Chapter 1

Noting Numbers Scientifically

In This Chapter

- Crunching numbers in scientific and exponential notation
- Telling the difference between accuracy and precision
- Doing math with significant figures

Chemistry is a science. This means that like any other kind of scientist, a chemist tests hypotheses by doing experiments. Better tests require more reliable measurements, and better measurements are those that have more accuracy and precision. This explains why chemists get so giggly and twitchy about high-tech instruments; those instruments make better measurements. How do chemists report their precious measurements? What's the difference between accuracy and precision in those measurements? How do chemists do math with measurements? These questions may not keep you awake at night, but knowing the answers to them will keep you from making embarrassing, rookie errors in chemistry. So we address them in this chapter.

Using Exponential and Scientific Notation to Report Measurements

Because chemistry concerns itself with ridiculously tiny things like atoms and molecules, chemists often find themselves dealing with extraordinarily small or extraordinarily large numbers. Numbers describing the distance between two atoms joined by a bond, for example, run in the ten-billionths of a meter. Numbers describing how many water molecules populate a drop of water run into the trillions of trillions.

To make working with such extreme numbers easier, chemists turn to *scientific notation,* which is a special kind of exponential notation. *Exponential notation* simply means writing a number in a way that includes exponents. Every number is written as the product of two numbers, a coefficient and a power of 10. In plain old exponential notation, a coefficient can be any value of a number multiplied by a power with a base of 10 (such as 10^4). But scientists have rules for coefficients in scientific notation. In scientific notation, a coefficient is always at least 1 and always less than 10 (such as 7, 3.48, or 6.0001).

REMEMBER

To convert a very large or very small number to *scientific notation,* position a decimal point between the first and second digits. Count how many places you moved the decimal to the right or left, and that's the power of 10. If you moved the decimal to the left, the power is positive; to the right is negative. (You use the same process for exponential notation, but you can position the decimal anywhere.)

TIP

In scientific notation, the coefficients should be greater than 1 and less than 10, so look for the first digit other than 0.

To convert a number written in scientific notation back into decimal form, just multiply the coefficient by the accompanying power of 10.

EXAMPLE

Q. Convert 47,000 to scientific notation.

A. **47,000 = 4.7×10^4.** First, imagine the number as a decimal:

47,000.00

Next, move the decimal between the first two digits:

4.7000

Then count how many positions to the left you moved the decimal (four, in this case), and write that as a power of 10: 4.7×10^4.

Q. Convert 0.007345 to scientific notation.

A. **0.007345 = 7.345×10^{-3}.** First, move the decimal between the first two nonzero digits:

7.345

Then count how many positions to the right you moved the decimal (three, in this case), and write that as a power of 10: 0.007345 = 7.345×10^{-3}.

1. Convert 200,000 into scientific notation.

Solve It

2. Convert 80,736 into scientific notation.

Solve It

3. Convert 0.00002 into scientific notation.

Solve It

4. Convert 6.903×10^2 from scientific notation into decimal form.

Solve It

Multiplying and Dividing in Scientific Notation

A major benefit of presenting numbers in scientific notation is that it simplifies common arithmetic operations. (Another benefit is that, among the pocket-protector set, numbers with exponents just look way cooler.) The simplifying powers of scientific notation are most evident in multiplication and division. (As we describe in the next section, addition and subtraction benefit from exponential notation, but not necessarily from strict scientific notation.)

REMEMBER

To multiply two numbers written in scientific notation, multiply the coefficients, and then add the exponents. To divide two numbers, simply divide the coefficients, and then subtract the exponent of the *denominator* (the bottom number) from the exponent of the *numerator* (the top number).

EXAMPLE

Q. Multiply, using the "shortcuts" of scientific notation: $(1.4 \times 10^{2}) \times (2.0 \times 10^{-5})$.

A. $\mathbf{2.8 \times 10^{-3}}$**.** First, multiply the coefficients:

$1.4 \times 2.0 = 2.8$

Next, add the exponents of the powers of 10:

$10^{2} \times 10^{-5} = 10^{2+(-5)} = 10^{-3}$

Finally, join your new coefficient to your new power of 10:

2.8×10^{-3}

Q. Divide, using the "shortcuts" of scientific notation: $(3.6 \times 10^{-3}) / (1.8 \times 10^{4})$.

A. $\mathbf{2.0 \times 10^{-7}}$**.** First, divide the coefficients:

$3.6 / 1.8 = 2.0$

Next, subtract the exponent of the denominator from the exponent of the numerator:

$10^{-3} / 10^{4} = 10^{-3-4} = 10^{-7}$

Then, join your new coefficient to your new power of 10:

