1

WHAT IS AN ANTENNA AND HOW DOES IT WORK?

1.0 SUMMARY

An antenna is a structure that is made of material bodies that can be composed of either conducting or dielectric materials or may be a combination of both. Such a structure should be matched to the source of the electro-magnetic energy so that it can radiate or receive the electromagnetic fields in an efficient manner. The interesting phenomenon is that an antenna displays selectivity properties not only in frequency but also in space. In the frequency domain an antenna is capable of displaying a resonance phenomenon where at a particular frequency the current density induced on it can be sufficiently significant to cause radiation of electromagnetic fields from that structure. An antenna also possesses an impulse response that is a function of both the azimuth and elevation angles. Thus, an antenna displays spatial selectivity as it generates a radiation pattern that can selectively transmit or receive electromagnetic energy along certain spatial directions. As a receiver of electromagnetic fields, an antenna also acts as a spatial sampler of the electromagnetic fields propagating through space. The voltage induced in the antenna is related to the polarization and the strength of the incident electromagnetic fields. The objective of this chapter is to illustrate how the impulse response of an antenna can be determined. Another goal is to demonstrate that the impulse response of an antenna when it is transmitting is different from its response when the same structure operates in the receive mode. This is in direct contrast to antenna properties in the frequency domain as the transmit radiation pattern is the same as the receive antenna pattern. An antenna provides the matching necessary between the various electrical components associated with the transmitter and receiver and the free space where the electromagnetic wave is propagating. From a functional perspective an antenna is thus related to a loudspeaker, which matches the acoustic generation/receiving devices to the open space. However, in acoustics, loudspeakers and microphones are bandlimited devices and so their impulse responses are well behaved. On the other hand, an antenna is a high pass device and therefore the transmit and the receive impulse responses are not the same; in fact, the former is the time

derivative of the latter. An antenna is like our lips, whose instantaneous change of shapes provides the necessary match between the vocal cord and the outside environment as the frequency of the voice changes. By proper shaping of the antenna structure one can focus the radiated energy along certain specific directions in space. This spatial directivity occurs only at certain specific frequencies, providing selectivity in frequency. The interesting point is that it is difficult to separate these two spatial and temporal properties of the antenna, even though in the literature they are treated separately. The tools that deal with the dual-coupled space-time analysis are *Maxwell's equations*. We first present the background of Maxwell's equations and illustrate how to solve for them analytically. Then we utilize them in the subsequent sections and chapters to illustrate how to obtain the impulse responses of antennas both as transmitting and receiving elements and illustrate their relevance in the saga of smart antennas.

1.1 HISTORICAL OVERVIEW OF MAXWELL'S EQUATIONS

In the year 1864, James Clerk Maxwell (1831–1879) read his "Dynamical Theory of the Electromagnetic Field" [1] at the Royal Society (London). He observed theoretically that electromagnetic disturbance travels in free space with the velocity of light [1–7]. He then conjectured that light is a transverse electromagnetic wave by using dimensional analysis [7]. In his original theory Maxwell introduced 20 equations involving 20 variables. These equations together expressed mathematically virtually all that was known about electricity and magnetism. Through these equations Maxwell essentially summarized the work of Hans C. Oersted (1777–1851), Karl F. Gauss (1777–1855), André M. Ampère (1775–1836), Michael Faraday (1791–1867), and others, and added his own radical concept of displacement *current* to complete the theory.

Maxwell assigned strong physical significance to the magnetic vector and electric scalar potentials $\boldsymbol{A}$ and ψ, respectively (bold variables denote vectors; italic denotes that they are function of both time and space, whereas roman variables are a function of space only), both of which played dominant roles in his formulation. He did not put any emphasis on the sources of these electromagnetic potentials, namely the currents and the charges. He also assumed a hypothetical mechanical medium called *ether* to justify the existence of displacement currents in free space. This assumption produced a strong opposition to Maxwell's theory from many scientists of his time. It is well known that Maxwell's equations, as we know them now, do not contain any potential variables; neither does his electromagnetic theory require any assumption of an artificial medium to sustain his displacement current in free space. The original interpretation given to the displacement current by Maxwell is no longer used; however, we retain the term in honor of Maxwell. Although modern Maxwell's equations appear in modified form, the equations introduced by Maxwell in 1864 formed the foundation of electromagnetic theory, which together is popularly referred to as *Maxwell's electromagnetic theory* [1–7].

Maxwell's original equations were modified and later expressed in the form we now know as Maxwell's equations independently by Heinrich Hertz (1857–1894) and Oliver Heaviside (1850–1925). Their work discarded the requirement of a medium for the existence of displacement current in free space, and they also eliminated the vector and scalar potentials from the fundamental equations. Their derivations were based on the impressed sources, namely the current and the charge. Thus, Hertz and Heaviside, independently, expressed Maxwell's equations involving only the four field vectors ***E***, ***H***, ***B***, and ***D***: the electric field intensity, the magnetic field intensity, the magnetic flux density, and the electric flux density or displacement, respectively. Although priority is given to Heaviside for the vector form of Maxwell's equations, it is important to note that Hertz's 1884 paper [2] provided the Cartesian form of Maxwell's equations, which also appeared in his later paper of 1890 [3]. Thus, the coordinate forms of the four equations that we use nowadays were first obtained by Hertz [2,7] in scalar form and then by Heaviside in 1888 in vector form [4,7].

It is appropriate to mention here that the importance of Hertz's theoretical work [2] and its significance appear not to have been fully recognized [5]. In this 1884 paper [2] Hertz started from the older action-at-a-distance theories of electromagnetism and proceeded to obtain Maxwell's equations in an alternative way that avoided the mechanical models that Maxwell used originally and formed the basis for all his future contributions to electromagnetism, both theoretical and experimental. In contrast to the 1884 paper, in his 1890 paper [3] Hertz postulated Maxwell's equations rather than deriving them alternatively. The equations, written in component forms rather than in vector form as done by Heaviside [4], brought unparalleled clarity to Maxwell's theory. The four equations in vector notation containing the four electromagnetic field vectors are now commonly known as Maxwell's equations. However, Einstein referred to them as *Maxwell–Heaviside–Hertz equations* [6,7].

Although the idea of electromagnetic waves was hidden in the set of 20 equations proposed by Maxwell, he had in fact said virtually nothing about electromagnetic waves other than light, nor did he propose any idea to generate such waves electromagnetically. It has been stated [6, Ch. 2, p. 24]: "*There is even some reason to think that he [Maxwell] regarded the electrical production of such waves as impossibility.*" There is no indication left behind by him that he believed such was even possible. Maxwell did not live to see his prediction confirmed experimentally and his electromagnetic theory fully accepted. The former was confirmed by Hertz's brilliant experiments, his theory received universal acceptance, and his original equations in a modified form became the language of electromagnetic waves and electromagnetics, due mainly to the efforts of Hertz and Heaviside [7].

