## PART I

## OPTICS TECHNOLOGY FOR DEFENSE SYSTEMS

## CHAPTER 1

## OPTICAL RAYS

Geometric or ray optics [16] is used to describe the path of light in free space in which propagation distance is much greater than the wavelength of the light-normally microns (see Section 1.2.3 for more exact conditions). Note that we cannot apply ray theory if the media properties vary noticeably in distances comparable to wavelength; for such cases, we use more computationally demanding finite approximation techniques such as finite-difference time domain (FDTD) [154] or finite elements [78, 79]. Ray theory postulates rays that are at right angles to wave fronts of constant phase. Such rays describe the path along which light emanates from a source and the rays track the Poynting vector of power in the wave. Geometric or ray optics provides insight into the distribution of energy in space with time. The spread of neighboring rays with time enables computation of attenuation, which provides information analogous to that provided by diffraction equations but with less computation. Ray optics is extensively used for the passage of light through optical elements, such as lenses, and inhomogeneous media for which refractive index (or dielectric constant) varies with position in space.

In Section 1.1, we derive the paraxial equation that reduces dimensionality when light stays close to the axis. In Section 1.2, we study geometric or ray optics: Fermat's principle, limits of ray theory, the ray equation, rays through quadratic media, and matrix representations. In Section 1.3, we consider thin lens optics for launching and/or receiving beams: magnification, beam expanders, beam compressors, telescopes, microscopes, and spatial filters.

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FIGURE 1.1 Illustrates the paraxial approximation.

### 1.1 PARAXIAL OPTICS

In 1840, Gauss proposed the paraxial approximation for propagation of beams that stay close to the axis of an optical system. In this case, propagation is, say, in the $z$ direction and the light varies in transverse $x$ and $y$ directions over only a small distance relative to the distance associated with the radius of curvature of a spherically curved surface in $x$ and $y$ (Figure 1.1). The region of the spherical surface near the axis can be approximated by a parabola. The spherical surface of curvature $R$ is

$$
\begin{equation*}
x^{2}+y^{2}+z^{2}=R^{2} \quad \text { or } \quad z=R \sqrt{\left(1-\frac{x^{2}+y^{2}}{R^{2}}\right)} \tag{1.1}
\end{equation*}
$$

Using the binomial theorem to eliminate the square root,

$$
\begin{equation*}
z=R\left(1-\frac{x^{2}+y^{2}}{2 R^{2}}\right) \quad \text { or } \quad R-z=\frac{x^{2}+y^{2}}{2 R} \tag{1.2}
\end{equation*}
$$

which is the equation for a parabola.

### 1.2 GEOMETRIC OR RAY OPTICS

### 1.2.1 Fermat's Principle

In 1658, Fermat introduced one of the first variational principles in physics, the basic principle that governs geometrical optics [16]: A ray of light will travel between points $P_{1}$ and $P_{2}$ by the shortest optical path $L=\int_{P_{1}}^{P_{2}} n \mathrm{~d} s$; no other path will have a shorter optical path length. The optical path length is the equivalent path length in air for a path through a medium of refractive index $n$. Equivalently, because the refractive index is $n=c / v$ ( $v$ is the phase velocity, and $c$ is the velocity of light), $n \mathrm{~d} s=c \mathrm{~d} t$, this is also the shortest time path. As the optical path length or time differs for each path, our optimization to determine the shortest (a minimum extremum) is that of a length or time function among many path functions, that is a function of a function (a functional), and this requires the use of calculus of variations [42]. Fermat's principle is written for minimum optical path length or, equivalently, for minimum time:

$$
\begin{equation*}
\delta L=\delta \int_{P_{1}}^{P_{2}} n \mathrm{~d} s=0 \quad \text { or } \quad \delta L=\delta \int_{P_{1}}^{P_{2}} c \mathrm{~d} t=0 \tag{1.3}
\end{equation*}
$$

Fermat's principle lends itself to geometric optics in which light is considered to be rays that propagate at right angles to the phase front of a wave, normally in the direction of the Poynting power vector. Note that electromagnetic waves are transverse, and the electric and magnetic fields in free space oscillate at right angles to the direction of propagation and hence to the ray path. When valid, a wave can be represented more simply by a single ray.

