CHAPTER 1

products and markets: equities, commodities, exchange rates, forwards and futures

The aim of this Chapter...

...is to describe some of the basic financial market products and conventions, to slowly introduce some mathematics, to hint at how stocks might be modeled using mathematics, and to explain the important financial concept of 'no free lunch.' By the end of the chapter you will be eager to get to grips with more complex products and to start doing some proper modeling.

In this Chapter...

- an introduction to equities, commodities, currencies and indices
- the time value of money
- fixed and floating interest rates
- futures and forwards
- no-arbitrage, one of the main building blocks of finance theory

1.1 INTRODUCTION

This first chapter is a very gentle introduction to the subject of finance, and is mainly just a collection of definitions and specifications concerning the financial markets in general. There is little technical material here, and the one technical issue, the 'time value of money,' is extremely simple. I will give the first example of 'no arbitrage.' This is important, being one part of the foundation of derivatives theory. Whether you read this chapter thoroughly or just skim it will depend on your background.

1.2 EQUITIES

The most basic of financial instruments is the **equity**, **stock** or **share**. This is the ownership of a small piece of a company. If you have a bright idea for a new product or service then you could raise capital to realize this idea by selling off future profits in the form of a stake in your new company. The investors may be friends, your Aunt Joan, a bank, or a venture capitalist. The investor in the company gives you some cash, and in return you give him a contract stating how much of the company he owns. The **shareholders** who own the company between them then have some say in the running of the business, and technically the directors of the company are meant to act in the best interests of the shareholders. Once your business is up and running, you could raise further capital for expansion by issuing new shares.

This is how small businesses begin. Once the small business has become a large business, your Aunt Joan may not have enough money hidden under the mattress to invest in the next expansion. At this point shares in the company may be sold to a wider audience or even the general public. The investors in the business may have no link with the founders. The final point in the growth of the company is with the quotation of shares on a regulated stock exchange so that shares can be bought and sold freely, and capital can be raised efficiently and at the lowest cost.

Figures 1.1 and 1.2 show screens from Bloomberg giving details of Microsoft stock, including price, high and low, names of key personnel, weighting in various indices, etc. There is much, much more info available on Bloomberg for this and all other stocks. We'll be seeing many Bloomberg screens throughout this book.

In Figure 1.3 I show an excerpt from *The Wall Street Journal Europe* of 14th April 2005. This shows a small selection of the many stocks traded on the New York Stock Exchange. The listed information includes highs and lows for the day as well as the change since the previous day's close.

The behavior of the quoted prices of stocks is far from being predictable. In Figure 1.4 I show the Dow Jones Industrial Average over the period January 1950 to March 2004. In Figure 1.5 is a time series of the Glaxo–Wellcome share price, as produced by Bloomberg.

If we could predict the behavior of stock prices in the future then we could become very rich. Although many people have claimed to be able to predict prices with varying degrees of accuracy, no one has yet made a completely convincing case. In this book I am going to take the point of view that prices have a large element of randomness. This does *not* mean that we cannot model stock prices, but it does mean that the modeling must be done in a probabilistic sense. No doubt the reality of the situation lies somewhere between

MSFT US $ C 95+1/16 Q Q194 15/16/95Q DL18 Equity DES
As of Sep10 DELAYED Vol 17,227,500 Op 95 1/16 Q Hi 95 5/16 Q Lo 94 Q

DESCRIPTION Page 1 /10

MSFT US MICROSOFT CORP
Computer Software
CUSIP 594918104

12) CN All News/Research
13) CWP Company Web Page
14) HH Hoover's Handbook

Microsoft Corporation develops, manufactures, licenses, sells, and supports software products. The Company offers operating system software, server application software, business and consumer applications software, software development tools, and Internet and intranet software. Microsoft also develops the MSN network of Internet products and services.

	STOCK DATA	Round Lot	100
1)GPO	Current Price	USD	95
	52Wk High 7/19/1999	USD	100 3/4
	52Wk Low 10/ 8/1998	USD	43 7/8
	YTD Chng (37.00%)	USD	25 31/32
2)TRA	1 Yr Total Return		82.25%
3)CH1	Shares Out as of 4/30		5103.859M
	Market Cap	USD	484866.63M
Float 3576.78M	Short Int		24.823M
5)BETA	Beta vs. SPX		1.25
6)OCM	Options avail & Stk Marginable		

8)DVD	DIVIDENDS – None		
	Indicated Gross Yld		
	Dividend Growth		
	Ex-Date	Type	Amt
	3/29/99	Split	2 for 1
EARNINGS	– Ann Date 10/20/99 (Est)		
9)ERN	Trailing 12mo EPS	USD	1.395
10)EE	Est EPS 6/2000	USD	1.566
11)GE	P/E 68.10	Est P/E	60.66
LT Growth	25.21	Est PEG	2.41

Par Value = .0000125

Bloomberg

Figure 1.1 Details of Microsoft stock. Source: Bloomberg L.P.

Page DL18 Equity DES
Hit 1 <GO> for a more detailed company management profile (MGMT).

MSFT US MICROSOFT CORP Page 2 /10

One Microsoft Way T:425-882-8080 F:425-936-8000
Bldg 8 Southwest D http://www.microsoft.com/msft/
Redmond,WA 98052-6399 TR AG ChaseMellon Shareholder Services
United States # OF EMPLOYEES 27,055

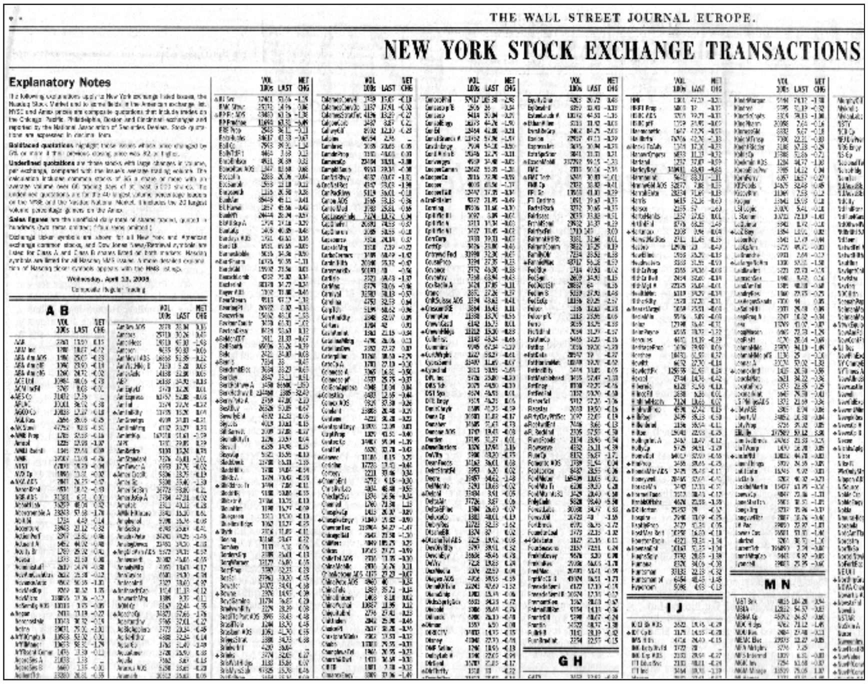
WILLIAM H GATES III	CHAIRMAN/CEO
STEVEN A BALLMER	PRESIDENT
ROBERT J HERBOLD	EXEC VP/COO
GREGORY B MAFFEI	SENIOR VP/CFO
TIM HALLADAY	INVESTOR RELATIONS CONTACT
STEVE SCHIRO	VP:CONSUMER CUSTOMER UNIT

Type	Common Stock	PAR $ .00001
PRIMARY EXCHANGE	NASDAQ N-Mkt	
COUNTRY	United States	
FISCAL YEAR END	JUNE	
SIC Code	7372 PREPAKG SOFTW	
VALOREN	000951692	
WPK Number	870747	
SEDOL	2588173	
Sicovam	903099	
ISIN	US5949181045	

3)WGT MEMBER	TICKER	WEIGHT
S&P 500 INDEX	SPX	4.368%
NASDAQ 100 STOCK	NDX	14.287%
S&P 100 INDEX	OEX	8.752%
TRIB WORLD INDEX	TRIB	5.245%
AMEX INSTITUTION	XII	6.540%
AMEX COMPUTER TE	XCI	23.453%
PHILA NATIONAL O	XOC	21.223%
CBOE TECHNOLOGY	TXX	4.157%
S&P INDUSTRIALS	SPXI	5.316%
S&P CAPITAL GOOD	SPCAPC	15.140%

Bloomberg

Figure 1.2 Details of Microsoft stock continued. Source: Bloomberg L.P.


THE WALL STREET JOURNAL EUROPE.