Hertz discovered electromagnetic waves around the year 1888 [8]; the results of his epoch-making experiments and his related theoretical work (based on the sources of the electromagnetic waves rather than on the potentials) confirmed Maxwell's prediction and helped the general acceptance of Maxwell's electromagnetic theory. However, it is not commonly appreciated that

"*Maxwell's theory that Hertz's brilliant experiments confirmed was not quite the same as the one Maxwell left at his death in the year 1879*" [6]. It is interesting to note how the relevance of electromagnetic waves to Maxwell and his theory prior to Hertz's experiments and findings are described in [6]: "*Thus Maxwell missed what is now regarded as the most exciting implication of his theory, and one with enormous practical consequences. That relatively long electromagnetic waves or perhaps light itself, could be generated in the laboratory with ordinary electrical apparatus was unsuspected through most of the 1870's.*"

Maxwell's predictions and theory were thus confirmed by a set of brilliant experiments conceived and performed by Hertz, who generated, radiated (transmitted), and received (detected) electromagnetic waves of frequencies lower than light. His initial experiment started in 1887, and the decisive paper on the finite velocity of electromagnetic waves in air was published in 1888 [3]. After the 1888 results, Hertz continued his work at higher frequencies, and his later papers proved conclusively the optical properties (reflection, polarization, etc.) of electromagnetic waves and thereby provided unimpeachable confirmation of Maxwell's theory and predictions. English translation of Hertz's original publications [9] on experimental and theoretical investigation of electric waves is still a decisive source of the history of electromagnetic waves and Maxwell's theory. Hertz's experimental setup and his epoch-making findings are described in [10].

Maxwell's ideas and equations were expanded, modified, and made understandable after his death mainly by the efforts of Heinrich Hertz, George Francis Fitzgerald (1851–1901), Oliver Lodge (1851–1940), and Oliver Heaviside. The last three have been christened as "the Maxwellians" by Heaviside [2, 11].

Next we review the four equations that we use today due to Hertz and Heaviside, which resulted from the reformulation of Maxwell's original theory. Here in all the expressions we use SI units (Système International d'unités or International System of Units).

1.2 REVIEW OF MAXWELL–HEAVISIDE–HERTZ EQUATIONS

The four Maxwell's equations are among the oldest sets of equations in mathematical physics, having withstood the erosion and corrosion of time. Even with the advent of relativity, there was no change in their form. We briefly review the derivation of the four equations and illustrate how to solve them analytically [12]. The four equations consist of Faraday's law, generalized Ampère's law, generalized Gauss's law of electrostatics, and Gauss's law of magnetostatics, respectively.

1.2.1 Faraday's Law

Michael Faraday (1791–1867) observed that when a bar magnet was moved near a loop composed of a metallic wire, there appeared to be a voltage induced between the terminals of the wire loop. In this way, Faraday showed that a magnetic field produced by the bar magnet under some special circumstances can indeed generate an electric field to cause the induced voltage in the loop of wire and there is a connection between the electric and magnetic fields. This physical principle was then put in the following mathematical form:

$$V = -\oint_L \boldsymbol{E} \cdot \mathbf{d}\ell = -\frac{\partial \Phi_m}{\partial t} = -\frac{\partial}{\partial t} \iint_S \boldsymbol{B} \cdot \mathbf{d}s \tag{1.1}$$

where:

V	=	voltage induced in the wire loop of length L,
$\mathbf{d}\ell$	=	differential length vector along the axis of the wire loop,
$\boldsymbol{E}$	=	electric field along the wire loop,
Φ_m	=	magnetic flux linkage with the loop of surface area S,
$\boldsymbol{B}$	=	magnetic flux density,
S	=	surface over which the magnetic flux is integrated (this surface is bounded by the contour of the wire loop),
L	=	total length of the loop of wire,
$\bullet$	=	scalar dot product between two vectors,
$\mathbf{d}s$	=	differential surface vector normal to the surface.

This is the integral form of Faraday's law, which implies that this relationship is valid over a region. It states that the line integral of the electric field is equivalent to the rate of change of the magnetic flux passing through an open surface S, the contour of which is the path of the line integral. In this chapter, the variables in italic, for example $\boldsymbol{B}$, indicate that they are functions of four variables, x, y, z, t. This consists of three space variables (x, y, z) and a time variable, t. When the vector variable is written as **B**, it is a function of the three spatial variables (x, y, z) only. This nomenclature between the variables denoted by italic as opposed to roman is used to distinguish their functional dependence on spatial-temporal variables or spatial variables, respectively.

To extend this relationship to a point, we now establish the differential form of Faraday's law by invoking Stokes' theorem for the electric field. Stokes' theorem relates the line integral of a vector over a closed contour to a surface integral of the curl of the vector, which is defined as the rate of spatial change of the vector along a direction perpendicular to its orientation (which provides a rotary motion, and hence the term curl was first introduced by Maxwell), so that

$$\oint_L \boldsymbol{E} \bullet \mathbf{d}\ell = \iint_S (\nabla \times \boldsymbol{E}) \bullet \mathbf{d}s \tag{1.2}$$

where the curl of a vector in the Cartesian coordinates is defined by

$$\nabla \times \boldsymbol{E}\,(x,y,z,t) = determinant\ of \begin{bmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x & E_y & E_z \end{bmatrix}$$
$$= \hat{\boldsymbol{x}}\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] + \hat{\boldsymbol{y}}\left[\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right] + \hat{\boldsymbol{z}}\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right] \tag{1.3}$$

Here $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, and $\hat{\boldsymbol{z}}$ represent the unit vectors along the respective coordinate axes, and E_x, E_y, and E_z represent the x, y, and z components of the electric field intensity along the respective coordinate directions. The surface S is limited by the contour L. ∇ stands for the operator $\left[\hat{\boldsymbol{x}}(\partial/\partial x) + \hat{\boldsymbol{y}}(\partial/\partial y) + \hat{\boldsymbol{z}}(\partial/\partial z)\right]$. Using (1.2), (1.1) can be expressed as

$$\oint_L \boldsymbol{E} \bullet \mathbf{d}\ell = \iint_S (\nabla \times \boldsymbol{E}) \bullet \mathbf{d}s = -\frac{\partial}{\partial t}\iint_S \boldsymbol{B} \bullet \mathbf{d}s \tag{1.4}$$

If we assume that the surface S does not change with time and in the limit making it shrink to a point, we get Faraday's law at a point in space and time as

$$\nabla \times \boldsymbol{E}(x, y, z, t) = \frac{1}{\varepsilon}\nabla \times \boldsymbol{D}(x, y, z, t)$$
$$= -\frac{\partial \boldsymbol{B}(x, y, z, t)}{\partial t} = -\mu \frac{\partial \boldsymbol{H}(x, y, z, t)}{\partial t} \tag{1.5}$$

where the constitutive relationships between the flux densities and the field intensities are given by

$$\boldsymbol{B} = \mu \boldsymbol{H} = \mu_0 \mu_r \boldsymbol{H} \tag{1.6a}$$

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} = \varepsilon_0 \varepsilon_r \boldsymbol{E} \tag{1.6b}$$

$\boldsymbol{D}$ is the electric flux density and $\boldsymbol{H}$ is the magnetic field intensity. Here, ε_0 and μ_0 are the permittivity and permeability of vacuum, respectively, and ε_r and μ_r are the relative permittivity and permeability of the medium through which the wave is propagating.

Equation (1.5) is the point form of Faraday's law or the first of the four Maxwell's equations. It states that at a point the negative rate of the temporal variation of the magnetic flux density is related to the spatial change of the electric field along a direction perpendicular to the orientation of the electric field (termed the curl of a vector) at that same point.