### 1.2.2 Fermat's Principle Proves Snell's Law for Refraction

Fermat's principle can be used to directly solve problems of geometric optics as illustrated by our proof of Snell's law of refraction, the bending at an interface between two media of different refractive indices $n_{1}=\sqrt{\mu_{1} \epsilon_{1}}$ and $n_{2}=\sqrt{\mu_{2} \epsilon_{2}}$, where $\epsilon$ is the dielectric constant and $\mu$ is the relative permeability (Figure 1.2). From Fermat's principle, the optical path from $P_{1}$ to $P_{2}$ intercepts the dielectric interface at $R$ so that the optical path length through $R$ is the least for all possible intercepts at the interface. Because at an extremum the function in equation (1.3) has zero gradient, moving the intercept point a very small variational distance $\delta x$ along the interface to $Q$ will not change the optical path length. From Figure 1.2, the change in optical path length when moving from the path through $R$ to the path through $Q$ is

$$
\begin{equation*}
\delta s-\delta s^{\prime}=n \delta x \sin \theta-n^{\prime} \delta x \sin \theta^{\prime}=0 \tag{1.4}
\end{equation*}
$$

which gives Snell's law

$$
\begin{equation*}
n \sin \theta=n^{\prime} \sin \theta^{\prime} \tag{1.5}
\end{equation*}
$$



FIGURE 1.2 Deriving Snell's law from Fermat's principle.

When light passes through an inhomogeneous medium for which refractive index $n(\mathbf{r})=n(x, y, z)$ varies with position, a ray will no longer be straight. The divergence of adjacent rays provides an estimate of the attenuation as a function of distance along the ray.

### 1.2.3 Limits of Geometric Optics or Ray Theory

Rays provide an accurate solution to the wave equation only when the radius of curvature of the rays and the electric field vary only slowly relative to wavelength, which is often the case for light whose wavelength is only in micrometers. When light rays come together as in the focus region from a convex lens, rapid changes in field can occur in distances comparable to a wavelength. Hence, rays are inaccurate representations for the solution to the wave equation at so-called caustics (the envelope formed by the intersection of adjacent rays).

As light travels straight in a constant medium, we can discretize our region into small regions of different but fixed refractive index in a finite-difference technique. The rays across a small region are then coupled into adjacent regions using Snell's law. The steps in refractive index between small regions can cause spurious caustics in the ray diagrams, which can be minimized by switching to a piecewise linear refractive index approximation, as in a first-order polynomial finite-element approach [78, 79]. To plot a ray from a source to a target, we can draw multiple rays starting out at difference angles from the source until we find one that passes through the target; this is a two-point boundary problem.

### 1.2.4 Fermat's Principle Derives Ray Equation

The ray equation, critical in geometric optics, describes the path of an optical ray through an inhomogeneous medium in which refractive index changes in 3D space $[16,148,176]$. The optical path length for use in Fermat's principle, equation
(1.3), may be written by factoring out $\mathrm{d} z$ from $\mathrm{d} s=\sqrt{\mathrm{d} x^{2}+\mathrm{d} y^{2}+\mathrm{d} z^{2}}$ :

$$
\begin{align*}
\delta \int_{P_{1}}^{P_{2}} n \mathrm{~d} s & =\delta \int_{z_{1}}^{z_{2}} n(x, y, z) \sqrt{\left(\frac{\mathrm{d} x}{\mathrm{~d} z}\right)^{2}+\left(\frac{\mathrm{d} y}{\mathrm{~d} z}\right)^{2}+1} \mathrm{~d} z \\
& =\delta \int_{z_{1}}^{z_{2}} n(x, y, z) \sqrt{x^{\prime 2}+y^{\prime 2}+1} \mathrm{~d} z \tag{1.6}
\end{align*}
$$

where prime indicates $\mathrm{d} / \mathrm{d} z$ and $\mathrm{d} s=\sqrt{x^{\prime 2}+y^{\prime 2}+1} \mathrm{~d} z$. Equation (1.6) can be written as $\delta \int_{z_{1}}^{z_{2}} F \mathrm{~d} z$, where the integrand $F$ has the form of a functional (function of functions)

$$
\begin{equation*}
F\left(x^{\prime}, y^{\prime}, x, y, z\right) \equiv n(x, y, z) \sqrt{x^{\prime 2}+y^{\prime 2}+1} \tag{1.7}
\end{equation*}
$$

