chapter 1

The Basics

Algebra is a very logical way to solve problems—both theoretically and practically. You need to know a number of things. You already know arithmetic of whole numbers. You will review the various properties of numbers, as well as using powers and exponents, fractions, decimals and percents, and square and cube roots. I am a firm believer in a picture being worth a thousand words, so I illustrate anything it's possible to illustrate and make extensive use of color. Each chapter concludes with practice exercises to help you to reinforce your skills. The solutions to these exercises can be found at the end of the chapter, just before the Glossary.

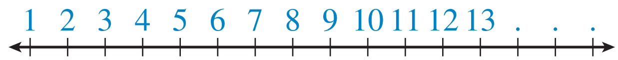
The most practical chapter in this book is undoubtedly the last one, because it brings together all the skills covered in earlier chapters and helps you put them to work solving practical word problems. You may not believe me right now, but algebra is all about solving word problems—some of them very practical problems, such as, "How much tax will I pay on a purchase? How big a discount is 35% off of something already selling for 25% off? And how good is the gas mileage my motor vehicle is getting?"

Groups of Numbers

Before beginning to review or to learn algebra, it is important to feel comfortable with some pre-algebra concepts including the various groups or realms of numbers with which you will work and the commonly used mathematical symbols and conventions. The first ones you look at are the various families of numbers.

COUNTING NUMBERS

Counting numbers are also known as *natural numbers*. You use these numbers to count things. In the time of cave people, counting numbers probably consisted of one, two, and many and were used to describe how many woolly mammoths had just gone by. As people settled into lives based on agriculture, the need for more specifics arose, so more modern folks got the familiar 1, 2, 3, 4, 5, and so on, as shown here.



Counting Numbers

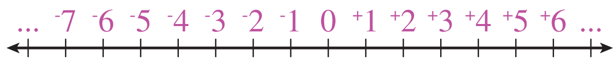
WHOLE NUMBERS

Notice the two arrowheads at either end of the Counting Numbers line. They indicate that the line continues for an infinite length in both directions—infinite meaning "without end." Adding a 0 to the left of the 1 takes you into the realm of **whole numbers**. Whole numbers are not very different from counting numbers. As the number line shows, Whole Numbers = Counting Numbers + 0.

Whole Numbers

INTEGERS

Integers are the next realm. **Integers** are numbers into which 1 can be divided without any remainder being left. Whether or not you've ever done algebra before, you have come across positive and negative values. You've seen them on a thermometer. The temperature inside your house should always be positive, and around 68 degrees, but in the winter, the reported outside temperature in Alaska (or outside your home) often falls below 0. Those are also known as *negative temperatures.* Similarly, the stock market may close up or down on any given day. If it makes gains for the day, it's positive; if the market loses value, it's negative.



Integers

Notice that every integer has a sign attached to it except for the integer 0. Zero is neither positive nor negative, but separates one group from the other. Additionally, notice that +4 is exactly the same distance from 0 on its right as –4 is on its left.

Groups of Numbers *(continued)*

RATIONAL NUMBERS

You might think from the name that rational numbers are numbers that make sense—and of course, they do—but that's not why they're called rational numbers. A **ratio**, from which the name comes, is a comparison between two quantities.

Say that you have two automobiles, but your neighbor has three. Then the ratio of your cars to your neighbor's is $\frac{2}{3}$. The ratio of your neighbor's cars to yours, on the other hand, is $\frac{3}{2}$.

If you're thinking "I've seen those before, but I called them 'fractions,'" you're absolutely right. A ratio is a form of fraction. Any number that can be expressed as a fraction is a **rational number**. That includes terminating decimal fractions, such as 0.25; percent fractions, such as 25% (each of which could be expressed as $\frac{1}{4}$); repeating decimals; and any integer (since 5 can be expressed as a fraction: $\frac{5}{1}$). So the realm of rational numbers includes all realms that have come before—including $0\left(\frac{0}{8} = 0\right)$. You will examine ratios and rational numbers much more fully later.