NEW YORK STOCK EXCHANGE TRANSACTIONS

Explanatory Notes

Figure 1.3 *The Wall Street Journal Europe* of 14th April 2005.

complete predictability and perfect randomness, not least because there have been many cases of market manipulation where large trades have moved stock prices in a direction that was favorable to the person doing the moving. Having said that, I will digress slightly in Appendix B where I describe some of the popular methods for supposedly predicting future stock prices.

See the simulation on the CD

To whet your appetite for the mathematical modeling later, I want to show you a simple way to simulate a random walk that looks something like a stock price. One of the simplest random processes is the tossing of a coin. I am going to use ideas related to coin tossing as a model for the behavior of a stock price. As a simple experiment start with the number 100 which you should think of as the price of your stock, and toss a coin. If you throw a head multiply the number by 1.01, if you throw a tail multiply by 0.99. After one toss your number will be either 99 or 101. Toss again. If you get a head multiply your *new* number by 1.01 or by 0.99 if you throw a tail. You will now have either $1.01^2 \times 100$, $1.01 \times 0.99 \times 100 = 0.99 \times 1.01 \times 100$ or $0.99^2 \times 100$. Continue this process and plot your value on a graph each time you throw the coin. Results of one particular experiment are shown in Figure 1.6. Instead of physically tossing a coin, the series used in this plot was generated on a spreadsheet like that in Figure 1.7. This uses the Excel spreadsheet function `RAND()` to generate a uniformly distributed random number between 0 and 1. If this number is greater than one half it counts as a 'head' otherwise a 'tail.'

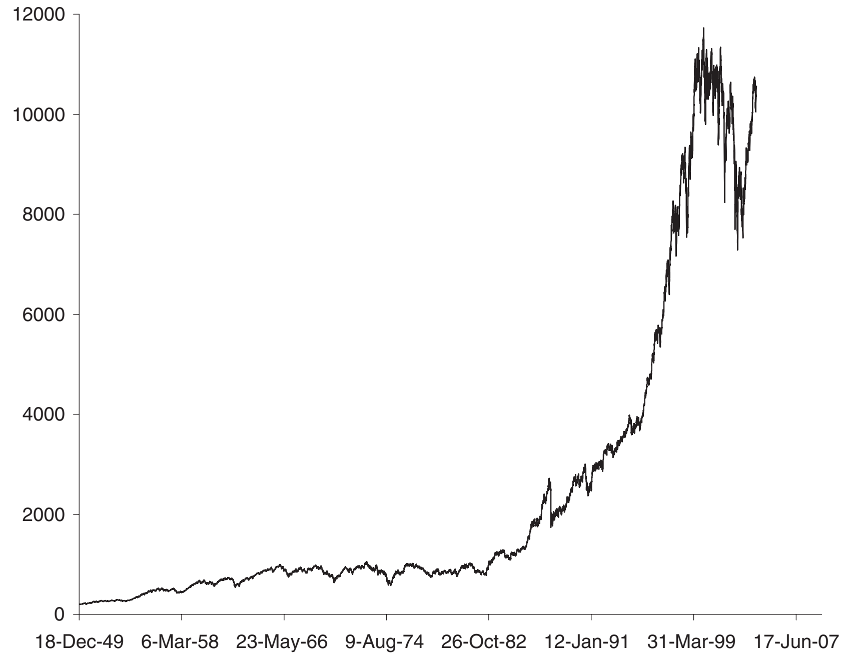


Figure 1.4 A time series of the Dow Jones Industrial Average from January 1950 to March 2004.

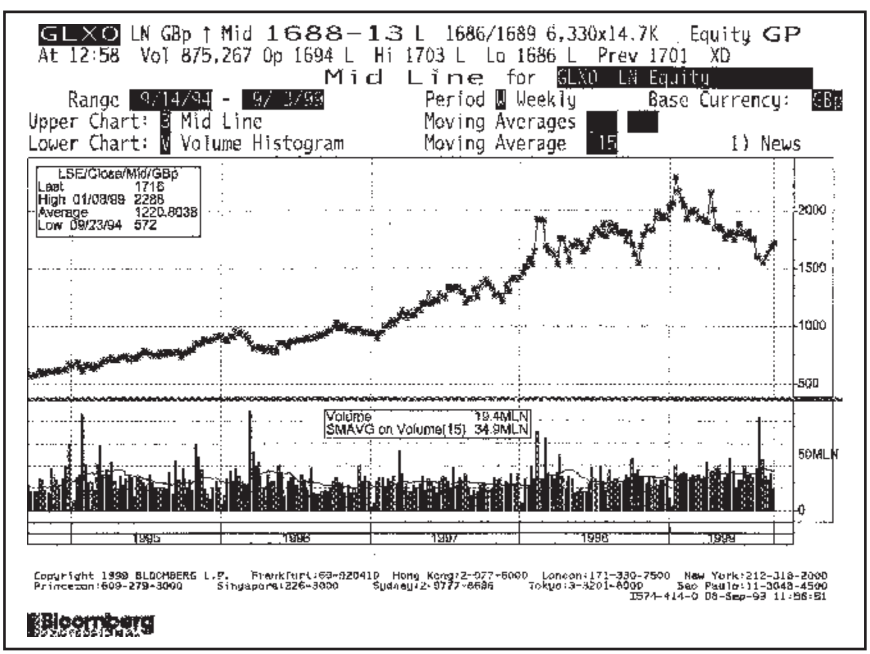


Figure 1.5 Glaxo−Wellcome share price (volume below). Source: Bloomberg L.P.

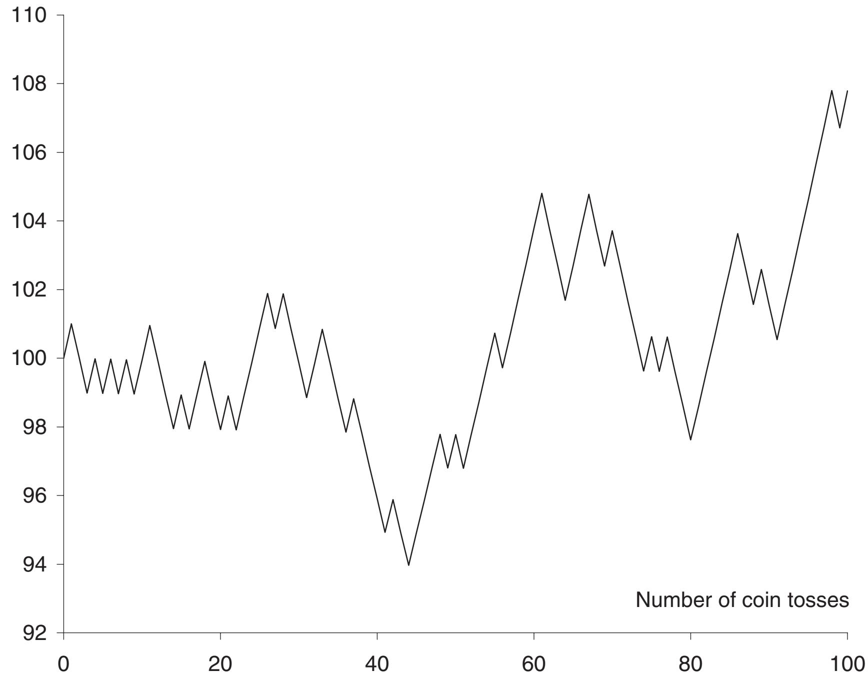


Figure 1.6 A simulation of an asset price path?

Time Out...

More about coin tossing

Notice how in the above experiment I've chosen to *multiply* each 'asset price' by a factor, either 1.01 or 0.99. Why didn't I simply *add* a fixed amount, 1 or −1, say? This is a very important point in the modeling of asset prices; as the asset price gets larger so do the changes from one day to the next. It seems reasonable to model the asset price changes as being proportional to the current level of the asset, they are still random but the magnitude of the randomness depends on the level of the asset. This will be made more precise in later chapters, where we'll see how it is important to model the return on the asset, its percentage change, rather than its absolute value. And, of course, in this simple model the 'asset price' cannot go negative.

If we use the multiplicative rule we get an approximation to what is called a **lognormal random walk**, also **geometric random walk**. If we use the additive rule we get an approximation to a **Normal** or **arithmetic random walk**.

As an experiment, using Excel try to simulate both the arithmetic and geometric random walks, and also play around with the probability of a rise in asset price; it doesn't have to be one half. What happens if you have an arithmetic random walk with a probability of rising being less than one half?

1.2.1 Dividends

The owner of the stock theoretically owns a piece of the company. This ownership can only be turned into cash if he owns so many of the stock that he can take over the company and keep all the profits for himself. This is unrealistic for most of us. To the average investor the value in holding the stock comes from the **dividends** and any growth in the stock's value. Dividends are lump sum payments, paid out every quarter or every six months, to the holder of the stock.

