Part One

Math Skill Development Games

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Chapter One

Paper-and-Pencil Games

Paper-and-pencil games are popular among children and require little preparation. The games in this book offer new ideas and variations on old ones. And they are not limited to paper and pencil. Students can play them on a chalkboard with chalk, a sidewalk with chalk, a laminated piece of paper or cardboard with crayons, or a whiteboard with dry erase markers. These alternatives help avoid the use of large amounts of paper.

Count

Object: Players take turns writing and saying the numbers from 1 to 45. Each player may claim one, two, or three numbers in sequence, starting where the other player left off. The player who claims 45 wins.

Skills: counting forward and counting back, place value, writing and saying numbers, problem solving, communication, representation, reasoning and proof

Number of players: 2

Grades: 1 to 5

Materials: paper and pencils

Preparation: none

Playing:

1. The players take turns writing and saying the numbers from 1 to 45. They write the numbers on a sheet of paper and say them aloud as they write them. The numbers are written and said in sequential order.

2. The first player starts by writing and saying one of the following: "one," "one, two," or "one, two, three." Thereafter, each player may write and say one, two, or three numbers in sequence, starting from where the other player left off counting.

Winning: The player who writes and says 45 is the winner.

Playing variations:

- Instead of using the numbers from 1 to 45, the players can use the numbers from *x* to *n*, where *x* and *n* are any counting numbers. For older children, 985 and 1005 work well.
- The player who says the last number can lose rather than win.
- Players can simply count aloud during the game, and forgo writing the numbers.

Skill variations:

Counting back: Instead of counting forward, players count backward from 45 to 0. The player who says 0 wins.

Fractions: Instead of using counting numbers, players can use a sequence of fractions (for example, the fractions from ¼ to 5 with a counting increment of ¼ can be used, as in ¼, ½, ¾, 1, and so on).

Note: This game has a winning strategy that children can discover if they search for it. Ask children to explain why their strategy works.

Capture the Fort

Object: Each player has 50 points to bet during a series of rounds. Whoever bets the most points wins a round and advances.

Skills: subtraction, inequalities, probability, problem solving, connections

Number of players: 2

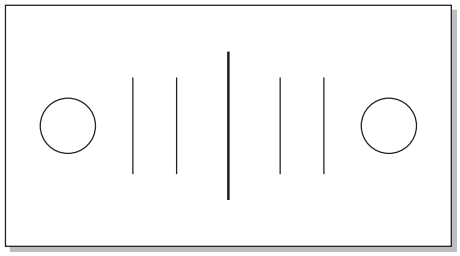
Grades: 1 to 5

Materials: paper, pencils, and a counter (such as a coin or button)

Preparation: Have players draw a larger version of the playing board shown here on a sheet of paper, or make and duplicate copies for players.

Playing:

1. Place the playing board between the two players with a circle in front of each player. The circle nearest each player is that player's fort. The five lines between the circles are the positions on which the players can move the counter. The thick center line is the boundary between the players' territories.



2. The game starts by placing the counter on the center boundary line. Subsequently the counter is moved in either direction, one line at a time, as determined by the betting. The object is to move the counter into the opponent's fort and thus capture it.
3. Moves are determined by making bets. Each player has 50 points to bet for the entire game.
4. The game is played as a series of rounds. During a round, each player first decides how many points to bet and secretly writes that number on a piece of paper. The players then simultaneously show each other their bets. The player with the larger bet wins the round and moves his or her counter one space toward the opponent's fort. If both players bet the same number, the counter is not moved. A new round then begins.
5. Players use up betting points as the game proceeds. Before play begins, they write 50 at the top of another small piece of paper. They subtract their first-round bet from 50, their second-round bet from the remainder of the first subtraction, and so on.
6. If a player runs out of points, he or she must bet zero points during each remaining round of the game, until the opponent either wins or runs out of points.

Winning: The first player to move the counter into the opposing fort wins. If neither player has sufficient points left to reach the opponent's fort, the game ends in a tie.

Playing variations:

- A player can be awarded a half-win if a tie results and the counter is in the opponent's territory.
- Players can avoid revealing their bets to each other by showing them instead to a third person, who announces only the winner of each round.

Foreheaded

Object: Each player writes a three-digit number on a slip of paper and attaches it to the forehead of the player on the left. Players then are given clues and try to guess their numbers.

