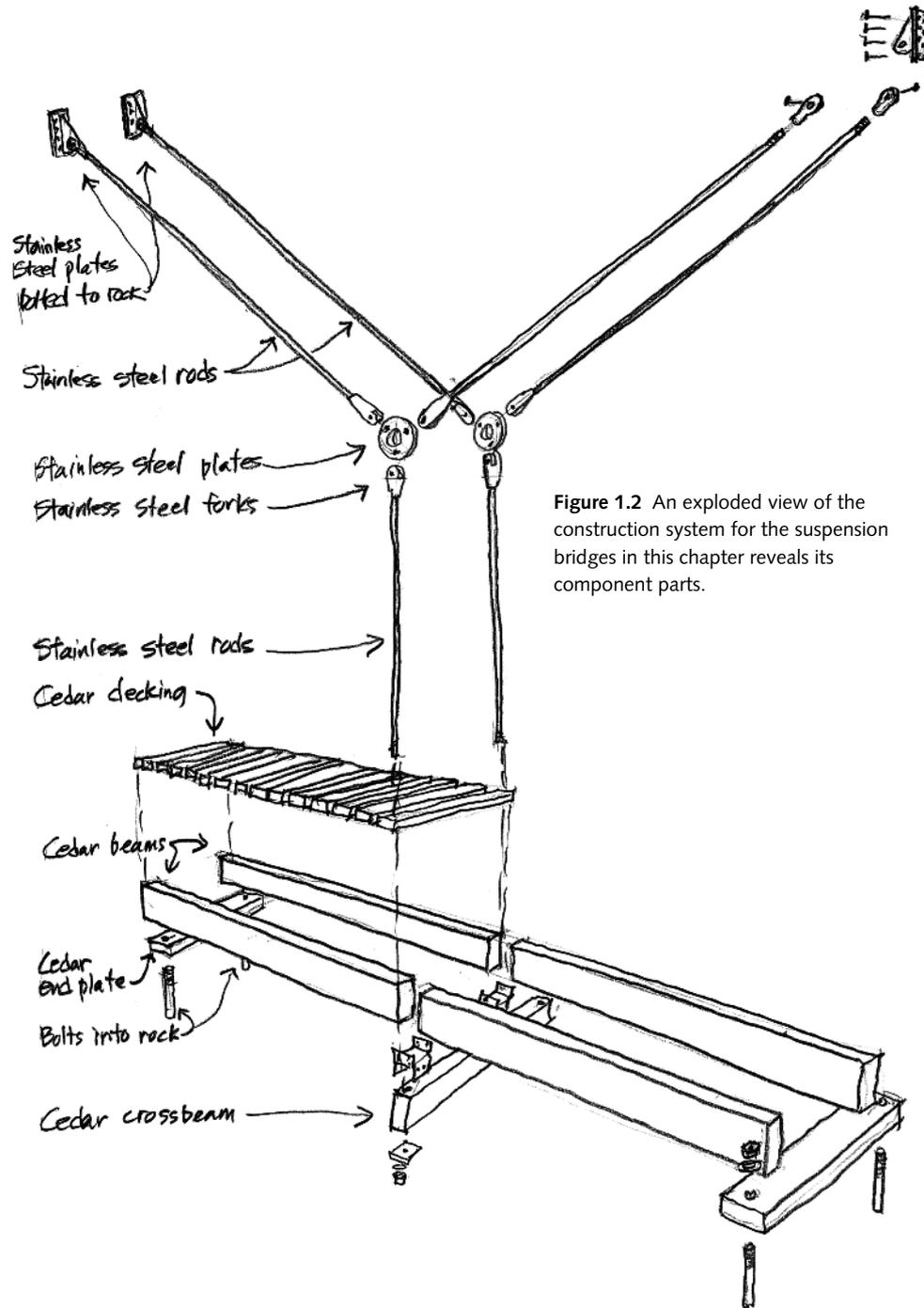


Figure 1.2 An exploded view of the construction system for the suspension bridges in this chapter reveals its component parts.

narrow rock ledges, as much of the work as possible will be done on components in a contractor's workshop. They will then be trucked to the site, where a construction crew will assemble them.

The beams of each bridge are made of wood and will be brought to the site in 20-ft lengths. Shorter lengths can be cut as needed to adjust bridge lengths to particular sites. We will support the ends of the beams that occur over the empty space of the canyon on short wood crossbeams that hang from steel rods. The rods will transmit their forces to steel plates anchored into the rock of the canyon walls. The beams, crossbeams, and rods will be used as a modular system to build *suspension bridges* of any necessary length for this trail (Figure 1.3). Suspension bridges tend to be lighter in weight than any other kind of bridge, which makes them particularly appropriate for the remote, difficult sites where they must be built in this park.

We have selected rods rather than cables because these bridges are small and the loads to be supported are correspondingly low. Cables, if we were to use them, would be very small in diameter, which would make them vulnerable to vandalism. Because the steel from which the rods are made is not as strong as that used in cables, the rods that we will use will be somewhat larger in diameter than cables of the same capacity. Rods are also easier and less costly to connect than cables. We will learn about cables and their connections in Chapter 2, in the context of a structure with much longer spans, where cables are appropriate and economical.

The beams and deck boards for the bridges are sawn from red cedar logs. We will use them in their rough, unplanned state, in keeping with the rustic nature of the canyon. Furthermore, rough beams are enough larger in dimension than planed ones that they have substantially larger structural capacities. Red cedar contains natural substances that are toxic to decay fungi, making the wood resistant to rot. However, cedar is a very soft, low-density wood that erodes gradually when exposed to severe winter weather, losing a few hundredths of an inch from each

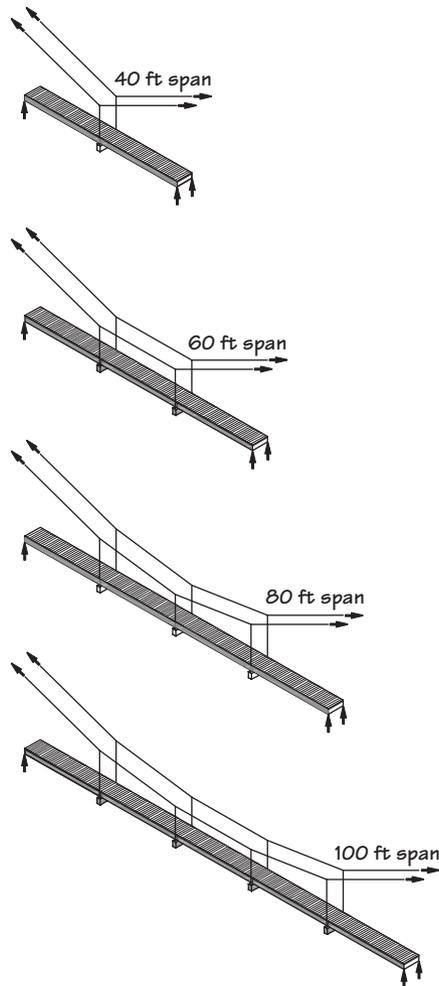


Figure 1.3 The construction system is modular, enabling it to be used for bridges of various spans.

surface every year. We have specified larger beams than are structurally necessary in order to provide for a few decades of erosion before they will have to be replaced.

With *stainless steel* rods and fittings, which do not rust, and cedar beams, which do not decay, the bridges will need no paint or chemical preservatives and should last for decades with little maintenance.

THE CHALLENGE

Our challenge is to design the bridges for this trail individually, taking into account for each of them the length of its span and the places above it where the rock is sufficiently solid to support the steel anchor plates. All the bridge sites have been surveyed to provide us with this information. As part of our design work, we need to determine the shapes that the rods will take in each situation, and the forces that they must transmit. This will enable us to specify the required lengths and diameters of the rods.

CONSTRUCTION DETAILS

We have given considerable thought to how the bridges will be put together. Figure 1.4 shows details of the major features. The rods are connected to the rock walls of the canyon with stainless steel anchor plates. Each anchor plate is secured with stainless steel bolts that are inserted into holes drilled in the rock and embedded there with *grout*, a fine-grained, high-strength mixture of sand, portland cement, and chemical admixtures that increase its strength and limit its shrinkage. The grout is poured into the holes in the form of a paste that hardens to unite the bolts securely with the rock. The rods, anchor plate assemblies, and other metallic components are all custom-fabricated for each bridge. We will determine the diameters of the rods in accordance with the amount of force each must carry when the bridge is fully loaded.

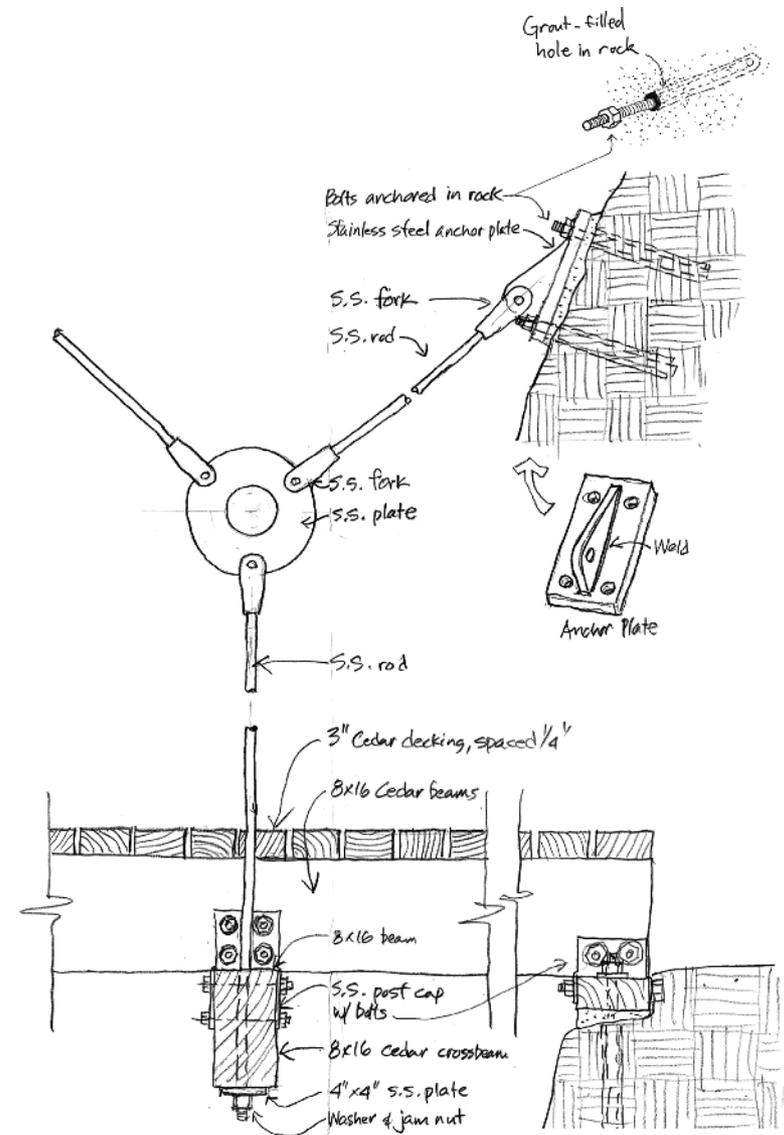


Figure 1.4 Typical details of the suspension bridge construction system.

The *fabricator* of the steel rods will prepare them to the exact lengths needed for each bridge. A *jaw fitting* (also called a *fork* or *clevis*) will be used wherever a rod joins a plate. Where a rod supports a wood beam, it will pass vertically through a hole drilled in the beam and a matching hole in a stainless steel plate 3/8 in. thick on the bottom of the beam (Figure 1.4). A circular *washer* and stainless steel *jam nut* transmit the force in the rod securely to the plate. The plate spreads this force over a large enough area of wood that the wood is not crushed. The jam nut is designed to develop a high degree of friction against the threaded rod so that it will not unscrew accidentally. The screw threads and nuts allow for easy assembly and fine-scale adjustment of the vertical positions of the beams, as well as easy disassembly when needed.

Where a vertical rod from a beam meets the sloping rods from the rock anchors, we will attach the rods with forks to a circular steel *plate* connector, as shown in Figure 1.4. A jaw fitting at the bottom of the plate will transfer force from the vertical rod to the plate, and the plate will pass this same force to the sloping rods and thence to the *anchor plates* and the rock walls of the canyon.

The details of the crossbeam, beams, and decking are straightforward. The deck boards will be nailed to the

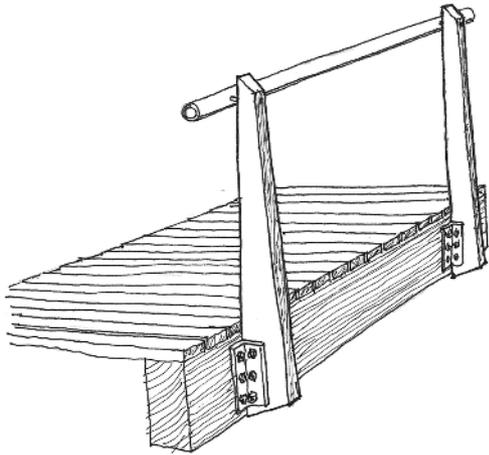


Figure 1.5 A sketch design for the handrail.

beams and spaced about 1/2 in. apart so that they will not retain water by capillary action after a rain or snow melt. The ends of each bridge deck will be held down by bolts anchored with grout into holes drilled in the rock.

A handrail must be provided on one side to give hikers something to hold as they cross the bridge. Because the trails that lead to the bridge are precarious and have steep drop-offs unprotected by railings, full railings and balusters are not required here as they are in buildings. Nevertheless, the handrail must be sturdy and reliable. We propose a steel pipe railing supported by tapered wood posts. The posts are fastened securely to the outside face of the deck beam with stainless steel angles and bolts (Figure 1.5).

FINDING FORCES IN BRIDGE #1

Figure 1.6 is a section through the canyon at a place where a bridge of 40-ft span is required. The bridge is drawn accurately to scale. The center of the bridge, where the beams join one another, is supported by a system of rods on each side that connects with the crossbeam, as shown. We need to determine the maximum forces that are likely to occur in these rods in order to assign sizes.

To accomplish this, we will estimate the total maximum weight that each set of rods must support then determine how much force this weight creates in each of the rods. Once we know the forces, a table in the rod manufacturer's catalog will tell us how large a rod is needed for each member.

Estimating the Load Borne by the Rods

A *load* is a weight or other force that must be supported or resisted by a structure. How much load does each system of rods in this bridge have to carry? We estimate this load by adding together the *dead load*, which is the weight of the bridge itself, and the *live load*, which is the maximum probable total weight of hikers that might be on the bridge at one time. The

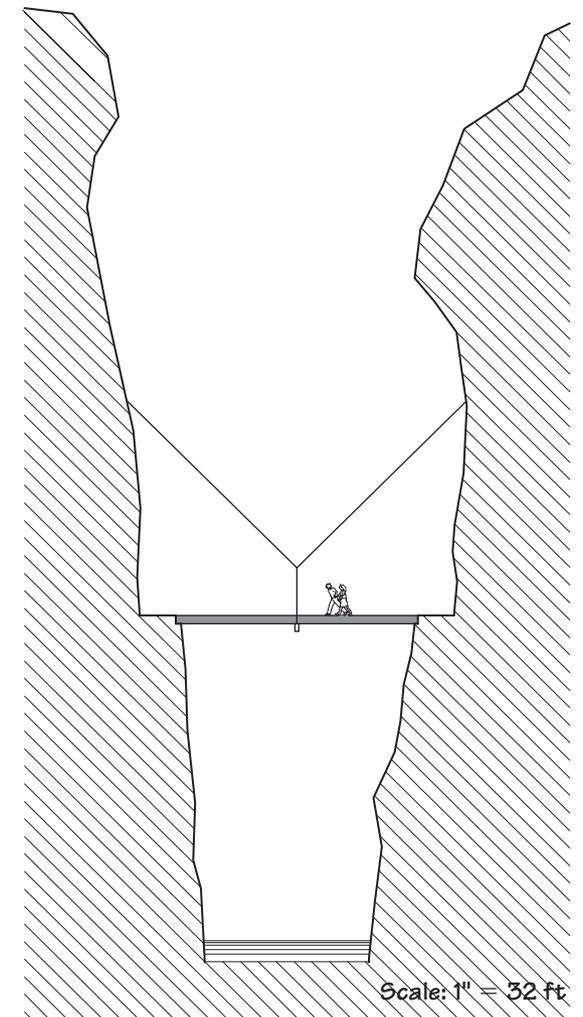


Figure 1.6 Bridge #1 utilizes two 20-ft lengths of deck beams to span 40 ft.

dead load is unchanging and can be predicted fairly accurately. The live load varies from zero, when the bridge is unoccupied, to the weight of a large band of hikers all squeezing together on the bridge for a photograph, so it is much more difficult to predict with certainty than the dead load.

Dead Load

In this bridge, the dead load consists mainly of the weights of the cedar beams and crossbeam and the deck boards. Each beam is 16 in. deep and 8 in. wide. The density of red cedar is about 30 lb per cubic foot. Thus the weight of a one-foot length of beam is figured as follows:

$$\begin{aligned} \text{Weight of beam per foot of length} &= \\ \frac{(8 \text{ in.})(16 \text{ in.})}{144 \text{ in}^2 / \text{ft}^2} (30 \text{ lb /ft}^3) &= 27 \text{ lb/ft} \end{aligned}$$

The weight of the beams for the entire bridge is calculated by multiplying this weight per foot by the total length of the beams in the bridge. There are two beams, each 40 ft long and made up of two 20-ft lengths:

$$\text{Total weight of beams} = 2(40 \text{ ft})(27 \text{ lb/ft}) = 2,160 \text{ lb}$$

The crossbeam on which the beams rest is only 5 ft long. Because its span is so short, it, too, can be made of the same 8 in. by 16 in. cedar stock, even though it carries more weight than a longitudinal beam. Its weight is 5 ft times 27 lb per foot, which is 135 lb.

The decking will be made of 3-in.-thick cedar boards that are fastened to the beams with stainless steel nails. Three inches is one-quarter of a foot, so a square foot of decking will weigh about one-quarter of 30 lb per cubic foot, the density of the cedar, which comes to 7.5 lb. The total weight of the bridge decking is equal to this weight per square foot times the surface area of the bridge:

$$\begin{aligned} \text{Total weight of decking} &= \\ (4 \text{ ft})(40 \text{ ft}) (7.5 \text{ lb/ft}^2) &= 1,200 \text{ lb} \end{aligned}$$

We will assume for the moment that the weight of the steel rods and fasteners is negligible in comparison to these weights, an assumption that we will check later when we know the diameters and lengths of the rods. The dead load for the entire bridge is figured as follows:

Decking	1,200 lb
Beams	2,160 lb
Crossbeam	135 lb
<hr/>	
Total dead load	3,495 lb, which we round to 3,500 lb

The average dead load per square foot of surface is 3,500 lb divided by the surface area of the bridge, 160 square ft, which is about 22 lb per square ft.