Just for the record, an example of a repeating decimal is the fraction $\frac{1}{6}$. When it is expressed as a decimal, it is 0.16666666..., which never stops repeating. For simplification, you put a bar over the number that repeats and simplify it to $0.1\overline{6}$. I have not attempted to include a number line for rational numbers for a very good reason: An infinite quantity of rational numbers exists between any two integers.

IRRATIONAL NUMBERS

Don't believe that irrational numbers are so angry that they are unable to think rationally. **Irrational numbers** come about when a number cannot be expressed as either an integer or a rational number. Two examples of irrational numbers are $\sqrt{2}$ (the square root of 2) and π (pi). The value of π has been worked out by computer to be 3 followed by 256 or more decimal places with no repeat occurring. As was the case with rational numbers, an infinite quantity of irrational numbers can fit between any two integers.

PRIME NUMBERS

A **prime number** is a number that has *exactly* two factors, itself and 1. (**Factors** is a name given to numbers that are multiplied together.) The first prime number is 2, with factors of 1 and 2. Next is 3, with factors of 1 and 3. Can you think of any other numbers you could multiply together to make 2 or 3? What are the next two prime numbers? Four is not prime, since it has three factors: 1, 2, and 4. In fact, 2 is the only even prime number. (We examine that further in a paragraph or so.) The next two primes after 3 are 5 and 7. Then come 11, 13, 17, 19, 23, 29, 31, 37, . . .

Why is it important to know prime numbers? The best answer I can give is it's a timesaver—especially when simplifying fractions. If you recognize 19 as a factor of a fraction, and you realize that 19 is a prime number, then you won't waste time trying to simplify it further.

EVEN NUMBERS

Every second number is an **even** one: 2, 4, 6, 8, 10, . . . Counting by 2s is something you probably could do even before you could add or subtract. Any number with a 2, 4, 6, 8, or 0 in its ones place is an even number. Note also that all even numbers contain 2 as a factor. That's why no even number except the first can be prime. Also note that 0 acts like an even number. The realm of even numbers is infinite.

ODD NUMBERS

Every second number is an **odd** one. Huh?! Didn't I just say that every second number is an even one? Strange as it may seem, that's not a contradiction. The only difference is the starting point. To name even numbers you start with 2; you start naming odd numbers with 1. If the ones place of a number contains a 1, 3, 5, 7, or 9, it's an odd number. The main feature of odd numbers is that dividing them by 2 will *not* result in a whole number.

Ways of Showing Things in Algebra

SHOWING MULTIPLICATION

Prior to algebra, multiplication has always been shown with a $\times$ sign. In algebra there are many different ways to show multiplication, but $\times$ is not one of them.

One of the more popular ways to indicate multiplication is with a **multiplication dot**, as shown at right.

$$2 \cdot 3 = 2 \times 3$$

There are also three different ways (see right) to show multiplication of two numbers using parentheses.

$$(3)(5) = 15$$
$$3(5) = 15$$
$$(3)5 = 15$$

We haven't yet studied variables, but I'm sure it's no secret to you that algebra often uses letters to stand for numbers.

Let's use the variables a, b, and c to stand for three different numbers. We could multiply these variable numbers by constant numbers in any of the ways shown above, or we could use the following notations:

$3a$ means 3 times a.

$4b$ means 4 times b.

$5c$ means 5 times c.

We could also use ac to indicate a times c, ab to indicate a times b, or bc to indicate b times c.

Do you think we could show 2 times 3 by writing 23? Sorry, but that configuration is already known as the quantity twenty-three.

COMMON MATH SYMBOLS

Certain mathematical symbols are used throughout this book. It is important that you are familiar with and feel comfortable with them.