2.0×10^{-7}

5. Multiply $(2.2 \times 10^{9}) \times (5.0 \times 10^{-4})$.

Solve It

6. Divide $(9.3 \times 10^{-5}) / (3.1 \times 10^{2})$.

Solve It

7. Using scientific notation, multiply 52×0.035.

Solve It

8. Using scientific notation, divide $0.00809 / 20.3$.

Solve It

Using Exponential Notation to Add and Subtract

Addition or subtraction gets easier when your numbers are expressed as coefficients of identical powers of 10. To wrestle your numbers into this form, you might need to use coefficients less than 1 or greater than 10. So, scientific notation is a bit too strict for addition and subtraction, but exponential notation still serves us well.

REMEMBER

To add two numbers easily by using exponential notation, first express each number as a coefficient and a power of 10, making sure that 10 is raised to the same exponent in each number. Then add the coefficients. To subtract numbers in exponential notation, follow the same steps, but subtract the coefficients.

EXAMPLE

Q. Use exponential notation to add these numbers: $3{,}710 + 2.4 \times 10^2$.

A. $\mathbf{39.5 \times 10^2}$**.** First, convert both numbers to the same power of 10:

37.1×10^2 and 2.4×10^2

Next, add the coefficients:

$37.1 + 2.4 = 39.5$

Finally, join your new coefficient to the shared power of 10:

39.5×10^2

Q. Use exponential notation to do this subtraction: 0.0743 – 0.0022.

A. **7.21×10^{-2}.** First, convert both numbers to the same power of 10:

7.43×10^{-2} and 0.22×10^{-2}

Next, subtract the coefficients:

$7.43 - 0.22 = 7.21$

Then join your new coefficient to the shared power of 10:

7.21×10^{-2}

9. Add $398 \times 10^{-6} + 147 \times 10^{-6}$.

Solve It

10. Subtract $7.685 \times 10^{5} - 1.283 \times 10^{5}$.

Solve It

11. Using exponential notation, add 0.00206 + 0.0381.

Solve It

12. Using exponential notation, subtract 9,352 – 431.

Solve It

Distinguishing between Accuracy and Precision

Accuracy and precision . . . precision and accuracy . . . same thing, right? Chemists everywhere gasp in horror, reflexively clutching their pocket protectors — accuracy and precision are different!

- *Accuracy* describes how closely a measurement approaches an actual, true value.
- *Precision*, which we discuss more in the next section, describes how close repeated measurements are to one another, regardless of how close those measurements are to the actual value. The bigger the difference between the largest and smallest values of a repeated measurement, the less precision you have.

The two most common measurements related to accuracy are *error* and *percent error.*

- **Error** measures accuracy, the difference between a measured value and the actual value:

  Actual value – Measured value = Error
- **Percent error** compares error to the size of the thing being measured:

  |Error| / Actual value = Fraction error

  Fraction error × 100 = Percent error

Being off by 1 meter isn't such a big deal when measuring the altitude of a mountain, but it's a shameful amount of error when measuring the height of an individual mountain climber.

Q. A police officer uses a radar gun to clock a passing Ferrari at 131 miles per hour (mph). The Ferrari was really speeding at 127 mph. Calculate the error in the officer's measurement.

A. **–4 mph.** First, determine which value is the actual value and which is the measured value:

Actual value = 127 mph; measured value = 131 mph

Then calculate the error by subtracting the measured value from the actual value:

Error = 127 mph – 131 mph = –4 mph

Q. Calculate the percent error in the officer's measurement of the Ferrari's speed.

A. **3.15%.** First, divide the absolute value (the size, as a positive number) of the error by the actual value:

|–4 mph| / 127 mph = 4 mph / 127 mph = 0.0315

Next, multiply the result by 100 to obtain the percent error:

Percent error = 0.0315 × 100 = 3.15%

13. Two people, Reginald and Dagmar, measure their weight in the morning by using typical bathroom scales, instruments that are famously unreliable. The scale reports that Reginald weighs 237 pounds, though he actually weighs 256 pounds. Dagmar's scale reports her weight as 117 pounds, though she really weighs 129 pounds. Whose measurement incurred the greater error? Whose incurred a greater percent error?

Solve It

14. Two jewelers were asked to measure the mass of a gold nugget. The true mass of the nugget was 0.856 grams (g). Each jeweler took three measurements. The average of the three measurements was reported as the "official" measurement with the following results:

Jeweler A: 0.863g, 0.869g, 0.859g

Jeweler B: 0.875g, 0.834g, 0.858g

Which jeweler's official measurement was more accurate? Which jeweler's measurements were more precise? In each case, what was the error and percent error in the official measurement?