1.2.2 Generalized Ampère's Law

André M. Ampère observed that when a wire carrying current is brought near a magnetic needle, the magnetic needle is deflected in a very specific way determined by the direction of the flow of the current with respect to the magnetic needle. In this way Ampère established the complementary connection with the magnetic field generated by an electric current created by an electric field that is the result of applying a voltage difference between the two ends of the wire. Ampère first illustrated how to generate a magnetic field using the electric field or current. Ampère's law can be stated mathematically as

$$I = \oint_L \boldsymbol{H} \bullet \mathbf{d}\ell \tag{1.7}$$

where I is the total current encircled by the contour. We call this the *generalized Ampère's law* because we use the total current, which includes the displacement current due to Maxwell and the conduction current. In principle, Ampère's law is connected strictly with the conduction current. Since we use the term *total current*, we use the prefix *generalized* as it is a sum of both the conduction an displacement currents. Therefore, the line integral of $\boldsymbol{H}$, the magnetic field intensity along any closed contour L, is equal to the total current flowing through that contour.

To obtain a point form of Ampère's law, we employ Stokes' theorem to the magnetic field intensity and integrate the current density $\boldsymbol{J}$ over a surface to obtain

$$\begin{aligned} I &= \iint_S \boldsymbol{J} \bullet \mathbf{d}s = \oint_L \boldsymbol{H} \bullet \mathbf{d}\ell = \iint_S (\nabla \times \boldsymbol{H}) \bullet \mathbf{d}s \\ &= \frac{1}{\mu} \iint_S (\nabla \times \boldsymbol{B}) \bullet \mathbf{d}s \end{aligned} \tag{1.8}$$

This is the integral form of Ampère's law, and by shrinking S to a point, one obtains a relationship between the electric current density and the magnetic field intensity at the same point, resulting in

$$\boldsymbol{J}(x,y,z,t) = \nabla \times \boldsymbol{H}(x,y,z,t) \tag{1.9}$$

Physically, it states that the spatial derivative of the magnetic field intensity along a direction perpendicular to the orientation of the magnetic field intensity is related to the electric current density at that point. Now the electric current density $\boldsymbol{J}$ may consist of different components. This may include the conduction current (current flowing through a conductor) density $\boldsymbol{J}_c$ and displacement current density (current flowing through air, as from a transmitter to a receiver without any physical connection, or current flowing through the dielectric between the plates of a capacitor) $\boldsymbol{J}_d$, in addition to an externally applied impressed current density $\boldsymbol{J}_i$. So in this case we have

$$\boldsymbol{J} = \boldsymbol{J}_i + \boldsymbol{J}_c + \boldsymbol{J}_d = \boldsymbol{J}_i + \sigma \boldsymbol{E} + \frac{\partial \boldsymbol{D}}{\partial t} = \nabla \times \boldsymbol{H} \tag{1.10}$$

where $\boldsymbol{D}$ is the electric flux density or electric displacement and σ is the conductivity of the medium. The conduction current density is given by *Ohm's law*, which states that at a point the conduction current density is related to the electric field intensity by

$$\boldsymbol{J}_c = \sigma \boldsymbol{E} \tag{1.11}$$

The displacement current density introduced by Maxwell is defined by

$$\boldsymbol{J}_d = \frac{\partial \boldsymbol{D}}{\partial t} \tag{1.12}$$

We are neglecting the convection current density, which is due to the diffusion of the charge density at that point. We consider the impressed current density as the source of all the electromagnetic fields.

1.2.3 Generalized Gauss's Law of Electrostatics

Karl Friedrich Gauss established the following relation between the total charge enclosed by a surface and the electric flux density or displacement $\boldsymbol{D}$ passing through that surface through the following relationship:

$$\oint\!\!\oint_S \boldsymbol{D} \bullet \mathbf{d}s = Q \tag{1.13}$$

where integration of the electric displacement is carried over a closed surface and is equal to the total charge Q enclosed by that surface S.

We now employ the divergence theorem. This is a relation between the flux of a vector function through a closed surface S and the integral of the divergence of the same vector over the volume V enclosed by S. The divergence of a vector is the rate of change of the vector along its orientation. It is given by

$$\oint\!\!\oint_S \boldsymbol{D} \bullet \mathbf{d}s = \iiint_V \nabla \bullet \boldsymbol{D} \; dv \tag{1.14}$$

Here dv represents the differential volume. In Cartesian coordinates the divergence of a vector, which represents the rate of spatial variation of the vector along its orientation, is given by

$$\begin{aligned} \nabla \bullet \boldsymbol{D} &= \left[\hat{\boldsymbol{x}} \frac{\partial}{\partial x} + \hat{\boldsymbol{y}} \frac{\partial}{\partial y} + \hat{\boldsymbol{z}} \frac{\partial}{\partial z} \right] \bullet \left[\hat{\boldsymbol{x}} D_x + \hat{\boldsymbol{y}} D_y + \hat{\boldsymbol{z}} D_z \right] \\ &= \frac{\partial D_x(x,y,z,t)}{\partial x} + \frac{\partial D_y(x,y,z,t)}{\partial y} + \frac{\partial D_z(x,y,z,t)}{\partial z} \end{aligned} \tag{1.15}$$

So the divergence $(\nabla \bullet)$ of a vector represents the spatial rate of change of the vector along its direction, and hence it is a scalar quantity, whereas the curl $(\nabla \times)$ of a vector is related to the rate of spatial change of the vector perpendicular to its orientation, which is a vector quantity and so possesses both a magnitude and a direction. All of the three definitions of *grad*, *Div* and *curl* were first introduced by Maxwell.

By applying the divergence theorem to the vector $\boldsymbol{D}$, we get

$$\oiint_S \boldsymbol{D} \bullet \mathbf{d}s = \iiint_V \nabla \bullet \boldsymbol{D}\ dv = Q = \iiint_V q_v\ dv \tag{1.16}$$

Here q_v is the volume charge density and V is the volume enclosed by the surface S. Therefore, if we shrink the volume in (1.16) to a point, we obtain

$$\begin{aligned} \nabla \bullet \boldsymbol{D} &= \frac{\partial D_x(x,y,z,t)}{\partial x} + \frac{\partial D_y(x,y,z,t)}{\partial y} + \frac{\partial D_z(x,y,z,t)}{\partial z} \\ &= q_v(x,y,z,t) \end{aligned} \tag{1.17}$$

This implies that the rate change of the electric flux density along its orientation is influenced only by the presence of a free charge density at that point.

1.2.4 Generalized Gauss's Law of Magnetostatics

Gauss's law of magnetostatics is similar to the law of electrostatics defined in Section 1.2.3. If one uses the closed surface integral for the magnetic flux density $\boldsymbol{B}$, its integral over a closed surface is equal to zero, as no free magnetic charges occur in nature. Typically, magnetic charges appear as pole pairs. Therefore, we have

$$\oiint \boldsymbol{B} \bullet \mathbf{d}s = 0 \tag{1.18}$$

From the application of the divergence theorem to (1.18), one obtains

$$\iiint_V \nabla \bullet \boldsymbol{B}\ \mathbf{d}v = 0 \tag{1.19}$$

which results in

$$\nabla \bullet \boldsymbol{B} = 0 \tag{1.20}$$

Equivalently in Cartesian coordinates, this becomes

$$\frac{\partial B_x(x,y,z,t)}{\partial x} + \frac{\partial B_y(x,y,z,t)}{\partial y} + \frac{\partial B_z(x,y,z,t)}{\partial z} = 0 \tag{1.21}$$

This completes the presentation of the four equations, which are popularly referred to as Maxwell's equations, which really were developed by Hertz in scalar form and cast by Heaviside into the vector form that we use today. These four equations relate all the spatial-temporal relationships between the electric and magnetic fields.

