FIRST, THERE WAS CLASSICAL PHYSICS 1

If I have seen further it is by standing on the shoulders of Giants.
Issac Newton, Letter to Robert Hooke, February 1676

Figure 1.0. "Nicolas Poussin (1594–1665): Blind Orion Searching for the Rising Sun (24.45.1)". Date: 1658. In *Heilbrunn Timeline of Art History*. New York: Courtesy of The Metropolitan Museum of Art, 2000.

In this picture, based on Greek mythology, blind Orion a hunter has set Cadelion, a servant, on his shoulders as a Guide to the East where the rays of the Sun would restore his eyesight. *Dwarfs standing on the shoulders of giants (Latin: nanos gigantium humeris insidentes)* is a Western metaphor with a modern-time interpretation: "One who develops future intellectual pursuits by understanding the research and works created by

Quantum Physics for Scientists and Technologists, by Paul Sanghera

notable thinkers of the past." This metaphor, first recorded in the twelfth century and attributed to Bernard of Chartres, a twelfth century French Platonist philosopher, was famously used by seventeenth-century scientist Isaac Newton. Newton himself was rather modest about his own achievements, when in his famous letter to Robert Hooke in February 1676, he wrote "If I have seen further it is by standing on the shoulders of Giants".

In This Chapter

1.1	Introduction	2
1.2	Physics and Classical Physics	3
1.3	The Classical World of Particles	10
1.4	Physical Quantities	12
1.5	Newton's Laws of Motion	15
1.6	Rotational Motion	18
1.7	Superposition and Collision of Particles	22
1.8	Classical World of Waves	26
1.9	Reflection, Refraction, and Scattering	30
1.10	Diffraction and Interference	32
1.11	Equation of Wave Motion	35
1.12	Light: Particle or Wave?	38
1.13	Understanding Electricity	39
1.14	Understanding Magnetism	45
1.15	Understanding Electromagnetism	49
1.16	Maxwell's Equations	52
1.17	Confinement, Standing Waves, and Wavegroups	55
1.18	Particles and Waves: The Big Picture	62
1.19	The Four Fundamental Forces of Nature	63
1.20	Unification: A Secret to Scientific and Technological Revolutions	69
1.21	Special Theory and Relativity	72
1.22	Classical Approach	75
1.23	Summary	77
1.24	Additional Problems	78

1.1 INTRODUCTION

Physics is a discipline in natural science, the branch of science that relies the most on mathematics to create an explanation of the universe we live in. The word *science* has its origin in a Latin word that means "to know". Science is the body of knowledge of the natural world organized in a rational and verifiable way. The word *physics* has its origin in the Greek word that means nature.

Physics is that branch, or discipline, of science that deals with understanding the universe and the systems in the universe to all levels of depth from planets to fundamental constituents of matter, such as atoms, electrons, and quarks. The core part of physics is to understand the universe and everything in it in terms of the fundamental constituents of matter and the interactions between those constituents. The interactions are commonly called forces.

As human, we are macroorganisms unable to observe the microobjects and phenomenon with our naked eyes. However, what worked for us as a species that we are cognizant of phenomena in and around the range we live in. In other words, we have the capability of studying and understanding something that is beyond our intuition. The distance between two points that we resolve with our eyes is on the order of one-tenth of a millimeter (mm), and the smallest time between two instances that we can measure without the help of sophisticated tools is on the order of one-tenth of a second(s). Therefore, the journey of physics, and hence science, began by studying the macroobjects and systems. The physics of these macrosystems is called classical physics. Therefore it is important to understand and appreciate classical physics before we can understand quantum physics. This chapter presents a high-level review of the important concepts in classical physics in a concise and cohesive fashion. If you are not sure of any concept covered in this chapter, consult an introductory physics book for help from the list presented at the end of this book.

Classical physics divides the physical world into two types of physical entities: particles and waves. Your main goal in this chapter is to grasp the classical approach of physics in terms of particles and waves being different kinds of entities. To that end, we will explore three avenues: particles, waves, and forces.

1.2 PHYSICS AND CLASSICAL PHYSICS

As mentioned earlier, at its core physics is that branch of science that deals with understanding the universe and the systems in the universe in terms of fundamental constituents of matter, such as atoms, electrons, and the interactions among those constituents.

> *Note*: When physicists use the word micro, they usually mean all sizes nonmacro, including micro (10^{-6}), nano (10^{-9}), pico (10^{-12}), and smaller. In this sense, the microscale word includes nanoscale. We also use the micro word in this book in this sense unless stated otherwise.

Physics, the most fundamental science, deals with (discovering and exploring) the fundamental principles that are subsequently applied to many other disciplines of science and technology, such as biology, chemistry, material science, electronics, engineering, and nanotechnology. Think of basic physics principles being used in building practical devices and systems, such as radio, television, cellular phone, or an radio frequency identification (RFID) system, and think of the whole field of physical chemistry and biophysics. Physics, undoubtedly, has been the foundation of all engineering and technology. Understanding and application of physics laws is necessary from designing a mousestrap to designing and building a flat screen TV, a sports car, and a spacecraft. Depending on the history of their development, some fields, such as chemistry and engineering, have been ahead of other fields, such as biology, in making use of physics. It is expected that in the coming years, physics, especially quantum physics, will be used enormously in the process of understanding entities and phenomena in various fields of biosciences, including biochemistry, molecular biology, genetics, and even evolution.

Despite its sophisticated theories, physics, at the end of the day, is an experimental science. Physicists observe the phenomena of nature, find the patterns and relationships among those phenomena, and try to explain this in terms of models and theories, which after rigorous experimental tests are established as physical laws or principles.

> *Caution*: The notion that a theory is just a theory, just a random thought, an abstract idea, or an unproven concept, is an incorrect notion. Scientists use this word as follows: A theory is an explanation of a natural phenomenon or a set of phenomena based on observation. By definition, a theory is falsifiable and faces the tests of experiments for its acceptance among the scientific community. The theory of gravitation, the theory of unification of forces, and the theory of biological evolution, known as Darwin's theory of evolution, are all examples of such theories, also called scientific theories.

The development of physical theories and principles is an endless process of back-and-forth between ideas and experiments. Scientists never regard any theory as final or as the ultimate truth. On the contrary, they are always looking for new observations that will require us to revise a theory or to discard it for a better one. By its very nature, a scientific theory can be disproved by finding phenomena and behavior that are inconsistent with it, but a theory can never be proved to be always correct. As you will see in this book, the development of quantum physics is a good example of this process of developing scientific theories.

Before quantum physics, we had classical physics, which is reviewed or overviewed in this chapter. In order to get through this chapter smoothly, let us review some basic concepts related to physics:

Particle. A particle is a small object that behaves as a whole unit in terms of its motion, properties, and behavior. Although one usually thinks of a particle as a very small object, there is no size limit on what can be treated as a particle. A point particle is an idealized and simplified representation of an entity. It can be fully described as having a spatial extent of zero (size zero), and therefore its position is completely defined by one set of coordinates (x, y, z), called Cartesian coordinates. Given it is a point, it obviously has no internal structure. While its geometry is simple, it can, however, have properties associated with it, such as mass and electrical charge. In many cases, this approximation works well in understanding the overall behavior of a system and getting out reasonable quantitative results. As seen later in this book, this concept can also be used to represent the center of mass of a system, a reference frame.

So, if one point in an object can be located to determine its position in space, in order to simplify the discussion of its motion we can treat this object as a point particle, also referred to as a particle for brevity. Any object can be considered a particle as long as we are not interested in its internal structure and rotational motion. In some cases, large objects, such as the Sun, can be modeled as point particles for the determination of certain characteristics or quantities. For example, to determine the orbit of the Earth around the Sun it is reasonable to assume that all of the Sun's mass is concentrated at one point, its geometric center. It is a good approximation for determining the force exerted on the Earth and thus determining the Earth's orbit. As another example, the molecules in the kinetic molecular theory of gases are considered as particles. Once you start considering the rotation of molecules and the fact that that they are made of atoms (internal structure), you can no more treat them as particles (i.e., point particles).

When we want to study the internal structure and dynamics of an object, the object is no more a particle, it is a system.

Note: Over centuries, physicists have invented several coordinate systems to describe the position and motion of a physical entity in space. Depending on the nature of the entity and the problem, one coordinate system may be more convenient than others. For example, you know from your introductory physics course that translational motion is usually better described in a Cartesian coordinate system, and circular or rotational motion is usually better described in polar coordinates.

System. A system is a set of distinct entities often interacting with one another. A system has a structure defined by its constituents, which have a structural as well as functional relationship with one another. A system as a whole has some characteristics and a certain behavior. For example, an atom is a system constituted of electrons, protons, and possible neutrons. Similarly, planets revolving around the Sun make up a system called the solar system. An organism, such as a human being, is also a system composed of organs; and a deoxyribonucleic acid (DNA) molecule inside an organism is a system composed of smaller components called nucleotides.

Physical Quantity. A measurable observable, for example energy of an entity, such as a particle or a system, is called a physical quantity. A physical quantity describes an aspect of an object or a phenomenon quantitatively. In physics, we understand the universe, the systems in the universe, and their behavior in terms of physical quantities, as well as the relationships among these physical quantities. In other words, laws of physics are usually expressed in terms of relationships among the physical quantities. Mass, electric charge, length, time, speed, force, energy, and temperature are some examples of physical quantities. This book will use the term observable and physical quantity interchangeably.

Unit. A physical quantity is measured in numbers of a basic amount called a unit. The measurement of a quantity contains a number and a unit, for example, in 15 miles, the mile is a unit of distance (or length). Similarly, a kilogram (kg) is a unit of mass.

Force. The influence that an object exerts on another object to cause some change is the force. Where there is change, there is some force behind it. Motion of an object is an example of change. The change, for example, could be in the physical properties, such as the speed or position (location) of the object. The exact nature of this change will be determined by some physical principles, such as Newton's laws of motion, which is described later in this chapter.

Interaction. A mutual force between two objects through which they influence each other is an interaction. For example, two particles attract each other due to an attractive force or repel each other due to a repulsive force between them. Sometimes the words *interaction* and *force* are used synonymously. There are four known fundamental interactions or forces: (1) gravitational force; (2) electromagnetic force; (3) strong nuclear force; and (4) weak nuclear force.

For example, most of the forces that chemistry students learn about, such as covalent bonds, ionic bonds, hydrogen bonds, and London dispersion force, are different manifestations of the electromagnetic (EM) force.

Where there is a force there is energy, or the potential for energy, called potential energy. Force acting on a particle results in a change in the kinetic or potential energy of the particle.

Speed. The speed describes the motion of a particle. To be precise it is the change in position with time. For example, your car is moving at a speed of 70 miles per hour (mph). When you specify the direction of the speed, it becomes velocity. For example, the velocity of your car at a given moment could be 70 mph toward the North.

Energy. The measure of the ability of a force to do work is energy. There are different kinds of energies corresponding to different forces, such as EM energy corresponding to EM force, nuclear energy corresponding to nuclear forces, and so on. All of these energies have the same units. Flow of energy, that is, energy transfer, is a key to maintaining order in the universe and in our bodies. For example, energy flows from the Sun to the plants, and from the plants to our bodies in the form of food. Energy can be converted from one form to another, but it can neither be created nor destroyed. This is called the principle of conservation of energy. For example, in your microwave oven, the EM energy that you buy from your power company is converted into heat energy that warms (or cooks) your food; the form is changed, not the total content of the energy.

Work. A measure of the amount of change produced by a force acting on an object is work. For example, gravitational force between the Sun and the Earth is making the Earth revolve around the Sun. In doing so, Earth is doing some work. But how is it possible that two objects separated from each other can exert force on each other? This is where the concept of field comes into the picture.

Power. The energy transferred per unit time or the amount of work done by a force per unit time is power.

Field. The fundamental forces of nature can work between two objects without the objects physically touching each other. For example, Sun and Earth attract each other through gravitational force without touching each other. Two charged particles attract or repel each other through EM force without touching each other. This effect is called *action at a distance* and is explained in physics by the concept of a *field*. Each of the two objects that, for example, attract or repel each other from a distance, create a field in the space. This is the field that exerts the force on the other object. For example, there is a gravitational field corresponding to the gravitational force and an EM field corresponding to the EM force, and so on.

For the purpose of visualization, the force applied through a field is often represented in terms of field lines. For example, Figure 1.1 illustrates electric field lines around positively and negatively charged particles. As shown in this figure, electric field lines always point away from a positively charged particle and point toward a negatively charged particle. The symbol $\vec{E}$ stands for an electric field, and the arrow on E means the electric field is a vector.

A physical quantity can be a scalar or a vector.

Scalars and Vectors. A scalar is a quantity that has a magnitude, but no direction. For example, a speed of 70 mph is a scalar. A vector is a quantity that has both magnitude and direction. For example, a speed of

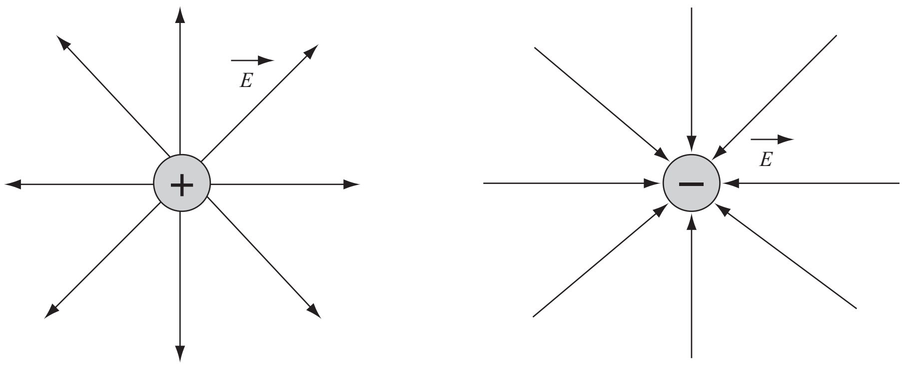


Figure 1.1. Electric field lines around a particle with electric charge.

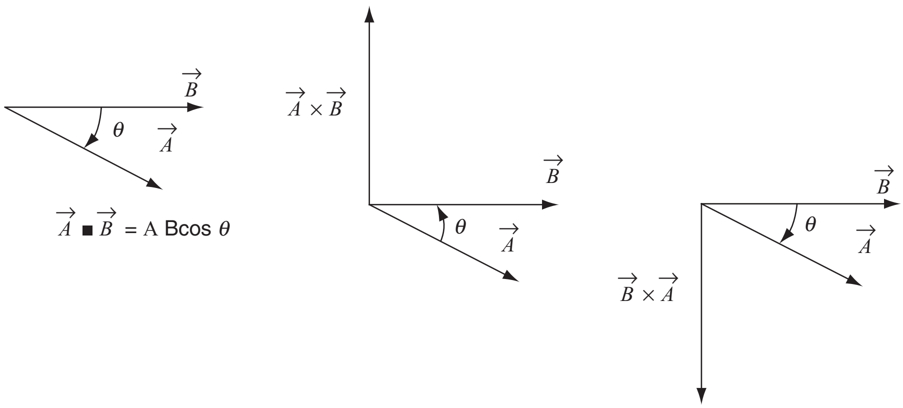


Figure 1.2. Illustration of a scalar and a vector product.

70 mph toward the North is a vector. Speed in a specified direction is called velocity.

Scalar quantities can be multiplied just like numbers, whereas vector quantities can undergo any of two kinds of multiplication called scalar product and vector product. The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is denoted as $\vec{A}.\vec{B}$, and due to the dot in this notation it is also called a dot product. As the name suggests, the scalar product of two vectors is a scalar obtained by multiplying the magnitude of a vector with the magnitude of the component of the other vector in the direction of this vector:

$$\vec{A}.\vec{B} = \mathrm{AB}\cos\theta \tag{1.1}$$

where θ is the angle between the two vectors $\vec{A}$ and $\vec{B}$, as shown in Figure 1.2, which also illustrates the vector product.

A vector product of two vectors $\vec{A}$ and $\vec{B}$ is denoted as $\vec{A}\times\vec{B}$, and due to this notation it is also called a cross product. As the name suggests, the vector product of two vectors $\vec{A}$ and $\vec{B}$ is also a vector, say $\vec{C}$ perpendicular to the plane of $\vec{A}$ and $\vec{B}$, as illustrated in Figure 1.2. The magnitude of $\vec{C}$ is given by

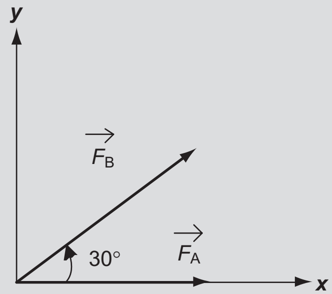
$$\left|\vec{C}\right| = \left|\vec{A}\times\vec{B}\right| = \mathrm{AB}\sin\theta \tag{1.2}$$

Also,

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

PROBLEM 1.1

Forces $\vec{F}_{\mathrm{A}}$ and $\vec{F}_{\mathrm{B}}$ with magnitudes of 3 and 5 Ns, respectively, are acting on a particle. The force $\vec{F}_{\mathrm{A}}$ is acting along the *x*-axis, and the force $\vec{F}_{\mathrm{B}}$ is making an angle of 30° with the x-axis in the x–y plane.



A. Calculate the scalar product of these two forces.
B. Calculate the vector product of these two forces.

Solution:

A.

$$\vec{F}_{\mathrm{A}}.\vec{F}_{\mathrm{B}} = F_{\mathrm{A}}F_{\mathrm{B}}\cos\theta = 3\times5\times\cos30^\circ = 12.99\,\mathrm{N}$$

B.

$$\vec{F}_{\mathrm{A}}\times\vec{F}_{\mathrm{B}} = F_{\mathrm{A}}F_{\mathrm{B}}\sin\theta = 3\times5\times\sin30^\circ = 7.5\,\mathrm{N}$$

The force as a result of the cross product will be along the *z*-axis

> *Note*: A material body or particle is a physical entity that contains mass and occupies space.

Classical physics looks at the universe in terms of material bodies and waves. These material bodies, their structures, and their behavior are understood in terms of what are called physical quantities, which were explained earlier. Many physical quantities of material bodies can be studied by treating them as point particles. So, you can say that classical physics divides the physical world into particles and waves.

1.3 THE CLASSICAL WORLD OF PARTICLES

The Greek philosopher Aristotle is considered by many to be the first physicist. What is known as classical physics or Newtonian physics today is a result of the scientific work of an enormous number of scientists across several centuries, starting from Aristotle (384–322 BC) to Galileo Galilei (1564–1642), to Isaac Newton (1643–1727), to James Clerk Maxwell (1831–1879). The goal of this chapter is not to present a full review of the classical physics, but rather briefly explore a few concepts in a cohesive way in order to establish the big cohesive picture of classical physics to capture the key characteristics of the classical approach. It is important for you to understand and appreciate the classical approach to physics in order to fully understand how this approach was challenged by experimental results and how this challenge was met with the quantum approach, which developed into quantum physics, also called quantum mechanics, as opposed to classical mechanics.