Common Math Symbols	
Symbol	**Meaning**
=	is equal to
≠	is not equal to
; or ≈	is approximately equal to
>	is greater than
≥	is greater than or equal to
<	is less than
≤	is less than or equal to

The last four symbols above are the ones with which people often have trouble. I know of some people who remember them as representing alligator jaws, always ready to snap on the larger of the two quantities. In the case of the two types of greater-than symbols, the number on the left is greater; for the two less-than symbols, the greater quantity is on the right, so reading from left to right, the first number is less than the second. A number that is greater than or less than another is a group of numbers that excludes both named quantities from the group. A number that is greater than or equal to a number includes the lower number in the group. One that is less than or equal to a number includes the upper number in the group.

GROUPING SYMBOLS

When writing algebraic expressions and or mathematical sentences (which you'll learn more about in Chapter 4), it is often essential to group certain numbers together, either for the sake of clarity, or in order to specify the sequence in which those numbers should be operated upon. Certain symbols and rules govern these groupings.

Ways of Showing Things in Algebra *(continued)*

PARENTHESES

The most commonly used grouping symbols are **parentheses** (). The equation at the right is a true statement, as you'll see if you solve what's in parentheses on both sides of the = sign:

$$(4 + 3) = (11 - 4)$$
$$7 = 7$$

BRACKETS

When it is necessary to group numbers in parentheses with another group of numbers in parentheses, you use **brackets** [ ]. When you use grouping symbols, you must first clear them by combining the innermost numbers, then working your way outward. In the sentence (below, right) the innermost numbers are $(3 + 4)$ and $(2 + 2)$.

Is the equation a true statement? By combining the numbers in the parentheses you get: $3[7 - 4] = 9$. Next, you perform the indicated subtraction, to get $3[3] = 9$. Brackets perform the same function as parentheses, so $3[3]$ means the same thing as $3(3)$, which you should recognize as indicating multiplication. That leaves your fully interpreted mathematical sentence to read $9 = 9$. Thus, $3[(3 + 4) - (2 + 2)] = 9$ has proven to be a true statement.

$$3[(3 + 4) - (2 + 2)] = 9$$
$$3[7 - 4] = 9$$
$$3[3] = 9$$
$$9 = 9$$

BRACES

Sometimes, although rarely, it is necessary to group numbers together that already are in bracketed parentheses. Then we use grouping symbols known as **braces** { }. Consider the equation at the right.

Let's find out whether this mathematical sentence is true or not. As always, start inside the parentheses first, then move to the brackets, and finally the braces.

First solve $(6 - 4)$ and $(7 - 5)$. Next, add the 2s. Then combine the $3 + 4$. Finally, multiply.

$$6\{3 + [(6 - 4) + (7 - 5)]\} = 42$$
$$6\{3 + [2 + 2]\} = 42$$
$$6\{3 + 4\} = 42$$
$$6\{7\} = 42$$
$$42 = 42$$

Remember, the order of use is always parentheses, then brackets, then braces: {[( )]}. Sometimes larger parentheses are used as well:

({[( )]})

Properties and Elements

There are certain properties that apply to addition and multiplication but do not apply to subtraction or division. You probably came across them in elementary school math, but they are unlikely to be fresh in your memory, so let's review them here.

COMMUTATIVE PROPERTY

The **commutative property** applies to addition and multiplication. It deals with order. Formally stated, it says that when two numbers are added together or when two numbers are multiplied together, the order in which they are added or multiplied does not affect the result. Here's a shorter statement:

The commutative property for addition: $(a + b) = (b + a) = a + b$
The commutative property for multiplication: $(a \cdot b) = (b \cdot a) = ab$

At this point in your algebra career, you might better understand it as follows:

The commutative property for addition: $(5 + 3) = (3 + 5) = 8$
The commutative property for multiplication: $(5 \cdot 3) = (3 \cdot 5) = 15$

