Chapter 1

We've Got Your Numbers

In This Chapter

- Understanding how place value turns digits into numbers
- Rounding numbers to the nearest ten, hundred, or thousand
- Calculating with the Big Four operations (adding, subtracting, multiplying, and dividing)
- Getting comfortable with long division

In this chapter, I give you a review of basic math, and I do mean *basic*. I bet you know a lot of this stuff already. So consider this a trip down memory lane, a mini-vacation from whatever math you may be working on right now. With a really strong foundation in these areas, you'll find the chapters that follow a lot easier.

First, I discuss how the number system you're familiar with — called the *Hindu-Arabic number system* (or decimal numbers) — uses digits and place value to express numbers. Next, I show you how to round numbers to the nearest ten, hundred, or thousand.

After that, I discuss the Big Four operations: adding, subtracting, multiplying, and dividing. You see how to use the number line to make sense of all four operations. Then I give you practice doing calculations with larger numbers. To finish up, I make sure you know how to do long division both with and without a remainder.

REMEMBER

Algebra often uses the dot (·) in place of the times sign (×) to indicate multiplication, so that's what I use in this book.

Getting in Place with Numbers and Digits

The number system used most commonly throughout the world is the *Hindu-Arabic number system*. This system contains ten *digits* (also called *numerals*), which are symbols like the letters *A* through *Z*. I'm sure you're quite familiar with them:

1 2 3 4 5 6 7 8 9 0

Like letters of the alphabet, individual digits aren't very useful. When used in combination, however, these ten symbols can build numbers as large as you like using *place value*. Place value assigns each digit a greater or lesser value depending upon where it appears in a number. Each place in a number is ten times greater than the place to its immediate right.

Although the digit 0 adds no value to a number, it can act as a placeholder. When a 0 appears to the right of *at least one* non-zero digit, it's a placeholder. Placeholders are important for giving digits their proper place value. In contrast, when a 0 isn't to the right of any nonzero digit, it's a *leading zero*. Leading zeros are unnecessary and can be removed from a number.

Q. In the number 284, identify the ones digit, the tens digit, and the hundreds digit.

A. The ones digit is 4, the tens digit is 8, and the hundreds digit is 2.

Q. Place the number 5,672 in a table that shows the value of each digit. Then use this table and an addition problem to show how this number breaks down digit by digit.

A.

Millions	*Hundred Thousands*	*Ten Thousands*	*Thousands*	*Hundreds*	*Tens*	*Ones*
			5	6	7	2

The numeral 5 is in the thousands place, 6 is in the hundreds place, 7 is in the tens place, and 2 is in the ones place, so here's how the number breaks down:

$5{,}000 + 600 + 70 + 2 = 5{,}672$

Q. Place the number 040,120 in a table that shows the value of each digit. Then use this table to show how this number breaks down digit by digit. Which 0s are placeholders, and which are leading zeros?

A.

Millions	*Hundred Thousands*	*Ten Thousands*	*Thousands*	*Hundreds*	*Tens*	*Ones*
	0	4	0	1	2	0

The first 0 is in the hundred-thousands place, 4 is in the ten-thousands place, the next 0 is in the thousands place, 1 is in the hundreds place, 2 is in the tens place, and the last 0 is in the ones place, so

$0 + 40{,}000 + 0 + 100 + 20 + 0 = 40{,}120$

The first 0 is a leading zero, and the remaining 0s are placeholders.

1. In the number 7,359, identify the following digits:

a. The ones digit

b. The tens digit

c. The hundreds digit

d. The thousands digit

Solve It

2. Place the number 2,136 in a table that shows the value of each digit. Then use this table to show how this number breaks down digit by digit.

Millions	*Hundred Thousands*	*Ten Thousands*	*Thousands*	*Hundreds*	*Tens*	*Ones*

Solve It

3. Place the number 03,809 in a table that shows the value of each digit. Then use this table to show how this number breaks down digit by digit. Which 0 is a placeholder and which is a leading zero?

