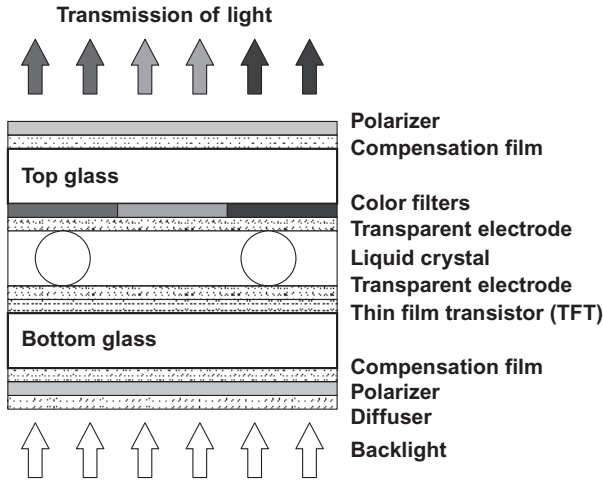


# 1

## Preliminaries

Electronic display of information is becoming increasingly important in many aspects of our lives. Electronic mails and e-commerce are two examples. In an information-dominated age, the display of information is an essential element for the interface (or bridge) between machine and human. The interface requires visual display systems utilizing cathode ray tubes (CRTs), electroluminescence (EL) devices, plasma display devices, field emission devices, light-emitting diodes (LEDs), flat CRTs, vacuum fluorescence, liquid crystal displays (LCDs), or flexible displays using organic LEDs. Among these systems, LCD is one of the most important optical display systems for high-performance flat panel display of information. The importance arises from many of its advantages, including flat panel, light weight, large area, high definition, high viewing quality, low driving voltages, and low power consumption. Generally speaking, a display panel consists of a two-dimensional array of pixels (picture elements). Each of the pixels can be turned on or off independently for the display of two-dimensional images. For direct displays (e.g., LED panels), each of the pixels is an emitter of light. For indirect displays (e.g., LCD panels), each of the pixels is a light valve that controls the transmission, reflection, or scattering of light. In this chapter, we will briefly discuss the function of various key elements in an LCD panel, including a brief introduction to the physical properties of liquid crystal materials. Figure 1.1 shows a schematic drawing of a basic element (pixel) of an LCD panel.

Referring to the figure, we note that the basic element of an LCD system includes a thin layer of liquid crystal sandwiched between a pair of sheet polarizers. To control the optical transmission of the display element electronically, the liquid crystal layer is placed between transparent electrodes [e.g., indium tin oxide (ITO)]. In most high-quality displays, thin optical films of birefringent materials are employed to improve the contrast ratios and colors at large viewing angles. The sheet polarizers, thin optical films, and the electrodes are cemented on the surfaces of the glass plates. The thickness of the glass plates can be from 0.5 mm up to a few millimeters to maintain a uniform liquid crystal layer and the structural integrity of the panel. The thickness of the liquid crystal layer (also known as the cell gap) is kept uniform by



**Figure 1.1.** Schematic drawing of a basic element of a liquid crystal display.

using spacers made of glass fibers or transparent plastic microspheres. In some LCDs, an array of micron-high posts is employed to maintain a uniform cell gap. Typical cell gaps are in the range of  $5\mu\text{m}$ . By applying a voltage across the transparent electrodes, an electric field inside the liquid crystal can be obtained to control the orientation of the liquid crystal molecules and thus to change the optical property (e.g., phase retardation) of the liquid crystal layer. This leads to a change of the transmission of light when the liquid crystal cell is sandwiched between a pair of polarizers. To achieve the display of information, we need a two-dimensional array of these electrodes. These electrodes can be driven electrically for data input by using two sets ( $x, y$ ) of parallel arrays of electrodes. In active matrix (AM) displays, the two-dimensional arrays of electrodes are controlled electronically via the use of an array of thin film transistors (TFTs). To improve the uniformity of viewing, diffusers are employed to ensure that the backlight is spread uniformly over the entire panel and the viewing angles. In what follows, we will briefly describe each of the optical components. More detailed discussions of the optical properties of the components can be found in later chapters.

## 1.1. BASIC COMPONENTS OF LCDs

### 1.1.1. Polarizers

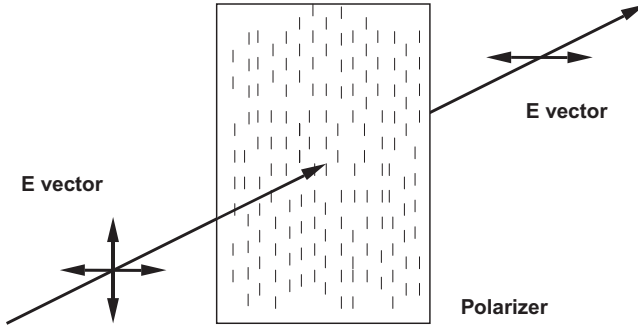
A polarizer is an optical device that produces a beam of polarized light from a beam of unpolarized light. The subject of polarization states of light will be discussed in Chapter 2. Polarizers are key components in most high-quality LCDs. Virtually all light sources used in LCDs are unpolarized. A beam of polarized light is described by its direction of propagation, frequency, and

vector amplitude (electric field vector  $\mathbf{E}$ ). The vector amplitude is related to the intensity of the beam and is perpendicular to the direction of propagation. Given a direction of propagation, there are two independent (orthogonal) components of the vector amplitude  $\mathbf{E}$ . For a beam of light propagating along the  $z$ -axis, these two independent components can be  $E_x$  and  $E_y$ .

A beam of light is regarded as unpolarized provided two conditions are met. First, the time-averaged intensity of the transmitted beam through a polarizer is independent of the orientation of the absorption axis of the polarizer. Second, any two orthogonal components of the vector amplitude are completely uncorrelated. Thus, if we decompose the vector amplitude of a beam of unpolarized light into two mutually orthogonal transverse components (e.g.,  $E_x$ ,  $E_y$  for light propagating along the  $z$ -axis), these two components will have a random relative phase relationship between them. The random relative phase leads to a resultant vector amplitude that varies rapidly in time in a random manner.

Polarizers are made of anisotropic media that transmit one component and redirect or absorb the other component. It is known that prisms made of anisotropic crystals provide methods of obtaining plane polarized light by double refraction where one of the components can be removed by reflection or mechanical obstruction. Although prism polarizers (e.g., Rochon prisms, Wollaston prisms) can provide excellent extinction ratios, they are not suitable for flat panel applications. The extinction ratio of a polarizer is a measure of the quality of the polarizer and is defined as the ratio of the maximum transmission intensity to the minimum transmission intensity.

In most LCDs, the polarizers are made of a thin sheet of materials that transmit one polarization component and absorb the other component. This effect is known as optical dichroism. Natural dichroism of a material is, like natural birefringence, due to its anisotropic molecular structure. The ratio of the two absorption coefficients for two linear orthogonal polarizations is called the dichroic ratio. One of the best known materials of this kind is tourmaline, a naturally occurring mineral that exhibits uniaxial birefringence and a dichroic ratio of about 10. In this crystal, the ordinary polarization component (electric field vector perpendicular to the  $c$ -axis of crystal) is much more strongly absorbed (about 10 times) than the extraordinary polarization component (electric field vector parallel to the  $c$ -axis of crystal). A single crystal plate of such material cut parallel to the  $c$ -axis can be employed as a polarizer. For LCDs, the polarizers must be as big as the panel. Large plates of single crystal are not easily available. Most sheet polarizers are, however, synthesized by stretching films that contain ultrafine rodlike or needlelike dichroic materials (e.g., iodine crystals). The stretching provides a uniaxial alignment of the microcrystals or molecules (see Fig. 1.2). Dichroic ratios of greater than 100 can be easily obtained. A beam of polarized light with the vector amplitude parallel to the molecular axis (axis of iodine crystals) may suffer more material absorption. Thus, given enough thickness, the stretched films transmit one polarization component, which is perpendicular to the direction of alignment,



**Figure 1.2.** Schematic drawing of a sheet polarizer that transmits the polarization component with **E** vector perpendicular to the direction of alignment and absorbs the polarization component with **E** vector parallel to the direction of alignment (absorption axis).

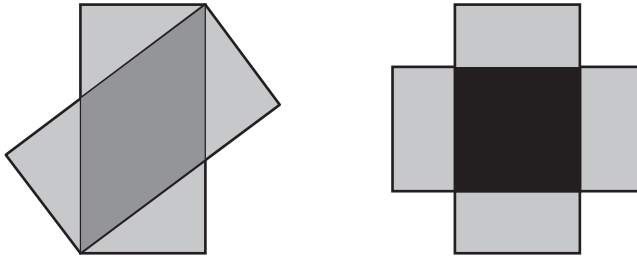
and absorb the other polarization component, which is parallel to the direction of alignment. These sheet polarizers provide excellent extinction ratios while simultaneously accommodating a large angular aperture and an enormously large linear aperture. These are desirable features for large flat panel applications.

Another method of producing dichroic materials is to embed oriented ultrafine metallic needles that are not dichroic by themselves in a medium. The composite material then is said to have form dichroism, just as a composite medium containing oriented dielectric needles exhibits form birefringence [1]. An extreme example is a parallel array of ultrafine metallic wires. Such an array is known as a wire grid polarizer. The wire grid will reflect incoming light with polarization parallel to the wires and transmit incoming light with polarization perpendicular to the wires, provided the period of the wires is much less than the wavelength of light. To function as a polarizer, the wires must be aligned in the same direction. If the wires are connected in the direction of the long axes, then the wire grid polarizer can also function as a conductor in the direction of the long axes.

Generally speaking, 50% of the incident energy of a beam of unpolarized light is absorbed by an ideal polarizer. This energy loss of about 3 dB accounts for the absorption of one of the polarization components resulting in a beam of polarized light. When two identical sheet polarizers are in series, the intensity transmission is approximately proportional to  $\cos^2 \theta$ , where  $\theta$  is the angle between the directions of alignment (absorption axes) of the two polarizers. Thus, a pair of cross-polarizers ( $\theta = 90^\circ$ ) can block the transmission of light regardless of the polarization state of the incident beam (see Fig. 1.3). Further coverage of polarizers can be found later in the book.

### 1.1.2. Transparent Electrodes

To allow the transmission of light, the electrodes of the LCD element (pixel) must be transparent in the visible spectral regime. This requires transparent



**Figure 1.3.** Transmission of unpolarized light through two polarizers in series.

materials with a good electrical conductivity. For example, ITO is a transparent material that also exhibits a good electrical conductivity. Generally speaking, all transparent semiconductors exhibit a finite electrical conductivity. The transparency requires semiconductors with an energy bandgap greater than 3.5 eV so that all optical radiation in the visible spectrum is transmitted. The electrical conductivity in semiconductors can also be increased by an appropriate doping of impurity atoms. The doping leads to an increase of charge carriers (electrons or holes) in the semiconductors. In recent LCD applications, ITO is widely used as the transparent conductive layer because of its higher electrical conductivity. ITO used in LCDs is a solid solution of about 90% indium oxide ( $\text{In}_2\text{O}_3$ ) and 10% tin oxide ( $\text{SnO}_2$ ). The introduction of tin atoms in indium oxide creates additional electrons in the conduction band (note: A tin atom has one more valence electron than an indium atom.). All transparent conductors exhibit a small absorption of light due to the presence of electrons in the conduction band. Thus, the electrodes must be thin enough to allow adequate optical transmission, yet thick enough to provide adequate electrical conductivity. Typical ITO material has an index of refraction of approximately 1.96. As a result of the index mismatch between the glass substrate and the conducting electrode materials, a Fabry–Perot optical cavity is formed by each of the electrodes sandwiched between glass and the liquid crystal. A proper choice of the thickness of the ITO layer (e.g.,  $\lambda/2n$ , where  $\lambda$  is the wavelength and  $n$  is the index of refraction) can ensure a constructive interference to maximize the transmission of light. In typical electrodes, the ITO thickness is in the range of 100–300 nm.

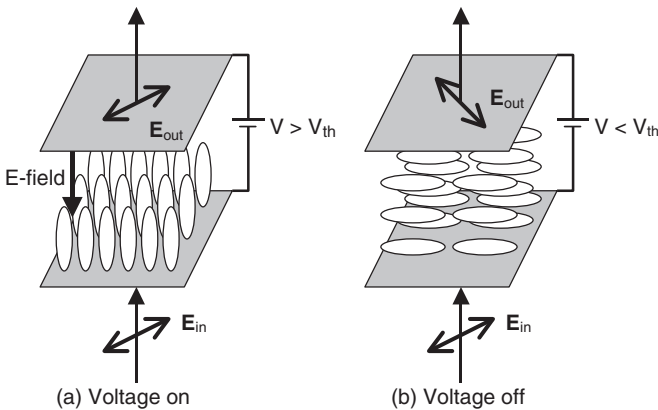
Although ITO is currently the most widely used transparent conducting materials in LCDs, it can be inadequate for many device applications due to its brittle nature and material processing requirements (e.g., high temperature). Other transparent oxide materials are being investigated for electrode applications. These include aluminum-doped zinc oxide (ZnO) and gallium-doped zinc oxide. In addition, multilayer combinations of ITO and metals (e.g., ITO/Ag/ITO) are being studied to improve the flexibility of the conducting layers. Another approach involves the use of parallel arrays of submicron metallic wires (wire grid) for electrode applications. As discussed earlier, the parallel array of metallic wires will function as a reflective polarizer. Since the

light arriving at the wires is already polarized (by the sheet polarizer), the metallic wires do not cause additional energy loss. To avoid energy loss due to diffraction, the wires and the spacing must be much less than the optical wavelength (e.g., wire spacing  $\sim \lambda/10$ ).

### 1.1.3. Liquid Crystal Cell

The space between the conducting electrodes is filled with liquid crystal material. The thickness of the liquid crystal layer is kept uniform by using glass fibers or plastic balls as spacers. Typical thicknesses of the cell are in the range of few micrometers. There are several different liquid crystal configurations that can be employed for display applications. For the purpose of illustrating the principle of operation of a basic element of LCD, we consider the case of a parallel-aligned (PA) nematic liquid crystal cell where all the rodlike liquid crystal molecules are aligned parallel to the surface of the glass plates.

In a liquid crystal cell without any external field, the ordering of the molecules is usually determined by the anisotropy of the boundary. The surfaces of the electrodes are usually coated with a thin alignment layer (e.g., polyimide). Rubbing the surface of the alignment layer has been widely employed to align the liquid crystal molecules in LCD panel fabrication. The liquid crystal molecules in Figure 1.4*b* are aligned on the rubbed surface with their long axes parallel to the rubbing direction, especially for those molecules immediately next to the boundary. Depending on the boundary conditions



**Figure 1.4.** Liquid crystal cell can be controlled electrically to modify the polarization state and the intensity of the transmitted beam of light. (a) The liquid crystal molecules are aligned perpendicular to the glass plates when the voltage is on. The polarization state of the transmitted light remains unchanged. (b) The liquid crystal molecules are aligned parallel to the glass plates when the voltage is off. The polarization state of the transmitted light undergoes a rotation of  $90^\circ$ . This is an example of a liquid crystal with a positive dielectric anisotropy so that  $\Delta\epsilon = \epsilon_{||} - \epsilon_{\perp} > 0$ . The double arrows indicate the polarization states of the linearly polarized light.

and the molecular interaction between the liquid crystal molecules, a long-range ordering of the molecular orientation may exist throughout the liquid crystal cell. As a result of the ordering of the rodlike molecules, nematic liquid crystals exhibit a strong dielectric anisotropy ( $\epsilon_{\parallel} \neq \epsilon_{\perp}$ ). By virtue of its dielectric anisotropy, the ordering and the orientation of the molecules can be distorted (or controlled) electrically by applying an external field. In the presence of an applied electric field, rodlike molecules (with  $\epsilon_{\parallel} > \epsilon_{\perp}$ ) are aligned with their long axes parallel to the applied electric field to minimize the electrostatic energy.

As a result of the ordering of the rodlike molecules, nematic liquid crystals exhibit a strong optical birefringence. In other words, there are two modes of optical propagation in liquid crystals each with a unique phase velocity and a corresponding polarization state. The difference in the phase velocity leads to a phase retardation between these two modes as the optical beam transmits through the cell. As a consequence, the polarization state of an incoming beam of polarized light can be modified. An example is shown in Figure 1.4*b* where an incoming beam of linearly polarized light is converted into another beam of linearly polarized light with a rotation of  $90^{\circ}$  in the polarization state due to a properly chosen cell gap and orientation of the liquid crystal molecules. If the axes of all the molecules are aligned parallel to the propagation direction of the incoming beam by applying an external electric field, the polarization state will remain unchanged. This is illustrated in Figure 1.4*a*. When such a liquid crystal cell is sandwiched between a pair of crossed polarizers (or parallel polarizers), the intensity of the transmitted beam can be controlled electrically.

There are several different types of liquid crystal cells. These include PA nematic cells, vertically aligned (VA) nematic cells, in-plane switching (IPS) nematic cells, and twisted nematic (TN) cells. These cells and their applications in LCDs will be discussed later in the book.