Live Load

As noted, the *live load*, the maximum possible total weight of hikers on the bridge, is more difficult to estimate with certainty than the dead load. An average hiker—taking into account males and females, adults and children—weighs about 160 lb and carries a pack weighing 30 lb, for a total of 190 lb. With backpack, a hiker occupies an area about 2 ft by 2 ft, which is 4 sq ft. Assuming that a group of hikers has crowded closely together on the bridge for a photograph, the maximum live load per square foot of bridge deck is 190 lb per hiker divided by 4 sq ft per hiker, which is 47.5 lb per sq ft. The total load per square foot, including both dead and live loads, is 22 lb dead load plus 47.5 lb live load, which is 69.5 lb, which we round up to 70. The total expected load on the bridge is thus estimated as:

$$\text{Total load on bridge} = (160 \text{ ft}^2)(70 \text{ lb/ft}^2) = 11,200 \text{ lb}$$

How much of this load will be borne by each of the two vertical rods? In any symmetrically loaded beam, half of the load is conducted to each end. Referring to the top view of the bridge in Figure 1.7, each piece of decking will transmit half its load to one beam and half to the other, so that each line of beams carries half of the total load. Each beam is symmetrically loaded and made up of two pieces, each 20 ft long. For each of these pieces, half the load will be transmitted to the foundation at the end of the bridge and half to the crossbeam and the vertical rod. This means that each rod supports a *tributary area* that is 20 ft long (half the

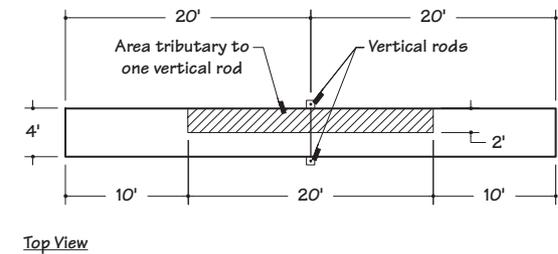


Figure 1.7 The tributary area for one vertical rod of Bridge #1 is shaded in this top view.

length of the bridge) and 2 ft wide (half the width), a total of 40 sq ft. The load per vertical rod is 40 sq ft times 70 lb per sq ft, or 2,800 lb.

FUNDAMENTAL CONCEPTS

Having estimated the force borne by each of the vertical rods, we need to develop a set of tools that we can use to determine the forces in all the rods, both vertical and sloping, that support the bridge. These tools are built on some basic concepts:

- **Force:** A force is a push or pull. Force tends to cause motion. Because we do not want our buildings and bridges to move, we design them in such a way that every force is balanced against an equal and opposite force that prevents motion. However, a force, even when it is balanced against another force, will cause *stresses* and *deformations* within the object, phenomena that we will begin to explore in Chapter 13.
- **Characteristics of a force:** A force has three primary characteristics (Figure 1.8):
 1. Its *magnitude*, in units of pounds or kips (A kip is a “kilopound,” or 1,000 lb)
 2. The *direction* in which it acts
 3. The location of its *line of action*, a line along the centerline of the force that extends indefinitely in both directions

Every force is exerted at a *point of application* somewhere on its line of action.

Figure 1.8 The characteristics of a force vector. On this diagram, the magnitude of the force is given by a number alongside the vector.

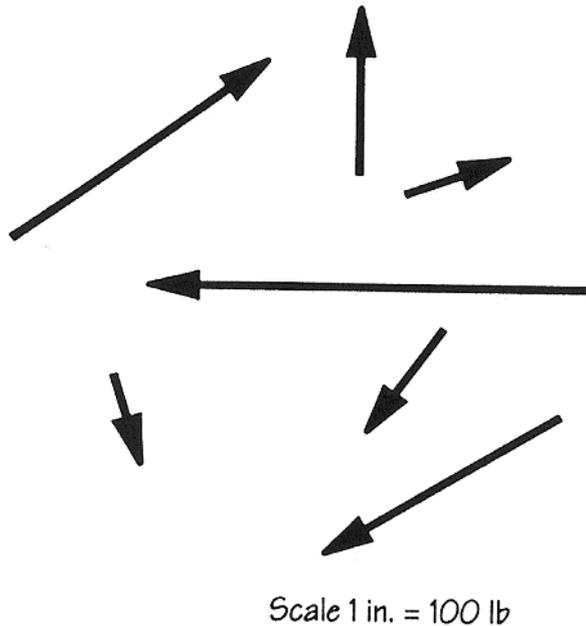
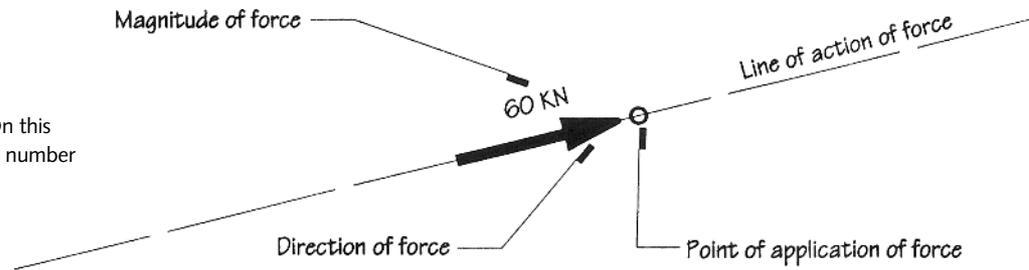


Figure 1.9 Force vectors. The magnitudes of the forces on this diagram are represented by the lengths of the arrows.



Figure 1.10 Tension is pulling; compression is pushing. The rope at the left can resist only tension. The stack of small wooden blocks to the right cannot resist tension but can resist compression, even though the blocks are not connected to one another except by friction.

- **Vectors:** A force is a vector quantity, which means that it has both magnitude and direction. (A quantity that has only magnitude and not direction is a *scalar* quantity. A sum of money is an example of a scalar quantity. So is a volume of water or a person's age.)

We represent force vectors with arrows (Figure 1.9). The direction of an arrow indicates the direction of the force that it represents. The length of the arrow, including its head, is often used to indicate the magnitude of the force. Alternatively, magnitude may

be indicated by a number alongside the arrow that gives the quantity and units of the force.

- **Tension and compression:** There are only two fundamental types of forces: pulls and pushes. Pulling is called tension, and pushing is compression (Figure 1.10). For purposes of analysis, every structural action, no matter how complex, can be reduced to pushes and pulls. Figure 1.11 shows how tension and compression are indicated symbolically with vectors.

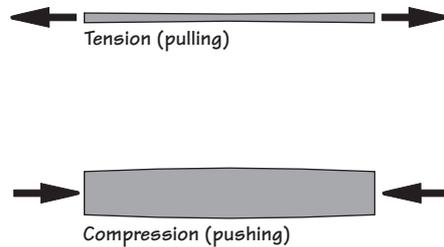


Figure 1.11 Tension and compression represented as bodies acted on by vectors.

“The remarkable, inherent simplicity of nature allows the structure to perform its task through two elementary actions only: pulling and pushing.”

—MARIO SALVADORI

- **Transmissibility:** The effect of a force on a body as a whole is independent of where the force is applied along its line of action (Figure 1.12). This is called the *principle of transmissibility*.
- **Concurrent forces:** Forces whose lines of action pass through a common point are said to be concurrent (Figure 1.13). Forces that do not pass through a common point are *nonconcurrent*.
- **Free-body diagram:** The examination of any system of forces is easier and less subject to errors if it is done with reference to a simple, unambiguous sketch called a free-body diagram (FBD). An FBD is a simple picture of a portion of a structure that we want to study (the *free body*), completely detached as if it were floating in space. Vector arrows are added to indicate the external forces that **act upon** the free body. The free body may be a whole structure or a piece that is hypothetically unfastened, cut, or torn from the structure, whichever suits our purposes.

When using graphical methods of solution, as we will do throughout this book, we draw the free body and vector arrows with drafting instruments or CAD software to depict the directions and magnitudes of

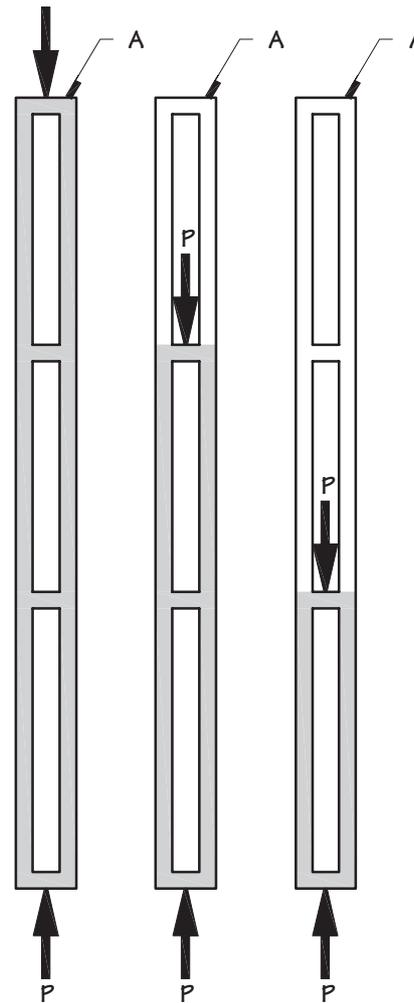


Figure 1.12 The principle of transmissibility: With respect to the external effects of a force on a body, the force P can be considered to act anywhere on its line of action. The internal effect of force P on body A , however, is dependent on the point of application of P . In the case of a force applied within the body, this effect is confined to the shaded area.

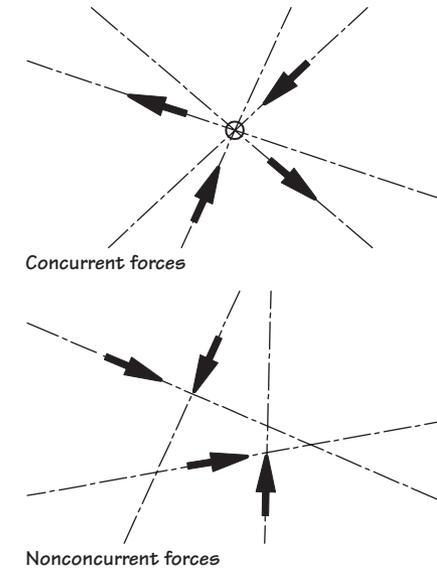


Figure 1.13 Concurrent and nonconcurrent forces. There are pairs of concurrent forces in the lower diagram, but there is no concurrence of more than two forces.

the forces as accurately as possible. Often, however, it is sufficient to draw the FBD freehand as reference for thought or discussion, or as the basis for a numerical calculation. Figure 1.14b is a free-body diagram of a *node* where three rods join one another in the bridge that we are about to examine. It is isolated from the rest of the structure by imaginary cutting of the three steel rods. Notice that it includes only forces that act **on** the free body, not forces exerted **by** the free body.

- **Static equilibrium:** A body at rest is said to be in a state of static equilibrium. “Static” means that the body is at rest; “equilibrium” means that the forces that act on the body balance one another exactly, leaving no net force remaining to set it in motion. We want buildings and bridges to remain at rest, so we design them to be in static equilibrium.

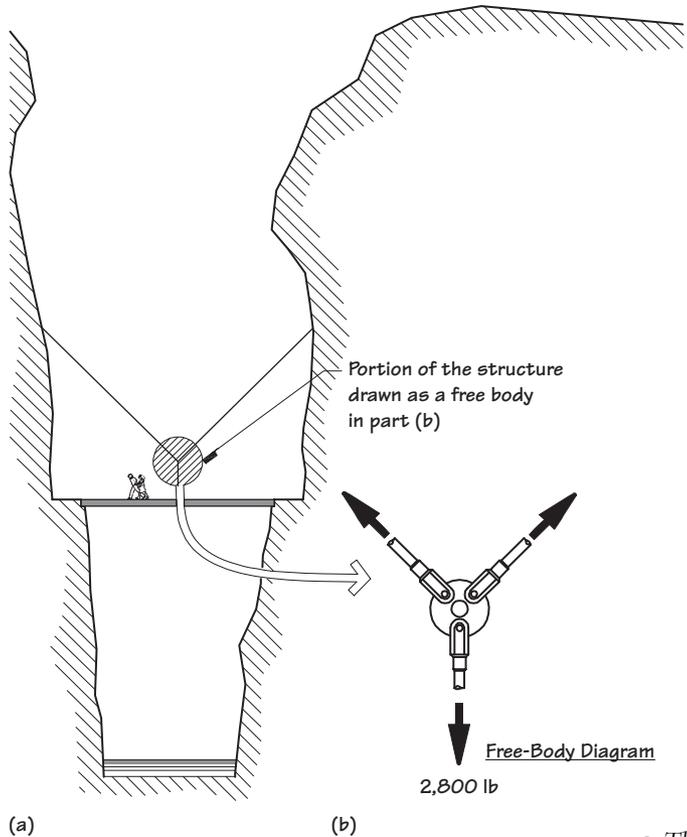


Figure 1.14 In part (a), a free-body diagram has been constructed of the rod intersection or node in Bridge #1. The intersection is cut imaginarily from the larger structure (a) and represented as a free body, isolated from the rest of the structure. Vectors are added to indicate all the external forces that act upon the free body. We know the magnitude and direction of the downward force on the free body (2,800 lb) but we know only the directions of the other two forces.

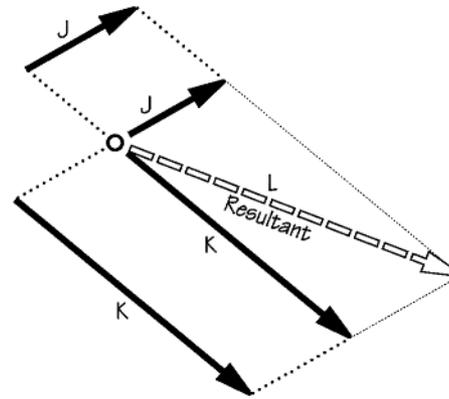


Figure 1.15 The parallelogram law. The resultant of vectors J and K is found by bringing them together as the adjacent sides of a parallelogram. The diagonal of the parallelogram, L , is their resultant, a single force that has the same effect as J and K acting together.

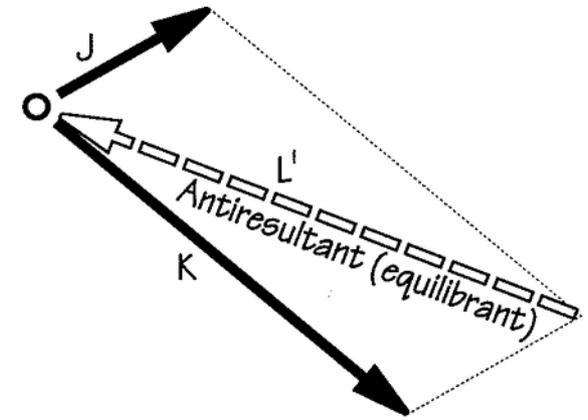


Figure 1.16 If the direction of the resultant in Figure 1.15 is reversed, it becomes an antiresultant or equilibrant, and the three forces are in static equilibrium.

- *The parallelogram law:* Often we need to add force vectors to find a single vector that will have the same effect as two other vectors acting together. This single vector is called the *resultant*. The parallelogram law states that the resultant of any two forces is a vector that is the diagonal of a parallelogram formed on their vectors (Figure 1.15). The parallelogram law is an axiom, which is another way of saying that it is always true but that it can't be proven mathematically.

If the direction of the resultant of any system of forces is reversed, it becomes the *equilibrant*, also sometimes called the *antiresultant*, a single force that will balance the other forces in the system in a state of static equilibrium (Figure 1.16).

- *Tip-to-tail addition of vectors:* It follows from the parallelogram law that we can find the resultant of any two forces by connecting their vectors tip to tail and drawing a single vector, the resultant, from the tail of the first vector to the tip of the second one (Figure 1.17).

If we wish to find the magnitude and direction of the resultant of more than two forces, we can connect their vectors tip to tail in a chain. The resultant is a vector whose tail lies at the tail of the first vector and whose tip lies at the tip of the last vector in the chain. The order in which the vectors are connected doesn't matter—the resultant will always be the same in both direction and magnitude (Figure 1.18). Any number of vectors may be added by this

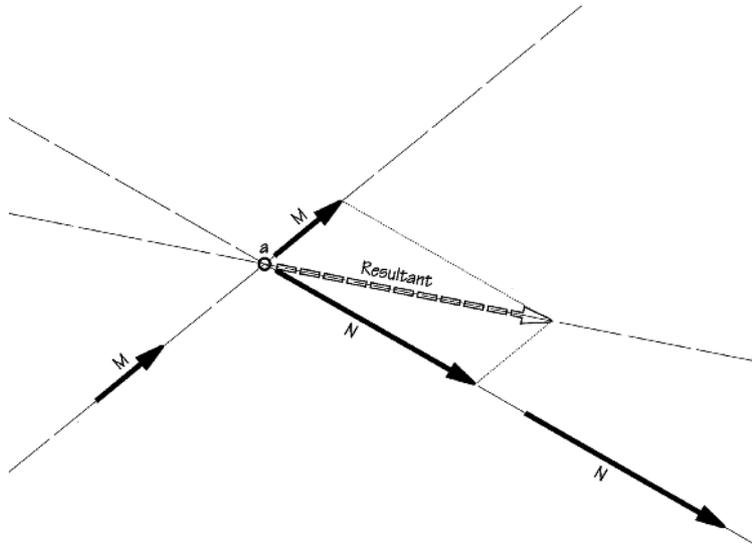


Figure 1.17 The resultant of two forces may also be found by connecting their vectors tip to tail and drawing a vector from the tail of the first force vector to the tip of the second. The line of action of the resultant passes through the intersection of the lines of action of the two original forces.

method. The diagram of tip-to-tail vectors and their resultant is called a *force polygon*. The resultant may be reversed in direction to serve as an equilibrant.

- *Equilibrium of concurrent forces:* If the chain of tip-to-tail vectors for a group of concurrent forces closes exactly upon itself, with the last tip at the same point as the first tail, the resultant has a value of zero (Figure 1.19). This means that the group of vectors exerts no net force and that, therefore, the group of forces is in static equilibrium. (Nonconcurrent forces may be in static equilibrium as well as concurrent forces, but their formation of a closed chain of vectors is insufficient by itself to prove equilibrium. We will take up the determination of equilibrium of groups of nonconcurrent forces in Chapter 5).

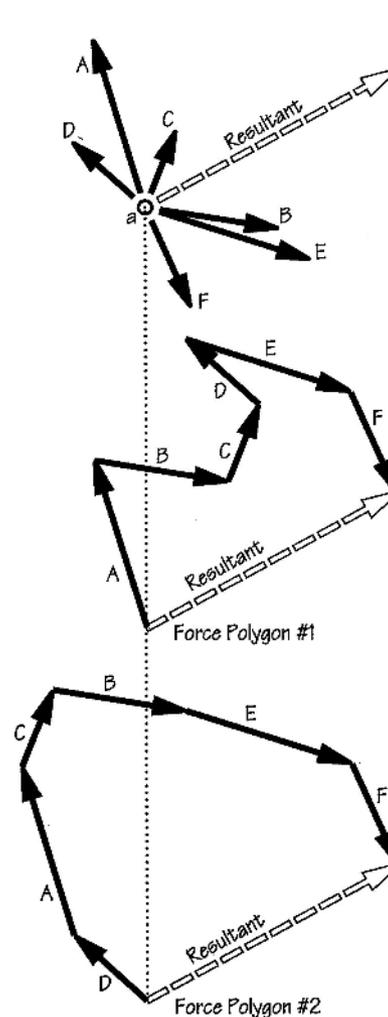


Figure 1.18 Tip-to-tail addition can be applied to any number of forces. The forces in this example have been connected in two different orders to show that the resultant is the same regardless of the order of connection. The line of action of the resultant passes through the point of concurrence of the vectors in their original locations.