1.2.5 Equation of Continuity

Often, the equation of continuity is used in addition to equations (1.18)–(1.21) to relate the impressed current density $\boldsymbol{J}_i$ to the free charge density q_v at that point. The equation of continuity states that the total current is related to the negative of the time derivative of the total charge by

$$I = \frac{-\partial Q}{\partial t} \tag{1.22}$$

By applying the divergence theorem to the current density, we obtain

$$I = \oiint_S \boldsymbol{J} \bullet \mathrm{d}s = \iiint_V (\nabla \bullet \boldsymbol{J})\, dv = \frac{-\partial}{\partial t} \iiint_V q_v \, dv \tag{1.23}$$

Now shrinking the volume V to a point results in

$$\nabla \bullet \boldsymbol{J} = \frac{-\partial q_v}{\partial t} \tag{1.24}$$

In Cartesian coordinates this becomes

$$\frac{\partial J_x(x,y,z,t)}{\partial x} + \frac{\partial J_y(x,y,z,t)}{\partial y} + \frac{\partial J_z(x,y,z,t)}{\partial z} = -\frac{\partial q_v(x,y,z,t)}{\partial t} \tag{1.25}$$

This states that there will be a spatial change of the current density along the direction of its flow if there is a temporal change in the charge density at that point. Next we obtain the solution of Maxwell's equations.

1.3 SOLUTION OF MAXWELL'S EQUATIONS

Instead of solving the four coupled differential Maxwell's equations directly dealing with the electric and magnetic fields, we introduce two additional variables $\boldsymbol{A}$ and ψ. Here $\boldsymbol{A}$ is the magnetic vector potential and ψ is the scalar electric potential. The introduction of these two auxiliary variables facilitates the solution of the four equations.

We start with the generalized Gauss's law of magnetostatics, which states that

$$\nabla \bullet \boldsymbol{B}(x, y, z, t) = 0 \tag{1.26}$$

Since the divergence of the curl of any vector $\boldsymbol{A}$ is always zero, that is,

$$\nabla \bullet \nabla \times \boldsymbol{A}(x,y,z,t) = 0 \tag{1.27}$$

one can always write

$$\boldsymbol{B}(x,y,z,t) = \nabla \times \boldsymbol{A}\,(x,y,z,t) \tag{1.28}$$

which states that the magnetic flux density can be obtained from the curl of the magnetic vector potential $\boldsymbol{A}$. So if we can solve for $\boldsymbol{A}$, we obtain $\boldsymbol{B}$ by a simple differentiation. It is important to note that at this point $\boldsymbol{A}$ is still an unknown quantity. In Cartesian coordinates this relationship becomes

$$\begin{aligned}
\boldsymbol{B}(x,y,z,t) &= \hat{\boldsymbol{x}}\, B_x(x,y,z,t) + \hat{\boldsymbol{y}}\, B_y(x,y,z,t) + \hat{\boldsymbol{z}}\, B_z(x,y,z,t) \\
&= \textit{determinant} \text{ of } \begin{bmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix} \\
&= \hat{\boldsymbol{x}} \left[\frac{\partial A_z(x,y,z,t)}{\partial y} - \frac{\partial A_y(x,y,z,t)}{\partial z} \right] + \\
&\quad \hat{\boldsymbol{y}} \left[\frac{\partial A_x(x,y,z,t)}{\partial z} - \frac{\partial A_z(x,y,z,t)}{\partial x} \right] + \\
&\quad \hat{\boldsymbol{z}} \left[\frac{\partial A_y(x,y,z,t)}{\partial x} - \frac{\partial A_x(x,y,z,t)}{\partial y} \right]
\end{aligned} \tag{1.29}$$

Note that if we substitute $\boldsymbol{B}$ from (1.28) into Faraday's law given by (1.5), we obtain

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} = -\frac{\partial}{\partial t}\left[\nabla \times \boldsymbol{A}\right] = -\nabla \times \frac{\partial \boldsymbol{A}}{\partial t} \tag{1.30}$$

or equivalently,

$$\nabla \times \left[\boldsymbol{E} + \frac{\partial \boldsymbol{A}}{\partial t} \right] = 0 \tag{1.31}$$

If the curl of a vector is zero, that vector can always be written in terms of the gradient of a scalar function ψ, since it is always true that the curl of the gradient of a scalar function ψ is always zero, that is,

$$\nabla \times \nabla\, \psi(x,y,z,t) = 0 \tag{1.32}$$

where the gradient of a vector is defined through

$$\nabla \psi = \left(\hat{\boldsymbol{x}} \frac{\partial}{\partial x} + \hat{\boldsymbol{y}} \frac{\partial}{\partial y} + \hat{\boldsymbol{z}} \frac{\partial}{\partial z} \right) \psi(x,y,z,t) \tag{1.33}$$

We call ψ the electric scalar potential. Therefore, we can write the following (we choose a negative sign in front of the term on the right-hand side of the equation for convenience):

$$\boldsymbol{E} + \frac{\partial \boldsymbol{A}}{\partial t} = -\nabla\psi \tag{1.34}$$

or

$$\boldsymbol{E}(x,y,z,t) = -\frac{\partial \boldsymbol{A}(x,y,z,t)}{\partial t} - \nabla\psi(x,y,z,t) \tag{1.35}$$

This states that the electric field at any point can be given by the time derivative of the magnetic vector potential and the gradient of the scalar electric potential. So we have the solution for both $\boldsymbol{B}$ from (1.28) and $\boldsymbol{E}$ from (1.35) in terms of $\boldsymbol{A}$ and ψ. The problem now is how we solve for $\boldsymbol{A}$ and ψ. Once $\boldsymbol{A}$ and ψ are known, $\boldsymbol{E}$ and $\boldsymbol{B}$ can be obtained through simple differentiation, as in (1.35) and (1.28), respectively.