#### 1.1.4. Birefringent Thin Film Compensators

In simple LCDs, the liquid crystal cell is sandwiched between a pair of polarizers. Such basic displays can provide high-quality display of information only for viewing along the normal direction (i.e., perpendicular to the panel). Poor contrast ratios and color instability occur at large viewing angles. These problems are primarily due to the leakage of light at the dark state of the display. The main sources of leakage of light in LCDs include (1) imperfection in polarizers, (2) a pair of ideal crossed polarizers at oblique viewing angles, and (3) the birefringence of the liquid crystal cell, which produces elliptical polarization states at oblique viewing angles. In the discussion on polarizers (Subsection 1.1.1 and Fig. 1.3), we indicated that a pair of crossed polarizers can be employed to stop the transmission of light. Although a pair of ideal crossed polarizers can indeed completely extinguish the transmission of light, we must emphasize that this happens at normal incidence only. A leakage of

light actually occurs at oblique viewing angles. This leakage of light at oblique viewing angles is fundamental. Even though the absorption axes of the crossed polarizers are mutually perpendicular, the polarization states of the transmission modes in the polarizers are not mutually orthogonal at oblique viewing angles. The deviation from  $90^\circ$  of the polarization states at oblique viewing angles leads to a leakage of light. The leakage can be as high as 4% at a viewing angle of  $80^\circ$  for a beam of unpolarized light. Thin films of birefringent materials can be placed after the first polarizers to provide a rotation of the polarization state at oblique viewing angles so that the beam of light is completely extinguished by the second polarizer. These films are known as polarization compensation films.

The elliptical polarization state of light in the LCDs is also a fundamental source of leakage. Most polarizers employed in LCDs are linear polarizers. To be completely extinguished by a linear polarizer, the beam of light must be linearly polarized in a direction perpendicular to the transmission axis of the polarizer. A beam of elliptically polarized light cannot be completely extinguished by a linear polarizer. In most LCDs, the polarization state of light upon transmitting through the liquid crystal cell is elliptically polarized, particularly at large viewing angles. The elliptical polarization state is due to the positive birefringence of the liquid crystal layer. Thin films of negative birefringence can be inserted between the liquid crystal cell and a polarizer. Under the appropriate conditions, the elliptical polarization state can be converted into a linear polarization state that is parallel to the absorption axis of the polarizer. This leads to an elimination of the leakage of light. These films are also known as birefringence compensation films. Detailed discussion of compensation films will be covered later in the book.

### 1.1.5. Color Filters

Most liquid crystal cells in display systems function as light valves. A light valve controls the transmission of light via an application of an electric voltage. An array of light valves can be employed for monochrome displays that display information with bright and dark pixels (picture elements or image elements). To display color information, an array of color filters (e.g., red, green, blue) can be placed after the array of light valves. Color filters are optical devices or materials that deliberately change the spectral distribution of a beam of light incident upon them. For colored LCDs, an array of color filters is coated on the inner surface of the glass plate facing the viewer. Each individual pixel of the display is divided into three cells, or subpixels, which are colored red, green, and blue (RGB), by color filters (e.g., pigment filters, dye filters, and metal oxide filters). These color filters have transmission bands centered at primary colors red, green and blue. Each subpixel can be controlled independently to yield thousands or millions of possible colors for each pixel. A black matrix, consisting of black stripes, is interlaced between the color pixels to avoid leakage of light through the edge of the color pixels. The



black matrix also protects the array of TFTs from exposure to external light [e.g., ultraviolet (UV)] that can produce photoelectrons in TFTs. The black mask material must have a low reflectance to minimize the glare. Generally speaking, the black matrix material can be organic or inorganic (e.g., chromium, chromium oxide, or black resins) [2–5].

The color filters can be based on material absorption or optical interference in thin film structures. Filters based on absorption are the most widely used in LCDs. The main advantages of absorption filters are that they are simple to use and fabricate, insensitive to angle of viewing, and relatively low cost to manufacture. Color filters consist of active ingredients, known as colorants or coloring agents, in a neutral host material. The colorants are either dyes that can dissolve in liquids or pigments that are tiny solid particles dispersed in the host material. For manufacturing via photolithography process, the host materials are usually photosensitive resins that involve dichromate in materials consisting of macromolecules such as gelatin, glue, and casein, or synthetic materials of macromolecules such as polyvinyl alcohol. The array of color filters is usually coated on the glass plate via photolithography, printing, or electroplating. The color filters are usually covered with transparent acryl resin, polyimide resin, or polyurethane resin. The overcoat layer is employed for the reduction of the color filter's thickness variation and to ensure the durability against sputtering and chemical resistance. Color filters involving colorant dye allow fine patterning and high freedom of color tone. The disadvantages of the dyeing method are inferior heat and light resistance and difficulty in reducing cost, since several complicated processes are involved. Using pigments, the color filters can provide high heat and light resistance. The particle size (nanoparticles) must be small enough to minimize scattering and depolarization.

The color filters in LCDs must have certain important characteristics. These include high transmission (light throughput), high color purity, zero or minimum phase retardation, zero or minimum depolarization, low scattering, low reflectance, high stability against heat, light, and chemicals, and sufficient reliability to withstand the LCD panel manufacturing processes. The filter surfaces must be as smooth as possible to ensure a uniform liquid crystal layer thickness. In addition, they must adhere strongly to the pixel surface [2–5].

The main disadvantage of absorption filters is the energy loss due to absorption. For example, a red filter transmits only the red spectral portion of backlight. The blue and green spectral portion of backlight are absorbed by the red filter, leading to an energy loss of about 70% or more. To improve the brightness of displays, research is being carried out to develop lossless filters, including dielectric interference filters and polarization interference filters.

### **1.1.6. Brightness Enhancement Films (BEFs)**

For information displays, the brightness of the displays is one of the most important characteristics. To introduce the brightness, we must first define

some terms (in the following table) that have similar meanings. Table 1.1 shows the relationship between physical power (in units of watt) and visible power (in units of lumen).

Human eyes respond only to the visible spectrum of the electromagnetic radiation. The visual response (known as photopic response) is centered at green light (at around  $\lambda = 555\text{nm}$ ) with a bell-shaped response curve. The visual response tapers off at infrared and UV. Table 1.2 shows the visible power in units of lumen for a beam of light with a physical power of 1W at various wavelengths.

For visual displays, the brightness is defined as visible power per unit area per unit solid angle. In most displays, the backlight (lighting source at the back of the panel) is usually made of cold cathode fluorescence lamp (CCFL) or LEDs. For uniformity in the brightness of the display, the light must be spread uniformly in both the panel area and the viewing angles. In other words, the brightness (in units of lumen per unit area per unit solid angle) must be uniform over the panel area and the viewing angles. This requires the employment of diffusers between the backlight and the liquid crystal cell. A brief discussion of diffusers is found in Subsection 1.1.8.

In some LCD panels (e.g., notebook computers), a thin layer of BEF (also known as a field-of-view restrictor) can be placed on top of the diffuser. The field-of-view restrictor restricts the output radiation pattern to a predetermined range of angles (e.g.,  $\pm 30^\circ$ ). As a result, the display panel appears brighter in the desired angular viewing zone. These films are usually made of an array of microprisms or microlenses. These microprisms and microlenses

**Table 1.1. Physical Brightness and Visual Brightness**

	Physical Measurement	Unit	Visual Perception	Unit
Power	Physical power (power)	Watt	Visible power	Lumen
Intensity	Irradiance (intensity)	Watt/m <sup>2</sup>	Illuminance	Lumen/m <sup>2</sup>
Brightness	Radiance	Watt/m <sup>2</sup> /sr	Luminance	Lumen/m <sup>2</sup> /sr

**Table 1.2. Visible Power (in Units of Lumen) of 1W of Light at Various Wavelengths**

Wavelength (nm)	Physical Power (Watt)	Visible Power (Lumen)
450	1	25.9
500	1	220.0
550	1	679.0
555	1	683.0
600	1	430.0
650	1	73.0

can concentrate the angular distribution of luminance, leading to a higher luminance over a selected portion of the viewing angles. In some cases, micro-optical structure, such as a holographic diffuser or a binary diffractive diffuser, can be employed to perform similar functions. The BEF is particularly important for notebook computers, which are designed for a single user. In this case, the viewing is primarily normal viewing. An ideal diffuser will spread the light over the entire hemisphere with a solid angle of  $2\pi$ . For a notebook user with viewing angles concentrated at  $\pm 30^\circ$  around the normal (with a solid angle of approximately  $0.3\pi$ ), this is an enormous waste of energy. Under the appropriate condition, it is possible to concentrate the illuminance over a viewing cone of  $\pm 30^\circ$  or less for notebook computers by using BEFs. Concentrating the illuminance (visual intensity) over a smaller solid angle leads to an enhancement of the brightness (or luminance). For the case of an array of microprisms, the BEF transmits only those light rays from the backlight that satisfies certain incidence angle criteria with respect to the top surface of the display panel. All other light rays are reflected back into the backlight unit (BLU) toward the bottom, side, or end surfaces where they are reflected by the high-reflectance diffusive reflector. In effect, the reflected rays are “recycled” until they are incident on the BEF at an angle that permits them to pass through the BEF. If the BEFs are designed for concentrating in one dimension only, then two crossed BEFs can be employed in sequence to provide the concentration in two dimensions.

### **1.1.7. AM and TFTs (Active Matrix and Thin Film Transistors)**

For the display of information, the liquid crystal cell for each of the pixels in an LCD must be controlled electrically. For low information content (e.g., calculators, fuel pump displays), each of the image elements can be wired electrically for the application of voltages. In this case, independent voltages can be applied to each of the image elements. For high information content (e.g., notebook computers, TVs), a typical display consists of millions of pixels. Independent wiring for each of the image elements is impractical and virtually impossible. To solve this problem, the image elements are addressed via rows and columns of electrodes. This scheme reduces the electric wiring from millions to thousands.

In the early days of LCDs of  $M \times N$  pixels,  $M$  rows of electrodes and  $N$  columns of electrodes are employed in a multiplexed addressing. In a multiplexed addressing, the voltage applied at one pixel is dependent on the voltage of the neighboring pixels. For example, an electrical pulse is applied to switch the image element (101, 202) via a positive voltage at row 101 and a negative voltage at column 202. The image element (101, 202) gets the largest applied voltage and is switched electrically. However, all the neighboring image elements on row 101 and column 202 get a fraction of the applied voltage and can be partially switched. This leads to a poor contrast ratio. The limitation of multiplexed addressing will be discussed later in this book.

A transistor [e.g., field-effect transistor (FET)] can be employed at each of the image elements to isolate each of the image elements from its neighboring elements on the same row and column. The transistor (e.g., FET) functions like an electrical gate. Once a voltage is applied to the image element and charges are built up on the capacitor containing the liquid crystal, the gate is closed. This creates an electrical isolation during the addressing of neighboring image elements. Such a scheme is known as AM addressing. Virtually all high-quality displays employ AM addressing, which requires an array of transistors (e.g., FETs) for the array of image elements. These transistors are fabricated on a thin film of silicon deposited on a glass panel. They are known as TFTs. The transistors are relatively small and occupy a small fraction of the pixel area. A large portion of the silicon film can thus be etched away in the remaining areas, allowing the transmission of light. By using the unique properties of the FET, it is possible to maintain an applied voltage at any pixel, independent of the voltages of the neighboring pixels. Further discussion of TFTs will be covered later in this book.

### **1.1.8. Diffusers**

A diffuser is an optical element that spreads and homogenizes the visible power (in units of lumens) of an incoming light source over a large area and angles, creating a uniformly lit panel at the output with a uniform brightness (luminance, in units of lumen/m<sup>2</sup>-s.r.). In flat panel displays, the backlight must be spread uniformly over the entire panel and over all viewing angles. In other words, the luminance (brightness) must be uniform over the entire panel and over all viewing angles. The homogenization function of any optical diffuser is related to the random distribution of its scattering centers, which constitute the active ingredient responsible for redirecting light into the various directions within the spread of the diffuser. The scattering centers can be due to the rough surface of a thin film or due to the suspension of particles or voids inside the film. So, any optical diffuser film is capable of homogenizing a non-uniform light source as long as the distribution of scattering centers provides an adequate randomization and ray-bending strength compared with the spatial feature of the light source. An ideal diffuser must produce an output panel whose brightness is uniform over the entire panel area and over all viewing angles. In addition, the efficiency or the amount of light transmitted (or reflected) by the diffuser is particularly important.

To achieve a uniform brightness over the entire display panel, it is essential to employ waveguide reflectors in addition to the diffusers in the BLU. Light emitted from the light source (e.g., LED or CCFL) is injected into the waveguide underneath the panel. This ensures the delivery of light to every part of the panel. Diffusers and reflectors are also employed so that the light is directed toward the front of the panel into all possible viewing angles. The waveguide is usually made of a thin layer of transparent dielectric material. The index of refraction of the layer must be higher than its surroundings to

ensure the guiding of light waves in the layer. The bottom surface of the waveguide can be slanted or curved so as to ensure uniform distribution of light in the panel. An air gap can exist between the bottom surface of the waveguide and a high-reflectance diffusive reflector. The waveguide is usually made of materials that are transparent in the visible spectrum, such as transparent polymeric materials. These materials include acrylic, polycarbonate, and silicone. The waveguide can be manufactured by various well-known methods, such as machining or injection molding. In some cases, the waveguide may be formed by a set of diffusive reflective surfaces that enclose an optical cavity containing a light-transmissive medium, such as air. Coupling gel can be employed to ensure efficient coupling (butt coupling) of light from the light sources (e.g., LEDs) to the waveguide.

A conventional optical diffuser used for a BLU of an LCD consists of a transparent polyester (or polycarbonate) support and an optical diffusing layer coated on the surface of the support. The diffusing layer consists of inorganic particles (matting agents) and a binder. An optical diffuser can also be made of a combination of an organic polymer binder and organic polymer particles. The diffusive reflective material can be a coating of one or more layers of diffusely reflective tape, which can have a reflectivity of approximately 97%–99.5%, depending on the thickness of the material and the wavelength of the light. Some of these tapes are made of a fluorocarbon-based polymer known as polytetrafluoroethylene (PTFE). The microporous structure in the tape is responsible for the high reflectance. Alternatively, the reflective material can be a paint or coating such as white house paint or a more exotic material, such as barium sulfate paint. The reflectivity of a typical good white house paint is approximately 90%. Barium sulfate paint's reflectivity is considerably higher, reaching as high as 98%. A diffusive reflective material, known as cyclocac (manufactured by General Electric), has a reflectivity of approximately 94%.

Once injected into the waveguide, the light propagates toward the center of the display panel. A thin layer of diffuser is placed on the top surface of the waveguide. The diffuser is usually made of a translucent material, typically a thin plastic surface or a volume diffuser, both of which are characterized by very low absorption and minimum energy loss. The diffuser may be supplemented or replaced with a grooved or scratched pattern on the top surface of the waveguide that is formed by abrading the top surface of the waveguide with an abrasive, such as sandpaper. The diffuser layer and/or the scratch pattern are employed to ensure the output coupling of light from the waveguide into the viewing directions. The light eventually achieves the correct angles of incidence to exit the waveguide through the top surface of the waveguide. The abrading is preferably nonuniform such that the density of the scratches or grooves increases with distance from the light source (e.g., LEDs, CCFLs). The proper design of the waveguide, diffusive reflector, the thin diffuser, and the scratch pattern can lead to a uniformly lit panel. A diffuser that can provide a radiance that follows the cosine law variation with the viewing angles is known as a Lambertian diffuser (see Problem 1.10).

More discussion on the scattering of light in diffusers will be covered in Appendix H.

### 1.1.9. Backlights

For the display of information, either ambient light and/or backlight is needed. Most indoor displays (e.g., notebook computers, TVs) require a BLU consisting of light sources (fluorescent lamps and LEDs), electronic controllers and drivers, and various optical elements, including light guides, diffusers, and reflectors. The BLU is a vital part of the LCD system, consuming most of the electric power. Generally speaking, there are two configurations for the positioning of the light source. For notebook displays, the edge lit configuration is desirable for minimum thickness consideration of the display panel. For TVs, the array lit (or backlit) configuration is often employed where the thickness requirement is not an issue. In the edge lit configuration, the light source is positioned at the edge of the display panel. A thin plate of light guide (also known as waveguide) is employed to deliver the light through the backside of the panel. The light guide in conjunction with the diffuser will then redirect the light out of the panel for display viewing.

The light source in the BLU can be LED, electroluminescence panel (ELP), cold cathode fluorescence lamp (CCFL), or even an incandescent lamp. The choice of the light source depends on applications, brightness, power consumption, lifetime, cost, etc. CCFL backlight is, by far, the most popular backlighting for large screen displays, and LED backlight is the most popular backlighting for small and medium LCDs. Recently, LEDs have become increasingly important in various LCDs. Both phosphor-based LEDs and RGB (Red Green Blue) LEDs can be employed for white backlight illumination.