ASSOCIATIVE PROPERTY

The **associative property** also applies to addition and multiplication. It deals with grouping. In order to understand it, you must first understand that all arithmetic operations are **binary**. That means that you can operate on only two numbers at any one time. This may seem to fly in the face of what you learned about column addition. Remember the columns of four or five numbers, say 3 + 5 + 4 + 9 + 8 all stacked up over a line segment, as shown here:

$$\begin{array}{c} 3 \\ 5 \\ 4 \\ 9 \\ \underline{8} \end{array}$$

Properties and Elements *(continued)*

Well, the fact of the matter is, you never added more than two of those numbers together at a time. If you tried adding that column right now, you'd probably go $3 + 5 = 8$; $8 + 4 = 12$; $12 + 9 = 21$; $21 + 8 = 29$. No contradiction there! You add numbers two at a time, no matter how many there are to add, which brings you back to the associative property for both addition and multiplication. They say that when three (or more) numbers are to be added together or when three or more numbers are multiplied together, the order in which they are grouped for addition or multiplication does not affect the result.

The associative property for addition: $(a + b) + c = a + (b + c) = a + b + c$

The associative property for multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c) = abc$

That may also be shown as:

The associative property for addition: $(5 + 3) + 2 = 5 + (3 + 2) = 10$

The associative property for multiplication: $(5 \cdot 3) \cdot 2 = 5 \cdot (3 \cdot 2) = 30$

Remember, the commutative property deals with order, the associative property deals with grouping.

IDENTITY ELEMENTS

The **identity element** is the number that may be combined with another number without changing its value. For addition, the identity element is 0.

$0 + 3 = 3$

$n + 0 = n$

What do you suppose the identity element is for multiplication? That's right, it's 1. It's demonstrated here:

$5 \cdot 1 = 5$

$1 \cdot y = y$

For subtraction, the identity element is once again 0.

$7 - 0 = 7$

$n - 0 = n$

For division, the undoing of multiplication, the identity element is once again 1. Remember, the fraction line symbolizes division.

$\frac{5}{1} = 5$

$\frac{g}{1} = g$

DISTRIBUTIVE PROPERTY

The **distributive property** is used to remove grouping symbols (usually parentheses) by sharing a multiplier outside the grouped numbers with those on the inside. It always involves numbers that are added together or subtracted within the grouping symbols. For that reason, it is often referred to as the **distributive property of multiplication over addition**.

In the first example at the right, notice that the 3 was distributed.

$3(5) + 3(2) = 15 + 6$

The second example involves subtraction.

$4(6) - 4(3) = 24 - 12$

Exponents and Powers

An exponent is a number written small to the right and high off the line next to another number. In 2^3, 3 is the exponent, and 2 is the base. An exponent expresses the power to which a number is to be raised (or lowered). (I discuss the lowering power of certain exponents in a later chapter.)

What They Do

An exponent after a number tells the number of times the base number is to be multiplied by itself (see examples at right).

$$4^2 = 4 \cdot 4$$
$$2^3 = 2 \cdot 2 \cdot 2$$
$$6^4 = 6 \cdot 6 \cdot 6 \cdot 6$$

Many would argue that an exponent and a power are the same thing, but they really are not. An exponent is a symbol for the power (how many of the base number are multiplied together). The first example would be read *four to the second power*, or *four squared* (more about that in a moment). The second depicts *two to the third power*, or *two cubed* (more about that in a moment, as well). The third shows *six to the fourth power*.

The exponents 2 and 3 have special names, based upon a plane and a solid geometric figure. Because the square's area is found by multiplying one side by itself, raising something to the second power is called **squaring**. Since the volume of a cube is found by multiplying one side by itself and by itself again, raising something to the third power is called **cubing**. These terms are in common use and chances are that you've heard them before. A list of the first 12 **perfect squares** (squares of whole numbers) is shown at the right.