The amount of the dividend varies from time to time depending on the profitability of the company. As a general rule companies like to try to keep the level of dividends about the same each time. The amount of the dividend is decided by the board of directors of the company and is usually set a month or so before the dividend is actually paid.

When the stock is bought it either comes with its entitlement to the next dividend (**cum**) or not (**ex**). There is a date at around the time of the dividend payment when the stock goes from cum to ex. The original holder of the stock gets the dividend but the person who buys it obviously does not. All things being equal a stock that is cum dividend is better than one that is ex dividend. Thus at the time that the dividend is paid and the stock goes ex dividend there will be a drop in the value of the stock. The size of this drop in stock value offsets the disadvantage of not getting the dividend.

This jump in stock price is in practice more complex than I have just made out. Often capital gains due to the rise in a stock price are taxed differently from a dividend, which is often treated as income. Some people can make a lot of risk-free money by exploiting tax 'inconsistencies.'

	A	B	C	D	E
1	**Initial stock price**	**100**		**Stock**	
2	**Up move**	**1.01**		100	
3	**Down move**	**0.99**		101	
4	**Probability of up**	**0.5**		99.99	
5				98.9901	
6				99.98	
7				98.9802	
8				99.97	
9				[illegible]	
10				99.96001	
11				98.96041	
12				99.95001	
13				100.9495	
14				99.94001	
15				98.94061	
16				97.95121	
17				98.93072	
18				97.94141	
19				98.92083	
20				99.91004	
21				98.91094	
22				97.92183	
23				98.90104	
24				97.91203	
25				98.89115	
26				99.88007	
27				100.8789	
28				101.8877	
29				100.8688	
30				101.8775	
31				100.8587	

= B1

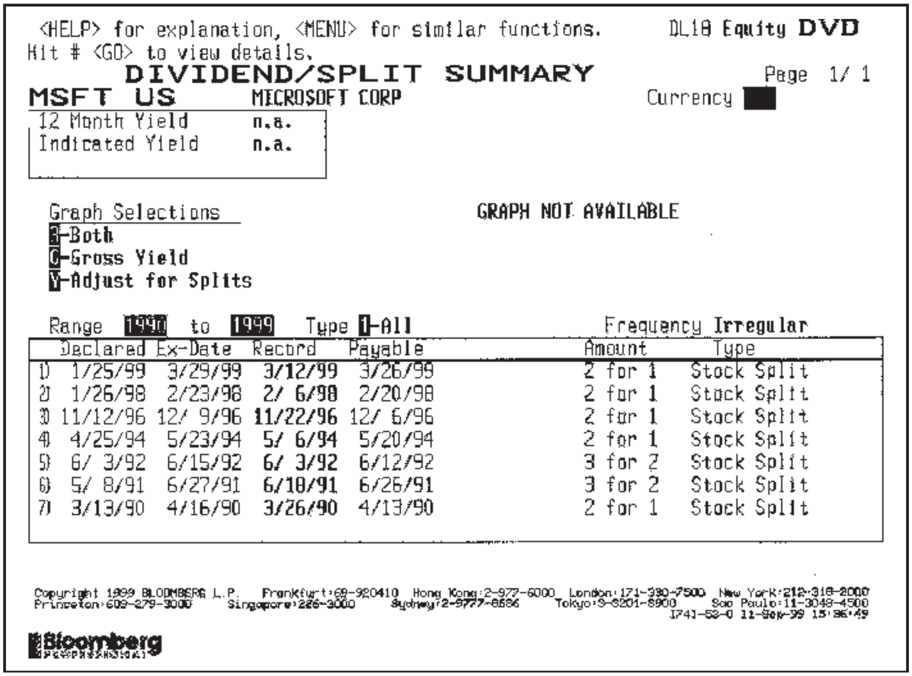
=D6*IF(RAND()>1-B4,B2,B3)

Figure 1.7 Simple spreadsheet to simulate the coin-tossing experiment.

1.2.2 Stock splits

Stock prices in the US are usually of the order of magnitude of $100. In the UK they are typically around £1. There is no real reason for the popularity of the number of digits, after all, if I buy a stock I want to know what percentage growth I will get, the absolute level of the stock is irrelevant to me, it just determines whether I have to buy tens or thousands of the stock to invest a given amount. Nevertheless there is some psychological element to the stock size. Every now and then a company will announce a **stock split**. For example, the company with a stock price of $90 announces a three-for-one stock split. This simply means that instead of holding one stock valued at $90, I hold three valued at $30 each.[1]

[1] In the UK this would be called a two-for-one split.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DIVIDEND/SPLIT SUMMARY Page 1/ 1
MSFT US MICROSOFT CORP Currency
12 Month Yield n.a.
Indicated Yield n.a.

Graph Selections
B-Both
G-Gross Yield
Y-Adjust for Splits

GRAPH NOT AVAILABLE

Range 1990 to 1999 Type 1-All Frequency Irregular

	Declared	Ex-Date	Record	Payable	Amount	Type
1)	1/25/99	3/29/99	3/12/99	3/26/99	2 for 1	Stock Split
2)	1/26/98	2/23/98	2/ 6/98	2/20/98	2 for 1	Stock Split
3)	11/12/96	12/ 9/96	11/22/96	12/ 6/96	2 for 1	Stock Split
4)	4/25/94	5/23/94	5/ 6/94	5/20/94	2 for 1	Stock Split
5)	6/ 3/92	6/15/92	6/ 3/92	6/12/92	3 for 2	Stock Split
6)	5/ 8/91	6/27/91	6/10/91	6/26/91	3 for 2	Stock Split
7)	3/13/90	4/16/90	3/26/90	4/13/90	2 for 1	Stock Split

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Princeton:609-279-3000 Singapore:226-3000 Sydney:2-9777-8686 Tokyo:3-3201-8900 Sao Paulo:11-3048-4500
1741-53-0 11-Sep-99 15:36:49

Bloomberg

Figure 1.8 Stock split info for Microsoft. Source: Bloomberg L.P.

1.3 COMMODITIES

Commodities are usually raw products such as precious metals, oil, food products, etc. The prices of these products are unpredictable but often show seasonal effects. Scarcity of the product results in higher prices. Commodities are usually traded by people who have no need of the raw material. For example they may just be speculating on the direction of gold without wanting to stockpile it or make jewelry. Most trading is done on the futures market, making deals to buy or sell the commodity at some time in the future. The deal is then closed out before the commodity is due to be delivered. Futures contracts are discussed below.

Figure 1.9 shows a time series of the price of pulp, used in paper manufacture.

1.4 CURRENCIES

Another financial quantity we shall discuss is the **exchange rate**, the rate at which one currency can be exchanged for another. This is the world of **foreign exchange**, or **Forex** or **FX** for short. Some currencies are pegged to one another, and others are allowed to float freely. Whatever the exchange rates from one currency to another, there must be consistency throughout. If it is possible to exchange dollars for pounds and then the pounds for yen, this implies a relationship between the dollar/pound, pound/yen and dollar/yen exchange rates. If this relationship moves out of line it is possible to make **arbitrage profits** by exploiting the mispricing.

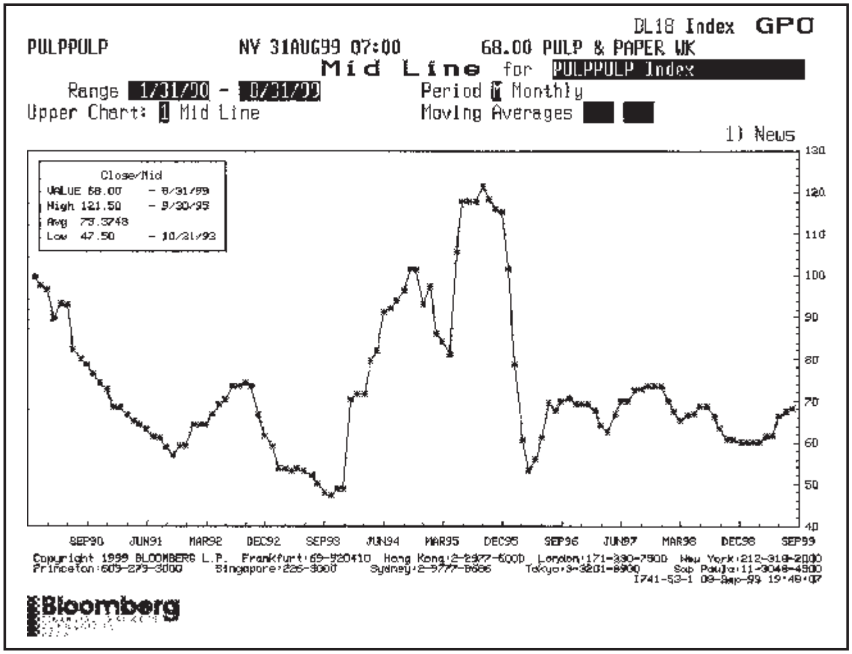
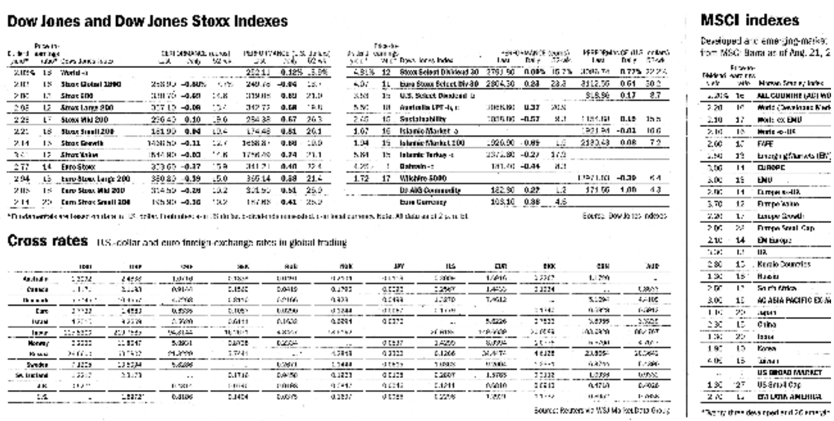