Millions	*Hundred Thousands*	*Ten Thousands*	*Thousands*	*Hundreds*	*Tens*	*Ones*

Solve It

4. Place the number 0,450,900 in a table that shows the value of each digit. Then use this table to show how this number breaks down digit by digit. Which 0s are placeholders and which are leading zeros?

Millions	*Hundred Thousands*	*Ten Thousands*	*Thousands*	*Hundreds*	*Tens*	*Ones*

Solve It

Rollover: Rounding Numbers Up and Down

Rounding numbers makes long numbers easier to work with. To round a two-digit number to the nearest ten, simply bring it up or down to the nearest number that ends in 0:

- When a number ends in 1, 2, 3, or 4, bring it down; in other words, keep the tens digit the same and turn the ones digit into a 0.
- When a number ends in 5, 6, 7, 8, or 9, bring it up; add 1 to the tens digit and turn the ones digit into a 0.

To round a number with more than two digits to the nearest ten, use the same method, focusing only on the ones and tens digits.

After you understand how to round a number to the nearest ten, rounding a number to the nearest hundred, thousand, or beyond is easy. Focus only on two digits: The digit in the place you're rounding to and the digit to its immediate right, which tells you whether to round up or down. All the digits to the right of the number you're rounding to change to 0s.

Occasionally when you're rounding a number up, a small change to the ones and tens digits affects the other digits. This is a lot like when the odometer in your car rolls a bunch of 9s over to 0s, such as when you go from 11,999 miles to 12,000 miles.

Q. Round the numbers 31, 58, and 95 to the nearest ten.

A. **30, 60, and 100.**

The number 31 ends in 1, so round it down:

$31 \rightarrow 30$

The number 58 ends in 8, so round it up:

$58 \rightarrow 60$

The number 95 ends in 5, so round it up:

$95 \rightarrow 100$

Q. Round the numbers 742, 3,820, and 61,225 to the nearest ten.

A. **740, 3,820, and 61,230.**

The number 742 ends in 2, so round it down:

$7\underline{42} \rightarrow 7\underline{40}$

The number 3,820 already ends in 0, so no rounding is needed:

$3{,}8\underline{20} \rightarrow 3{,}8\underline{20}$

The number 61,225 ends in 5, so round it up:

$61{,}2\underline{25} \rightarrow 61{,}2\underline{30}$

5. Round these two-digit numbers to the nearest ten:

a. 29

b. 43

c. 75

d. 95

Solve It

6. Round these numbers to the nearest ten:

a. 164

b. 765

c. 1,989

d. 9,999,995

Solve It

7. Round these numbers to the nearest hundred:

a. 439

b. 562

c. 2,950

d. 109,974

Solve It

8. Round these numbers to the nearest thousand:

a. 5,280

b. 77,777

c. 1,234,567

d. 1,899,999

Solve It

Using the Number Line with the Big Four

The *number line* is just a line with numbers marked off at regular intervals. You probably saw your first number line when you were first figuring out how to count to ten. In this section, I show you how to use this trusty tool to perform the Big Four operations (adding, subtracting, multiplying, and dividing) on relatively small numbers.

The number line can be a useful tool for adding and subtracting small numbers:

- When you add, move *up* the number line, to the right.
- When you subtract, move *down* the number line, to the left.

To multiply on the number line, start at 0 and count by the *first number* in the problem as many times as indicated by the *second number.*

To divide on the number line, first block off a segment of the number line from 0 to the *first number* in the problem. Then divide this segment evenly into the number of pieces indicated by the *second number.* The length of each piece is the answer to the division.