Next we substitute the solution for both $\boldsymbol{E}$ [using (1.35)] and $\boldsymbol{B}$ [using (1.28)] into Ampère's law, which is given by (1.10), to obtain

$$\boldsymbol{J}_i(x,y,z,t) + \sigma\boldsymbol{E}(x,y,z,t) + \frac{\partial \boldsymbol{D}(x,y,z,t)}{\partial t} = \nabla\times\boldsymbol{H}(x,y,z,t) \tag{1.36}$$

Since the constitutive relationships are given by (1.6) (i.e., $\boldsymbol{D} = \varepsilon\boldsymbol{E}$ and $\boldsymbol{B} = \mu\boldsymbol{H}$), then

$$\boldsymbol{J}_i + \sigma\boldsymbol{E} + \varepsilon\frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{\mu}\nabla\times\nabla\times\boldsymbol{A} \tag{1.37}$$

Here we will set $\sigma = 0$, so that the medium in which the wave is propagating is assumed to be free space, and therefore conductivity is zero. So we are looking for the solution for an electromagnetic wave propagating in a non-conducting medium. In addition, we use the following vector identity:

$$\nabla\times\nabla\times\boldsymbol{A} = \nabla(\nabla\bullet\boldsymbol{A}) - (\nabla\bullet\nabla)\boldsymbol{A} \tag{1.38}$$

By using (1.38) in (1.37), one obtains

$$\begin{aligned}\nabla\times\nabla\times\boldsymbol{A} &= \nabla(\nabla\bullet\boldsymbol{A}) - (\nabla\bullet\nabla)\boldsymbol{A}\\ &= \mu\boldsymbol{J}_i + \mu\varepsilon\frac{\partial}{\partial t}\left[-\frac{\partial \boldsymbol{A}}{\partial t} - \nabla\psi\right]\\ &= \mu\boldsymbol{J}_i - \mu\varepsilon\frac{\partial^2\boldsymbol{A}}{\partial t^2} - \mu\varepsilon\nabla\frac{\partial\psi}{\partial t}\end{aligned} \tag{1.39}$$

or equivalently,

$$(\nabla\bullet\nabla)\boldsymbol{A} - \mu\varepsilon\frac{\partial^2\boldsymbol{A}}{\partial t^2} + \mu\boldsymbol{J}_i = \nabla\left[\nabla\bullet\boldsymbol{A} + \mu\varepsilon\frac{\partial\psi}{\partial t}\right] \tag{1.40}$$

Since we have introduced two additional new variables, $\boldsymbol{A}$ and ψ, we can without any problem impose a constraint between these two variables or these two potentials. This can be achieved by setting the right-hand side of the expression in (1.40) equal to zero. This results in

$$\nabla \bullet \boldsymbol{A} + \mu \varepsilon \frac{\partial \psi}{\partial t} = 0 \tag{1.41}$$

which is known as the *Lorenz gauge condition* [13]. It is important to note that this is not the only constraint that is possible between the two newly introduced variables $\boldsymbol{A}$ and ψ. This is only a particular assumption, and other choices will yield different forms of the solution of the Maxwell–Heaviside–Hertz equations. Interestingly, Maxwell in his treatise [1] chose the Coulomb gauge [7], which is generally used for the solution of static problems.

Next, we observe that by using (1.41) in (1.40), one obtains

$$(\nabla \bullet \nabla)\ \boldsymbol{A} - \mu \varepsilon \frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\mu \boldsymbol{J}_i \tag{1.42}$$

In summary, the solution of Maxwell's equations starts with the solution of equation (1.42) first, for $\boldsymbol{A}$ given the impressed current $\boldsymbol{J}_i$. Then the scalar potential ψ is solved for by using (1.41). Once $\boldsymbol{A}$ and ψ are obtained, the electric and magnetic field intensities are derived from

$$\boldsymbol{H} = \frac{1}{\mu} \boldsymbol{B} = \frac{1}{\mu} \nabla \times \boldsymbol{A} \tag{1.43}$$

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla \psi \tag{1.44}$$

This completes the solution in the time domain, even though we have not yet provided an explicit form of the solution. We now derive the explicit form of the solution in the frequency domain and from that obtain the time domain representation. We assume the temporal variation of all the fields to be time harmonic in nature, so that

$$\boldsymbol{E}(x,y,z,t) = \mathbf{E}(x,y,z)\, e^{j\omega t} \tag{1.45}$$

$$\boldsymbol{B}(x,y,z,t) = \mathbf{B}(x,y,z)\, e^{j\omega t} \tag{1.46}$$

where $\omega = 2\pi f$ and f is the frequency (Hertz) of the electromagnetic fields. By assuming a time variation of the form $e^{j\omega t}$, we now have an explicit form for the time differentiations, resulting in

$$\frac{\partial}{\partial t}\left[\mathcal{A}(x,y,z,t)\right] = \frac{\partial}{\partial t}\left[\mathbf{A}(x,y,z)\, e^{j\omega t}\right] = j\omega\, \mathbf{A}(x,y,z)\, e^{j\omega t} \tag{1.47}$$

Therefore, (1.43) and (1.44) are simplified in the frequency domain after eliminating the common time variations of $e^{j\omega t}$ from both sides to form

$$\mathbf{H}(x,y,z) = \frac{1}{\mu}\mathbf{B}(x,y,z) = \frac{1}{\mu}\nabla\times\mathbf{A}(x,y,z) \tag{1.48}$$

$$\mathbf{E}(x,y,z) = -j\omega\mathbf{A}(x,y,z) - \nabla\psi(x,y,z) \tag{1.49}$$

Furthermore, in the frequency domain (1.41) transforms into

$$\nabla \bullet \mathbf{A} + j\omega\mu\varepsilon\psi = 0$$

or equivalently,

$$\psi = -\frac{\nabla \bullet \mathbf{A}}{j\omega\mu\varepsilon} \tag{1.50}$$

In the frequency domain, (1.42) transforms into

$$\nabla^2\mathbf{A} + \omega^2\mu\varepsilon\mathbf{A} = -\mu\mathbf{J}_i \tag{1.51}$$

The solution for $\mathbf{A}$ in (1.51) can now be written explicitly in an analytical form through [12]

$$\mathbf{A}(x,y,z) = \frac{\mu}{4\pi}\int_V \frac{\mathbf{J}_i(x',y',z')\, e^{-jkR}}{R}\, \mathrm{d}v \tag{1.52}$$

where

$$\mathbf{r} = \hat{x}x - \hat{y}y + \hat{z}z \tag{1.53}$$

$$\mathbf{r}' = \hat{x}x' + \hat{y}y' + \hat{z}z' \tag{1.54}$$

$$R = |\mathbf{r} - \mathbf{r}'| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \tag{1.55}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \sqrt{\omega^2\mu\varepsilon} = \sqrt{(2\pi)^2 f^2\mu\varepsilon} \tag{1.56}$$

$$c = \text{velocity of light in the medium} = \frac{1}{\sqrt{\mu\varepsilon}} \tag{1.57}$$

$$\lambda = \text{wavelength in the medium} \tag{1.58}$$

In summary, first the magnetic vector potential $\mathbf{A}$ is solved for in the frequency domain given the impressed currents $\mathbf{J}_i(\mathbf{r})$ through

$$\mathbf{A}(\mathbf{r}) = \mathbf{A}(x,y,z) = \frac{\mu}{4\pi} \iiint_V \frac{\mathbf{J}_i(\mathbf{r}')\; e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \mathrm{d}\mathbf{r}' \tag{1.59}$$

then the scalar electric potential ψ is obtained from (1.50). Next, the electric field intensity **E** is computed from (1.49) and the magnetic field intensity **H** from (1.48).

