PART I
Principles of Industrial Lasers

1 Laser Generation

In this chapter, we outline the basic principles underlying the generation of a laser beam. The term laser is an acronym for light amplification by stimulated emission of radiation, and thus a laser beam, like all other light waves, is a form of electromagnetic radiation. Light may be simply defined as electromagnetic radiation that is visible to the human eye. It has a wavelength range of about 0.37 – 0.75 μm, between ultraviolet and infrared radiation, with a median wavelength and frequency of 0.55 μm and 10^{15} Hz, respectively. Lasers, however, may have wavelengths ranging from 0.2 to 500 μm, that is, from X-ray, through ultraviolet and visible, to infrared radiation. Figure 1.1 illustrates the electromagnetic spectrum, which indicates that the visible spectrum is only a minute portion of the entire spectrum. The colors associated with the various wavelengths in the visible range are listed in Table 1.1. The various colors are characterized by specific wavelength ranges. However, white light has the same amplitude over all wavelengths in the visible light region.

In its simplest form, laser generation is the result of energy emission associated with the transition of an electron from a higher to a lower energy level or orbit within an atom. Thus, before proceeding with our discussion on laser generation, we first look at the basic structure of an atom. This is followed by a discussion on atomic transitions and associated absorption, spontaneous emission, and stimulated emission, which form the cornerstone of laser generation. To gain some insight into the timescale associated with transitions, the lifetime or time constant of an excited atom is briefly discussed. The absorption of a beam as it propagates through a medium is then presented to lay the foundation for a discussion on two criteria that are necessary for sustaining laser oscillation, that is, population inversion and threshold gain. Finally, the concept of two-photon absorption is introduced.

1.1 BASIC ATOMIC STRUCTURE

From basic chemistry, we know that an atom consists of a nucleus that is surrounded by electrons and that the nucleus itself is composed of protons and neutrons. The electrons are negatively charged, and the protons positively charged, while the neutrons are electrically neutral. In a simplified description of the atomic structure based on the Bohr model, the electrons are considered to move in circular orbits of

---

Principles of Laser Materials Processing, by Elijah Kannatey-Asibu, Jr.

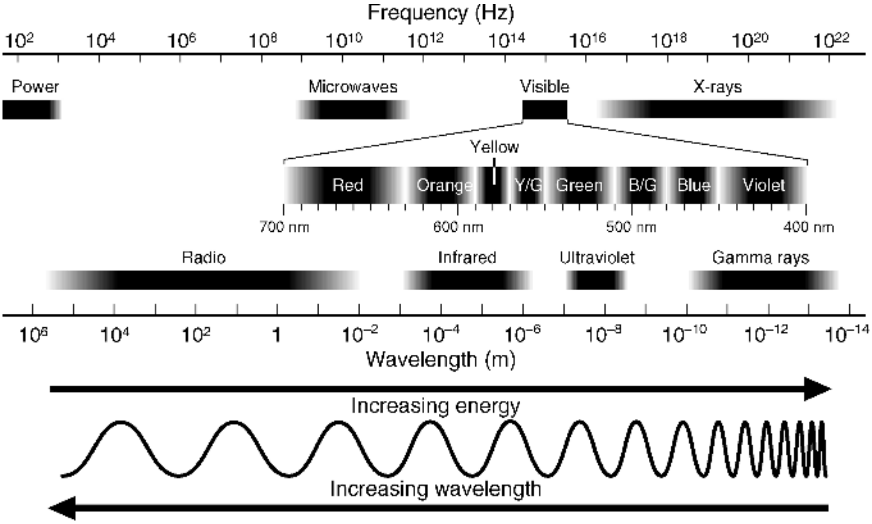

FIGURE 1.1 The electromagnetic spectrum.

specific radii corresponding to discrete energy states, with the nucleus as the center. The atom may then be viewed as consisting of circular shells of electrons, where the lowest shell corresponds to the lowest energy state, with the energy of the higher level shells or energy states being integral multiples, m ($m = 1, 2, 3, \ldots$), of the lowest state (Fig. 1.2). This is a simple description of atomic structure using classical mechanics.

The actual motion of an electron in an atom is best characterized using quantum mechanics rather than classical mechanics due to the uncertainty associated with specifying the position and velocity of the electron, a consequence of the Heisenberg uncertainty principle. Classical mechanics is based on a deterministic description of the motion, that is, assumes precise knowledge of the position and velocity. Quantum mechanics, on the contrary, is based on stochastic theory, which indicates the

TABLE 1.1 Wavelengths Associated with the Visible Spectrum

Wavelength Range (nm)	Color
400–450	Violet
450–480	Blue
480–510	Blue–green
510–550	Green
550–570	Yellow–green
570–590	Yellow
590–630	Orange
630–700	Red

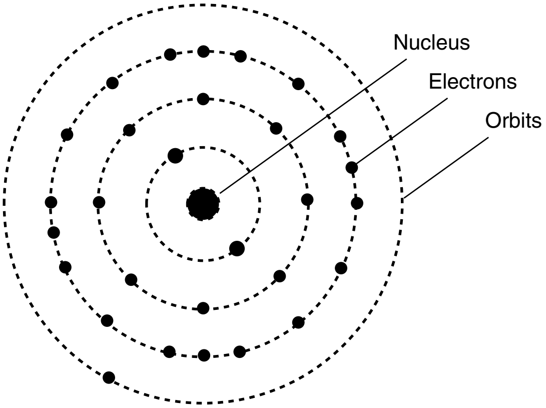

FIGURE 1.2 Schematic of atomic structure (copper).

probability of finding the electron at a certain location rather than a precise specification of the location. This is illustrated in Fig. 1.3. On the basis of this, the motion of electrons in an atom is described by the Schrodinger wave equation, which for the one-dimensional case is given by

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m_e}{{h_p}^2}(E - V_e)\psi = 0 \tag{1.1}$$

where ψ is the wave function of the electron, h_p is the Planck's constant $= 6.625 \times 10^{-34}$ J – s, m_e is the mass of electron $= 9.11 \times 10^{-31}$ kg, E is the total energy of electron (J), V_e is the potential energy of electron (J), and x is the electron position (m).