Q. Add 6 + 7 on the number line.

A. **13.** The expression 6 + 7 means *start at 6, up 7,* which brings you to 13 (see Figure 1-1):

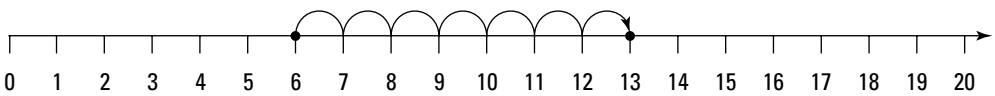


Figure 1-1: Adding 6 + 7 = 13 on the number line.

Q. Subtract 12 – 4 on the number line.

A. **8.** The expression 12 – 4 means *start at 12, down 4,* which brings you to 8 (see Figure 1-2):

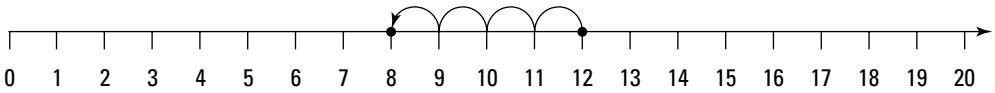
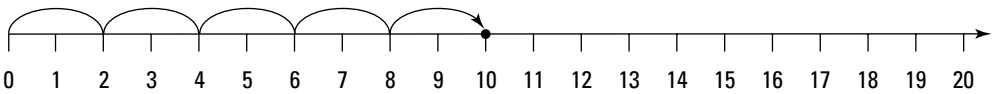


Figure 1-2: Subtracting 12 – 4 = 8 on the number line.

Q. Multiply 2 · 5 on the number line.

A. **10.** Starting at 0, count by twos a total of five times, which brings you to 10 (see Figure 1-3).

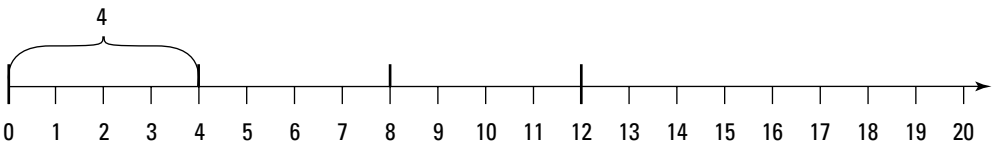
Figure 1-3: Multiplying 2 · 5 = 10 on the number line.



Q. Divide 12 ÷ 4 on the number line.

A. **3.** Block off the segment of the number line from 0 to 12. Now divide this segment evenly into three smaller pieces, as shown in Figure 1-4. Each of these pieces has a length of 4, so this is the answer to the problem.

Figure 1-4: Dividing 12 ÷ 3 = 4 on the number line.



9. Add the following numbers on the number line:

a. 4 + 7 = ?

b. 9 + 8 = ?

c. 12 + 0 = ?

d. 4 + 6 + 1 + 5 = ?

Solve It

10. Subtract the following numbers on the number line:

a. 10 – 6 = ?

b. 14 – 9 = ?

c. 18 – 18 = ?

d. 9 – 3 + 7 – 2 + 1 = ?

Solve It

11. Multiply the following numbers on the number line:

a. 2 · 7

b. 7 · 2

c. 4 · 3

d. 6 · 1

e. 6 · 0

f. 0 · 10

Solve It

12. Divide the following numbers on the number line:

a. 8 ÷ 2 = ?

b. 15 ÷ 5 = ?

c. 18 ÷ 3 = ?

d. 10 ÷ 10 = ?

e. 7 ÷ 1 = ?

f. 0 ÷ 2 = ?

Solve It

The Column Lineup: Adding and Subtracting

To add or subtract large numbers, stack the numbers on top of each other so that all similar digits (ones, tens, hundreds, and so forth) form columns. Then work from right to left. Do the calculations vertically, starting with the ones column, then going to the tens column, and so forth:

- When you're adding and a column adds up to 10 or more, write down the ones digit of the result and carry the tens digit over to the column on the immediate left.
- When you're subtracting and the top digit in a column is less than the bottom digit, borrow from the column on the immediate left.