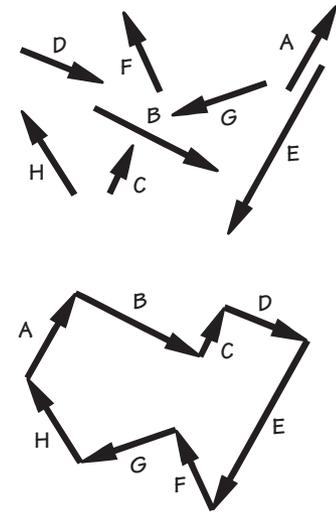


Figure 1.19 If the tip-to-tail polygon of forces closes upon itself, the resultant is zero. If all the forces are concurrent, the closure of the force polygon means that they are in static equilibrium.

GRAPHICAL SOLUTIONS

We will use a *graphical solution* to find the forces in the rods that support the bridge we are designing. In a graphical solution, we work with accurately drawn diagrams of vectors whose lengths we can measure to determine the magnitudes of the forces that they represent. The alternative to a graphical solution is a *numerical solution*, which employs numbers rather than scaled diagrams.

Each solution method has its advantages and disadvantages. A numerical solution is usually more compact than a graphical one and can be carried out to as many decimal places as the designer wishes. It is easily programmed for solution by a calculator or computer.

Drawing Parallel Lines*

Graphical analysis techniques require the drawing of many lines that are accurately parallel to other lines. What are some efficient, precise ways of drawing parallel lines on a drawing?

The traditional method is to use two drafting triangles, placing the hypotenuse of one tightly against the hypotenuse of the other (Fig. A). A leg of the upper triangle is aligned with the original line. Then the lower triangle is held firmly against the paper while the upper triangle is slid along its hypotenuse to a new location that allows a parallel line to be constructed through the desired point. It often takes a bit of experimentation to find the best way of arranging the triangles for a particular pair of lines. If the operation must occur over a long distance, it sometimes works better to substitute a long straightedge for the lower triangle (Fig. B).

An adjustable triangle (Fig. C), used in conjunction with a T-square or parallel rule, offers a means of drawing parallel lines that many designers find more convenient and secure than two triangles.

A drafting machine is an ideal instrument for graphical constructions, because it facilitates rapid, very accurate construction of parallel lines at any location on a sheet (Fig. D).

A rolling ruler offers a simple, easy way of ruling parallel lines, even if the paper is not fastened to a drawing board (Fig. E).

Computer-assisted drafting (CAD) offers graphical constructions that give results identical to those of numerical computations that are carried to the same number of decimal places. Every CAD program has a simple routine for drawing

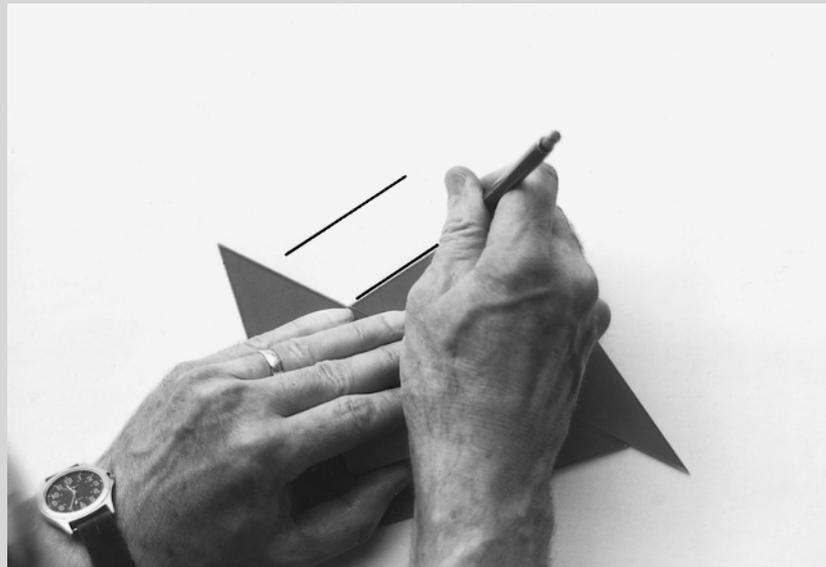


Fig. A

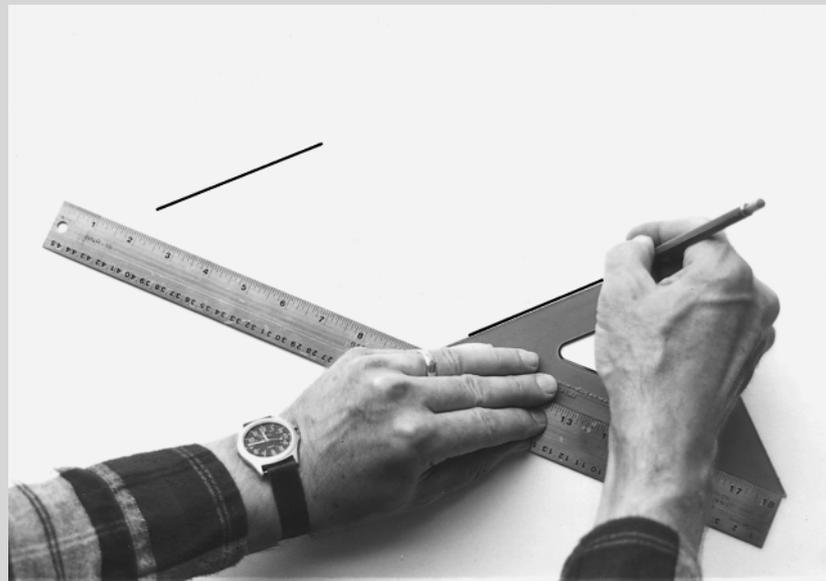


Fig. B

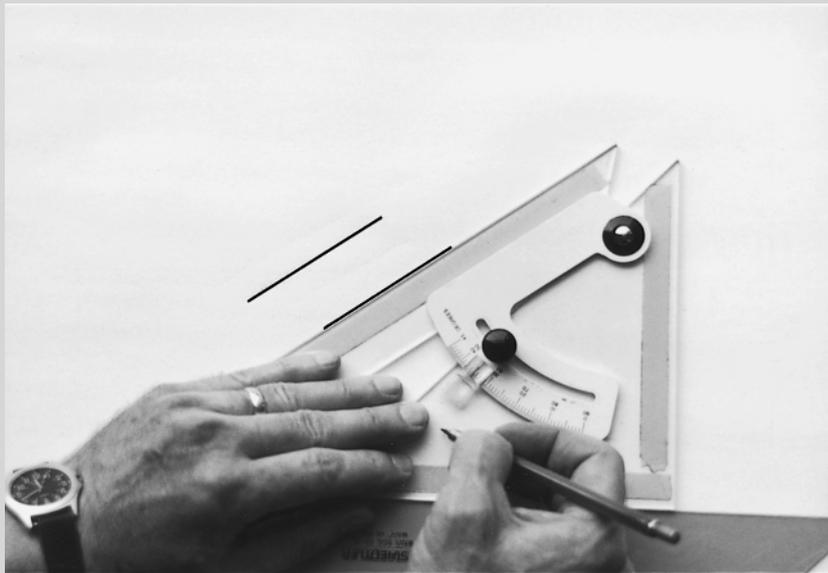


Fig. C

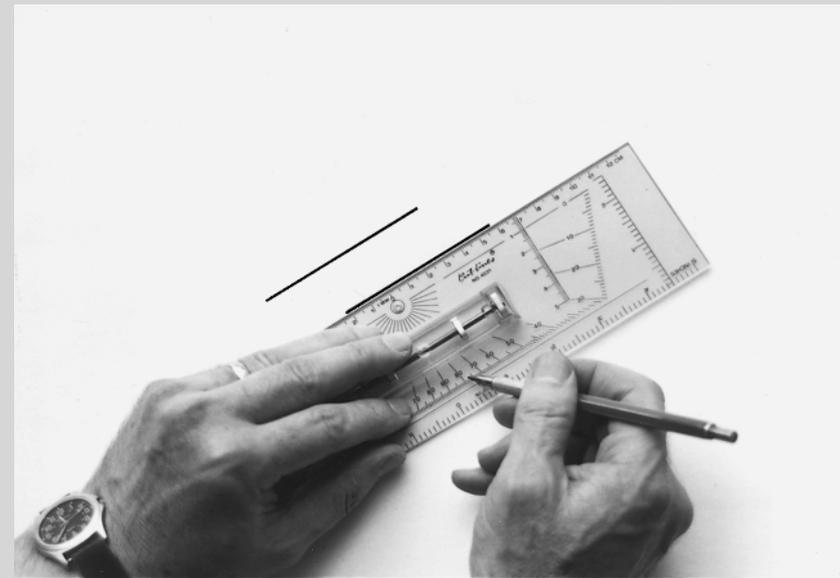


Fig. E

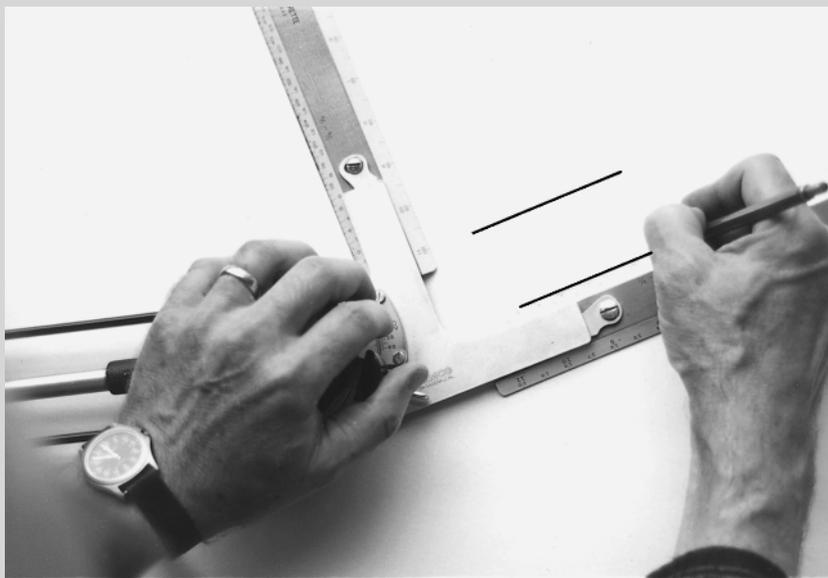


Fig. D

parallel lines, as well as another routine for scaling the lengths of line segments with extreme precision. As we noted earlier, however, extreme numerical accuracy is not usually required, and CAD construction of a force polygon is generally appropriate only when the entire process is being carried out by computer. If the structure is being designed on paper, it is easiest and more natural to analyze it on the same sheet of paper.

*Photos by Gregory D. Thomson from *Shaping Structures: Statics* by Waclaw Zalewski and Edward Allen (New York: John Wiley & Sons, Inc.), 1998. Reprinted with permission.

However, the apparent accuracy of numerical methods often masks the fact that the data on which a solution is based have a very low degree of precision, as is the case with our estimate of live loads on the bridge. Additionally, numerical methods do little to help us visualize what is going on in a structure. Furthermore, many people are so fearful of mathematics that they find it difficult to learn and remember numerical solutions.

A graphical solution drawn manually is less precise than a numerical one. Its imprecision seldom exceeds 1 percent, which is generally far better than the precision of the data on which it is based. A graphical solution done on a computer is as precise as a numerical solution and gives an identical answer, because numerical methods are based on graphical understandings such as the parallelogram law.

Most people find that graphical solutions are faster and less subject to human error than numerical solutions. They are much more helpful than numerical methods in visualizing how a structure works and how its form may be improved. Graphical solutions may be used to find efficient forms for structures. For these reasons, we will use graphical solutions for most of the projects in this book.

Bow's Notation

We are about to begin an analysis of the forces that act on the node where a vertical rod intersects two sloping rods in the bridge structure. To keep track of the forces that act on the node, we will add to our free-body diagram a labeling system called *Bow's notation*, after Robert Bow, whose invention of it was published in 1873. Starting at any arbitrarily chosen location, we place uppercase letters **in the spaces between the lines of action** of the forces (Figure 1.20). Each force is named by the letters that lie on either side of it. The force to the upper right lies between spaces *A* and *B* on the free-body diagram. It may be called either force *ab* or force *ba*. The downward force is

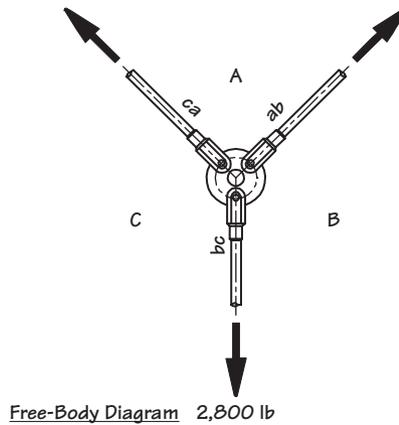


Figure 1.20 Bow's notation is applied to the free-body diagram of the intersection of the rods in Bridge #1. In practice, we would not draw the details of the intersection, because they have no effect on the determination of the exterior forces.

either *bc* or *cb*, and the force to the upper left either *ca* or *ac*. Notice that uppercase letters are used to label the spaces between the forces on the free-body diagram, while corresponding lowercase letters designate the forces. Points or nodes may be designated by the letters that surround them in sequence: this node may be called *ABC* or *BCA* or *CAB*. By convention, we name nodes using consistent clockwise readings of letters.

FINDING THE FORCES IN THE BRIDGE STRUCTURE

Constructing the Free-Body Diagram

We begin our graphical solution at the upper left of a sheet of paper by drawing accurately with drafting tools or a computer-aided drafting (CAD) program the free-body diagram of the node where the vertical rod joins the two inclined rods (Figure 1.21). Three forces act on this node: the 2,800-lb vertical load that

we have estimated, and two inclined forces of known direction but unknown magnitude. These three forces must be in static equilibrium in order to assure the stability of the bridge. Using Bow's notation, we place the uppercase letters *A*, *B*, and *C* in the spaces between these three forces.

Constructing the Force Polygon

Next to the free-body diagram, we draw a force polygon for this system of three forces. A force polygon, as discussed previously, is a tip-to-tail diagram of the vectors of the forces in a system. It is drawn accurately to any convenient scale of length to force. In this case, we choose a scale of 1 in. equals 1,000 lb, which will produce a diagram that fits comfortably on the page.

We start construction of the force polygon by drawing a vertical line called the *load line* to represent the vertical direction of the external load, which is force *bc* (Figure 1.21). To this line we add two horizontal cross ticks exactly 2.8 in. apart. This distance between the ticks represents 2,800 lb, the magnitude of force *bc* at the given scale of 1 in. equals 1,000 lb. (The cross ticks permit more accurate measurement than a representation in which only a line is used to represent the value. Similarly, although this line is a vector, we do not add an arrowhead, because the head often makes accurate measurement difficult.)

This force could be called either *bc* or *cb*. By convention, however, we read clockwise around the point of concurrence to determine the name of the force, which is thus *bc*. As we read clockwise from space *B* to space *C* around the point of concurrence in the free-body diagram, force *bc* acts downward, from the top of the page toward the bottom. Turning our attention to the force polygon, we label the upper tick on the load line *b* and the lower tick *c*, so that as we read from *b* to *c* on this line, we are reading downward, the direction indicated by the clockwise reading of the letters on the free-body diagram.

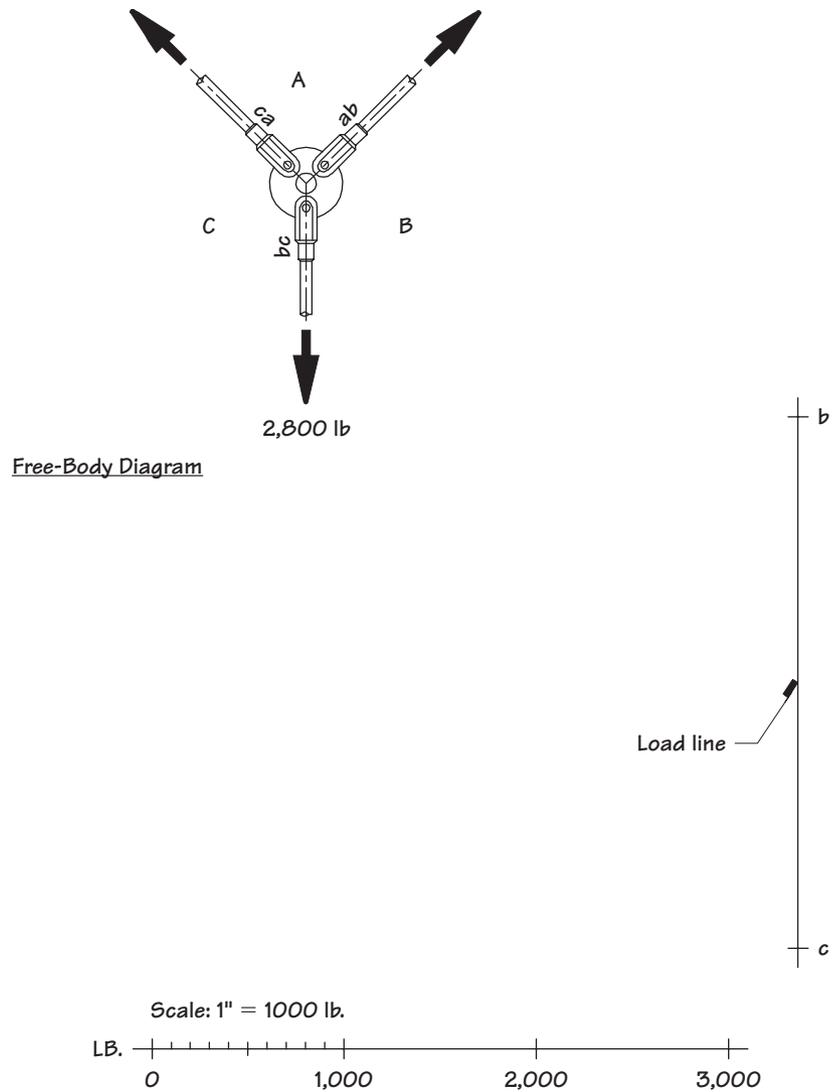


Figure 1.21 The first step in finding the magnitudes of the unknown rod forces is to represent the known force, bc , as a line segment whose length is equal to the magnitude of the force and whose direction is parallel to the force. Line bc is a force vector, but is generally drawn without an arrowhead. Any convenient scale of length to force may be used in this construction. There is no relationship between the scale of the free-body diagram and the scale of the force polygon.