In the time domain the equivalent solution for the magnetic vector potential $\boldsymbol{A}$ is then given by the time-retarded potentials:

$$\boldsymbol{A}(\mathbf{r},t) = \boldsymbol{A}(x,y,z,t) = \frac{\mu}{4\pi} \iiint_V \frac{\boldsymbol{J}_i\left(\mathbf{r}', t - \frac{|\mathbf{r}-\mathbf{r}'|}{c}\right)}{|\mathbf{r}-\mathbf{r}'|} \mathrm{d}\mathbf{r}' \tag{1.60}$$

It is interesting to note that the time and space variables are now coupled and they are not separable. That is why in the time domain the spatial and temporal responses of an antenna are intimately connected and one needs to look at the complete solution. From the magnetic vector potential we obtain the scalar potential ψ by using (1.41). From the two vector and scalar potentials the electric field intensity $\boldsymbol{E}$ is obtained through (1.44) and the magnetic field intensity $\boldsymbol{H}$ using (1.43).

We now use these expressions to calculate the impulse response of some typical antennas in both the transmit and receive modes of operations. The reason that impulse response of an antenna is different in the transmit mode than in the receive mode is because the reciprocity principle in the time domain contains an integral over time. The reciprocity theorem in the time domain is quite different from its counterpart in the frequency domain. For the former a time integral is involved, whereas for the latter no such relationship is involved. Because of the frequency domain reciprocity theorem, the antenna radiation pattern when in the transmit mode is equal to the antenna pattern in the receive mode. However, this is not true in the time domain, as we shall now see through examples.

1.4 RADIATION AND RECEPTION PROPERTIES OF A POINT SOURCE ANTENNA IN FREQUENCY AND IN TIME DOMAIN

1.4.1 Radiation of Fields from Point Sources

In this section we first define what is meant by the term *radiation* and then observe the nature of the fields radiated by point sources and the temporal nature of the voltages induced when electromagnetic fields are incident on them. In contrast to the acoustic case (where an isotropic source exists), in the electromagnetic case there are no isotropic point sources. Even for a point source, which in the electromagnetic case is called a *Hertzian dipole*, the radiation pattern is not isotropic, but it can be omnidirectional in certain planes.

We describe the solution in both the frequency and time domains for such classes of problems.

Any element of current or charge located in a medium will produce electric and magnetic fields. However, by the term *radiation* we imply the amount of finite energy transmitted to infinity from these currents. Hence, radiation is related to the far fields or the fields at infinity. This will be discussed in detail in Chapter 2. A static charge may generate near fields, but it does not produce radiation, as the field at infinity due to this charge is zero. Therefore, radiated fields or far fields are synonymous. We will also explore the sources of a radiating field.

1.4.1.1 Far Field in Frequency Domain of a Point Radiator. If we consider a delta element of current or a Hertzian dipole located at the origin represented by a constant J times a delta function δ (0. 0, 0), the magnetic vector potential from that current element is given by

$$\mathbf{A}(x,y,z) = \frac{\mu}{4\pi}\,\frac{e^{-jkR}}{R}\,\mathbf{J}_i \tag{1.61}$$

where

$$R = \sqrt{x^2+y^2+z^2} \tag{1.62}$$

Here we limit our attention to the electric field. The electric field at any point in space is then given by

$$\begin{aligned}\mathbf{E}(x,y,z) &= -j\omega\mathbf{A} - \nabla\psi = -j\omega\mathbf{A} + \frac{\nabla(\nabla\bullet\mathbf{A})}{j\omega\mu\varepsilon}\\ &= \frac{1}{j\omega\mu\varepsilon}\left[k^2\mathbf{A} - \nabla(\nabla\bullet\mathbf{A})\right]\end{aligned} \tag{1.63}$$

In rectangular coordinates, the fields at any point located in space will be

$$\mathbf{E}(x,y,z) = \frac{1}{j\omega\mu\varepsilon}\left[\begin{array}{l} k^2\,\mathbf{A}(x,y,z) + \left\{\hat{x}\,\dfrac{\partial}{\partial x} + \hat{y}\,\dfrac{\partial}{\partial y} + \hat{z}\,\dfrac{\partial}{\partial z}\right\} \\ \times\left\{\dfrac{\partial A_x(x,y,z)}{\partial x} + \dfrac{\partial A_y(x,y,z)}{\partial y} + \dfrac{\partial A_z(x,y,z)}{\partial z}\right\}\end{array}\right] \tag{1.64}$$

However, some simplifications are possible for the far field (i.e., if we are observing the fields radiated by a source of finite size at a distance of $2D^2/\lambda$ from it, where D is the largest physical dimension of the source and λ is the wavelength – the physical significance of this will be addressed in chapter 2.). For a point source, everything is in the far field. Therefore, for all practical purposes, observing the fields at a distance $2D^2/\lambda$ from a source is equivalent to observing the fields from the same source at infinity. In that case, the far fields can be obtained from the first term only in (1.63) or (1.64). This first term due to the magnetic vector potential is responsible for the far field and there is no contribution from the scalar electric potential ψ. Hence,

$$\mathbf{E}_{far}(x,y,z) = -j\,\omega\,\mathbf{A} = -j\,\frac{\omega\,\mu}{4\pi}\,\mathbf{J}_i\,\frac{e^{-jkR}}{R} \tag{1.65}$$

and one obtains a spherical wavefront in the far field for a point source. However, the power density radiated is proportional to E_θ and that is clearly zero along $\theta = 0°$ and is maximum in the azimuth plane where $\theta = 90°$. The characteristic feature is that the far field is polarized and the orientation of the field is along the direction of the current element. It is also clear that one obtains a spherical wavefront in the far field radiated by a point source.

The situation is quite different in the time domain, as the presence of the term ω in the front of the expression of the magnetic vector potential will illustrate.

1.4.1.2 Far Field in Time Domain of a Point Radiator. We consider a delta current source at the origin of the form

$$J_i\,\delta(0,0,0,t) = \hat{z}\,\delta(0,0,0)\,f(t) \tag{1.66}$$

where $\hat{z}$ is the direction of the orientation of the elemental current element and $f(t)$ is the temporal variation for the current fed to the point source located at the origin. The magnetic vector potential in this case is given by

$$\boldsymbol{A}(\mathbf{r},t) = \frac{\mu}{4\pi}\,\frac{\hat{z}\,f(t - |R|/c)}{R} \tag{1.67}$$

There will be a time retardation factor due to the space-time connection of the electromagnetic wave that is propagating, where R is given by (1.62).

Now the transient far field due to this impulsive current will be given by

$$\boldsymbol{E}(\mathbf{r},t) = -\frac{\partial \boldsymbol{A}(\mathbf{r},t)}{\partial t} = -\frac{\mu\,\hat{z}}{4\pi R}\,\frac{\partial f(t - |R|/c)}{\partial t} \tag{1.68}$$

Hence, the time domain field radiated by a point source is given by the time derivative of the transient variation of the elemental current element. Therefore, a time-varying current element will always produce a far field and hence will cause radiation. However, if the current element is not changing with time, there will be no radiation from it. Equivalently, the current density $\boldsymbol{J}_i$ can be expressed in terms of the flow of charges; thus it is equivalent to $\rho\boldsymbol{\upsilon}$, where ρ is the charge density and $\boldsymbol{\upsilon}$ is its velocity. Therefore, radiation from a time-varying current element in (1.68) can occur if any of the following three scenarios occur:

1. The charge density ρ may change as a function of time.
2. The direction of the velocity vector $\boldsymbol{\upsilon}$ may change as a function of time.
3. The magnitude of the velocity vector $\boldsymbol{\upsilon}$ may change as a function of time, or equivalently, the charge is accelerated or decelerated.