Skills: reading, writing, and saying numbers using place value language; problem solving; reasoning and proof; communication

Number of players: 3 to 6

Grades: 1 to 5

Materials: paper, pencils, and tape (or sticky notes)

Preparation: Cut 1 × 2 inch (3 × 5 cm) slips of paper for each player. Sticky notes (about 1.5 × 2 inches) are ideal.

Playing:

1. The players sit in a circle.
2. Each player secretly writes a three-digit number on a slip of paper and then tapes it to the forehead of the player to the left. (If sticky notes are used, simply write on them and post them on a forehead.) The numbers are taped facing out so that every player sees all the opponents' numbers but not his or her own number.
3. Players take turns in clockwise rotation.
4. On each turn, a player must guess the number on his or her forehead. If a player guesses correctly, he or she wins. If a player guesses incorrectly, two things happen. First, opponents must tell him or her how many digits in the number he or she guessed are correct (the correct digit in the correct place). Second, the player must give every opponent a clue. This involves telling each opponent a three-digit number, using place value terminology, that contains at least one digit that is the same and in the same position as a digit in the opponent's number. For example, a player might give the clue 135 to an opponent with the number 237 taped to his or her head. Players should write down the clues they are given, for future reference.
5. If a player gives another player the exact same number as a clue that was already given on a previous turn by any player, that player must give another clue.
6. The game ends when a player guesses the number on his or her forehead.

Winning: The winner is the first player to guess his or her number.

Playing variations:

- Two-digit or four-digit numbers may be used instead of three-digit numbers.
- Foreheaded can be played so that a player is told both how many digits are completely correct (correct digit in the correct place) and how many digits are partially correct (correct digit in the wrong place). For example, if a player with 237 on his or her forehead guessed 532, he or she would be told, "One digit completely correct and one digit partially correct."

Googol

Object: One player secretly writes eight numbers on slips of paper. Another player tries to guess which slip has the largest number. Points are awarded for guessing and ordering numbers.

Skills: writing and reading numbers, inequalities, ordering numbers
Number of players: 2
Grades: 1 to 5
Materials: paper and pencils
Preparation: none
Playing:

1. There are ten rounds in the game. During each round, one player is the writer and the other is the picker. Players switch roles after each round. Thus, in ten rounds each player will be writer five times and picker five times.
2. To start a round, the writer tears a sheet of paper into eight small slips and secretly writes a different positive number on each one. The writer then turns them face down on the playing surface. The numbers written may range from small fractions to a googol (a 1 followed by a hundred 0s); they may be in fractional and decimal forms.
3. During the round, the picker turns the slips face up one at a time. The picker stops turning over slips when he or she believes that the one containing the largest of the eight numbers is turned face up. The picker cannot go back to a previously turned-up slip; the decision to turn over another slip is final, even though it is made without knowing what is written on all the slips. If the picker turns over all eight slips, the last one turned over is considered to be the chosen one.
4. To end a round, the picker turns all eight slips face up and arranges them in order from smallest to largest.
5. Score each round as follows: three points to a picker who chooses the largest number; one point to a picker who correctly arranges the slips in numerical order from smallest to largest.

Winning: The player with the highest total score after ten rounds is the winner.
Playing variation:

- The following scoring rule can be added: If the writer incorrectly marks slips of paper (with a negative number, the same number twice, and so on), the picker is awarded one point.

Number-Tac-Toe

Object: Players add (or multiply) numbers to get sums (or products) equal to numbers on a Tic-Tac-Toe grid. The first player to get three numbers in a row (as in Tic-Tac-Toe) wins.

Skills: addition (or multiplication), reasoning and proof, connections

Number of players: 2

Grades: 1 to 5

Materials: paper and pencils

Preparation: Create a Number-Tac-Toe grid by writing sums (or products) of the numbers from 1 to 9 in the nine cells of a regular Tic-Tac-Toe grid. (See the sample for addition or use the grids supplied. A blank playing sheet for reproduction is also provided.)