ψ is a probability function such that $|\psi|^2 dx$ is the probability of the electron being in an interval dx. There are only specific or discrete values of E for which a solution is obtained for the fully three-dimensional form of equation (1.1), indicating that the

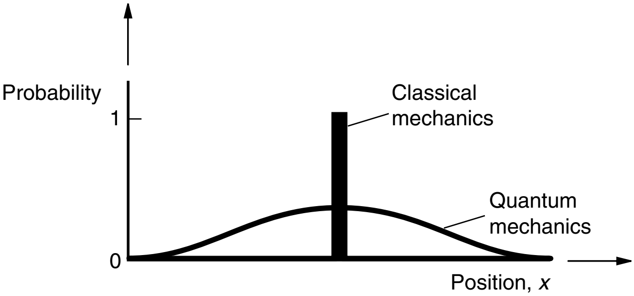

FIGURE 1.3 Classical and quantum mechanics descriptions of electron position in an atom.

electron can only have discrete energy states. This leads to the quantized nature of atoms.

In the more realistic description of the atomic structure given by the Schrodinger equation, the electron orbit is more complex, and not necessarily circular. The motion of the electrons is then described by a set of four quantum numbers indicated by n, l, m, s.

- n: This is the principal quantum number and determines the size of the electron orbit. It takes on integer values $n = 1, 2, 3, 4, \ldots$, with each number corresponding to a specific orbit. These are commonly referred to as the $K, L, M, N, \ldots$ shells. The energy differences between contiguous n values are much greater than those for the other quantum numbers.
- l: This is the quantum number characterizing the orbital angular momentum of the electron and takes on the integer values $l = 0, 1, 2, 3, \ldots, n - 1$, which are commonly referred to as the s, p, d, f, ... states. The angular momentum of the s state electrons is zero, giving them a spherical orbit since they move in all directions with the same probability. The other states do not have zero angular momentum, and thus have some directionality to their orbital motion.
- m: This is the magnetic quantum number and indicates the spatial orientation of the angular momentum. It takes on the integer values $-l \leq m \leq l$. It affects the energy of the electron only when an external magnetic field is applied.
- s: This is the quantum number associated with the spin of the electron itself about its own axis and indicates the angular momentum of the electron. It is either $+\frac{1}{2}$ or $-\frac{1}{2}$.

When atoms that have the same n and l values but different m and s values have the same amount of total energy, they are said to be degenerate. The presence of a magnetic field, however, changes the energy of each degenerate level, depending on m and s, thereby removing the degeneracy.

On the basis of the foregoing discussion, we find that each electron in an atom is identified by a set of quantum numbers that uniquely defines its motion, and according to the *Pauli exclusion principle*, no two electrons in an atom can have the same set of quantum numbers. Thus, there cannot be more than one electron in a quantum state. The quantum states of an atom are occupied starting with the lowest level, that is, $n = 1$, and increasing as each level is filled. Within each given level, say $n = 2$, occupation starts with the $l = 0$ or s sublevel. Thus for, say, copper with an atomic number of 29 (i.e., 29 electrons), the atomic structure (Fig. 1.2), will be

$$1\mathrm{s}^2 2\mathrm{s}^2 2\mathrm{p}^6 3\mathrm{s}^2 3\mathrm{p}^6 3\mathrm{d}^{10} 4\mathrm{s}$$

where the principal quantum numbers are indicated by the integers, while the superscripts indicate the number of electrons that have the same principal and orbital quantum numbers.

After briefly reviewing the basic structure of the atom, we now turn our attention to atomic transitions or changes in the energy levels of electrons that form the basis for laser generation. The discussion starts with the selection rules that determine what transitions can occur. This is followed by a discussion on how atoms are distributed among different energy levels. The different forms of transition are then presented, along with the Einstein coefficients, which constitute an integral part of the equations governing the transitions.

1.2 ATOMIC TRANSITIONS

Under the right conditions, electrons within an atom can change their orbits. Light or energy is emitted as an electron moves from a higher level or outer orbit to a lower level or inner orbit and is absorbed when the reverse transition takes place. The emission and absorption of light is explained by the fact that light generally consists of photons, which are small bundles or quanta of energy or particles. A photon is able to impart its energy to a single electron, enabling it to overcome the force of attraction restraining it to the atom surface (the work function) and also providing it with initial kinetic energy that enables it to move from a lower to a higher orbit. There is a specific quantum of energy (a photon), ΔE, of specific wavelength or frequency associated with each transition from one orbit or energy level to another and is given by

$$\Delta E = \frac{h_{\mathrm{p}} c}{\lambda} = h_{\mathrm{p}} \nu \tag{1.2}$$

where c is the velocity of light = 3×10^8 (exactly 299, 792, 458) m/s, λ is the wavelength (m), ν is the frequency of transition between the energy levels (Hz), and ΔE is the energy difference between the levels of interest.

Since there are a number of discrete orbits, there are a variety of different transitions possible, and thus many different frequencies that can be emitted.

1.2.1 Selection Rules

As we learned in Section 1.1, the motion of an electron in an atom can only be described using probability theory. In much the same way, the transition of an electron from one energy level to another can only be described using probability theory. The transitions that have a very high probability of occurring are said to be the allowed transitions. Other transitions have a very low probability of occurring (almost nonexistent). These are the forbidden transitions. The selection rules determine which transitions are permitted. One common rule is that, during a transition, there cannot be a change in the total spin of an atom (which is the sum of the individual electron spins). Let us now take a look at the distribution of atoms at various energy levels under equilibrium conditions.

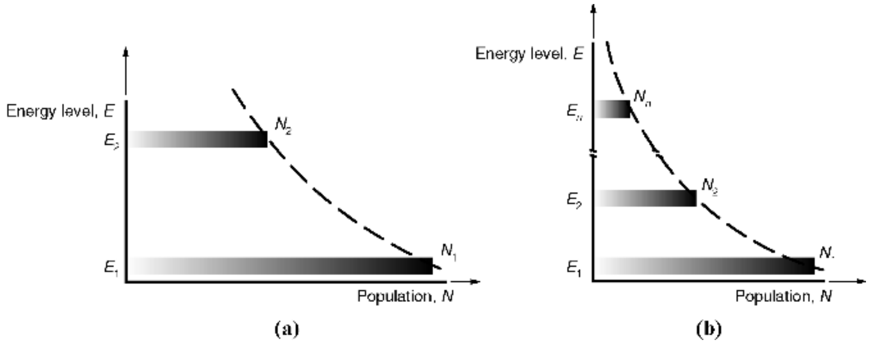

FIGURE 1.4 Schematic of Boltzmann's law. (a) Two-level system. (b) More general case for a multilevel system.