Solve It

Expressing Precision with Significant Figures

After you know how to express your numbers in scientific notation and how to distinguish precision from accuracy (we cover both topics earlier in this chapter), you can bask in the glory of a new skill: using scientific notation to express precision. The beauty of this system is that simply by looking at a measurement, you know just how precise that measurement is.

REMEMBER

When you report a measurement, you should only include digits if you're really confident about their values. Including added digits in a measurement means something — it means that you really know what you're talking about — so we call the included digits *significant figures.* The more significant figures in a measurement, the more precise that measurement must be. The last significant figure in a measurement is the only figure that includes any uncertainty. Here are the rules for deciding what is and what isn't a significant figure:

- **Any nonzero digit is significant.** So, 6.42 seconds (s) contains three significant figures.
- **Zeros sandwiched between nonzero digits are significant.** So, 3.07s contains three significant figures.
- **Zeros on the left side of the first nonzero digit are *not* significant.** So, 0.0642s and 0.00307s each contain three significant figures.
- **When a number is greater than 1, all digits to the right of the decimal point are understood to be significant.** So, 1.76s has three significant figures, while 1.760s has four significant figures. The "6" is uncertain in the first measurement, but is certain in the second measurement.

- **When a number has no decimal point, any zeros after the last nonzero digit *may or may not* be significant.** So, in a measurement reported as 1,370s, you can't be certain if the "0" is a certain value, or if it's merely a placeholder.

  TIP

  Be a good chemist. Report your measurements in scientific notation to avoid such annoying ambiguities (see the earlier section, "Using Exponential and Scientific Notation to Report Measurements").
- **Numbers from *counting* (for example, 1 kangaroo, 2 kangaroos, 3 kangaroos . . .) or from *defined quantities* (that is to say, 60 seconds per 1 minute) are understood to have an unlimited number of significant figures;** in other words, these values are completely certain.
- **If a number is already written in scientific notation, then all the digits in the coefficient are significant, and none others.**

TIP

So, the number of significant figures you use in a reported measurement should be consistent with your certainty about that measurement. If you know your speedometer is routinely off by 5 miles per hour, then you have no business protesting to a policeman that you were only going 63.2 miles per hour.

EXAMPLE

Q. How many significant figures are in the following three measurements?

20,175 yards, 1.75×10^5 yards, 1.750×10^5 yards

A. **Five, three, and four significant figures, respectively.** In the first measurement, all digits are nonzero, except for a 0 that is sandwiched between nonzero digits, which counts as significant. The second measurement contains only nonzero digits. The third measurement contains a 0, but that 0 is the final digit and to the right of the decimal point, and is therefore significant.

15. Modify the following three measurements so that each possesses the indicated number of significant figures (SF) and is expressed properly in scientific notation.

76.93×10^{-2} meters (1 SF), 0.0007693 meters (2 SF), 769.3 meters (3 SF)

Solve It

16. In chemistry, the potential error associated with a measurement is often reported alongside the measurement, as in: 793.4 ±0.2 grams. This report indicates that all digits are certain except the last, which may be off by as much as 0.2 grams in either direction. What, then, is wrong with the following reported measurements?

893.7 ±1 gram, 342 ±0.01 gram

Solve It

Doing Arithmetic with Significant Figures

Doing chemistry means making a lot of measurements. The point of spending a pile of money on cutting-edge instruments is to make really good, really precise measurements. After you've got yourself some measurements, you roll up your sleeves, hike up your pants, and do math with the measurements.

REMEMBER

When doing that math, you need to follow some rules to make sure that your sums, differences, products, and quotients honestly reflect the amount of precision present in the original measurements. You can be honest (and avoid the skeptical jeers of surly chemists) by taking things one calculation at a time, following a few simple rules. One rule applies to addition and subtraction, and another rule applies to multiplication and division.

- **When adding or subtracting, round the sum or difference to the same number of decimal places as the measurement with the fewest decimal places.** Rounding like this is honest, because you acknowledge that your answer can't be any more precise than the least precise measurement that went into it.
- **When multiplying or dividing, round the product or quotient so that it has the same number of significant figures as the least precise measurement — the measurement with the fewest significant figures.**

Notice the difference between the two rules. When you add or subtract, you assign significant figures in the answer based on the number of decimal places in each original measurement. When you multiply or divide, you assign significant figures in the answer based on the total number of significant figures in each original measurement.