Playing:

1. Players decide who will mark X's and who will mark O's. If not provided a playing sheet, they draw a Number-Tac-Toe grid on a sheet of paper, and each list the numbers from 1 to 9 in a column on either side of the grid. (See the sample playing sheet.)
2. The first player crosses out any one number in his or her column of nine numbers. Beginning with the second player, the game continues as follows.
3. During a turn, a player crosses out any one number in his or her column of nine numbers that has not yet been crossed out. The player then adds (or multiplies) that number to (or by) the last number the opponent crossed out. If the sum (or product) is on the Number-Tac-Toe grid and if it is not yet marked, the player marks an X or O over it (depending on whether the player is marking X's or O's). Players should plan ahead.
4. The game ends when any of the following occurs:
   - Three of a player's marks are in a row (as in Tic-Tac-Toe).
   - All of the numbers on the grid are marked X or O.
   - All nine numbers in each player's column of numbers are crossed out.

X Player				O Player
1				1
2	5	7	12	2
3				3
4	16	9	15	4
5				5
6	2	6	8	6
7				7
8				8
9				9

Winning: The player who gets three marks in a row wins, as in Tic-Tac-Toe. If neither player gets three marks in a row, a tie is declared.

Playing variation:

- Players can construct their own Number-Tac-Toe grids by taking turns placing numbers in the nine cells of the grid at the start of a game. The Number-Tac-Toe grids shown here are samples that can be used to guide players in thinking about which numbers are appropriate to put in the cells of the grid.

Add-Tac-Toe Grids

5	7	12
16	9	15
2	6	8

14	3	12
8	16	6
2	18	10

7	9	15
16	11	5
3	13	4

6	4	10
12	14	8
16	9	5

13	8	9
15	5	11
17	10	7

15	12	9
16	7	5
2	8	6

Multiply-Tac-Toe Grids

25	15	56
12	36	16
49	20	8

24	6	56
8	35	14
63	8	15

8	10	5
48	24	3
36	6	30

12	24	4
35	27	18
15	21	5

3	36	63
15	30	9
18	24	8

15	12	36
16	25	49
20	8	56

Number-Tac-Toe

X Player		O Player
1		1
2		2
3		3
4		4
5		5
6		6
7		7
8		8
9		9

Ask and Give

Object: In this variation of Go Fish, players use place value skills to trade numbers.

Skills: place value, reading and writing large numbers, addition, subtraction, communication

Number of players: 2
Grades: 3 to 5
Materials: paper and pencils
Preparation: none
Playing:

1. At the top of separate sheets of paper, each player secretly writes a six-digit counting number, containing no 0s and no identical digits. Players keep their papers and numbers hidden from each other for the entire game.
2. As in Go Fish, players take turns being asker and giver. The game ends when each player has had five turns as asker. Their objective is to increase the size of their numbers by taking digits from each other.
3. A turn begins when the asker says, "Give me your ___'s," where ___ can be any digit from 1 to 9. (For example, "Give me your 6's.")
4. If that digit is in the giver's number, the giver announces its place value in the number. (For example, "You get 600.") If that digit is not in the giver's number, the giver announces this. (For example, "You get 0.") Note that the value of a digit that is asked for depends on its position in the giver's number. For example, if 6 is asked for and the giver's number is 512,639, then the giver responds, "You get 600." But if the giver's number is 561,243, then the giver responds, "You get 60,000."

	Player A		Player B
	621,845		**297,613**
A says, "Give me your 2's."	+ 200,000	←	− 200,000
	821,845		97,613
B says, "Give me your 8's."	− 800	→	+ 800
	821,045		98,413
A says, "Give me your 4's."	+ 400	←	− 400
	821,445		98,013

5. As soon as the giver responds with a number, the asker adds that amount to his or her number (for example, + 600), and the giver subtracts that amount from his or her number (for example, − 600).
6. Each player's number changes with each new addition or subtraction. Players always use the most recent form of their numbers when adding, subtracting, or announcing the positional value of a digit. Players keep track of their changing number by adding to and subtracting from their original number and its successors directly under the original number. (See the sample game.)

7. If the same digit appears two or more times in the giver's number during the play, the giver may announce either of its values. For example, in 621,063 the giver may say, "You get 60" and say nothing about the 600,000.
8. The game ends after each player has five turns as asker. Players then check each other's additions, subtractions, and final numbers.

Winning: The player with the largest number at the end of the game wins. If either player's paper contains an error, that player automatically loses.

Hangmath

Object: This is a variation of Hangman. One player creates an arithmetic problem involving multidigit addition, subtraction, multiplication, or division. The other player tries to reconstruct the problem before being "hanged" (that is, within fourteen guesses).