In Section 1.2, we explored some physical quantities; many more can be listed, and it can become very overwhelming. However, here is the good news: Recall all introductory physics courses that you might have taken, and it will not be hard to realize that most of the physics is based on the following three fundamental concepts:

Existence of an Entity. A physical entity, such as a material particle, exists with some defining physical characteristics, such as mass and charge, pertaining to the existence of the entity. Physical entities can also be waves, which we will introduce later in this chapter.

Position in Space. At a given moment, the particle exists at some point in a three-dimensional (3D) space. This point is represented by using some coordinate system, such as a Cartesian coordinate system with rectangular coordinates *x, y*, and *z*, as shown in Figure 1.3.

Time. The particle exists at a specific point in space and at some specific time.

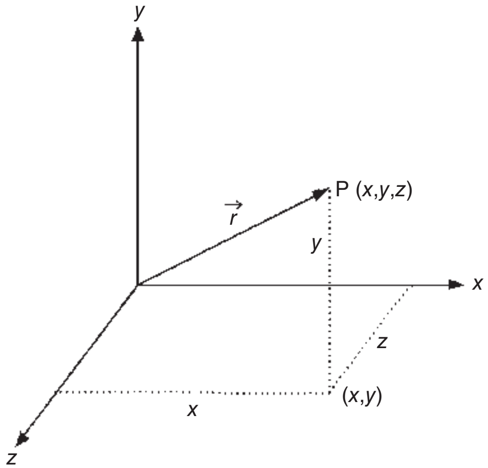


Figure 1.3. The Cartesian coordinate system used to represent the position of a particle in space.

It is important to realize that before describing the motion of a particle in space, we first must be able to determine the particle's position at a given time.

As mentioned earlier, physics relies mostly on mathematics to develop scientific explanations for entities and their behavior: the phenomena. It is time to demonstrate this phenomena by putting the physical quantities described earlier in their mathematical form. In addition, we will also derive these quantities from the three fundamental concepts just described.

One of these concepts, the position, (see Fig. 1.3) of a particle at a point P at a certain moment, is described by a vector $\vec{r}$ that goes from the origin of the coordinate system to point P. The position vector, as it is called, can be resolved into three component vectors along the x-, y-, and z-axis:

$$\vec{r} = \vec{x} + \vec{y} + \vec{z} = \hat{\imath}x + \hat{\jmath}y + \hat{k}z \tag{1.3}$$

where x, y, and z are the magnitudes of the position vector in the x, y, and z direction; and $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ are the unit vectors along the x, y, and z direction. The magnitude of the position vector is given by

$$r = \sqrt{x^2 + y^2 + z^2} \tag{1.4}$$

If you are considering motion in a straight line or in just one dimension, you can treat the vector quantities as scalar, because the direction is known and fixed. This is exactly what we are doing for most of the quantities in the rest of this section for simplicity.

So, the most fundamental issue in physics is the existence of a physical entity at a specific point in space and time, and the rest of physics can be derived by throwing in a time-related concept called change, for example, the change of position of the entity in time. This change creates motion, which in turn gives rise to quantities like momentum and energy.

> *Note*: Change may also apply to other fundamental properties of an entity. For example, later in this chapter you will see, that change in electric charge at a point with respect to time generates electric current.

The classical world of particles is explained in terms of some physical quantities derived from the three fundamental concepts discussed here.

1.4 PHYSICAL QUANTITIES

Physical entities (material bodies and waves) exist in space defined, for example, by the spatial coordinates (x, y, z) in the Cartesian coordinate system and by temporal coordinate, t. Figure 1.4 illustrates an example of how other physical quantities can be derived from three basic quantities: mass m of the particle, position x of the particle, and time t corresponding to three fundamental concepts discussed earlier:

- *Distance.* The distance (displacement) d that the particle has traveled during a time interval can be measured as the change in position (x) in this time duration:

$$d = x_2 - x_1 \tag{1.5}$$

  Meter (m) is a unit of distance; so is a mile.

- *Velocity.* Velocity, v, of a particle at a given moment is simply the rate of change of position of the particle with time represented by

$$v = \frac{dx}{dt} \tag{1.6}$$

  Velocity is a vector quantity, that is, it has a magnitude and a direction. The magnitude alone is called speed. Meters per second (m/s) is a unit of velocity or speed; so is mile/s.

- *Momentum.* The momentum p of a particle is the product of its mass and velocity:

$$p = mv = m\frac{dx}{dt} \tag{1.7}$$

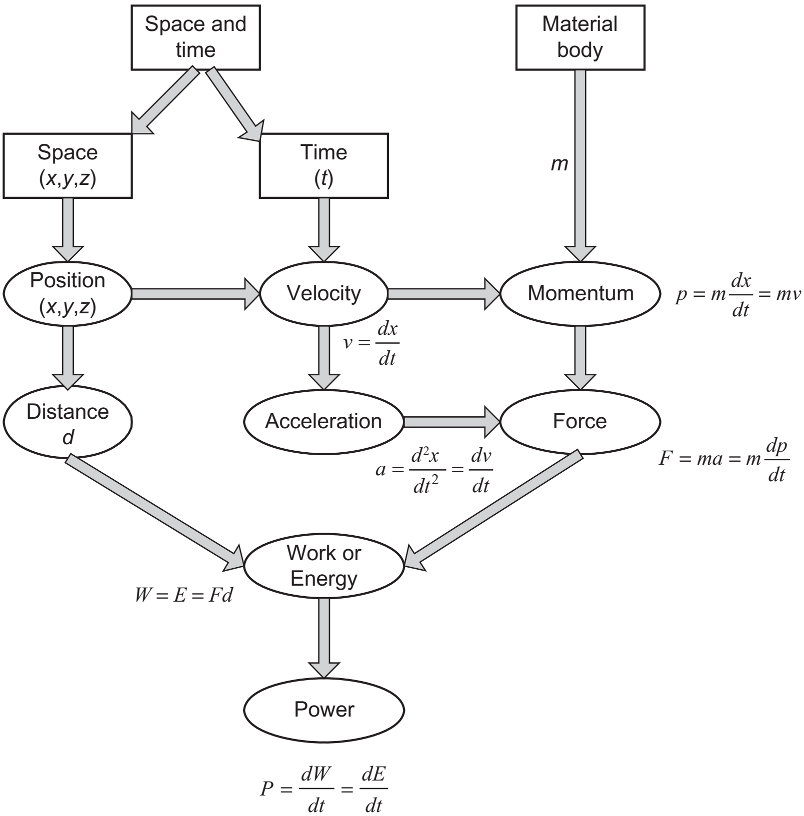


Figure 1.4. Physical quantities of material particles derived from the three fundamental concepts: existence (mass), space, and time.

- *Acceleration.* The acceleration of a particle at a given time is simply the rate of change in velocity with time:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \tag{1.8}$$

- *Force.* The force applied on a particle is measured as the product of its mass and acceleration it is experiencing as a result of the force:

$$F = ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2} \tag{1.9}$$

The newton (N) is a unit of force, which is given by

$$1\,\mathrm{N} = 1\,\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}$$

- *Work.* The work W is measured as a product of force and distance:

$$W = Fd = F(x_2 - x_1) \quad (1.10)$$

In general, the work performed by a force F on an object moving along a path S is given by

$$W = \int_{s_1}^{s_2} \vec{F}.d\vec{S} \quad (1.11)$$

where s_1 and s_2 are the start and end points of the curve. The joule (J) is a unit of work, which is given by

$$1\,\mathrm{J} = 1\,\mathrm{N\,m} = 1\,\mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}^2}$$

Energy has the same unit as work.

Energy. As mentioned earlier, the ability of a force to do work on an object is called energy. The work done by a force on an object is stored in the object in some form of energy. For example, assume you lift a body of mass m from the ground straight up and put it on a platform that is at a height h from the ground. In doing so, you have done work, W_g, against the gravitational force, $F_g = mg$, where g is the acceleration due to gravity. The work done by you has been stored into the body as its potential energy, E_p:

$$E_p = W_g = F_g h = mgh \quad (1.12)$$

When an external force works on a body at rest and gives it a speed of v, the work done on the body is equal to the kinetic energy, E_k, achieved by the body given by

$$E_k = \frac{1}{2}mv^2 \quad (1.13)$$

In general, the work done on a body is equal to the energy gained by the body and the work done by the body is equal to the energy lost by the body. For example, when you lifted up an object the energy was lost by your body and gained by the object that was lifted up. Overall, energy stayed conserved.

Power. Power is the rate of work performed or the rate of change of energy with time:

$$P = \frac{dW}{dt} = \frac{dE}{dt} \tag{1.14}$$

The watt (W) is a unit of power, which is equivalent to Joules per second (J/s).

Caution: Many quantities discussed here, such as velocity, acceleration, force, and momentum, are vector quantities; that is, they have magnitude as well as direction. For simplicity, in our treatment of these quantities, for the purpose at hand, we are only considering the magnitude in writing the equations, or alternatively considering these quantities in only one dimension, along the x-axis.

STUDY CHECKPOINT 1.1

Demonstrate how the physical quantity, called kinetic energy, can be derived from the three fundamental concepts of physics discussed in this section.

Solution:

Kinetic energy E_k is given by

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2$$

So, observe that the kinetic energy can be derived from m, x, and t.

In a nutshell, motion is caused by the change in position of an entity. But, what causes the change and exactly how does it occur? The work of several scientists over centuries elegantly put the answers to this question into three laws of motion by Issac Newton.

1.5 NEWTON'S LAWS OF MOTION

In an introductory physics course, you learn about these physical quantities by using point particles, that is, material entities with their mass concentrated at one point. Most of the physical quantities discussed so far describe the motion, or mechanics, of particles, and therefore belong to the branch of classical physics called classical mechanics. Classical mechanics is based on Newton's three laws of motion discussed in the following.

First Law. A physical body, or object, will continue in its state of rest or of uniform motion in a straight line until acted upon by an external force.

> *Note*: Newton's first law answers this question: If the position and momentum of a particle is known at time $t = 0$, what will be the position and momentum of the particle at some later time t in the absence of any external force?

In other words, the first law states that in the absence of any external force the motion of a body does not change: The body stays in a state of equilibrium. However, it does not necessarily mean that no force is acting on the body. For the body to be in equilibrium, the net result, that is, the vector sum, of all the forces acting on a body should be zero:

$$\sum \vec{F} = 0 \tag{1.15}$$

What if there is some nonzero net force acting on the body? This is when the second law of motion comes into the picture.

Second Law. If a net external force acts on a body, the body accelerates in the same direction as the force. The magnitude of the acceleration a is directly proportional to the force F that causes it:

$$a \infty F$$

This means

$$a = CF$$

where C is the proportionality constant and is equal to the inverse of the mass of the object: $C = 1/m$.

Therefore,

$$F = ma \tag{1.16}$$

It can also be written by using Eqs. 1.7 and 1.8 as:

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt} \tag{1.17}$$

Equation 1.17 is the fundamental equation of Newtonian mechanics, where p, the product of mass and speed of an entity, is called its momentum.

Note: Newton's first and second law answer this question: If the position and momentum of a particle is known at time $t = 0$, what will be the position and momentum of the particle at some later time t under the influence of a net force force F. If $F = 0$, we are talking about the first law.

The second law states how an object's motion changes when an external force is applied on it. What else does an object do in response to an external force? This question is answered by the third law of motion.

Third Law. When a body A exerts a force on a body B, then as a reaction, body B exerts back an equal and opposite force on body A. Equal and opposite means that the two forces are equal in magnitude and opposite in direction. It is also stated as: Action and reaction are equal and opposite. Therefore, this law is also known as the law of action and reaction, which means forces always occur in pairs. It can mathematically be represented as:

$$\vec{F}_{21} = -\vec{F}_{21} \tag{1.18}$$

Caution: Do not think that because these forces are equal and opposite, they will cancel out. Remember that these forces are being applied on two different bodies. They would have canceled out if they were working on the same body at the same point.

PROBLEM 1.2

Derive the first law of motion from the second law of motion.

Solution:

From the second law of motion:

$$F = ma = m\frac{dv}{dt} = \frac{dp}{dt}$$

This means that if F is zero, $a = 0$, which in turn means if no force is applied on the object, the object will keep moving with the uniform (constant) velocity v with which it is currently moving, or it will stay at rest if it is currently at rest.

It can be proved that the second law is the fundamental law from which the first and third laws can be derived.

STUDY CHECKPOINT 1.2

Specify if each quantity below is a scalar or a vector:

A. Velocity
B. Energy
C. Momentum
D. Acceleration
E. Mass
F. Temperature

Solution:

Scalars: B, E, F
Vectors: A, C, D

Note: Newton's three laws of motion based on the work of several scientists, including Copernicus, Kepler, Galileo Galilei, and Newton himself, represent a great intellectual feat of humans on this planet. These three rather simple laws can be applied to any system until certain limits are hit with respect to size and speed. For very small sizes, Newtonian physics is replaced with quantum physics, and for very high speeds (closer to the speed of light), Newtonian physics is replaced with relativistic physics based on Einstein's theory of relativity. More on this later in this chapter.

So far, we have defined the concepts and some basic physical quantities related to motion in a straight line. However, Newton's laws also apply to motion that is not in a straight line, rotational motion.

1.6 ROTATIONAL MOTION

Motion can also occur in a curvy or circular fashion called rotation. As illustrated in Figure 1.5, the position of a particle in circular motion (in two dimensions, 2D) can be determined by the radius, r, of the circle, and the angle θ that it makes with a fixed direction by extending an arc of length s. The relationship between these three quantities is given by

$$\theta = \frac{s}{r} \tag{1.19}$$

The units of angle θ determined this way are in radians (rad) and are related to degrees as follows:

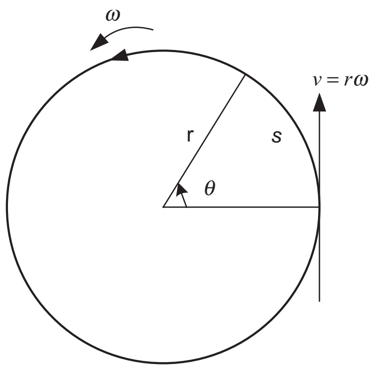


Figure 1.5. Circular motion for an arc length s corresponding to an angle θ: $s = r\,\theta$.

$$1\ \text{rad} = \frac{360^\circ}{2\pi} = 57.3^\circ$$

This is an example of a polar coordinate system in 2D. The basic quantities related to rotation are derived in the following:

Angular Velocity. Because the angle θ specifies the position of a particle in polar coordinates at a given moment, the rotational motion (velocity) of the particle can be defined in terms of the rate of change of θ. This means the angular velocity, ω, of a particle moving in a circle of radius, r, is defined as the rate of change in the angle θ with time:

$$\omega = \frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt} = \frac{v}{r} \tag{1.20}$$

Therefore,

$$v = r\omega \tag{1.21}$$

To derive Eq. 1.20, we have used the definition of $\theta = s/r$. Equation 1.21 gives the relationship between the linear velocity, v, along the length of the curve and the angular velocity ω along the curve. The direction of the linear velocity v is tangent to the curve. Note that even if the linear velocity v is constant in magnitude, its direction is continuously changing, so it is under acceleration in that sense. Hence, $\vec{v}$ is changing. The acceleration of a particle moving in a circle can be presented in terms of centripetal and tangent components.

Centripetal Acceleration. The component of the acceleration along the radius of the circle and directed toward the center of the circle, called centripetal acceleration, is given by

$$a_r = \frac{v^2}{r} \tag{1.22}$$

For example, in the case of a satellite revolving around the earth, this acceleration means that the satellite is being attracted by the Earth with a force F_s given by

$$F_s = \frac{mv^2}{r} \tag{1.23}$$

where m is the mass of the satellite. The satellite is pulling the Earth toward itself with the gravitational force F_g given by

$$F_g = G\frac{Mm}{r^2} \tag{1.24}$$

where M is the mass of the Earth and the satellite is revolving around the Earth in a circular orbit with radius r. If the two forces are equal, they will balance and the satellite will stay in the circular orbit. Therefore, the condition for the satellite to stay in orbit is

$$G\frac{Mm}{r^2} = \frac{mv^2}{r}$$

which means

$$v = \sqrt{\frac{GM}{r}} \tag{1.25}$$

this value of v is called the escape velocity.

Tangential Acceleration. This is the angular acceleration, that is, the change in angular velocity with time, and can be written as:

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \frac{1}{r}\frac{d^2 s}{dt^2} = \frac{a_t}{r} \tag{1.26}$$

where a_t is the component of acceleration along the tangent to the circle.

Angular Momentum. The angular momentum, L, of a particle in a circular motion is given by

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = I\vec{w} \tag{1.27}$$

where

$$I = mr^2 \tag{1.28}$$

is a physical quantity called the moment of inertia, which is related to the geometric structure of the entity.

Therefore, the magnitude of angular momentum can be written as:

$$L = I\omega = mr^2\omega \tag{1.29}$$

The rotational kinetic energy can be described as:

$$E_k = \frac{1}{2}I\omega^2 \tag{1.30}$$

which can be shown as equivalent to the definition of kinetic energy for linear motion:

$$E_k = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(mr^2\right)\left(\frac{v^2}{r^2}\right) = \frac{1}{2}mv^2 \tag{1.31}$$

Isaac Newton (1643–1727)

Isaac Newton, the great physicist and mathematician, was born on 4 January 1643 in Woolsthorpe-by-Colsterworth, England. He studied and did most of his work at Cambridge, where he became professor of mathematics. His 1687 publication of the *Philosophiæ Naturalis Principia Mathematica* (usually called the *Principia*) in 1687 is among the most influential books in the history of science. It laid the groundwork for most of classical mechanics. In this work, Newton presented three laws of motion that make up the core of classical mechanics. In addition, his work includes the universal law of gravitation, principles of conservation of momentum and angular momentum, and theory of colors. He developed the first reflecting telescope.

Newton shares the credit with Gottfried Leibniz for the development of differential and integral calculus. Newton died in his sleep in London on 31 March 1727.

In a nutshell, using three laws of motion and the physical quantities that we have derived from the three fundamental concepts (mass, position, and time), we can describe complex motion of physical objects or particles.

So far, we have treated particles pretty much separated from each other. However, it is an interesting world and interesting things happen when particles (like people) interact with each other. The physical interaction with direct touch is called collision.

1.7 SUPERPOSITION AND COLLISION OF PARTICLES

It is convenient to illustrate some physical concepts by using just one particle. However, in the real world particles hardly ever exist alone. They tend to exist as colonies or communities, and therefore they interact with each other. They collide with each other and scatter, their properties superpose. Scattering and superposition are important characteristics of particle interactions.