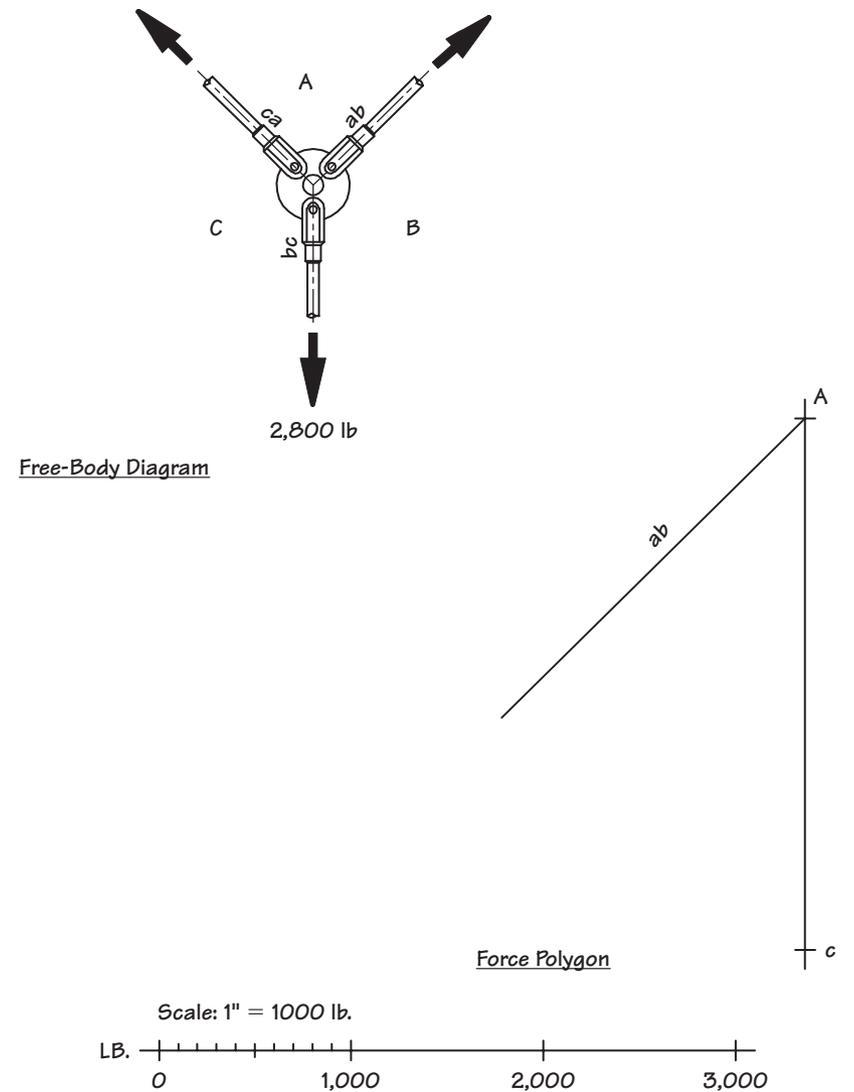


Figure 1.22 The second step in finding the magnitudes of the unknown rod forces is to draw a line on the force polygon to represent the direction of one of the forces. Bow's notation helps us to know where to connect this line to the line that represents the known force: The unknown force is ab and there is no letter a on the force polygon at the moment; therefore the new line must connect at b .

You will have noticed that uppercase letters that designate **spaces** on the free-body diagram become lowercase letters that designate **points** on the force polygon. This will seem strange at first, but as you will see, this system of notation is a powerful tool for making sure that each vector ends up in its proper place.

So far, we have represented on the force polygon only one force of the three that act on the node where the three rods come together. Now we will add the other two. Reading clockwise, the force that pulls to the upper right lies between letters *A* and *B* on the free-body diagram. Thus its name is *ab*. Of these two letters, only *b* appears on the load line at this stage. The *b* in the name of this force tells us that its vector will pass through point *b* on the load line. It must also pass through a point *a* that is not on the load line, but we don't yet know where *a* is. We draw line *ab* of indefinite length through *b* on the force polygon, accurately parallel to vector *AB* on the free-body diagram (Figure 1.22). We do not yet know the length of line *ab*, but we do know that *a*, which will mark its other end, will lie somewhere on it.

Lastly, we draw the vector for the remaining force, *ca* (Figure 1.23). Of these two letters, only *c* appears on the force polygon at this stage. The *c* in its name tells us that its vector must pass through point *c* on the force polygon and be parallel to the corresponding vector *CA* on the free-body diagram. The *a* in its name tells us that point *a* will lie somewhere on it. Line *ca* intersects line *ab*. Because the names of both lines contain the letter *a*, we name this intersection *a*.

The force polygon is now complete. We can measure its sides to determine that the force in member *AB* has a magnitude of 1,980 lb and the force in member *CA* is also 1,980 lb. If this solution is drawn by hand, the accuracy of these figures is plus or minus 1 percent at the small scale of our drawing, which is far better than the degree of certainty of our live load estimates. If greater accuracy is required, the force

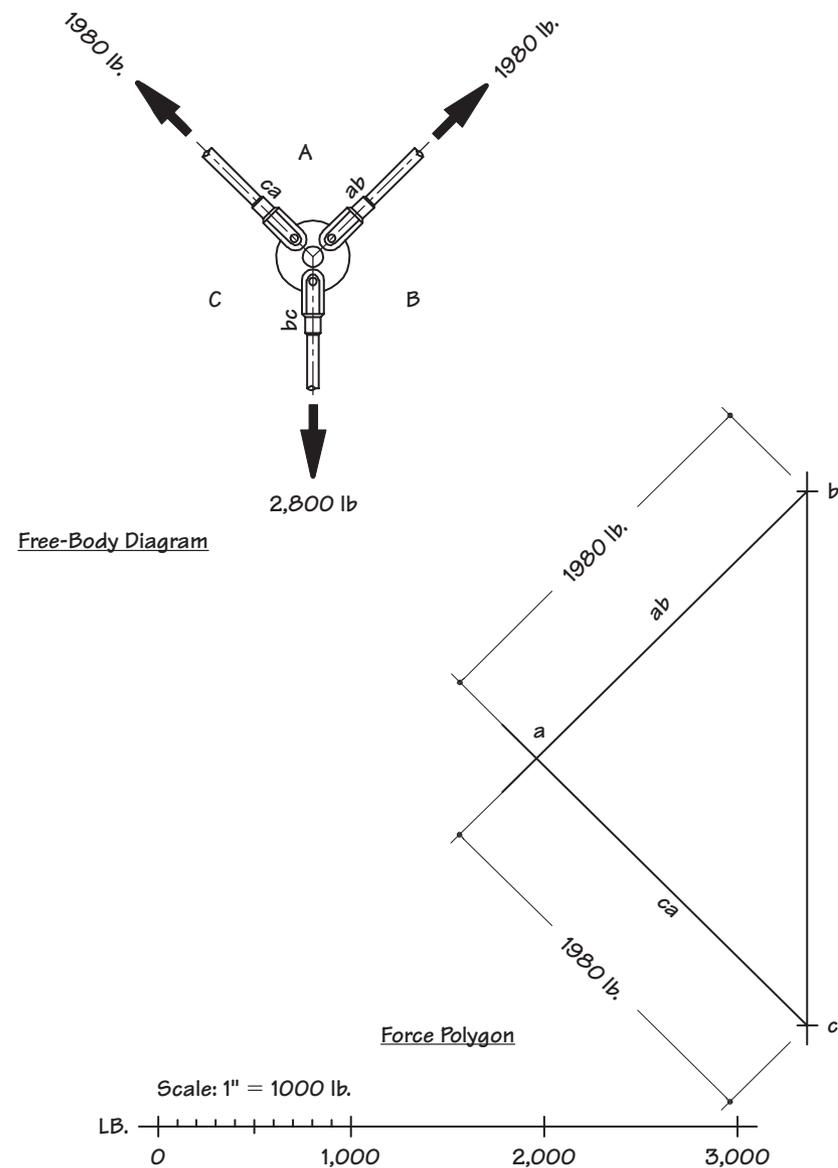


Figure 1.23 The force polygon is completed by repeating the previous step for the remaining unknown force. *a* is the intersection of the two lines that represent the unknown forces. Lines *ab* and *ca* are vectors that may be measured at the given scale to determine the magnitudes of the unknown forces.

polygon can be constructed at a larger scale, or in a CAD program.

Determining Directions of Forces

Slender rods are effective only in resisting tension, because if they are subjected to compression, they *buckle* sideways and have no resistance. How can we be certain that forces *ab* and *ca* are tensile? Bow's notation helps us find the answer. On the free-body diagram, the upper portion of Figure 1.23, we read the sequence of letters around the point of concurrence in clockwise order: The force to the upper left lies between spaces *C* and *A*, in that order, as we make that reading. We now direct our attention to the force polygon below: On the force polygon, as we move from point *c* to point *a*, we move from lower right to upper left. Returning to the FBD above, a force that moves along line *ca* from lower right to upper left is pulling away from the point of concurrence, which indicates that *ca* is a tensile force. We can follow the same procedure to find that *ab* is also in tension. We now know the directions and magnitudes of the three forces at this node. This completes our analysis of the forces in the three rods.

Sizing the Rods

Now that we know the forces in the rods under the estimated maximum load on the bridge, we are able to determine how large the rods must be in order to sustain these forces safely.

We work from a table of allowable strengths that is based on data furnished by a manufacturer of stainless steel rods (Figure 1.24). The *yield strength* of each diameter of rod, which is the tensile force at which the rod would begin to stretch irreversibly, has been divided by a *factor of safety* of 1.67 to arrive at the values in this table. If there were a full load on the bridge, each rod would be loaded to $1/1.67$, or 0.6 of its yield strength. The remaining 0.4 is reserve strength, a portion of which may come into play under unexpected events such as floods and tornadoes.

Tensile Strengths of Medium-Strength Stainless Steel Rods

Rod Diameter		Yield Strength (F_y)		Allowable Strength ($0.6F_y$)	
Inches	mm	Pounds	kN	Pounds	kN
0.125	3.2	1,350	6.0	810	3.6
0.188	4.8	3,000	13.3	1,800	8.0
0.225	5.7	4,400	19.6	2,640	11.8
0.250	6.4	5,400	24.0	3,240	14.4
0.330	8.4	9,400	41.8	5,640	25.1
0.375	9.5	12,100	53.8	7,260	32.3
0.437	11.1	16,500	73.3	9,900	44.0
0.500	12.7	21,600	96.0	12,560	57.6
0.625	15.9	33,000	147	19,800	88.2

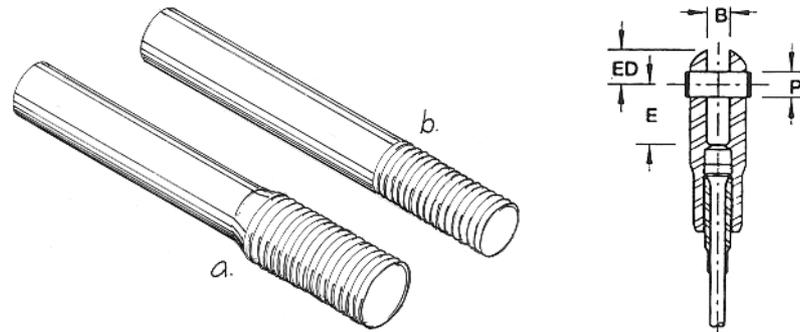
Figure 1.24 This table gives dimensions and allowable loads for stainless steel rods of the type that we will use in our bridge.

The bottom end of the vertical rod is *threaded*, which means that it has been given a helical ridge that will engage a nut or a fitting with a similar ridge. Threads are usually produced by removing material from a rod, but this reduces the diameter and strength of the rod. The manufacturer of these particular rods places the end of the rod in a machine that squeezes and re-forms it to produce an end region with a larger diameter. When threads are formed in this *upset end* detail, the minimum diameter at the roots of the threads exceeds slightly the diameter of the rod, and maintains the full strength of the rod even where it is threaded (Figure 1.25).

For our bridge we need rods to sustain safely tensile forces of 2,800 lb and 1,980 lb in the vertical and inclined rods, respectively. Starting from the top of the table in Figure 1.24, a 0.125-in. (1/8-in.) diameter rod can safely sustain a force of 810 lb. This is not enough for either of the rods in this bridge. A 0.188-in. diameter rod can carry 1,800 lb; again, not enough. A 0.225-in. rod can carry 2,640 lb, which is

larger than the 1,980 lb carried by the sloping rods, so it is suitable for these members, but is not strong enough for the vertical members. The next larger rod is 0.250 in. (1/4 in.) in diameter and can carry 3,240 lb, which is larger than the forces in the vertical rods of our bridge. For ruggedness, better appearance, and simplicity of fabrication and maintenance, we decide to use quarter-inch rods throughout the structure. The use of quarter-inch fittings and rods for all the members of the bridge will increase the cost slightly, but it eliminates a potential source of construction error and reduces the number of parts that must be brought to the site.

We need to review our earlier assumption that the weight of the rods themselves is negligible in comparison to the overall dead and live loads on the bridge. The catalog data for the rods tell us that quarter-inch rods weigh 0.167 lb per foot of length. The two vertical rods total about 22 ft in length, taking into account their 8-ft exposed length and the additional length needed to pass through the crossbeam.



Rod Diameter Inch (mm)	Pin Diameter (P) Inch (mm)	Jaw Gap (B) Inch (mm)	Throat Depth (E) Inch (mm)	Edge Distance (ED) Inch (mm)	Take-up Inch (mm)
0.188 (4.8)	0.31 (8.0)	0.28 (7.1)	0.58 (14.7)	0.44 (11.2)	0.75 (19.1)
0.225 (5.7)	0.38 (9.5)	0.28 (7.1)	0.69 (17.5)	0.50 (12.7)	1.00 (25.4)
0.250 (6.4)	0.50 (12.7)	0.40 (10.2)	0.92 (23.4)	0.63 (15.9)	1.00 (25.4)
0.330 (8.4)	0.56 (14.3)	0.40 (10.2)	1.04 (26.4)	0.65 (16.5)	1.00 (25.4)
0.375 (9.5)	0.63 (15.9)	0.53 (13.5)	1.16 (29.5)	0.75 (19.1)	1.25 (31.8)
0.437 (11.1)	0.75 (19.1)	0.53 (13.5)	1.39 (35.3)	0.88 (22.4)	1.25 (31.8)
0.500 (12.7)	0.88 (22.2)	0.65 (16.5)	1.62 (41.1)	1.10 (27.9)	1.25 (31.8)
0.625 (15.9)	1.13 (28.6)	0.78 (19.8)	2.08 (52.8)	1.50 (38.1)	1.75 (44.5)

Figure 1.25 Dimensions of the jaw fitting for 1/2-in. rods. At the upper left, upset end (a) and plain threaded end rods.

Multiplying 22 ft by 0.167 lb/ft, we find that the vertical rods weigh less than 4 lb. The four inclined rods are each about 34 ft long, for a total length of 136 ft. Their total weight is about 23 lb. The total weight of all the rods is 23 plus 4, or 27 lb. This is less than 1 percent of the weight of the beams and decking. We have chosen to use quarter-inch rods, which have an allowable tensile strength of 3,240 lb, to resist total forces that are unlikely to exceed 2,800 lb. This oversizing of the rods gives them an excess capacity of 440 lb, which is 16 times the weight of the rods

themselves. Our design is sufficiently strong—and much more.

Selecting Fittings

The rod manufacturer furnishes detailed information on a wide variety of fittings for stainless steel rod structures. We select fittings that are attached to the tension rods by cold-heading the end of the rod. (*Cold-heading* is a room-temperature process of squeezing the end of the rod into a flared shape.) Adjustment is provided by a threaded bushing (nose)

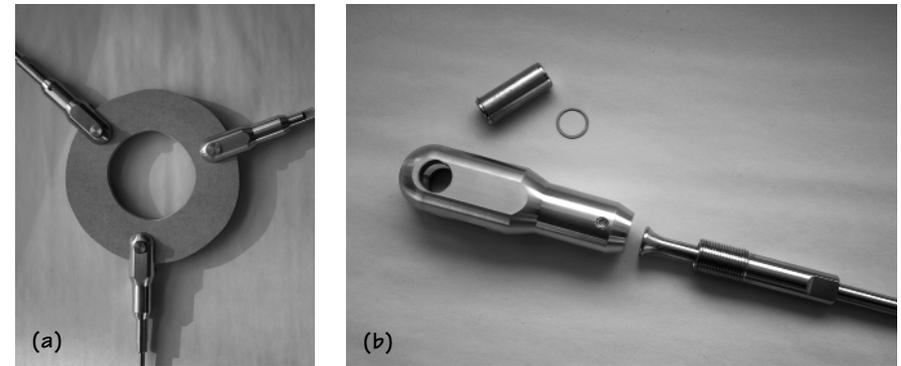


Figure 1.26 (a) A mockup of the main rod connection for the disk. For the sake of speed and ease of fabrication, the circular plate is modeled with medium-density fiberboard rather than stainless steel. (b) How the rod connection works: The stainless steel rod is cut to length. The end of the rod is cold-headed into a flared shape that seats snugly into a matching cavity in the end of the threaded bushing (right). The bushing, which rotates freely around the rod, is inserted into the fork fitting (left) and rotated clockwise to engage the threads. The pin (top) is inserted into one side of the fork, through the connecting plate, and out the other side. It is held in position by a small, flat stainless steel ring that snaps into a matching groove near the small end of the pin. Using a wrench on the flat end of the bushing, the assembly is tightened as needed to adjust the length of the rod and provide tension if necessary. A recessed screw in the side of the fork fitting is tightened to maintain the bushing in the desired position. This ingenious assembly avoids the need for a turnbuckle by building adjustability into the end fittings.

Photo: Edward Allen, The stainless steel components are manufactured by TriPyramid Structures, Westford, MA.

that is captured by the head on the rod. This bushing is free to rotate about the rod, threading into or out of the fitting to provide length adjustment. Threads are completely concealed when the fitting is properly adjusted (see Figure 1.26b).

Detailing the Rods

Figure 1.25 shows dimensions for a quarter-inch jaw fitting that we will use to attach the quarter-inch rods to the stainless steel plates. The opening of the jaws is 0.40 in. This suggests that we use 0.375 in. (3/8 in.)

plates for the connections, which will leave a *clearance* of 0.025 in., which will make it easy for workers to install the plates and fittings. These fittings are designed to be stronger than the quarter-inch rod with which they are used. In Figure 1.26a, we have mocked up the typical rod components at full scale.

BRIDGE #2

Bridge #2 is identical to #1 in every particular except that areas of weak, fractured rock in the canyon walls require that the cable anchorages be placed lower with respect to the deck of the bridge. Whereas the rods in Bridge #1 are inclined at about 45° to the horizontal, those in Bridge #2 are inclined at only 30° . What is the effect of the lower angle?

We find the answer to this question with the diagram in Figure 1.27. The force polygon for Bridge #1 is drawn with gray lines and that for Bridge #2 with solid black lines. The forces in the inclined rods are 2,800 lb in #2, as compared with 1,980 lb in #1.

We consult Figure 1.24 to find that the required rod size for the sloping members is still only $1/4$ in. This is also the size we need for the vertical members.

BRIDGE #3

In Bridge #3, the span remains the same, except that the anchorages must be located higher on the cliffs, which increases the steepness of the inclined rods (Figure 1.28). The result is that while the inclined rods are substantially longer, the forces in the rods are only 1,530 lb, compared to 1,980 lb in #1, and 2,800 lb in #2.

From these first three bridges we can conclude that, in general, the steeper the inclination of the rods, the lower their forces will be. As the rods grow steeper, the overall structure of the bridge grows deeper. In all structures—beams, trusses, hanging cables, and arches—other factors being equal, a deeper structure (that is, one with a greater ratio of depth to span) will have lower internal forces.