1.2.2 Population Distribution

For simplicity, let us focus our initial discussion on a single frequency, which corresponds to two specific energy levels or orbits, E_1 and E_2, where E_1 is the lower energy level and E_2 is a higher energy level, that is, $E_2 > E_1$. Furthermore, we let the *population* or number of atoms (or molecules or ions) per unit volume at level 1 be N_1 and that at level 2 be N_2. We also assume conditions of nondegeneracy. Degeneracy exists when there is more than one level with the same energy. For example, when atoms with the same values of n and l but different values of m and s have the same total energy.

Now let us consider the distribution of atoms among the energy levels under normal or thermal equilibrium conditions. Under such conditions of thermal equilibrium, the lower energy levels are more highly populated than the higher levels, and the distribution is given by Boltzmann's law that relates N_1 and N_2 as

$$\frac{N_2}{N_1} = e^{-\frac{E_2 - E_1}{k_B T}} = \mathrm{e}^{-\frac{h_p \nu}{k_B T}} \tag{1.3}$$

where k_B is Boltzmann's constant = 1.38×10^{-23} J/K and T is the absolute temperature of the system (K).

This is illustrated in Fig. 1.4a. Figure 1.4b illustrates the equilibrium distribution for the more general case. Boltzmann's law holds for thermal equilibrium conditions, and as such, N_2 will always be less than N_1 under equilibrium conditions. What this means is that the number of atoms with electron configurations corresponding to the excited or higher energy level will be less than those corresponding to the lower energy level. We now look at the various transitions associated with laser generation.

1.2.3 Absorption

Now consider atoms that are in a lower energy state, E_1. Generally, this would be the ground state. When such atoms are *excited* or *stimulated*, that is, they are subjected to some external radiation or photon with the same energy as the energy difference

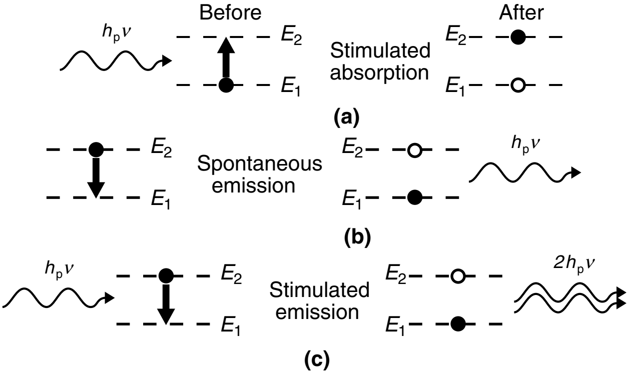

FIGURE 1.5 Schematic of (a) absorption, (b) spontaneous emission, and (c) stimulated emission. (From O'Shea, D. C., Callen, W. R., and Rhodes, W. T., 1977, *Introduction to Lasers and Their Applications*. Reprinted by permission of Pearson Education, Inc.)

between the lower state and a higher state or level, say E_2, the atoms will change their energy level (electrons raised to a higher energy level or molecular vibrational energy increased). If electromagnetic radiation of frequency ν is incident on these atoms, the atoms will absorb the radiation energy and change their energy level to E_2 (Fig. 1.5a) in correspondence with equation (1.2). This process is called *absorption* or more specifically, *stimulated absorption*. The rate at which energy is absorbed by the atoms will be proportional to the number of atoms at the lower energy level and also to the energy density of the incident radiation. Thus,

$$n_{\text{abs}} = B_{12} N_1 e(\nu) \tag{1.4}$$

where B_{12} is a proportionality constant referred to as the Einstein coefficient for stimulated absorption, or stimulated absorption probability per unit time per unit spectral energy density (m^3 Hz/Js), N_1 is the population of level 1 (per m^3), $e(\nu)$ is the energy density (energy per unit volume) at the frequency ν ($\text{J/m}^3\text{Hz}$), and n_{abs} is the absorption rate (number of absorptions per unit volume per unit time).

Once the atom has been excited to a higher energy level, it can make a subsequent transition to a lower energy level, accompanied by the emission of electromagnetic radiation. The emission process can occur in two ways, by *spontaneous emission* and/or *stimulated emission*. Each absorption removes a photon, and each emission creates a photon.

1.2.4 Spontaneous Emission

Spontaneous emission occurs when transition from the excited state to the lower energy level is not stimulated by any incident radiation but occurs more or less naturally (Fig. 1.5b). This happens because the excited atoms want to go back down to their ground state, and if left alone, it is just a matter of time before they do. If the atom was completely stable in its excited state, there would be no spontaneous emission.

The transition between energy levels E_2 and E_1 results in the emission of a photon of energy given by

$$\Delta E = E_2 - E_1 = h_p \nu \tag{1.5}$$

where ν is the frequency of the emitted photon. In spontaneous emission, the rate of emission per unit volume, n_{sp}, to the lower energy level is only proportional to the population, N_2, at the higher energy level and is independent of radiation energy density. Thus, we have

$$n_{sp} = A_e N_2 \tag{1.6}$$

where A_e is the Einstein coefficient for spontaneous emission, or spontaneous emission probability per unit time. The photons emitted by individual atoms under spontaneous emission are independent of each other, and thus there is neither a phase nor directional relationship between them.

1.2.5 Stimulated Emission

If the atom in energy level 2 is subjected to electromagnetic radiation or photon of frequency ν corresponding to the energy difference $\Delta E = E_2 - E_1$ between levels 1 and 2, the photon will stimulate the atom to undergo a transition to the lower energy level. The energy emitted as a result of this transition, which is in the form of an electromagnetic wave or a photon, is the same as the stimulating photon and is superimposed on the incident photon, thereby reinforcing the emitted light (Fig. 1.5c). This results in stimulated emission, where the incident and emitted photons have the same characteristics and are in phase, resulting in a high degree of coherence, and the direction, frequency, and state of polarization of the emitted photon are essentially the same as those of the incident photon. The two photons can generate yet another set, with a resulting avalanche of photons. This is illustrated schematically in Fig. 1.6. The rate of emission per unit volume, n_{st}, in the case of stimulated emission is also proportional to the population at level 2, as well as the energy density, and is given by

$$n_{st} = B_{21} N_2 e(\nu) \tag{1.7}$$

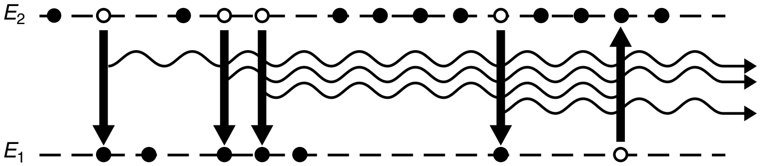

FIGURE 1.6 Illustration of the process of stimulated emission. (From Chryssolouris, G., 1991, *Laser Machining: Theory and Practice*. By permission of Springer Science and Business Media.)

where B_{21} is the Einstein coefficient for stimulated emission, or stimulated emission probability per unit time per unit energy density ($\mathrm{m^3\ Hz/J\ s}$).