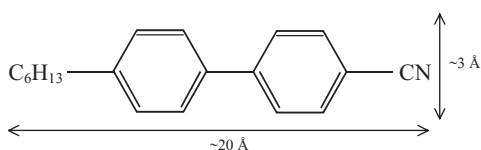
CCFL backlights offer a very bright white light at relatively low power consumption (in terms of lumens per watt). It is important to note that CCFLs require an electrical inverter to supply high voltages needed by the CCFL tube. Typical lifetimes are in the range of 10,000–50,000 h. The advantages of LED backlighting include low cost, long lifetime (over 50,000 h), immunity to vibration, low operational voltages, and the possibility to control its intensity. The major drawback is its higher power consumption. In addition to white light, LED backlights also come in a variety of colors with high color purity. This leads to the possibility of displaying more colors (higher color saturation). Being a class of semiconductor devices, LED backlights do not need inverters and require only low voltages (e.g., 5V). EL is an electro-optical phenomenon that uses phosphors in solids to generate light via electric excitation. EL backlights are very thin and lightweight and provide an even light. They are available in a variety of colors, with white being the most popular for use with LCDs. While their power consumption is fairly low, they require an inverter to provide the high voltages needed. ELPs also have a limited life of 3,000–5,000 h to half brightness. This is probably the biggest drawback of ELP backlights. Incandescent lamp backlighting is used only in applications where cost is a major factor. While incandescent lights are very bright, they generate a

significant amount of heat (which can cause problems at high temperatures). They also have relatively short life spans and consume a significant power for the brightness required [6].

## 1.2. PROPERTIES OF LIQUID CRYSTALS

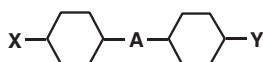
Liquid crystal is a state of matter that is intermediate between the crystalline solid and the amorphous liquid. It may also be viewed as a liquid in which an ordered arrangement of molecular orientation exists. Liquid crystals arise under certain conditions in organic substances having sharply anisotropic molecules, that is, highly elongated (rodlike) molecules or flattened (disklike) molecules. A direct consequence of the ordering of the anisotropic molecules is the anisotropy of mechanical, electric, magnetic, and optical properties. This intermediate state was first observed in 1888 in cholesteryl benzoate, a crystalline solid. It becomes a turbid cloudy liquid, or liquid crystal, when heated to 145 °C; on further heating to 179 °C, the liquid becomes isotropic and clear. The sequence is reversed when the substance is cooled. The cloudy intermediate phase contains domains that seem to have a crystal-like molecular structure. Color changes occur on both heating and cooling. Many organic compounds exhibiting this behavior, for example, hexyl-cyanobiphenyl (6CB) and sodium benzoate, are known and used extensively in electric and electronic displays, electronic clocks, calculators, and similar devices dependent on temperature determination. Table 1.3 lists a number of liquid crystal materials.

The compound 6CB is a nematic liquid crystal in the temperature range  $15^{\circ}\text{C} < T < 29^{\circ}\text{C}$ . When 6CB is heated at room temperature, a nematic-isotropic transition occurs at  $29^{\circ}\text{C}$ . The material “then becomes suddenly completely clear.” 6CB has the following rodlike molecular structure:





















which consists of two benzene rings and two terminal groups. In the nematic phase, only the long axes of the molecules have a statistical preferential orientation. As illustrated previously, 6CB has a molecular length of approximately  $20 \text{ \AA}$  and a transverse dimension of approximately  $3 \text{ \AA}$ . The rodlike molecular shape is responsible for the anisotropic liquid crystal properties.

Referring to Table 1.3, we note that a typical rodlike liquid crystal molecule has a general molecular structure shown in the following figure:



where X and Y are terminal groups and A is a linking group between two or more ring systems. In the following, we discuss the contribution of each of the groups.

**Table 1.3. Liquid Crystal Materials**

Name	Formula	Nematic Range °C
BCH-5	$C_5H_{11}$ —  —  —  —CN	96–219
PAA	$CH_3O$ —  — $N=N$ —  — $OCH_3$   O	118–135.5
EBBA	$CH_3CH_2O$ —  — $C=N$ —  — $C_4H_9$   O	35–77
MBBA	$CH_3O$ —  — $C=N$ —  — $C_4H_9$   O	22–47
CCH-501	$C_5H_{11}$ —  —  — $OCH_3$	29–36.8
5CB	$C_5H_{11}$ —  —  —CN	24–35
6CB	$C_6H_{13}$ —  —  —CN	15–29
T15	$C_5H_{11}$ —  —  —  —CN	131–240

BCH-5	biphenyl cyclohexane
PAA	polyacrylic acid
EBBA	p-ethoxybenzylidene-p'-n-butylaniline
MBBA	p-methoxybenzylidene-p'-n-butylaniline
CCH-501	trans-(trans)-4-methoxy-4'-n-pentyl-1,1'-bicyclohexyl
5CB	4-pentyl-4'-cyanobiphenyl
6CB	4-hexyl-4'-cyanobiphenyl
T15	4-pentyl-4'-cyanoterphenyl

### Ring Systems

The rings can be either benzene rings (unsaturated) or cyclohexanes (saturated), or a combination of both. The presence of the ring system is essential in most liquid crystals that contain at least one ring (either phenyl or cyclohexyl). The benzene ring  $C_6H_6$  is known to have a planar hexagonal structure. Each carbon in the ring is attached to two other carbons and one hydrogen. Each carbon-carbon bond consists of one full  $\sigma$ -bond and half a  $\pi$ -bond. The three  $\pi$ -bonds are actually delocalized over all six carbons. The cyclohexane ring  $C_6H_{12}$  is actually a nonplanar hexagonal molecule. Each carbon-carbon bond consists of one full  $\sigma$ -bond only. So,  $\pi$ -electrons exist only in phenyl liquid crystal compounds. The presence of the rings provides the short-range intermolecular forces needed to form the nematic phase. They also affect the absorption, dielectric anisotropy, birefringence, elastic constants, and viscos-



ity. The  $\sigma \rightarrow \sigma^*$  electronic transitions occur at vacuum UV regime ( $\lambda < 180$  nm). Thus, the absorption of a saturated liquid crystal compound is negligible in the visible spectral regime. In unsaturated liquid crystal compounds such as 5CB, the  $\pi \rightarrow \pi^*$  transitions occur at  $\lambda \sim 210$  nm, 280 nm with a strong dichroic ratio at 280 nm. The strong dichroic ratio is also responsible for the large dielectric anisotropy and optical birefringence in a nematic phase.

### Terminal Group X (Also Called Side Chain)

There are three common X-terminal groups: (1) alkyl chain  $C_nH_{2n+1}$ , (2) alkoxy chain  $C_nH_{2n+1}O$ , and (c) alkenyl chain, which contains a double bond. The length of the chain can strongly influence the elastic constants of the nematic phase as well as the phase transition temperature. For short chains with one or two carbon atoms ( $n = 1, 2$ ), the molecules are too rigid to exhibit liquid crystal phases. Terminal groups with medium chain length (e.g.,  $n = 3-8$ ) are most suitable for nematic phases (explained later in this section). Compounds with even longer chain length can exhibit smectic phases. Generally speaking, the melting point decreases as the chain length increases. However, some compounds are irregular and unpredictable. The nematic to isotropic transition temperature (known as the clearing point or N-I point) is a smooth function of  $n$  for both odd carbon number  $n$  and even carbon number  $n$ . Generally speaking, the N-I points (clearing points) for liquid crystal compounds with an even number of carbon atoms are lower than those of compounds with an odd number of carbon atoms. This is known as the “odd-even effect.” Although liquid crystal compounds with longer chain length can offer a lower melting point, they also exhibit a higher viscosity. In LCDs, it is desirable to have a low viscosity in order to have a higher frame rate (fast switching).

### Linking Group A

The linking group can be a single bond between the two rings. The generic name of these liquid crystals is biphenyl. In case of an additional ring as the linking group, the generic name of these liquid crystals is terphenyl. Some common linking groups are  $C_2H_4$  (diphenylethane),  $C_2H_2$  (stilbene),  $-C=C-$  (tolane),  $-N=N-$  (azobenzene),  $-CH=N-$  (Schiffs base), and COO (phenyl benzoate, ester).

### Terminal Group Y

The terminal group Y plays an important role in determining the dielectric constants  $\epsilon$  and its anisotropy  $\Delta\epsilon$ . In an LCD, the operation voltage is usually a multiple of the threshold voltage, which is the minimum voltage required to cause a reorientation of the liquid crystal molecules. It is also known that the threshold voltage is inversely proportional to the square root of the dielectric anisotropy. Thus, it is desirable to have a large  $\Delta\epsilon$  for low-voltage operations. The terminal group Y can be either polar or nonpolar. A nonpolar terminal group Y, such as the alkyl chain  $C_nH_{2n+1}$ , has very little effect on the dielectric anisotropy. On the other hand, polar terminal groups such as CN can contribute significantly to the dielectric anisotropy. Some common polar terminal

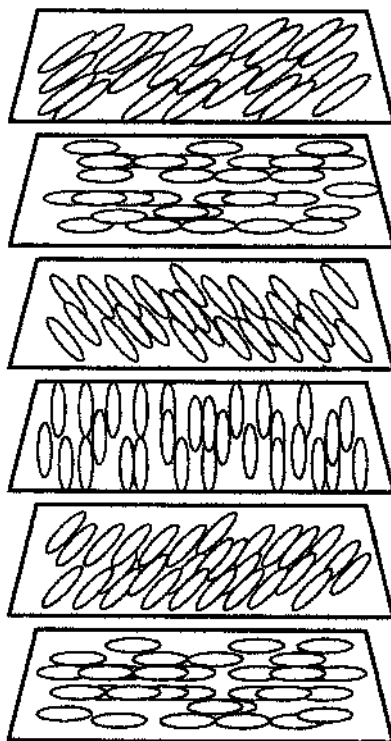
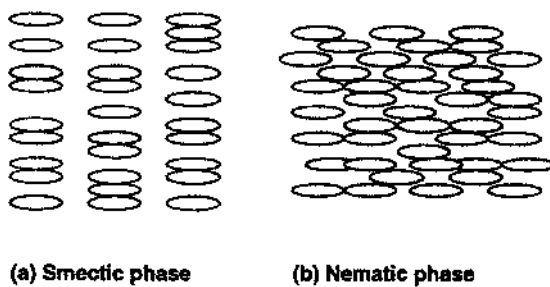
groups Y include CN, F, and Cl. Among them, CN has the highest polarity, leading to a high dielectric anisotropy and optical birefringence. On the other hand, liquid crystal compounds containing CN terminal group Y also exhibit high viscosity, insufficient resistivity, and stability problems under UV illumination. These physical properties can affect the operation of TFT-LCD. For example, liquid crystal compounds with CN terminal group Y are not suitable for high-temperature operations such as those used in projection displays. In addition, liquid crystal molecules with CN terminal group Y can be dissociated under the illumination of UV. On the contrary, liquid crystal molecules with F terminal group Y exhibit low viscosity, high resistivity, and relatively high stability under UV illumination. Being relatively lower in polarity, these liquid crystal materials exhibit relatively smaller dielectric anisotropy and optical birefringence.

### **Eutectic**

A single liquid crystal compound that can fulfill all liquid crystal specifications for most display applications is not available at the moment. For example, 5CB has a melting point of 24 °C and a clearing point of 35.3 °C. It is clear that such a small temperature window for the nematic phase is not adequate for most industrial applications with operation temperature from -20 to 80 °C. It is known that the melting point of a binary mixture of compounds is less than either of its constituent compounds. The melting point of the mixture depends on the mixture ratio. At the eutectic point, the melting point reaches its minimum. The clearing point of the liquid crystal mixture is usually the linear average of the composition. Thus, a mixture of two liquid crystal compounds can offer a much larger temperature range that exhibits the nematic phase for LCD applications. For example, the eutectic mixture of MBBA and EBBA (see Table 1.3) can provide a nematic range from 0 to 60 °C. In addition to the melting point, many other physical properties, such as dielectric constants, dielectric anisotropy, elastic constants, birefringence, and viscosity, also depend on the mixture ratio. Many of the liquid crystal mixtures are designed to provide optimum physical parameters for LCD applications.

### **Liquid Crystal Phases**

Generally speaking, there are three phases of liquid crystals: smectic, nematic, and cholesteric. For the sake of clarity, we assume that the liquid crystals are made of rodlike molecules. Figure 1.5*a* illustrates the smectic phase in which a one-dimensional translational order as well as an orientational order exist. The orientation of the molecular axis is known as the director. Figure 1.5*b* illustrates the nematic phase in which only a long-range orientational order of the molecular axes exists. The cholesteric phase is also a nematic type of liquid crystal except that it is composed of chiral molecules. As a consequence, the structure acquires a spontaneous twist about a helical axis normal to the directors. The twist may be right- or left-handed depending on the molecular



(c) Cholesteric phase

Figure 1.5. Phases of liquid crystals.

conformation. Figure 1.5c illustrates the cholesteric phase of a liquid crystal by viewing the distribution of molecules at several planes that are perpendicular to the helical axis. Notice that the liquid crystal is in a nematic phase in each of the perpendicular planes.

A smectic liquid crystal is closest in structure to solid crystals. It is interesting to note that in substances that form both a nematic phase and a smectic phase, the sequence of phase changes on rising temperature is

solid crystal  $\rightarrow$  smectic liquid crystal  $\rightarrow$  nematic liquid crystal  
 $\rightarrow$  isotropic liquid

Although the smectic phase possesses the highest degree of order, it is the nematic and cholesteric phases that have the greatest number of electro-optical applications. In the nematic phase, the medium may look milky if the orientation order exists in many different domains. The milky appearance is a result of scattering of light due to the index discontinuity at the domain boundaries. The nematic liquid crystal is clear only when a long-range order exists in the whole medium. At the nematic-isotropic transition temperature, the medium becomes isotropic and looks clear and transparent. Thus, the temperature is also known as the clearing point. In the following, we discuss various important physical properties that are the result of the orientational order.

### 1.2.1. Orientational Order Parameter

The orientation of a molecule with a rigid molecular structure can be described by using a unit vector in space plus an azimuth angle describing the transverse orientation around the unit vector. The polar angles of the unit vector plus the azimuth angle are known as the Eulerian angles of the rigid body in classical mechanics. For a rodlike molecule, the direction of the long axis is often chosen as the unit vector representing the orientation of the molecule. Strictly speaking, the molecular structure may not be symmetric with respect to a rotation around the long axis. In the nematic phase of most liquid crystals, the long axes are aligned approximately parallel to one another. This leads to a longitudinal orientational order. The transverse orientation of the molecules remains random. Thus, at any point in the medium, we can define a unit vector  $\mathbf{n}$  to represent the preferred orientation (or the averaged direction of the long axes) in the immediate neighborhood of the point. This vector is known as the director. In a homogeneous nematic liquid crystal, the director is a constant throughout the medium. In an inhomogeneous nematic liquid crystal, the director  $\mathbf{n}$  can change from point to point and is, in general, a function of space  $(x, y, z)$ . If we define a unit vector to represent the long axis of each molecule, then the director  $\mathbf{n}$  is the statistical average of the unit vectors over a small volume element around the point.

The longitudinal order parameter  $S$  of a liquid crystal is defined as

$$S = \frac{1}{2} \langle 3 \cos^2 \theta - 1 \rangle \quad (1.2-1)$$

where  $\theta$  is the angle between the long axis of an individual molecule and the director  $\mathbf{n}$ , and the angular brackets denote a statistical average. For perfectly parallel alignment,  $S = 1$ , while for totally random orientations,  $S = 0$ . In the nematic phase, the order parameter  $S$  has an intermediate value that is strongly

temperature dependent. It is evident that  $S = 0$  at the clearing point. Typical values of the order parameter  $S$  are in the range between 0.6 and 0.4 at low temperatures. As the clearing point is approached, the order parameter  $S$  drops abruptly to zero. The value of  $S$  also depends on the structure of the molecules. Experimental observation indicates that liquid crystals based on cyclohexane rings exhibit higher values of  $S$  than aromatic systems.

### 1.2.2. Dielectric Constants

Because of the orientational ordering of the rodlike molecules, the smectic and nematic liquid crystals are uniaxially symmetric, with the axis of symmetry parallel to the averaged direction of the molecules (director  $\mathbf{n}$ ). As a result of the uniaxial symmetry, the dielectric constants differ in value along the preferred axis ( $\epsilon_{\parallel}$ ) and perpendicular to this axis ( $\epsilon_{\perp}$ ). The dielectric anisotropy is defined as

$$\Delta\epsilon = \epsilon_{\parallel} - \epsilon_{\perp} \quad (1.2-2)$$

The sign and magnitude of the dielectric anisotropy  $\Delta\epsilon$  are of the utmost importance in the applicability of the liquid crystal material in LCDs using one of the various electro-optic effects. To illustrate this, we consider an applied electric field along the  $z$ -axis in a homogeneous nematic liquid crystal. As a result of the anisotropy, the induced dipole moment of the molecules is not parallel to the applied electric field, except when the molecular axis is parallel or perpendicular to the electric field. This creates a net torque that tends to align the molecules along the direction of the electric field for most rodlike molecules. The macroscopic electrostatic energy density of a liquid crystal medium can be written as

$$U = \frac{1}{2} \mathbf{D} \cdot \mathbf{E} \quad (1.2-3)$$

where  $\mathbf{E}$  is the electric field vector and  $\mathbf{D}$  is the displacement field vector. In a homogeneous medium, the displacement field vector  $\mathbf{D}$  is independent of the orientation of the liquid crystal. Let  $\theta$  be the angle between the director and the  $z$ -axis; the  $z$ -component of the displacement field vector can be written as

$$D_z = (\epsilon_{\parallel} \cos^2 \theta + \epsilon_{\perp} \sin^2 \theta) E \quad (1.2-4)$$

Thus, the electrostatic energy can be written (see Appendix A) as

$$U = \frac{1}{2} \frac{D_z^2}{\epsilon_{\parallel} \cos^2 \theta + \epsilon_{\perp} \sin^2 \theta} \quad (1.2-5)$$

For liquid crystals with positive dielectric anisotropy ( $\epsilon_{\perp} < \epsilon_{\parallel}$ ), lowest electrostatic energy occurs at  $\theta = 0$  when the director is parallel to the applied electric field.