TIP

Caught up in the breathless drama of arithmetic, you may sometimes perform multi-step calculations that include addition, subtraction, multiplication, and division, all at once. No problem. Follow the normal order of operations, doing multiplication and division first, followed by addition and subtraction. At each step, follow the simple rules previously described, and then move on to the next step.

EXAMPLE

Q. Express the following sum with the proper number of significant figures:

35.7 miles + 634.38 miles + 0.97 miles = ?

A. **671.1 miles.** Adding the three values yields a raw sum of 671.05 miles. However, the 35.7 miles measurement extends only to the tenths place; the answer must therefore be rounded to the tenths place, from 671.05 to 671.1 miles.

Q. Express the following product with the proper number of significant figures:

27 feet × 13.45 feet = ?

A. **3.6×10^2 feet^2.** Of the two measurements, one has two significant figures (27 feet) and the other has four significant figures (13.45 feet). The answer is therefore limited to two significant figures. The raw product, 363.15 feet^2, must be rounded. You could write 360 feet^2, but doing so implies that the final 0 is significant and not just a placeholder. For clarity, express the product in scientific notation, as 3.6×10^2 feet^2.

17. Express this difference using the appropriate number of significant figures:

127.379 seconds – 13.14 seconds + 1.2×10^{-1} seconds = ?

Solve It

18. Express the answer to this calculation using the appropriate number of significant figures:

345.6 feet × (12 inches / 1 foot) = ?

Solve It

19. Report the difference using the appropriate number of significant figures:

3.7×10^{-4} minutes – 0.009 minutes = ?

Solve It

20. Express the answer to this multistep calculation using the appropriate number of significant figures:

87.95 feet × 0.277 feet + 5.02 feet – 1.348 feet / 10.0 feet = ?

Solve It

Answers to Questions on Noting Numbers Scientifically

The following are the answers to the practice problems presented in this chapter.

1 **2×10^5.** Move the decimal point immediately after the 2 to create a coefficient between 1 and 10. Because this means moving the decimal point five places to the left, multiply the coefficient of 2 with the power 10^5.

2 **8.0736×10^4.** Move the decimal point immediately after the 8 to create a coefficient between 1 and 10. This involves moving the decimal point four places to the left, so multiply the coefficient of 8.0736 with the power 10^4.

3 **2×10^{-5}.** Move the decimal point immediately after the 2 to create a coefficient between 1 and 10. This means moving the decimal point five spaces to the right, so multiply the coefficient of 2 with the power 10^{-5}.

4 **690.3.** This question requires you to understand the meaning of scientific notation in order to reverse the number back into "regular" decimal form. Because 10^2 equals 100, multiply the coefficient 6.903 with 100. This moves the decimal point two spaces to the right.

5 **1.1×10^6.** The raw calculation yields 11×10^5, which converts to the given answer when expressed in scientific notation.

6 **3.0×10^{-7}.** The ease of math with scientific notation shines through in this problem. Dividing the coefficients yields a coefficient quotient of 3.0, while dividing the powers yields a quotient of 10^{-7}. Marrying the two quotients produces the given answer, already in scientific notation.

7 **1.82.** First, convert each number to scientific notation: 5.2×10^1 and 3.5×10^{-2}. Next, multiply the coefficients: $5.2 \times 3.5 = 18.2$. Then add the exponents on the powers of 10: $10^{1+(-2)} = 10^{-1}$. Finally, join the new coefficient with the new power: 18.2×10^{-1}. Expressed in scientific notation, this answer is $1.82 \times 10^0 = 1.82$.

8 **3.99×10^{-4}.** First, convert each number to scientific notation: 8.09×10^{-3} and 2.03×10^1. Then divide the coefficients: $8.09 / 2.03 = 3.99$. Next, subtract the exponent on the denominator from the exponent of the numerator to get the new power of 10: $10^{-3-1} = 10^{-4}$. Join the new coefficient with the new power: 3.99×10^{-4}. Finally, express gratitude that the answer is already conveniently expressed in scientific notation.

9 **545×10^{-6}.** Because the numbers are each already expressed with identical powers of 10, you can simply add the coefficients: $398 + 147 = 545$. Then join the new coefficient with the original power of 10.

10 **6.402×10^5.** Because the numbers are each expressed with the same power of 10, you can simply subtract the coefficients: $7.685 - 1.283 = 6.402$. Then join the new coefficient with the original power of 10.

11 **40.16×10^{-3} (or an equivalent expression).** First, convert the numbers so they each use the same power of 10: 2.06×10^{-3} and 38.1×10^{-3}. Here, we used 10^{-3}, but you can use a different power, so long as the same power is used for each number. Next, add the coefficients: $2.06 + 38.1 = 40.16$. Finally, join the new coefficient with the shared power of 10.