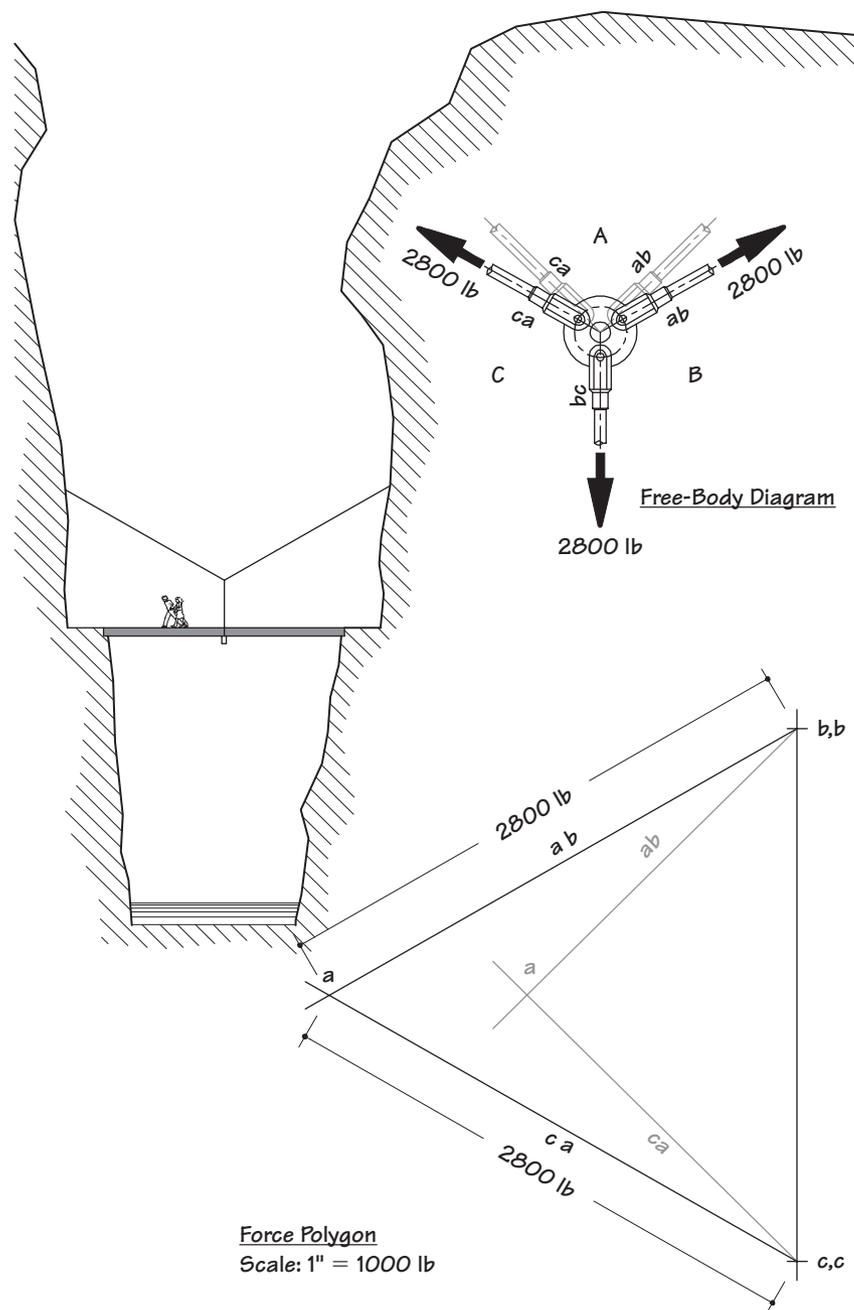


Figure 1.27 In Bridge #2, the slopes of the inclined rods are decreased, which causes the forces in the rods to rise. The lighter lines are the vectors for the original slopes; the solid lines represent the new slopes.

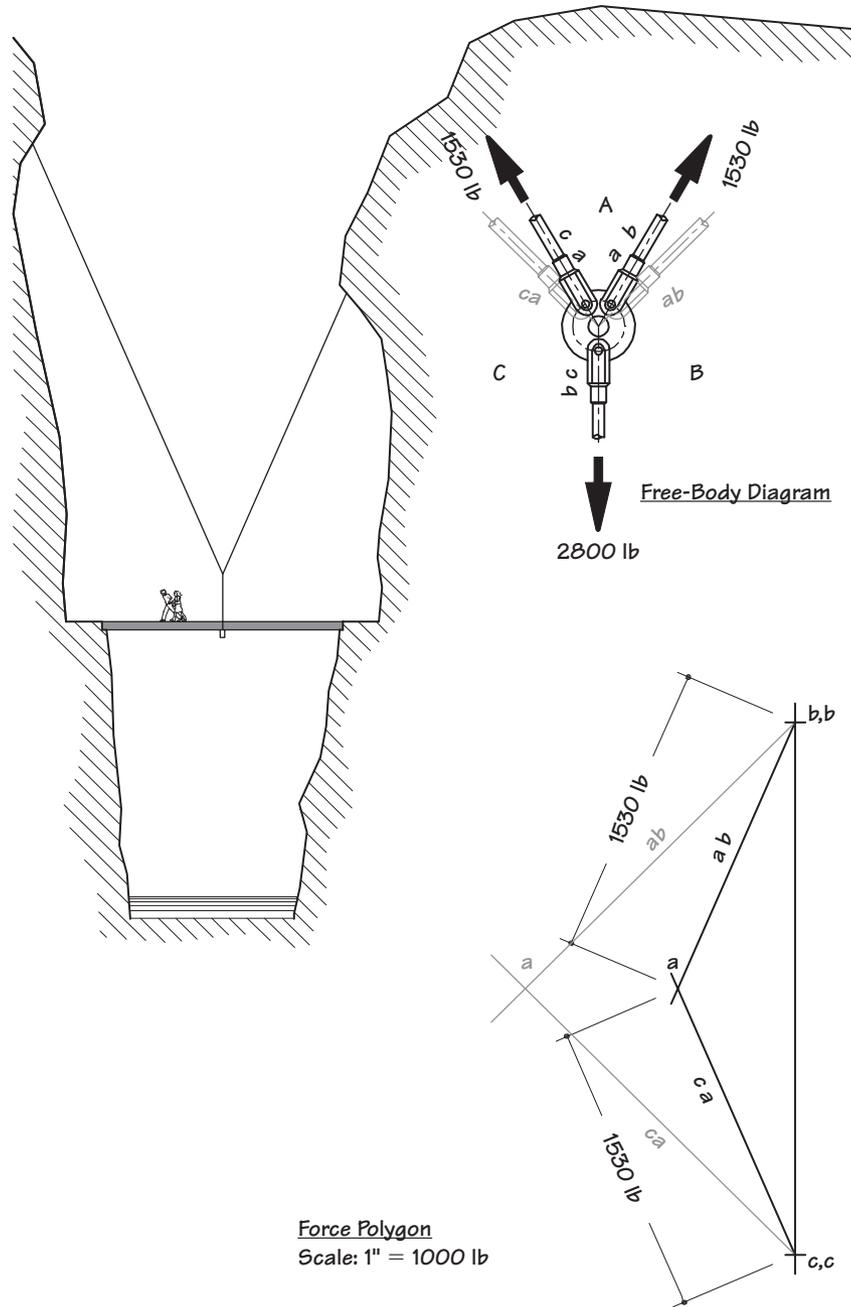


Figure 1.28 If the rods are sloped more steeply, as they are in Bridge #3, the forces in them decrease.

BRIDGE #4

Two areas of fractured rock on the site of Bridge #4 require us to relocate one of the anchorages. We test two alternative designs (Figures 1.29, 1.30). One has the anchorage higher than the top of the vertical rod and one has it lower. Both designs increase the force in the rod whose angle is unchanged from that of Bridge #1. Either design will work, but the higher of the two anchorage locations produces lower rod forces.

BRIDGE #5

Bridge #5 must span 60 ft instead of the 40 ft of the previous four bridges. We will do this by using three beams end-to-end, each supported at interior points by crossbeams and vertical rods (Figure 1.31). The tributary area of each vertical rod remains the same as before, but there are now four vertical rods, so the total load that must be supported by the inclined rods is doubled. We isolate as a free body the two nodes of the rod system and the horizontal rod between them. In Figure 1.31, we construct a force polygon for the node *abc*. In Figure 1.32, we construct a force polygon for the node *acd*. In doing this, we recognize that line *ac* occurs in both force polygons. Rather than draw this line twice, we simply attach the second force polygon to the first along line *ac*. This works out well: The two loads, *bc* and *cd*, form a single load line with point *c* common to both. We find that the accuracy of our construction of the force polygon is checked automatically—when we draw the last vector, *da*, on the force polygon, it must intersect both point *d* and point *a* with reasonable precision. If it does not, we must start the diagram over again. The forces in the two inclined rods measure as 3,960 lb, and the force in the horizontal segment is 2,800 lb. Reference to Figure 1.24 tells us that we must use rods with a diameter of 0.330 in. for the inclined tension members in this bridge.

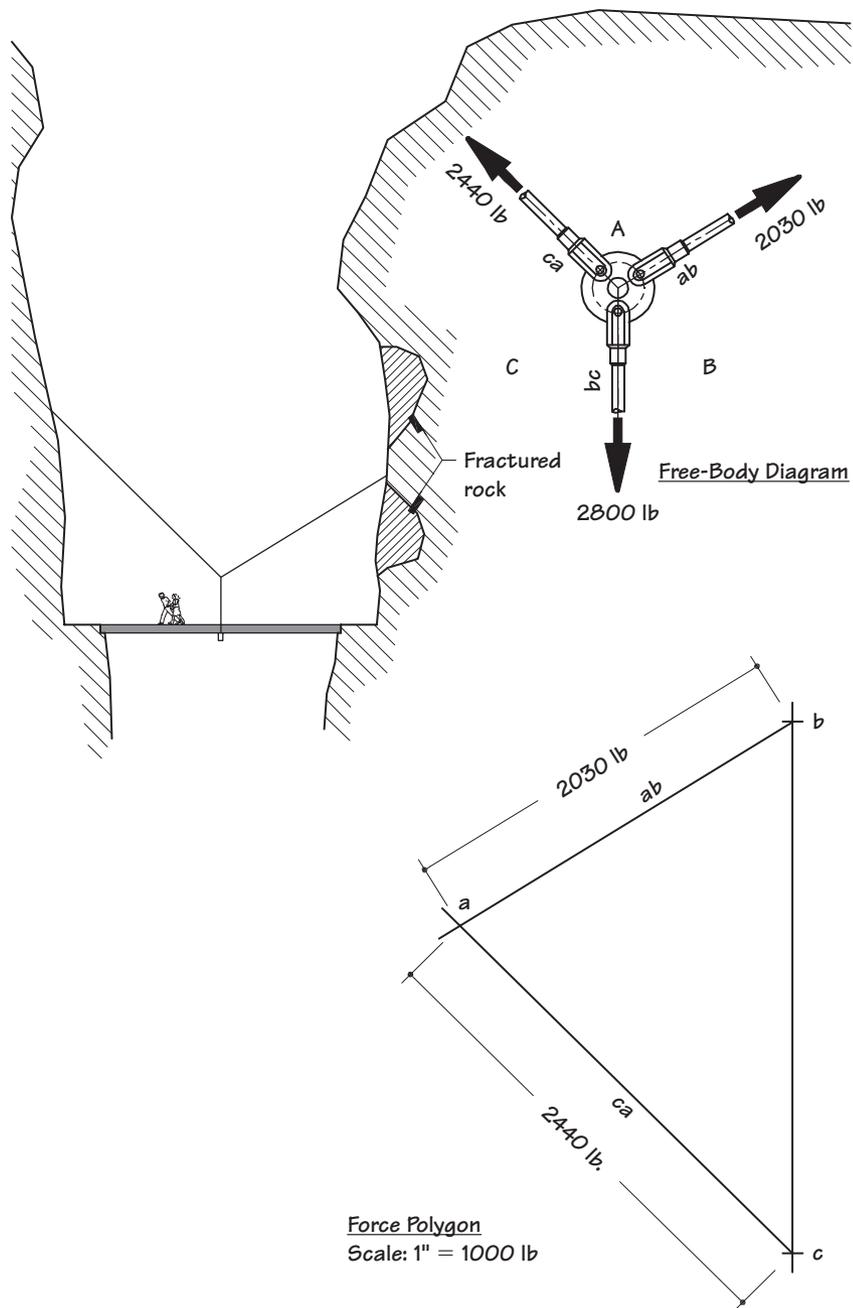


Figure 1.29 The outer end of one of the inclined rods in Bridge #4 is lowered to avoid areas of fractured rock. The magnitudes of the forces in the rods rise, but the bridge remains in static equilibrium.

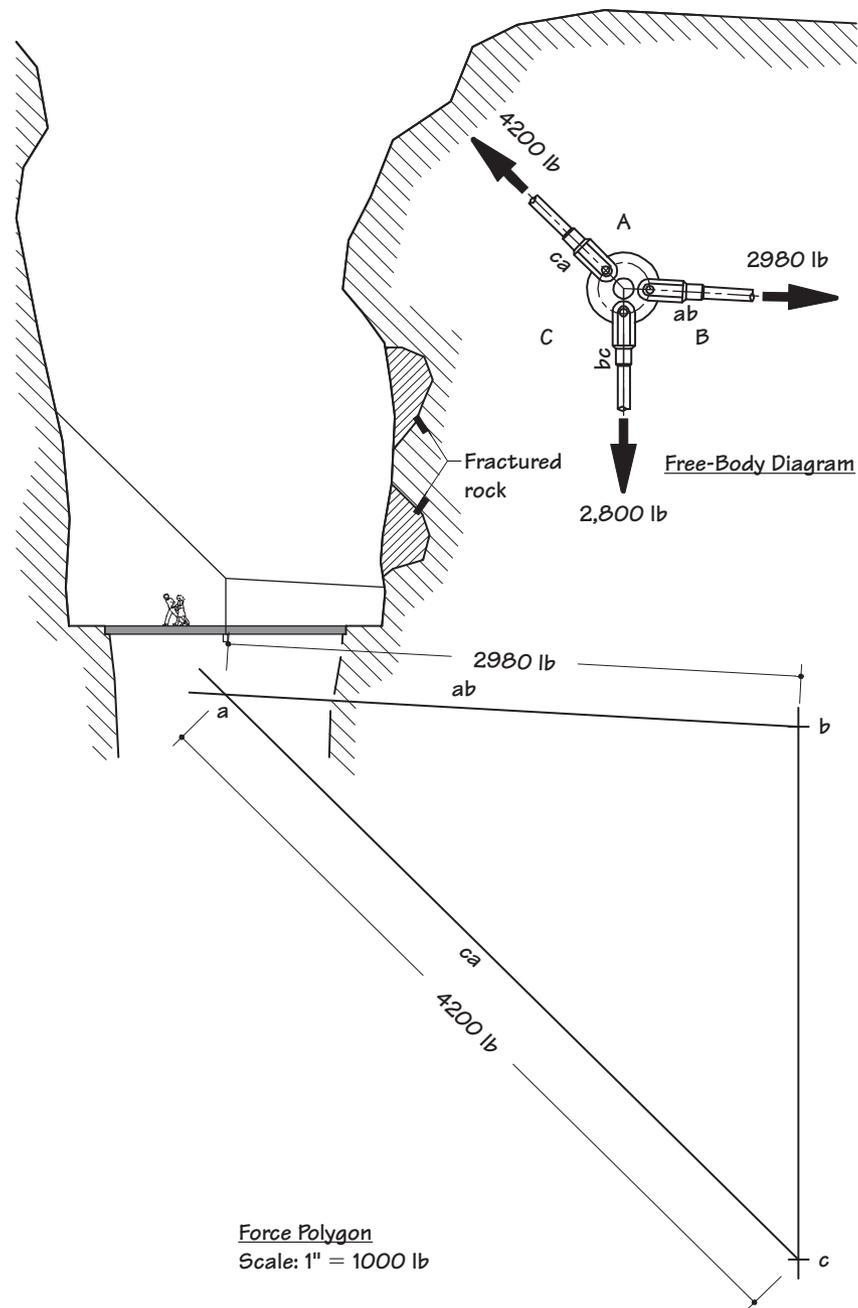


Figure 1.30 Static equilibrium is possible even with the outer end of one rod lower than its inner end.

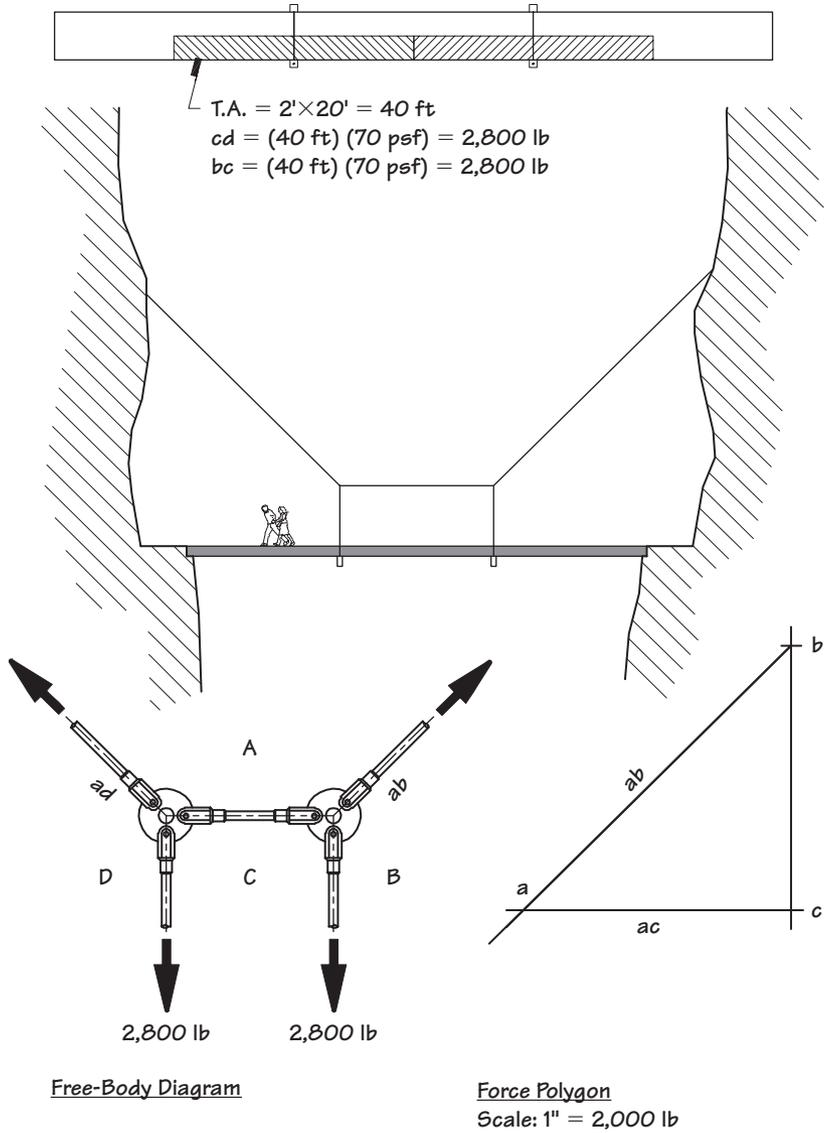


Figure 1.31 Finding forces in Bridge #5, which has a span of 60 ft. Step 1: A force polygon is constructed for the right node, *abc*.