In classical dielectric theory, the macroscopic dielectric constant is often proportional to the molecular polarizability. For rodlike molecules, the longitudinal polarizability (parallel to molecular axis) is often greater than the transverse polarizability (perpendicular to axis). So even in nonpolar liquid crystals with rodlike molecules, the dielectric anisotropy is positive ( $0 < \Delta\epsilon$ ). In the case of polar liquid crystal compounds, there is an additional contribution to the dielectric constant due to the partial alignment of permanent dipoles. Depending on the angle between the dipole moment and the molecular axis, the dipole contribution can cause an increase or a decrease of  $\Delta\epsilon$ , leading eventually to a negative value of  $\Delta\epsilon$  (e.g., MBBA). In practice, dielectric anisotropy varies between  $-4\epsilon_0$  and  $+15\epsilon_0$ . Liquid crystal compounds with a large dielectric anisotropy can be synthesized by substitution of a strongly polar group (e.g., cyanide group) in specific positions. Table 1.4 lists properties of liquid crystal compounds with various dielectric anisotropy. The dielectric anisotropy also depends on the temperature and approaches zero abruptly at the clearing point. Beyond the clearing point, the dielectric constant becomes the mean dielectric constant:

$$\bar{\epsilon} = \frac{\epsilon_{\parallel} + 2\epsilon_{\perp}}{3}$$

It is important to note that the dielectric constants ( $\epsilon_{\parallel}$  and  $\epsilon_{\perp}$ ) are actually not constants. They depend on the frequency of the electric field. This is known as dispersion. The frequency dependence can be described from the physical origin of the induced polarization of the molecules. Generally speaking, there are at least three contributions to the induced polarization of the liquid crystal medium: orientational, electronic, and vibrational polarization. Orientational polarization is due to the partial alignment of the permanent dipoles. The extent to which dipoles can be oriented by an applied electric field was calculated by Debye using the Boltzmann distribution law. The second contribution (electronic) is from the field-induced displacement of the averaged position of electrons relative to the nuclei of the molecule. The ease with which liquid crystal molecules become polarized by displacement of electron cloud differs greatly. The magnitude of the induced dipole moment depends on the orientation of the molecule in the electric field. The third contribution (vibrational) is from the deformation of the nuclear skeleton of the liquid crystal molecule by the electric field. At low frequencies (e.g., 1 kHz), all three contributions participate in the overall polarization. As the frequency is increased above the radio frequency range, the dielectric constant decreases and the orientation polarization eventually becomes negligible because there is insufficient time for molecular orientation to occur before the electric field is reversed. For

**Table 1.4 Properties of Liquid Crystals**

	$T$ (°C)	$\lambda$ (nm)	$n_e$	$n_o$	$\epsilon_{  }$	$\epsilon_{\perp}$	$(10^{-12}N)$			Nematic Range (°C)	Rotational Viscosity (mPa·s)	References	
							$k_1$	$k_2$	$k_3$				
MBBA	22	589	1.769	1.549	4.7	5.4	6.2	3.8	8.6	20	47	152.6 (20 °C)	[7,8]
PCH-5			1.605	1.493	17.7	4.8	9.60	6.50	19.4			141	[9]
PCH-5	20	436	1.6408	1.5048	17.5	4.8				30	55		[9]
		509	1.6229	1.4954									
		546	1.6173	1.4924									
		589	1.6122	1.4895									
		633	1.608	1.4872									
	25	436	1.6408	1.5038	17.4	4.9	9.1	5.3	17.9				[10]
		509	1.6194	1.4939									
		546	1.6137	1.4913									
		589	1.6089	1.4884									
		633	1.6049	1.4863									
K15(5CB)	20	577	1.7304	1.5335	17.9	6.9	4.00	2.64	4.85	24	35.3	125	[11-13]
K21(7CB)	22	577	1.7115	1.5203	15.7	6	4.65	1.74	5	30	42.8	81 (32 °C)	[12,14,15]
M15(SOCB)	50	589	1.7187	1.5259	17.9	6.7	3.00	3.74	3.85	48	68		[12,16,17]
M21(7OCB)	50	589	1.7019	1.5122	16.3	6.5	3.40	3.65	3.65	54	74		[12,18]
M24(8OCB)	50	589	1.6983	1.5028	14.7	6.2	4.50	2.90	4.75	67	80		[12]
T15	20	598	1.891	1.542	26.4	4.3				131	240		[16]
E5	20	577	1.736	1.5228	19	5.9				-8	50.5		[9]
E7	20	577	1.75	1.5231	19.6	5.1	11	10.2	16.9	-10	60.5	250	[11]
ZLI-1646	20	589	1.558	1.478	10.6	4.6	7.7	4	12.2	-20	60	160	[11]
ZLI-4792	20	589	1.573	1.479	8.3	3.1	13.2	6.5	18.3	-40	92	134	[19,20]
MLC-7026-000	20	589.3	1.5577	1.4755	3.5	7.2	14.9	15.1	15.1	80	126		[9]
MLC-7026-100	20	589.3	1.5924	1.4833	3.7	7.6	17.1	15.3	14.2	80.5	142		
MLC-7029	20	589.3	1.6157	1.4892	3.6	7.2	16.1	15.0	15.0	95	175		
MLC-6608	20	589	1.5578	1.4748	3.6	7.8	16.7	18.1	18.1	<-30	90	186	
MLC-6810	20	589	1.5824	1.4828	3.5	6.6	14.6	16.5	16.5	<-30	80	148	
MLC-6882	20	589.3	1.5816	1.4838	3.6	6.7	13.1	12.8	12.8	69	108		
MLC-6886	20	589.3	1.5701	1.4802	3.6	7.4	13.8	14.8	14.8	<-30	75	146	
MBBA			p-methoxybenzylidene-p'-n-butylamine		M24(8OCB)		4-octyloxy-4'-cyanobiphenyl						
PCH-5			4-(trans-4-Pentylcyclohexyl)-benzomitrile		T15		4-pentyl-4'-cyanobiphenyl						
K15(5CB)			4-pentyl-4'-cyanobiphenyl		E5		45%K15+24%K21+10%M15+9%M21+12%M24						
K21(7CB)			4-heptyl-4'-cyanobiphenyl		E7		47%K15+25%K21+18%M24+10%T15						
M15(SOCB)			4-pentyl-4'-cyanobiphenyl		ZLI-xxxx		liquid crystal mixtures manufactured by Merck						
M21(7OCB)			4-heptyloxy-4'-cyanobiphenyl		MLC-xxxx		liquid crystal mixtures manufactured by Merck						

conventional liquid crystals consisting of rodlike molecules, the orientational contribution of permanent dipoles to the polarization is negligible for  $\epsilon_{\perp}$  due to the hindered rotational motion around the short axis. As a result,  $\epsilon_{\perp}$  remains constant up to microwave frequencies. On the other hand, the orientational contribution of permanent dipoles to the polarization is significant for  $\epsilon_{\parallel}$ . According to Debye's theory of polar molecules, the frequency dependence of the dielectric constant in this frequency region can be written as

$$\epsilon_{\parallel} = \epsilon_{\parallel H} + \frac{(\epsilon_{\parallel L} - \epsilon_{\parallel H})}{1 + (2\pi f\tau)^2} = \epsilon_{\parallel H} + \frac{(\epsilon_{\parallel L} - \epsilon_{\parallel H})}{1 + (f/f_R)^2} \quad (1.2-6)$$

$$\epsilon_{\perp} \approx \text{constant}$$

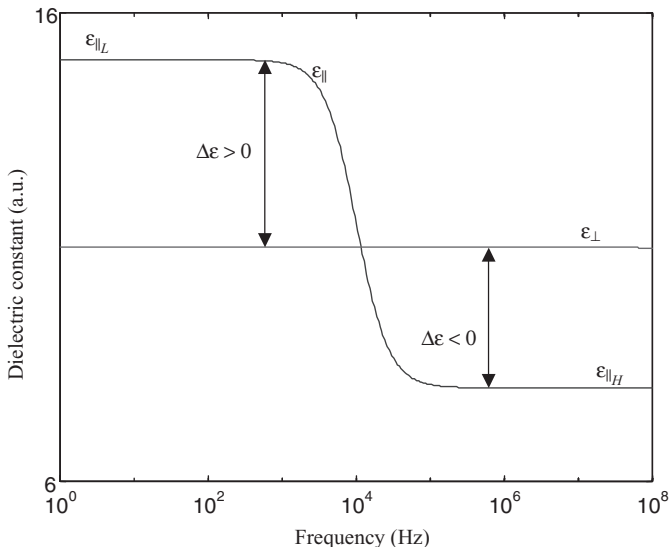
where  $\epsilon_{\parallel H}$  is the dielectric constant at high frequencies ( $f_R \ll f$ ) and  $\epsilon_{\parallel L}$  is the dielectric constant (with  $\epsilon_{\parallel L} > \epsilon_{\parallel H}$ ) at low frequencies ( $0 < f \ll f_R$ ). The relaxation frequency  $f_R = 1/(2\pi\tau)$  is related to the relaxation time  $\tau$  of the physical process (e.g., orientational motion of permanent dipoles). We note that  $\epsilon_{\parallel}$  is a decreasing function of frequency (see Fig. 1.6).

For liquid crystals with  $\epsilon_{\parallel H} < \epsilon_{\perp} < \epsilon_{\parallel L}$ , there exists a frequency  $f_c$  around the relaxation frequency where  $\Delta\epsilon$  is zero and becomes negative for frequency beyond  $f_c$ . This frequency is known as the crossover frequency. For example, liquid crystal mixture MLC-2048 exhibits a positive dielectric anisotropy of  $\Delta\epsilon = 3.2\epsilon_0$  at  $f = 1$  kHz, and negative dielectric anisotropy of  $\Delta\epsilon = -3.1\epsilon_0$  at  $f = 50$  kHz, with a crossover frequency of  $f_c = 31$  kHz [21]. Liquid crystal materials with such a reversal of dielectric anisotropy are known as dual-frequency materials. They can be employed for improving the switching speed of displays via dual-frequency addressing. Further discussion on this can be found later in this section.

### 1.2.3. Refractive Index

In a glass flask, nematic liquid crystals often appear as an opaque milky fluid. The scattering of light is due to the random fluctuation of the refractive index of the sample. With no proper boundaries to define the preferred orientation, the sample consists of many domains of nematic liquid crystal. As discussed earlier, the discontinuity of the refractive index at the domain boundaries is the main cause of the scattering leading to the milky appearance. Under the proper treatment (e.g., rubbing the alignment layer on the glass substrate), a slab of nematic liquid crystal can be obtained with a uniform alignment of the director. Such a sample exhibits uniaxial optical symmetry with two principal refractive indices  $n_o$  and  $n_e$ . The ordinary refractive index  $n_o$  is for light with electric field polarization perpendicular to the director, and the extraordinary refractive index  $n_e$  is for light with electric field polarization parallel to the director. The birefringence (or optical anisotropy) is defined as





**Figure 1.6.** Frequency dependence of dielectric constants.

$$\Delta n = n_e - n_o \tag{1.2-7}$$

If  $n_o < n_e$ , the liquid crystal is said to be positive birefringent, whereas if  $n_e < n_o$ , it is said to be negative birefringent. In classical dielectric theory, the macroscopic refractive index is related to the molecular polarizability at optical frequencies. The existence of the optical anisotropy is due mainly to the anisotropic molecular structures. Most liquid crystals with rodlike molecules exhibit positive birefringence ranging from 0.05 to 0.45. The optical polarizability is mainly due to the presence of delocalized electrons not participating in chemical bonds and of  $\pi$ -electrons. This is the reason that liquid crystal molecules composed of benzene rings have higher values of  $\Delta n$  than the respective cyclohexane counterparts. As mentioned earlier, the substitution of a cyano terminal group (CN) can cause an increase in the optical anisotropy. Another effective way of increasing  $\Delta n$  or  $n_e$  is to employ triple bonds (tolane). For most liquid crystals, the ordinary refractive index is around 1.5. The various components employed in the synthesis of liquid crystals do not markedly affect  $n_o$ . On the other hand, the extraordinary refractive index can be higher by as much as 0.45 for some diphenyldiacetylenic compounds or only by 0.06 for bicyclohexanes. The optical anisotropy plays an essential role in changing the polarization state of light in liquid crystals. For liquid crystals with both positional order and orientational order, the medium can exhibit optical biaxial properties. In other words, some liquid crystals can be optically biaxial with three principal indices of refraction,  $n_x$ ,  $n_y$ , and  $n_z$ .

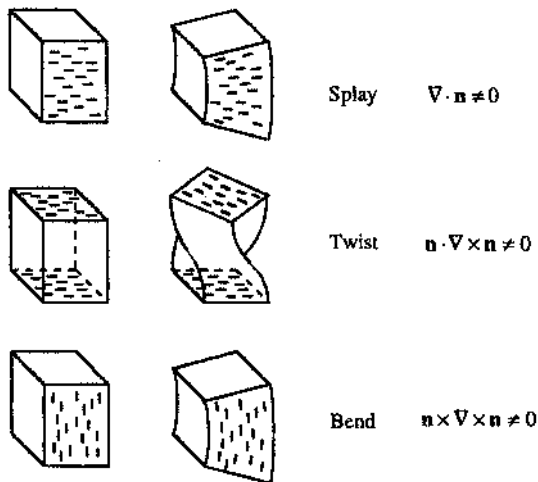
### 1.2.4. Elastic Constants

Like most liquids and solids, liquid crystals exhibit curvature elasticity. The elastic constants of a liquid crystal determine the restoring torques that arise when the system is perturbed from its equilibrium configuration. The elastic restoring torques are usually very weak compared with those of solids. In LCDs, electric field is often applied to cause a reorientation of the molecules. It is the balance between the electric torque and the elastic restoring torque that determines the static deformation pattern of a liquid crystal. Any static deformation of liquid crystals can be divided into a combination of three basic deformations. These are splay, twist, and bend, as illustrated in Figure 1.7. In liquid crystals, the deformation is described in terms of the director  $\mathbf{n}(x, y, z)$  as a function of position  $(x, y, z)$ . The variation of  $\mathbf{n}$  with position leads to various forms of deformation described in Figure 1.7.

It can be shown that for an isothermal deformation in an incompressible fluid, the free energy (elastic energy) can be written as a quadratic function of the curvature strain tensor. Following the notation of the Oseen–Frank theory, the elastic energy density of a deformed liquid crystal can be written [22,23] as

$$F = \frac{1}{2}k_1(\nabla \cdot \mathbf{n})^2 + \frac{1}{2}k_2(\mathbf{n} \cdot \nabla \times \mathbf{n})^2 + \frac{1}{2}k_3(\mathbf{n} \times \nabla \times \mathbf{n})^2 \quad (1.2-8)$$

where  $k_1, k_2, k_3$  are the splay, twist, and bend elastic constants, respectively.



**Figure 1.7.** Schematic drawing of splay, twist, and bend in a liquid crystal.

Like many other physical properties, the elastic constants are strongly temperature dependent. For most liquid crystal compounds, the elastic constants are in the range from 3 to 25 pN ( $10^{-12}$  N). The ratio for the elastic constants varies from 0.7 to 1.8 for  $k_3/k_1$  and varies from 1.3 to 3.2 for  $k_3/k_2$ . Table 1.4 lists the elastic constants for some typical liquid crystal materials.

### 1.2.5. Viscosity—Rotational Viscosity

The viscosity of fluid is an internal resistance to flow, defined as the ratio of shearing stress to the rate of shear. It arises from the intermolecular forces in the fluid. The viscous behavior of liquid crystals has a profound effect on the dynamical behavior of LCD systems. Like most liquids, the viscosity increases at low temperatures due to lower molecular kinetic energy. This can severely limit the operations of LCDs. An important parameter is the rotational viscosity coefficient  $\gamma_1$ , which provides a resistance to the rotational motion of the liquid crystal molecules. In most LCDs, the directors are reoriented by the application of an electric field. The switching time is approximately proportional to  $\gamma_1 d^2$ , where  $d$  represents the cell spacing. For most nematic liquid crystals used in displays, the magnitude of the rotational viscosity is in the range from 0.02 to 0.5 Pa·s (comparable to light machine oils). As a reference, water at 20 °C has a viscosity of 1.002 mPa·s. The viscosity unit of pascal second corresponds to 1 N·s/m<sup>2</sup> in SI (Système International) unit, or 10 P (1 P = 1 dyn·s/cm<sup>2</sup> in cgs unit). It is known experimentally that molecules with a higher number of rings or longer alkyl chains are characterized by an increasing viscosity. In addition, liquid crystals with high values of  $\Delta\epsilon$  usually exhibit higher viscosities. This is possibly due to the stronger polar interaction between the molecules. More discussion on the effect of rotational viscosity on the rotational motion of liquid crystal molecules can be found later in this chapter.