Skills: place value, addition, subtraction, multiplication, division, problem solving, communication

Number of players: 2

Grades: 3 to 5

Materials: paper and pencils

Preparation: none

Playing:

1. Players take turns being hangman and guesser.
2. On one piece of paper, the hangman secretly writes a multidigit addition, subtraction, multiplication, or division problem (see sample).
3. On another piece of paper, the hangman makes a playing board that shows the type of problem and position of the digits in the problem (see sample). Problems up to the following sizes are suitable: four-digit addition and subtraction; three-digit by three-digit multiplication; and two-digit into four-digit division.
4. The hangman keeps the paper with the problem on it hidden, but gives the guesser the playing board.
5. The guesser tries to reconstruct the problem by guessing which digits belong where on the playing board. Guesses have the following format: "Is there a ___ in the ___ column?" (For example, "Is there a 5 in the ones column?")

Hangman's Problem

```
   43
 × 25
  215
 860
 1075
```

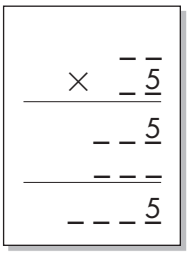
Guesser's Playing Board

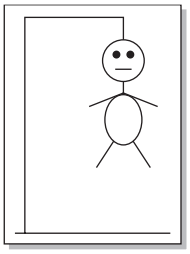
```
   __
 × __
  ___
 ___
 ____
```

6. If the guesser guesses a correct digit in the correct column, the hangman must indicate every place the digit occurs in that column. (See the sample for the answer to the guess "Is there a 5 in the ones column?")
7. If the digit guessed does not appear in the specified column, the hangman begins or adds a line to the picture of a hanged man. The hanged man consists of fourteen lines drawn in this order: base of gallows, upright post, cross beam, rope, head, neck, body, leg, leg, arm, arm, eye, eye, mouth.
8. As the game progresses, the guesser tries to use information obtained from previous guesses to guide further guesses.
9. The game ends when either the hangman completes the picture or the guesser reconstructs the problem. Then guesser and hangman switch roles.

Winning: If the hangman picture is completed, the hangman wins. If the arithmetic problem is completed, the guesser wins.

Is there a five in the ones column?





Factors

Object: Players take turns giving and receiving numbers to factor. They score points equivalent to the numbers given and the factors identified.

Skills: factoring, multiples, division, multiplication, prime and composite numbers, addition, problem solving, communication

Number of players: 2 (or whole class in two teams)

Grades: 3 to 5

Materials: paper and pencils

Preparation: Have the players make a playing board or provide them with copies of the playing board on the next page. The playing board has a scoring column for each player on opposite sides of the paper, and the numbers from 1 to 30 in an array in the center.

Playing:

1. Players alternate roles as picker and factorer.
2. On a turn as picker, a player crosses out any legal number on the playing board. (A legal number is any number that is not crossed out and that has at least one factor that also has not been crossed out.) The picker writes that number in his or her scoring column.
3. On a turn as factorer, a player crosses out any remaining factors of the number the picker crossed out. The factorer writes all these numbers in his or her scoring column.
4. Neither player can reuse a number that is crossed out.

5. The factorer can cross out all possible factors of a number in one turn. For example, if the picker crosses out 30 as the first play of the game, the factorer can cross out 1, 2, 3, 5, 6, 10, and 15. The factorer may choose not to cross out a factor of a number.

Emilio	Factors	Sonya
30	~~1~~ ~~2~~ ~~3~~ 4 ~~5~~ ~~6~~ 7 8 9 ~~10~~ 11 12 13 14 ~~15~~ 16 17 18 19 20 21 22 23 24 25 26 27 28 29 ~~30~~	1 2 3 5 6 10 15

6. Players switch roles after each round.
7. The game ends when there are no more legal numbers to cross out. Players then add the numbers in their columns to find their total score.

Winning: The player with the highest total score wins.

Playing variations:

- Have players keep cumulative sums of the numbers they acquire, rather than waiting until the end of the game to find the sum of the numbers.
- Use other numbers on the playing board—for example, 1 to 20, 1 to 35, or 1 to 40.
- Allow the picker to cross out illegal numbers (numbers that have no remaining factors on the playing board). Add the rule that if the picker crosses out an illegal number, then the factorer (who now has no numbers to cross out) gets two consecutive turns as picker, with the other player being factorer after each turn.

Factors

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

Get One

Object: Players choose a number and then take turns, first subtracting a factor of the number from itself, and then factors of the resultant differences from those differences, until one player leaves the other a difference of l.