1.7.1 Superposition

To understand superposition, you first must understand the resolution of a physical vector quantity, such as velocity, into its components. As demonstrated in Figure 1.6 by using velocity as an example, the component of a vector in a given direction is the projection of the vector on a line in that direction. Figure 1.6 shows the projections of a velocity vector $\vec{v}$ on the axes

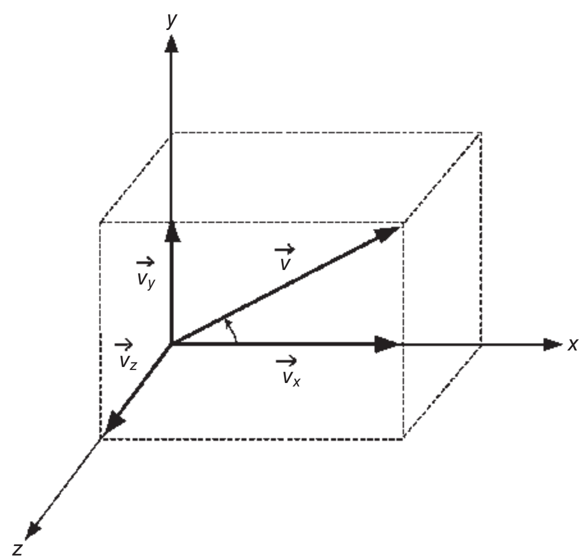


Figure 1.6. Resolving components of a vector as projections to the x-, y-, and z-axis.

of the rectangular (Cartesian) coordinate system: x, y, z. These projections are called rectangular components of velocity and are denoted as v_x, v_y, v_z:

$$\vec{v} = \overrightarrow{v_x} + \overrightarrow{v_y} + \overrightarrow{v_z} = \hat{\imath} v_x + \hat{\jmath} v_y + \hat{k} v_z \tag{1.32}$$

where v_x, v_y, v_z are the magnitudes of the vector $\vec{v}$ in the x, y, and z directions, respectively. The magnitude of the vector $\vec{v}$ can be written in terms of the magnitudes of its components:

$$v^2 = v_x^2 + v_y^2 + v_z^2 \tag{1.33}$$

or

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \tag{1.34}$$

Note that we have combined the magnitudes of the components to obtain the magnitude of the original vector. Similarly, you can also combine the components of two vectors to get a single vector called the resultant vector. This property is called superposition of vectors.

As you learned in your introductory physics course, superposition of vector physical quantities, such as velocity and force, is of utmost importance. For example, any number of forces applied at a particle (or at a point of a body) has the same effect as a single force that is equal to the vector sum of all the individual forces. In other words, the forces combine or superpose. Figure 1.7a illustrates an example of two forces $\overrightarrow{F_1}$ *and* $\overrightarrow{F_2}$ acting at a point on a box in an x–y plane. The effect of these forces is the same as the force $\overrightarrow{F_R}$, which is the vector sum of the two forces. As shown in Figure 1.7b, the two individual forces can be vectorially added by first resolving into their components.

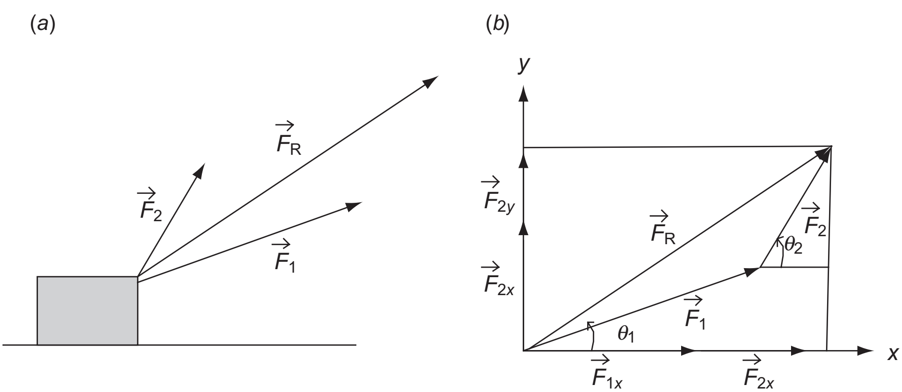


Figure 1.7. Superposition of two forces $\overrightarrow{F_1}$ *and* $\overrightarrow{F_2}$ resulting in one force $\overrightarrow{F_R}$.

$$\overrightarrow{F_1} = \hat{\imath} F_{1_x} + \hat{\jmath} F_{1_y}$$
$$\overrightarrow{F_2} = \hat{\imath} F_{2_x} + \hat{\jmath} F_{2_y}$$
$$\overrightarrow{F_R} = \overrightarrow{F_1} + \overrightarrow{F_2} = \hat{\imath} F_{1_x} + \hat{\jmath} F_{1_y} + \hat{\imath} F_{2_x} + \hat{\jmath} F_{2_y} = \hat{\imath}\left(F_{1_x} + F_{2_x}\right) + \hat{\jmath}\left(F_{1_y} + F_{2_y}\right)$$

where

$$\overrightarrow{F_{R_x}} = \hat{\imath}\left(F_{1_x} + F_{2_x}\right)$$
$$\overrightarrow{F_{R_y}} = \hat{\jmath}\left(F_{1_y} + F_{2_y}\right)$$

Therefore,

$$F_R^2 = F_{R_x}^2 + F_{R_y}^2$$

or

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} \tag{1.35}$$

We have demonstrated superposition in a 2D plane, but the extension into 3D space, and thereby using all the three coordinates (*x, y*, and *z*) is straightforward. Then, we will also have *z* components of each force, which we add to obtain F_{R_z}, and the magnitude of the resultant force becomes

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2} \tag{1.36}$$

Also, there could be more than two forces acting on a particle. You decompose each of the forces acting on a particle into three coordinates, and then recombine their components along each coordinate to obtain the resultant force. By now, you should be able to realize that a number of forces acting on a particle may result in a net force of zero, that is, the components of all the forces along each of the three coordinates add up to zero. A particle (or a body) with zero resultant force acting on it is a particle in equilibrium. Zero force means zero acceleration, not necessarily zero speed; it may have a constant speed or a uniform velocity. Equilibrium of a particle with zero velocity is called static equilibrium and the equilibrium of a particle with uniform velocity is called dynamic equilibrium.

Of course, you can apply superposition to other vector physical quantities as well, such as velocity.

> *Note*: In the context of equilibrium, you can restate Newton's first law of motion as: A particle in equilibrium will stay in equilibrium until and unless acted upon by an external force.

1.7.2 Collision and Scattering

A car accident on a freeway and balls colliding on a billiard table are both examples of collisions. So are the two beams of protons colliding at a gigantic particle collider at CERN (a particle physics lab near Geneva, Switzerland).

However, for our purpose, consider a simple system of two particles A and B, as shown in Figure 1.8, moving with momentum $\vec{P}_\mathrm{A}$, *and* $\vec{P}_\mathrm{B}$; kinetic energy T_A, *and* T_B; and total energy E_A, and E_B. So the momentum and energy of the system before collision are given by

$$\vec{P} = \vec{P}_\mathrm{A} + \vec{P}_\mathrm{B}$$

$$T = T_\mathrm{A} + T_\mathrm{B}$$

$$E = E_\mathrm{A} + E_\mathrm{B}$$

Now, the two particles collide and in general may have a different velocity and direction of motion after the collision, and therefore different energy and momentum. Assume that after the collision, the two particles are moving with momentum $\vec{P}'_\mathrm{A}$, and $\vec{P}'_\mathrm{B}$; kinetic energy T'_A, *and* T'_B; and total energy E'_A, *and* E'_B. So the momentum and energy of the system after collision are given by

$$\vec{P}' = \vec{P}'_\mathrm{A} + \vec{P}'_\mathrm{B}$$

$$T' = T'_\mathrm{A} + T'_\mathrm{B}$$

$$E' = E'_\mathrm{A} + E'_\mathrm{B}$$

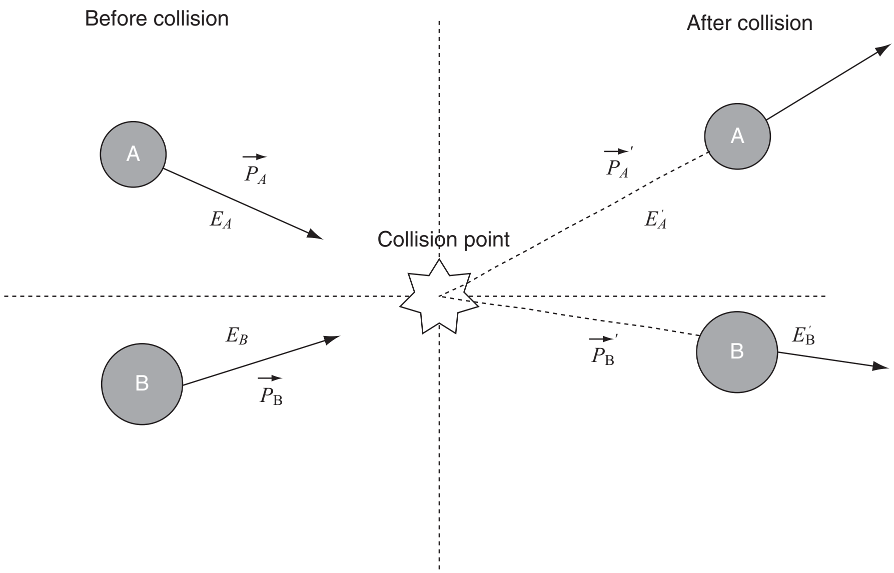


Figure 1.8. Illustration of collision between two particles.

Assume there is no external force acting on the system. Then, according to the principle of momentum conservation, the momentum of the system after the collision will be equal to the momentum of the system before the collision:

$$\vec{P}' = \vec{P}$$

Also, according to the principle of energy conservation, the total energy of the system after the collision will be equal to the total energy of the system before the collision:

$$E' = E$$

Even though the energy is conserved, its form can change, and it is the toal energy that is conserved. Therefore, one specific form of energy, such as kinetic energy, does not have to be conserved. If the kinetic energy of the system after the collision is the same as before the collision, the collision is called elastic. If the total kinetic energy of the system changes as a result of the collision, the collision is called an inelastic collision.

Because the particles as a result of collision deviate from their original path, they can be considered as scattered, and the process is called scattering. You can imagine that when there are a large number of particles, such as molecules, colliding with each other in an elastic fashion, they will be deflected from their path quite frequently. As a result, they will be moving in a zigzag path. Such a motion is called Brownian motion. Yes, the billiard ball model of collision discussed in this section works well even at the microscopic scale of molecules and subatomic particles, such as protons and electrons; well enough to predict the behavior of participants to some degree of accuracy. Chapter 2 discusses an example of collision (and hence scattering) called Compton scattering, one of the few phenomenon that pushed physicists toward quantum mechanics.

In a nutshell, collision, scattering, and superposition are important characteristics of particles. You will see that the concept of scattering and superposition also apply to waves, but in a different way.

The motion of particles can be considered as transportation of energy and momentum from one point to another in space. However, this leaves out a class of natural phenomena and physical entities, which can only be explained by using a concept called a wave.

1.8 CLASSICAL WORLD OF WAVES

A wave is a disturbance of some sort that propagates through space and transfers some kind of energy from one point to another without transporting mass, which occurs with particles. For example, when you speak to a person face to face, the sound wave travels from your mouth to the ear of the listener. The propagating disturbance here is the change of pressure in the air. As long as the wave is traveling through a point, the air pressure at that point does not stay constant over time, and your ear and brain perceives the pressure change as a sound. You must also be familiar with the water waves (ripples) that you can see by throwing a rock in a pond with still water. As another familiar example, the light that you need to see anything is a result of an EM wave. The disturbance in an EM wave is the change in the electric and magnetic field. The wave can be looked upon as propagation of this disturbance. The EM wave is a wave that carries EM energy. You will learn more about these waves later in this chapter. One of the interesting things about EM waves is that they can travel through a vacuum. This means that they do not need a material medium to travel through, whereas sound waves and water waves are mechanical disturbances that require a physical medium such as air or water, to travel through. The EM radiation is an example of periodic waves.

1.8.1 Periodic Waves

Figure 1.9 illustrates a traveling sinusoidal wave by presenting snapshots of a wave at different times $t = 0.5\,T$, $1\,T$, $1.5\,T$, and $2\,T$, where T is the period of the wave, which is the time in which the wave travels a distance equal to its wavelength.

Wavelength is one of the defining characteristics of a wave.

The wave illustrated in Figure 1.9 is called a periodic wave because the shape of the wave is the repetition of a pattern stretched from one crest to the next or from one trough to the next.

We can use the periodic wave to define some wave characteristics.

1.8.2 Defining Wave Characteristics

As shown in Figure 1.10, you can describe a wave in terms of some parameters, such as amplitude, frequency, and wavelength, as described in the following:

Wavelength. Denoted by the symbol λ, this is the distance between two consecutive crests or two consecutive troughs of a periodic wave. The distance equal to the wavelength makes one cycle of change.

Amplitude. Amplitude is the maximum amount of disturbance during one wave cycle.

Velocity. The velocity or speed, v, of a wave is the speed with which the disturbance defining the wave travels in space. As illustrated in Figure 1.11, the wave pattern with length λ travels with velocity v and covers the distance λ in time T, the period of the wave. Therefore,

$$v = \frac{\lambda}{T} = f\lambda \tag{1.37}$$

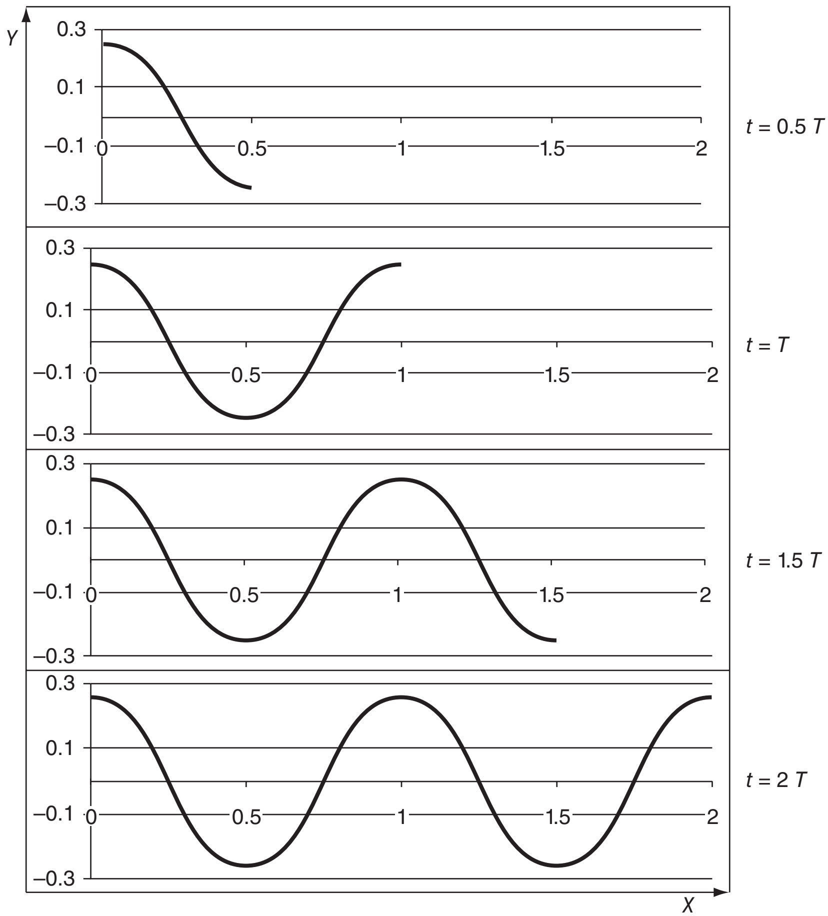


Figure 1.9. Propagation of a sinusoidal wave in time.

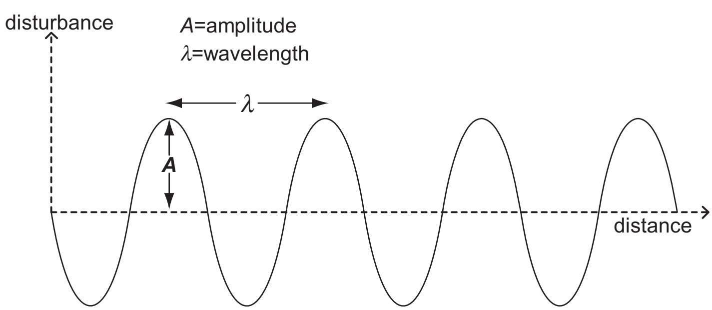


Figure 1.10. Illustration of a wave.

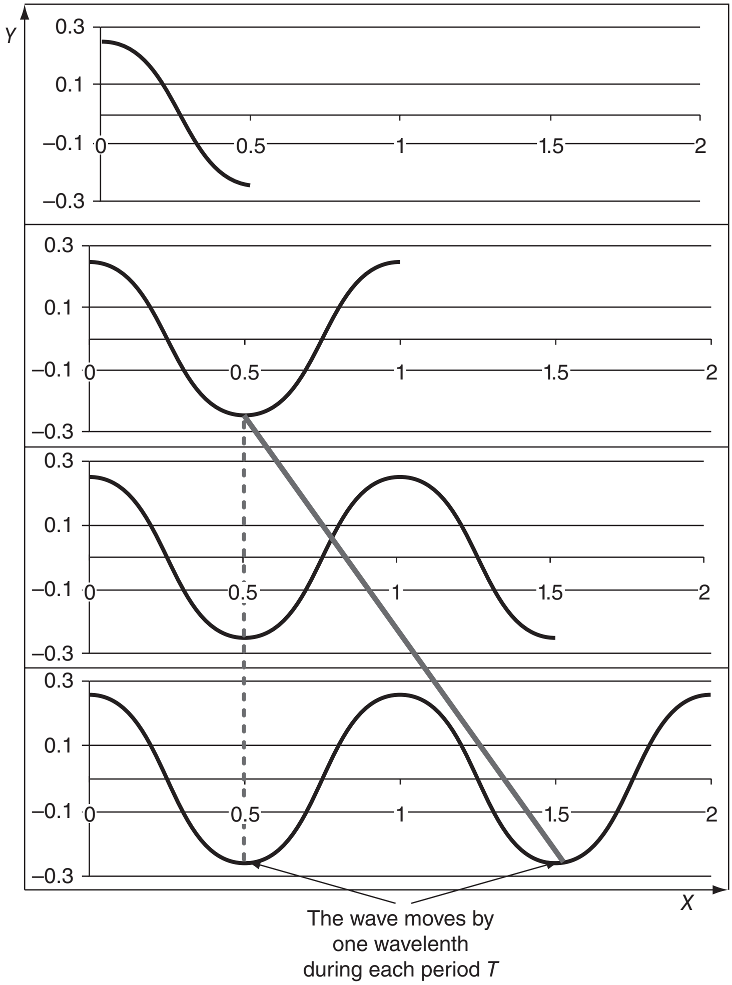


Figure 1.11. An illustration of a wave propagation.

The speed of a mechanical wave depends on the properties of the medium in which it propagates. The speed of EM waves in a vacuum is constant, which is the speed of light, c, given by

$$c = 2.998 \times 10^8 \ m/s$$

Frequency. The frequency, f, is the number of cycles per unit time a wave repeats at a given point:

$$f = \frac{1}{T} \tag{1.38}$$

where T is the time to complete one cycle and is called a period. This translates to

$$f = \frac{v}{\lambda} \tag{1.39}$$

Therefore, the frequency of an EM wave, f, propagating through free space (vacuum) is calculated by using the following equation:

$$f = \frac{c}{\lambda} \tag{1.40}$$

The frequency is measured in units of the hertz (Hz) and 1 cycle per second (cps) is 1 Hz.

