1 Why Periodic Structures Cannot Synthesize Negative Indices of Refraction

1.1 INTRODUCTION

1.1.1 Overview

In this chapter we first list some of the features that are widely accepted as being facts regarding metamaterials with simultaneously negative μ and ε:

1. The index of refraction is negative.
2. The phase of a signal advances as it moves away from the source.
3. The evanescent waves increase as they get farther away from the source.
4. Whereas the E- and H-fields in an ordinary material form a right-handed triplet with the direction of phase propagation, in a material with negative μ and ε, they form a left-handed triplet.

Such materials have never been found in nature. However, numerous researchers have suggested ways to produce them artificially. Periodic structures of elements varying from simple straight wires to very elaborate concoctions have been claimed to produce a negative index of refraction. Nevertheless, we show here that according to a well-known theory based on expansion into inhomogeneous plane waves, it does not seem possible to obtain the characteristic features that are listed above for materials with negative μ and ε. Thus, it seems logical to reexamine Veselago's original paper. We find that it is mathematically correct. However, when used in certain practical applications such as the well-known flat lens, it may lead to negative time. Although such a solution might be acceptable mathematically, it would violate the causality principle from a physical

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point of view. So it should not surprise us that, so far, we have encountered difficulties when trying to create materials with negative μ and ε: in particular, a negative index of refraction.

1.1.2 Background

When in 1968 Veselago published his now-famous paper [1], he posed the question: What would happen if a material had both negative permittivity ε and negative permeability μ? Perhaps his most striking conclusion was that a negative sign must be chosen for the index of refraction:

$$n_1 = -\sqrt{\mu\varepsilon} \tag{1.1}$$

This observation led to significant new concepts. We list the most important in Section 1.2. We emphasize that at this point we neither endorse nor condone these new concepts. However, subsequently, in Sections 1.4 to 1.6, we investigate whether it is feasible to synthesize Veselago's material by the use of periodic structures made with special elements. We will find this to be highly unlikely. In view of this, in Section 1.10 we investigate whether Veselago's conclusion violates fundamental physical principles.

Further, in Section 1.9 we examine the dispersion of a cable terminated in a complex load. We show that in that case it is indeed possible to partially eliminate dispersion over a limited frequency band. This is equivalent to the mixture of forward- and backward-traveling waves deemed essential to achieve the special features of Veselago's medium. However, it is erroneous to conclude that a new exotic material has been created. It will simply lose its features if the load impedance is, for example, purely imaginary. More specifically, we have merely used old tricks from broadband matching techniques.

1.2 CURRENT ASSUMPTIONS REGARDING VESELAGO'S MEDIUM

1.2.1 Negative Index of Refraction

In his original paper, Veselago [1] concluded that the index of refraction n_1 between an ordinary medium and one with negative ε and μ would be negative. Thus, as illustrated in Figure 1.1, the refraction angle θ_r would, according to Snell's law, have the same sign as the angle of incidence θ_i when $n_1 > 0$, whereas it would be negative for $n_1 < 0$. Veselago's original proof is discussed in Section 1.10.

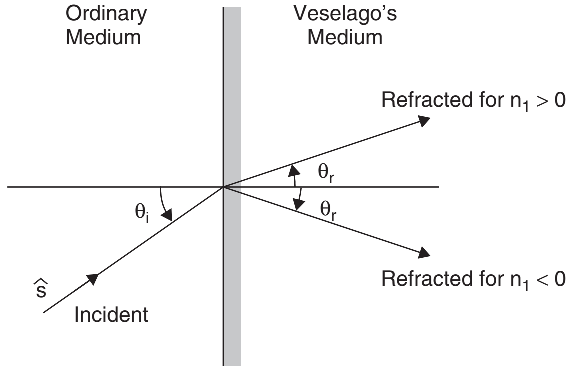


Figure 1.1 Snell's law for an ordinary medium adjacent to Veselago's medium for index of refraction $n_1 > 0$ and $n_1 < 0$, respectively.