Figure 1.9 Pulp price. Source: Bloomberg L.P.



Dow Jones and Dow Jones Stoxx Indexes

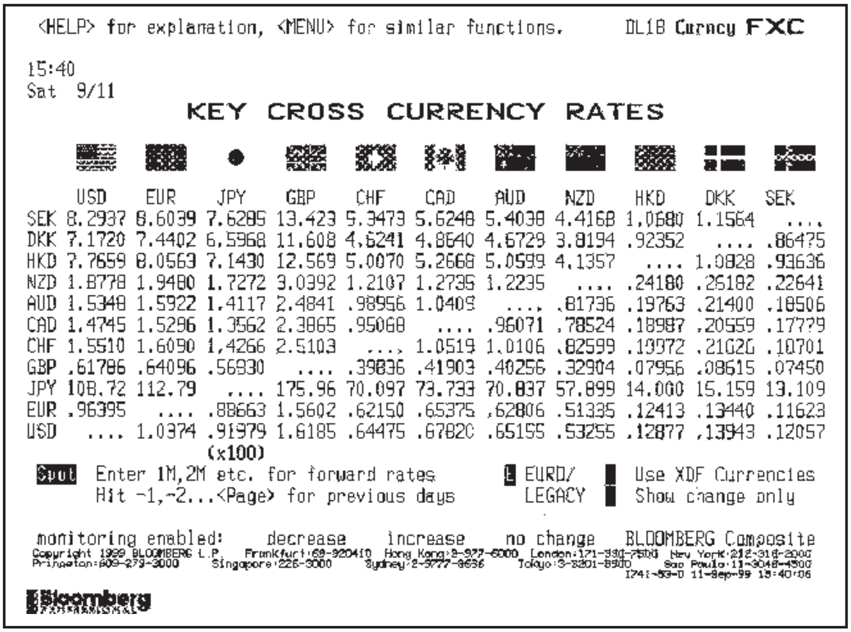
MSCI indexes

Cross rates U.S.-dollar and euro foreign-exchange rates in global trading

Figure 1.10 *The Wall Street Journal Europe* of 22nd August 2006, currency exchange rates.

Figure 1.10 is an excerpt from *The Wall Street Journal Europe* of 22nd August 2006. At the bottom of this excerpt is a matrix of exchange rates. A similar matrix is shown in Figure 1.11 from Bloomberg.

Although the fluctuation in exchange rates is unpredictable, there is a link between exchange rates and the interest rates in the two countries. If the interest rate on dollars is raised while the interest rate on pounds sterling stays fixed we would expect to see sterling depreciating against the dollar for a while. Central banks can use interest rates as a tool for manipulating exchange rates, but only to a degree.



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KEY CROSS CURRENCY RATES

	USD	EUR	JPY	GBP	CHF	CAD	AUD	NZD	HKD	DKK	SEK
SEK	8.2937	8.6039	7.6285	13.423	5.3473	5.6248	5.4038	4.4168	1.0680	1.1564	....
DKK	7.1720	7.4402	6.5968	11.608	4.6241	4.8640	4.6729	3.8194	.92352	....	.86475
HKD	7.7659	8.0563	7.1430	12.569	5.0070	5.2668	5.0599	4.1357	....	1.0828	.93636
NZD	1.8778	1.9480	1.7272	3.0392	1.2107	1.2735	1.2235	....	.24180	.26182	.22641
AUD	1.5348	1.5922	1.4117	2.4841	.98956	1.0409	....	.81736	.19763	.21400	.18506
CAD	1.4745	1.5296	1.3562	2.3865	.95068	....	.96071	.78524	.18987	.20559	.17779
CHF	1.5510	1.6090	1.4266	2.5103	....	1.0519	1.0106	.82599	.19972	.21026	.18701
GBP	.61786	.64096	.56830	....	.39836	.41903	.40256	.32904	.07956	.08615	.07450
JPY	108.72	112.79	....	175.96	70.097	73.733	70.837	57.899	14.000	15.159	13.109
EUR	.96395	....	.88663	1.5602	.62150	.65375	.62806	.51335	.12413	.13440	.11623
USD	....	1.0374	.91979	1.6185	.64475	.67820	.65155	.53255	.12877	.13943	.12057
			(x100)								

Spot Enter 1M,2M etc. for forward rates E EURO/ LEGACY Use XDF Currencies
Hit -1,-2...<Page> for previous days Show change only

monitoring enabled: decrease increase no change BLOOMBERG Composite
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Princeton:609-279-3000 Singapore:226-3000 Sydney:2-9777-8636 Tokyo:3-3201-8900 Sao Paulo:11-3048-4500
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Bloomberg

Figure 1.11 Key cross currency rates. Source: Bloomberg L.P.

At the start of 1999 Euroland currencies were fixed at the rates shown in Figure 1.12.

1.5 INDICES

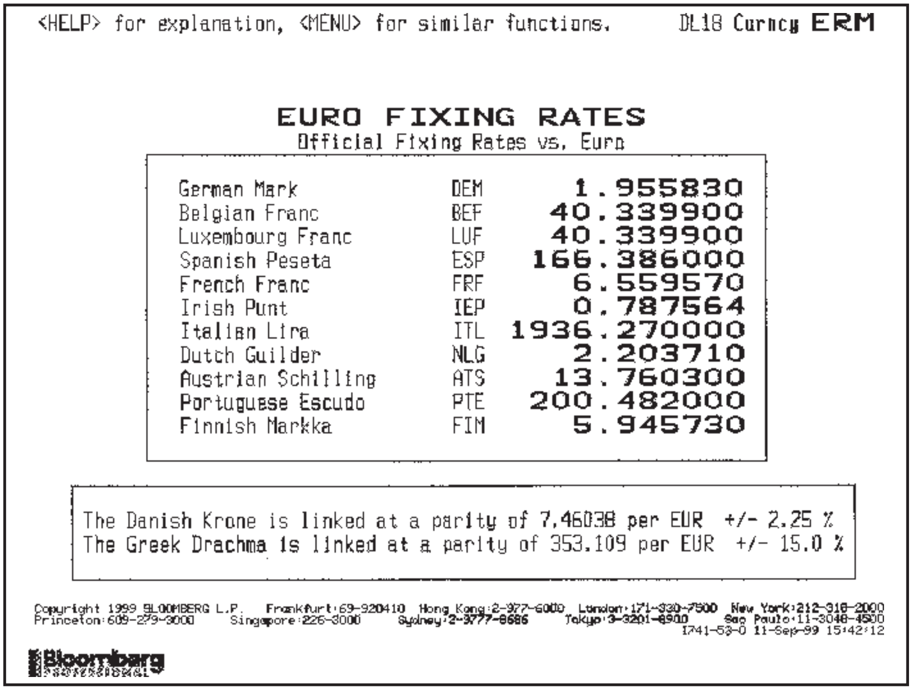
For measuring how the stock market/economy is doing as a whole, there have been developed the stock market **indices**. A typical index is made up from the weighted sum of a selection or **basket** of representative stocks. The selection may be designed to represent the whole market, such as the Standard & Poor's 500 (S&P500) in the US or the Financial Times Stock Exchange index (FTSE100) in the UK, or a very special part of a market. In Figure 1.4 we saw the DJIA, representing major US stocks. In Figure 1.13 is shown JP Morgan's Emerging Market Bond Index.