Phase. A phase is the current position in the cycle of change in a wave.

Wavefront. The concept of wavefront is often used to describe the propagation of a wave. A wavefront is the locus, such as a line for a light ray in one dimension (ID) or a surface for a wave propagating in 3D, of points having the same phase. The locus is a mathematical entity (Latin for "place") that represents a collection of points with a shared property. For example, in 2D, a circle is the locus of points that are at an equal distance from a single point, the center of the circle. In 3D, the surface of a sphere is a locus of points that are at an equal distance from one point, the center of the sphere. A sphere makes a spherical wavefront for the wave propagation in 3D space.

Some concepts, such as velocity and energy, are common to both particles and waves. Other concepts, such as frequency and wavelength, are specific to waves, that is, in the framework of classical physics. Reflection of waves is very similar to the elastic collision of a material object in which the object strikes a wall and bounces back. Some other properties of waves, such as refraction and diffraction, are specific to waves. In classical physics, the terms of diffraction and refraction are not applied to particles. Although reflection and scattering actually do apply to particle interactions. These and some other wave characteristics are discussed next.

1.9 REFLECTION, REFRACTION, AND SCATTERING

As a wave, such as a light wave, travels through a medium, it can scatter. Also, when a wave, such as a light wave, strikes an interface between two transparent materials, such as air and glass, it can go under reflection and refraction. These phenomena are described in the following.

Reflection. Reflection is defined as the abrupt change in direction of a wavefront at an interface between two dissimilar media, so that the wavefront returns into the medium from which it hits the interface. For example, radio frequency (rf) waves, also called radio waves, are reflected when they strike the objects much larger than their wavelength, such as the floor, ceiling, and support beam. Metals are an obstruction to the signal because they are good at reflecting radio waves.

Refraction. Refraction is defined as the change in direction of a wavefront at an interface between two dissimilar media, but the wavefront does not return back to the medium from which it hit the interface. For example, the radio waves bend when they pass from one medium into another. Figure 1.12 illustrates reflection and refraction.

Scattering. Scattering is the phenomenon of absorbing a wave and reradiating it in various directions. Thereby it changes its direction of propagation from the original direction. Note that no absorption happens in particle scattering. As an example of wave scattering, consider the reflection of an EM wave, which is actually a scattering. For example, when a rf wave is scattered, it results in the loss of the signal or dispersion of the wave, as shown in Figure 1.13. It happens due to the interaction of the wave with the medium at the molecular level.

An application of scattering is a radar that uses scattering to detect metallic objects, such as airplanes or speeding cars.

Two other important characteristics of waves are diffraction and interference, discussed next.

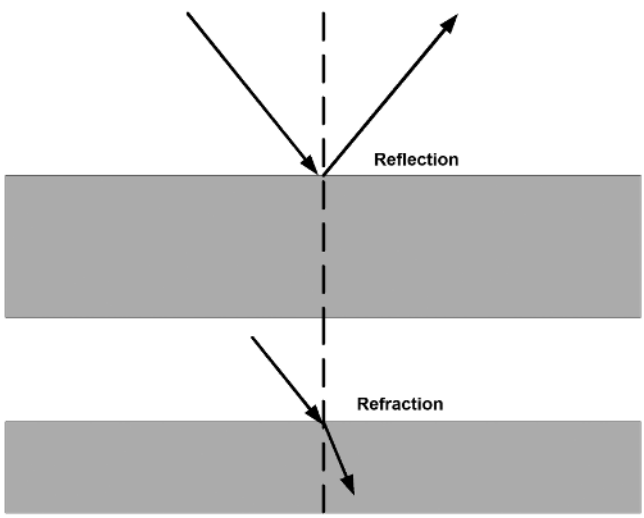


Figure 1.12. Reflection and refraction of waves; only the direction of propagation is shown.

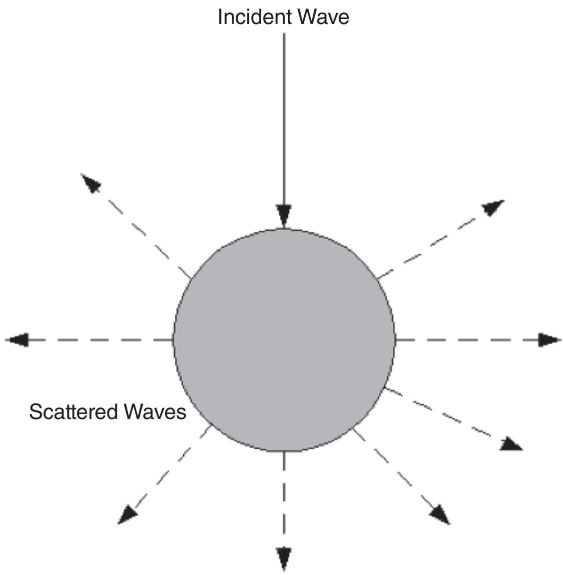


Figure 1.13. An illustration of wave scattering; only the direction of propagation is shown.

1.10 DIFFRACTION AND INTERFERENCE

It will come as no surprise if I tell you that even if the person who is talking to you turns his back on you, you can still listen, because sound is a wave, and waves exhibit a characteristic (or a phenomenon) called diffraction. If sound was carried by a particle, you would never be able to listen to a person whose back is turned to you.

Diffraction and a related property called interference are described in the following sections.

1.10.1 Diffraction

Diffraction refers to the bending of a wave when it strikes sharp edges or when it passes through narrow gaps. This makes propagation of waves quite different from the propagation of particles. The barrier will completely stop a particle coming at it as illustrated in Figure 1.14*a*, whereas waves will bend around it to keep going as illustrated in Figure 1.14*b*. This shows that in classical physics, the propagation of waves is very different from the propagation of a beam of particles. In the case of waves, the narrow opening in the barrier acts as a source.

Diffraction is related to a phenomenon called interference.

Note: One of the main objectives of this chapter is to help you understand how waves and particles are treated as different types of physical entities in classical physics. If you do not feel comfortable with any of the concepts discussed here, you should review them by using a basic physics book before you proceed; a list of books is provided in the Bibliography section at the end of this book.

STUDY CHECKPOINT 1.3

Diffraction is the process in which a wave bends around an edge or a narrow opening. Why does this bending happen in waves and not in particles?

Solution:

The slit acts as a source of waves, whereas it does not act as a source of a beam of particles. This is a unique characteristic of waves that each wavefront act as a source; this is how waves propagate.

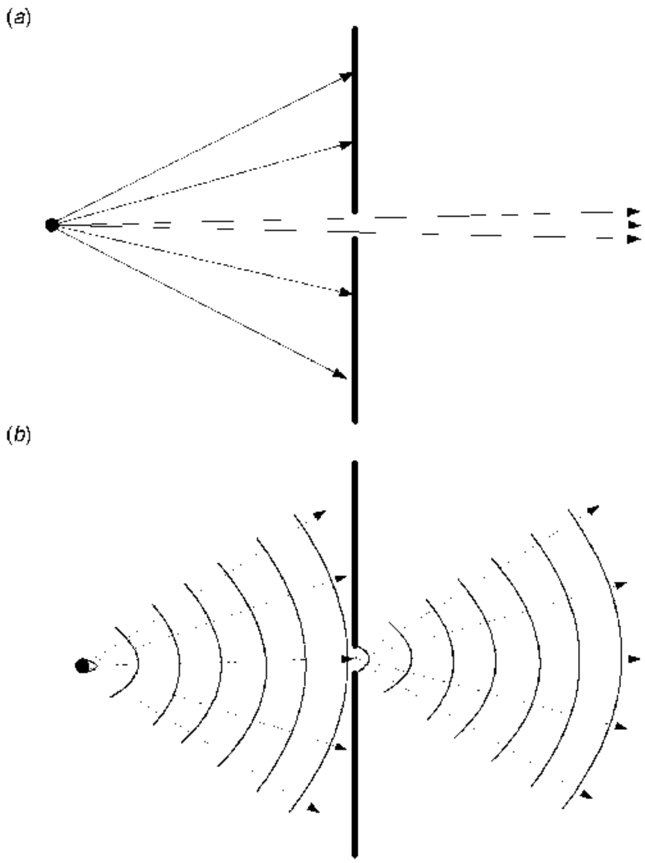


Figure 1.14. Illustration of (*a*) a beam of particles passing through a narrow opening in a barrier and (*b*) a wave transmitting through a narrow opening in a barrier.

1.10.2 Interference

Interference is defined as the interaction between two waves. Thomas Young in 1802 formulated the interference of light waves in terms of the principle of superposition: If two light waves arrive simultaneously at the same place, their effect is identical to that of the third wave, whose amplitude is the algebraic sum of the individual amplitudes of the component waves. An example of interference as a result of light from a source passing through a double slit is illustrated in Figure 1.15, where light waves passing through two different slits interfere with each other. The interference can be constructive, in which case the resultant wave has a larger amplitude, that is, the individual amplitudes are added to each other; or destructive, in which case the resultant wave has a smaller amplitude than the original wave, that is, the individual amplitudes are subtracted from each other. This constructive and destructive interference creates a pattern on the screen with high-intensity spots corresponding to constructive interference and very low intensity (dark) spots corresponding to destructive interference.

Thomas Young (1773–1829)

Thomas Young was born on June 13, 1773 in Milverton, Somerset, England into a large Quaker family. He began studying medicine in 1792, and also became interested in pure science. Young was a true polymath for his time, with interests ranging from physics to physiology and medicine, to music, to languages, and egyptology; he studied and contributed to all these fields. He obtained the degree of doctor of physics in 1796 from the University of Göttingen, Lower Saxony, Germany.

In 1801, Young was appointed as a lecturer at the recently formed Royal Institution in London, where he gave a series of lectures on a variety of topics. At that time, there was evidence for both pictures of light: waves and particles, and Newton was the proponent of the particle theory. In 1801, Thomas Young was intellectually bold enough to present a serious challenge to Newton's ideas on the topic, even though Newton was so greatly revered as a scientist that it was nearly impossible for anyone to dispute his theory.

Thomas Young died on May 10, 1829 at the age of 55 in London, England.

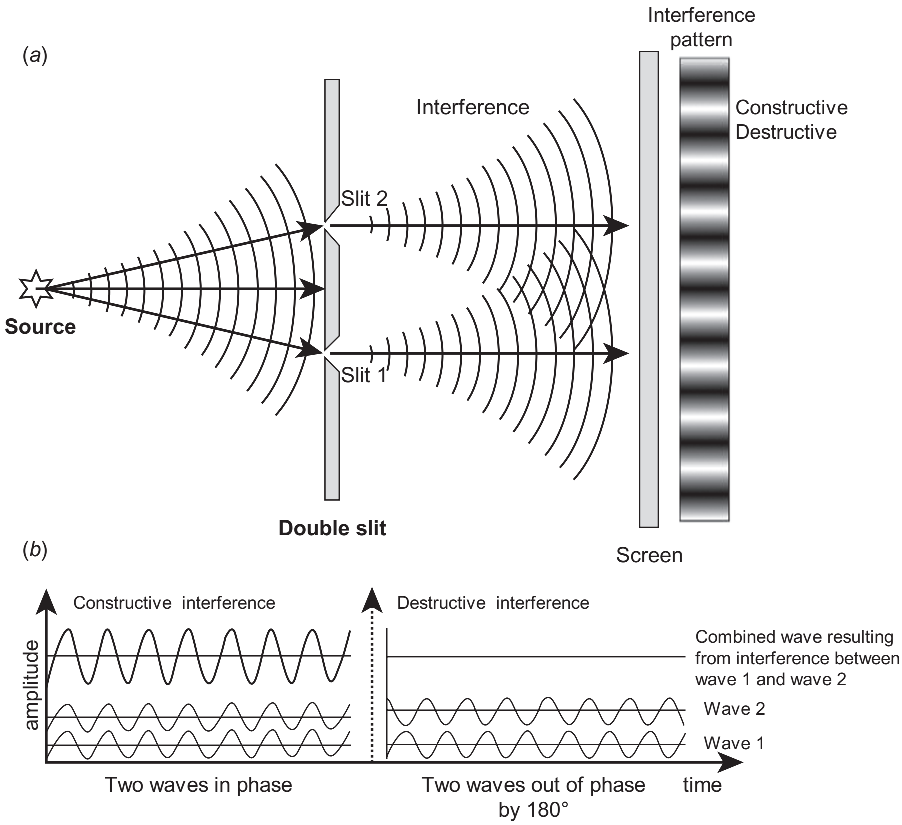


Figure 1.15. Interference of two waves produced by a double slit.

In order to avoid confusion, it is important to understand the relationship among diffraction, scattering, and interference. Diffraction is bending of a wave around small obstacles or the spreading out of waves through small openings. Consider an obstacle providing multiple closely spaced openings. A wave passing through it will split into components coming from different openings, which will interfere with each other through a process called interference or superposition. This interference gives rise to a pattern with maxima and minima of wave intensity, and is called an interference or diffraction pattern. One may also look at the bending of waves as scattering with a particle view in mind.

Newton's second law describes the motion of a particle. What describes the motion of a wave?

1.11 EQUATION OF WAVE MOTION

Newton's second law of motion, represented by Eq 1.17, provides the fundamental equation of motion for Newtonian mechanics to describe the motion of a particle. Is there any fundamental equation of motion for a wave? The answer is yes, and here it is in 1D.

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2} \tag{1.41}$$

In general, this equation describes a wave that has a variable quantity Ψ, called a wave function, which propagates in the x direction with velocity v. For example, for the wave on a string, Ψ represents the displacement of the string from the x-axis; for a sound wave, Ψ represents the pressure difference; and for an EM wave, Ψ is the magnitude of an electric or magnetic field. For mechanical waves, Eq. 1.41 can be derived from Newton's second law of motion, and for EM waves, it can be derived from Maxwell's equations.

> *Note*: As demonstrated in Problem 1.3, the wave equation 1.41 is obeyed by any wave in 1D that is propagated without dispersion or change in shape. It is satisfied by any function of $(x - vt)$ or of $(x + vt)$.

It can be proved that a general solution of the wave equation 1.41 is given as:

$$\Psi(x,t) = \Psi_0 \sin(kx - \omega t) \tag{1.42}$$

where Ψ_0 is the maximum displacement called the wave amplitude and ω is the angular velocity given by

$$\omega = kv = \frac{2\pi}{\lambda} v \tag{1.43}$$

where k is called wavenumber.

PROBLEM 1.3

Prove that any function of $x - vt$ or of $x + vt$ will satisfy the wave equation.

Solution:

Substitute:

$$f = x - vt$$

which implies

$$\frac{\partial f}{\partial x} = 1 \tag{1}$$

$$\frac{\partial f}{\partial t} = -v \tag{2}$$

Therefore,

$$\Psi = \Psi(x - vt) = \Psi(f)$$

$$\frac{\partial \Psi}{\partial x} = \frac{\partial \Psi}{\partial f}\frac{\partial f}{\partial x} = \frac{\partial \Psi}{\partial f} \tag{3}$$

We made use of Eq. 1.

$$\frac{\partial \Psi}{\partial t} = \frac{\partial \Psi}{\partial f}\frac{\partial f}{\partial t} = -v\frac{\partial \Psi}{\partial f} \tag{4}$$

We made use of Eq. 2.
From Eqs. 3 and 4:

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{\partial^2 \Psi}{\partial f^2} \tag{5}$$

$$\frac{\partial^2 \Psi}{\partial t^2} = -v\frac{\partial^2 \Psi}{\partial f^2}\frac{\partial f}{\partial t} = v^2\frac{\partial^2 \Psi}{\partial f^2} \tag{6}$$

This means:

$$\frac{\partial^2 \Psi}{\partial f^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}$$

Substitute this in Eq. 5:

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}$$

This is the wave equation.
Hence, the proof.

Just like particle motion, wave motion can be considered as the transportation of energy and momentum from one point to another in space, but without the transportation of matter. In mechanical waves, such as sound, water, and waves on a string, the momentum and energy are transported through a disturbance that is propagated through the medium. Some waves do not need any medium to travel through. For example, EM waves transport momentum and energy through electric and magnetic fields, which propagate through a vacuum.

In this book, we will be dealing with the EM waves in order to introduce quantum physics. So, it is relevant to review some concepts of EM, which is also an important part of classical physics. While motion of mass (material objects) can be described with Newtonian mechanics, the motion of electric charge (also referred to as just charge for brevity) is described by EM.

Again, while reviewing the basic concepts of EM, pay attention to how all these concepts are based on (or can be derived from) three fundamental concepts: charge instead of mass in Newtonian mechanics, position of charge in space, and change of position of charge with time. So the three fundamental concepts are again generally the same: entity, space, and time. An example of an EM wave visible to us is visible light.

1.12 LIGHT: PARTICLE OR WAVE?

Light has intrigued human beings for centuries. The struggle to understand the nature of light and the centuries-long controversy over it is not only one of the most interesting chapters in the history of science, but it also provides grounds to develop quantum physics. Early theories considered light as a stream of particles that emanated from a source and created a sensation of vision by entering the eyes. Newton during his time was the most influential proponent of the particle (also called corpuscular) theory of light, whereas Christian Huygens and Robert Hooks were among the major proponents of the wave theory of light. The phenomena of reflection and refraction of light known at the time were easily explained using the particle theory. Although Huygens was able to explain reflection and refraction by using wave theory, Newton rejected the wave theory of light based on the lack of evidence for diffraction. Even when the evidence for light diffraction became available, Newton's followers successfully argued that diffraction was due to the scattering of light particles from the edges of slits.

The wave theory of light made a comeback in 1801 when Thomas Young showed that sound waves and light both exhibit interference; showing that it is not possible for particles to create interference. Although Young's work went largely unnoticed by the scientific community for about a decade, Augustin Fresnel put wave theory on a firm mathematical basis, while performing extensive experiments on diffraction and interference. A big blow to Newton's particle theory came in 1850, when Jean Foucault showed by his measurements that the speed of light in water is less than that in air. In 1860, James Clerk Maxwell published his theory of EM, which predicted the speed of EM waves to be 3×10^8 m/s. He suggested that this agreement with the speed of light was not coincidental, that light was in fact an EM wave.

Maxwell's theory was experimentally confirmed by experiments performed by Hertz in 1887, who successfully generated and detected EM waves in the laboratory by using electrical means, such as a spark gap in a tuned circuit. He used the spark gap to in a tuned circuit to generate the wave and another circuit to detect it. The irony of the situation is that in the same year when Hertz's experiment confirmed Maxwell's wave theory, Hertz also discovered a phenomenon called the photoelectric effect, which could only be explained by Einstein using the particle model of light. However, we are getting ahead of ourselves now. These latter two points will be discussed in the forthcoming chapters. For now, understand that toward the end of the nineteenth century, when some experiments challenged classical physics, light was largely viewed as a wave: the EM wave.

To understand EM waves, it is important to understand electricity.