$1^2 = 1$	$5^2 = 25$	$9^2 = 81$
$2^2 = 4$	$6^2 = 36$	$10^2 = 100$
$3^2 = 9$	$7^2 = 49$	$11^2 = 121$
$4^2 = 16$	$8^2 = 64$	$12^2 = 144$

To cube a number, multiply it by itself and then do it again. A list of the first 12 **perfect cubes** (cubes of whole numbers) is shown at the right.

$1^3 = 1$	$5^3 = 125$	$9^3 = 729$
$2^3 = 8$	$6^3 = 216$	$10^3 = 1000$
$3^3 = 27$	$7^3 = 343$	$11^3 = 1331$
$4^3 = 64$	$8^3 = 512$	$12^3 = 1728$

Comparing squares and cubes to whole numbers, you should note that squares get big very quickly, but not nearly as quickly as cubes.

Any number raised to a power of 1 is that number itself: $3^1 = 3$; $13^1 = 13$.

Any number raised to a power of 0 is equal to 1: $5^0 = 1$; $18^0 = 1$. This is explained further in a following section.

Operations Using Exponents and Powers

It is possible to perform all four arithmetic operations on numbers raised to powers, but some very specific rules apply. You can examine them as they apply to each of the operations, starting with multiplication.

MULTIPLYING

To multiply numbers with the same base, all you have to do is keep the base the same and add the exponents:

$3^2 \cdot 3^2 = 3^{2+2} = 3^4$	$3^4 = 81$	Check it: $\rightarrow 3^2 \cdot 3^2 = 9 \cdot 9 = 81$
$5^3 \cdot 5^3 = 5^{3+3} = 5^6$	$5^6 = 15{,}625$	Check it: $\rightarrow 5^3 \cdot 5^3 = 125 \cdot 125 = 15{,}625$
$2^4 \cdot 2^3 = 2^{4+3} = 2^7$	$2^7 = 128$	Check it: $\rightarrow 2^4 \cdot 2^3 = 16 \cdot 8 = 128$

Suppose that the bases are different, such as 2^3 and 4^2? When the bases are different, each number must be expanded. First find 2^3 which is $2 \cdot 2 \cdot 2 = 8$; 4^2 is $4 \cdot 4 = 16$. Then multiply the results to get $8 \cdot 16 = 128$.

2^3 and $4^2 =$ ____

$2 \cdot 2 \cdot 2 = 8$

$4 \cdot 4 = 16$

$8 \cdot 16 = 128$

You'll find practice examples at the end of the chapter on page 18.

Exponents and Powers *(continued)*

DIVIDING

To divide numbers with the same base, all you have to do is keep the base the same and subtract the exponents. Does that sound like the exact opposite of multiplication? You bet:

$3^3 \div 3^2 = 3^{3-2} = 3^1$	$3^1 = 3$	Check it: $\rightarrow 3^3 \div 3^2 = 27 \div 9 = 3$
$5^4 \div 5^2 = 5^{4-2} = 5^2$	$5^2 = 25$	Check it: $\rightarrow 5^4 \div 5^2 = 625 \div 25 = 25$
$2^6 \div 2^3 = 2^{6-3} = 2^3$	$2^3 = 8$	Check it: $\rightarrow 2^6 \div 2^3 = 64 \div 8 = 8$

Here's a bonus explanation. It's the one promised two sections back. This is why any number raised to the 0 power = 1. Let n stand for any number, or if you'd prefer, you can substitute any number you like for n. The results will be the same. In the first line to the right, you see $n^3 \div n^3$. But that is a number divided by itself. Any number divided by itself = 1, so n^0, which you see in the second line, equals 1. Pretty cool, don't you think?

$$n^3 \div n^3 = n^{3-3}$$
$$n^{3-3} = n^0$$

As with multiplication, if the bases are different, you have to expand each value before combining. For example, to find $4^2 \div 2^3$, you first find the values of each, 16 and 8; then you divide: $16 \div 8 = 2$.