Therefore, in theory any one of these three scenarios can cause radiation. For example, in a dipole the current goes to zero at the ends of the structure and hence the charges decelerate when they come to the end of a wire. That is why radiation seems to emanate from the ends of the wire and also from the feed point of a dipole where a current is injected or a voltage is applied and where the charges are induced and hence accelerated. Current flowing in a loop of wire can also radiate as the direction of the velocity is changing as a function of time even though its magnitude is constant. So a current flowing in a loop of wire may have a constant angular velocity, but the temporal change in the orientation of the velocity vector may cause radiation. To maintain the same current along a cross-section of the wire loop, the charges located along the inner circumference of the loop have to decelerate, whereas the charges on the outer boundary have to accelerate. This will cause radiation. In a klystron, by modulating the velocity of the electrons, one can have bunching of the electrons or a change of the electron density with time. This also causes radiation. In summary, if any one of the three conditions described above occurs, there will be radiation.

By observing (1.68), we see that a transmitting antenna acts as a differentiator of the transient waveform fed to its input. The important point to note is that an antenna impulse response on transmit is a differentiation of the excitation on transmit. Therefore in all baseband broadband simulations the differential nature of the point source must be taken into account. This implies that if the input to a point radiator is a pulse, it will radiate two impulses of opposite polarities – a derivative of the pulse. Therefore, when a baseband broadband signal is fed to an antenna, what comes out is the derivative of that pulse. It is rather unfortunate that very few simulations dealing with baseband broadband signals really take this property of an isotropic point source antenna into account in analyzing systems.

1.4.2 Reception Properties of a Point Receiver

On receive, an antenna behaves in a completely different way than on transmit. We observed that an isotropic point antenna acts as a differentiator on transmit. On receive, the voltage received at the terminals of the antenna is given by

$$V = \int \boldsymbol{E} \cdot \mathbf{d}\ell \tag{1.69}$$

where the path of the integral is along the length of the antenna. Equivalently, this voltage, which is called the *open-circuit voltage* V_{oc}, is equivalent to the dot product of the incident field vector and the effective height of the antenna and is given by [14, 15]

$$V_{oc} = \mathbf{E} \bullet \mathbf{H}_{eff}$$

The effective height of an antenna is defined by

$$H_{eff} = \int_0^H I(z)\ dz = H\, I_{av} \qquad (1.70)$$

where H is the length of the antenna and it is assumed that the maximum value of the current along the length of the antenna $I(z)$ is unity. I_{av} then is the average value of the current on the antenna. This equation is valid at only a single frequency. Therefore, when an electric field E^{inc} is incident on a small dipole of total length L from a broadside direction, it induces approximately a triangular current on the structure [15]. Therefore, the effective height in this case is $L/2$ and the open-circuit voltage induced on the structure in the frequency domain is given by

$$V_{oc}(\omega) = -\frac{L\, E^{inc}(\omega)}{2} \qquad (1.71)$$

and in the time domain as the effective height now becomes an impulse-like function, we have

$$V_{oc}(t) = -\frac{L\, E^{inc}(t)}{2} \qquad (1.72)$$

Therefore, in an electrically small receiving antenna called a *voltage probe* the induced waveform will be a replica of the incident field provided that the frequency spectrum of the incident electric field lies mainly in the low-frequency region, so that the concept of an electrically small antenna is still applicable.

In summary, the impulse response of an antenna on transmit given by (1.65) is the time derivative of the impulse response of the same antenna when it is operating in the receive mode as given by (1.72). In the frequency domain as we observe in (1.65) the term $j\omega$ is benign as it merely introduces a purely imaginary scale factor at a particular value of ω. However, the same term when transferred to the time domain represents a time derivative operation. Hence, in frequency domain the transmit radiation antenna pattern is identical to the antenna pattern when it is operating in the receive mode. In time domain, the transmit impulse response of the same antenna is the time derivative of the impulse response in the receive mode for the same antenna. At this point, it may be too hasty to jump to the conclusion that something is really amiss as it does not relate quite the same way to the reciprocity theorem which in the frequency domain has shown that the two patterns in the transmit–receive modes are identical. This is because the mathematical form of the reciprocity theorem is quite different in the time and in the frequency domains. Since the reciprocity theorem manifests itself as a product of two quantities in the frequency domain, in the time domain then it becomes a convolution. It is this phenomenon that makes the impulse response of the transmit and the receive modes different. We use another example, namely a dipole, to illustrate this point further.

1.5 RADIATION AND RECEPTION PROPERTIES OF FINITE-SIZED DIPOLE-LIKE STRUCTURES IN FREQUENCY AND IN TIME

In this section we describe the impulse responses of transmitting and receiving dipole-like structures whose dimensions are comparable to a wavelength. Therefore, these structures are not electrically small. Detailed analysis of these structures will be done in Chapter 3. In this section, the main results are summarized. The reason for choosing finite-sized structures is that the impulse responses of these wire-like structures are quite different from the cases described in the preceding section. For a finite-sized antenna structure, which is comparable to the wavelength at the frequency of operation, the current distribution on the structure can no longer be taken to be independent of frequency. Hence the frequency term must explicitly be incorporated in the expression of the current.

1.5.1 Radiation Fields from Wire-like Structures in the Frequency Domain

For a finite-sized dipole, the current distribution that is induced on it can be represented mathematically to be of the form [14]

$$I(z) = \sin\left[k\left(L/2-|z|\right)\right] \tag{1.73}$$

where L is the wire antenna length. We here assume that the current distribution is known. However, in a general situation we have to use a numerical technique to solve for the current distribution on the structure before we can solve for the far fields. This is particularly important when mutual coupling effects are present or there are other near-field scatterers. For a current distribution given by (1.73), the far fields can be obtained [14] as

$$E_\theta = \frac{I_0\,\eta\, e^{j\omega[t-r/c]}}{2\pi r}\left[\frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\frac{kL}{2}}{\sin\theta}\right] \tag{1.74}$$

where η is the characteristic impendence of free space and I_0 represents the maximum value of the current. Here L is the length of the antenna, k is the free-space wavenumber and is equal to $2\pi/\lambda = \omega/c$, where c is the velocity of light in that medium. It is important to note that only along the broadside direction and in the azimuth plane of $\theta = \pi/2 = 90°$ is the radiated electric field omni-directional in nature.

1.5.2 Radiation Fields from Wire-like Structures in the Time Domain

When the current induced on the dipole is a function of frequency, the far-zone time-dependent electric field at a spatial location r is given approximately by [15]

$$E_\theta(t) = \frac{\eta}{2\pi r \sin\theta} \left\{ \begin{aligned} I\left(t - \frac{r}{c}\right) + I\left[t - \frac{r}{c} - \frac{L}{c}\right] \\ - I\left[t - \frac{r}{c} - \frac{L}{2c}(1 + \cos\theta)\right] \\ - I\left[t - \frac{r}{c} - \frac{L}{2c}(1 - \cos\theta)\right] \end{aligned} \right\} \quad (1.75)$$

where $I(t)$ is the transient current distribution on the structure. It is interesting to note that for L/c small compared to the pulse duration of the transient current distribution on the structure, then from [15] the approximate far field can be written as

$$E_\theta \simeq \frac{\eta}{2\pi r}\left(\frac{L}{2c}\right)^2 \frac{\partial^2 I\left(t - \frac{r}{c}\right)}{\partial t^2} \sin\theta \quad (1.76)$$

that is, the far-field now is proportional to the second temporal derivative of the transient current on the structure.