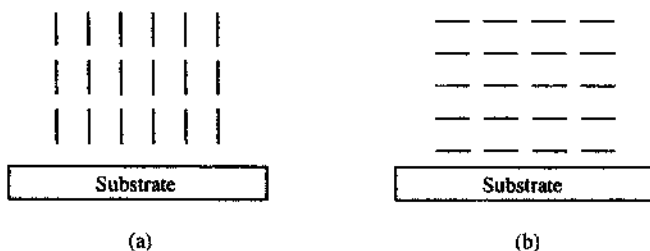
### 1.2.6. Surface Alignment and Rubbing

As a result of the rodlike or disk-like nature, the distribution of the orientation of liquid crystal molecules plays an essential role in the optical properties of liquid crystal in display applications. Uniform or well-prescribed orientation of the liquid crystal molecules is required in most LCDs. The role of the surface is to ensure a single domain. Without special treatments of the surfaces (either physical or chemical), the liquid crystal will generally have many domains and many disinclinations or discontinuities in orientations. These domains and discontinuities can cause a severe scattering of light, leading to a cloudy appearance.

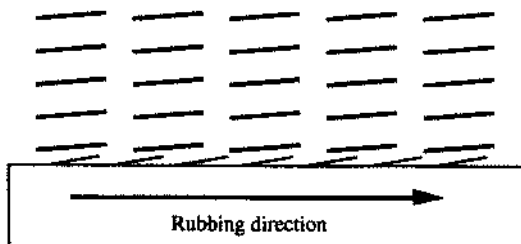
The alignment of a liquid crystal by the surface treatment of a substrate has been one of the least understood aspects of the behavior of liquid crystals. It depends on both the nature of the liquid crystal and the surface. The most important factors include dipolar interactions, chemical and hydrogen bonding,

van der Waals interactions, steric factors, surface topography, and the elasticity of liquid crystal molecules. For example, it is possible to deposit molecules known as “silane coupling agents” onto a glassy surface. These molecules may promote the adhesion of liquid crystal molecules, in a vertical manner, onto the surface. This is known as “homeotropic alignment” when the liquid crystal director is normal to the surface (see Fig. 1.8*a*).

To achieve a “parallel homogeneous alignment” where the liquid crystal director is uniformly parallel to the surface of the substrate (see Fig. 1.8*b*), one must provide a preferred orientation to the alignment by physically or chemically treating the surface. Rubbing the surface has been a simple and effective way of achieving a preferred orientation. Typical rubbing materials include linen cloth and lens paper. Although the technique of rubbing to achieve parallel homogeneous alignment is still very much an art, it is generally believed that physically rubbing the surface produces a uniform and unidirectional tilt of the dangling bonds or side chains on the surface (see Fig. 1.9). It is also possible that the rubbing creates a parallel array of micro-scratches on the surface of the alignment layer. The unidirectional tilt of these surface molecules may lead to the parallel homogeneous alignment of the liquid crystal molecules. Notice that the liquid crystal director is usually tilted up in the direction of rubbing.



**Figure 1.8.** (a) Homeotropic (vertical) alignment. (b) Parallel homogeneous alignment.

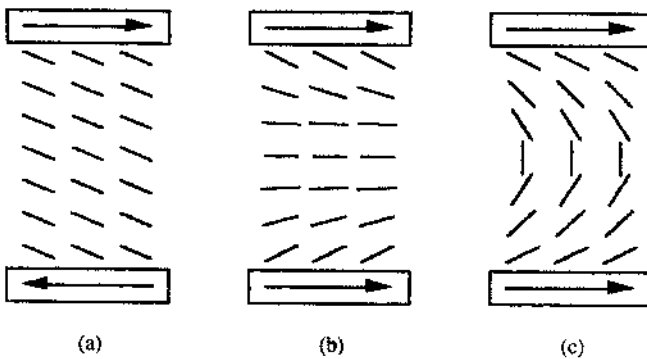


**Figure 1.9.** Schematic drawing of the anchoring of rodlike liquid crystal molecules near the surface of a rubbed substrate.

**Pretilt Angle**

As a result of the surface rubbing, a small tilt angle exists for the liquid crystal director. The pretilt angle is very important in electro-optic applications when an electric field is applied to the cell to reorient the liquid crystal director. Having no polarity, the rodlike liquid crystal molecules can turn in two ways following the application of an electric field, if the electric field is initially perpendicular to the molecular axis. A small tilt angle of the molecular axis would greatly facilitate the turning of the molecule under the application of an electric field and would ensure a single way of turning. A single way of turning for all molecules in the cell will ensure a single domain, which is essential in most display applications. Referring to Figure 1.10, we consider the distribution of the director in a parallel cell of nematic liquid crystal. A uniform tilt angle can exist in a parallel cell provided the rubbing directions in the inner surfaces are antiparallel (see Fig. 1.10a). If, on the other hand, the rubbing directions are parallel, as shown in Figure 1.10b,c, a nonuniform distribution of the director orientation exists with a zero tilt or a 90° angle at the midlayer. These are the splay or bend cells. For small pretilt angles, the splay cell has the lowest elastic energy density. This is, however, an undesirable configuration for electro-optic applications, as multidomains may occur when an electric field is applied. The bend cell is also known as the Pi-cell, which was originally developed by Bos et al. for an electrically controllable wave plate [24].

A small pretilt angle not only ensures a single domain upon switching, but also speeds up the switching process. Without a pretilt angle, the molecular axis is perpendicular to the electric field (or parallel in VA cells). In this configuration, the electrostatic torque is zero. A small pretilt angle will ensure a



**Figure 1.10.** Tilt of liquid crystal molecular axes due to the rubbing of the inner surfaces of the liquid crystal cell. The arrows indicate the direction of rubbing. (a) Parallel alignment occurs when the rubbings are in opposite directions. Splay cell (b) or bend cell (c) can occur when the rubbings are in the same direction.

finite electrostatic torque for the reorientation of the molecules. A discussion in Subsections 1.2.7 and 1.2.8 will show that the switching time of the liquid crystal cells depends on the pretilt angle. Generally speaking, a larger pretilt angle will lead to a larger electrostatic torque and thus a faster switching time.

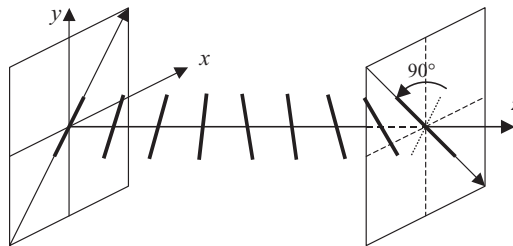
### ***Twisted Nematic Liquid Crystal (TN-LC) Cell***

The pretilt angle also plays an important role in TN-LC cells. Referring to Figure 1.11, we consider a liquid crystal cell with a pair of orthogonal rubbing directions. The rubbing directions are important to prepare a TN-LC cell with a total twist angle of  $90^\circ$ . Having no polarity and no chirality, the rodlike molecules can twist in either direction in the absence of a small pretilt angle. In other words, both right- and left-handed twists of  $90^\circ$  are possible to match the boundary conditions imposed by the rubbing. When a small pretilt angle exists, only the right-handed twist (with a uniform tilt angle) is possible in the cell described in Figure 1.11. In the cell shown in Figure 1.11, the upper parts ( $+y$ ) of liquid crystal molecular axes are tilted toward the right side ( $+z$  direction). The total twist angle of liquid crystal molecules in a TN cell can be greater or less than  $90^\circ$ . In most displays employing TN-LC cells, the twist angle is either  $90^\circ$  or approximately  $90^\circ$ .

#### **1.2.7. Field-Induced Distortion—Electro-Optical Effect**

By virtue of its dielectric anisotropy, a liquid crystal molecule can be reoriented via an application of an electric field. For rodlike molecules with  $\epsilon_\perp < \epsilon_\parallel$ , minimum electrostatic energy occurs when the long axes of the molecules are aligned with the electric field. The reorientation of the liquid crystal molecules leads to a change in the birefringent property of the liquid crystal cell, and thus a change in the transmission property of the liquid crystal cell.

For the purpose of illustrating the electro-optical effect, we consider a PANematic liquid crystal cell under the influence of an applied electric field along

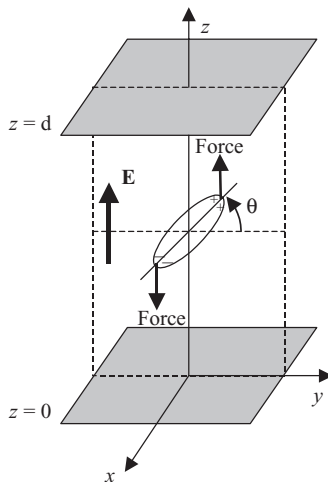


**Figure 1.11.** Distribution of liquid crystal molecular axes in a  $90^\circ$  TN-LC cell with a right-handed twist. The arrows indicate the rubbing directions at the boundaries. The rods indicate the distribution of director orientations in the cell.

the  $z$ -axis (see Fig. 1.12). The director  $\mathbf{n}$  is initially parallel to the  $y$ -axis. As a result of the dielectric anisotropy, the liquid crystal molecules can be reoriented in the  $yz$ -plane. For free molecules with  $\epsilon_{\perp} < \epsilon_{\parallel}$ , the minimum electrostatic energy occurs when the long axis of the molecules is parallel to the electric field (i.e., with  $\theta = \pi/2$ ). In a liquid crystal cell, the molecules are not exactly free. The orientation of the molecules at the boundaries are anchored with the surface as a result of the surface rubbing. The reorientation of the molecules inside the cell creates a distortion of the director distribution that leads to an elastic restoring torque that tends to resist the reorientation. As a result, the tilt angle depends not only on the applied electric field but also on the elastic property of the liquid crystal as well as the molecular interaction at the boundaries. Thus, the tilt angle  $\theta$  is an increasing function of the applied field provided  $\epsilon_{\perp} < \epsilon_{\parallel}$ , approaching  $\theta = \pi/2$  only when the electric field is very strong. The actual distribution of the liquid crystal director  $\mathbf{n}(z)$  is determined by the balance between the electrostatic torque and the elastic torque, subject to the boundary conditions.

Here, we describe the redistribution of the director  $\mathbf{n}$  due to the presence of an electric field. Let the director  $\mathbf{n}$  be written as a row vector in the  $xyz$ -coordinate:

$$\mathbf{n} = (0, \cos \theta, \sin \theta) \tag{1.2-9}$$



**Figure 1.12.** A nematic liquid crystal cell showing a liquid crystal with its long axis oriented at an angle  $\theta$  measured from the  $xy$ -plane. An electric field ( $\mathbf{E}$ ) is present along the direction of the  $z$ -axis. As a result of the induced dipole moment, the molecule experiences a torque that tends to reorient the molecule.

where the tilt angle  $\theta$  is, in general, a function of  $z$ . We assume that the liquid crystal molecules at the boundaries are anchored parallel to the surface ( $\theta = 0$ ). So, the boundary condition can be written as

$$\mathbf{n}(z = 0) = \mathbf{n}(z = d) = \mathbf{n} = (0, 1, 0) \quad (1.2-10)$$

where we recall that  $d$  is the liquid crystal cell gap. In other words, the long axes of molecules at the boundaries are parallel to the  $y$ -axis. The electromagnetic field energy density due to the presence of an electric field can be written, according to Eq. (A-19) in Appendix A, as

$$\Delta U_{EM} = \frac{1}{2} \frac{D_z^2}{(\epsilon_{\parallel} \sin^2 \theta + \epsilon_{\perp} \cos^2 \theta)} - \frac{1}{2} \frac{D_z^2}{\epsilon_{\perp}} \quad (1.2-11)$$

where  $D_z$  (=constant) is the  $z$ -component of the displacement field vector. For a cell that is uniform in the  $xy$ -plane, the dielectric tensor  $\epsilon$  is a function of  $z$  only. By applying an electric voltage drop across the cell, an electric field inside the cell is created. The electric field is a function of  $z$  due to the variation of the dielectric tensor  $\epsilon$  in the cell. However, the  $z$ -component of the displacement field  $D_z$  is a constant (see Problem 1.9), regardless of the orientation of the liquid crystal molecules. We note that the second term is a constant independent of the director orientation  $\theta(z)$ . The elastic energy density in this case is given by, according to Eq. (A-5) in Appendix A,

$$U_{EL} = \frac{1}{2} (k_1 \cos^2 \theta + k_3 \sin^2 \theta) \left( \frac{d\theta}{dz} \right)^2 \quad (1.2-12)$$

We note that the elastic energy density is proportional to the square of the slope of the tilt angle. The elastic energy density is zero in a uniformly oriented cell. The electrostatic torque density in the cell acting on the liquid crystal molecules is related to the electromagnetic energy density by the following relationship (see Appendix I):

$$(\tau_x)_{EM} = -\frac{\partial}{\partial \theta} \Delta U_{EM} + \frac{d}{dz} \frac{\partial}{\partial \theta'} \Delta U_{EM} \quad (1.2-13)$$

where we note that  $\theta' = (d\theta/dz)$ . Using Eq. (1.2-11) for the electromagnetic energy density and carrying out the differentiation, we obtain the following expression for the electrostatic torque:

$$(\tau_x)_{EM} = \frac{D_z^2 (\epsilon_{\parallel} - \epsilon_{\perp})}{(\epsilon_{\parallel} \sin^2 \theta + \epsilon_{\perp} \cos^2 \theta)^2} \sin \theta \cos \theta \quad (1.2-14)$$



or, equivalently,

$$(\tau_x)_{EM} = (\epsilon_{\parallel} - \epsilon_{\perp}) E^2 \sin \theta \cos \theta \quad (1.2-15)$$

where we note that the displacement field  $D_z$  is a constant and the electric field  $E$  is a function of  $z$ . We notice that the electrostatic torque is proportional to  $(\epsilon_{\parallel} - \epsilon_{\perp}) E^2$ , which is independent of the polarity of the field. In other words, reversing the direction of the electric field does not change the direction of the torque. So, even an AC electric field can create the same electrostatic torque. The torque is also proportional to the dielectric anisotropy  $\Delta\epsilon = (\epsilon_{\parallel} - \epsilon_{\perp})$ .

The elastic torque density experienced by the liquid crystal molecules due to the deformation is related to the elastic energy density by the following relationship (see Appendix I):

$$(\tau_x)_{EL} = -\frac{\partial}{\partial \theta} U_{EL} + \frac{d}{dz} \frac{\partial}{\partial \theta'} U_{EL} \quad (1.2-16)$$

Using Eq. (1.2-12) for the elastic energy density and carrying out the differentiation, we obtain

$$(\tau_x)_{EL} = (k_3 - k_1) \sin \theta \cos \theta \left( \frac{d\theta}{dz} \right)^2 + (k_1 \cos^2 \theta + k_3 \sin^2 \theta) \frac{d^2 \theta}{dz^2} \quad (1.2-17)$$

This is the elastic restoring torque density.

In the equilibrium state, the liquid crystal molecules are oriented so that the total torque acting on the molecules must vanish. In other words,

$$(\tau_x)_{EM} + (\tau_x)_{EL} = 0 \quad (1.2-18)$$

The liquid crystal molecules are in an orientation distribution where the electrostatic torque exactly balances with the elastic restoring torque. Using Eqs. (1.2-15) and (1.2-17), the equilibrium condition becomes

$$(\epsilon_{\parallel} - \epsilon_{\perp}) E^2 \sin \theta \cos \theta + (k_3 - k_1) \sin \theta \cos \theta \left( \frac{d\theta}{dz} \right)^2 + (k_1 \cos^2 \theta + k_3 \sin^2 \theta) \frac{d^2 \theta}{dz^2} = 0 \quad (1.2-19)$$

This is the equation for the director distribution  $\theta(z)$  at a static equilibrium of the liquid crystal cell. This is a nonlinear differential equation in that it can only be solved numerically. Here we consider a special case of interest.