ADDING AND SUBTRACTING

To add or subtract numbers with exponents, whether the bases are the same or not, each expression must be evaluated (expanded) and then the addition or subtraction found. Answers are at the end of the chapter.

Try These

1. $2^3 + 3^2 =$ ___
2. $2^4 + 5^3 =$ ___
3. $3^3 + 4^2 =$ ___
4. $6^2 - 3^2 =$ ___
5. $9^2 - 3^3 =$ ___
6. $12^2 - 4^3 =$ ___

RAISING TO ANOTHER EXPONENT

When a number with an exponent is raised to another exponent, simply keep the base the same as it was, but multiply the exponents.

$$(3^4)^3 = 3^{12} \quad (2^3)^3 = 2^9 \quad (4^2)^2 = 4^4 \quad (7^3)^8 = 7^{24} \quad (5^6)^3 = 5^{18}$$

Square Roots and Cube Roots

Operating with square roots and cube roots in algebra is explored later in this book. For now, I am concerned with defining them, identifying them, and simplifying them.

IDENTIFYING SQUARE ROOTS AND CUBE ROOTS

The **square root** of a number is the number that when multiplied by itself gives that number. The square root or radical sign looks like this: $\sqrt{}$. You read $\sqrt{25}$ as "the square root of 25." A list of the first 12 **perfect (whole number) square roots** is given at right.

$$\sqrt{0} = 0 \qquad \sqrt{1} = 1 \qquad \sqrt{4} = 2$$
$$\sqrt{9} = 3 \qquad \sqrt{16} = 4 \qquad \sqrt{25} = 5$$
$$\sqrt{36} = 6 \qquad \sqrt{49} = 7 \qquad \sqrt{64} = 8$$
$$\sqrt{81} = 9 \qquad \sqrt{100} = 10 \qquad \sqrt{121} = 11$$

Square roots may also be identified using a fractional exponent $\frac{1}{2}$. For example, an alternative to writing the square root of 144 is shown at right.

$$\sqrt{144} = 144^{\frac{1}{2}} = 12$$

The **cube root** of a number is the number that when multiplied by itself twice gives that number. The cube root bracket looks like this: $\sqrt[3]{}$. You read $\sqrt[3]{27}$ as "the cube root of 27." A list of the first 12 **perfect (whole number) cube roots** is given at right.

$$\sqrt[3]{0} = 0 \qquad \sqrt[3]{1} = 1 \qquad \sqrt[3]{8} = 2$$
$$\sqrt[3]{27} = 3 \qquad \sqrt[3]{64} = 4 \qquad \sqrt[3]{125} = 5$$
$$\sqrt[3]{216} = 6 \qquad \sqrt[3]{343} = 7 \qquad \sqrt[3]{512} = 8$$
$$\sqrt[3]{729} = 9 \qquad \sqrt[3]{1000} = 10 \qquad \sqrt[3]{1331} = 11$$

Cube roots may also be identified using a fractional exponent $\frac{1}{3}$. For example, an alternative to writing the cube root of 1728 is shown at right.

$$\sqrt[3]{1728} = 1728^{\frac{1}{3}} = 12$$

SIMPLIFYING SQUARE ROOTS

Not all numbers have perfect square roots; in fact, most of them do not. Some square roots, however, can be simplified. Take, for example, $\sqrt{40}$. It is possible to do some factoring beneath the radical sign.

$$\sqrt{40} = \sqrt{4 \cdot 10} = 2\sqrt{10}$$

Note that you can factor a perfect square (4) out of the 40. Then you can remove the square 4 from under the radical sign, writing its square root in front of the sign, so $\sqrt{40}$ becomes 2 times $\sqrt{10}$.