Q. Add 35 + 26 + 142.

A. **203.** Stack the numbers and add the columns from right to left:

$$\begin{array}{r} {}^{1\,1} \\ 35 \\ 26 \\ +142 \\ \hline 203 \end{array}$$

Notice that when I add the ones column (5 + 6 + 2 = 13), I write the 3 below this column and carry the 1 over to the tens column. Then, when I add the tens column (1 + 3 + 2 + 4 = 10), I write the 0 below this column and carry the 1 over to the hundreds column.

Q. Subtract 843 – 91.

A. **752.** Stack the numbers and subtract the columns from right to left:

$$\begin{array}{r} {}^{7}\cancel{8}\,{}^{1}43 \\ -\,91 \\ \hline 752 \end{array}$$

When I try to subtract the tens column, 4 is less than 9, so I borrow 1 from the hundreds column, changing the 8 to 7. Then I place this 1 in front of the 4, changing it to 14. Now I can subtract 14 – 9 = 5.

13. Add 129 + 88 + 35 = ?

Solve It

14. Find the following sum: 1,734 + 620 + 803 + 32 = ?

Solve It

15. Subtract 419 – 57.

Solve It

16. Subtract 41,024 – 1,786.

Solve It

Multiplying Multiple Digits

To multiply large numbers, stack the first number on top of the second. Then multiply each digit of the bottom number, from right to left, by the top number. In other words, first multiply the top number by the ones digit of the bottom number. Then write down a 0 as a placeholder and multiply the top number by the tens digit of the bottom number. Continue the process, adding placeholders and multiplying the top number by the next digit in the bottom number.

When the result is a two-digit number, write down the ones digit and carry the tens digit to the next column. After multiplying the next two digits, add the number you carried over.

Add the results to obtain the final answer.

EXAMPLE

Q. Multiply 742 · 136.

A. **100,912.** Stack the first number on top of the second:

$$\begin{array}{r} 742 \\ \times 136 \\ \hline \end{array}$$

Now multiply 6 by every number in 742, starting from the right. Because 2 · 6 = 12, a two-digit number, you write down the 2 and carry the 1 to the tens column. In the next column, you multiply 4 · 6 = 24 and add the 1 you carried over, giving you a total of 25. Write down the 5 and carry the 2 to the hundreds column. Multiply 7 · 6 = 42 and add the 2 you carried over, giving you 44:

$$\begin{array}{r} {}^{2\,1} \\ 742 \\ \times 136 \\ \hline 4452 \end{array}$$

Next, write down a 0 all the way to the right in the row below the one that you just wrote. Multiply 3 by every number in 742, starting from the right and carrying when necessary:

$$\begin{array}{r} {}^{1} \\ 742 \\ \times 136 \\ \hline 4452 \\ 22260 \end{array}$$

Write down two 0s all the way to the right of the row below the one that you just wrote. Repeat the process with 1:

$$\begin{array}{r} 742 \\ \times 136 \\ \hline 4452 \\ 22260 \\ \underline{74200} \end{array}$$

To finish, add up the results:

$$\begin{array}{r} 742 \\ \times 136 \\ \hline 4452 \\ 22260 \\ 74200 \\ \hline 100912 \end{array}$$

So 742 · 136 = 100,912.

17. Multiply 75 · 42.

Solve It

18. What's 136 · 84?

Solve It

19. Solve 1,728 · 405.

Solve It

20. Multiply 8,912 · 767.

Solve It

Cycling through Long Division

To divide larger numbers, use *long division.* Unlike the other Big Four operations, long division moves from left to right. For each digit in the *divisor,* the number you're dividing, you complete a cycle of division, multiplication, and subtraction.

In some problems, the number at the very bottom of the problem isn't a 0. In these cases, the answer has a *remainder,* which is a leftover piece that needs to be accounted for. In those cases, you write *r* followed by whatever number is left over.