1.2.6 Einstein Coefficients: A_e, B_{12}, B_{21}

Under conditions of thermal equilibrium, the rates of upward ($E_1 \rightarrow E_2$) and downward ($E_2 \rightarrow E_1$) transitions must be the same. Thus, we have

$$n_{1\rightarrow 2} = n_{2\rightarrow 1} \tag{1.8}$$

In other words,

$$\text{Stimulated absorption rate} = \text{stimulated emission rate} + \text{spontaneous emission rate} \tag{1.9}$$

or from equations (1.4), (1.6), and (1.7),

$$B_{12}N_1 e(\nu) = B_{21}N_2 e(\nu) + A_\mathrm{e}N_2 \tag{1.10}$$

This gives the energy density as

$$e(\nu) = \frac{A_\mathrm{e}}{B_{12}\frac{N_1}{N_2} - B_{21}} \tag{1.11}$$

Substituting equation (1.3) gives the energy density as

$$e(\nu) = \frac{A_\mathrm{e}}{B_{12}\mathrm{e}^{\frac{h_\mathrm{p}\nu}{k_\mathrm{B}T}} - B_{21}} \tag{1.12}$$

This can be compared with the energy density expression resulting from Planck's law on blackbody radiation, which is given by

$$e(\nu) = \frac{8\pi h_\mathrm{p}\nu^3}{c^3}\frac{1}{\mathrm{e}^{\frac{h_\mathrm{p}\nu}{k_\mathrm{B}\mathrm{T}}} - 1} \tag{1.13}$$

Since equations (1.12) and (1.13) express the same energy density $e(\nu)$ in two different forms, we find that they can be equivalent only if

$$B_{12} = B_{21} = B$$

and

$$A_\mathrm{e} = B\frac{8\pi h_\mathrm{p}\nu^3}{c^3} \tag{1.14}$$

The Einstein coefficients B_{12} and B_{21} give the respective probabilities per unit time per unit spectral energy density that a stimulated transition will occur, while A_e is the probability per unit time that a spontaneous transition will occur. The equality of B_{12} and B_{21} indicates that the stimulated absorption and emission have the same probabilities of occurring between the same energy levels. Stimulated and spontaneous emissions, however, are related under equilibrium conditions by

$$\frac{A_e}{B} = (e^{\frac{h_p \nu}{k_B T}} - 1)e(\nu) = \frac{8\pi h_p \nu^3}{c^3} \tag{1.15}$$

Equation (1.15) indicates that for a given temperature, the rate of spontaneous emission is much greater than the rate of stimulated emission at high frequencies, whereas the opposite is true at relatively low frequencies.

Example 1.1

(a) Compare the rates of spontaneous and stimulated emission at room temperature ($T = 300$K) for an atomic transition where the frequency associated with the transition is about 3×10^{10} Hz, which is in the microwave region.

Solution:

From equation (1.15),

$$\frac{h_p \nu}{k_B T} = \frac{6.625 \times 10^{-34}(\text{J s}) \times 3 \times 10^{10}(\text{Hz})}{1.38 \times 10^{-23}(\text{J/K}) \times 300(\text{K})} \approx 5 \times 10^{-3}$$

Thus,

$$\frac{A_e}{B} = (e^{5\times 10^{-3}} - 1)e(\nu) \approx 0$$

This indicates that the stimulated emission rate is much greater than the spontaneous emission rate, and thus amplification is feasible in the microwave range at room temperature.

(b) Repeat Example 1.1a for a transition frequency in the optical region of $\nu = 10^{15}$.

Solution:

$$\frac{h_p \nu}{k_B T} = \frac{6.625 \times 10^{-34}(\text{J s}) \times 10^{15}(\text{Hz})}{1.38 \times 10^{-23}(\text{J/K}) \times 300(\text{K})} \approx 160$$

$$\Rightarrow \frac{A_e}{B} = (e^{160} - 1)e(\nu) \approx \infty$$

indicating that spontaneous emission is then predominant, resulting in incoherent emission from normal light sources. In other words, under conditions of thermal equilibrium, stimulated emission in the optical range is very unlikely.

(c) What will be the wavelength of the line spectrum resulting from the transition of an electron from an energy level of 40×10^{-20} J to a level of 15×10^{-20} J?

Solution:

From equation (1.2), we have

$$\Delta E = \frac{h_{\mathrm{p}}c}{\lambda} = h_{\mathrm{p}}\nu$$

$$\Rightarrow (40 - 15) \times 10^{-20} = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{\lambda}$$

Therefore,

$$\lambda = 0.792 \times 10^{-6}\ \mathrm{m} = 0.792\ \ \mu\mathrm{m}$$

1.3 LIFETIME

The time constant or *lifetime*, τ_{sp}, of atoms in an excited state is a measure of how long the atoms stay in that state, or the time period over which spontaneous transition occurs. Strictly speaking, this is how long it takes for the number of atoms in the excited state to reduce to $1/e$ of the initial value. To determine τ_{sp}, we consider the probability p that an atom will leave the excited state in the elemental time interval Δt as

$$p = p_t \Delta t \tag{1.16}$$

where p_t is the probability per unit time that a spontaneous transition will occur. Then if there are N atoms in that state at time t, the change ΔN in the number of atoms in the interval Δt is

$$\Delta N = -Np_t \Delta t$$

The negative sign is due to the fact that there is a reduction in the number of atoms. In the limit, we have

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -Np_t \tag{1.17}$$

which when integrated gives

$$N = N_0 \mathrm{e}^{-p_t t} \tag{1.18}$$

where N_0 is the number of atoms in the excited state at time $t = 0$. Thus for spontaneous emission, the lifetime or time constant is

$$\tau_{\text{sp}} = \frac{1}{p_t} \tag{1.19}$$

And since A_{e} is the probability per unit time that a spontaneous transition will occur,

$$\tau_{\text{sp}} = \frac{1}{p_t} = \frac{1}{A_{\text{e}}} \tag{1.20a}$$

or

$$A_{\text{e}} = \frac{1}{\tau_{\text{sp}}} \tag{1.20b}$$

We now discuss the absorption of a light beam as it propagates through an absorbing medium.