1.2.2 Phase Advance when $n_1 < 0$

If a lossless dielectric slab is placed in front of a ground plane, the input impedance Z_i for an ordinary material with $n_1 > 0$ will be obtained by a rotation $2\beta d = 2\beta_0 n_1 d$ in the clockwise direction, as shown in the Smith chart in Figure 1.2. Similarly, if $n_1 < 0$, Z_i is obtained by rotation in a counterclockwise direction. In other words, we experience a phase delay when $n_1 > 0$ and a phase advance when $n_1 < 0$. These statements are based on refs. 2 to 4. Note that loss is not necessary to obtain these features.

1.2.3 Evanescent Waves Grow with Distance for $n_1 < 0$

When propagating waves change into evanescent waves, it is usually because n_1 goes imaginary [5]. Thus, in view of the phase advance postulated above, it should not surprise us that Pendry [6] suggested that evanescent waves in a medium with $n_1 < 0$ would grow and not be attenuated as usual for $n_1 > 0$, as illustrated in Figure 1.3.

1.2.4 The Field and Phase Vectors Form a Left-Handed Triplet for $n_1 < 0$

Also shown by Veselago in his original paper [1] was that the field vectors $\bar{E}$ and $\bar{H}$ and the direction of phase propagation $\hat{s}$ form a left-handed triplet when $n_1 < 0$ (see Figure 1.4b). This feature is probably the least observed when performing experiments. However, as we shall see later, it is a theoretical point very powerful in determining whether or not we have a true Veselago medium.

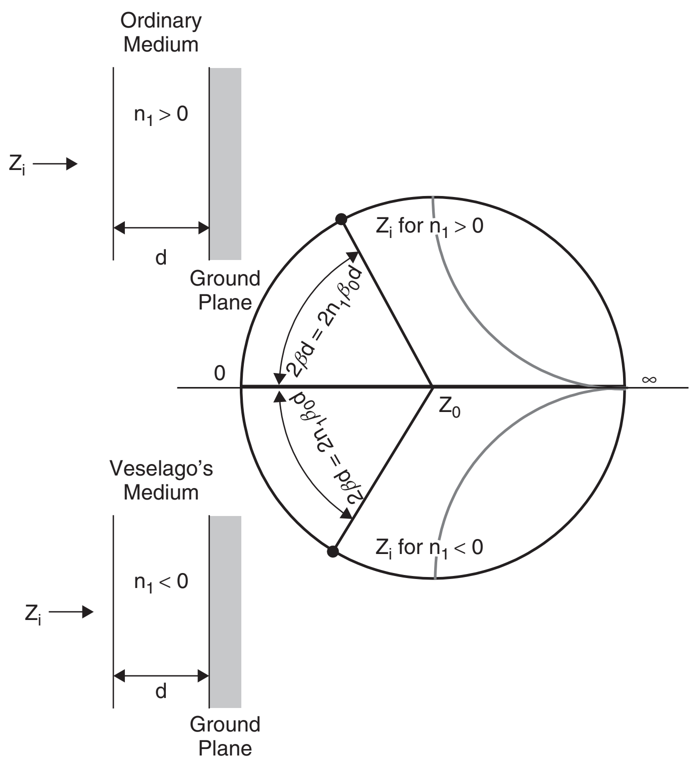


Figure 1.2 Perception of the input impedance Z_i as seen in a Smith chart of a dielectric slab in front of a ground plane for index of refraction $n_1 > 0$ (top) and for $n_1 < 0$ (bottom). For a discussion about causality for $n_1 < 0$, see equations (1.15) to and (1.17).