1.13 UNDERSTANDING ELECTRICITY

Electricity is the property of matter related to electric charge. Historically, the word electricity has been used by several scientists to mean electric charge. This property (electricity) is responsible for several natural phenomena, such as lightning, and is used in numerous industrial applications, such as electric power and the whole field of electronics. To understand electricity, you must understand the related concepts discussed in the following:

Electric Charge. Electric charge, also referred to as just charge, is a basic property of some particles of matter. There are two types of charge: positive and negative. For example, an electron has a negative charge and a proton has a positive charge, equal in magnitude to that of the electron. The charge of an electron (or proton) is used as a natural unit of charge. The standard symbol used to represent charge is q or Q. Two particles (or objects) with the same type of charge repel each other and two objects with the opposite types of charge attract each other. The charge is measured in units of the coulomb, denoted by C.

Charge is conserved: the algebraic sum of all the electric charges in a closed system is a constant. Note, it does not mean that charge cannot flow. Nor does it mean that charges cannot be created or destroyed. For example, consider a system of one particle called a photon, which can decay into a pair of electrons with a negative charge and a positron with a positive charge. Nevertheless, the algebraic sum of charges is still zero as before. Also, the electrically neutral objects are not neutral because they have no charge, but because they have an equal number of positive and negative charges. For example, all matter is made up of atoms and all atoms in their natural form contain an equal number of protons (positively charged particles) and electrons (negatively charged particles).

Electric Potential–Voltage. The electric potential difference V_{ab} between two points a and b is the work required to move one unit, C, called a coulomb, of charge from one point a to the other point b in space. This is commonly called an electric potential or voltage because it is measured in units of the volt, denoted by V.

Capacitance. Capacitance is the amount of charge stored in a system, called a capacitor, per unit of electric potential. In other words, the capacitance, C, is defined by the following equation:

$$C = \frac{Q}{V} \tag{1.44}$$

A capacitor is a device that stores charge and the resulting electric potential energy. Therefore, capacitance may also be looked upon as a measure of the ability of a device to store energy. For example, consider a parallel plate conductor shown in Figure 1.16, which contains two parallel plates as conductors. Assume that initially each conductor has a zero charge and therefore has a zero net charge. Now assume that electrons are transferred from conductor a to conductor b, so that conductor b acquires a $-Q$ charge and conductor a acquires a $+Q$ charge. The total

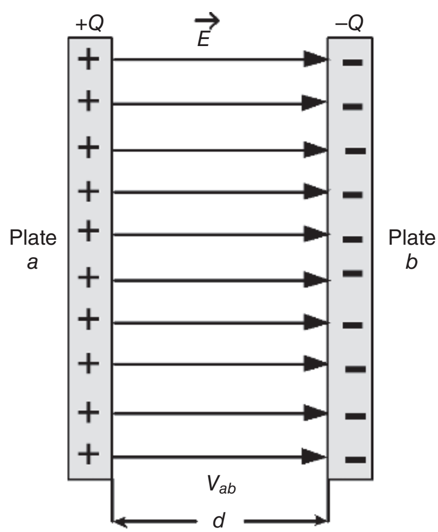


Figure 1.16. A parallel plate capacitor.

work performed for this transfer is stored as the potential energy in the capacitor. The voltage (potential difference) between the plates can be written as:

$$V_{ab} = Ed \tag{1.45}$$

where E is the electric field between the two plates (conductors), and d is the distance between the plates. Therefore, from Eqs. 1.44 and 1.45, the capacitance of the capacitor can be written as:

$$C = \frac{Q}{V_{ab}} = \frac{Q}{Ed} \tag{1.46}$$

This example of a capacitor is the so-called parallel plates capacitor: Two metallic plates separated from each other and each plate carrying equal and opposite charge, Q, with a potential difference between them, V. Capacitance is measured in units of the farad, denoted by F. For example, if the charge on each plate of a parallel plate capacitor is 1 C, and the voltage between them is 1 V, the capacitance of the capacitor will be 1 F.

Electric Current. Electric current is the rate of flow of an electric charge per unit time, and can be defined by the following equation:

$$I = \frac{Q}{t} \tag{1.47}$$

In this equation, I is the current and Q is the amount of charge that flowed past a point in time t. More generally, it can be represented in terms of the rate of the charge change with time:

$$I = \frac{dQ}{dt} \tag{1.48}$$

Current is measured in ampere units, (A). For example, one C of charge flowing past a point in 1 s represents 1 A of current. The materials, such as metals, that permit relatively free flow of charge are called conductors, whereas the materials, such as glass, that do not allow free flow of charge are called insulators.

Resistance. Resistance is a measure of opposition offered by a material to the flow of charge through it. The resistance can be measured by Eq. 1.49.

$$I = \frac{V}{R} \tag{1.49}$$

This means that the larger the resistance, the smaller the current. Resistance is measured in units of the ohm (Ω). For example, if the voltage of 1 V creates 1 A of current in a conductor, the resistance of the conductor is 1 Ω.

Electric Energy. The amount of work that can be done by an amount of electric charge across a potential difference is the electric energy. For example, the energy, E, of a charge Q across a voltage V is given by Eq. 1.50.

$$E = QV \tag{1.50}$$

Electric Power. The rate of work performed by an electric current is the electric power. In other words, it is the electric energy produced or consumed per unit time, and is given by Eq. 1.51.

$$P = \frac{E}{t} = \frac{QV}{t} = IV \tag{1.51}$$

The power is measured in units of the watt (W). For example, the power consumed to maintain a current of 1 A across a voltage of 1 V is 1 W.

PROBLEM 1.4

Show that electric power can also be expressed by the following equations:

$$P = I^2 R$$

$$P = V^2 / R$$

Solution:

We know that

$$P = IV$$

We also know that

$$I = V / R$$

Therefore,

$$P = IV = (V/R)V = V^2 / R$$

But,

$$I = V/R \quad \text{means} \quad V = IR$$

Therefore,

$$P = IV = I \times IR = I^2 R$$

Electric Field. The electric field is a field used by the charges at a distance to exert a force on each other. In other words, the charges at a distance interact with each other through their fields, called electric fields. Here is how it works. A particle as a result of its charge modifies the properties of the space around it. Any other charged particle, due to the charge it carries, experience that modification in terms of an electric force. As explained earlier, the fields are usually represented by field lines, as illustrated in Figure 1.17 for two charged particles interacting with each other through their electric fields.

Note: An electric force on a charged particle is exerted by the electric field created by other charged particles.

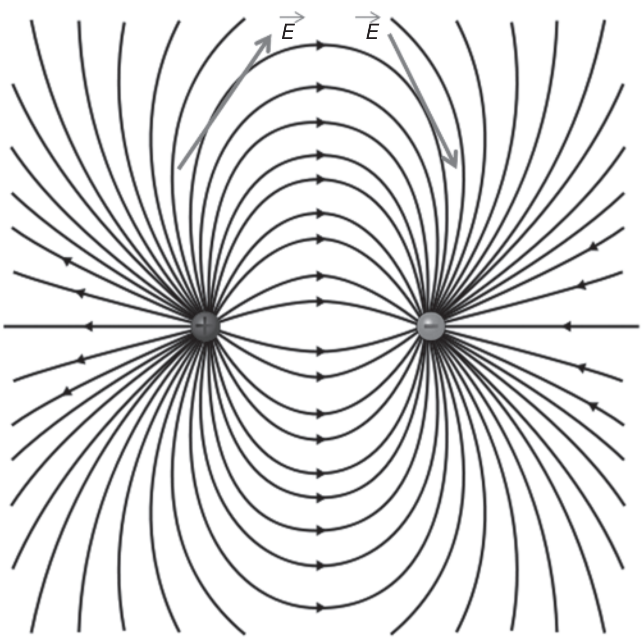


Figure 1.17. A particle with a positive and a negative electric charge applying force on each other through electric field lines. Courtesy of Sharayanan.

An electric field due to a particle with charge q at a distance r is defined as the force exerted on a unit charge at that point and is given as:

$$\vec{E} = k\frac{q}{r^2}\hat{r} \tag{1.52}$$

where k is called the Coulomb constant with a value of 8.988×10^9 N.m^2/C^2, and $\hat{r}$ is the unit vector along the line from the particle (considered to be at the origin) to the point where the electric field is being considered. The unit vector $\hat{r}$ is obviously the direction of the electric field and, as shown in Figure 1.17, it is always tangent to the field lines. The number of field lines passing through an area represent the strength (magnitude) of the field in that area. A quantity related to the number of electric field lines passing through an area is called an electric flux, denoted as Φ_E. Now, recall that an electric field at a point is a tangent to the field line (or curve) at that point. From differential calculus, you immediately know that the magnitude of the electric field will be given by the slope of the tangent line to the flux (as a function of area), that is, $\partial\Phi_E/\partial A$:

$$E = \frac{\partial\Phi_E}{\partial A} \tag{1.53}$$

Therefore,

$$\Phi_E = \int \vec{E}.d\vec{A} \tag{1.54}$$

It turns out that the flux through a closed surface is the same as the flux through the sphere around that surface with the source charge at the center of the sphere:

$$\Phi_E = \oint \vec{E}.d\vec{A} = \oint \vec{E}.\hat{n}dA = EA = k\frac{q}{r^2}4\pi r^2 = 4\pi kq = \frac{q}{\varepsilon_0} \tag{1.55}$$

Note that the radius of the sphere does not matter. We used the definition of E from Eq. 1.52 and

$$k = \frac{1}{4\pi\varepsilon_0} \tag{1.56}$$

Equation 1.55 is the mathematical representation of Gauss's law, which states that the total electric flux through a closed surface is equal to the total charge inside the surface divided by ε_0, a constant called the free permittivity of space.

Two charges of the same type exert a repulsive force on each other, and two charges of opposite types exert an attractive force on each other. This force is called an electric force. A charge in motion creates another kind of force called a magnetic force applied through a magnetic field.

1.14 UNDERSTANDING MAGNETISM

Magnetism is the property of a material that enables two objects to exert a specific kind of force on each other called a magnetic force, which is created by an electric charge in motion. To understand magnetism, you must understand the related concepts discussed in this section.

1.14.1 Magnetic Field

A magnetic field is a field produced by a moving charge that it used to exert a magnetic force on another moving charge. A magnetic field is also produced by some magnetic materials, such as permanent magnets. It turns out that a magnetic field in such materials is also produced due to moving charges, the circulating current on the atomic scale. So, in both cases a magnetic field is generated by circulating currents: In conventional electric circuits, the current circulates at the macroscale (the scale of the circuit), whereas in magnetic materials the current circulates at the atomic scale.

> *Note*: A moving charge (i.e., current) creates a magnetic field in the space around it. Note that a moving charge is still a charge; so it will generate an electric field as well.

What is common among a doorbell, an electric motor, an electromagnet, and a transformer? All contain coils of wires with a large number of turns spaced closely enough so that each turn closely approximates a planar circular loop of current. Each loop of current I generates a magnetic field B according to a rule called the right-hand rule, as illustrated in Figure 1.18. Imagine the fist made with your right hand with the thumb sticking straight up. If the fingers curl in the direction of $\vec{I}$, the thumb points in the direction of $\vec{B}$.

To reverse this situation, now consider a long straight current carrying conductor. The magnitude of the magnetic field at a distance r from this wire carrying current I is defined as:

$$B = \mu_0 \frac{I}{2\pi r} \tag{1.57}$$

where the direction of the magnetic field B is again defined by the right-hand rule depicted in Figure 1.19. As illustrated by this figure, the magnetic flux

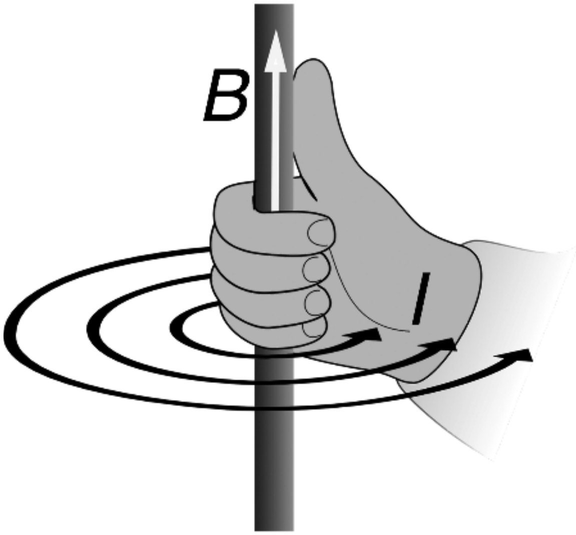


Figure 1.18. The right-hand rule for the magnetic field *B* generated by a current in a loop. Based on images by José Ferando. Courtesy of Wikipedia Commons.

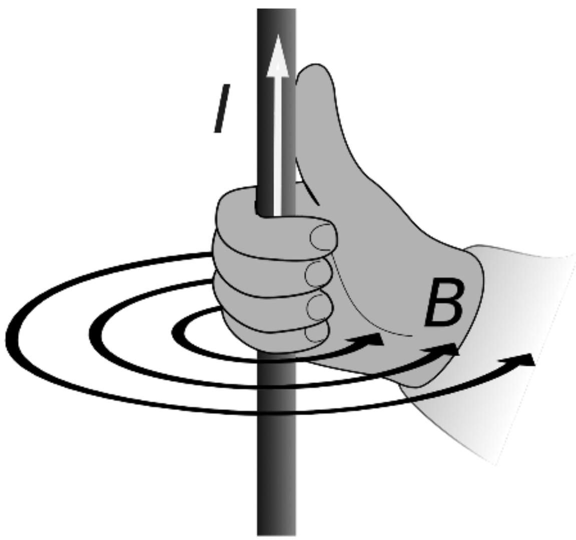


Figure 1.19. The right-hand rule for the magnetic field *B* generated by a current running through a long straight conductor. Based on images by José Ferando. Courtesy of Wikipedia Commons.

lines are coaxial circles with a radius r, and thereby with a circumference of $2\pi r$. The line integral of B along this closed path can be taken as;

$$\oint \vec{B}.d\vec{l} = \frac{\mu_0 I}{2\pi r}\oint d\vec{l} = \frac{\mu_0 I}{2\pi r}2\pi r = \mu_0 I \tag{1.58}$$

As you can see, the end result is independent of the radius of the circle (the closed path); the equation is true for any closed path that encloses a steady current I. This equation is the mathematical form of Ampere's law, which states that the line integral of a magnetic field along any closed path is equal to μ_0 times the net current I that crosses the area bounded by the path.

Just like an electric field, a magnetic field is also represented by field lines, the magnetic field lines, which give rise to the concept of a magnetic flux.

1.14.2 Magnetic Flux

Analogous to the electric flux described earlier, a magnetic flux is a measure of the quantity of the magnetic field lines through a certain area. It is proportional to the strength of the magnetic field and the surface area under consideration. For example, the current running through a wire in a circuit will create the magnetic field, and hence the magnetic flux in the area around it. Flux is a general property of any field, and is not limited to only magnetic and electric fields.

In a somewhat analogous way to an electric flux, we can explore the mathematical representation of a magnetic flux. The magnitude of a magnetic field at a point is equal to the magnetic flux per unit area across the area perpendicular to the field. This means that the relationship between the magnetic flux and the magnetic field $\vec{B}$ is given by

$$\frac{\partial \Phi_B}{\partial A_\perp} = B \tag{1.59}$$

Therefore,

$$\Phi_B = \oint \vec{B}.d\vec{A}_\perp = \oint \vec{B}_\perp.d\vec{A} = 0 \tag{1.60}$$

Note: The direction of an area is given by the unit vector $\hat{n}$ perpendicular to the surface of the area. Therefore $\vec{B}.\vec{A} = BA\cos\theta$, where θ is the angle that $\vec{B}$ makes with $\hat{n}$.

Equation 1.60 is the mathematical form of Gauss's law for magnetism. This integral is zero because any flux (field) line that enters a closed area exits that area too. So the net flux through the surface is zero, because unlike an electric charges, there is no such thing as isolated magnetic charges. Theoretically they are called magnetic monopoles. In other words, unlike electric field lines that begin and end on electric charges, magnetic field lines never have end points, they always form closed loops.

Faraday's law for a magnetic flux states that the change in magnetic flux creates an electromotive force (emf), which is practically a voltage. In other words, the changing magnetic flux through a circuit will induce a current in the circuit. Recall that the magnetic flux can be created by the current in a circuit according to Amperes law. Faraday's law says the reverse: The change in magnetic flux can create a current.

Analogous to the electric flux, magnetic flux Φ_B through a surface with area A is defined as:

$$\Phi_B = \int \vec{B}.d\vec{A}$$

$$\varepsilon = -\frac{\partial \Phi_B}{\partial t} \tag{1.61}$$

As mentioned earlier the change in this flux induces an emf ε:

This is Faraday's law of induction, which states that the induced emf in a closed loop equals the negative of the rate of change of a magnetic flux through the loop. The induced emf is actually the electric force represented by the electric field E:

$$\varepsilon = \oint \vec{E}.d\vec{l} \tag{1.62}$$

the line integral of $\vec{E}$ around the closed path. Therefore, Faraday's law can also be written as:

$$\oint \vec{E}.d\vec{l} + \frac{\partial \Phi_B}{\partial t} = 0 \tag{1.63}$$

To sum up, consider two electric circuits next to each other. There will be a magnetic flux through the second circuit due to the current in the first circuit: Ampere's law. If you change the current in the first circuit, it will change the magnetic flux through the second circuit, and the change in magnetic flux will create the current through the second circuit according to Faraday's law. So, you see that electricity and magnetism are related to each other and can be looked upon as two facets of what is called electromagnetism.

1.15 UNDERSTANDING ELECTROMAGNETISM

Electromagnetism is the unified framework to understand electricity, magnetism, and the relationship between them. In other words, to understand the electric field, magnetic field, and the relationship between them. To see this relationship, first recall that a charge creates an electric field, and the same charge, when it starts moving, creates a magnetic field. The electric field exerts an electric force, whereas a magnetic field exerts a magnetic force: Both originate from the electric charge. Therefore, they are intimately related: A changing electric field produces a magnetic field and a changing magnetic field produces an electric field. Due to this intimacy, the electric and magnetic force are considered two different manifestations of the same unified force called an EM force. The unified form of an electric and magnetic field is called an EM field, and an electric and magnetic field are considered its components. In other words, an EM force is exerted by an EM field.

Where there is a force, there is energy. The energy corresponding to an EM force is called EM energy or radiation. This energy is transferred from one point in space to another through what are called EM waves.

So, what is the frequency of EM waves? Electromagnetic waves cover a wide spectrum of frequencies, and the ranges of these frequencies are one way to define different types of EM waves.

1.15.1 Types of Electromagnetic and Other Waves

Waves can be grouped according to the direction of disturbance in them and according to the range of their frequency. Recall that a wave transfers energy from one point to another in space. That means there are two things going on: The disturbance that defines a wave, and the propagation of a wave. In this context, the waves are grouped into the following two categories:

Longitudinal Waves. A wave is called a longitudinal wave when the disturbances in the wave are parallel to the direction of propagation of the wave. For example, sound waves are longitudinal waves because the change of pressure occurs parallel to the direction of wave propagation.

Transverse Waves. A wave is called a transverse wave when the disturbances in the wave are perpendicular (at right angles) to the direction of propagation of the wave.