From calculus of variations [16], the solutions for extrema (maximum or minimum) with integrand of the form of equation (1.7) are the Euler equations

$$
\begin{equation*}
F_{x}-\frac{\mathrm{d}}{\mathrm{~d} z} F_{x^{\prime}}=0, \quad F_{y}-\frac{\mathrm{d}}{\mathrm{~d} z} F_{y^{\prime}}=0 \tag{1.8}
\end{equation*}
$$

where subscripts refer to partial derivatives. From equation (1.7) and $x^{\prime}=\mathrm{d} x / \mathrm{d} z$,

$$
\begin{equation*}
F_{x}=\frac{\partial n}{\partial x} \mathrm{~d} s=\frac{\partial n}{\partial x} \sqrt{x^{\prime 2}+y^{\prime 2}+1}=\frac{\partial n}{\partial x} \frac{\mathrm{~d} s}{\mathrm{~d} z} \tag{1.9}
\end{equation*}
$$

and

$$
\begin{equation*}
F_{x^{\prime}}=n \frac{1}{2 \sqrt{x^{\prime 2}+y^{\prime 2}+1}} 2 x^{\prime}=n \frac{\mathrm{~d} x}{\mathrm{~d} z} \frac{\mathrm{~d} z}{\mathrm{~d} s}=n \frac{\mathrm{~d} x}{\mathrm{~d} s} \tag{1.10}
\end{equation*}
$$

Similar equations apply for $F_{y}$ and $F_{y^{\prime}}$. Substituting equations (1.9) and (1.10) into equation (1.8) gives

$$
\begin{equation*}
\frac{\partial n}{\partial x} \frac{\mathrm{~d} s}{\mathrm{~d} z}-\frac{\mathrm{d}}{\mathrm{~d} z}\left(n \frac{\mathrm{~d} x}{\mathrm{~d} s}\right)=\frac{\partial n}{\partial x}-\frac{d z}{d s} \frac{\mathrm{~d}}{\mathrm{~d} z}\left(n \frac{\mathrm{~d} x}{\mathrm{~d} s}\right)=0 \tag{1.11}
\end{equation*}
$$

The resulting equations for the ray path are

$$
\begin{equation*}
\frac{\mathrm{d}}{\mathrm{~d} s}\left(n \frac{\mathrm{~d} x}{\mathrm{~d} s}\right)=\frac{\partial n}{\partial x}, \quad \frac{\mathrm{~d}}{\mathrm{~d} s}\left(n \frac{\mathrm{~d} y}{\mathrm{~d} s}\right)=\frac{\partial n}{\partial y}, \quad \frac{\mathrm{~d}}{\mathrm{~d} s}\left(n \frac{\mathrm{~d} z}{\mathrm{~d} s}\right)=\frac{\partial n}{\partial z} \tag{1.12}
\end{equation*}
$$

where the last equation is obtained by reassigning coordinates, by analogy, or by additional algebraic manipulation [16]. These equations can be written in vector form
for the vector ray equation

$$
\begin{equation*}
\frac{\mathrm{d}}{\mathrm{~d} s}\left(n \frac{\mathrm{~d} \mathbf{r}}{\mathrm{~d} s}\right)=\nabla n \tag{1.13}
\end{equation*}
$$

Another derivation [16] for the ray equation provides a different perspective. The derivation generates, from Maxwell's equations or from the wave equation, an equivalent to Fermat's principle, the eikonal equation.

$$
\begin{equation*}
(\nabla S)^{2}=n^{2} \quad \text { or } \quad\left(\frac{\partial S}{\partial x}\right)^{2}+\left(\frac{\partial S}{\partial y}\right)^{2}+\left(\frac{\partial S}{\partial z}\right)^{2}=n^{2}(x, y, z) \tag{1.14}
\end{equation*}
$$

The eikonal equation relates phase fronts $S(\mathbf{r})=$ constant and refractive index $n$. A ray $n \mathbf{s}$ is in the direction at right angles to the phase front, that is, in the direction of the gradient of $S(\mathbf{r})$, or

$$
\begin{equation*}
n \mathbf{s}=\nabla S \quad \text { or } \quad n \frac{\mathrm{~d} \mathbf{r}}{\mathrm{~d} s}=\nabla S \tag{1.15}
\end{equation*}
$$

By taking the derivative of equation (1.15) with respect to $s$, we obtain the ray equation (1.13).