The EMBI+ is an index of emerging market debt instruments, including external-currency-denominated Brady bonds, Eurobonds and US dollar local markets instruments. The main components of the index are the three major Latin American countries, Argentina, Brazil and Mexico. Bulgaria, Morocco, Nigeria, the Philippines, Poland, Russia and South Africa are also represented.

Figure 1.14 shows a time series of the MAE All Bond Index which includes Peso and US dollar denominated bonds sold by the Argentine Government.

1.6 THE TIME VALUE OF MONEY

The simplest concept in finance is that of the **time value of money**; $1 today is worth more than $1 in a year's time. This is because of all the things we can do with $1 over the



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EURO FIXING RATES

Official Fixing Rates vs. Euro

German Mark	DEM	1.955830
Belgian Franc	BEF	40.339900
Luxembourg Franc	LUF	40.339900
Spanish Peseta	ESP	166.386000
French Franc	FRF	6.559570
Irish Punt	IEP	0.787564
Italian Lira	ITL	1936.270000
Dutch Guilder	NLG	2.203710
Austrian Schilling	ATS	13.760300
Portuguese Escudo	PTE	200.482000
Finnish Markka	FIM	5.945730

The Danish Krone is linked at a parity of 7.46038 per EUR +/- 2.25 %
The Greek Drachma is linked at a parity of 353.109 per EUR +/- 15.0 %

Figure 1.12 Euro fixing rates. Source: Bloomberg L.P.

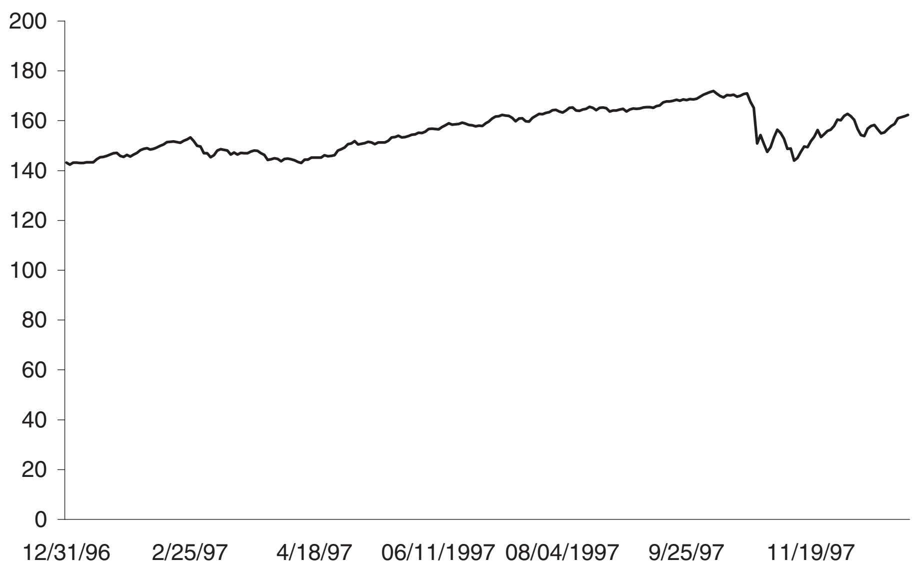


Figure 1.13 JP Morgan's EMBI +.

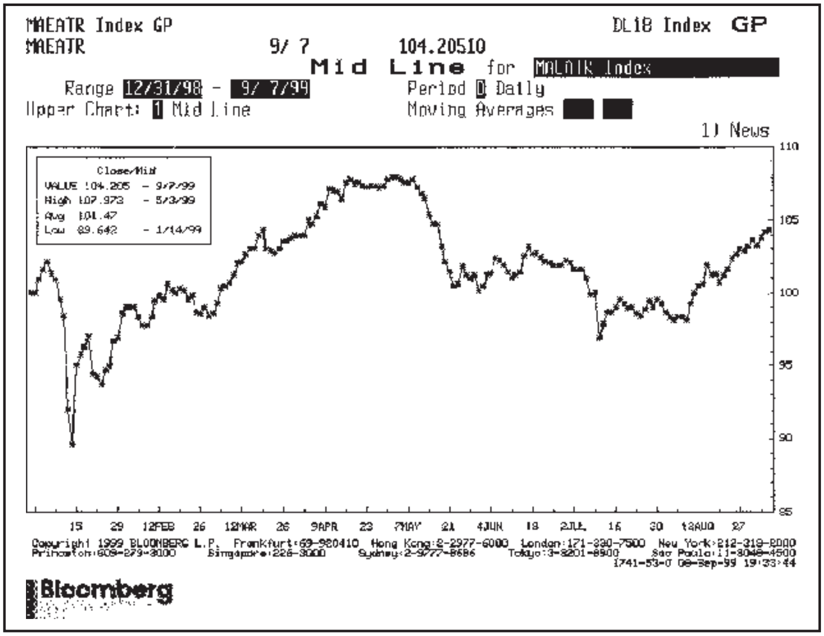
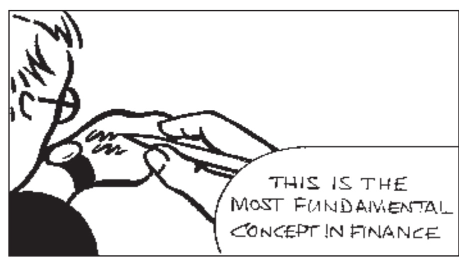


Figure 1.14 A time series of the MAE All Bond Index. Source: Bloomberg L.P.



next year. At the very least, we can put it under the mattress and take it out in one year. But instead of putting it under the mattress we could invest it in a gold mine, or a new company. If those are too risky, then lend the money to someone who is willing to take the risks and will give you back the dollar with a little bit extra, the **interest**. That is what banks do, they borrow your money and invest it in various risky ways, but by spreading their risk over many investments they reduce their overall risk. And by borrowing money from many people they can invest in ways that the average individual cannot. The banks compete for your money by offering high interest rates. Free markets and the ability to quickly and cheaply change banks ensure that interest rates are fairly consistent from one bank to another.

Time Out...

Symbols

It had to happen sooner or later, and the first chapter is as good as anywhere. Our first mathematical symbol is nigh.

Please don't be put off by the use of symbols if you feel more comfortable with numbers and concrete examples. I know that math is the one academic subject that can terrify adults, just because of poor teaching in schools. If you fall into this category, just go with the flow, concentrate on the words, the examples and the Time Outs, and before you know it...

I am going to denote interest rates by r. Although rates vary with time I am going to assume for the moment that they are constant. We can talk about several types of interest. First of all there is **simple** and **compound interest**. Simple interest is when the interest you receive is based only on the amount you initially invest, whereas compound interest is when you also get interest on your interest. Compound interest is the only case of relevance. And compound interest comes in two forms, **discretely compounded** and **continuously compounded**. Let me illustrate how they each work.

Suppose I invest \$1 in a bank at a discrete interest rate of r paid once *per annum*. At the end of one year my bank account will contain

$$1 \times (1 + r).$$

If the interest rate is 10% I will have one dollar and ten cents. After two years I will have

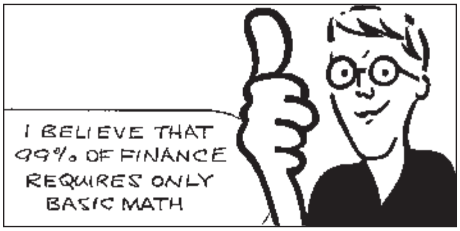
$$1 \times (1 + r) \times (1 + r) = (1 + r)^2,$$

or one dollar and twenty-one cents. After n years I will have $(1 + r)^n$. That is an example of discrete compounding.

Now suppose I receive m interest payments at a rate of r/m *per annum*. After one year I will have

$$\left(1 + \frac{r}{m}\right)^m. \tag{1.1}$$

Now I am going to imagine that these interest payments come at increasingly frequent intervals, but at an increasingly smaller interest rate: I am going to take the limit $m \to \infty$. This will lead to a rate of interest that is paid continuously. Expression (1.1) becomes[2]



$$\left(1 + \frac{r}{m}\right)^m = e^{m \log\left(1+\frac{r}{m}\right)} \sim e^r.$$

That is how much money I will have in the bank after one year if the interest is continuously compounded. And similarly, after a time t I will have an amount

$$\left(1 + \frac{r}{m}\right)^{mt} \sim e^{rt} \tag{1.2}$$

in the bank. Almost everything in this book assumes that interest is compounded continuously.