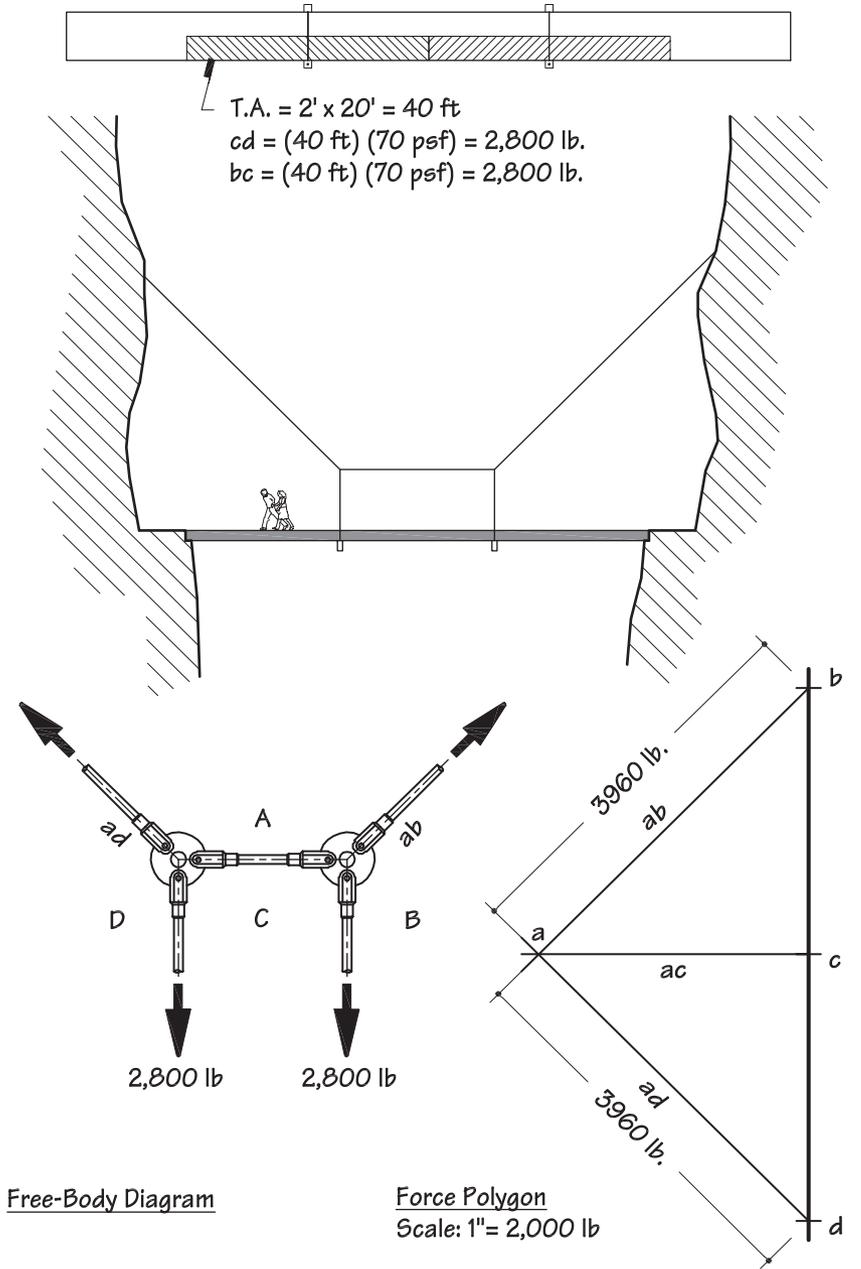


Figure 1.32 Step 2: The analysis is completed by constructing a force polygon for the left node as an extension of the force polygon for the right node.

Deformation by Nonuniform Loads

When the deck is supported at two points along each beam rather than one, a heavy load such as a group of hikers applied at one of the points can cause that point to sag and the other point to rise (Figure 1.33). This is a problem that occurs in any hanging structure that supports two or more loads. (You may find it helpful to your understanding to model this situation with a piece of string that supports two weights a short distance apart.) If nothing is done to accommodate this behavior, the bridge will feel unstable and the connections between beams and crossbeams may flex excessively, leading to eventual failure.

There are a number of ways of counteracting this problem, two of which are illustrated in Figure 1.34. In sketch (a), inclined *stay rods* have been installed below the deck at each crossbeam, anchored to the rock, and

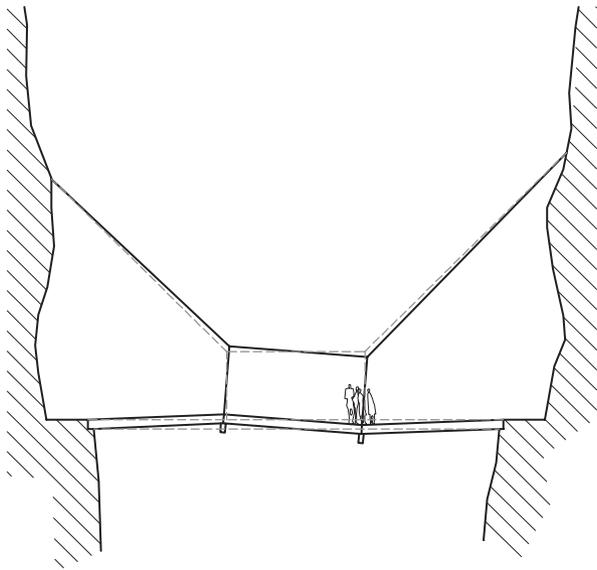


Figure 1.33 A heavy live load concentrated at one of the vertical rods can cause serious disruption of the shape of the bridge.

tightened. These hold the crossbeams down so that they cannot rise. They cannot sag, either, because in order for one of them to sag, the other would have to rise, and its stay rod will not allow it to do that. Sketch (b) in Figure 1.34 shows another way of dealing with this problem. It indicates symbolically that the joints

have been eliminated from the wood deck beams, making the deck stiff throughout its length. The stiff deck allows only very slight rises and sags in the cables.

Figure 1.35 shows that a stiff deck could be created in this bridge by building up each beam from thinner beams placed side by side and nailed together.

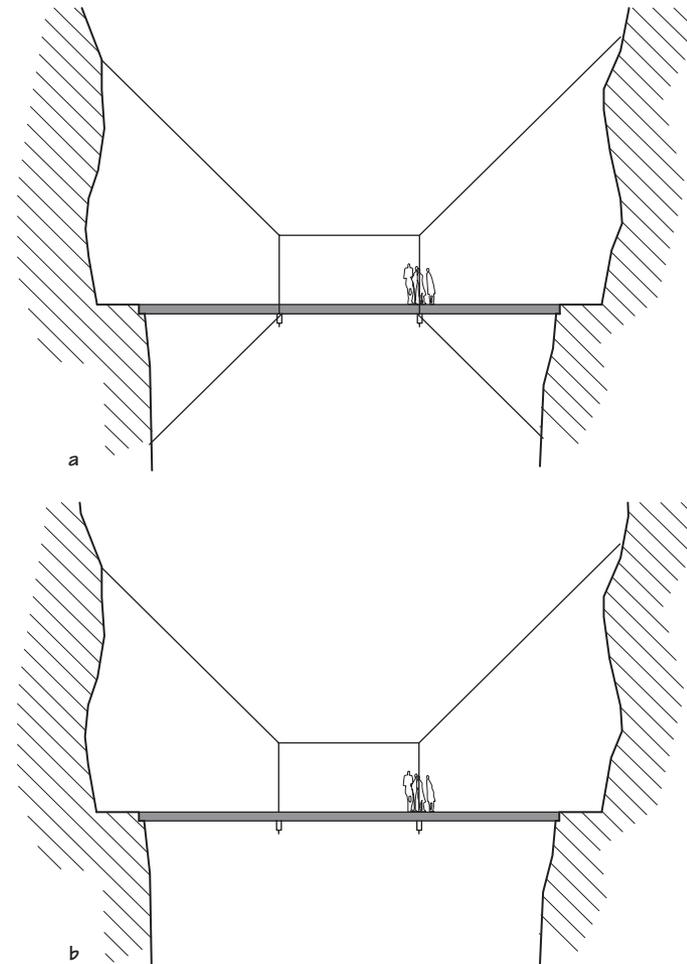


Figure 1.34 Two ways of avoiding or minimizing the problem illustrated in Figure 1.33: (a) Rods below the deck restrain the bridge against upward movement. (b) The deck is stiffened so that it is resistant to abrupt changes in shape at the supporting cables.

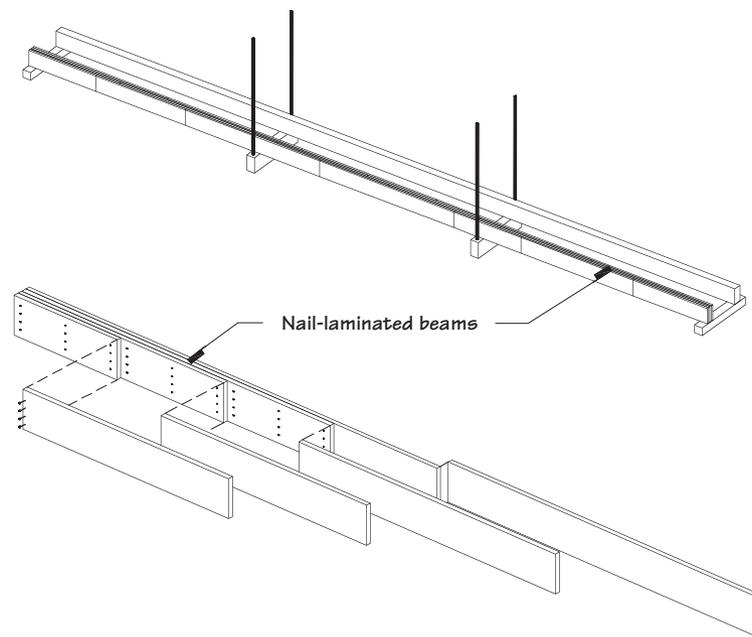


Figure 1.35 A stiff deck could be created for this bridge by nail-laminating thin planks together to form each of the beams, staggering the joints in the various layers so there are no points of major weakness.

The joints in the thinner beams would be staggered to avoid creating weak spots. The result would be a thick beam that is stiff all along its length and thus suitable for the strategy outlined in the previous paragraph. We could, of course, simply order very long beams direct from the sawmill but they would be so unwieldy that they would be almost impossible to install in the cramped space of the canyon. The nail-laminated beams would be much easier to put in place.

BRIDGE #6

The sixth bridge in the series also spans 60 ft, the same as #5, but the right anchorage must be lower to avoid an area of fractured rock in the cliff. We will

attempt to do this while retaining the remainder of the bridge's geometry as it was in #5 (Figure 1.36).

We discover that this strategy doesn't work. The force polygon does not close, which indicates that this design is not in static equilibrium. Thus we discover that we can't give an arbitrary shape to a hanging rod or cable. When designing a hanging structure, we must learn what shape the structure itself wants to take. We can find this by using the force polygon to determine the geometry of the rods. We can start by establishing arbitrarily the inclination of any two rods of the three, but the inclination of the third rod must then be found by constructing its parallel ray on the force polygon in such a way that the force polygon closes. In Figure 1.37, we have chosen to incline rod ad at an angle of 45° and ab at 30° . Their corresponding vectors

on the force polygon intersect to establish the location of a . Thus the inclination of vector ac is fixed because it must pass through points a and c on the force polygon. We draw rod segment ac parallel to these vectors and connect the ends of the rod segments to complete the picture of the bridge. We can scale the lines in the force polygon to find the forces in all the rods. You may wish to find the shape the rods must take if the center rod is horizontal and the inclination of one of the sloping rods is given.

The form that we have found for the upper rods of this bridge is a *funicular* form, one that would be taken by a rope, cable, chain, or string under a similar set of loads similarly placed. The word comes from the Latin *funiculus*, which means "string." Funicular forms experience only *axial* forces, pushes or pulls that act along the longitudinal axis of the element, when loaded in the pattern for which they are shaped. Axial forces can be resisted with the absolute minimum amount of structural material. If the form of a structure is not funicular for its loading, it will experience *bending* forces. Later in this book we will work with bending actions in structures, and we will find that a structure that acts in bending uses many times as much material to resist a force as a structure that acts axially.

Slender structural elements such as rods, cables, ropes, wires, and strings take funicular forms because they are very flexible, with no significant capability to resist bending forces. In a very real way, they are intelligent: In response to any new loading condition they always reshape themselves in such a way that they experience only axial tension. Because of this capability, funicular forms can span much farther than any other elements in the designer's repertoire.

BRIDGE #7

The seventh bridge has a span of 80 ft. This requires four 20-ft beams laid end-to-end on each side of the bridge, their ends supported at three points by

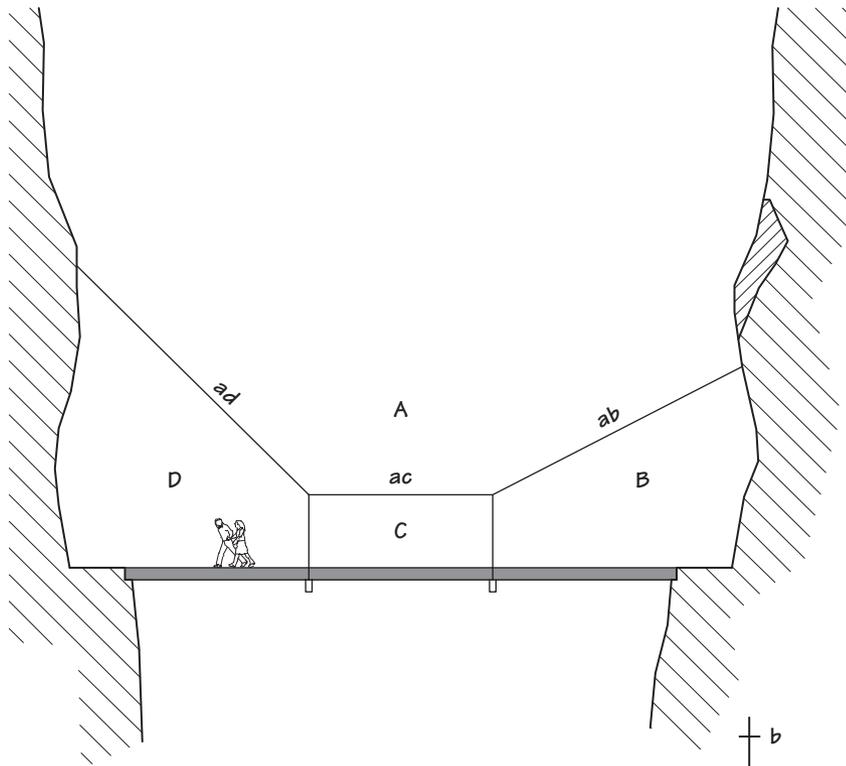


Figure 1.36 If we attempt to change the slope of one of the inclined rods, the force polygon will not close. This indicates that this form is not funicular for the two vertical loads. A stable bridge cannot be built in this form.

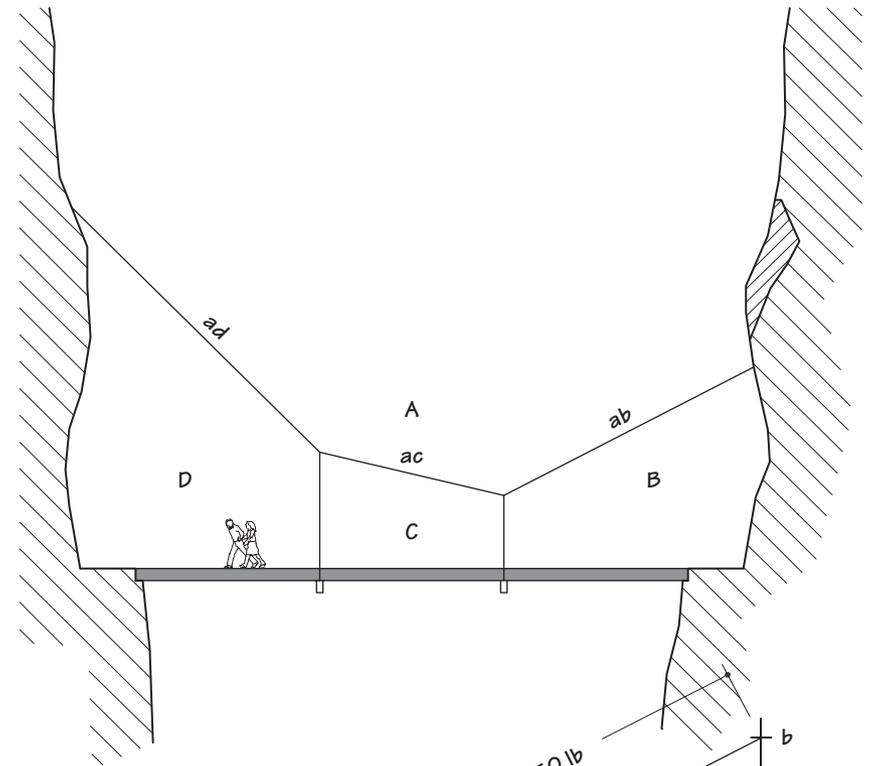
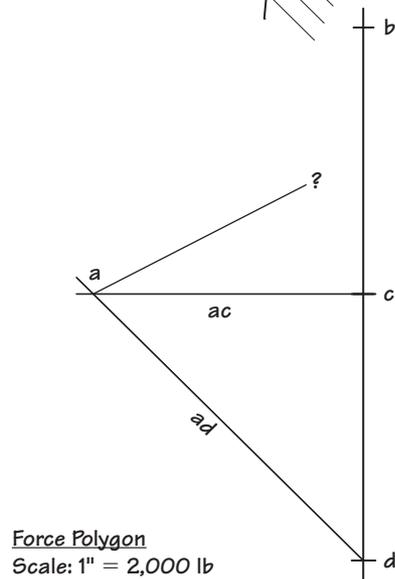
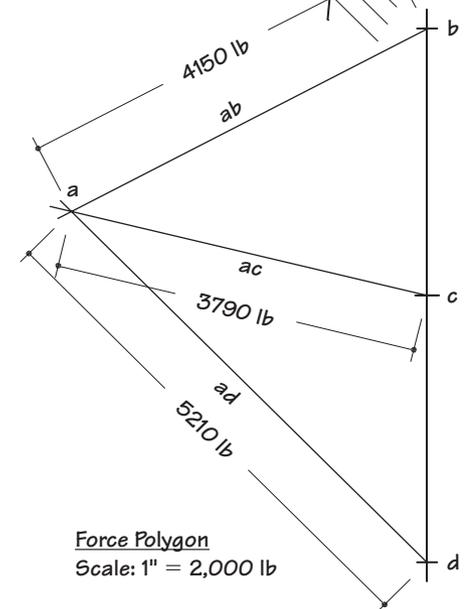


Figure 1.37 We can use the force polygon to find a funicular form for the bridge in Figure 1.36. Slopes are assumed in this case for rods da and ac . This determines the location of point a on the force polygon, which then gives us the slope of rod ab . We transfer these slopes back to the drawing of the bridge to find the form that we are seeking.



crossbeams and vertical rods (Figure 1.38). We have decided to make the arrangement of inclined rods symmetrical, with its low point 8 ft above the bridge deck. The Park Service has rods left over from an earlier project that can safely sustain a tensile force of 5,940 lb, and would like to use them in this bridge. We will use the force polygon to generate the shape of the inclined rods to meet these given conditions.

The load line is made up of three vertical segments, each of them 2,800 lb, that represent the forces in the three vertical rods on each side of the bridge. Because the arrangement of rods is symmetrical, the vectors that represent their forces on the force polygon will come together at a point that lies on a horizontal line through the center of the load line. We construct this line, but we do not yet know the location of the point on it where the vectors will converge.

To utilize the rods that the Park Service is offering from stock, the longest vector on the force polygon must have a length of 5,940 lb. Inspection of the force polygon tells us that either the topmost inclined vector or the bottommost will be the longest—each will be 5,940 lb long. Using a scale ruler and/or compass, we find the inclination of a line 5,940 lb long that has one end at point *b* and the other end on the horizontal line that we have just drawn. The point where it intersects the horizontal line is *a*.