If we assume a small distortion so that  $\theta \ll 1$ , the equation becomes approximately

$$k_1 \frac{d^2\theta}{dz^2} + (\varepsilon_{\parallel} - \varepsilon_{\perp}) E^2 \theta = 0 \quad (1.2-20)$$

This is a linear differential equation. A solution can be written as

$$\theta(z) = \theta_0 \sin\left(\frac{\pi}{d} z\right) \quad (1.2-21)$$

where  $\theta_0$  is a constant ( $\theta_0 \ll 1$ ) and  $d$  is the cell gap. This solution also satisfies the boundary condition

$$\theta(0) = \theta(d) = 0 \quad (1.2-22)$$

Substitution of the solution Eq. (1.2-20) back into the nonlinear equation (1.2-19), we obtain

$$(\varepsilon_{\parallel} - \varepsilon_{\perp}) E^2 \sin\theta \cos\theta + (k_3 - k_1)(\theta_0^2 - \theta^2) \sin\theta \cos\theta - (k_1 \cos^2\theta + k_3 \sin^2\theta) \left(\frac{\pi}{d}\right)^2 \theta = 0 \quad (1.2-23)$$

Keeping only the first-order term (note:  $\theta \leq \theta_0 \ll 1$ ), we have

$$(\varepsilon_{\parallel} - \varepsilon_{\perp}) E^2 \sin\theta \cos\theta - k_1 \left(\frac{\pi}{d}\right)^2 \theta = 0 \quad (1.2-24)$$

For all practical ranges of interest,  $\sin\theta \cos\theta < \theta$ . Thus, the nonlinear differential equation can support nontrivial solution only when

$$E^2 > \frac{k_1}{(\varepsilon_{\parallel} - \varepsilon_{\perp})} \left(\frac{\pi}{d}\right)^2 \equiv E_{c1}^2 \quad (1.2-25)$$

This is the critical field needed to create a distortion in a parallel nematic liquid crystal cell. Since the voltage across the cell is  $V = Ed$  (assuming small distortion with  $\theta \ll 1$ ), the critical voltage can be written as

$$V_{c1} = \pi \frac{\sqrt{k_1}}{\sqrt{\varepsilon_{\parallel} - \varepsilon_{\perp}}} \quad (\text{PA cell}) \quad (1.2-26)$$

where  $k_1$  is the splay elastic constant. The critical voltage (or threshold voltage) is proportional to the square root of elastic constant  $k_1$  and is inversely proportional to the square root of dielectric anisotropy  $\Delta\varepsilon = \varepsilon_{\parallel} - \varepsilon_{\perp}$ . A larger elastic constant  $k_1$  corresponds to a stiffer medium that requires a higher electrostatic field to distort the medium. Similarly, a larger anisotropy  $\Delta\varepsilon = \varepsilon_{\parallel} - \varepsilon_{\perp}$  corresponds to a stronger electrostatic torque at a given applied

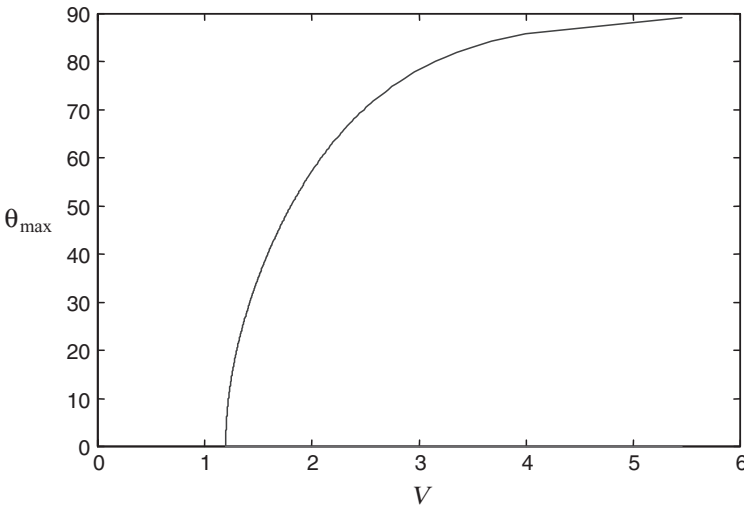
field. The critical voltage is derived for the case when the initial orientation of the director of liquid crystal (with  $\Delta\varepsilon = \varepsilon_{\parallel} - \varepsilon_{\perp} > 0$ ) is perpendicular ( $\theta = 0$ ) to the applied electric field. If the initial director orientation is not perpendicular to the applied electric field, then the critical voltage disappears. In other words, there is no critical voltage when an initial finite tilt angle exists. Further discussion of this will be covered in Chapter 5.

We note that for a small distortion, the tilt angle  $\theta(z)$  follows a sinusoidal solution as given by Eq. (1.2-21). The tilt angle remains zero at the boundaries and increases monotonically toward the center of the cell, reaching a maximum  $\theta_{\max} = \theta_0$  at midlayer ( $z = d/2$ ). When the applied field is much higher than the critical field, the distortion can be significant. The maximum tilt angle  $\theta_{\max}$  at midlayer is an increasing function of the applied field. The maximum tilt at midlayer reaches a saturation value of  $\pi/2$ . A numerical solution is needed to obtain the director distribution. For this purpose, Eq. (1.2-19) can be integrated and rewritten as

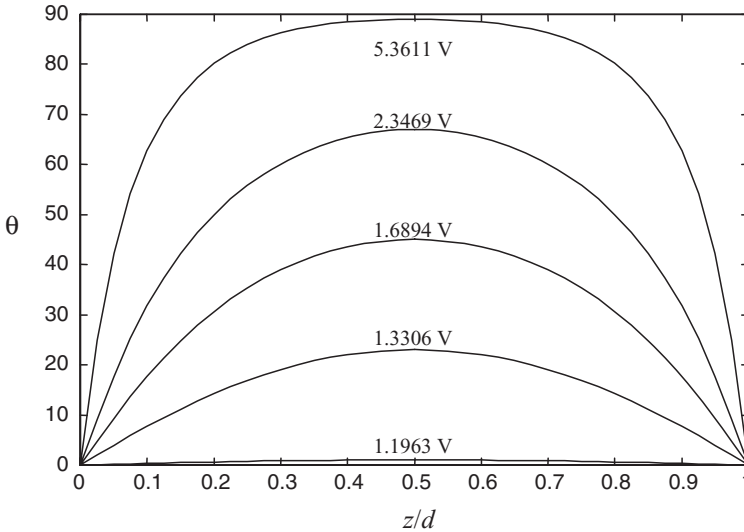
$$(k_1 \cos^2 \theta + k_3 \sin^2 \theta) \left( \frac{d\theta}{dz} \right)^2 = \frac{D_z^2}{(\varepsilon_{\parallel} \sin^2 \theta + \varepsilon_{\perp} \cos^2 \theta)} - \frac{D_z^2}{(\varepsilon_{\parallel} \sin^2 \theta_{\max} + \varepsilon_{\perp} \cos^2 \theta_{\max})} \tag{1.2-27}$$

where  $\theta_{\max}$  is the maximum tilt angle at midlayer ( $z = d/2$ ). This equation can be easily integrated numerically.

As an example, we consider a parallel liquid crystal cell of ZLI-1646 with a cell gap of  $d = 3 \mu\text{m}$ . Using the material parameters from Table 1.4, a numerical integration of Eq. (B-21) in Appendix B yields the following maximum tilt angle as a function of applied voltage. This is illustrated in Figure 1.13. We



**Figure 1.13.** Maximum tilt angle at midlayer of a parallel nematic liquid crystal cell of ZLI-1646 with a cell gap of  $d = 3 \mu\text{m}$ .



**Figure 1.14.** Director tilt angle distribution of a parallel liquid crystal cell of ZLI-1646 with a cell gap of  $3\mu\text{m}$ , at various applied voltages.

notice that the midlayer tilt angle is an increasing function of applied voltage when  $V_{cl} < V$ . The midlayer tilt angle reaches a saturation value of  $90^\circ$  when the applied voltage is much higher than the threshold voltage ( $V_{cl} \ll V$ ).

For a given midlayer tilt angle  $\theta_{\max}$ , Eq. (1.2-27) can be integrated numerically for the tilt angle distribution  $\theta(z)$ . Figure 1.14 shows the tilt angle distribution  $\theta(z)$  of the ZLI-1646 cell. We note the distortion starts when the applied voltage is greater than the threshold voltage of 1.1961V. At an applied voltage of 5.3611V, the midlayer tilt angle is near  $90^\circ$ .

As another example, we consider a VA liquid crystal cell. The director is initially oriented at a tilt angle of  $\theta = \pi/2$ . Due to the negative dielectric anisotropy ( $\Delta\epsilon = \epsilon_{\parallel} - \epsilon_{\perp} < 0$ ) of the liquid crystal medium (e.g., MLC-6886), an applied field along the  $z$ -axis in the liquid crystal cell can reorient the liquid crystal molecules to a new direction to minimize the electrostatic energy. According to Eq. (1.2-11), minimum electrostatic energy occurs at  $\theta = 0$  for a medium with negative dielectric anisotropy ( $\Delta\epsilon = \epsilon_{\parallel} - \epsilon_{\perp} < 0$ ). As a result of the presence of an elastic restoring torque, the director distribution must satisfy the equilibrium condition [Eq. (1.2-19)].

For VA cells, we can write

$$\theta(z) = \frac{\pi}{2} - \sigma(z) \quad (1.2-28)$$

where a finite  $\sigma$  corresponds to field-induced distortion. The equilibrium condition [Eq. (1.2-19)] becomes

$$(\epsilon_{\parallel} - \epsilon_{\perp})E^2 \sin \sigma \cos \sigma + (k_3 - k_1) \sin \sigma \cos \sigma \left( \frac{d\sigma}{dz} \right)^2 - (k_1 \sin^2 \sigma + k_3 \cos^2 \sigma) \frac{d^2 \sigma}{dz^2} = 0 \quad (1.2-29)$$

Again, we consider the special case of small distortion with  $\sigma \ll 1$ . The equation becomes

$$-k_3 \frac{d^2 \sigma}{dz^2} + (\epsilon_{\parallel} - \epsilon_{\perp}) E^2 \sigma = 0 \quad (1.2-30)$$

Equation (1.2-30) supports a solution of the form

$$\sigma(z) = \sigma_0 \sin\left(\frac{\pi}{d} z\right), \quad (\sigma_0 \ll 1) \quad (1.2-31)$$

where  $d$  is the cell gap. The solution satisfies the condition that  $\theta = \pi/2$  at the boundaries of a VA cell. We now substitute Eq. (1.2-31) into Eq. (1.2-29) and keep only the first-order terms in  $\sigma$ . We obtain

$$k_3 \left( \frac{\pi}{d} \right)^2 \sigma + (\epsilon_{\parallel} - \epsilon_{\perp}) E^2 \sin \sigma \cos \sigma = 0 \quad (1.2-32)$$

Since  $\sin \sigma \cos \sigma < \sigma < 1$ , a nontrivial solution of Eq. (1.2-32) exists only when

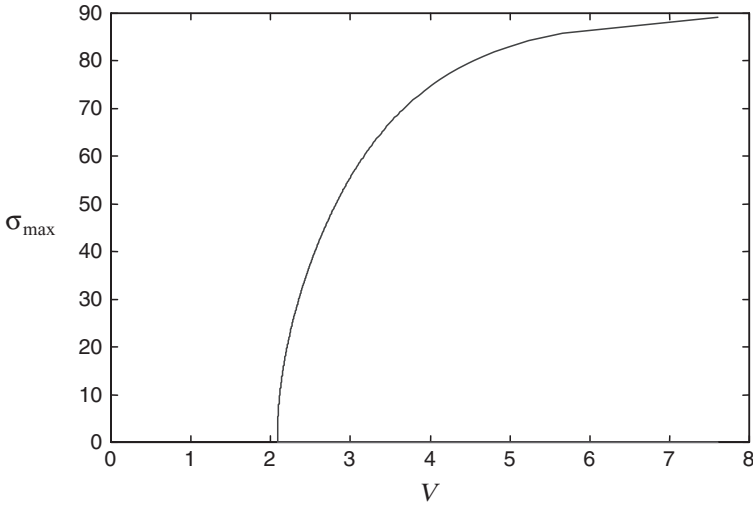
$$E^2 > E_{c3}^2 = \frac{k_3}{(\epsilon_{\perp} - \epsilon_{\parallel})} \left( \frac{\pi}{d} \right)^2 \quad (\text{VA cell}) \quad (1.2-33)$$

This is the critical field to create a distortion in a VA liquid crystal cell. Since the voltage across the cell is  $V = Ed$ , the critical voltage can be written as

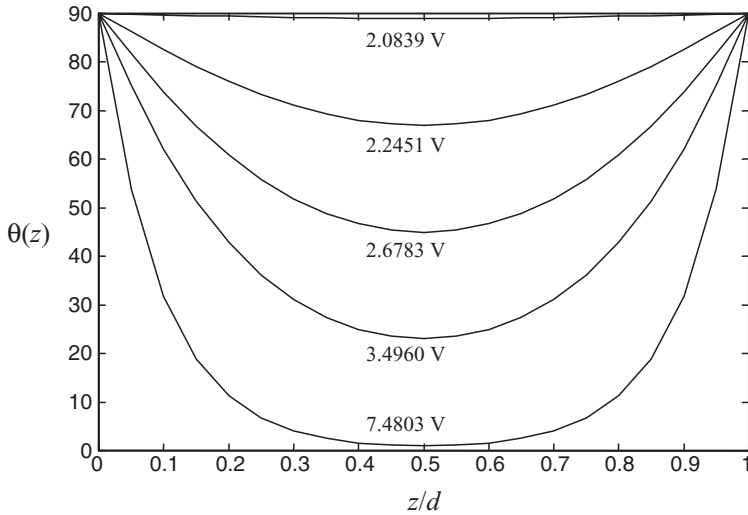
$$V_{c3} = \pi \frac{\sqrt{k_3}}{\sqrt{\epsilon_{\perp} - \epsilon_{\parallel}}} \quad (\text{VA cell}) \quad (1.2-34)$$

where  $k_3$  is the bend elastic constant. The critical voltage (or threshold voltage) is proportional to the square root of elastic constant  $k_3$  and is inversely proportional to the square root of dielectric anisotropy  $|\Delta\epsilon| = |\epsilon_{\parallel} - \epsilon_{\perp}|$ . A larger elastic constant  $k_3$  corresponds to a stiffer medium that requires a higher electrostatic field to distort the medium. Similarly, a larger anisotropy  $|\Delta\epsilon| = |\epsilon_{\parallel} - \epsilon_{\perp}|$  corresponds to a stronger electrostatic torque at a given applied field.

For a more general field-induced distortion, we need to solve Eq. (B-37) or Eq. (B-40) for the midlayer tilt angle, and Eq. (1.2-27) or Eq. (B-26) for the director distribution  $\theta(z)$ . These can be done only via numerical integration. The results are shown in Figures 1.15 and 1.16. Using the parameters of



**Figure 1.15.** Midlayer field-induced distortion of a VA cell (MLC-6886) with a cell gap of  $3\mu\text{m}$ , as a function of applied voltage.



**Figure 1.16.** Director distribution  $\theta(z)$  of a VA cell (MLC-6886) with a cell gap of  $3\mu\text{m}$ , at various applied voltages.

MLC-6886 in Table 1.4, we obtain a threshold voltage of  $2.0836\text{V}$ . Again, we find the field-induced distortion (in terms of midlayer rotation  $\sigma_{\max}$ ) is an increasing function of applied voltage. At a large applied voltage ( $V_{c3} \ll V$ ), the midlayer distortion  $\sigma_{\max}$  reaches a saturation value of  $\pi/2$ .

For small applied voltages above the threshold voltage, the director distribution  $\theta(z)$  follows a sinusoidal variation as given by Eqs. (1.2-28) and (1.2-31). The distortion is an increasing function of the applied voltage. At a large applied voltage ( $V_{c3} \ll V$ ), the director is turned almost  $90^\circ$ , especially around the midpoint of the VA cell.

A similar analysis yields the following critical voltages for IPS (In-Plane Switching) and TN cells:

$$V_{c2} = E_{c2}d = \pi \frac{\sqrt{k_2}}{\sqrt{\epsilon_\perp - \epsilon_\parallel}} \quad (\text{IPS cells})$$

$$V_T = E_{cT}d = V_{c1} \left\{ 1 + \frac{(k_3 - 2k_2)}{k_1} \left( \frac{\Phi}{\pi} \right)^2 \right\}^{1/2} \quad (\text{TN cells})$$

where  $k_2$  is the twist elastic constant;  $E_{c2}$  and  $E_{cT}$  are the critical fields needed to create a distortion of the director distribution in the liquid crystal cell;  $d$  is the cell gap of the liquid crystal cells; and  $\Phi$  is the total twist angle of the TN cell. Further discussions on the critical voltages of liquid crystal cells can be found in the appendices.

In what follows, we consider the dynamics of the molecular rotation under the influence of an applied field. This is important for the understanding of the rise time and decay time when the liquid crystal cell is switched on and off. The rise time and decay time play an essential role in the frame rate of LCDs.

### 1.2.8. Dynamics of Molecular Rotation—Rise Time and Decay Time

In the last section, we discussed the director distribution under the condition of static equilibrium. Here we consider the rotational motion of liquid crystal molecules under the influence of an applied electric field. The equation of motion for the rotation of the molecules can be written as

$$I \frac{d^2\theta}{dt^2} = (\tau_x)_{EM} + (\tau_x)_{EL} - \gamma \frac{d\theta}{dt} \quad (1.2-35)$$

where  $I$  is the density of moment of inertia of the liquid crystal molecules; the first two terms on the right side of the equation are the torques acting on the molecules; and the last term represents density of rotational viscosity torque with  $\gamma$  being the rotational viscosity coefficient (see Table 1.4). The viscosity torque is a material resistance (friction) to the rotation. If we assume negligible angular acceleration (the left-hand side), then the equation of motion can be written approximately as

$$\gamma \frac{d\theta}{dt} = (\tau_x)_{EM} + (\tau_x)_{EL} \quad (1.2-36)$$

Substituting Eqs. (1.2-15) and (1.2-17) for the torques into Eq. (1.2-36), the equation of motion becomes

$$\gamma \frac{d\theta}{dt} = (\varepsilon_{\parallel} - \varepsilon_{\perp}) E^2 \sin \theta \cos \theta + (k_3 - k_1) \sin \theta \cos \theta \left( \frac{d\theta}{dz} \right)^2 + (k_1 \cos^2 \theta + k_3 \sin^2 \theta) \frac{d^2 \theta}{dz^2} \quad (1.2-37)$$

This is a nonlinear differential equation of  $\theta(z, t)$ . It can only be solved numerically. For the purpose of investigating the rise time and decay time, we consider the following two cases of interest.