### 1.2.5 Useful Applications of the Ray Equation

We illustrate the ray equation for rays propagating in a $z-y$ plane of a slab, where $z$ is the propagation direction axis for the paraxial approximation and refractive index varies transversely in $y$. For a homogeneous medium, $n$ is constant and $\nabla n=\partial n / \partial y=$ 0 . Then the ray equation (1.13) becomes $\mathrm{d}^{2} y / \mathrm{d} z^{2}=0$. After integrating twice, $y=$ $a z+b$, a straight line in the $z-y$ plane. Therefore, in a numerical computation, we discretize the refractive index profile into piecewise constant segments in $y$ and obtain a piecewise linear optical ray path in plane $z-y$.

For a linearly varying refractive index in $y, n=n_{0}+a y$ with $n \approx n_{0}, \partial n / \partial y=$ $a$, the ray equation (1.13) becomes $\mathrm{d}^{2} y / \mathrm{d} z^{2} \approx a / n_{0}$. After two integrations, $y=$ $\left(a / n_{0}\right) z^{2}+\left(b / n_{0}\right) z+d$, which is a quadratic in the $z-y$ plane and can be represented to first approximation by a spherical arc. Therefore, if we discretize the refractive index profile into piecewise linear segments, we obtain a ray path of joined arcs that is smoother than the piecewise linear optical ray path for a piecewise constant refractive index profile. The approach is extrapolatable to higher dimensions.

Another useful refractive index profile is that of a quadratic index medium, in which the refractive index smoothly decreases radially out from the axis of a cylindrical body (Figure 1.3a):

$$
\begin{equation*}
n^{2}=n_{0}^{2}\left(1-(g r)^{2}\right) \quad \text { with } \quad r^{2}=\left(x^{2}+y^{2}\right) \tag{1.16}
\end{equation*}
$$



FIGURE 1.3 Ray in quadratic index material: (a) refractive index profile and (b) ray path.
where $g$ is the strength of the curvature and $g r \ll 1$. Material doping creates such a profile in graded index fiber to replace step index fiber. In a cylindrical piece of glass, such an index profile will act as a lens, called a GRIN lens [47]. A GRIN lens can be attached to the end of an optical fiber and can match the fiber diameter to focus or otherwise image out of the fiber. In the ray equation, for the paraxial approximation, $\mathrm{d} / \mathrm{d} s=\mathrm{d} / \mathrm{d} z$, and from equation (1.16), $\partial n / \partial r=-n_{0} g^{2} r$. So the ray equation reduces to

$$
\begin{equation*}
\frac{\mathrm{d}^{2} r}{\mathrm{~d} z^{2}}+g^{2} r=0 \tag{1.17}
\end{equation*}
$$

which has $\sin$ and cos solutions. A solution to equation (1.17) with initial conditions $\left(r_{0}\right)_{\text {in }}$ and $\left(\mathrm{d} r_{0} / \mathrm{d} z\right)_{\text {in }}=\left(r_{0}^{\prime}\right)_{\text {in }}$ is

$$
\begin{equation*}
r=\left(r_{0}\right)_{\text {in }} \cos (g z)+\left(r_{0}^{\prime}\right)_{\text {in }} \frac{\sin (g z)}{g} \tag{1.18}
\end{equation*}
$$

which can be verified by substituting into equation (1.17). A ray according to equation (1.18) for a profile, equation (1.17), is shown in Figure 1.3b.

### 1.2.6 Matrix Representation for Geometric Optics

The ability to describe paraxial approximation propagation in the $z$ direction through circularly symmetric optical components using a location and a slope in geometric optics allows for a $2 \times 2$ matrix representation [44, 132, 176].