Because all four segments of the rod have *a* in their names, all their vectors will pass through point *a* on the force polygon. We decided at the outset that the low point of the rods will be 8 ft above the bridge deck; this will occur at the middle of the span.

On the force polygon, we construct inclined vector *ac* first, because we know that the corresponding rod segment must pass through point *ACD*, which is 8 ft above the center of the span. We draw this segment parallel to vector *AC* on the picture of the bridge.

Vector *ab* is already drawn on the force polygon. We draw rod segment *ab* parallel to it through node *abc* on the picture of the bridge. To complete this construction, we draw vector *ad* on the force polygon, and rod segment *ad* parallel to it on the picture of the bridge.

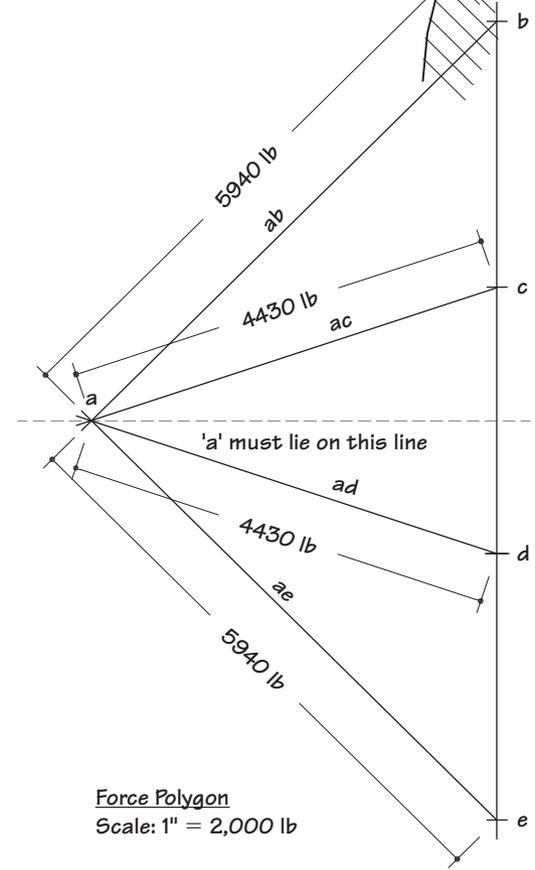
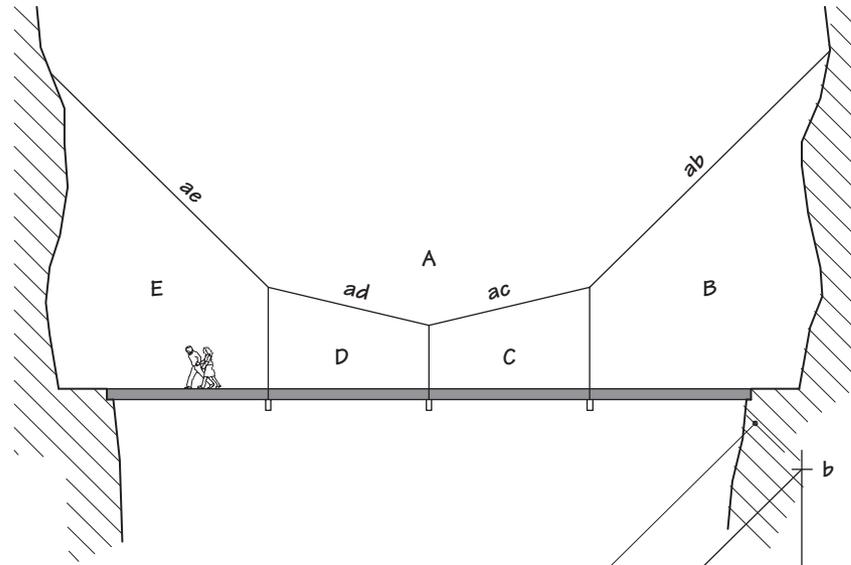


Figure 1.38 Bridge #7. Available rods have an allowable strength of 5,940 lb, so we make the longest sloping lines on the force polygon, *ab* and *ae*, 5,940 lb long at the scale we have adopted. The intersection of these two lines determines the location of *a*, which allows us to complete the force polygon. From it, we transfer parallel lines to the drawing of the bridge, starting with *ad* and *ac*, both of which go through given point *acd*. Lines *ab* and *ae* are connected to the ends of these two lines at nodes *abc* and *ade*.

Then, through node *ade*, we draw rod segment *ae* parallel to vector *ae* on the force polygon.

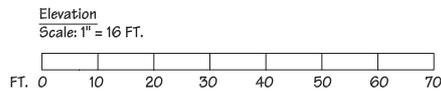
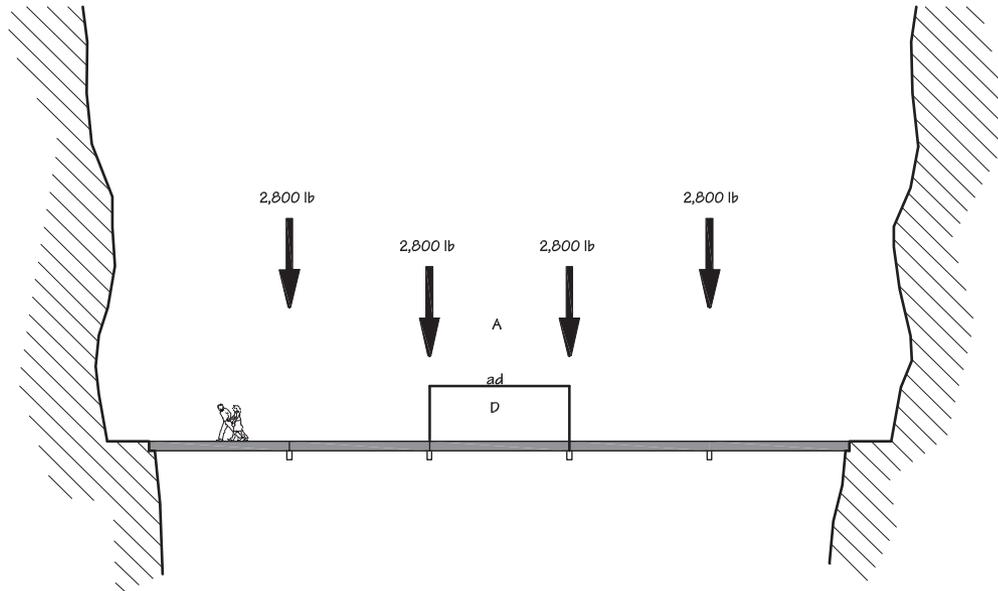
We scale the vectors on the force polygon to learn that *ab* and *ae* represent forces of 5,940 lb in their corresponding rod segments, and rods *ac* and *ad* carry 4,500 lb each. In theory, we could use smaller rods for these two segments than for the more

heavily loaded segments. In practice, it is much simpler and will save labor to use the same diameter rod throughout.

This bridge, like its three-beam-span predecessor, will tend to distort in response to asymmetrical loading patterns, and we will have to provide a stiff deck or inclined stay cables from below to restrain such distortions.

BRIDGE #8

Bridge #8, with its 100-ft span is the longest of the group (Worksheet 1A, shown in Figure 1.39). It requires five lengths of beams, as well as vertical rod supports (often called *hangers*) at four locations. You have been assigned to find the form of this bridge

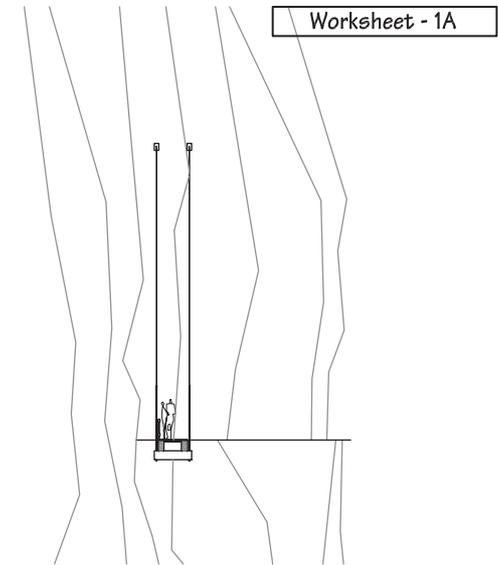
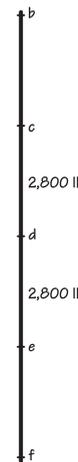


5 Bay Span

- Here we are given a bridge with:
- 5 spans = 100 ft.
 - Maximum force in rod = 9,600 lbs
 - Direction of rod segment *ad*

- Find:
- Direction and forces in rods *ab*, *ac*, *ae*, *af*

Force Polygon
Scale: 1" = 2800 lbs



Typical Section
Scale: 1" = 16 ft.

Figure 1.39 Worksheet 1A, where you are asked to find the form that the rods will take in Bridge #8, the longest in the group. The location and direction of rod *ad* is given. No rod may experience a force greater than 9,900 lb under the assumed loading. This figure has been reduced in scale from the worksheet and will not give accurate forces.

and the forces in all its rods. The group of designers working on the bridges has agreed that junctions of rods should never be less than 8 ft above the deck, so as to minimize vandalism, and the maximum force in the rods cannot exceed 9,900 lb (the allowable strength of a 0.437-in. rod). Given this information and the site as shown on Worksheet 1A, find the form of this bridge and the forces in all its rod segments. This bridge is the last of the series of suspension bridges for which we have been asked to prepare preliminary designs.

“Geometry is the mathematics of structural imagination.”

—WACŁAW ZALEWSKI

ERECTING THE BRIDGES

An important aspect of the design of any structure is to design the way in which it will be built. If we can't figure out at least one practical, economical way to build our bridges, it is unlikely that they will get built.

Figure 1.40 is a sequence of six sketches that we have prepared to suggest a construction method. In sketch *a*, workers in safety harnesses have been let down the canyon wall on ropes to drill bolt holes and install anchor plates on both sides. They stand on a small wooden platform called a *float* that hangs on ropes from the top of the cliff.

While this work is going on, a soft weight such as a small sack filled with sand is tied to the end of a long, sturdy cord and propelled across the top of the canyon with a slingshot, archery bow, or other throwing device. This cord is used to pull successively heavier cords and ropes across the top of the canyon until a rope sufficiently strong for the next step of construction has been installed. In sketch *b*, each of the rod assemblies is lowered down the cliff by a small *winch* on the canyon rim. The rope from the other rim is tied to the left top end of each assembly to act as a *tag line* that is used to pull it into place

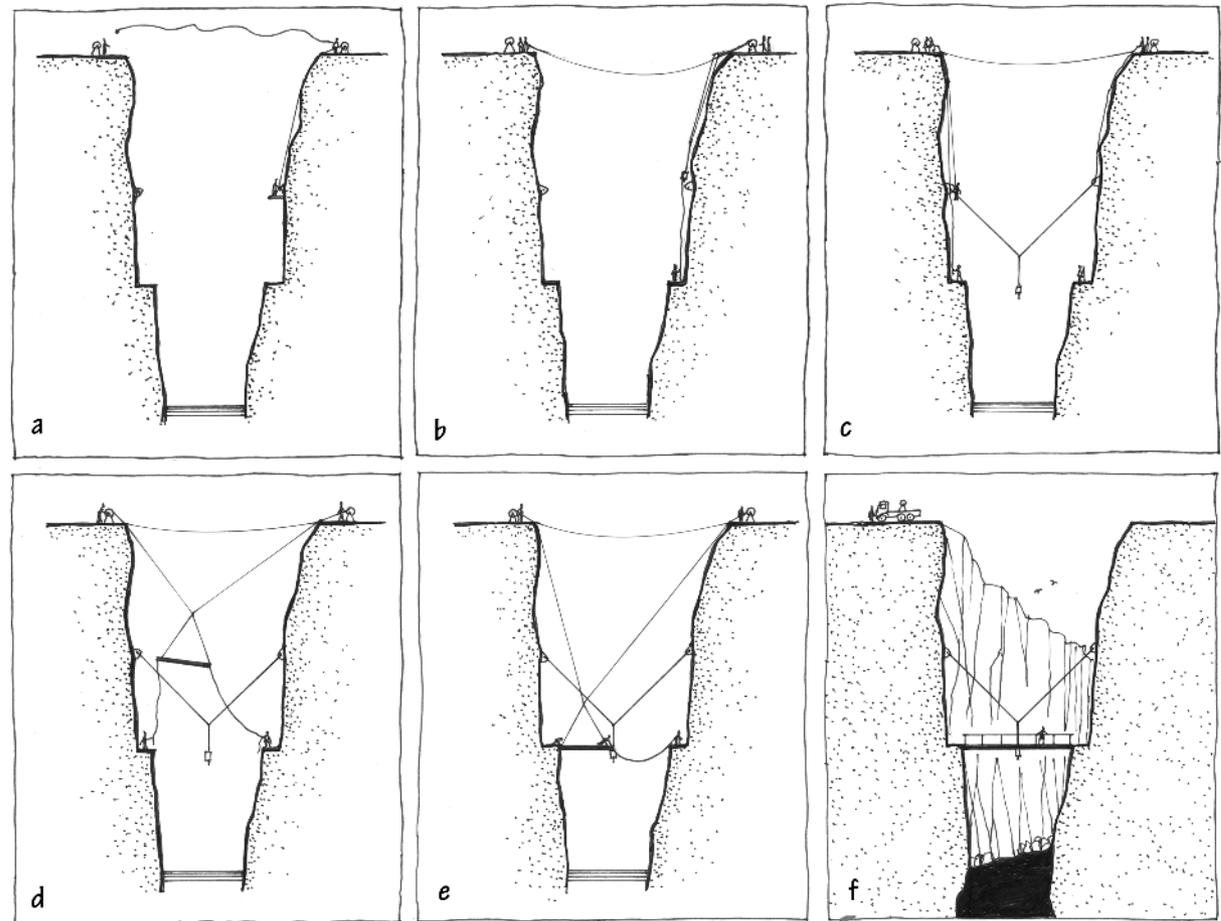


Figure 1.40 A proposed erection sequence for Bridge #1.

at the left anchorage. Workers attach the jaw fittings at the ends of the rod assemblies to the anchorages (sketch *c*).

The first beam is lowered by winches on both sides of the canyon working in coordination. Workers positioned below guide both ends of the beam with tag lines (*d*). In sketch *e*, the beam is bolted into its

final position. This entire process is repeated for all the other components. The decking and railing are installed, and in sketch *f*, the last of the construction equipment is cleared away, leaving the bridge ready for service.

The bridge may not be erected in exactly this way. The workers and supervisors who do the actual work

High-Tech Architecture

During the late 1960s and 1970s, a group of architects and engineers centered in London developed a particular approach to structural expressionism that led to what is known generically as “high-tech architecture.” This broad category, characterized by elaborately exposed, exquisitely finished structural members, mechanical equipment and services, and other typically hidden features, found resonance among designers such as Norman Foster, Renzo Piano, Richard Rogers, Nicholas Grimshaw, Anthony Hunt, and Meinhard von Gerkan. Peter Rice and Ted Happold, who both worked at the seminal interdisciplinary office founded by Danish-British engineer Ove Arup (1895–1988) in London, left to found independent engineering consulting practices (RFR and Buro Happold, respectively), which further expanded related design approaches. Buildings such as the Centre Pompidou (1972–1977) by Piano, Rogers, and Rice, with its cantilevered external tubular escalators and ducts, demonstrated how the “high-tech” look made possible some of the machine-oriented visions for futuristic architecture drawn by members of Archigram in London only a decade earlier. Many of these firms have grown as international practices even after changes in leadership or the death of their founders (e.g., Arup, Happold, Rice). Generally speaking, the Hi-Tech designers are structurally knowledgeable and have produced buildings whose structures are logically formed.

may have better ideas and methods. We will consult with them as soon as possible to seek agreement on how the work will be done, using this sequence of sketches as a starting point. It is likely that the method will be further modified during the construction process in response to problems and ideas that occur on the site.

RESISTING LATERAL AND UPLIFT FORCES

In addition to the downward *gravity forces* that we have been considering, the bridges also must resist wind forces from the side and from below, which are classified as *lateral forces* and *uplift forces*, respectively. *Lateral force resistance* is provided in most bridges by making the deck act as a beam lying on its side. A common way of doing this is to add *diagonal members*

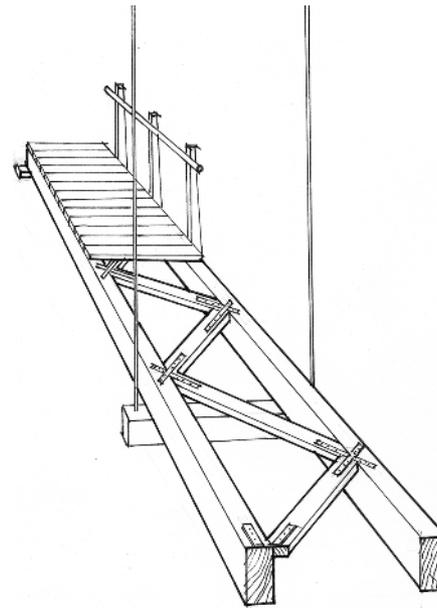


Figure 1.41 Diagonal bracing in a horizontal plane just below the deck creates a truss that prevents lateral distortion of the deck by wind loads.

in a horizontal plane within the deck structure to create a *truss*. In our wooden bridges, the diagonals would be easy to install and highly effective in resisting lateral forces (Figure 1.41).

Uplift resistance is created automatically in many bridges: the dead weight of the bridge is substantially larger than the expected uplift forces. In our bridges, with their relatively lightweight wood decks, the dead weight is insufficient for this purpose. An appropriate solution would be to add steel rods under the bridge to pull it down. If these rods lie in two planes that are sufficiently tilted from the vertical, they can serve several purposes simultaneously: to resist both lateral and uplift forces created by wind, and to restrain the main supporting rods from changing shape in response to variable loading conditions (Figures 1.42, 1.43).

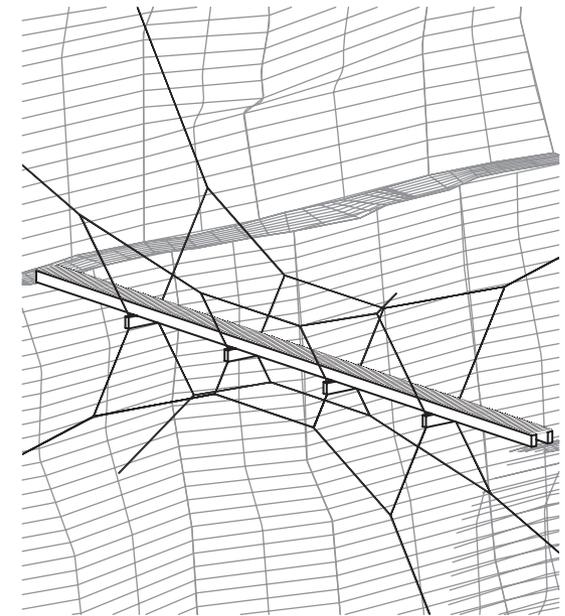


Figure 1.42 Funicular arrangements of rods could be used to resist both lateral forces and uplift forces on all the bridges. They would also help to restrain the bridge against changes of shape from concentrated loads.



Figure 1.43 The Royal Gorge Suspension Bridge, designed by George Cole and constructed in 1929, spans 268.2 m (880 ft) over the Arkansas River in Colorado. The river lies 321 m (1,053 ft) below. Two dramatic funicular cables pull both down and out on the deck to stabilize the narrow bridge. One is visible as a white arc to the lower left.

U.S. Air Force photo by Staff Sgt. Donald Branum.