### Case I: Relaxation Time (Decay Time)

In this case, we assume a given initial director distribution  $\theta(z, t = 0)$  in a static equilibrium. This distortion is supported by a given applied field that balances the elastic restoring force. The tilt angle can be any of the curves in Figures 1.14 and 1.16. We then turn off the field suddenly. The director distribution will relax to its field-off distribution. The time evolution of the relaxation satisfies the following equation:

$$\gamma \frac{d}{dt} \theta(z, t) = (k_3 - k_1) \sin \theta \cos \theta \left( \frac{d\theta}{dz} \right)^2 + (k_1 \cos^2 \theta + k_3 \sin^2 \theta) \frac{d^2 \theta}{dz^2} \quad (1.2-38)$$

subject to the initial condition  $\theta(z, t = 0)$ . The solution of this equation again requires numerical integration. For the case of a small distortion in a parallel nematic cell with an initial distortion of

$$\theta(z, t = 0) = \theta_0 \sin \left( \frac{\pi}{d} z \right) \quad (1.2-39)$$

where  $\theta_0 \ll 1$  is a constant, the equation for the director relaxation can be written approximately as

$$\gamma \frac{d\theta}{dt} = -k_1 \left( \frac{\pi}{d} \right)^2 \theta \quad (1.2-40)$$

We note the distortion decays exponentially as

$$\theta(t) = \theta(0) \exp(-t/t_{\text{decay}}) \quad (1.2-41)$$

where decay time constant is given by

$$t_{\text{decay}} = \frac{\gamma}{k_1} \left( \frac{d}{\pi} \right)^2 = \frac{\gamma}{(\varepsilon_{\parallel} - \varepsilon_{\perp}) E_{c1}^2} \quad (\text{PA cell}) \quad (1.2-42)$$

where  $k_1$  is the splay elastic constant and  $E_{c1}$  is the critical field needed to create a finite splay distortion of the PA cell [see Eq. (1.2-25)]. We note that the decay time constant is proportional to the product of the rotational viscos-



ity and the square of the cell gap. For faster decay (higher frame rate), a small cell gap is often chosen for LCDs. The decay time constant is inversely proportional to the splay elastic constant  $k_1$ . A stiffer medium with a larger  $k_1$  corresponds to a faster relaxation of PA cells.

For VA cells with a small initial sinusoidal distortion,

$$\theta(z, t = 0) = \frac{\pi}{2} - \sigma(z, t = 0) = \frac{\pi}{2} - \sigma_0 \sin\left(\frac{\pi}{d} z\right) \quad (1.2-43)$$

where  $\sigma_0 \ll 1$  is a constant, the equation of motion for the director relaxation can be written approximately as

$$\gamma \frac{d\sigma}{dt} = -k_3 \left(\frac{\pi}{d}\right)^2 \sigma \quad (1.2-44)$$

We note the distortion in VA cell decays exponentially as

$$\sigma(t) = \sigma(0) \exp(-t/t_{\text{decay}}) \quad (1.2-45)$$

where decay time constant is given by

$$t_{\text{decay}} = \frac{\gamma}{k_3} \left(\frac{d}{\pi}\right)^2 = \frac{\gamma}{(\epsilon_{\parallel} - \epsilon_{\perp}) E_{c3}^2} \quad (\text{VA cell}) \quad (1.2-46)$$

where  $k_3$  is the bend elastic constant and  $E_{c3}$  is the critical field needed to create a finite bend distortion of the VA cell [see Eq. (1.2-33)]. We note that the decay time constant is proportional to the product of the rotational viscosity and the square of the cell gap. For faster decay (higher frame rate), a small cell gap is often chosen for LCDs. The decay time constant is inversely proportional to the bend elastic constant  $k_3$ . A stiffer medium with a larger  $k_3$  corresponds to a faster relaxation of VA cells.

The expressions for the decay time [Eqs. (1.2-42) and (1.2-46)] are derived by assuming a small distortion. In real displays, the liquid crystals are reoriented by a large angle close to  $90^\circ$ . The distortion actually includes both splay and bend in VA and PA cells. So, the actual decay time is likely to involve both the decay times [Eqs. (1.2-42) and (1.2-46)] and can be several times larger, as the molecules are rotated over a large range of angles (e.g., from  $85^\circ$  to  $5^\circ$  in a PA cell).

A similar analysis yields the following decay time constant for an IPS cell in which the liquid crystal molecules undergo twist reorientation,

$$t_{\text{decay}} = \frac{\gamma}{k_2} \left(\frac{d}{\pi}\right)^2 = \frac{\gamma}{(\epsilon_{\parallel} - \epsilon_{\perp}) E_{c2}^2} \quad (\text{IPS cell}) \quad (1.2-47)$$

where  $k_2$  is the twist elastic constant and  $E_{c2}$  is the critical field needed to create a finite twist distortion in the liquid crystal cell [see Eq. (C-19)]. We note that all three decay constants [Eqs. (1.2-42), (1.2-46), and (1.2-47)] have similar dependence on the cell gap, rotational viscosity, and the elastic constants.

A similar analysis yields the following decay time constant for a TN cell in which the liquid crystal molecules undergo both twist and tilt reorientation,

$$t_{\text{decay}} = \frac{\gamma}{k_1} \left( \frac{d}{\pi} \right)^2 \left\{ 1 + \frac{(k_3 - 2k_2)}{k_1} \left( \frac{\Phi}{\pi} \right)^2 \right\}^{-1} \quad (\text{TN cells}) \quad (1.2-48)$$

where  $\Phi$  is the total twist angle of the TN cell and  $k_1$ ,  $k_2$ , and  $k_3$  are the elastic constants. For TN cells with  $\Phi = \pi/2$ , the decay time becomes

$$t_{\text{decay}} = \frac{\gamma}{k_1} \left( \frac{d}{\pi} \right)^2 \left\{ 1 + \frac{(k_3 - 2k_2)}{4k_1} \right\}^{-1} \quad (\text{TN cells}) \quad (1.2-49)$$

We note that all decay constants [Eqs. (1.2-42), (1.2-46), (1.2-47), and (1.2-49)] have similar dependence on the cell gap, rotational viscosity, and the elastic constants. They are all proportional to  $\gamma d^2$ . So, small cell gaps are often employed to achieve fast time response.

The next case of interest is the rise time of the LCDs.

### Case II: Rise Time

For investigating the rise time, we examine the equation of motion by assuming an initially undistorted liquid crystal cell. The equation of motion is given by

$$\gamma \frac{d\theta}{dt} = (\epsilon_{\parallel} - \epsilon_{\perp}) E^2 \sin \theta \cos \theta + (k_3 - k_1) \sin \theta \cos \theta \left( \frac{d\theta}{dz} \right)^2 + (k_1 \cos^2 \theta + k_3 \sin^2 \theta) \frac{d^2 \theta}{dz^2} \quad (1.2-50)$$

subject to the initial condition of

$$\theta(z, t = 0) \approx 0 \quad (1.2-51)$$

for a PA cell. The solution of Eq. (1.2-50) with the initial condition of Eq. (1.2-51) again requires numerical integrations. For the case of small distortions, we may assume a solution of the form

$$\theta(z, t) = \theta_0(t) \sin \left( \frac{\pi}{d} x \right) \quad (1.2-52)$$

with  $\theta_0(t = 0) \ll 1$ . Under the circumstance, the equation of motion becomes approximately

$$\gamma \frac{d\theta}{dt} = (\varepsilon_{\parallel} - \varepsilon_{\perp}) E^2 \theta - k_1 \left( \frac{\pi}{d} \right)^2 \theta = (\varepsilon_{\parallel} - \varepsilon_{\perp}) (E^2 - E_{c1}^2) \theta \quad (1.2-53)$$

The distortion now grows exponentially as

$$\theta_0(t) = \theta_0(0) \exp(t/t_r) \quad (1.2-54)$$

where  $t_r$  is the rise time constant of the PA cell,

$$t_r = \frac{\gamma}{(\varepsilon_{\parallel} - \varepsilon_{\perp})(E^2 - E_{c1}^2)} = \frac{\gamma}{k_1} \left( \frac{d}{\pi} \right)^2 \frac{1}{\left( \frac{E^2}{E_{c1}^2} - 1 \right)} \approx \left( \frac{E_{c1}}{E} \right)^2 t_{\text{decay}} = \left( \frac{V_{c1}}{V} \right)^2 t_{\text{decay}} \quad (\text{PA cells}) \quad (1.2-55)$$

Since  $(E/E_{c1})^2$  is often much larger than 1, the rise time can be much shorter than the decay time. Both rise time and decay time are proportional to  $\gamma d^2$ . So, small cell gaps are often employed to achieve fast time response.

Similar analyses yield the following rise time constants for various liquid crystal cells:

$$t_r = \frac{\gamma}{(\varepsilon_{\parallel} - \varepsilon_{\perp})(E^2 - E_{c3}^2)} = \frac{\gamma}{k_3} \left( \frac{d}{\pi} \right)^2 \frac{1}{\left( \frac{E^2}{E_{c3}^2} - 1 \right)} \approx \left( \frac{E_{c3}}{E} \right)^2 t_{\text{decay}} = \left( \frac{V_{c3}}{V} \right)^2 t_{\text{decay}} \quad (\text{VA cells}) \quad (1.2-56)$$

$$t_r = \frac{\gamma}{(\varepsilon_{\parallel} - \varepsilon_{\perp})(E^2 - E_{c2}^2)} = \frac{1}{\left( \frac{E^2}{E_{c2}^2} - 1 \right)} \frac{\gamma}{k_2} \left( \frac{d}{\pi} \right)^2 \approx \frac{\gamma}{k_2} \left( \frac{d}{\pi} \right)^2 \left( \frac{E_{c2}}{E} \right)^2 \quad (\text{IPS}) \quad (1.2-57)$$

$$t_r = \frac{\gamma}{(\varepsilon_{\parallel} - \varepsilon_{\perp})(E^2 - E_{cT}^2)} = \frac{\gamma}{k_1} \left( \frac{d}{\pi} \right)^2 \left\{ 1 + \frac{k_3 - 2k_2}{k_1} \left( \frac{\Phi}{\pi} \right)^2 \right\}^{-1} \frac{1}{\left( \frac{E^2}{E_{cT}^2} - 1 \right)} \approx \left( \frac{V_T}{V} \right)^2 t_{\text{decay}} \quad (\text{TN cells}) \quad (1.2-58)$$

where we note that the approximate equality “ $\approx$ ” holds for the case when the driving field is much higher than the critical field and that  $V_T$  is the threshold voltage needed to create a finite distortion in a TN cell with zero pretilt [see Eq. (D-54) in Appendix D].

**Example 1.** Consider a PA cell made of a ZLI-1646 liquid crystal. The data are

$$n_e = 1.558, n_o = 1.478, \Delta n = 0.08, \varepsilon_{\parallel} = 10.6\varepsilon_0, \varepsilon_{\perp} = 4.6\varepsilon_0 \\ k_1 = 7.7 \text{ pN}, k_3 = 12.2 \text{ pN}, \gamma = 160 \text{ mPa}\cdot\text{s} = 0.16 \text{ Pa}\cdot\text{s} \quad (\text{MKS})$$

Assume a cell gap of half-wave retardation centered at  $\lambda = 550$  nm. The cell gap is given by  $d = \lambda/(2\Delta n) = 550 \text{ nm}/0.18 = 3.4 \mu\text{m}$ .

According to Eq. (1.2-26), the critical voltage is given by

$$V_{c1} = E_{c1}d = \pi \sqrt{\frac{k_1}{(\epsilon_{\parallel} - \epsilon_{\perp})}} = \pi \sqrt{\frac{7.7 \times 10^{-12}}{6.0 \times 8.854 \times 10^{-12}}} = 1.1961 \text{ V}$$

The decay time constant is, according to Eq. (1.2-42),

$$t_{\text{decay}} = \frac{\gamma}{k_1} \left( \frac{d}{\pi} \right)^2 = \frac{0.16}{7.7 \times 10^{-12}} \left( \frac{3.4 \times 10^{-6}}{3.14159} \right)^2 = 0.024 \text{ s} = 24 \text{ ms}$$

**Example 2.** Consider a VA cell made of MLC-6886 (a Merck liquid crystal mixture). The data are  $n_e = 1.570$ ,  $n_o = 1.480$ ,  $\Delta n = 0.09$ ,  $\epsilon_{\parallel} = 3.6\epsilon_0$ ,  $\epsilon_{\perp} = 7.4\epsilon_0$ ,  $k_1 = 13.8 \text{ pN}$ ,  $k_3 = 14.8 \text{ pN}$ , and  $\gamma = 146 \text{ mPa}\cdot\text{s} = 0.146 \text{ Pa}\cdot\text{s}$  (MKS).

Assume a cell gap of half-wave retardation centered at  $\lambda = 550$  nm. The cell gap is written as

$$d = \lambda/(2\Delta n) = 550 \text{ nm}/0.18 = 3.1 \mu\text{m}$$

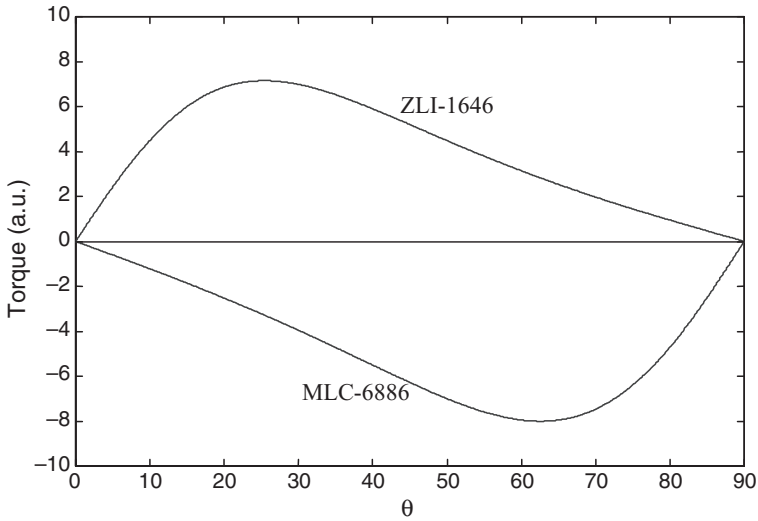
The critical voltage is, according to Eq. (1.2-34),

$$V_{c3} = E_{c3}d = \pi \sqrt{\frac{k_3}{(\epsilon_{\perp} - \epsilon_{\parallel})}} = \pi \sqrt{\frac{14.8 \times 10^{-12}}{3.8 \times 8.854 \times 10^{-12}}} = 2.08 \text{ V}$$

The decay time constant is, according to Eq. (1.2-46),

$$t_{\text{decay}} = \frac{\gamma}{k_3} \left( \frac{d}{\pi} \right)^2 = \frac{0.146}{14.8 \times 10^{-12}} \left( \frac{3.1 \times 10^{-6}}{3.14159} \right)^2 = 0.009 \text{ s} = 9 \text{ ms}$$

In the above discussion, we assume a small distortion for the sake of simplicity in introducing the decay time constant and the rise time constant. In actual LCDs, the angle of rotation can be much larger. The angular rotation at large distortion (e.g., around  $45^\circ$ ) can be much faster. This is illustrated as follows. We examine the electrostatic torque as given by Eqs. (1.2-14) and (1.2-15). According to Eq. (1.2-15), the electrostatic torque is maximum at  $\theta = 45^\circ$  for a given E field. However, it is important to note that the electric field E is actually a function of  $\theta$  at a given applied voltage. Figure 1.17 shows the electrostatic torque acting on the molecules (ZLI-1646 and MLC-6886) as functions of tilt angle  $\theta$ , according to Eq. (1.2-14) in which  $D_z$  is a constant. We notice that the electrostatic torques are relatively small at small distortions. For ZLI-1646 (with  $\Delta\epsilon = 6\epsilon_0 > 0$ ), maximum torque occurs at a tilt angle of around  $25^\circ$ . For MLC-6886 (with  $\Delta\epsilon = -3.8\epsilon_0 < 0$ ), maximum torque occurs at a tilt angle of around  $63^\circ$ . The speed of the angular rotation of molecules



**Figure 1.17.** Electrostatic torque in arbitrary unit as function of tilt angle  $\theta$  (measured from the  $xy$ -plane) at a given applied voltage.

is proportional to the torque acting on them in the presence of viscosity resistance. So, the molecules are actually rotating faster at these large tilt angles. The same argument applies to the decay time constant when the field is suddenly turned off. Before the field is turned off, the elastic torque is exactly the same as the electrostatic torque in the equilibrium state. So, at large tilt angles (e.g., around  $45^\circ$ ), the elastic restoring torques are also larger. They basically follow the same angular dependence as those of the electrostatic torques, except with an opposite sign. So, the relaxation rotational motions of molecular reorientation are also much faster at these large tilt angles. In the extreme orientations (e.g., around  $0^\circ$  and  $90^\circ$ ), the angular rotations are relatively slow due to the small electrostatic torque and elastic torque at these orientations. Since the rise time and decay time are the integrated time duration of the overall rotation, they are limited by the time needed for rotation at these extreme orientations. Therefore, the actual time needed for the rotation in LCDs is likely to be a multiple of the sum of the decay time and the rise time.