We consider a material of constant refractive index and width $d$. For this medium, light propagates in a straight line (Section 1.2.3), and a ray path does not change slope, $r_{\text {out }}^{\prime}=r_{\text {in }}^{\prime}$. The ray location changes after passing through width $d$ of this medium according to

$$
\begin{equation*}
r(z)_{\text {out }}=r(z)_{\text {in }}+r^{\prime}(z)_{\text {in }} d \tag{1.19}
\end{equation*}
$$

where after traveling a distance $d$ at slope $r^{\prime}$, location has changed by $r^{\prime}(z)_{\text {in }} d$.
Hence, we can write a matrix equation relating the output and the input for a position and a slope vector $\left[r(z), r^{\prime}(z)\right]^{\mathrm{T}}$ :

$$
\left[\begin{array}{c}
r(z)  \tag{1.20}\\
r^{\prime}(z)
\end{array}\right]_{\text {out }}=\left[\begin{array}{ll}
1 & d \\
0 & 1
\end{array}\right]\left[\begin{array}{c}
r(z) \\
r^{\prime}(z)
\end{array}\right]_{\mathrm{in}}
$$

Similarly, the ray can be propagated through a change in refractive index from $n_{1}$ to $n_{2}$ with

$$
\left[\begin{array}{c}
r(z)  \tag{1.21}\\
r^{\prime}(z)
\end{array}\right]_{\mathrm{out}}=\left[\begin{array}{cc}
1 & 0 \\
0 & \frac{n_{1}}{n_{2}}
\end{array}\right]\left[\begin{array}{c}
r(z) \\
r^{\prime}(z)
\end{array}\right]_{\mathrm{in}}
$$

where position does not change and from Snell's law for small angles, for which slope $\tan \theta \approx \sin \theta$, the slope changes by $n_{1} / n_{2}$.

Another common matrix is that for passing through a lens of focal length $f$ :

$$
\left[\begin{array}{c}
r(z)  \tag{1.22}\\
r^{\prime}(z)
\end{array}\right]_{\text {out }}=\left[\begin{array}{cc}
1 & 0 \\
\frac{1}{-f} & 1
\end{array}\right]\left[\begin{array}{c}
r(z) \\
r^{\prime}(z)
\end{array}\right]_{\mathrm{in}}
$$

where a lens changes the slope of a ray by $-r(z) / f$.
A useful case is the propagation of rays through a quadratic medium. From equation (1.18), a $2 \times 2$ matrix can be written and verified by substituting

$$
\left[\begin{array}{c}
r(z)  \tag{1.23}\\
r^{\prime}(z)
\end{array}\right]_{\text {out }}=\left[\begin{array}{cc}
\cos (g z) & \frac{\sin (g z)}{g} \\
-g \sin (g z) & \cos (g z)
\end{array}\right]\left[\begin{array}{c}
r(z) \\
r^{\prime}(z)
\end{array}\right]_{\text {in }}
$$

Other matrices are illustrated in Ref. [176]. The advantage of the $2 \times 2$ representation is that for a string (or sequence) of circularly symmetric components, the matrices can be multiplied together to achieve a single $2 \times 2$ matrix for transmission through the complete string. The property is that the determinant of any matrix is zero. We will use in Section 2.1.2 the $2 \times 2$ notation with matrix elements labeled clockwise from top left as ABCD to compute the effect of propagating a Gaussian beam through the corresponding optical element.

### 1.3 OPTICS FOR LAUNCHING AND RECEIVING BEAMS

Ray tracing allows modeling of simple optics for launching and receiving beams. Beam expanders, beam compressors, telescopes, microscopes, and spatial filters are frequently used in military optical systems to change the beam diameter, view an object at different levels of magnification, or improve the beam spatial coherence. These systems can be constructed with two thin refractive lenses [61]. More complex lens designs can be performed with commercial software such as Code V. A single thin lens system and a magnifier are discussed first.

### 1.3.1 Imaging with a Single Thin Lens

1.3.1.1 Convex Lens for Imaging The focal length of a convex (positive) lens is the distance $f^{\prime}$ at which parallel rays (a collimated beam) are focused to a point $F^{\prime}$, (Figure 1.4a) [61]. A single lens can be used for imaging, that is, to create a copy


FIGURE 1.4 Focusing a collimated parallel beam: (a) with a convex lens, (b) with a concave lens, (c) with a concave mirror, and (d) with a convex mirror.
of an input object to an output image of different size and location (Figure 1.5). An object $U_{0}$ is at distance $d_{\mathrm{o}}$ (o for object) in front of the lens $L$ of focal length $f$. A copy, called the image $U_{\mathrm{i}}$ (i for image), is located at a distance $d_{\mathrm{i}}$ behind the lens (o and i are not to be confused with output and input). For a sharp image, the lens equation must be satisfied.

$$
\begin{equation*}
\frac{1}{d_{\mathrm{o}}}+\frac{1}{d_{\mathrm{i}}}=\frac{1}{f} \quad \text { and } \quad m=\frac{-d_{\mathrm{i}}}{d_{\mathrm{o}}} \tag{1.24}
\end{equation*}
$$



FIGURE 1.5 Imaging with a single thin lens.