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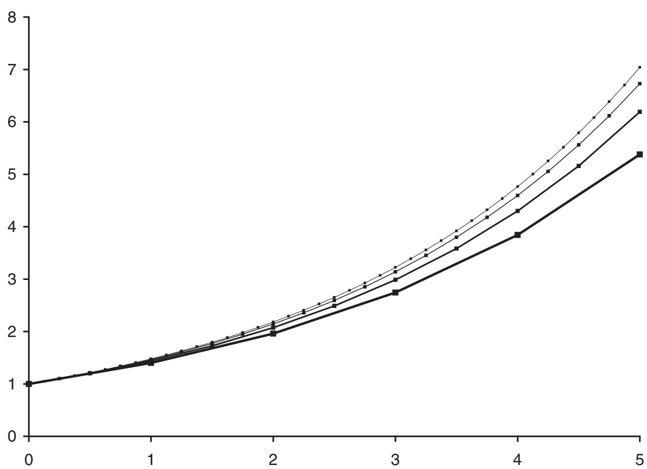
[2] The symbol $\sim$, called 'tilde,' is like 'approximately equal to,' but with a slightly more technical, in a math sense, meaning. The symbol $\to$ means 'tends to.'

Time Out...

The math so far

Let's see *m* getting larger and larger in an example. I produced the next figure in Excel.



These functions are all plotted on a spreadsheet

As *m* gets larger and larger, so the curve seems to get smoother and smoother, eventually becoming the exponential function. We'll be seeing this function a lot. In Excel the exponential function e^x (also written exp(x)) is EXP( ).

What mathematics have we seen so far? To get to (1.2) all we needed to know about are the two functions, the **exponential function** *e* (or exp) and the **logarithm** log, and Taylor series. Believe it or not, you can appreciate almost all finance theory by knowing these three things together with 'expectations.' I'm going to build up to the basic Black–Scholes and derivatives theory assuming that you know all four of these. Don't worry if you don't know about these things, in Appendix A I review these requisites.

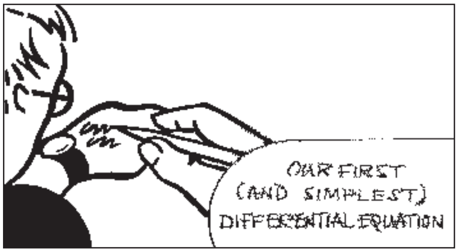
En passant, what would the above figures look like if interest were simple rather than compound? Which would you prefer to receive?

Another way of deriving the result (1.2) is via a differential equation. Suppose I have an amount $M(t)$ in the bank at time t, how much does this increase in value from one day to the next? If I look at my bank account at time t and then again a short while later, time $t + dt$, the amount will have increased by

$$M(t + dt) - M(t) \approx \frac{dM}{dt} dt + \cdots,$$

where the right-hand side comes from a Taylor series expansion. But I also know that the interest I receive must be proportional to the amount I have, M, the interest rate, r, and the time step, dt. Thus

$$\frac{dM}{dt}dt = rM(t)\,dt.$$



Dividing by dt gives the ordinary differential equation

$$\boxed{\frac{dM}{dt} = rM(t)}$$

the solution of which is

$$M(t) = M(0)\, e^{rt}.$$

If the initial amount at $t = 0$ was \$1 then I get (1.2) again.

Time Out...

Differential equations

This is our first **differential equation**; hang on in there, it'll become second nature soon. Whenever you see *d* something over *d* something else you know you're looking at a slope, or gradient, also known as rate of change or sensitivity. So here we've got the rate of change of money with time, i.e. rate of growth of money in the bank. You don't need to know how I solved this differential equation really. In Appendix A I explain all about slope, sensitivities and differential equations.

This first differential equation is an example of an **ordinary differential equation**, there is only one **independent variable** t. M is the **dependent variable**, its value depends on t. We'll also be seeing **partial differential equations** where there is more than one independent variable. And we'll also see quite a few **stochastic differential equations**. These are equations with a random term in them, used for modeling the randomness in the financial world. For the next few chapters there will be no more mention of differential equations. Whew.

This equation relates the value of the money I have now to the value in the future. Conversely, if I know I will get one dollar at time T in the future, its value at an earlier time t is simply

$$\frac{1}{e^{r(T-t)}} = e^{-r(T-t)}.$$

I can relate cashflows in the future to their **present value** by multiplying by this factor. As an example, suppose that r is 5%, i.e. $r = 0.05$, then the present value of $1,000,000 to be received in two years is

$$\$1{,}000{,}000 \times e^{-0.05\times 2} = \$904{,}837.$$

The present value is clearly less than the future value.

Interest rates are a very important factor determining the present value of future cashflows. For the moment I will only talk about one interest rate, and that will be constant. In later chapters I will generalize.

1.7 FIXED-INCOME SECURITIES

In lending money to a bank you may get to choose for how long you tie your money up and what kind of interest rate you receive. If you decide on a fixed-term deposit the bank will offer to lock in a fixed rate of interest for the period of the deposit, a month, six months, a year, say. The rate of interest will not necessarily be the same for each period, and generally the longer the time that the money is tied up the higher the rate of interest, although this is not always the case. Often, if you want to have immediate access to your money then you will be exposed to interest rates that will change from time to time, since interest rates are not constant.

These two types of interest payments, **fixed** and **floating**, are seen in many financial instruments. **Coupon-bearing bonds** pay out a known amount every six months or year, etc. This is the **coupon** and would often be a fixed rate of interest. At the end of your fixed term you get a final coupon and the return of the **principal**, the amount on which the interest was calculated. **Interest rate swaps** are an exchange of a fixed rate of interest for a floating rate of interest. Governments and companies issue bonds as a form of borrowing. The less creditworthy the issuer, the higher the interest that they will have to pay out. Bonds are actively traded, with prices that continually fluctuate.

1.8 INFLATION-PROOF BONDS

A very recent addition to the list of bonds issued by the US Government is the **index-linked bond**. These have been around in the UK since 1981, and have provided a very successful way of ensuring that income is not eroded by inflation.

In the UK inflation is measured by the **Retail Price Index** or **RPI**. This index is a measure of year-on-year inflation, using a 'basket' of goods and services including mortgage interest payments. The index is published monthly. The coupons and principal of the index-linked bonds are related to the level of the RPI. Roughly speaking, the amounts of the coupon and principal are scaled with the increase in the RPI over the period from the issue of the bond to the time of the payment. There is one slight complication in that the actual RPI level used in these calculations is set back *eight months*. Thus the base measurement is eight months before issue and the scaling of any coupon is with respect to the increase in the RPI from this base measurement to the level of the RPI eight months before the coupon is paid. One of the reasons for this complexity is that the initial estimate of the RPI is usually corrected at a later date.

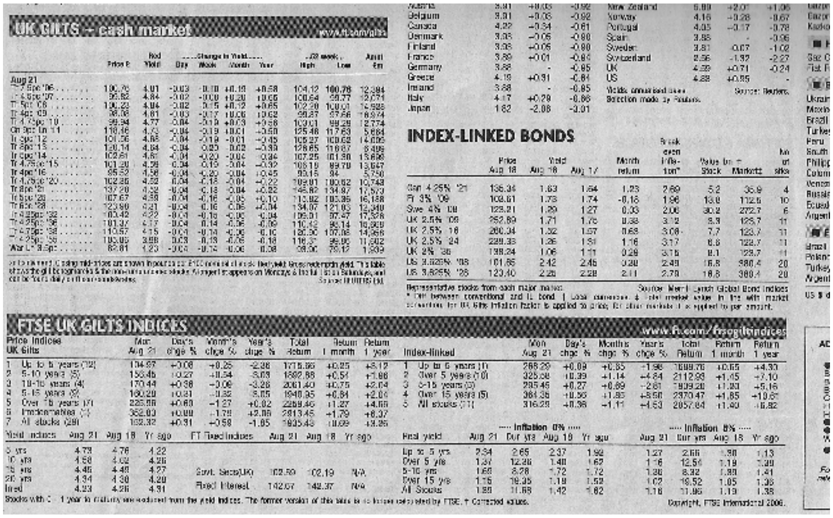


Figure 1.15 UK gilts prices from *The Financial Times* of 22nd August 2006.