12 **89.21×10^2 (or an equivalent expression).** First, convert the numbers so each uses the same power of 10: 93.52×10^2 and 4.31×10^2. Here, we picked 10^2, but any power is fine so long as the two numbers have the same power. Then subtract the coefficients: $93.52 - 4.31 = 89.21$. Finally, join the new coefficient with the shared power of 10.

13 **Reginald's measurement incurred the greater magnitude of error, while Dagmar's measurement incurred the greater percent error.**

Reginald's scale reported with an error of 256 pounds – 237 pounds = 19 pounds. Dagmar's scale reported with an error of 129 pounds – 117 pounds = 12 pounds. Comparing the *magnitudes* of error, we see that 19 pounds > 12 pounds. However, Reginald's measurement had a percent error of 19 pounds / 256 pounds $\times$ 100 = 7.4%, while Dagmar's measurement had a percent error of 12 pounds / 129 pounds $\times$ 100 = 9.3%.

14 Jeweler A's "official" average measurement was 0.864g, while Jeweler B's official measurement was 0.856g; thus, **Jeweler B's official measurement is more *accurate* because it's closer to the actual value of 0.856g.**

However, **Jeweler A's measurements were more *precise*** because the differences between A's measurements were much smaller than the differences between B's measurements. Despite the fact that Jeweler B's average measurement was closer to the actual value, the *range* of his measurements (that is, the difference between the largest and the smallest measurements) was 0.041g. The range of Jeweler A's measurements was 0.010g.

This example shows how low precision measurements can yield highly accurate results through averaging of repeated measurements. In the case of Jeweler A, the error in the official measurement was 0.864g – 0.856g = 0.008g. The corresponding percent error was 0.008g / 0.856g $\times$ 100 = 0.9%. In the case of Jeweler B, the error in the official measurement was 0.856g – 0.856g = 0.000g. Accordingly, the percent error was 0%.

15 With the correct number of significant figures and expressed in scientific notation, the measurements should read as follows: **8×10^{-1} meters, 7.7×10^{-4} meters, 7.69×10^{2} meters.**

16 **"893.7 ± 1 gram" is an improperly reported measurement because the reported value, 893.7, suggests that the measurement is certain to within a few tenths of a gram.** The reported error is known to be greater, at ±1 gram. The measurement should be reported as "894 ±1 gram."

"342 ±0.01 gram" is improperly reported because the reported value, 342, gives the impression that the measurement becomes uncertain at the level of grams. The reported error makes clear that uncertainty creeps into the measurement only at the level of hundredths of a gram. The measurement should be reported as "342.00 ±0.01 gram."

17 **1.1436×10^{2} seconds.** The trick here is remembering to convert all measurements to the same power of 10 before comparing decimal places for significant figures. Doing so reveals that 1.2×10^{-1} seconds goes to the hundredths of a second, despite the fact that the measurement contains only two significant figures. The raw calculation yields 114.359 seconds, which rounds properly to the hundredths place (taking significant figures into account) as 114.36 seconds, or 1.1436×10^{2} seconds in scientific notation.

18 **4.147×10^{3} inches.** Here, you must recall that defined quantities (1 foot is defined as 12 inches) have unlimited significant figures. So, our calculation is limited only by the number of significant figures in the 345.6 feet measurement. When you multiply 345.6 feet by 12 inches per foot, the feet cancel, leaving units of inches. The raw calculation yields 4,147.2 inches, which rounds properly to four significant figures as 4,147 inches, or 4.147×10^{3} inches in scientific notation.

19 **-9×10^{-3} minutes.** Here, it helps here to convert all measurements to the same power of 10 so you can more easily compare decimal places in order to assign the proper number of significant figures. Doing so reveals that 3.7×10^{-4} minutes goes to the hundred-thousandths of a minute, while 0.009 minutes goes to the thousandths of a minute. The raw calculation yields –0.00863 minutes, which rounds properly to the thousandths place (taking significant figures into account) as –0.009 minutes, or -9×10^{-3} minutes in scientific notation.

20 **2.93×10^{1} feet.** Following standard order of operations, this problem can be executed in two main steps, first performing multiplication and division, and then performing addition and subtraction.

Following the rules of significant figure math, the first step yields: 24.4 feet + 5.02 feet – 0.135 feet. Each product or quotient contains the same number of significant figures as the number in the calculation with the fewest number of significant figures.

The second step yields 29.3 feet, or 2.93×10^{1} feet in scientific notation. The final sum goes only to the tenths place, because the number in the calculation with the fewest decimal places went only to the tenths place.