Electromagnetic waves are transverse waves, which means that the electric and magnetic fields change (oscillate) in a plane that is perpendicular to the direction of propagation of the wave. Also, note that the electric and magnetic field in an EM wave are perpendicular to each other.

Note: Electric and magnetic fields (E and B) in an EM wave are perpendicular to each other and are also perpendicular to the direction of propagation of the wave.

STUDY CHECKPOINT 1.4

Are the waves shown in Figures 1.9 and 1.10 longitudinal or transverse waves?

Solution:

Transverse waves.

Because electric and magnetic fields change in a plane that is perpendicular to the direction of wave propagation, the direction of change still has some freedom. Different ways of using this freedom provide another criterion for classifying EM waves into the following:

Linearly Polarized Waves. If the electric field (and hence the magnetic field) changes in such a way that its direction remains parallel to a line in space as the wave travels, the wave is said to be linearly polarized.

Circularly Polarized Waves. If the change in the electric field occurs in a circle or in an ellipse, the wave is said to be circularly or elliptically polarized. Therefore the polarization of a transverse wave determines the direction of disturbance (oscillation) in a plane perpendicular to the direction of wave propagation.

Caution: Only transverse waves can be polarized, because in a longitudinal wave the disturbance is always parallel to the direction of wave propagation.

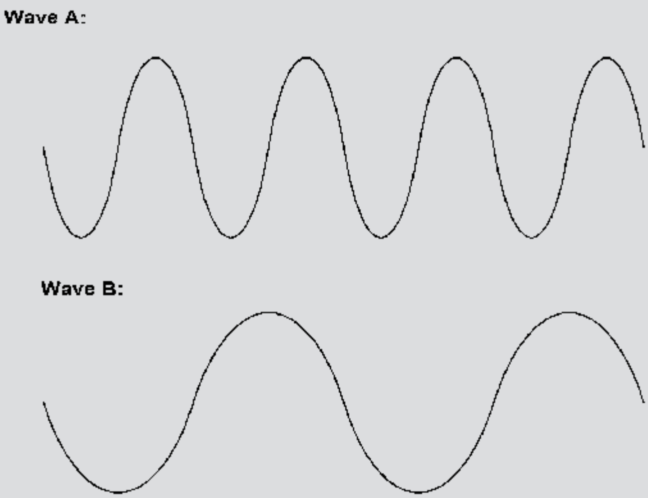
So, you can classify EM waves based on the direction of disturbance in them (polarization). The other criterion to classify EM waves is the frequency. This classification gives rise to what is called the EM spectrum.

1.15.2 Electromagnetic Spectrum

Have you ever seen EM waves with your naked eye? The answer, of course, is yes! Visible light is an example of EM waves. In addition to visible light, EM waves include radio waves, ultraviolet (UV) radiation, and X-rays; which, of course, are not visible to the naked eye. These different kinds of EM waves only differ in their frequency, and therefore the wavelength. The whole frequency range of EM waves is called the EM spectrum, which is shown in Figure 1.20 and Table 1.1 along with the names associated with different frequency ranges or regions within the spectrum. The human eye has evolved to detect visible light that happens to be in the range where the Sun puts out most of its light energy that hits Earth.

PROBLEM 1.5

Consider two EM waves represented by A and B:



A. Which wave has the higher wavelength?
B. Which wave has the higher frequency?
C. If one of these waves represents visible light and the other one UV radiation, which wave is which?

Solution:

A. Wave B has longer or higher wavelength.
B. Wave A has higher frequency because it has smaller wavelength.
C. Wave B represents visible light and wave A represents UV radiation, because UV radiation has a higher frequency than that of a wave of visible light.

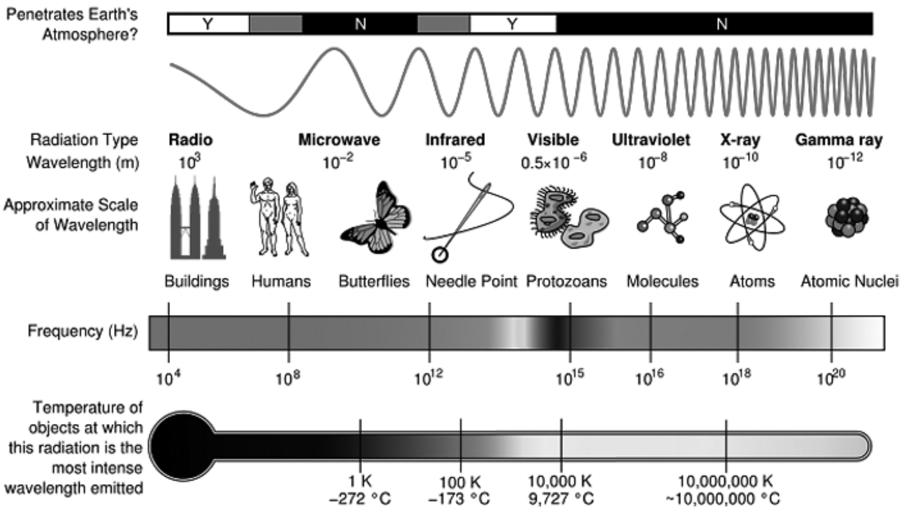


Figure 1.20. Electromagnetic spectrum. Courtesy of NASA.

TABLE 1.1. Spectrum of Electromagnetic Radiation or Waves

Wave Region	Wavelength (m)	Frequency (Hz)	Energy (eV)
Radio	>0.1	$<3 \times 10^9$	$<10^{-5}$
Microwave	0.1–0.0001	3×10^9–3×10^{12}	10^{-5}–0.01
Infrared	0.001–7×10^{-7}	3×10^{12}–4.3×10^{14}	0.01–2
Visible	7×10^{-7}–4×10^{-7}	4.3×10^{14}–7.5×10^{14}	2–3
Ultraviolet	4×10^{-7}–10^{-9}	7.5×10^{14}–3.0×10^{17}	3–10^3
X-Rays	10^{-9}–10^{-11}	3×10^{17}–3.0×10^{19}	10^3–10^5
Gamma Rays	$<10^{-11}$	$>3.0 \times 10^{19}$	$>10^5$

All the fundamental principles of EM can be boiled down to four equations called Maxwell's equations.

1.16 MAXWELL'S EQUATIONS

In the early nineteenth century, when electric and magnetic forces were thought to be two different forces, the physical quantity charge was organized into two different units in electrostatic phenomena and in magnetic phenomena resulting in two different physical dimensions. To their surprise, physicists noticed that the ratio of these two different units had the unit of velocity and its value was measured to be precisely equal to the velocity of light. At that time, physicists had no idea how to explain this and took it as a remarkable coincidence. As often happens in science, the questions put to bed by one generation of scientists are often confronted sooner or later by a future generation, if not the same.

When the electric and the magnetic fields do not vary with time, we can analyze them independent of each other. However, when they do vary with time, they are interrelated. For example, Faraday's law states that a magnetic field varying with time generates an electric field, for example, as demonstrated by induced emfs in inductors and transformers. In the reverse, Ampere's law states that an electric field varying with time generates a magnetic field. James Clerk Maxwell (1831–1879) realized the disturbance element in the fact that the variation in either an electric or magnetic field in time induces the other in the adjacent regions of space. So, he considered the possibility of an EM disturbance caused by time varying electric and magnetic fields propagating through space as a wave. The mystery of the ratio of electric and magnetic charges being equal to the speed of light was solved in 1865, when Maxwell showed that an EM disturbance (wave) would propagate in space with the speed of light, and hence light was a wave and was electromagnetic in nature. Furthermore, Maxwell realized that the basic principles of electromagnetism can be expressed in terms of four equations, which display the mutual interaction between electric and magnetic fields discussed above.

Although Maxwell developed the concept of displacement current, he did not discover Maxwell's equations single handedly. His genius was to put them together and recognize their importance especially in terms of EM waves. Maxwell's equations are presented in Table 1.2 in both differential and integrals forms, where ρ is the electric charge density, J is the electric current density, Q is the charge enclosed within the surface, I_c is the conducting current, Φ_E is the electric flux, and Φ_B is the magnetic flux. Furthermore, $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field, and ε_0 and μ_0 are called vacuum electric and vacuum magnetic permeability, respectively.

TABLE 1.2. Maxwell's Equations in Differential and Integral Forms

Differential Form	Integral Form	Origin
$\vec{\nabla}.\vec{E}=\dfrac{\rho}{\varepsilon_0}$	$\oint \vec{E}.d\vec{A}=\dfrac{Q}{\varepsilon_0}$	Gauss's law for an electric field
$\vec{\nabla}.\vec{B}=0$	$\oint \vec{B}.d\vec{A}=0$	Gauss's law for a magnetic field
$\vec{\nabla}\times\vec{E}+\dfrac{\partial\vec{B}}{\partial t}=0$	$\oint \vec{E}.d\vec{l}+\dfrac{\partial\Phi_B}{\partial t}=0$	Faraday's law of induction
$\vec{\nabla}\times\vec{B}=\mu_0\vec{J}+\varepsilon_0\mu_0\dfrac{\partial\vec{E}}{\partial t}$	$\oint \vec{B}.d\vec{l}=\mu_0\left(I_c+\dfrac{\partial\Phi_E}{\partial t}\right)$	Ampere's law of circuit

The differential forms of Maxwell's equations presented in Table 1.2 involve a vector $\vec{\nabla}$, which is given by

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k} \tag{1.64}$$

which means, for example,

$$\vec{\nabla}.\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \tag{1.65}$$

and

$$\nabla^2 E = \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} \tag{1.66}$$

To start from the top in Table 1.2, the integral forms of the first two Maxwell equations involve the integrals of electric and magnetic fields over a closed surface. The first equation represents Gauss's law for electric fields, which states that he surface integral of an electric field $E_\perp$ over any closed surface is equal to the charge enclosed by the surface divided by ε_0. Analogously, Gauss's law for magnetic fields expressed in the second equation states that the surface integral of the magnetic field $B_\perp$ over a closed surface is always zero. The third Maxwell's equation in the table expresses Faraday's law of EM induction, which shows how a time varying magnetic field generates circulating electric field. Finally, the fourth Maxwell equation expresses Amperes law by showing how an electric current can generate a magnetic field. The second term on the right-hand side of the equation was added by Maxwell, in analogy with the Faraday's law, to account for the fact that a time varying electric field also generates a circulating magnetic field.

In the absence of charges and currents, Maxwell's equations can be transformed into the following 3D wave equations:

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0 \tag{1.67}$$

and

$$\nabla^2 \vec{B} - \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} = 0 \tag{1.68}$$

where c is a constant given by

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 2.9979 \times 10^8 \text{ m/s} \tag{1.69}$$

Note: The permittivity of vacuum ε_0 is also called the electric constant and permeability of vacuum μ_0 is also called the magnetic constant. The relationship between the speed of light, the electric constant, and the magnetic constant are given by

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

which symbolizes the unification of electric and magnetic forces into an EM force.

This turns out to be the speed of light. As a matter of fact, solutions to these differential equations represent transverse electric and magnetic waves traveling with the speed of light.

To ease your way into quantum mechanics, you need to be comfortable with a few more concepts related to waves, namely, confinement, standing waves, and wavegroups.

1.17 CONFINEMENT, STANDING WAVES, AND WAVEGROUPS

This chapter established that most of the physics is based on three fundamental concepts: physical entity, space, and time. Time represents the interval over which there is a change, such as motion of the physical entity. As you know, Newton's first law states that an object (or a particle) that is moving with a velocity v, will continue moving with this velocity until an external force is applied to it. The world and the universe is full of external forces. Under the influence of these forces, the motion of objects is restricted or constrained in a certain fashion. For example, the motion of earth around the Sun is constrained in an orbit. The motion of a pendulum is constrained as well. Such constraints are called confinement.

1.17.1 Confinement

Both particles and waves travel, that is, move in time. Confinement refers to the situation in which the motion of a particle or a wave is constrained (confined) to a certain region in space. The confinement on motion of a physical entity understandably results in constraints on the values of some observables of the entity, such as energy and momentum. For example, imagine a particle confined to a box: Its position is always contrained to be within the range of values that represent the dimension of the box.

It is important for you to understand standing waves, an example of confinement, because it is a good ice breaker into quantum mechanics.

1.17.2 Standing Waves

A standing wave is a wave that remains in a constant position. For that reason, it is also called a stationary wave. The first time I learned about standing waves was in terms of a string firmly attached to rigid walls on two ends, I wondered how I was going to use that knowledge. So, I stoked it in the backburner of my brain. However, I found its usefulness during my student life some time later when I started learning quantum mechanics. Furthermore, standing waves have several applications in about all fields of science and technology, and, of course, in music.

So, if you already have some knowledge of standing waves, recall it. Nevertheless, I will give you a brief refresher in this section. A standing wave will help you a great deal in understanding the Schrödinger equation, the defining heart of quantum mechanics, which you will deal with in this book. A standing wave is a wave that results from the interference of several waves confined in space. Waves on a piano string and in an organ pipe are examples of the situations, which corresponding to certain wave frequencies, can give rise to a stationary wave pattern called standing waves.

We will explore the concept of standing waves by considering a simple situation: A string tied to a tuning fork at one end A, and fixed at the other end B. Assume that the length of the string, stretched from A to B, is L. Further assume that the tuning fork is vibrating with frequency f, wavelength λ, and a small amplitude. Now, consider a wave traveling from end A (the fork) to end B. At end B, the wave is reflected back in an inverted form. After traveling a distance of $2L$, it is reflected back in an inverted form again from end A at the fork. Because it has been inverted twice, it will only differ in phase from the wave that the fork generates at this moment. However, if the wavelength is exactly equal to the length of the string, the wave that is reflected twice is now in phase with the wave created by the fork at the moment of second reflection. Therefore, the two waves interfere constructively. The amplitude of the wave resulting from interference is twice that of any of the two interfering waves. Following the same argument, you can envision another event of constructive interference after traveling a distance of the next $2L$, and so on. This way, the amplitude of the wave at the string continues to grow until it reaches a maximum value at which the energy provided by the fork to the string equals the energy lost to damping effects, such as imperfect flexibility of the string and the reflection phenomenon. At this point, the wave on the string has a constant position and energy and is called the standing wave.

> *Note*: Because the tuning form is vibrating with a small amplitude, the end A can be considered a fixed point for the purpose of reflection and interference.

When the wavelength λ of the wave generated by the tuning fork and transported to the string is equal to the length of the string L, the tuning fork is called in resonance with the string and the frequency of the fork (and the wave) is called the resonance frequency. Note that the constructive interference will happen if $2L$ is any integer times λ. In other words, the resonance frequency may have any of the values that result in the following values for the wavelength:

$$2L = n\lambda_n \tag{1.70}$$

Therefore,

$$\lambda_n = \frac{2L}{n} \tag{1.71}$$

where n is any positive integer. Accordingly, the resonance frequency can assume the following values:

$$f_n = \frac{v}{\lambda} = n\frac{v}{2L} = nf_1 \tag{1.72}$$

where f_1 is called the fundamental frequency. The different frequencies corresponding to the different n values are called harmonics. Therefore, a harmonic of a wave is a component frequency of the wave that is an integer multiple of the fundamental frequency.

The amplitude of a standing wave, like any other wave, is the amplitude of the maximum vertical displacement of the wave, and the displacement is represented by its wave function Ψ. Let us derive this wave function from the wave functions of the individual waves that the standing wave is composed of. The displacement of the wave traveling from A to B, is given by

$$\Psi_R = \Psi_0 \sin(kx - \omega \mathrm{t}) \tag{1.73}$$

where

$$k = \frac{2\pi}{\lambda} \tag{1.74}$$

is called the wavenumber and

$$\omega = 2\pi f \tag{1.75}$$

is called the angular frequency. Similarly, the displacement of the second wave on the string traveling from B to A is given by

$$\Psi_L = \Psi_0 \sin(kx + \omega t) \tag{1.76}$$

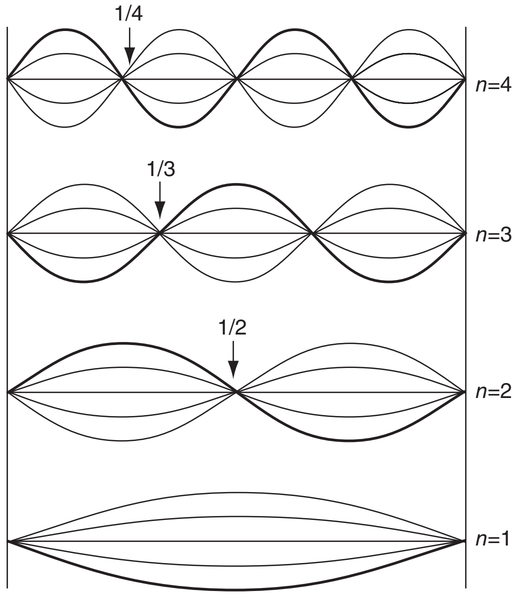


Figure 1.21. Standing waves on a string fixed at both ends.

The displacement of the standing wave, that is, the resultant of superposition of the two waves, is obtained by adding the corresponding wave functions:

$$\Psi = \Psi_R + \Psi_L = \Psi_0 \sin(kx - \omega t) + \Psi_0 \sin(kx + \omega t) \tag{1.77}$$

which can be simplified to be written as:

$$\Psi = 2\Psi_0 \cos \omega t \sin kx \tag{1.78}$$

The wave function for standing waves is illustrated in Figure 1.21 for the first four n values.

STUDY CHECKPOINT 1.5

From the expressions for Ψ_R and Ψ_L, prove that

$$\Psi = 2\Psi_0 \cos \omega t \sin kx$$

Solution:

$$\Psi = \Psi_R + \Psi_L = \Psi_0 \sin(kx - \omega t) + \Psi_0 \sin(kx + \omega t)$$

Use the trigonometric identity:

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$A + B = 2kx; \quad A - B = 2\omega t$$

which implies

$$\Psi = 2\Psi_0 \cos\omega t \sin kx$$

Note that the wave function of the standing wave not only has the information about the displacement of the wave, but also about other characteristics, such as the resonance condition that we derived earlier without considering the wave function. Problem 1.9 demonstrates how the resonance condition can be derived from the wave function, which is

$$\lambda_n = \frac{2L}{n} \tag{1.79}$$

and therefore,

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} \tag{1.80}$$

where v is the phase velocity and, also in this case, is the group velocity because we are assuming that the string is a perfectly flexible string.

In general, a single wave made of several different waves is called a wavepacket or a wavegroup. The phase velocity is the velocity of the individual component waves and the group velocity is the velocity of the wavepacket. Note that the two are not the same.

1.17.3 Wavegroups

When waves of slightly different frequencies interfere with each other, they form a pattern of wavegroups or wavepackets due to the superposition of the individual waves. Because the individual waves have different frequencies, they will be traveling with different speeds, but the group as a whole will be traveling with a speed called a group velocity. Figure 1.22 illustrates a simple example of the superposition of two waves A and B with frequencies f_1 and f_2 generating a third wave with frequency f that consists of wavegroups.