1.4 OPTICAL ABSORPTION

As a laser beam propagates through an absorbing medium, especially a gaseous medium, absorption by the medium results in the beam intensity diminishing as it propagates. In this section, we analyze the variation of the beam intensity with distance as it propagates. Consider a control volume $\Delta x \Delta y \Delta z$ of material through which a beam of specific frequency, ν, propagates in the x-direction (Fig. 1.7).

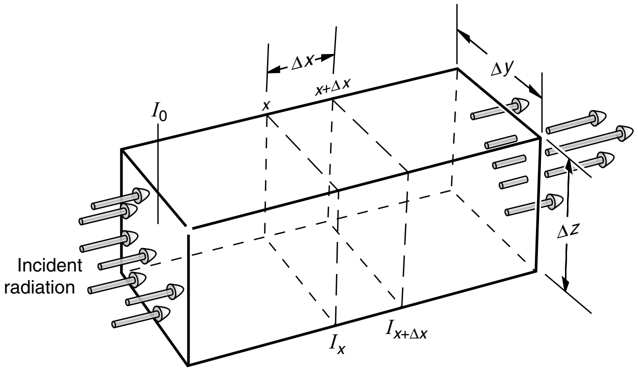

FIGURE 1.7 Propagation of a monochromatic beam in the x-direction.

From equation (1.4), the number of stimulated absorptions per unit time, R_{abs}, in the control volume due to the incident beam is given by

$$R_{\mathrm{abs}} = n_{\mathrm{abs}} \Delta x \Delta y \Delta z \tag{1.21}$$

And since the energy of each photon is $h_{\mathrm{p}}\nu$, and each photon results in one transition, the energy absorption rate in the control volume $\Delta x \Delta y \Delta z$ is

$$q_{\mathrm{abs}} = h_{\mathrm{p}}\nu R_{\mathrm{abs}} = n_{\mathrm{abs}} h_{\mathrm{p}}\nu \Delta x \Delta y \Delta z \tag{1.22}$$

Likewise, the energy rate of stimulated emission is given by

$$q_{\mathrm{st}} = n_{\mathrm{st}} h_{\mathrm{p}}\nu \Delta x \Delta y \Delta z \tag{1.23}$$

Since spontaneous emission is transmitted in all directions, its contribution in any specific direction is negligible. We thus consider only the contributions of stimulated emission and absorption. The net rate of energy absorption in the control volume at the frequency ν is then given by

$$q_a = [n_{\mathrm{abs}} - n_{\mathrm{st}}] h_{\mathrm{p}}\nu \Delta x \Delta y \Delta z \tag{1.24}$$

Let the irradiance or energy intensity of the beam (power per unit area) at a general location x be $I(x)$. Then the flux of energy (energy per unit time) into the element will be

$$q(x) = I(x) \Delta y \Delta z \tag{1.25}$$

and the flux out of the control volume will be

$$q(x + \Delta x) = I(x + \Delta x) \Delta y \Delta z = I(x) \Delta y \Delta z + \frac{\partial I(x)}{\partial x} \Delta x \Delta y \Delta z \tag{1.26}$$

Thus, the net flux of energy out of the control volume, q_{e}, is

$$q_{\mathrm{e}} = q(x + \Delta x) - q(x) = \frac{\partial I(x)}{\partial x} \Delta x \Delta y \Delta z \tag{1.27}$$

Now under steady-state conditions, the net energy absorbed into the control volume per unit time and that out of it per unit time must be equal in magnitude, but of opposite sign. Thus from equations (1.24) and (1.27), we have

$$\frac{\partial I(x)}{\partial x} \Delta x \Delta y \Delta z = -[n_{\mathrm{abs}} - n_{\mathrm{st}}] h_{\mathrm{p}}\nu \Delta x \Delta y \Delta z \tag{1.28}$$

Substituting for n_{abs} and n_{st} from equations (1.4), (1.7), (1.14), and (1.20a), we have

$$\frac{\partial I(x)}{\partial x} = -\frac{c^3}{8\pi\nu^2\tau_{\mathrm{sp}}} e(\nu)(N_1 - N_2) \tag{1.29}$$

But the intensity (energy per unit area per unit time) of a light beam propagating in a medium can also be expressed as the product of the propagating speed and energy density (energy per unit volume):

$$I = c_{\mathrm{m}} e(\nu) \tag{1.30}$$

where $c_{\mathrm{m}} = c/n$ is the velocity of the light beam in the medium and n is the refractive index of the medium. Equation (1.29) then becomes, for unidirectional propagation,

$$\frac{\mathrm{d}I(x)}{\mathrm{d}x} = -\alpha I(x) \tag{1.31}$$

where

$$\alpha = \frac{c^2 n}{8\pi\nu^2\tau_{\mathrm{sp}}}(N_1 - N_2) \tag{1.32}$$

Integration of equation (1.31) results in the following expression for the beam variation in the material:

$$I(x) = I_0 e^{-\alpha x} \tag{1.33}$$

where I_0 is the intensity of the incident beam (W/m^2), and α is the absorption coefficient (m^{-1}).

Equation (1.33) is known as the Beer–Lambert law and indicates that the beam intensity varies exponentially as it propagates into the medium.

Example 1.2 A medium absorbs 1% of the light incident on it over a distance of 1.5 mm into the medium. Determine

(i) The medium's absorption coefficient.
(ii) The length of the medium if it transmits 75% of the light.

Solution:

(i) From equation (1.33), we have

$$I(x) = I_0 e^{-\alpha x}$$

If 1% of the incident light is absorbed over a distance of 1.5 mm, then

$$0.99 \times I_0 = I_0 e^{-\alpha \times 1.5}$$

$$\Rightarrow \log_e 0.99 = -1.5 \times \alpha$$

Therefore,

$$\alpha = 6.7 \times 10^{-3}/\text{mm}$$

(ii) If 75% of the incident light is transmitted, then

$$0.75 \times I_0 = I_0 \mathrm{e}^{-6.7 \times 10^{-3} \times x}$$

$$\Rightarrow \log_e 0.75 = -6.7 \times 10^{-3} \times x$$

Therefore,

$$x = 42.9\,\text{mm}$$

1.5 POPULATION INVERSION

From equation (1.32), it is evident that α is positive if $N_1 > N_2$. The beam intensity then decreases exponentially with distance into the material. Since $N_1 > N_2$ under normal thermal equilibrium conditions, the beam will be attenuated as it propagates through the medium. α is then referred to as the *absorption coefficient* (with units: per unit length) and is positive.