The negative sign in the image or lateral magnification $m$ refers to the fact that the image is inverted. By wearing inverting glasses, it was shown that the brain inverts the image in the case of the human eye. Note that lens designers may use a different convention that changes the equations; for example, distances to the left of an element are often considered negative.

Note that figures can be reversed as light can travel in the opposite directions through lenses and mirrors. In a concave lens (Figure 1.4b), parallel rays are caused to diverge. A viewer at the right will think the light is emitted by a point source at $F^{\prime}$. This is a virtual point source as, unlike with a convex lens, a piece of paper cannot be placed at $F^{\prime}$ to see a real image. When using the lens equation (1.24) for a concave lens, the focal length $f$ for a convex lens is replaced by $-f$ for a concave lens.

A concave mirror (Figure 1.4c) performs a function similar to the convex lens in focusing parallel rays of light. But the light is folded back to focus on the left of the mirror instead of passing through. Mirrors may be superior to lenses because of less weight and small size owing to folding. Similarly, the convex mirror acts like a folded concave lens (Figure 1.4d).
1.3.1.2 Convex Lens as Magnifying Glass A single lens can be used as a simple microscope to increase the size of an object over the one that would be obtained without the magnifying lens. Such a system is used as an eyepiece in more complex systems. The closest a typical eye can come to an object for sharp focusing is the standard distinct image distance of $s^{\prime}=25 \mathrm{~cm}$. If it were possible to see an object closer to the eye, the image would occupy a larger area of the retina and the object would look larger. The magnifying glass allows the object to be brought closer than the minimum sharp distance of the eye, say to a distance $d_{\mathrm{o}}$ in front of the eye, by projecting a virtual image at the standard distance, $s^{\prime}$ (Figure 1.6) [61]. From the lens law, equation (1.24), using a negative sign for $s^{\prime}$ because it is on the opposite side of the lens relative to Figure 1.5,

$$
\begin{equation*}
\frac{1}{d_{\mathrm{o}}}=\frac{1}{s^{\prime}}+\frac{1}{f}=\frac{f+s^{\prime}}{f s^{\prime}} \tag{1.25}
\end{equation*}
$$



FIGURE 1.6 A magnifying glass.

The angle $\theta$ subtended by the object in the absence of a magnifying lens and the angle $\theta^{\prime}$ subtended with the magnifying glass are

$$
\begin{align*}
\tan \theta & =\frac{y}{s^{\prime}} \\
\tan \theta^{\prime}=\frac{y}{d_{0}} & =y \frac{f+s^{\prime}}{f s^{\prime}} \tag{1.26}
\end{align*}
$$

where the second equation used equation (1.25). Therefore, the angular or power magnification may be written for small angles, using equation (1.25) for $1 / d_{0}$, as

$$
\begin{equation*}
M=\frac{\theta^{\prime}}{\theta}=\frac{s^{\prime}}{d_{\mathrm{o}}}=\frac{s^{\prime}}{f}+1 \approx \frac{s^{\prime}}{f} \tag{1.27}
\end{equation*}
$$

For $f$ in centimeters, and minimum distinct distance of $s^{\prime}=25 \mathrm{~cm}$, magnifying power is $M=25 / f$. An upper case $M$ distinguishes from lateral magnification $m$ in equation (1.24).

### 1.3.2 Beam Expanders

Beam expanders are used to increase beam diameter for beam weapons and optical communications. Beam expansion reduces the effects of diffraction when propagating light through the atmosphere. A source with a larger beam diameter will spread less with distance than one with a smaller diameter (Section 3.3.5) or, for example, if $\Delta s$ in equation (3.20) increases, then $\Delta \theta$ decreases (Section 3.2.2). Therefore, when a beam is launched into the air for optical communications, to replace a microwave link, or for a power ray, the beam diameter is expanded to minimize beam spreading. A wider beam is less influenced by turbulence because of averaging across the beam (Chapter 5).