Q. Divide 956 ÷ 4.

A. **239.** Start off by writing the problem like this:

$$4\overline{)956}$$

To begin, ask how many times 4 goes into 9 — that is, what's 9 ÷ 4? The answer is 2 (with a little left over), so write 2 directly above the 9. Now multiply 2 · 4 to get 8, place the answer directly below the 9, and draw a line beneath it:

$$\begin{array}{r} 2 \\ 4\overline{)956} \\ 8 \\ \hline \end{array}$$

Subtract 9 – 8 to get 1. (***Note:*** After you subtract, the result should be less than the divisor (in this problem, the divisor is 4). Then bring down the next number (5) to make the new number 15.

$$\begin{array}{r} 2 \\ 4\overline{)956} \\ -8 \\ \hline 15 \end{array}$$

These steps are one complete cycle. To complete the problem, you just need to repeat them. Now ask how many times 4 goes into 15 — that is, what's 15 ÷ 4? The answer is 3 (with a little left over). So write the 3 above the 5, and then multiply 3 · 4 to get 12. Write the answer under 15.

$$\begin{array}{r} 23 \\ 4\overline{)956} \\ -8 \\ \hline 15 \\ -12 \\ \hline \end{array}$$

Subtract 15 – 12 to get 3. Then bring down the next number (6) to make the new number 36.

$$\begin{array}{r} 23 \\ 4\overline{)956} \\ -8 \\ \hline 15 \\ -12 \\ \hline 36 \end{array}$$

Another cycle is complete, so begin the next cycle by asking how many times 4 goes into 36 — that is, what's 36 ÷ 4? The answer this time is 9. Write down 9 above the 6, multiply 9 · 4 = 36, and place this below the 36.

$$\begin{array}{r} 239 \\ 4\overline{)\ 956} \\ -8 \\ \hline 15 \\ -12 \\ \hline 36 \\ 36 \end{array}$$

Now subtract 36 – 36 = 0. Because you have no more numbers to bring down, you're finished, and the answer (that is, the *quotient*) is the very top number of the problem:

$$\begin{array}{r} 239 \\ 4\overline{)956} \\ -8 \\ \hline 15 \\ -12 \\ \hline 36 \\ -36 \\ \hline 0 \end{array}$$

Q. Divide 3,042 ÷ 5.

A. **608 r 2.** Start off by writing the problem like this:

$$5\overline{)3042}$$

To begin, ask how many times 5 goes into 3. The answer is 0 — because 5 doesn't go into 3 — so write a 0 above the 3. Now you need to ask the same question using the first *two* digits of the divisor: How many times does 5 go into 30 — that is, what's 30 ÷ 5? The answer is 6, so place the 6 over the 0. Here's how to complete the first cycle:

$$\begin{array}{r} 06 \\ 5\overline{)3042} \\ -30 \\ \hline 04 \end{array}$$

Next, ask how many times 5 goes into 4. The answer is 0 — because 5 doesn't go into 4 — so write a 0 above the 4. Now bring down the next number (2), to make the number 42:

$$\begin{array}{r} 060 \\ 5\overline{)3042} \\ -30 \\ \hline 042 \end{array}$$

Ask how many times 5 goes into 42 — that is, what's 42 ÷ 5? The answer is 8 (with a little bit left over), so complete the cycle as follows:

$$\begin{array}{rl} 0608 & \leftarrow \text{quotient} \\ 5\overline{)3042} & \\ s\ -30 & \\ \hline 042 & \\ -40 & \\ \hline 2 & \leftarrow \text{remainder} \end{array}$$

Because you have no more numbers to bring down, you're finished. The answer (quotient) is at the top of the problem (you can drop the leading 0), and the remainder is at the bottom of the problem. So 3,042 ÷ 5 = 608 with a remainder of 2. To save space, write this answer as 608 r 2.

21. Divide 741 ÷ 3.

Solve It

22. Solve 3,245 ÷ 5.

Solve It

23. Figure out 91,390 ÷ 8.

Solve It

24. Find 792,541 ÷ 9.

Solve It

Solutions to We've Got Your Numbers

The following are the answers to the practice questions presented in this chapter.