Assume that the two individual waves A and B have the same amplitude Ψ_0, have wave numbers k_1 and k_2, and angular frequencies ω_1 and ω_2. From Eq. 1.73, we can write the wave functions for these two waves:

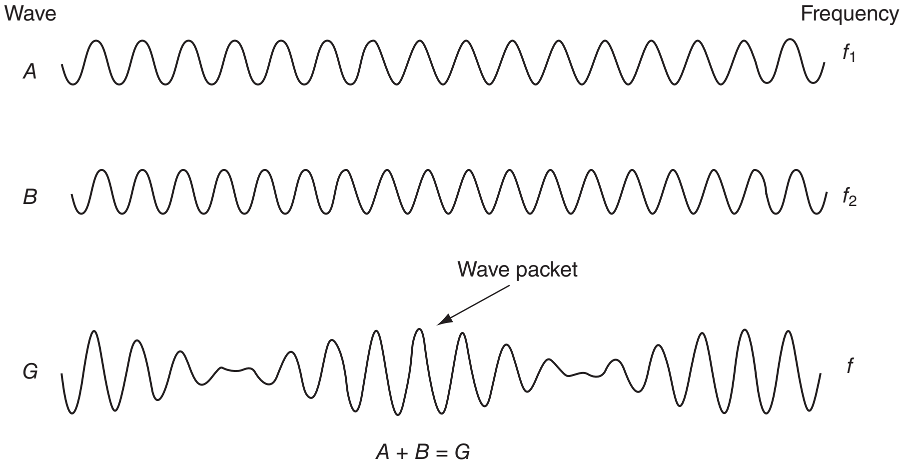


Figure 1.22. Superposition of two waves A and B with frequencies f_1 and f_2 generating a third wave with frequency f that has wavegroups.

$$\Psi_A = \Psi_0 \sin(k_1 x - \omega_1 t) \tag{1.81}$$

$$\Psi_B = \Psi_0 \sin(k_2 x - \omega_2 t) \tag{1.82}$$

The wave function, Ψ, of the resultant wave G in Figure 1.22 is given by

$$\Psi = \Psi_A + \Psi_B = \Psi_0 [\sin(k_1 x - \omega_1 t) + \sin(k_2 x - \omega_2 t)] \tag{1.83}$$

We use the trigonometric identity

$$\sin c + \sin d = 2 \sin \frac{c+d}{2} \cos \frac{c-d}{2} \tag{1.84}$$

to rewrite Eq. 1.83 as:

$$\Psi = 2\Psi_0 \sin\left[\frac{1}{2}(k_2 + k_1)x - \frac{1}{2}(\omega_2 + \omega_1)t\right] \cos\left[\frac{1}{2}(k_2 - k_1)x - \frac{1}{2}(\omega_2 - \omega_1)t\right] \tag{1.85}$$

By substituting

$$\bar{k} = \frac{1}{2}(k_2 + k_1)$$

$$\bar{\omega} = \frac{1}{2}(\omega_2 + \omega_1)$$

$$\Delta k = \frac{1}{2}(k_2 - k_1)$$

$$\Delta\omega = \frac{1}{2}(\omega_2 - \omega_1)$$

we can rewrite Eq. 1.85 in a simplified form:

$$\Psi = 2\Psi_0 \sin\left(\bar{k}x - \bar{\omega}t\right)\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right) \tag{1.86}$$

Use Eq. 1.86 to realize the relationship between Δk and Δx, where Δx is the width of the packet along the x-axis, which must be at least one-half of the wavelength. This means that the end points of the wavepacket (where the amplitude is zero), say x_1 and x_2, must have a phase difference of π. Mathematically, this means that the difference in the terms $\Delta k\, x/2$ in Eq. 1.86 at points x_1 and x_2 must be equal to π. This means

$$\frac{\Delta k\, x_2}{2} - \frac{\Delta k\, x_1}{2} = \pi$$

which means

$$\Delta k\left(\frac{x_2}{2} - \frac{x_1}{2}\right) = \frac{\Delta k\, \Delta x}{2} = \pi$$

which implies

$$\Delta k\, \Delta x = 2\pi \tag{1.87}$$

Also, Eq. 1.86 means that the group or wavepacket travels with wavenumber $\Delta k/2$ and angular frequency $\Delta\omega/2$. Therefore, the speed of the wavegroup is given by

$$v_g = \frac{\Delta\omega}{\Delta k} \approx \frac{d\omega}{dk} \tag{1.88}$$

For simplicity, we have only considered the wavegroup as a superposition of two waves. In general, a wavepacket can be thought of as a superposition of many waves with angular frequencies and wavenumbers distributed over limited ranges. As illustrated in Figure 1.23, superposition of several waves of different frequencies will give rise to a more localized interference pattern, the wavegroup.

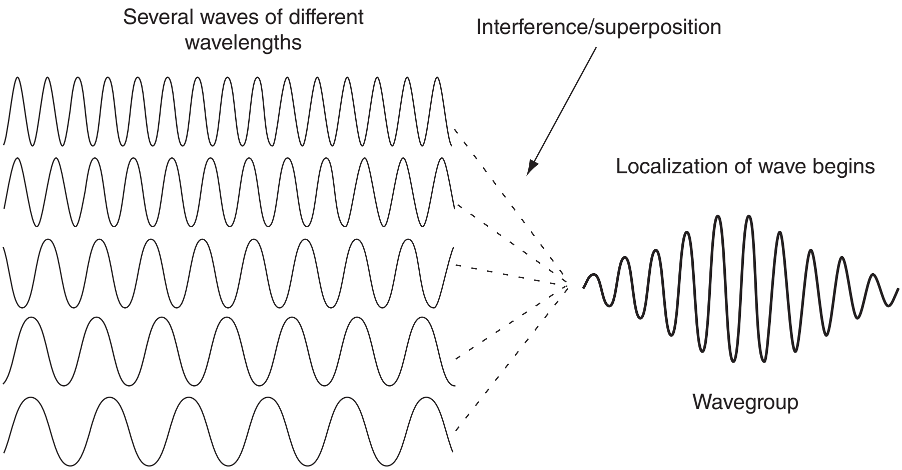


Figure 1.23. Localization of waves in the form of a wavegroup.

STUDY CHECKPOINT 1.6

As illustrated in Figure 1.23, a wavegroup is more localized, that is, the spread over space is less. How about the wavenumber?

Solution:

Wave number is given by

$$k = \frac{2\pi}{\lambda}$$

Because there are many wavelengths composing the wavegroup, therefore the wavenumber is more spread out.

So, in classical physics, a physical entity could be either a particle or a wave. We have reviewed important characteristics and the behavior of particles and waves. Now let us compare particles and waves in the context of these characteristics in order to distinguish particles from waves.

1.18 PARTICLES AND WAVES: THE BIG PICTURE

As mentioned earlier, classical physics treats particles and waves as two separate kinds of entities. We have discussed some of the characteristics that

TABLE 1.3. Comparison of Particles and Waves

Characteristic	Particles	Waves
Collision	Yes	No particle-like collision
Scattering	Yes Deviate direction after collision	Yes, but different from particle scattering: Reradiated in various directions after absorption.
Reflection	No. Although particles do deviate from the path after meeting an obstacle; scattering.	Yes
Refraction	No	Yes
Diffraction	No	Yes
Interference/ superposition	No Forces acting on particles can go under superposition, but particles themselves do not go under interference and superposition like waves do.	Yes Waves interfere and make an interference/diffraction pattern corresponding to constructive and destructive interference.
Energy	Transport energy	Transport energy

particles and waves have and some of the phenomena that they exhibit. Table 1.3 compares particles and waves in terms of phenomena they exhibit.

Classical physics is based on our perception of physical reality with our five senses. Particles are particles because they carry mass and they collide with each other as solid objects. Waves are waves because they have frequency, wavelength, and they go under phenomena, such as interference and diffraction.

Recall again the three fundamental concepts on which the physics is based: physical entity, space that specifies the position of the entity, and time that is used to specify the change in position of the entity. According to Newton's law, the change in the position of rest or in the state of uniform motion occurs due to external forces. There are countless forces all around and within us. However, all of those forces are different manifestations of the four fundamental forces of nature. Both kinds of entities, particles and waves, are subject to one or more of these interactions.

1.19 THE FOUR FUNDAMENTAL FORCES OF NATURE

Physicists study matter from elementary particles to galaxies and the universe, and all the fundamental forces that keep the universe and the systems in the universe in order. Although the fundamental forces acting on material systems are universal, the size of the material systems does matter in many ways in determining which force is significant for a given situation. For example, we as humans are macrobeings, and therefore we started our studies from the macroworld by using macrotools starting with our five senses. However, we are incapable of seeing the microworld with our naked eyes. Table 1.4 presents some examples of material systems on the size scale of macro to micro including nano and beyond nano.

Throughout the history of physics, physicists have discovered the following:

- All matter is composed of two kinds of a very few elementary particles called quarks and leptons: there are six types of quarks and six types of leptons. For example, a proton is made up of three quarks, as is a neutron. An electron is an example of a lepton. All atoms are made of protons, neutrons, and electrons.

TABLE 1.4. Examples of Material Systems on the Size Scale

Size	Scientific Notation (m)	Example (In This Neighborhood)	Observation Tools/ Techniques
1.7 m	1.7×10^{0}	Human height	Human eye
1 cm	1×10^{-2}	Wedding ring	Human eye
1 mm	1×10^{-3}	Thickness of a CD	Human eye Optical microscope
100 μm	1×10^{-4}	Plant cell	Human eye Optical microscope Electron microscope
10 μm	1×10^{-5}	Animal cell	Human eye Optical microscope Electron microscope
1 μm	1×10^{-6}	Bacterial cell	Optical microscope Electron microscope
100 nm	1×10^{-7}	Virus	Electron microscope
10 nm	1×10^{-8}	Virus, Protein molecule	Electron microscope
1 nm	1×10^{-9}	Protein molecule, aspirin molecule	Electron microscope
100 pm	1×10^{-10}	Water molecule	Electron microscope
10 pm	1×10^{-11}	Atom	Indirect observation by tools (e.g., cyclotrons)
0.001 pm	1×10^{-15}	Proton	Indirect observation by tools (e.g., cyclotrons)
<1 pm	$<1 \times 10^{-12}$	Other subatomic particles	Indirect observation by tools means (e.g., particle accelerators and particle colliders)

- Elementary particles are bound together to make atoms, molecules, and bulk matter through the presence of four fundamental forces: Strong and weak nuclear forces to form the nuclei of atoms, EM force to form atoms and molecules, and gravity to hold together large systems like planets, stars, and galaxies. These forces are expressed through particles, for example, EM force is expressed by an elementary particle called a photon. In other words, two charged particles interacting with each other via an EM force means they are exchanging photons. Photons are the mediators of EM force.
- These fundamental forces also govern the behavior and properties of particles and waves and of the bulk material. For example, chemistry students know that the stronger the intermolecular forces, which are some sort of EM forces, the higher is the melting and the boiling point of a substance.

STUDY CHECKPOINT 1.7

A. Consult with a chemistry textbook to write down the names of some types of intermolecular forces.
B. Intermolecular forces belong to which fundamental force?

Solution:

A. London dispersion forces, dipole–dipole-forces, and hydrogen bonding.
B. Electromagnetic force

Note: When we say EM force is expressed through photons, we mean that the two particles, for example, an electron and a proton, experience an EM force between them by exchanging photons. In other words, these two particles interact with each other and photons are the means of interaction; they intermediate the interaction. For this reason, an EM force and any other of the four forces are also called interactions.

In a nutshell, fundamental forces help form the material systems from basic building blocks, keep these systems intact, and hold this planet and the universe together in some order. There are four fundamental forces (gravitational, electromagnetic, weak and strong forces) that govern our universe. These forces are discussed next in Sections 1.19.1–1.19.4.

1.19.1 Gravitational Force

Also called gravity, this is the force of attraction between any two material bodies with nonzero mass. According to gravitational law, first formulated by

Newton in 1687, every material body attracts every other material body in this universe with a force that is directly proportional to the product of the masses of the two bodies and inversely proportional to the square of the distance between them. It is the gravitational force that keeps us on Earth and not floating randomly in space. Also, it is the gravitational force that keeps Earth revolving around the Sun. Gravity is the weakest of the fundamental forces.

The gravitational force in classical mechanics is given by Newton's law of gravitation, which states that two particles experience an attractive force F_g, that is directly proportional to the product of their masses m_1 and m_2, and inversely proportional to the square of the distance r between them:

$$F_g = G\frac{m_1 m_2}{r^2} \tag{1.89}$$

where G is the proportionality constant and is known as a gravitational constant. So, the gravitational field exerts a gravitational force on any particle with mass. You can see that mass is the origin of a gravitational force.

1.19.2 Electromagnetic Force

This is the force that an EM field exerts on any particle with an electric charge: Charge is the origin of an EM force. Electromagnetic force manifests itself through an electric and a magnetic force. The magnitude of an electric force between any two charged particles is given by Coulomb's law, which states that two charged particles experience a force F_e that is directly proportional to the product of their charges q_1 and q_2, and inversely proportional to the square of the distance r between them:

$$F_e = k\frac{q_1 q_2}{r^2} \tag{1.90}$$

where k is the proportionality constant and is known as the Coulomb constant. Notice the striking similarity between equations representing an electric and a gravitational force. However, note that gravitational force is only an attractive force because there are no negative mass particles, whereas an electric force is attractive between unlike charges and repulsive between like charges, because particles can have positive or negative charge. We can present Coulomb's law in terms of an electric field. The particles with charges q_1 and q_2 exert an electric force on each other through their electric fields. The electric field $\vec{E}_1$ at position $\vec{r}$ due to the particle with charge q_1 at the origin is defined as:

$$\vec{E}_1 = k\frac{q_1}{r^2}\hat{r} \tag{1.91}$$

where $\hat{r}$ is the unit vector along $\vec{r}$. This electric field is the force applied on any particle at the position $\vec{r}$ per unit charge of that particle. Therefore, the electric force applied by this electric field on a particle with charge q_2 is given by

$$\vec{F}_{1,2} = q_2\vec{E}_1 = k\frac{q_1q_2}{r^2}\hat{r} \tag{1.92}$$

which is Coulomb's law.

The magnetic force component $\vec{F}_B$ of an EM force is given by

$$\vec{F}_B = q\vec{v}\times\vec{B} = qvB\sin\theta \tag{1.93}$$

where q is the magnitude of charge of a particle moving with a velocity $\vec{v}$ in a magnetic field with magnetic induction $\vec{B}$ and θ is the angle made by the velocity vector with the magnetic induction vector. Therefore, both electric and magnetic forces can be written as one force, the EM force:

$$\vec{F}_{\text{EM}} = \vec{F}_E + \vec{F}_B = q\left(\vec{E} + \vec{v}\times\vec{B}\right) \tag{1.94}$$

This is the force experienced by a particle with a charge q and moving with a velocity $\vec{v}$ in an electric field $\vec{E}$ and a magnetic field $\vec{B}$.

Electromagnetic force is responsible for holding the electrons and protons together in an atom and also for binding atoms into molecules. This force plays an important role in all forms of chemical phenomenon, such as chemical reactions. Most of the forces we experience in our daily lives other than gravitational force during our interaction with ordinary objects, such as pushing, pulling, scrubbing, have their origin in an EM force at the molecular level, that is, the molecules in our body interact with the molecules of the body we interact with by using an EM force. Light is also associated with the EM force. Therefore, the macroscopic world is largely run by gravitational and EM forces.

1.19.3 Weak and Strong Nuclear Forces

There are two kinds of nuclear forces:

Weak Nuclear Force. This nuclear force is responsible for radioactive decay. It is 10^{13} times weaker than the strong nuclear force.

Strong Nuclear Force. This force keeps the subatomic particles intact. For example, it is responsible for keeping three quarks bound together to form a proton.

STUDY CHECKPOINT 1.8

Which fundamental force facilitates biochemical reactions and many processes occurring in our body?

Solution:

Electromagnetic force.

1.19.4 Four Fundamental Forces: The Big Picture

These four fundamental forces are also called interactions because particles and other systems of matter use these forces to interact with each other. Physicists use the terms force and interaction interchangeably. This universe starting from subatomic particles to galaxies is held together by these interactions. In other words, our universe is an expression of the greatest underlying orchestra being played by fundamental particles and the interactions among them. The basic characteristics of these forces, such as relative strength, and the most effective range in the context of the real world are presented in Table 1.5.

The strength of the electric force between two particles is inversely proportional to the square of the distance between them, that is, it decreases as the particles are separated from each other. Because theoretically the force between two charged particles never drops all the way to zero, you can say that the range of the EM force is infinite. However, practically speaking, the effective range of an EM force in matter only goes to $\sim 5 \times 10^{-10}$ m, that is, 0.5 nm. This is because of the cancellation effects of positive and negative charges in the matter, whereas the range of gravity stays infinite because it is

TABLE 1.5. Characteristics of Fundamental Forces of Nature

Force/Interaction	Intermediate Particle	Relative Strength	Most Effective Range (m)	Mostly Affected Matter or Process
Gravitational	Graviton	1	Infinitely large	Macrosystems (e.g., planets and objects on the planets)
Electromagnetic	Photon	10^{36}	10^{-9}	Microsystems and nanoparticles (e.g., molecules and subatomic particles)
Weak nuclear	W^+, W^-, Z^0	10^{25}	10^{-18}	Nuclear level: Radioactive decay
Strong nuclear	Gluon	10^{38}	10^{-15}	Nuclear level: Quarks

only an attractive force. Furthermore, it is the force that keeps protons and electrons bound together to form atoms, and atoms bound together to form molecules, crystals, and other materials, such as metals. Therefore, it is of specific interest to different fields in nanotechnology. Because quantum physics is needed to understand phenomena in the micro (and nano)-scale and the most effective force in this scale is the EM, force, we will be dealing with this force in this book.

Note: Out of the four fundamental interactions in the universe, the interaction that is relevant to fields discussed in this book (biology, chemistry, nanotechnology, and computer science) is the EM interaction, which exhibits itself in our world in many forms including electricity and magnetism. In chemistry classes, the EM force appears (or expresses itself) under different forms and names, such as electrostatic force, van der Wall's force, London dispersion force, hydrogen bonding, atomic bonding, and so on. You know from your introductory physics course that the EM force is also the force that leads to friction, lets your car move smoothly, and let you walk on earth and pick up things and hug your friends.

Although physics has made advances in exploring all of these four interactions, other sciences has mostly made use of EM forces. Physicists have learned repeatedly that the four fundamental forces appear to be a manifestation of a single unified force. As a matter of fact, the whole history of physics can be looked upon in terms of exploring unification.

1.20 UNIFICATION: A SECRET TO SCIENTIFIC AND TECHNOLOGICAL REVOLUTIONS

The whole history of the development of science can be explained in terms of one concept called unification. Most of the breakthroughs in science and technology have to do something with some kind of unification at some level. So, what is unification? Broadly speaking, unification is the process or act of uniting two or more apparently different things into one.