ALTERNATIVE POSITIONS FOR THE RODS

There are many possible variations on our bridge design that involve changing the relationship of the rods to the deck. Figure 1.44 shows a configuration that simplifies the design by omitting the vertical rods. The bridge in Figure 1.45 relocates the rods below the deck, where they support the middle of the span by means of a central compression strut. The deck beams in this design must act in compression to resist the inward pull of the inclined rods. This bridge has no need for tensile anchors to the cliffs; it needs only simple anchor bolts at each end to keep the ends of the beams from moving sideways.

All these variations have advantages with respect to saving steel, lowering the overall profile of the bridges, and simplifying their appearance.

The building in Figure 1.46 makes ingenious use of steel rods to support the roof of a warehouse and showroom building. Figure 1.47 makes clear how the rods are deployed, and in Figure 1.48 we adapt this configuration to our 80-ft-long suspension bridge.

LOOKING AHEAD

In this lesson we have developed the rudiments of a graphical technique that can be used to design suspension bridges and suspension roofs, ones that are supported by rods or cables that take funicular forms. It is also applicable to the design of inverted forms of these structures that work in compression. In the next chapter, we will develop this technique further as we apply it to the design of a suspended roof. In the third chapter, we will explore the inverted, compressive application. Before we move on to these chapters, however, you are invited to apply what you have learned to the task of designing an aerial walkway for an ecological park.

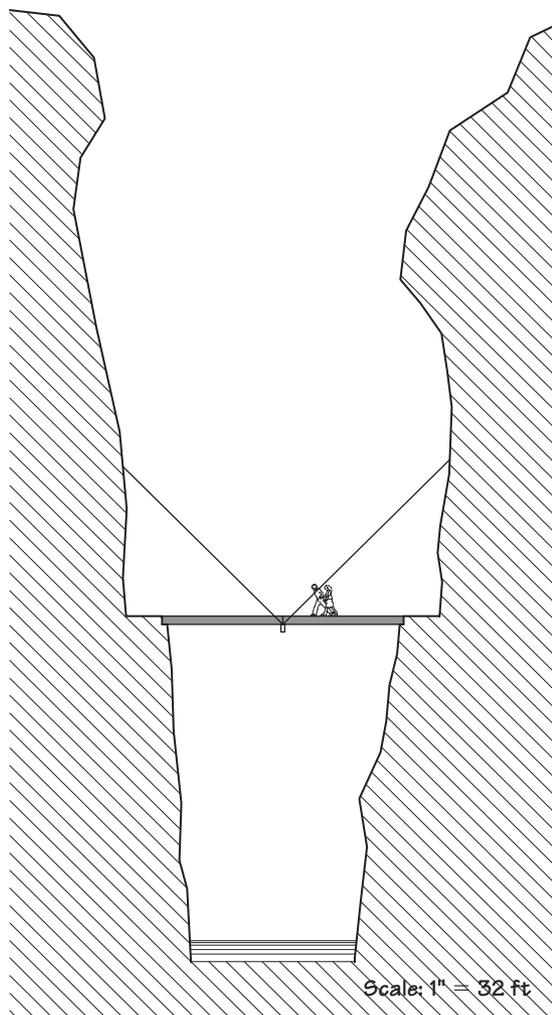


Figure 1.44 A bridge alternative that omits the vertical rods.

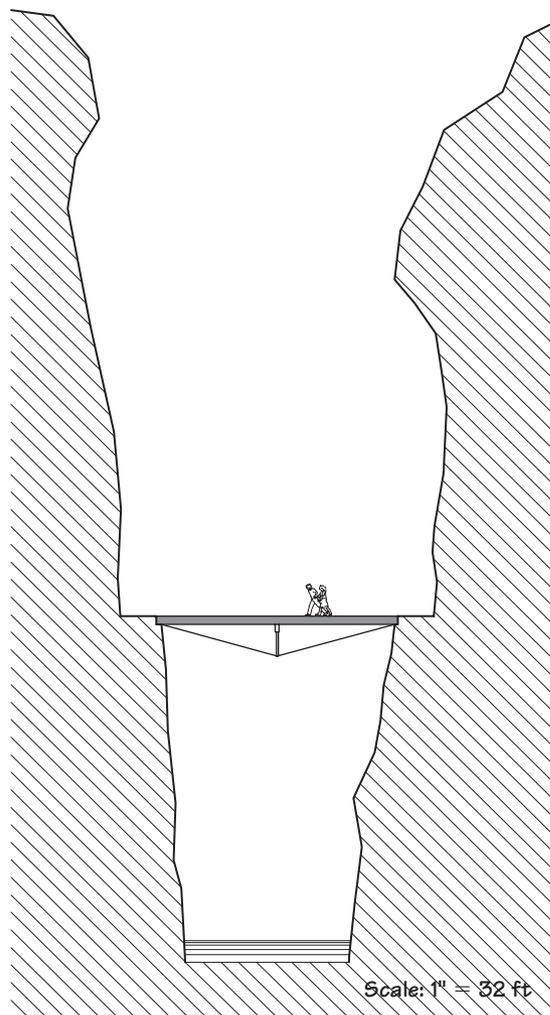


Figure 1.45 A bridge alternative with the rods completely below the deck.



Figure 1.46 The roof of the Renault Centre at Swindon, England, is supported by an ingenious arrangement of steel rods and beams. (Architect: Norman Foster. Structural engineer: Arup Associates.)

Photo courtesy of Richard Davies.

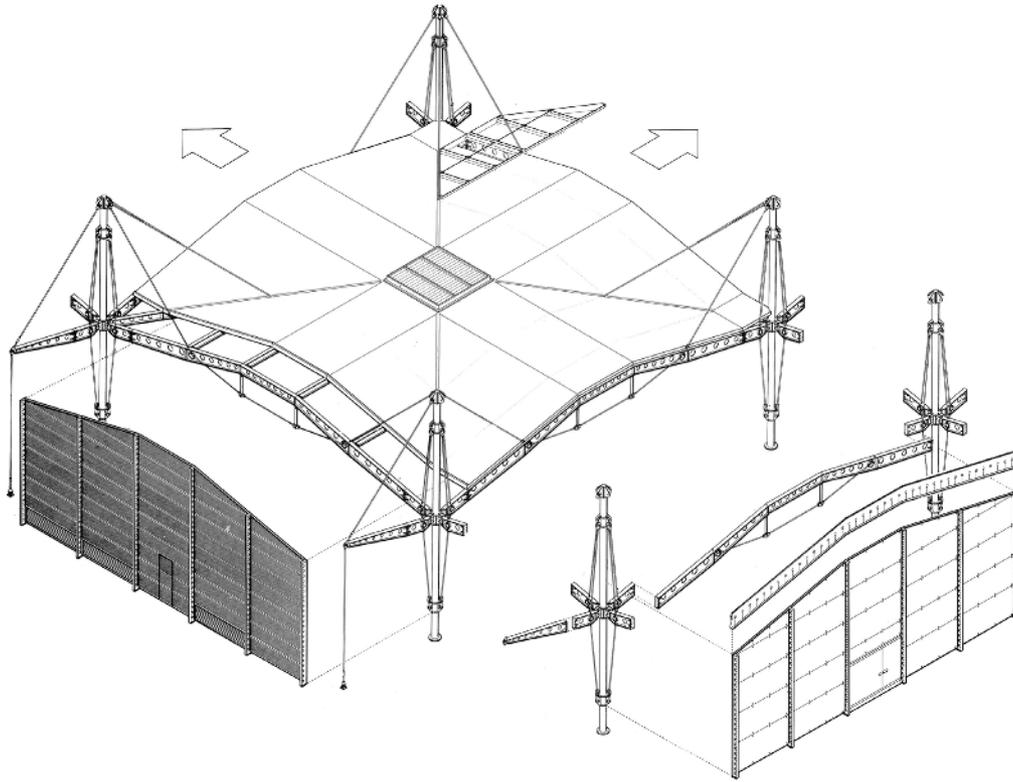


Figure 1.47 Given a measured drawing of the roof structure of the Renault Centre, how would you go about finding the forces in its members?

Image courtesy of Foster Associates, as shown in Chris Abel, *Renault Centre*, Norman Foster.

ANOTHER CHALLENGE: DESIGNING AN AERIAL WALKWAY

A new eco-park on the Pacific Coast of Oregon will feature an elevated walkway 8 ft wide that makes it possible for visitors to stroll through the treetops of mammoth Douglas firs and red cedars, where they can observe the birds, mammals, insects, fungi, lichens, and other life forms that make up the unique ecosystem of a forest canopy. Worksheet 1B (available on the supplemental website) is set up to make it easy for you to design this walkway (Figure 1.49). The structural

concept that has been suggested by the client's structural engineer is to erect a series of 100-ft-high cedar log frames 72 ft apart, using these to support the walkway at a height of approximately 60 ft above the ground. The available wooden beams can span no farther than 24 ft between the stainless steel rod supports that you will provide. The gravity load includes a live load estimate of 100 lb per square foot (psf) and a dead load of 24 psf. You must propose ways of resisting gravity loads, lateral forces, and uplift forces from wind. A large construction crane is available to help erect the walkway.

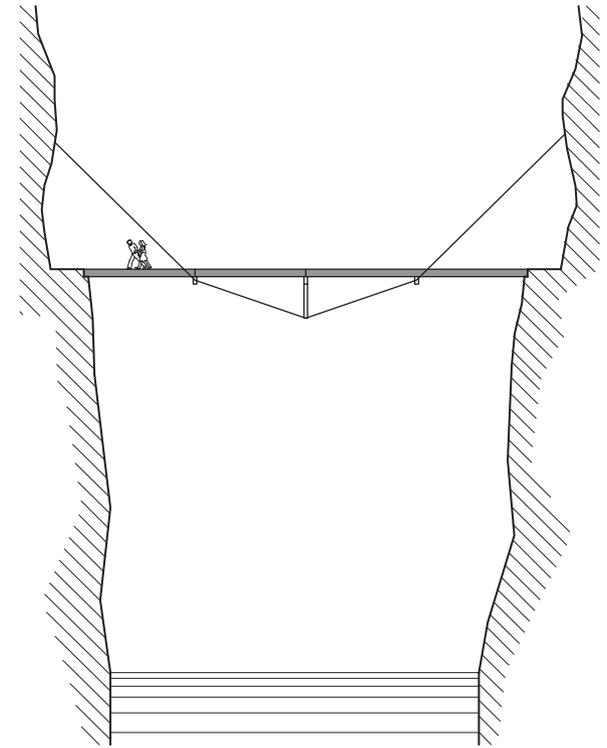


Figure 1.48 This configuration of rods and struts from the Renault Centre could be envisioned in modified form as a solution for the footbridge as well.

To present your structural concept, you are to complete an elevation drawing (side view) of the bridge and a section, both of which are shown partially completed on the worksheet. Notice that in the elevation, because of the zigzagging path of the walkway, only Span II appears in its true size and shape, so you should find the form and forces on this span before transferring the shapes to the other spans. You are to determine the forces in the rods that support the walkway and to show your means of finding these forces on your drawing. A load line location is shown as the starting point for a force polygon. The force

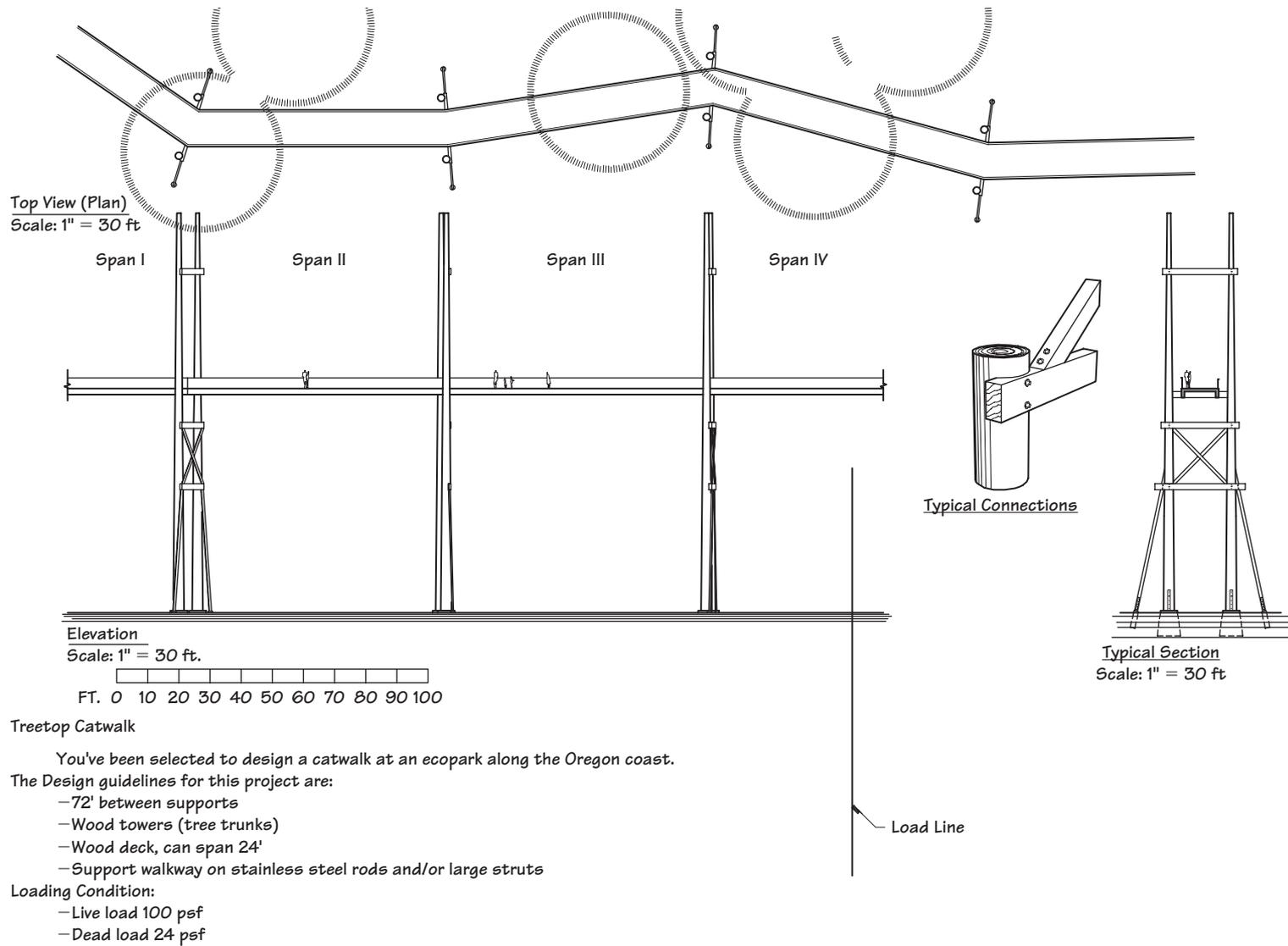


Figure 1.49 Worksheet 1B shows the project of designing an aerial walkway in an eco-park. As with all the worksheets, the full-scale worksheet is available on the supplemental site.

polygon will overlap the elevation of the walkway, as is often done to save space on a sheet of drawings. Find and indicate the required diameters of the rods.

The meandering plan of the walkway presents a problem: At most towers, the suspension rods for the two adjoining spans do not lie in the same plane. If they did, their horizontal pulls would balance one another. In this nonorthogonal walkway layout, however, components of these forces will tend to pull the towers over sideways. Work out a simple way of resisting this tendency and show it on your drawings.

► **Figure 1.50** The Menai Bridge (1818–1826) was designed and built by Thomas Telford to provide a crossing of the Menai Strait for the road connecting London and Holyhead. The eyebar links that make up the chains were made of wrought iron, which is much less brittle than cast iron, in the era before plentiful, economical steel was available. The span of the bridge, approximately 580 ft, was by far the longest in the world at the time of its construction. The bridge still carries vehicular traffic nearly two centuries later. For more information on Thomas Telford, see his biography on the overleaf.

Photo courtesy of Kyle Gann.



◄ **Figure 1.51** Schlaich Bergermann collaborated in 1988 with architect Eberhard Schunck to use funicular rods to support a pedestrian bridge over the railroad tracks at Bad Windsheim in Germany.

Photo courtesy of Schlaich Bergermann and Partners.



◀ **Figure 1.52** The engineers at Schlaich Bergemann used a similar arrangement of rods to suspend this large-span factory roof at Dachwig, Germany, providing an efficient solution and eye-catching profile to satisfy the client's desire for corporate identity. It is shown here during construction in 1993.

Photo courtesy of Schlaich Bergemann and Partners.

THOMAS TELFORD

Thomas Telford was born in Scotland in 1757. He began his career as a mason and builder, but gradually transitioned to work as a surveyor and then to design and construction of several dozen road bridges in Shropshire. He became involved in the creation of the Ellesmere Canal, including, in 1797, the dramatic Pontcysyllte Aqueduct, which carries a navigable cast-iron trough of water 126 ft above the River Dee. Between 1811 and 1826, he designed and constructed the 580-ft span Menai suspension bridge to enable travel between London and the port at Holyhead. This early suspension bridge was a major influence on the succeeding generation of British engineers. Among its notable features were its rock anchor design, its efficient depth of curvature, and its parallel bar wrought-iron chain and hangers. Nearly two centuries later it continues to carry traffic. Although Poet Laureate Robert Southey referred to Telford, in jest, as the “colossus of roads,” in fact, Telford’s professional output included docks, railway construction, buildings, travel writing, and even published poetry, in addition to his roadway, canal, and bridge projects. He became the first president of the Institute of Civil Engineers and served from 1820 until his death in 1834. He is buried in Westminster Abbey.

▶ **Figure 1.53** The Lowry Bridge, a lifting footbridge over the Manchester (England) Ship Canal at Salford Quays, was designed by engineers at the Parkman (now Mouchel) Group and built in 2000. Its 92-m (301-ft) span has rod and cable elements that suspend the deck from inclined arches, in an arrangement designed to resist racking and vibration.

Photo: David M. Foxe.



Key Terms and Concepts

span	point of application of force
deck	scalar quantity
suspension bridge	vector quantity
stainless steel	tension
grout	compression
jaw	principle of transmissibility
fork	concurrent forces
clevis	nonconcurrent forces
steel fabricator	free-body diagram (FBD)
washer	free body
jam nut	graphical methods
plate	numerical methods
anchor plate	node
load	static equilibrium
dead load	parallelogram law
live load	resultant
total load	equilibrant
pound	antirestant
tributary area	tip-to-tail addition of vectors
force	force polygon
push, pull	graphical solution
stress	numerical solution
deformation	Bow's notation
magnitude of force	load line
direction of force	clockwise reading of member names
line of action of force	buckling

deformation	axial force
yield strength	bending force
factor of safety	restraining funicular structures
threads	character of a force
upset end	gravity forces
right-hand and left-hand threads	lateral forces
cold heading	diagonals
clearance	truss
ultimate load	uplift forces
stay rods	hanger
funicular	winch
funicular form	tag line

Further Resources

Abel, Chris. *Renault Centre, Norman Foster*. London, UK: Architecture Design and Technology Press, 1994.

www.clevelandcityforge.com: A manufacturer of rod and cable hardware. See especially “upset forged threaded rods.”

www.ronstan.com: Another useful manufacturer's site; see the section on “Architectural rigging systems.”

www.tri pyramid.com: TriPyramid Structures is a firm that acts as a specialized consultant for designing and fabricating precision components, including architectural tension connections, cables, and rods. See the product and technical sections, in particular, for information about rod materials.