1 Identify the ones, tens, hundreds, and thousands digit in the number 7,359.

a. 9 is the ones digit.

b. 5 is the tens digit.

c. 3 is the hundreds digit.

d. 7 is the thousands digit.

2 **2,000 + 100 + 30 + 6 = 2,136**

Millions	*Hundred Thousands*	*Ten Thousands*	*Thousands*	*Hundreds*	*Tens*	*Ones*
			2	1	3	6

3 **0 + 3,000 + 800 + 0 + 9 = 3,809. The first 0 is the leading zero, and the second 0 is the placeholder.**

Millions	*Hundred Thousands*	*Ten Thousands*	*Thousands*	*Hundreds*	*Tens*	*Ones*
		0	3	8	0	9

4 **0 + 400,000 + 50,000 + 0 + 900 + 0 + 0 = 0,450,900. The first 0 is a leading zero, and the remaining three 0s are placeholders.**

Millions	*Hundred Thousands*	*Ten Thousands*	*Thousands*	*Hundreds*	*Tens*	*Ones*
0	4	5	0	9	0	0

5 Round to the nearest ten:

a. 29 → **30.** The ones digit is 9, so round up.

b. 43 → **40.** The ones digit is 3, so round down.

c. 75 → **80.** The ones digit is 5, so round up.

d. 95 → **100.** The ones digit is 5, so round up, rolling 9 over.

6 Round to the nearest ten:

a. 1<u>64</u> → **1<u>60</u>.** The ones digit is 4, so round down.

b. 7<u>65</u> → **7<u>70</u>.** The ones digit is 5, so round up.

c. 1,9<u>89</u> → **1,9<u>90</u>.** The ones digit is 9, so round up.

d. 9,999,99<u>5</u> → **10,000,0<u>00</u>.** The ones digit is 5, so round up, rolling all of the 9s over.

7 Focus on the hundreds and tens digits to round to the nearest hundred.

a. 439 → **400.** The tens digit is 3, so round down.

b. 562 → **600.** The tens digit is 6, so round up.

c. 2,950 → **3,000.** The tens digit is 5, so round up.

d. 109,974 → **110,000.** The tens digit is 7, so round up, rolling over all the 9s.

8 Focus on the thousands and hundreds digits to round to the nearest thousand.

a. 5,280 → **5,000.** The hundreds digit is 2, so round down.

b. 77,777 → **78,000.** The hundreds digit is 7, so round up.

c. 1,234,567 → **1,235,000.** The hundreds digit is 5, so round up.

d. 1,899,999 → **1,900,000.** The hundreds digit is 9, so round up, rolling over all the 9s to the left.

9 Add on the number line.

a. 4 + 7 = **11.** The expression 4 + 7 means *start at 4, up 7,* which brings you to 11.

b. 9 + 8 = **17.** The expression 9 + 8 means *start at 9, up 8,* which brings you to 17.

c. 12 + 0 = **12.** The expression 12 + 0 means *start at 12, up 0,* which brings you to 12.

d. 4 + 6 + 1 + 5 = **16.** The expression 4 + 6 + 1 + 5 means *start at 4, up 6, up 1, up 5,* which brings you to 16.

10 Subract on the number line.

a. 10 – 6 = **4.** The expression 10 – 6 means *start at 10, down 6,* which brings you to 4.

b. 14 – 9 = **5.** The expression 14 – 9 means *start at 14, down 9,* which brings you to 5.

c. 18 – 18 = **0.** The expression 18 – 18 means *start at 18, down 18,* which brings you to 0.

d. 9 – 3 + 7 – 2 + 1 = **12.** The expression 9 – 3 + 7 – 2 + 1 means *start at 9, down 3, up 7, down 2, up 1,* which brings you to 12.