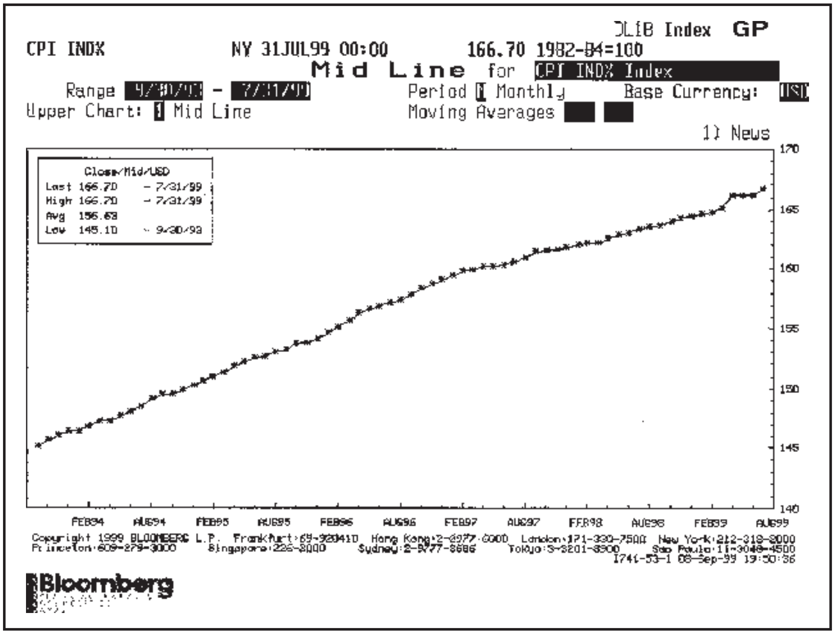


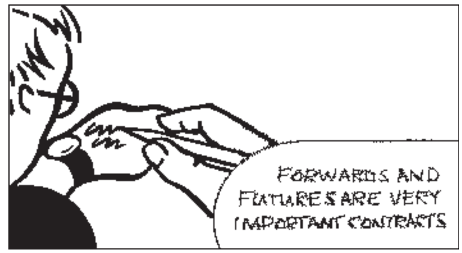
Figure 1.16 The CPI index. Source: Bloomberg L.P.

Figure 1.15 shows the UK gilts prices published in *The Financial Times* of 22nd August 2006. The index-linked bonds are on the right.

In the US the inflation index is the **Consumer Price Index** (**CPI**). A time series of this index is shown in Figure 1.16.

I will not pursue the modeling of inflation or index-linked bonds in this book. I would just like to say that the dynamics of the relationship between inflation and short-term interest rates is particularly interesting. Clearly the level of interest rates will affect the rate of inflation directly through mortgage repayments, but also interest rates are often used by central banks as a tool for keeping inflation down.

1.9 FORWARDS AND FUTURES



A **forward contract** is an agreement where one party promises to buy an asset from another party at some specified time in the future and at some specified price. No money changes hands until the **delivery date** or **maturity** of the contract. The terms of the contract make it an obligation to buy the asset at the delivery date, there is no choice in the matter. The asset could be a stock, a commodity or a currency.

The amount that is paid for the asset at the delivery date is called the **delivery price**. This price is set at the time that the forward contract is entered into, at an amount that gives the forward contract a value of zero initially. As we approach maturity the value of *this particular forward contract* that we hold will change in value, from initially zero to, at maturity, the difference between the underlying asset and the delivery price.

In the newspapers we will also see quoted the **forward price** for different maturities. These prices are the delivery prices for forward contracts of the quoted maturities, should we enter into such a contract *now*.

Try to distinguish between the value of a particular contract during its life and the specification of the delivery price at initiation of the contract. It's all very subtle. You might think that the forward price is the market's view on the asset value at maturity; this is not quite true as we'll see shortly. In theory, the market's expectation about the value of the asset at maturity of the contract is irrelevant.

A **futures contract** is very similar to a forward contract. Futures contracts are usually traded through an exchange, which standardizes the terms of the contracts. The profit or loss from the futures position is calculated every day and the change in this value is paid from one party to the other. Thus with futures contracts there is a gradual payment of funds from initiation until maturity.

Because you settle the change in value on a daily basis, the value of a futures contract at any time during its life is zero. The futures price varies from day to day, but must at maturity be the same as the asset that you are buying.

I'll show later that provided interest rates are known in advance, forward prices and futures prices of the same maturity must be identical.

Forwards and futures have two main uses, in speculation and in hedging. If you believe that the market will rise you can benefit from this by entering into a forward or futures contract. If your market view is right then a lot of money will change hands (at maturity or every day) in your favor. That is speculation and is very risky. Hedging is the opposite, it is avoidance of risk. For example, if you are expecting to get paid in yen in six months' time, but you live in America and your expenses are all in dollars, then you could enter into a futures contract to lock in a guaranteed exchange rate for the amount of your yen income. Once this exchange rate is locked in you are no longer exposed to fluctuations in the dollar/yen exchange rate. But then you won't benefit if the yen appreciates.

1.9.1 A first example of no arbitrage

Although I won't be discussing futures and forwards very much they do provide us with our first example of the **no-arbitrage** principle. I am going to introduce some more mathematical notation now, it will be fairly consistent throughout the book. Consider a forward contract that obliges us to hand over an amount $\$F$ at time T to receive the underlying asset. Today's date is t and the price of the asset is currently $\$S(t)$, this is the **spot price**, the amount for which we could get immediate delivery of the asset. When we get to maturity we will hand over the amount $\$F$ and receive the asset, then worth $\$S(T)$. How much profit we make cannot be known until we know the value $\$S(T)$, and we cannot know this until time T. From now on I am going to drop the '\$' sign from in front of monetary amounts.

We know all of F, $S(t)$, t and T, but is there any relationship between them? You might think not, since the forward contract entitles us to receive an amount $S(T) - F$ at expiry and this is unknown. However, by entering into a special portfolio of trades *now* we can eliminate all randomness in the future. This is done as follows.

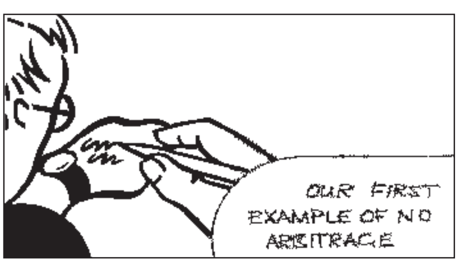
Enter into the forward contract. This costs us nothing up front but exposes us to the uncertainty in the value of the asset at maturity. Simultaneously sell the asset. It is called **going short** when you sell something you don't own. This is possible in many markets, but with some timing restrictions. We now have an amount $S(t)$ in cash due to the sale of the asset, a forward contract, and a short asset position. But our net position is zero. Put the cash in the bank, to receive interest.

When we get to maturity we hand over the amount F and receive the asset, this cancels our short asset position regardless of the value of $S(T)$. At maturity we are left with a guaranteed $-F$ in cash as well as the bank account. The word 'guaranteed' is important because it emphasizes that it is independent of the value of the asset. The bank account contains the initial investment of an amount $S(t)$ with added interest, this has a value at maturity of

$$S(t)e^{r(T-t)}.$$

Our net position at maturity is therefore

$$S(t)e^{r(T-t)} - F.$$



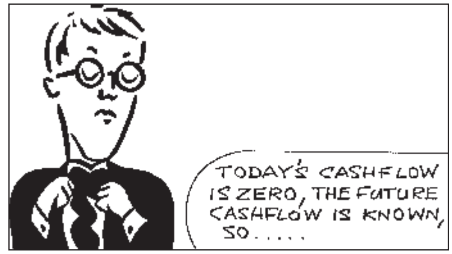
Since we began with a portfolio worth zero and we end up with a predictable amount, that predictable amount should also be zero. We can conclude that

$$F = S(t)e^{r(T-t)}. \tag{1.3}$$

This is the relationship between the spot price and the forward price. It is a linear relationship, the forward price is proportional to the spot price.

Table 1.1 Cashflows in a hedged portfolio of asset and forward.

Holding	Worth today (t)	Worth at maturity (T)
Forward	0	$S(T) - F$
$-$Stock	$-S(t)$	$-S(T)$
Cash	$S(t)$	$S(t)e^{r(T-t)}$
Total	0	$S(t)e^{r(T-t)} - F$



The cashflows in this special hedged portfolio are shown in Table 1.1.

Time Out...

No arbitrage again

Example: The spot asset price S is 28.75, the one-year forward price F is 30.20 and the one-year interest rate is 4.92%. Are these numbers consistent with no arbitrage?

$$F - Se^{r(T-t)} = 30.20 - 28.75e^{0.0492\times 1} = 0.0001.$$

This is effectively zero to the number of decimal places quoted.

If we know any three out of S, F, r and $T - t$ we can find the fourth, assuming there are no arbitrage possibilities. Note that the forward price in no way depends on what the asset price is expected to do, whether it is expected to increase or decrease in value.

In Figure 1.17 is a path taken by the spot asset price and its forward price. As long as interest rates are constant, these two are related by (1.3).

If this relationship is violated then there will be an arbitrage opportunity. To see what is meant by this, imagine that F is less than $S(t)e^{r(T-t)}$. To exploit this and make a riskless arbitrage profit, enter into the deals as explained above. At maturity you will have $S(t)e^{r(T-t)}$ in the bank, a short asset and a long forward. The asset position cancels when you hand over the amount F, leaving you with a profit of $S(t)e^{r(T-t)} - F$. If F is greater than that given by (1.3) then you enter into the opposite positions, going short the forward. Again you make a riskless profit. The standard economic argument then says that investors will act quickly to exploit the opportunity, and in the process prices will adjust to eliminate it.