1.5.3 Induced Voltage on a Finite-Sized Receive Wire-like Structure Due to a Transient Incident Field

For a finite-sized antenna of total length L, the effective height will be a function of frequency and it is given by

$$\mathrm{H}_{\mathrm{eff}}(\omega) = \int_{-L/2}^{L/2} \sin\left[k\left(\frac{L}{2} - |z|\right)\right] dz = \frac{2c}{\omega}\left[1 - \cos\left(\frac{kL}{2}\right)\right] \quad (1.77)$$

Hence the induced voltage for a broadside incidence will be given approximately by

$$\mathrm{V}_{\mathrm{oc}}(\omega) = -\mathrm{H}_{\mathrm{eff}}(\omega)\mathrm{E}^{\mathrm{inc}}(\omega) \quad (1.78)$$

In the time domain, the effective height will be given by

$$H_{\mathrm{eff}}(t) = jc \begin{cases} +1 & 0 < t < \dfrac{L}{2c} \\ -1 & \dfrac{-L}{2c} < t < 0 \end{cases} \quad (1.79)$$

Hence the transient received voltage in the antenna due to an incident field will result in the following convolution (defined by the symbol ✪) between the incident electric field and the effective height, resulting in

$$V_{oc}(t) = -E^{inc}(t) \circledast H_{eff}(t) \tag{1.80}$$

This illustrates that when (1.79) is used in (1.80), the received open-circuit voltage will be approximately the derivative of the incident field when L/c is small compared to the duration of the initial duration of the transient incident field. In Chapter 2, we study the properties of arbitrary shaped antennas in the frequency domain using a general purpose computer code described in [16]. Furthermore, we focus on the implications of near and far fields. The near/far field concepts are really pertinent in the frequency domain as they characterize the radiation properties of antennas. However, in the time domain this distinction is really not applicable as everything is near field unless we have a strictly band limited signal! In Chapter 3 the properties of arbitrary shaped antennas embedded in different materials are studied in the time domain using the methodology of [17].

1.6 CONCLUSION

The objective of this chapter has been to present the necessary mathematical formulations, popularly known as Maxwell's equations, which dictate the space-time behavior of antennas. Additionally, some examples are presented to note that the impulse response of antennas is quite complicated and the waveshapes depend on both the observation and the incident angles in azimuth and elevation of the electric fields. Specifically, the transmit impulse response of an antenna is the time derivative of the impulse response of the same antenna in the receive mode. This is in contrast to the properties in the frequency domain where the transmit antenna pattern is the same as the receive antenna pattern. Any broadband processing must deal with factoring out the impulse response of both the transmitting and receiving antennas. The examples presented in this chapter do reveal that the waveshape of the impulse response is indeed different for both transmit and receive modes, which are again dependent on both the azimuth and elevation angles. For an electrically small antenna, the radiated fields produced by it along the broadside direction are simply the differentiation of the time domain waveshape that is fed to it. While on receive it samples the field incident on it. However, for a finite-sized antenna, the radiated fields are proportional to the temporal double derivative of the current induced on it, and on receive, the same antenna differentiates the transient electric field that is incident on it. Hence all baseband broadband applications should deal with the complex problem of determining the impulse responses of the transmitting and receiving antennas. This is in contrast to spread spectrum methodologies where one deals with an instantaneous narrowband signals even when frequency hopping. For the narrowband case, determination of the impulse response is not necessary. The goal of this chapter is to outline the methodology that will be necessary to determine the impulse response of the transmit/receive antennas. By thus combining the electromagnetic analysis with the signal-processing algorithms, it will be possible to design better systems.

REFERENCES

[1] J. C. Maxwell, "A Dynamical Theory of the Electromagnetic Field", *Philosophical Transactions*, Vol. 166, pp. 459–512, 1865 (reprinted in the *Scientific Papers of James Clerk Maxwell*, Vol. 1, pp. 528–597, Dover, New York, 1952).

[2] H. Hertz, "On the Relations between Maxwell's Fundamental Equations of the Opposing Electromagnetics", (in German), *Wiedemann's Annalen*, Vol. 23, pp. 84–103, 1884. (English translation in [9, pp. 127–145]).

[3] H. Hertz, "On the Fundamental Equations of Electromagnetics for Bodies at Rest," in [9, pp. 195–240].

[4] P. J. Nahin, *Oliver Heaviside: Sage in Solitude*, IEEE Press, New York, 1988.

[5] C-T Tai and J. H. Bryant, "New Insights into Hertz's Theory of Electromagnetism", *Radio Science*, Vol. 29, No. 4, pp. 685–690, July–Aug. 1994.

[6] B. J. Hunt, *The Maxwellians*, Chap. 2, p. 24, Cornell University Press, Ithaca, NY, 1991.

[7] T. K. Sarkar, R. J. Mailloux, A. A. Oliner, M. Salazar-Palma, and D. L. Sengupta, *History of Wireless*, John Wiley and Sons, 2006.

[8] H. Hertz, "On the Finite Velocity of Propagation of Electromagnetic Action", *Sitzungsber ichte der Berliner Academic der Wissenschaften*, Feb. 2, 1888; *Wiedemann's Annalen*, Vol. 24, pp. 551, reprinted in H. Hertz (translated by D. E. Jones), *Electric Waves*, Chap. 7, pp. 107–123, Dover, New York, 1962.

[9] H. Hertz, *Electric Waves* (authorized English translation by D. E. Jones), Dover, New York, 1962.

[10] J. H. Bryant, *Heinrich Hertz: The Beginning of Microwaves*, IEEE Service Center, Piscataway, NJ, 1988.

[11] J. G. O'Hara and W. Pritcha, *Hertz and Maxwellians*, Peter Peregrinus, London, 1987.

[12] J. D. Kraus, *Electromagnetics*, McGraw-Hill, New York, 1980.

[13] R. Nevels and C. Shin, "Lorenz, Lorentz, and the Gauge", *IEEE Antennas and Propagat. Magazine*, Vol. 43, No. 3, pp. 70–72, June 2001.

[14] J. D. Kraus, *Antennas*, McGraw-Hill, New York, 1988.

[15] D. L. Sengupta and C. T. Tai, "Radiation and Reception of Transients by Linear Antennas", Chap. 4, pp. 182–234 in L. B. Felsen (ed.), *Transient Electromagnetic Fields*, Springer-Verlag, New York. pp. 182–234, 1976.

[16] B. M. Kolundzija, J. S. Ognjanovic, and T. K. Sarkar, *WIPL-D: Electromagnetic Modeling of Composite Metallic and Dielectric Structures*, Artech House, Norwood, MA, 2000.

[17] T. K. Sarkar, W. Lee, and S. M. Rao, "Analysis of Transient Scattering from Composite Arbitrarily Shaped Complex Structures", *IEEE Transactions on Antennas and Propagation*, Vol. 48, No. 10, pp. 1625–1634, Oct. 2000.