Figure 1.7 shows how two convex lenses $L_{1}$ and $L_{2}$ of different focal lengths, $f_{1}$ and $f_{2}$, can expand a collimated beam diameter from $d_{1}$ to $d_{2}$. By similar triangles,

$$
\begin{equation*}
\frac{d_{2}}{d_{1}}=\frac{f_{2}}{f_{1}} \tag{1.28}
\end{equation*}
$$



FIGURE 1.7 Beam expander to reduce effects of beam spreading in the atmosphere.


FIGURE 1.8 Beam expander made with a concave lens as the first lens.


FIGURE 1.9 Beam compressor to reduce collimated beam diameter.

A beam expander can also be made shorter by using a concave lens for the first lens $L_{1}$ (Figure 1.8).

### 1.3.3 Beam Compressors

A compressor is the reverse of the expander as shown in Figure 1.9. In a receiver for an optical communication link, an incoming collimated beam is reduced in diameter from $d_{1}$ to $d_{2}$ to match the size of an optical sensor. By similar triangles, the image or lateral magnification, $m$ is

$$
\begin{equation*}
m=\frac{d_{2}}{d_{1}}=\frac{f_{2}}{f_{1}} \tag{1.29}
\end{equation*}
$$

In a practical optical link, the beam expander on the transmit side forms a slightly converging beam. As the beam profile is normally Gaussian (Section 2.1), the propagation follows that described in Section 2.1.2.

### 1.3.4 Telescopes

The beam compressor (Section 1.3.3) has the form of a refractive telescope $L_{1}$ forms an image and $L_{2}$ reimages to $\infty$ for viewing by eye and the beam expander (Section 1.3.2) has the form of a reverse telescope. A more common drawing for a refractive telescope is shown in Figure 1.10, in which the real image at $Q^{\prime}$ is at the focal point of both lenses [61]. Parallel rays from an object at infinity arrive at an object field angle $\theta$ to the axis and form an image at $Q^{\prime} .2 \theta$ is called the field of view. The objective lens


FIGURE 1.10 Telescope.
acts as the aperture stop or the entrance pupil in the absence of a separate stop [61]. The second lens, usually called an eyepiece, magnifies the image at $Q^{\prime}$ so that a larger virtual image $Q^{\prime \prime}$ appears at infinity (Section 1.3.1.2). The virtual image subtends an angle $\theta^{\prime}$ at the eye. Angular magnification or magnifying power $M$ (reciprocal of lateral magnification) is

$$
\begin{equation*}
M=\frac{\theta^{\prime}}{\theta}=\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}} \tag{1.30}
\end{equation*}
$$

Large astronomical telescopes built with refractive lenses are limited to approximately 1 m diameter because of the weight of the lenses. Higher resolution telescopes with larger diameters use mirrors and are discussed next.
1.3.4.1 Cassegrain Telescope The Cassegrain telescope has a common dish appearance and is used in military systems to reduce weight and size relative to a refractive lens telescope for transmitting and receiving signals. (see Sections 16.2.5, 15.1.1 and 12.2). Figure 1.11a shows the inverted telescope as a beam expander for


FIGURE 1.11 Cassegrain antenna as (a) beam expander or inverted telescope and (b) telescope.
transmitting light beams. The input beam passes through a small hole in the large concave mirror to strike the small convex mirror. Comparing the Cassegrain inverted telescope with the lens beam expander in Figure 1.8, the first small concave lens, $L_{1}$, is replaced by a small convex mirror that spreads the light over a concave mirror that replaces the second lens $L_{2}$. The output aperture size is close to that of the large mirror diameter.