11 Multiply on the number line.

a. 2 · 7 = **14.** Starting at 0, count by twos a total of seven times, which brings you to 14.

b. 7 · 2 = **14.** Starting at 0, count by sevens a total of two times, which brings you to 14.

c. 4 · 3 = **12.** Starting at 0, count by fours a total of three times, which brings you to 12.

d. 6 · 1 = **6.** Starting at 0, count by sixes one time, which brings you to 6.

e. 6 · 0 = **0.** Starting at 0, count by sixes zero times, which brings you to 0.

f. 0 · 10 = **0.** Starting at 0, count by zeros a total of ten times, which brings you to 0.

12 Divide on the number line.

a. 8 ÷ 2 = **4.** Block off a segment of the number line from 0 to 8. Now divide this segment evenly into two smaller pieces. Each of these pieces has a length of 4, so this is the answer to the problem.

b. 15 ÷ 5 = **3.** Block off a segment of the number line from 0 to 15. Divide this segment evenly into five smaller pieces. Each of these pieces has a length of 3, so this is the answer to the problem.

c. 18 ÷ 3 = **6.** Block off a segment of the number line from 0 to 18 and divide this segment evenly into three smaller pieces. Each piece has a length of 6, the answer to the problem.

d. 10 ÷ 10 = **1.** Block off a segment of the number line from 0 to 10 and divide this segment evenly into ten smaller pieces. Each of these pieces has a length of 1.

e. 7 ÷ 1 = **7.** Block off a segment of the number line from 0 to 7 and divide this segment evenly into 1 piece (that is, don't divide it at all). This piece still has a length of 7.

f. 0 ÷ 2 = **0.** Block off a segment of the number line from 0 to 0. The length of this segment is 0, so it can't get any smaller. This shows you that 0 divided by *any* number is 0.

13 **252**

$$\begin{array}{r} {\scriptstyle 1\ 2} \\ 129 \\ 88 \\ +35 \\ \hline 252 \end{array}$$

14 **3,189**

$$\begin{array}{r} {\scriptstyle 2} \\ 1734 \\ 620 \\ 803 \\ +\ \ 32 \\ \hline 3189 \end{array}$$

15 **362**

$$\begin{array}{r} {\scriptstyle 3\ 1} \\ \cancel{4}19 \\ -\ 57 \\ \hline 362 \end{array}$$

16 **39,238**

$$\begin{array}{r} {\scriptstyle 3\ \ 10\ 9\ \ 11\ \ 1} \\ \cancel{4}\cancel{1}\cancel{0}\cancel{2}4 \\ -1786 \\ \hline 39238 \end{array}$$

17 **3,150**

$$\begin{array}{r} 75 \\ \times 42 \\ \hline 150 \\ 3000 \\ \hline 3150 \end{array}$$

18 **11,424**

$$\begin{array}{r} 136 \\ \times 84 \\ \hline 544 \\ 10880 \\ \hline 11424 \end{array}$$

19 **699,840**

$$\begin{array}{r} 1728 \\ \times 405 \\ \hline 8640 \\ 691200 \\ \hline 699840 \end{array}$$

20 **6,835,504**

$$\begin{array}{r} 8912 \\ \times\ 767 \\ \hline 62384 \\ 534720 \\ 6238400 \\ \hline 6835504 \end{array}$$

21 **247**

$$\begin{array}{r} 247 \\ 3\overline{)741} \\ -6 \\ \hline 14 \\ -12 \\ \hline 21 \\ -21 \\ \hline 0 \end{array}$$

22 **649**

```
   0649
5)3245
 −30
   24
  −20
    45
   −45
     0
```

23 **11,423 r 6**

```
   11423
8)91390
 − 8
   11
  −8
   33
  −32
    19
   −16
     30
    −24
      3
```

24 **88,060 r 1**

```
    088060
9)792541
 −72
     72
   −72
     054
    −54
      01
     −0
      1
```