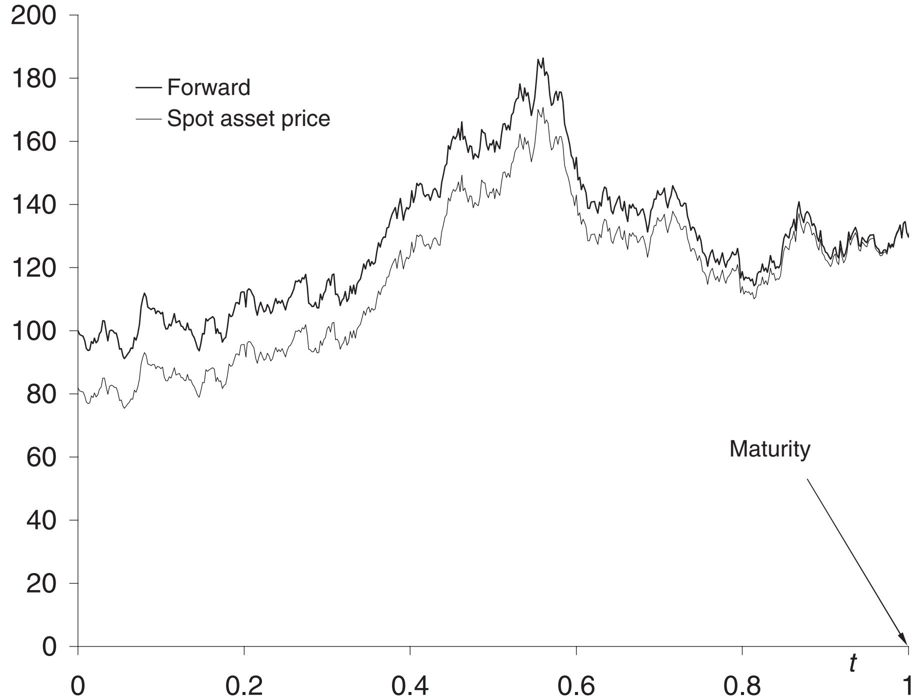


Figure 1.17 A time series of a spot asset price and its forward price.

1.10 MORE ABOUT FUTURES

Futures are usually traded through an exchange. This means that they are very liquid instruments and have lots of rules and regulations surrounding them. Here are a few observations on the nature of futures contracts.

Available assets A futures contract will specify the asset which is being invested in. This is particularly interesting when the asset is a natural commodity because of non-uniformity in the type and quality of the asset to be delivered. Most commodities come in a variety of grades. Oil, sugar, orange juice, wheat, etc. futures contracts lay down rules for precisely what grade of oil, sugar, etc. may be delivered. This idea even applies in some financial futures contracts. For example, bond futures may allow a range of bonds to be delivered. Since the holder of the short position gets to choose which bond to deliver he naturally chooses the cheapest.

The contract also specifies how many of each asset must be delivered. The quantity will depend on the market.

Delivery and settlement The futures contract will specify when the asset is to be delivered. There may be some leeway in the precise delivery date. Most futures contracts are closed out before delivery, with the trader taking the opposite position before maturity. But if the position is not closed then delivery of the asset is made. When the asset is another financial contract settlement is usually made in cash.

Margin I said above that changes in the value of futures contracts are settled each day. This is called **marking to market**. To reduce the likelihood of one party defaulting, being unable or unwilling to pay up, the exchanges insist on traders depositing a sum of money to cover changes in the value of their positions. This money is deposited in a **margin account**. As the position is marked to market daily, money is deposited or withdrawn from this margin account.

Margin comes in two forms, the **initial margin** and the **maintenance margin**. The initial margin is the amount deposited at the initiation of the contract. The total amount held as margin must stay above a prescribed maintenance margin. If it ever falls below this level then more money (or equivalent in bonds, stocks, etc.) must be deposited. The levels of these margins vary from market to market.

Margin has been much neglected in the academic literature. But a poor understanding of the subject has led to a number of famous financial disasters, most notably Metallgesellschaft and Long Term Capital Management. We'll discuss the details of these cases in Chapter 26, and we'll also be seeing how to model margin and how to margin hedge.

1.10.1 Commodity futures

Futures on commodities don't necessarily obey the no-arbitrage law that led to the asset/future price relationship explained above. This is because of the messy topic of storage. Sometimes we can only reliably find an upper bound for the futures price. Will the futures price be higher or lower than the theoretical no-storage-cost amount? Higher. The holder of the futures contract must compensate the holder of the commodity for his storage costs. This can be expressed in percentage terms by an adjustment s to the risk-free rate of interest.

But things are not quite so simple. Most people actually holding the commodity are benefiting from it in some way. If it is something consumable, such as oil, then the holder can benefit from it immediately in whatever production process they are engaged in. They are naturally reluctant to part with it on the basis of some dodgy theoretical financial calculation. This brings the futures price back down. The benefit from holding the commodity is commonly measured in terms of the **convenience yield** c:

$$F = S(t)e^{(r+s-c)(T-t)}.$$

Observe how the storage cost and the convenience yield act in opposite directions on the price. Whenever

$$F < S(t)e^{r(T-t)}$$

the market is said to be in **backwardation**. Whenever

$$F > S(t)e^{r(T-t)}$$

the market is in **contango**.

1.10.2 FX futures

There are no problems associated with storage when the asset is a currency. We need to modify the no-arbitrage result to allow for interest received on the foreign currency r_f. The result is

$$F = S(t)e^{(r-r_f)(T-t)}.$$

The confirmation of this is an easy exercise.

1.10.3 Index futures

Futures contracts on stock indices are settled in cash. Again, there are no storage problems, but now we have dividends to contend with. Dividends play a role similar to that of a foreign interest rate on FX futures. So

$$F = S(t)e^{(r-q)(T-t)}.$$

Here q is the dividend yield. This is clearly an approximation. Each stock in an index receives a dividend at discrete intervals, but can these all be approximated by one continuous dividend yield?

1.11 SUMMARY

The above descriptions of financial markets are enough for this introductory chapter. Perhaps the most important point to take away with you is the idea of no arbitrage. In the example here, relating spot prices to futures prices, we saw how we could set up a very simple portfolio which completely eliminated any dependence on the future value of the stock. When we come to value derivatives, in the way we just valued a forward, we will see that the same principle can be applied albeit in a far more sophisticated way.

FURTHER READING

- For general financial news visit `www.bloomberg.com` and `www.reuters.com`. CNN has online financial news at `www.cnnfn.com`. There are also online editions of *The Wall Street Journal*, `www.wsj.com`, *The Financial Times*, `www.ft.com` and *Futures and Options World*, `www.fow.com`.
- For more information about futures see the Chicago Board of Trade website `www.cbot.com`.
- Many, many financial links can be found at Wahoo!, `www.io.com/~gibbonsb/wahoo.html`.
- See Bloch (1995) for an empirical analysis of inflation data and a theoretical discussion of pricing index-linked bonds.
- In the main, we'll be assuming that markets are random. For insight about alternative hypotheses see Schwager (1990, 1992).
- See Brooks (1967) for how the raising of capital for a business might work in practice.
- Cox, *et al*. (1981) discuss the relationship between forward and future prices.

EXERCISES

1. A company makes a three-for-one stock split. What effect does this have on the share price?
2. A company whose stock price is currently S pays out a dividend DS, where $0 \le D \le 1$. What is the price of the stock just after the dividend date?

3. The dollar sterling exchange rate (colloquially known as 'cable') is 1.83, £1 = \$1.83. The sterling euro exchange rate is 1.41, £1 = €1.41. The dollar euro exchange rate is 0.77, \$1 = €0.77. Is there an arbitrage, and if so, how does it work?

4. You put \$1000 in the bank at a continuously compounded rate of 5% for one year. At the end of this first year rates rise to 6%. You keep your money in the bank for another eighteen months. How much money do you now have in the bank including the accumulated, continuously compounded, interest?

5. A spot exchange rate is currently 2.350. The one-month forward is 2.362. What is the one-month interest rate assuming there is no arbitrage?

6. A particular forward contract costs nothing to enter into at time t and obliges the holder to buy the asset for an amount F at expiry T. The asset pays a dividend DS at time t_d, where $0 \le D \le 1$ and $t \le t_d \le T$. Use an arbitrage argument to find the forward price $F(t)$.

   Hint: Consider the point of view of the writer of the contract when the dividend is reinvested immediately in the asset.