Do you see any way to simplify $\sqrt{45}$? Look for a perfect square that is a factor of 45. 9 is such a number. The square root of 9 is 3, so remove the 9 from under the radical sign and write a 3 in front of it. The simplest form of $\sqrt{45}$ is $3\sqrt{5}$.

$$\sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$$

APPROXIMATING SQUARE ROOTS

For the majority of numbers that are not perfect squares, it is sometimes necessary to *approximate* a square root. Take, for example, $\sqrt{57}$. $49 < 57 < 64$. That means 57 is between 49 and 64, or greater than 49 and less than 64. $\sqrt{57}$ should fall between 7 and 8, the square roots of 49 and 64, respectively. 57 is about halfway between 49 and 64, so try 7.5. $7.5^2 = 56.25$; try a little higher: $7.6^2 = 57.76$. The number you're looking for is about halfway between both of those results, so try halfway between. $7.55^2 = 57.0025$. Now that's pretty darned good.

Generally, the square roots of nonperfect squares can be found using a table or a calculator that has a square root function. You may want to commit these two to memory:

$$\sqrt{2} \approx 1.414 \qquad \sqrt{3} \approx 1.732$$

MULTIPLYING AND DIVIDING BY ZERO

0 • anything = 0. Maybe you already knew that, but I can't emphasize it too much. If you have 5 apples 0 times, how many apples do you have? The answer is 0. How much is 5,000 • 0? Anything times 0 is 0. Remember the commutative property for multiplication from earlier in this chapter. Division by zero does not exist. Anything divided by 0 is **undefined**. Since a fraction is, among other things, a division, **the denominator of a fraction can never equal 0**.

FUNDAMENTAL ORDER OF OPERATIONS

If parentheses, addition, subtraction, exponents, multiplication, and so on are all contained in a single problem, the order in which they are done matters. A mnemonic device to help remember that order is P.E.M.D.A.S., or Please Excuse My Dear Aunt Sally. The letters really stand for Parentheses Exponents Multiplication Division Addition Subtraction and represent the sequence in which the operations are to be performed—with certain provisos, as indicated below:

1. Parentheses
2. Exponents or square roots
3. Multiplication } Whichever comes first from left to right.
4. Division }
5. Addition } Whichever comes first from left to right.
6. Subtraction }

DIVISIBILITY TESTS

You can save time when factoring (or dividing) numbers by following these rules.

A number is divisible by	If
2	Its ones digit is divisible by 2.
3	The sum of its digits is divisible by 3.
4	The number formed by its two right-hand digits is divisible by 4.
5	Its ones digit is a 0 or a 5.
6	It is divisible by 2 and by 3 (use the rules above).
7	Sorry, there's no shortcut here.
8	The number formed by its three rightmost digits is divisible by 8.
9	The sum of its digits is divisible by 9.

Chapter Practice

Practice Questions

GROUPS OF NUMBERS

1. What is the meaning of the arrowheads on the end of the number lines representing each of the realms of numbers?
2. How do integers differ from rational numbers?
3. Name two irrational numbers.

COMMON MATH SYMBOLS

4. Write an is-greater-than symbol followed by an is-greater-than-or-equal-to symbol.
5. Name the conventional algebraic grouping symbols from innermost to outermost.
6. Represent 5 + the product of 7 and 4 in mathematical symbols.
7. Represent 5 is about equal to 4.9 in mathematical symbols.

COMMUTATIVE AND ASSOCIATIVE PROPERTIES

8. Demonstrate the commutative property for multiplication.
9. Demonstrate the associative property for addition.

IDENTITY ELEMENTS AND DISTRIBUTIVE PROPERTY

10. Name the identity element for subtraction.
11. Name the identity element for division.
12. Demonstrate the distributive property by filling in the blank: $a(b + c) =$ ________________
13. Demonstrate the distributive property by filling in the blank: $5(7 - 3) =$ ________________

POWERS AND EXPONENTS

14. What is the meaning of 2^4?
15. What is the meaning of 4^3?
16. Represent 9 squared.
17. Represent 7 cubed.

OPERATIONS USING POWERS AND EXPONENTS

Express each answer in exponential form.