However, if conditions are such that the number of atoms at the higher energy level is greater than those at the lower energy level, that is, $N_2 > N_1$, then α will be negative, in which case the beam intensity will increase exponentially as it propagates through the medium. In other words, the original radiation will be amplified. Equation (1.33) can then be written as

$$I(x) = I_0 \mathrm{e}^{\beta x} \tag{1.34}$$

$$\beta = \frac{c^2 n}{8\pi \nu^2 \tau_{\text{sp}}}(N_2 - N_1) \tag{1.35}$$

β is referred to as the *small-signal gain coefficient* and is positive. Such amplification of the original radiation results from stimulated emission that occurs when the right conditions exist.

The condition where $N_2 > N_1$ with more atoms existing at the higher energy level than at the lower energy level is referred to as *population inversion* (Fig. 1.8). From

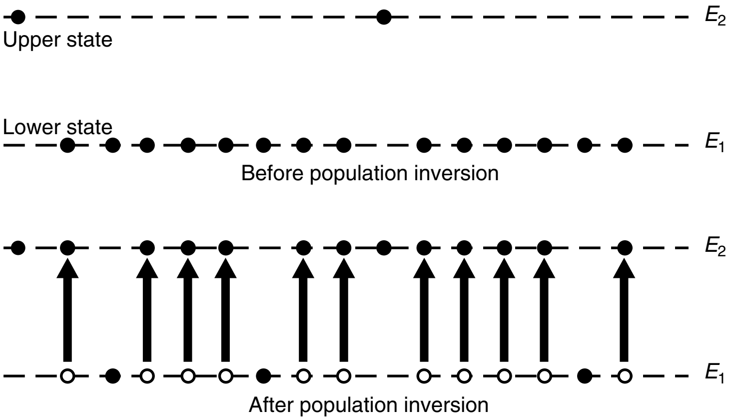

FIGURE 1.8 Population inversion. (From Chryssolouris, G., 1991, *Laser Machining: Theory and Practice*. By permission of Springer Science and Business Media.)

Boltzmann's equation, it is evident that population inversion does not occur under normal thermal equilibrium conditions. For it to be possible, atoms within the laser medium have to be excited or *pumped* to a nonequilibrium state (see Chapter 3). This is done with the fusion of a substantial quantity of energy into the medium using an external source. The process decreases the number of atoms at the lower energy level while increasing the number at the higher level. A material in which population inversion is induced is called an *active medium*.

Unfortunately, even though population inversion is a necessary condition for achieving laser action, it is not a sufficient condition, since a significant number of the excited atoms decay spontaneously to the lower energy level. Thus, there may be no laser action even when a population inversion is achieved. Compensation for the loss due to such decay is accomplished by introducing positive feedback into the system to amplify the laser beam. This is done using *optical resonators*, discussed in greater detail in chapter 2. However, we outline the basic concepts of optical resonators in the following section to gain insight into the other condition necessary for laser action, the threshold gain.

1.6 THRESHOLD GAIN

In an actual laser, the active medium is normally placed between two mirrors and these together constitute the resonator (Fig. 1.9a). Initially, spontaneous emission results in photons being generated in all directions (Fig. 1.9b). However, as stimulated emission becomes significant, an electromagnetic wave travelling along the axis of the resonator oscillates between the two mirrors (Fig. 1.9c). When population inversion exists, the radiation is amplified on each passage through the medium resulting in the buildup of signal intensity (Fig. 1.9d). A useful output beam is obtained by making

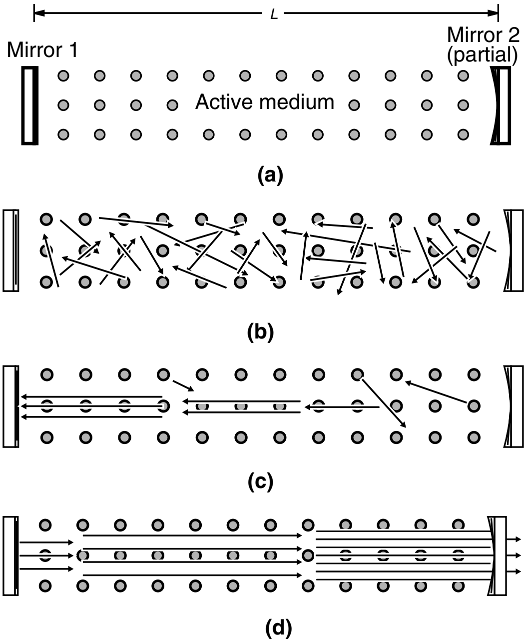

FIGURE 1.9 Illustration of laser amplification. (From Chryssolouris, G., 1991, *Laser Machining: Theory and Practice*. By permission of Springer Science and Business Media.)

one of the mirrors partially transparent. This output beam is the laser that comes out of the system. Such output coupling, along with absorption and scattering by the active medium, absorption by the mirrors, and so on, results in losses in the radiation intensity. These losses may be simply described by equation (1.33). If the losses encountered by the radiation during each passage are greater than the amplification or *gain* of the laser, equation (1.34), then the oscillations cannot continue, and the radiation intensity will eventually die down. Thus for oscillations to be maintained, the gain of the system must at least be equal to the losses in the system. This is accomplished when a *threshold gain*, β_{th}, is reached.

To determine the conditions under which the threshold for achieving laser oscillation is reached, we first note that the amplification factor for the beam during each passage through the medium is determined by the small-signal gain coefficient, β. Likewise, let the factor for losses due to absorption in the material, scattering, and so on in the medium be given by the absorption coefficient α. Equations (1.32) and (1.35) may indicate that α and β are equal and of opposite sign. These are simplifications. In reality, they have different values since α also includes losses due to scattering and other phenomena that were not considered in the development of equation (1.32). Likewise, β.