Skills: factoring, subtracting
Number of players: 2
Grades: 3 to 5
Materials: paper and pencils
Preparation: none
Playing:

1. Each player's objective is to leave the other player with the number 1.
2. To begin, the players jointly choose a positive whole number to start the game.
3. The first player subtracts any factor of this starting number (except the number itself) from the starting number to get a difference. The second player then subtracts any factor of this difference (except the number itself) from the difference to get a new difference. If a prime number comes up during the game (which has only factors of 1 and itself) the next player subtracts 1 (which is one of the factors of the number).
4. Players continue taking turns subtracting a factor of each new difference, always trying to leave the opponent a final difference of 1. (See the sample game.)
5. Both players do all subtracting on separate papers and check each other after each turn.

Starting number	32
Player 1	− 8
	24
Player 2	− 12
	12
Player 1	− 6
	6
Player 2	− 3
	3
Player 1	− 1
	2
Player 2	− 1
Player 2 wins.	1

Winning: The winner is the player who leaves the number 1 to the opponent.

Write It

Object: Each player secretly writes a number. If the sum of the players' numbers is a multiple of some predetermined number, then one player wins a point. If not, the other player wins a point.

Skills: multiples, factors, probability, addition, multiplication, division
Number of players: 2
Grades: 3 to 5
Materials: paper and pencils
Preparation: none

Playing:

1. Before beginning, both players decide which whole number *x* the game will be about. (Excellent numbers to use are 2, 3, and 4.)
2. There are two sets of eight rounds in the game. During the first eight rounds, one player tries to get numbers to sum to a multiple of the chosen number *x*, and the other player tries to prevent this. The players switch roles for the second eight rounds.
3. To start a set of eight rounds, each player cuts a sheet of paper into eight smaller pieces.
4. During a round, each player takes a piece of paper and secretly writes a whole number on it. (The numbers may be limited, for example, to those between 0 and 100.) The players then show each other their numbers and add them together. If the sum is a multiple of *x*, the player trying for a multiple of *x* gets one point; if not, the other player gets one point. For example, John and Sue are playing a game about the number 2. John is trying to get numbers to sum to a multiple of 2, and Sue is trying to prevent this. John writes a 25 on his sheet of paper and Sue writes the number 5 on her sheet of paper. The sum of the numbers is 30, so John scores one point because he is trying to get sums to be a multiple of 2.
5. After the winner of a round is determined and the point awarded, a new round then begins. After eight rounds, the players switch roles (in terms of who is trying to get numbers to sum to a multiple of *x* and who is trying to prevent this). Each player then cuts another sheet of paper into eight pieces, and eight more rounds take place.
6. The game ends after two sets of eight rounds, when sixteen points have been awarded.

Winning: The winner is the player with the most points.

Skill variation:

Prime numbers: Limit the secret numbers to those between 1 and 40. The goal is to make the sum of the numbers be a prime number.

Coordinate Tic-Tac-Toe

Object: Two players try to get four marks in a row, as in Tic-Tac-Toe. Players cannot touch the game board and must tell their moves to a referee.

Skill: coordinate graphing

Number of players: 3

Grades: 2 to 5

Materials: copies of the Coordinate Tic-Tac-Toe game board, paper, and pencils

Preparation: If desired, decorate and then laminate the game board and have players write on it with grease pencils or crayons (which can be erased with a paper towel after each game); otherwise, players write on the game board with pencils.

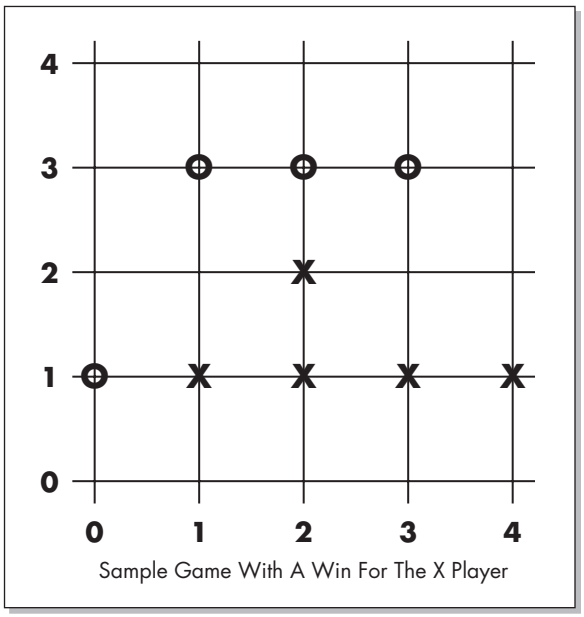
Playing:

1. The game requires two players (or teams, if played in groups) and one referee. One player uses X and the other player uses O. The players cannot touch the game board; they must tell the referee where they want their mark to be placed on the game board by specifying a coordinate for their mark. The referee records a player's coordinates and marks on the game board a corresponding X or O for the player.
2. X's and O's are marked on line intersections of a 5×5 coordinate grid.
3. Players attempt to get four of their X's or O's in a uninterrupted horizontal or vertical straight line (a tic-tac-toe four in a row), while blocking their opponent from doing so.
4. Players take turns. Players cannot write on the game board or point at the coordinate grid on the game board. When it is their turn, players tell the referee the coordinates of the line intersection on the game board where they want their mark (either an X or an O) placed. The coordinates are given in the form (horizontal, vertical) or (across, up). The referee records a player's coordinates in that player's coordinate column on the game board, then verifies with the player that that is where he or she wants his or her mark placed, and then records an X or O in that location. Only the referee can place X's or O's on the game board.
5. If a player specifies an illegal move (off the game board or where another X or O already exists), the referee records it, but then the player loses his or her turn.

Winning: The first player to get four of their marks (X or O) next to each other in an uninterrupted straight horizontal or vertical line wins the game. Draw games in which neither player wins are possible.

Playing variations:

- Allow players to get a tic-tac-toe on a diagonal.
- Play on a larger grid (for example, a 12×12 grid), and players must get five in a row to have a tic-tac-toe. (This is similar to the Japanese game of GoMoku.)



Sample Game With A Win For The X Player

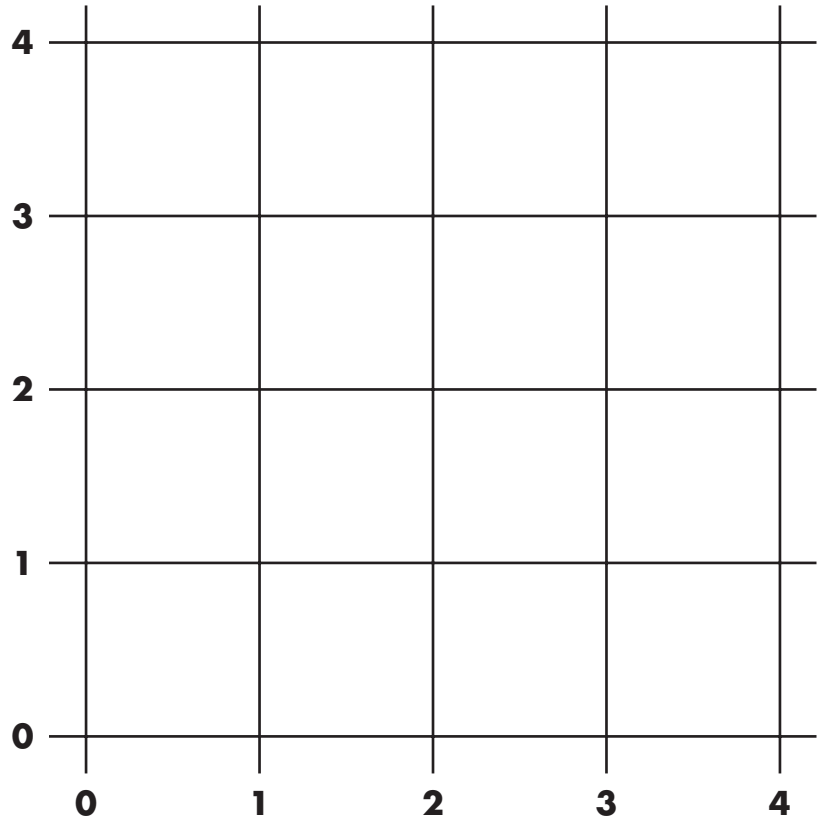
Coordinate Tic-Tac-Toe

Name of X Player:_____

Name of O Player:_____

Name of Referee:_____

X's Moves	O's Moves



Coordinate format: (across, up) or (horizontal, vertical)