18 $3^2 \cdot 3^5$

19 $4^3 \cdot 4^6$

20 $9^4 \div 9^3$

21 $12^8 \div 12^5$

22 $6^3 + 6^4$

Evaluate each of the following expressions.

23 $3^2 \cdot 5^4$

24 $6^3 + 6^4$

25 $5^3 + 2^5$

26 $6^3 - 3^4$

27 $5^2 - 4^3$

SQUARE ROOTS AND CUBE ROOTS

28 Name the first four perfect squares and their square roots.

29 Name the first four perfect cubes and their cube roots.

30 Express $\sqrt{50}$ in simplest terms using the radical sign.

31 Express $\sqrt{128}$ in simplest terms using the radical sign.

32 Approximate the square root of 42.

FUNDAMENTAL ORDER OF OPERATIONS AND DIVISIBILITY TESTS

33 What is the mnemonic device for remembering the order of operations?

34 Express in simplest form: $3 + [10(9 - 2^2)]$

35 Express in simplest form: $20 - 2 \cdot 6 + 12^2 + (9 - 1) \cdot 4$

36 When is a number divisible by 6?

37 Is 17,438,156,241 divisible by 9?

Chapter Practice
(continued)

Chapter Practice Answers

1. They indicate that the values continue infinitely in either direction.
2. Integers are whole numbers both positive and negative, and rational numbers include non-whole numbers, such as $\frac{1}{2}$ or $\frac{3}{4}$.
3. $\sqrt{2}$, $\sqrt{3}$, and π, to name a few.
4. $>$, $\geq$
5. parentheses, brackets, braces; ( ), [ ], { }
6. $5 + (7 \cdot 4)$
7. $5 \approx 4.9$
8. $(a \cdot b) = (b \cdot a)$ or (number 1 • number 2) = (number 2 • number 1)
9. $(a + b) + \mathrm{c} = a + (b + c)$ or (number 1 + number 2) + number 3 = number 1 + (number 2 + number 3)
10. 0
11. 1
12. $a(b + c) = ab + ac$
13. $5(7 - 3) = 35 - 15 = 20$
14. $2 \cdot 2 \cdot 2 \cdot 2 = 16$
15. $4 \cdot 4 \cdot 4 = 64$
16. 9^2
17. 7^3
18. 3^7
19. 4^9
20. 9

21. 12^3
22. $6^3 + 6^4$ — I hope you didn't fall for that one.
23. 5625
24. 1512
25. 157
26. 135
27. –39
28. $\sqrt{0} = 0$ $\sqrt{1} = 1$ $\sqrt{4} = 2$ $\sqrt{9} = 3$
29. $\sqrt[3]{0} = 0$ $\sqrt[3]{1} = 1$ $\sqrt[3]{8} = 2$ $\sqrt[3]{27} = 3$
30. $5\sqrt{2}$
31. $8\sqrt{2}$
32. 6.5
33. Please Excuse My Dear Aunt Sally.
34. First clear the parentheses by squaring the 2 and subtracting the resulting 4 from 9 to get 3 + [10(5)]; next multiply 10 times 5 to get 3 + 50; finally, add to get 53.
35. First clear parentheses by subtracting 1 from 9: $20 - 2 \cdot 6 + 12^2 + 8 \cdot 4$

    Next remove the exponent by squaring the 12: $20 - 2 \cdot 6 + 144 + 8 \cdot 4$

    Next do the multiplications from left to right: $20 - 12 + 144 + 32$

    Finally, take 12 from 20, and add everything else: $8 + 144 + 32 = 184$
36. When it is divisible by 3 and by 2.
37. $1 + 7 + 4 + 3 + 8 + 1 + 5 + 6 + 2 + 4 + 1 = 42$; 42 is not divisible by 9. No.

ANSWERS FROM P. 14

1. 17
2. 141
3. 43
4. 27
5. 54
6. 80