Now, let the reflectivities or reflection coefficients of the two mirrors be R_1 and R_2. If the length of the active medium is L, and the initial beam intensity as it leaves mirror

2 is I_0, then, on passage through the active medium, its intensity will be amplified by $\mathrm{e}^{\beta L}$ as a result of stimulated emission (see equation (1.34)). However, the beam will simultaneously be attenuated by a factor of $\mathrm{e}^{-\alpha L}$ because of medium losses (see equation (1.33)). Thus, the beam intensity as it leaves the medium will be

$$I = I_0\mathrm{e}^{(\beta-\alpha)L} \tag{1.36}$$

After reflection from mirror 1, the intensity is further reduced, as a result of reflection losses, to

$$I = I_0 R_1 e^{(\beta-\alpha)L} \tag{1.37}$$

Another transmission through the medium followed by a reflection from mirror 2 results in a beam intensity after one complete passage through the resonator of

$$I = I_0 G_\mathrm{g} = I_0 R_1 R_2 \mathrm{e}^{2(\beta-\alpha)L} \tag{1.38}$$

where G_g is the round-trip power gain. Oscillation (continued bouncing to and fro within the resonator) can be maintained only if the beam intensity does not diminish after each passage, that is, the amplification or gain must be sufficient to compensate for the energy lost. That means

$$I \geq I_0 \tag{1.39}$$

or

$$G_\mathrm{g} = R_1 R_2 \mathrm{e}^{2(\beta-\alpha)L} \geq 1 \tag{1.40}$$

This is the other condition for achieving oscillation in a resonator, and the threshold for oscillation is given by the lower bound. Under steady-state conditions, $I = I_0$. The small signal threshold gain is then given by

$$\beta_\mathrm{th} = \alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right) \tag{1.41}$$

The second term on the right-hand side of equation (1.41) reflects the losses due to useful output coupling. Substituting for α from equation (1.32) and considering only the threshold condition, we have

$$N_2 - N_1 = -\frac{8\pi\nu^2\tau_\mathrm{sp}}{c^2 n}\left(\beta_\mathrm{th} - \frac{1}{2L}\ln\frac{1}{R_1 R_2}\right) \tag{1.42}$$

That means it is not just sufficient to achieve population inversion for laser action to be initiated, but the population inversion must also attain a certain critical or threshold value given by equation (1.42).

Our discussion thus far has focused on the excitation of an atom from one energy level to another by the absorption of a photon of energy. There are a number of other situations where more than one photon is involved in the excitation, and this is discussed in the next section, focusing on the simpler case of two-photon absorption.

1.7 TWO-PHOTON ABSORPTION

Two-photon absorption is essentially a form of multiphoton absorption, and as such, a nonlinear optical phenomenon. To explain the concepts of nonlinear optics in simple terms, we consider the analogy between electromagnetic phenomena and mechanical oscillation. Excitation of a spring–mass system by an external force will cause the mass to oscillate. For relatively small displacements, the relationship between the force and displacement is linear. However, for large enough displacements, the relationship becomes nonlinear.

In electromagnetic wave theory, the equivalent to the driving force is the electric field that is applied, for example, a laser beam; the mass is equivalent to the electrons; and the displacement is equivalent to the polarization. The relationship between the electric field, E_l, and polarization, P_l, is linear when the strength of the electric field is relatively low, as obtained for ordinary light sources, and is given by

$$P_\mathrm{l}(t) = \chi E_\mathrm{l}(t) \tag{1.43}$$

where χ is the susceptibility of the dielectric medium and is independent of $E_\mathrm{l}(t)$, but is a function of the frequency.

The actual relationship between the electric field and polarization, however, is a power series given by

$$P_\mathrm{l}(t) = \chi^{(1)} E_\mathrm{l}(t) + \chi^{(2)} E_\mathrm{l}(t)^2 + \chi^{(3)} E_\mathrm{l}(t)^3 + \cdots \tag{1.44}$$

Here, $\chi^{(m)}$ is a tensor, and $\chi^{(1)} = \chi$. However, $\chi^{(2)}$, $\chi^{(3)}$, and so on are nonlinear susceptibilities and define the degree of nonlinearity. The elements of $\chi^{(m)}$ get smaller and smaller, the higher m gets. Thus for the effects of the higher order terms to be detected, the intensity of the light source has to be very high. This is normally achieved with pulsed lasers.

The nonlinear characteristic of electromagnetic radiation gives rise to several different phenomena. One of these is frequency doubling, which is essentially a $\chi^{(2)}$ effect and enables the frequency of an electromagnetic radiation to be doubled by passing it through a special crystal. For example, the frequency of an Nd:YAG laser

FIGURE 1.10 The two-photon absorption concept.

(infrared wavelength = 1.064 μm) can be doubled to produce a visible green beam of wavelength 0.532 μm after passing through a nonlinear crystal, say β-barium borate. Energy is conserved in the process. In general, higher order frequencies can also be generated.

With this general background, we now turn our attention to the specific phenomenon of two-photon absorption. Normal excitation of an atom or molecule from a lower energy level to a higher level involves absorption of a photon of a specific energy or wavelength defined by the energy levels (Section 1.2.3). Under normal circumstances, the excitation to a specific higher level cannot take place in steps. However, when the atom is exposed to radiation of a high enough intensity, the atom can simultaneously absorb two longer wavelength photons, resulting in the same effect as a single photon of half the wavelength (Fig. 1.10). The process is referred to as two-photon absorption. The combined energy of the two photons enables the atom to be excited to the higher energy level. In essence, each photon provides half the energy of the electronic transition. This occurs because each of the longer wavelength photons can excite the atom to a transient or virtual state that has a lifetime of a few femtoseconds. The virtual state is forbidden for a single-photon transition. Two-photon absorption can take place only if the second photon is absorbed before the virtual state decays. The concerted interaction of the two photons then results in a combined energy that enables the atom to be excited to a level equivalent to what would be induced by a single photon of half the wavelength or twice the energy (see equation (1.2)).

In essence, the absorption cannot occur sequentially. If it occurs one after the other, then the transition is equivalent to two single-photon transitions and not a two-photon transition. Since the two photons have to be absorbed simultaneously, it is essential for the laser source to be ultrashort pulsed and of high peak power or intensity. Femtosecond lasers have thus been found very effective in a number of applications. Rapid, repeated pulsing of the laser provides adequate instantaneous intensity for the process, while maintaining a low average power.

One of the distinguishing features of single-photon and two-photon absorption is the rate at which energy is absorbed by each process. The rate or probability for a two-photon absorption is proportional to the square of the beam intensity (or the fourth power of the electric field amplitude), while the rate for a single-photon absorption is directly proportional to the beam intensity. In general, the rate of n-photon absorption is proportional to the nth power of the photon flux density. To achieve the high intensity necessary for two-photon absorption to be initiated, it is necessary for the beam to be tightly focused. With a tightly focused beam, two-photon absorption is confined to the highly localized focal volume.

Two-photon absorption has found application in such areas as fluorescence microscopy, 3D optical data storage, lithography, photodynamic therapy, and microfabrication.