The reverse structure acts as a telescope (Figure 1.11b). The large concave mirror aperture determines the resolution of images. Light reflecting from the concave mirror focuses on the small convex mirror and then through a hole in the concave mirror onto a CCD image sensor. This configuration is used in the Geoeye imaging satellite 400 miles up (Figure 1.12) [125]. Such imaging satellites are critical for providing intelligence information for the military and data for commercial ventures such as Google. A Geoeye satellite, launched in 2008, as shown in Figure 1.13 [125], involves many other systems, solar panels, global positioning system (GPS), star tracker (together the star tracker and the GPS can locate objects to within 3 m ), image storage, and data antenna for transmitting signals back to earth when over designated ground stations. In the open literature as of 2009 , there are in orbit 51 imaging satellites with resolution between 0.4 and 56 m launched by 31 countries and 10 radar satellites launched by 18 countries [125]. The military and commercial sectors rely on these and classified satellites for intelligence relating to threat warnings of enemy activities and environmental issues, on global positioning satellites for guiding missiles and locating U.S. and allies personnel and vehicles, on communication satellites for battlefield communications,


FIGURE 1.12 Optics inside Geoeye using a Cassegrain telescope.


FIGURE 1.13 Geoeye imaging satellite.
and on classified antisatellite satellites aimed at interfering with other countries' satellites. Hence, the control of satellite space will be critical in future wars, although in recent wars control of air space was adequate. Most satellites are vulnerable to laser attack from the ground, aircraft, or other satellites. For example, imaging satellites can be blinded by glare from lasers and for most satellites the solar cell arrays can be easily damaged by lasers, which can disable their source of solar energy. Consequently, as discussed in Chapter 14, the military satellites should also have laser warning devices and protection such as their own lasers and electronic countermeasures.
1.3.4.2 Nasmyth Telescope Sometimes for convenience of mounting subsequent equipment, such as optical spectral analyzers, a variation of the Cassegrain telescope is used in which the light is brought out to one side using a third mirror, rather than through a hole in the primary mirror. This is referred to as a Nasmyth telescope (related to a Coudé telescope). Such an arrangement is shown diagrammatically in Figure 15.1.

### 1.3.5 Microscopes

A typical two-lens microscope (Figure 1.14) has a form similar to the beam expander (Section 1.3.2). A tiny object, in this case an arrow, is placed just inside the focal


FIGURE 1.14 Microscope.
length of the objective lens. According to the lens law, equation (1.24), an image is formed with magnification $m_{\mathrm{o}}=x^{\prime} / f_{\mathrm{o}}$. The eyepiece focal length $f_{\mathrm{e}}$ has magnification, equation (1.27) (Section 1.3.1.2), $M_{\mathrm{e}}=s^{\prime} / f_{\mathrm{e}}$, where the minimum distinct distance for the eye is $s^{\prime}=25 \mathrm{~cm}$. Consequently, the magnification is [61]

$$
\begin{equation*}
M=m_{\mathrm{o}} M_{\mathrm{e}}=\frac{x^{\prime}}{f_{\mathrm{o}}} \frac{s^{\prime}}{f_{\mathrm{e}}} \tag{1.31}
\end{equation*}
$$

### 1.3.6 Spatial Filters

Spatial filters are used to improve spatial coherence in interferometers and between power amplifier stages in a high-power laser (Chapter 8 and Section 13.2.1). A spatial amplifier looks like Figure 1.7 but has a pinhole of very small size, usually micrometers, placed exactly at the focal point of the two lenses (Figure 1.15). Alignment of the pinhole requires high precision to make sure the pinhole lines up exactly with the main power at the focus of the beams. The light to the right of the pinhole now appears to come from an almost perfect point source that produces an almost perfect spherical wave. The smaller the pinhole, the closer is the wave to perfectly spherical. Note that light will be lost if the pinhole is too small. A collimating lens following the pinhole as in Figure 1.7 converts the spherical wave into an almost perfect plane wave.


FIGURE 1.15 Spatial filter improves spatial coherence for higher quality beams.

Optical amplifiers cause distortion due to nonlinear effects at high power. Spatial correlation is degraded and the plane wave has regions pointing off axis. Hence, a spatial filter is often used to clean up the beam after amplification. In a series of power amplifiers (Section 13.2.1), a spatial filter after each amplifier will prevent distortion building up to unacceptable levels. The beam can even be passed back and forth through the same amplifier, as in the National Infrastructure Laser (Chapter 13), because the flash light duration is long enough for several passes of the beam to be amplified. An alternative method of improving beam quality by adaptive optics is described in Section 5.3.2 and used in the airborne laser in Section 12.2.2 and 12.2.3.


[^0]:    Military Laser Technology for Defense: Technology for Revolutionizing 21st Century Warfare, First Edition. By Alastair D. McAulay.
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