Based on Eqs. (1.2-55)–(1.2-58), the rise time can be significantly faster by using higher driving voltages. It is important to note that these equations are derived by assuming a small distortion. The actual rise times needed for the rotation in LCDs are likely to be a multiple of the predicted rise times. In the case when the driving voltages are much higher than the critical voltages, the equation of motion can be written, according to Eq. (1.2-50), as

$$\gamma \frac{d\theta}{dt} = (\epsilon_{\parallel} - \epsilon_{\perp}) E^2 \sin \theta \cos \theta \quad (1.2-59)$$

Notice that we have neglected the elastic restoring torque, which is much less than the electrostatic torque under the condition of high driving voltages. Equation (1.2-59) can be integrated to find the time needed to rotate from an initial tilt angle of  $\theta_{\text{initial}}$  to a final tilt angle of  $\theta_{\text{final}}$ . The result is

$$\Delta t = \frac{\gamma}{\Delta \epsilon E^2} \log \left| \frac{\tan \theta_{\text{final}}}{\tan \theta_{\text{initial}}} \right| \quad (1.2-60)$$

In arriving at Eq. (1.2-60), we assumed a given electric field. A more rigorous approach would require the use of Eq. (1.2-14) for the electrostatic torque (see Problem 1.8). An integration yields a similar result. For practical displays (e.g.,  $\theta_{\text{initial}} = 5^\circ$ ,  $\theta_{\text{final}} = 85^\circ$ ), the log term in Eq. (1.2-60) is about 5. The switching time is indeed inversely proportional to the square of driving field. In terms of the critical voltages, Eq. (1.2-60) can be written as

$$\Delta t = \begin{cases} \left( \frac{V_{c1}}{V} \right) t_{\text{decay}} \log \left| \frac{\tan \theta_{\text{final}}}{\tan \theta_{\text{initial}}} \right| & \text{(parallel cell)} \\ \left( \frac{V_{c3}}{V} \right) t_{\text{decay}} \log \left| \frac{\tan \theta_{\text{initial}}}{\tan \theta_{\text{final}}} \right| & \text{(vertical cell)} \end{cases} \quad (1.2-61)$$

In conventional displays, the rise times can be relatively fast via the use of higher addressing voltages. The decay times via natural relaxation, according to Eqs. (1.2-42) and (1.2-46)–(1.2-48), are relatively slow. To achieve a faster decay, we will need to employ an external field to force the molecules to reorient themselves back to the initial state. This is possible via the use of the dual-frequency materials discussed earlier. To illustrate this, we consider a PA liquid crystal cell containing a mixture MLC-2048 with  $\Delta \epsilon(1\text{kHz}) = 3.2\epsilon_0$  and  $\Delta \epsilon(50\text{kHz}) = -3.1\epsilon_0$  [21]. Initially, the liquid crystal molecules are parallel aligned with a tilt angle of about  $5^\circ$ . The molecules can be rapidly switched to a tilt angle of  $85^\circ$  with a high-voltage driving field at 1kHz. Once the molecular tilt angles reach  $85^\circ$ , the distortion can be sustained via a low-voltage field at the same frequency. The molecules can be switched back to the tilt angle of  $5^\circ$  via a high-voltage driving field at 50kHz. At this frequency, the liquid crystal becomes a negative dielectric medium. So, the molecules are forced to align parallel to the surface to reach a minimum energy. In this case, the molecules are switched via a high-voltage driving field at different frequencies. Both steps enjoy the speed enhancement factor of  $(V/V_{\text{critical}})^2$ . With a driving voltage of  $10V_{\text{critical}}$ , this enhancement factor can be 100. The higher driving voltage may not require higher driving electrical energy, as the switching time is reduced by the same factor.

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## PROBLEMS

- 1.1. (a) Show that 50% of the energy of a beam of unpolarized light transmits through an ideal polarizer regardless of the orientation of the polarizer.
- (b) As discussed in this chapter, the transmitted intensity through two polarizers in series is given by  $\frac{1}{2} \cos^2 \theta$ . Show that the fractional error in the intensity measurement due to an error in  $\Delta\theta$  is given by  $-2\Delta\theta \tan \theta$ .
- 1.2. *Polarizers in series:*
  - (a) It is known that a pair of crossed polarizers can stop the transmission of light. Show that optical transmission occurs when a third polarizer is inserted between the crossed polarizers with a transmission axis oriented at  $45^\circ$  with respect to those of the crossed polarizers. Find the transmission for unpolarized light (assuming the orientations of the crossed polarizers are at  $0^\circ$  and  $90^\circ$ ).



- (b) Consider the insertion of two polarizers oriented at  $30^\circ$  and  $60^\circ$  between the two crossed polarizers. Find the transmission for unpolarized light.
  - (c) By generalizing (a) and (b), we consider the insertion of  $N$  polarizers oriented at equal angular separations. Show that the transmission of unpolarized light approaches  $\frac{1}{2}$  when  $N$  tends to infinity. This is a situation when the electric field vector follows the direction of the transmission axis of the polarizers as the light propagates through the whole set of polarizers in series.
- 1.3. Derive Eqs. (1.2-4) and (1.2-5).
- 1.4. Let  $\alpha_\perp$  and  $\alpha_\parallel$  be the molecular polarizability of a liquid crystal molecule with  $\alpha_\perp < \alpha_\parallel$ . Under the application of an external electric field, the induced dipole moment of the liquid crystal molecule can be written as

$$p_\perp = \alpha_\perp E_\perp$$

$$p_\parallel = \alpha_\parallel E_\parallel$$

- (a) Show that, in general, the dipole moment is not parallel to the applied electric field.
  - (b) Show that a net torque is exerted on the molecule that tends to turn the molecule along the direction of the electric field. Evaluate the torque in terms of the polarizability.
  - (c) Show that the torque discussed in (b) tends to minimize the electrostatic energy in Eq. (1.2-5).
- 1.5. *Elastic energy density and electrostatic energy density:*
- (a) Consider a uniformly TN-LC with a director given by

$$\mathbf{n} = (\sin \phi, \cos \phi, 0)$$

where  $\phi$  is the twist angle given by  $\phi = \alpha z$ . Show that the twist elastic energy density is given by

$$U_t = \frac{1}{2} k_2 \alpha^2$$

- Evaluate the energy density in terms of  $\text{J/m}^3$  for the case of a  $5\text{-}\mu\text{m}$  cell of ZLI-1646 liquid crystal with a  $90^\circ$  twist. (Use  $k_2 = 4 \times 10^{-12} \text{N}$ .)
- (b) Using Eq. (1.2-4) or Eq. (1.2-5), show that the difference between the electrostatic energy density at  $\theta = 0$  and  $\pi/2$  can be written as

$$\Delta U = \frac{1}{2} \Delta \epsilon E^2$$

Evaluate  $\Delta U$  in terms of  $\text{J/m}^3$  for the case of a 5- $\mu\text{m}$  cell of ZLI-1646 liquid crystal with an applied voltage of 1 V. Show that the electrostatic energy density is much larger than the twist elastic energy density. (Use  $\Delta \epsilon = 6\epsilon_0$ .) This example illustrates the advantages of liquid crystals that can be easily reoriented by applying a small voltage.

- 1.6. Prove that in Eq. (1.2-8) the bend term can also be written as  $\frac{1}{2} k_3 (\mathbf{n} \cdot \nabla \mathbf{n})^2$ , in other words,  $(\mathbf{n} \times \nabla \times \mathbf{n})^2 = (\mathbf{n} \cdot \nabla \mathbf{n})^2$ .
- 1.7. *Director distribution in a parallel cell due to pretilt:*
- (a) Show that the elastic energy density due to a tilt angle distribution  $\theta(z)$  is given by

$$U_{EL} = \frac{1}{2} (k_1 \cos^2 \theta + k_3 \sin^2 \theta) \left( \frac{d\theta}{dz} \right)^2$$

where  $k_1$  and  $k_3$  are elastic constants.

- (b) Using variational calculus, show that minimum elastic energy occurs when

$$U_{EL} = \frac{1}{2} (k_1 \cos^2 \theta + k_3 \sin^2 \theta) \left( \frac{d\theta}{dz} \right)^2 = \text{constant}$$

The absolute minimum occurs when

$$\theta = \text{constant}$$

This is the case shown in Figure 1.9a.

- (c) Show that, in general, the tilt angle distribution  $\theta(z)$  for minimum elastic energy is given by

$$\frac{d\theta}{dz} = \frac{s\sqrt{k_1}}{\sqrt{k_1 \cos^2 \theta + k_3 \sin^2 \theta}}$$

where  $s$  is a constant. In the antisymmetrical case when  $\theta(0) = \theta_0$ ,  $\theta(d) = -\theta_0$ , the tilt angle is zero at midlayer ( $z = d/2$ ). In this case,  $s$  is the slope  $d\theta/dz$  at midlayer.

- (d) Integrate the equation in (c) and obtain an expression for  $s$ . Show that for small pretilt angle ( $\theta_0 \ll 1$ ), the slope can be written as

$$s = -\frac{2\theta_0}{d}$$

and the tilt angle distribution  $\theta(z)$  can be written as

$$\theta(z) = \theta_0 + sz$$

- (e) In a Pi-cell, the distribution  $\theta(z)$  for minimum elastic energy is again given by

$$\frac{d\theta}{dz} = \frac{s\sqrt{k_1}}{\sqrt{k_1 \cos^2 \theta + k_3 \sin^2 \theta}}$$

where  $s$  is a constant. In this case,  $\theta(0) = \theta_0$ ,  $\theta(d) = \pi - \theta_0$ , and the tilt angle is  $\pi/2$  at midlayer ( $z = d/2$ ). In this case,  $s$  is proportional to the slope  $d\theta/dz$  at midlayer. Show that

$$s = \sqrt{\frac{k_3}{k_1}} \frac{d\theta}{dz}$$

- (f) Explain why the splay cell has a lower elastic energy for cells with small pretilt angles.

1.8. *Molecular rotation with electrostatic torque:*

Consider the situation when the elastic torque is small compared with the electrostatic torque. This can happen when the applied voltage is much higher than the critical voltage, and when the equilibrium state is not reached. The equation of motion can be written as

$$\gamma \frac{d\theta}{dt} = \frac{D_z^2(\epsilon_{\parallel} - \epsilon_{\perp})}{(\epsilon_{\parallel} \sin^2 \theta + \epsilon_{\perp} \cos^2 \theta)^2} \sin \theta \cos \theta$$

where  $D_z$  is independent of  $\theta$ . Let  $x = \sin \theta$ , the equation of motion can be written as

$$\gamma \frac{dx}{dt} = \frac{D_z^2(\epsilon_{\parallel} - \epsilon_{\perp})}{[\epsilon_{\parallel} x^2 + \epsilon_{\perp}(1 - x^2)]^2} x(1 - x^2)$$

- (a) Show that the equation can be rewritten as

$$\left[ \epsilon_{\parallel}^2 \frac{x^3}{1 - x^2} + \epsilon_{\perp}^2 \frac{1 - x^2}{x} + 2\epsilon_{\parallel}\epsilon_{\perp}x \right] \gamma dx = D_z^2(\epsilon_{\parallel} - \epsilon_{\perp}) dt$$

- (b) Show that

$$-\epsilon_{\parallel}^2 \log \cos \theta + \epsilon_{\perp}^2 \log \sin \theta - \frac{1}{2}(\epsilon_{\parallel} - \epsilon_{\perp})^2 \sin^2 \theta = \frac{D_z^2(\epsilon_{\parallel} - \epsilon_{\perp})}{\gamma} t + \text{constant}$$

(c) Show that the time needed for rotation from  $\theta_{\text{initial}}$  to  $\theta_{\text{final}}$  is

$$\frac{D_z^2(\epsilon_{\parallel} - \epsilon_{\perp})}{\gamma} \Delta T = -\epsilon_{\parallel}^2 \log \frac{\cos \theta_{\text{final}}}{\cos \theta_{\text{initial}}} + \epsilon_{\perp}^2 \log \frac{\sin \theta_{\text{final}}}{\sin \theta_{\text{initial}}} - \frac{1}{2}(\epsilon_{\parallel} - \epsilon_{\perp})^2 (\sin^2 \theta_{\text{final}} - \sin^2 \theta_{\text{initial}})$$

(d) For high voltages in a PA cell, show that  $D_z$  can be written approximately as

$$D_z = \epsilon_{\parallel} V / d$$

(e) Show that

$$\Delta T = \left[ -\log \frac{\cos \theta_{\text{final}}}{\cos \theta_{\text{initial}}} + \frac{\epsilon_{\perp}^2}{\epsilon_{\parallel}^2} \log \frac{\sin \theta_{\text{final}}}{\sin \theta_{\text{initial}}} - \frac{1}{2} \left( 1 - \frac{\epsilon_{\perp}}{\epsilon_{\parallel}} \right)^2 (\sin^2 \theta_{\text{final}} - \sin^2 \theta_{\text{initial}}) \right] \left( \frac{V_{\text{critical}}}{V} \right)^2 t_{\text{decay}}$$

(f) For ZLI-1646, with  $\epsilon_{\parallel} = 10.6\epsilon_0$ ,  $\epsilon_{\perp} = 4.6\epsilon_0$ ,  $\theta_{\text{initial}} = 5^\circ$ , and  $\theta_{\text{final}} = 85^\circ$ , show that

$$\Delta T = 2.74 \left( \frac{V_{\text{critical}}}{V} \right)^2 t_{\text{decay}}$$

(g) For high voltages in a VA cell, show that  $D_z$  can be written approximately as

$$D_z = \epsilon_{\perp} V / d$$

(h) Show that

$$\Delta T = \left[ -\frac{\epsilon_{\parallel}^2}{\epsilon_{\perp}^2} \log \frac{\cos \theta_{\text{final}}}{\cos \theta_{\text{initial}}} + \log \frac{\sin \theta_{\text{final}}}{\sin \theta_{\text{initial}}} - \frac{1}{2} \left( \frac{\epsilon_{\parallel}}{\epsilon_{\perp}} - 1 \right)^2 (\sin^2 \theta_{\text{final}} - \sin^2 \theta_{\text{initial}}) \right] \left( \frac{V_{\text{critical}}}{V} \right)^2 t_{\text{decay}}$$

(i) For MLC-6886, with  $\epsilon_{\parallel} = 3.6\epsilon_0$ ,  $\epsilon_{\perp} = 7.4\epsilon_0$ ,  $\theta_{\text{initial}} = 5^\circ$ , and  $\theta_{\text{final}} = 85^\circ$ , show that

$$\Delta T = 2.88 \left( \frac{V_{\text{critical}}}{V} \right)^2 t_{\text{decay}}$$

1.9. *Displacement field in a liquid crystal cell:*

Consider a liquid crystal cell that is uniform in the  $xy$ -plane. The dielectric tensor  $\epsilon$  of the liquid crystal is a function of  $z$  only. The liquid crystal is sandwiched between a pair of conducting electrodes.

- (a) Using  $\nabla \cdot \mathbf{D} = 0$ , show that  $\mathbf{D}$  is a constant.
- (b) Show that  $E_x = E_y = 0$  for a liquid crystal cell that is infinite in the  $xy$ -plane.
- (c) For a liquid crystal with a director oriented at a tilt angle of  $\theta$  (measured from the  $xy$ -plane), show that

$$D_z = (\epsilon_{\parallel} \sin^2 \theta + \epsilon_{\perp} \cos^2 \theta)E$$

- (d) Show that the electrostatic energy density [Eqs. (A-18) and (A-19)] can be written as Eq. (1.2-11).

1.10. *Lambertian cosine law:*

The black body radiation from a surface element  $dA$  can be modeled as a small hole of area  $dA$  in the wall of a cavity at thermal equilibrium. Let  $U$  be the energy density of the black body radiation within the cavity.

- (a) Show that the energy flux ( $\text{W/m}^2$ ) arriving from direction  $\theta$  at the hole in a solid angle  $d\Omega$  is

$$cU \cos \theta \frac{d\Omega}{4\pi}$$

where  $\theta$  is the angle between the surface normal and the direction of the incoming energy flux. So, the energy flux (or intensity in units of  $\text{watts/m}^2$ ) emitted from the surface element  $dA$  can be written as

$$I(\theta) = I_0 \cos \theta$$

where  $I_0$  is the intensity emitted in the normal direction. This is the Lambertian cosine law.

- (b) Show that for a viewer in the direction  $\theta$  from normal, the surface element  $dA$  appears to have an apparent size of  $dA \cos \theta$ . In other words, the solid angle extended by the surface element  $dA$  is decreased by a factor of  $\cos \theta$ . Show that the luminance appears to be independent of the viewing angle.