Nature works the way it does. We, the human, have our limitations in understanding it. We develop our understanding, gather information, and obtain knowledge in pieces. Some pieces may look different from each other at the surface. However, they may be the different facets of the same thing at some other level. For example, nature did not create different disciplines of science, such as physics, chemistry, biology, and so on. These and the borders among them are our own creations representing our limitations. To see the underlying unity behind diversity has been the key to major breakthroughs in science. The history of Physics, the mother of all sciences, demonstrates this very well. According to physics, our universe is shaped and governed by four fundamental forces: gravitational, electromagnetic, weak nuclear, and strong nuclear forces.

Here are some examples of unification in Physics. The realization that it is the same force that keeps us on Earth and makes the Earth rotate around the Sun enabled the physicist to recognize one of the four fundamental forces of nature: the gravitational force. From this realization, Newton was able to write the general formula for this force:

$$F = G\frac{m_1 m_2}{r^2}$$

where F is the force with which two bodies with mass m_1 and m_2 separated by a distance r attract each other; G is the gravitational constant. The two bodies may be you and the Earth or the Earth and the Sun. This unification was nothing less than a revolution in classical physics. For example, the airplanes and the satellites make use of this unified concept.

As another example, the electric force is the force that repels two similarly charged particles away from each other and attracts two oppositely charged particles toward each other. The magnetic force is the force that makes magnets repel or attract each other. Electric and magnetic forces were supposed to be two different forces. In the nineteenth century, it was proved that electric and magnetic forces are two different manifestations of the same force called the EM force. James Clark Maxwell is given credit for this unification of electric and magnetic forces, and the equations in physics that illustrate this unification are called Maxwell's equations of electromagnetism. So, the fields of electricity and magnetism were unified into one field called electromagnetism. The great product of this unification is the realization (or discovery) that light is an EM wave.

In the second half of the twentieth century, Abdus Salam, Steven Weinberg, and Sheldon Glashow demonstrated that the EM force and the weak nuclear force are the two different manifestations of the same force called the electroweak force. They were awarded the Nobel Prize in physics (1979) for this unification. This unification predicted the existence of some fundamental particles unknown at the time. The existence of one of these particles was confirmed at several experiments at CERN, in which the author of this book also participated.

Physicists believe that the four fundamental forces that govern our universe today are the low-energy manifestations of the single unified force, say the super force, at high energy that existed at the moment the universe was created. The work is still in progress. The unification of forces is illustrated in Figure 1.24.

Some unification events are summarized in Table 1.6.

Unification is in the very spirit of scientific thinking: looking for a common principle to explain a multitude of observations and phenomena. This thinking has its root in our being cognizant of the environment around us and around our knowledge and theories. If we divide the world into macro and micro, we are macrobeings. However, if we consider the universal scale of size, we actually fall in the mid-range: We cannot see very small, such as atoms, and we

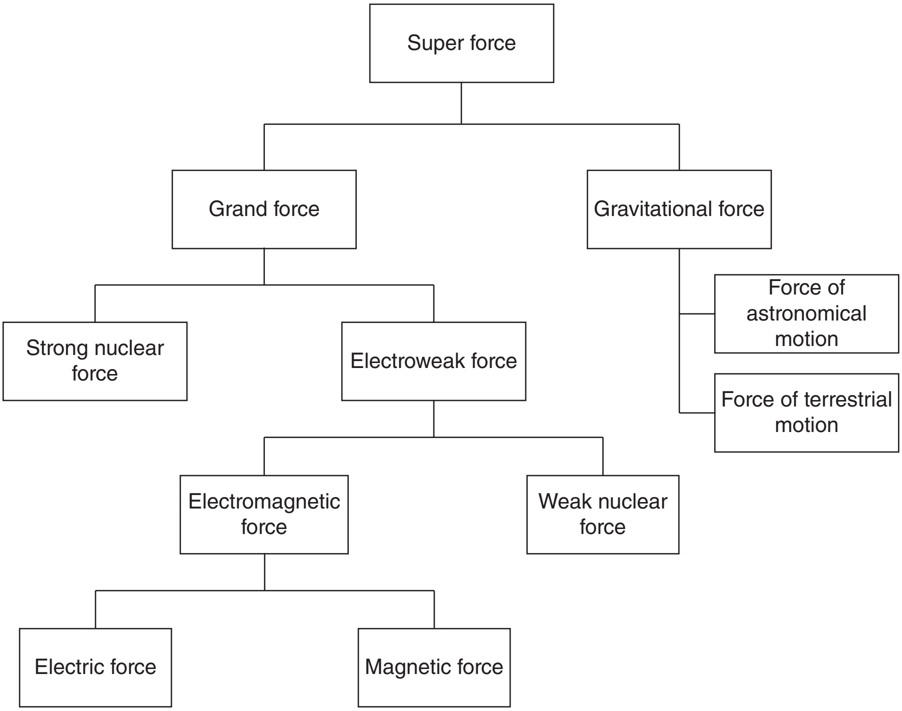


Figure 1.24. The unification of forces of nature.

TABLE 1.6. Some Examples of Unification Events

Unification Event	Time (s)	Result
Gravity	1700	Universal gravitation. Unifies the idea that the forces of attraction between material bodies that act outside of Earth are the same as those that act on Earth
Electricity and magnetism	1800	Light or photons Electromagnetic force
Electromagnetic force and weak nuclear force	1900	Electroweak force New particles W^+, W^-, Z^0
Theoretical efforts to unify electroweak force with the strong nuclear force and then with the gravity into one single force.	1900	Grand unification and super unification. Experimental evidences needed.

cannot see very large, such as the whole earth, entities at once or the universe in its entirety. Therefore our first observations and theories resulting from them also work well in the mid-range and breakdown on both ends of the spectrum. In this book, you will see how classical physics breaks down in the microworld, the world of the very small. However, keep in mind that classical physics also breaks down in some ways in the world of the very large. For example, classical physics fails to explain the behavior of objects moving at very large speed. Quantum physics is needed to explain phenomena in the microworld, whereas the special theory of relativity is needed to explain the world of very large speeds.

1.21 SPECIAL THEORY OF RELATIVITY

As we mentioned earlier, from the work of several physicists, including Young, Fresnel, and Maxwell, it was realized that light was a wave. As it often happens, with this achievement or solution, came another puzzle that eventually led to Einstein's special theory of relativity. At that time, all the known waves needed a medium to travel: sound waves travel through air, water waves travel through water, and so on. So the puzzle was What is the medium that light waves need to propagate through? It was known that light reaches us from outer space and possibly traveling through a vacuum on the way. This is an ideal example of how we (even the scientists) are prisoners of our past experience and knowledge. Great scientific discoveries are made by those who dare to break out of these self-made prisons. Continuing with the story, because all the waves discovered by then needed a medium to travel through, scientists assumed that light must need a medium too, even though it was known that light was apparently traveling through a vacuum to reach us from outer space.

So, driven by the need of a medium for light waves, scientists placed a hypothetical medium, called ether, in the vacuum, and everywhere else in the universe to be consistent. And off came the amazing properties of this mystic medium to fit in to our existing knowledge. It must have very low density so that the planets could move through it with no measurable change in their orbit positions and yet it must be so elastic that light, which was known to have an incredibly high speed, could propagate through it. However, the beauty of science is that you can theorize and fantasize as much as you can, but at the end of the day, your theory must withstand the test of experiments. It was not easy at the time to design an experiment to test the existence of ether. However, scientists tried for years by developing ever finer instruments and analysis methods. Eventually, there followed the experiment by Albert Michelson (1887) in collaboration with Edward Morley, which was designed to measure the motion of earth through ether. The experimental apparatus was sensitive enough to measure the ether drift due to the motion of the Earth, but to the surprise of the scientific world, no drift was detected.

Albert Einstein, a patent clerk at that time, interpreted the negative results of the Michelson–Morley experiment to develop what we know today as the special theory of relativity that he published in 1905. We are not covering this theory in this book, however, we present in this section some important features of this theory relevant to this book.

Energy Mass Equivalence. Mass, m, and energy, E, are equivalent and interconvertible according to the following equation:

$$E = mc^2 \tag{1.95}$$

where c is the speed of light through a vacuum, a constant; m is the relativistic mass; and E is the relativistic energy. For example, the Sun is powered by the conversion of mass into energy, which follows this formula. Given that $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$, the factor c^2 is a very big number. This finding leads to the potential to release tremendous amounts of energy.

Relativistic Mass. The mass, m, of an object moving at speed v is related to the mass, m_0, of the object at rest by the following equation:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1.96}$$

The mass m is called the relativistic mass and m_0 is called the rest mass.

Note that the concept of relativistic mass does not exist in Newtonian physics, in which the mass of a particle is always a constant and is equal to the rest mass. As shown in Study Checkpoint 1.9 and in Figure 1.25, this is a good approximation for an object moving substantially less than the speed of light; that is, we are dealing with speed, $v << c$.

STUDY CHECKPOINT 1.9

Show that the relativistic mass is equivalent to the rest mass at speeds substantially less than the speed of light.

Solution:

For

$$v \ll c$$

$$\frac{v^2}{c^2} \approx 0$$

Therefore,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = m_0$$

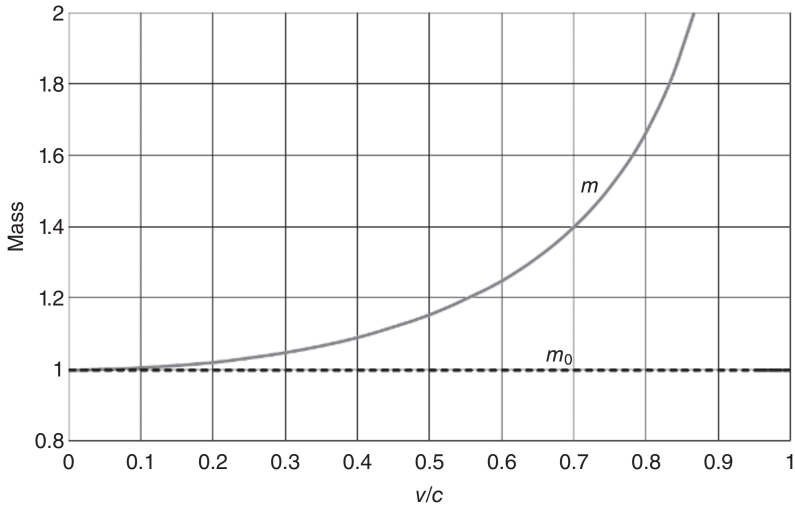


Figure 1.25. Comparison of rest mass (m_0) and relativistic mass m as a function of v/c for a particle with a rest mass of 1 unit.

Relativistic Energy and Momentum. We can write the relativistic energy in terms of the rest mass:

$$E = mc^2 = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1.97}$$

Similarly

$$p = mv = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1.98}$$

We can write the relativistic momentum in terms of the rest mass.

Let us examine how the total energy (i.e., the relativistic energy), the rest energy, and total momentum (relativistic momentum) are related. From Eqs. 1.97 and 1.98:

$$E^2 - p^2c^2 = \frac{m_0^2c^4}{1-\frac{v^2}{c^2}} - \frac{m_0^2v^2c^2}{1-\frac{v^2}{c^2}} = \frac{m_0^2c^4\left(1-\frac{v^2}{c^2}\right)}{1-\frac{v^2}{c^2}} = m_0^2c^4$$

Therefore,

$$E^2 = p^2c^2 + m_0^2c^4 \tag{1.99}$$

or

$$E = \sqrt{\left(p^2c^2 + m_0^2c^4\right)} \tag{1.100}$$

Equation 1.100 is the equation we will use in this book when we are dealing with relativistic energy or momentum. In most of the book, our treatment is nonrelativistic because this book covers nonrelativistic quantum mechanics.

We have covered a lot of material in this chapter with the intention of offering a brief review of classical physics. From this review, you should develop an understanding of the classical approach to physics. The key points of this classical approach emerging from the material covered in this chapter are summarized next in Section 1.22.

1.22 CLASSICAL APPROACH

Emerging from the review of classical physics presented here, you should see some core elements that define the classical approach of looking at the physical world. These elements are summarized in this section.

1.22.1 Separation of Particles and Waves: Either It Is a Particle or a Wave

In classical physics, particles and waves are two different types of entities. Particles are localized objects and can often be treated as point particles. Large particles and bulk matter are made of some fundamental particles and not waves. A moving particle transports energy by virtue of its moving mass. Waves, on the other hand, are spread out, a disturbance of some sort that transports energy from one point to another without transporting matter, and cannot be localized to a point. Particles can transport energy too, but by moving their mass. Waves have some unique characteristics, such as interference or superposition, in which the two waves can cancel each other out or add to each other. In other words, their amplitudes add or subtract depending on whether the interference is constructive or destructive. In the framework of classical physics, particles do not exhibit this interference phenomenon.

Other properties unique to waves include frequency, wavelength, refraction, and diffraction.

1.22.2 Either It Is Here or There: The Certainty

Knowing the current position and momentum of a particle under a given force, we can precisely predict its position and momentum at any time in the future.

Classical physics can make these predictions with full certainty; it is deterministic. Physical observables can be predicted and determined with certainty.

1.22.3 The World Is Continuous: Any Value Within a Range Is Possible

In the framework of classical physics, a particle can have any value within the acceptable range of values for its observables, such as position, momentum, and energy. In other words, the spectrum of possible values is a continuous spectrum. As an example, consider a ball of 1 kg mass. It will not be surprising if I say I can give any velocity to the ball between 0 and 25 $m\,s^{-1}$. If the kinetic energy E of the ball is $\frac{1}{2}mv^2$, all values of kinetic energy between 0 J and 312.5 J are possible. In other words, as shown in Figure 1.26, the possible values of E for this ball for velocities in the range of 0 and 25 m/s are represented by a continuous curve.

1.22.4 Common Grounds Among Particles and Waves: A Red Flag

As mentioned earlier, classical physics treats particles and waves as different types of entities. However, let us make a preliminary case to challenge that notion. While particles and waves have their own unique characteristics, they also share some properties or observables. For example, a moving particle transports energy from one place to another in the form of kinetic energy. Waves transport energy too. By definition, waves are traveling disturbances that carry energy. This finding is true about standing waves, discussed in this chapter, as well because standing waves are just superpositions of traveling waves. Furthermore, most waves, such as water and sound waves, need a

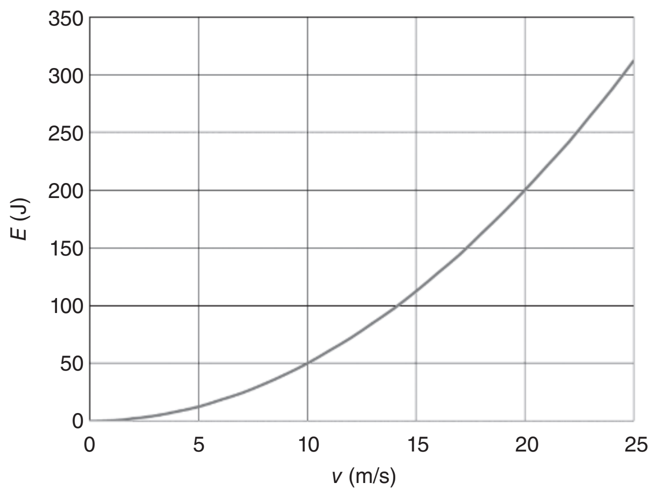


Figure 1.26. Kinetic energy as a continuous function of the velocity of a ball of 1 kg mass.

material medium to travel, and particles are material as well. Even the waves that do not need a medium to travel through, such as EM wave, need particles for their existence: You need charged particles to create EM waves.

Now here is a dilemma: Energy is transported in waves, but matter is not. However, as mentioned earlier, in the beginning of the twentieth century, Albert Einstein proposed in his special theory of relativity that matter (mass) and energy are equivalent according to Eq. 1.95:

$$E = mc^2 \tag{1.95}$$

where m is the mass of the particle and c is the speed of light.

If energy and mass are equivalent, then the waves can be considered transporting mass as well because they certainly transport energy. I will leave you with this dilemma, as an exercise to solve as you go through the forthcoming chapters. You can take it as a starting point in doubting classical physics theoretically. In Chapter 2, we will see how experimental results challenged the classical approach in a big way.

1.23 SUMMARY

Most of the concepts in classical physics can be derived from the three fundamental concepts: physical entity, space in which the entity exists, and time with which the entity changes its position in space. Classical physics, the physics of the macroworld, divides the physical world into two types of entities: particles and waves. Both particles and waves transport momentum and energy from one point to another in space. Waves do this transportation without transporting the matter. In classical physics, the motion of particles is governed by the fundamental equation of motion representing Newton's second law of motion:

$$F = ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}$$

Similarly, the propagation of waves is determined by the following fundamental equation:

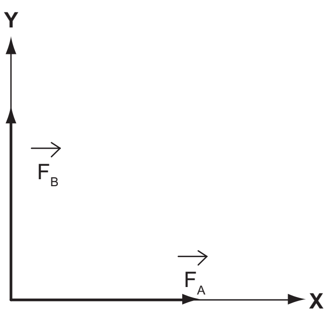
$$\frac{\partial^2\Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2}$$

The world and the universe is held together in an order due to the four fundamental forces of nature. These four forces encompass all the forces that cause change and motion in physical entities. These four forces are gravitational, electromagnetic, strong nuclear, and weak nuclear forces.

In a nutshell, this chapter has been a very brief review of classical physics aimed at making you realize that the classical approach divides the world of physical entities into two parts: particles and waves. This division mirrors the physical reality that we sense or experience as macrobeings. However, because the macroscopic world has been basically built from the objects of the microscopic world, such as molecules and atoms and subatomic particles, therefore the physical reality that we experience in the macroworld has its roots in the microworld. In the microscopic world, the standalone classical pictures of particles and waves collapse. This is where quantum mechanics comes into the picture with one promise: unification of two concepts: particles and waves. You will learn in Chapters 2 and 3 how this unification was historically achieved.

1.24 ADDITIONAL PROBLEMS

Problem 1.6 Two forces $\vec{F}_A$ and $\vec{F}_B$ with magnitudes 3 and 5 N, respectively, are acting on a particle. The force $\vec{F}_A$ is acting along the x-axis, and the force $\vec{F}_B$ is acting along the y-axis.



A. Calculate the scalar product of these two forces.
B. Calculate the vector product of these two forces.
C. Calculate the resultant force.

Problem 1.7 A ball is thrown up into the air with initial velocity u. The position of the ball as a function of time is given by the following equation:

$$x = ut + \frac{1}{2}gt^2$$

where g, the acceleration due to gravity, is a constant with a magnitude equal to 9.81 m/s^2. Calculate the instantaneous velocity of the ball at any time t.

Problem 1.8 If the ball in Problem 1.7 was thrown up with a velocity of 39.2 m/s, what will be its velocity at 4.0 s after the throw. Interpret your result.

Problem 1.9 The wave function of a standing wave is given by

$$\Psi = 2\Psi_0 \cos\omega t \sin kx$$

A. Derive the boundary conditions to represent the fact that the string is fixed at $x = 0$ and $x = L$.
B. Use the boundary conditions in A to derive the resonance condition:

$$\lambda_n = \frac{2L}{n}$$

Problem 1.10 Show that

$$\Psi(x,t) = \Psi_0 \sin(kx - \omega t) \tag{1}$$

is a solution of the wave equation:

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2} \tag{2}$$