1.8 SUMMARY

A laser is a form of electromagnetic radiation with wavelengths ranging from X-ray to infrared radiation. It is the result of energy emission associated with the transition of an electron from a higher to a lower energy level or orbit within an atom. This starts with excitation of an atom to a higher energy state as it absorbs a photon. When the excited atom is stimulated by a photon, it also releases another photon as it undergoes a transition to the lower state. This results in stimulated emission, where the incident and emitted photons have the same characteristics and are in phase, resulting in a high degree of coherence. Otherwise the photon is released spontaneously. Under conditions of thermal equilibrium, the rates of upward and downward transitions are the same. Again, under thermal equilibrium conditions, the distribution of atoms at the various energy levels is given by Boltzmann's law, where the number of atoms at the higher energy levels is lower than that at lower energy levels. For a given temperature, the rate of spontaneous emission is much greater than the rate of stimulated emission at high frequencies, while the opposite is true at relatively low frequencies.

As a laser beam propagates through an absorbing medium, especially a fluid medium, absorption by the medium results in the beam intensity diminishing exponentially, according to the Beer–Lambert law, as it propagates. For the beam intensity to increase as it propagates through the active medium, there has to be population inversion, with the number of atoms at the higher energy levels being higher than that at lower energy levels. For it to be possible, atoms within the laser medium have to be excited or pumped to a nonequilibrium state. In addition, it is necessary that the population inversion should attain a certain critical or threshold value.

Under conditions of very high energy density, multiphoton absorption can take place, where more than one photon is involved in exciting an atom to a higher energy level. In addition to the high energy density required, the incident radiation also has to be ultrashort pulsed, since the multiple photons have to be absorbed almost simultaneously. For two-photon absorption specifically, the atom simultaneously absorbs two longer wavelength photons, resulting in the same effect as a single photon of half the wavelength.

After looking at the conditions under which a laser beam is generated, we now look, in the next chapter, at the basic principle of the resonator, the device in which the radiation is generated and amplified, and how that affects the characteristics of the resulting beam.

REFERENCES

Barrett, C. R., Nix, W. D., and Tetelman, A. S., 1973, *The Principles of Engineering Materials*, Prentice Hall, Englewood Cliffs, NJ.

Berlien, H.-P., and Muller, G. J., 2003, *Applied Laser Medicine*, Springer-Verlag, Berlin.

Chryssolouris, G., 1991, *Laser Machining: Theory and Practice*, Springer-Verlag, Berlin.

Goeppert-Mayer, M., 1931, *Annalenden Physik*, Vol. 9, p. 273.

Henderson, A. R., 1997, *A Guide to Laser Safety*, Chapman and Hall, London.

Laud, B. B., 1985, *Lasers and Non-Linear Optics*, Wiley Eastern Limited, New Delhi.

Luxon, J. T., and Parker, D. E., 1985, *Industrial Lasers and Their Applications*, Prentice Hall, Englewood Cliffs, NJ.

O'Shea, D. C., Callen, W. R., and Rhodes, W. T., 1977, *Introduction to Lasers and Their Applications*, Addison-Wesley, Reading, MA.

Shimoda, K., 1986, *Introduction to Laser Physics*, 2nd edition, Springer-Verlag, Berlin.

Svelto, O., 1989, *Principles of Lasers*, 3rd edition, Plenum Press, New York.

Thyagarajan, K., and Ghatak, A. K., 1981, *Lasers, Theory and Applications*, Plenum Press, New York.

Wilson, J., and Hawkes, J. F. B., 1987, *Lasers: Principles and Applications*, Prentice Hall, New York.

APPENDIX 1A

List of symbols used in the chapter.

Symbol	Parameter	Units
A_e	Einstein coefficient for spontaneous emission	/s
B_{12}	Einstein coefficient for stimulated absorption	Sr m^2/J s
B_{21}	Einstein coefficient for stimulated emission	Sr m^2/J s
$B = B_{12} = B_{21}$	Einstein coefficient	Sr m^2/J s
G_g	round-trip power gain	–
n_{abs}	absorption rate (number of absorptions/unit volume/unit time)	$/m^3$ s
n_{sp}	spontaneous emission rate	$/m^3$ s
n_{st}	stimulated emission rate	$/m^3$ s
N_0	population (number of atoms) of excited state at time $t = 0$	$/m^3$
P_l	electric polarization	C m^2
q_{abs}	energy absorption rate	W
q_{st}	energy rate of stimulated emission	W
R_{abs}	number of stimulated absorptions per unit time	/s
V_e	potential energy of electron	J
x	electron position in X-direction	m
β	small-signal gain coefficient	/m
β_{th}	small-signal threshold gain coefficient	/m
χ	electric susceptibility	–
ψ	wave function of electron	–

PROBLEMS

1.1. Consider the energy levels E_1 and E_2 of a two-level system. Determine the population ratio of the two levels if they are in thermal equilibrium at room temperature, 27°C, and the transition frequency associated with this system is at 10^{15} Hz.

1.2. For the system in Problem 1.1, determine the fraction of atoms that will be in the lower state at room temperature, 27° C, if the transition wavelength associated with the two energy levels is (a) $\lambda = 1060$ nm and (b) $\lambda = 488$ nm.

1.3. If the population ratio, N_2/N_1, for the energy levels in Problem 1.1 under thermal equilibrium conditions at room temperature is $1/e^2$, determine the transition frequency and the type of electromagnetic radiation associated with this transition.

1.4. The oscillating wavelengths of the He–Ne, Nd:YAG, and CO_2 lasers are 0.6328, 1.06, and 10.6 μm, respectively. Determine the corresponding oscillating frequencies. What energy is associated with each transition?

1.5. Determine the stimulated emission probability, B, for a transition of wavelength $\lambda = 250$ nm, if its spontaneous emission rate, A, is 5×10^5/s. For the stimulated emission probability to be 400% that of spontaneous emission, what must be the irradiance at the given wavelength within the cavity at room temperature, 27°C?

1.6. A medium absorbs 0.5% of the light that passes through it for each millimeter of medium length. Determine its absorption coefficient and the percentage of light that will be transmitted if the overall length of the medium is 120 mm.

1.7. (a) Neglecting any losses, determine β for an active medium of length 2 m if the irradiance of light passing through it increases by 100%.

(b) Determine β if a 20% increase in irradiance occurred for the same length of active medium.

1.8. Determine the radiation density emitted by a blackbody at a frequency of 10^{15} Hz, if the temperature is 1200 K, using Planck's theory.

1.9. What should be the net gain coefficient, $\beta - \alpha$, for an active medium of length 0.1 m to result in a round-trip gain of 10%, assuming 100% reflection coefficients for both mirrors?

1.10. If the mirror reflection coefficients for a laser resonator of length 5 m are 98.5% and 60%, and there are no losses, determine the cavity threshold gain.

1.11. Would you expect the absorption coefficient α of a material to increase or decrease with temperature? Explain.