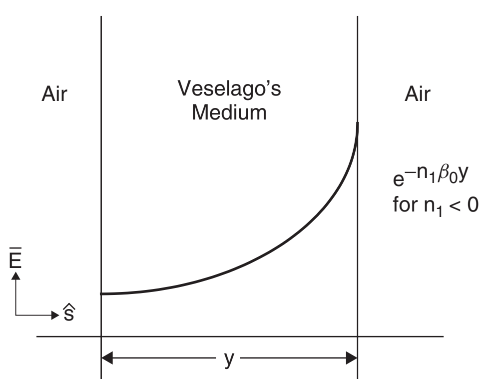


Figure 1.3 Normally, an evanescent wave is attenuated as it moves away from its source. In Veselago's medium it is believed to grow.

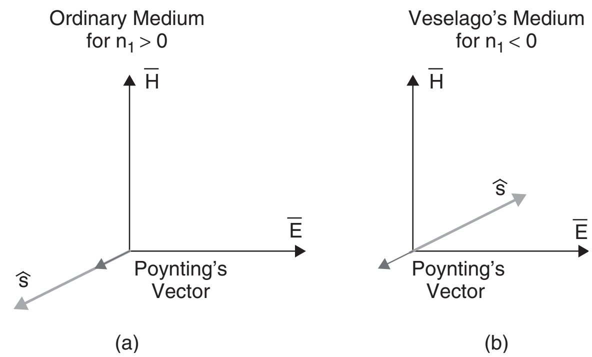


Figure 1.4 (a) In an ordinary medium, $\bar{E}, \bar{H}$, and the direction of propagation $\hat{s}$ form a right-handed triplet; (b) in Veselago's medium, $\bar{E}, \bar{H}$, and the direction of propagation $\hat{s}$ form a left-handed triplet. However, Poynting's vector always points in the same direction.

1.3 FANTASTIC DESIGNS COULD BE REALIZED IF VESELAGO'S MATERIAL EXISTED

When this author started to design high-precision antennas more than 50 years ago, he quickly realized that if the input impedance of a transmission line could go backward in the Smith chart with increasing frequency, matching antennas would, in general, be trivial. He also quickly observed that such components were just not available. However, that was the essence of what the Veselago material promised (if realized). Thus, it is no wonder that an avalanche of papers appeared (mostly simulated), all based on the assumption that Veselago's material was indeed possible to realize.

The most prominent concept was probably the flat lens, discussed in Section 1.10.3. Further, when Pendry later suggested that the evanescent waves at the source would arrive more strongly at the image (see Figure 1.3), the enthusiasm almost boiled over. The possibility of obtaining an optical system that could exceed the traditional diffraction limits was undoubtedly one of the greatest factors that kept funding going for years.

Similarly, Engheta gave a paper in 2001 in Torino [3] in which he considered the resonance frequency of a cavity between two ground planes. He suggested that the space was filled partly with ordinary dielectric with $n_1 > 0$ and the remainder with material with $n_1 < 0$. It was also stated that consultation of Figure 1.2 would readily show that the resonance frequency could potentially remain constant from dc to broad daylight! (The two ground-plane impedances could essentially cancel each other, regardless of frequency.) When the present author pointed out from the floor at the meeting that the rotation in the Smith chart for $n_1 < 0$ violated Foster's reactance theorem, it by no means settled the issue. In fact, it resulted in another showing that for $n_1 < 0$, a modified Foster's reactance theorem would indeed indicate counterclockwise rotation in accordance with Figure 1.2 [4]. The question was, and is, of course: Is there a material with $n_1 < 0$? Veselago himself was quick to point out that his material had never been found in nature. And he added, prudently, that there were perhaps profound reasons for its absence.

1.4 HOW VESELAGO'S MEDIUM IS ENVISIONED TO BE SYNTHESIZED USING PERIODIC STRUCTURES

For almost 30 years after Veselago published his original paper, there was little evidence of any particular interest in his material. However, in the mid-1990s, Pendry postulated that a negative ε could be produced by a periodic structure of strips, as shown in Figure 1.5a. Actually, such a surface is usually found to be inductive [5, Chap. 1]. However, an inductor

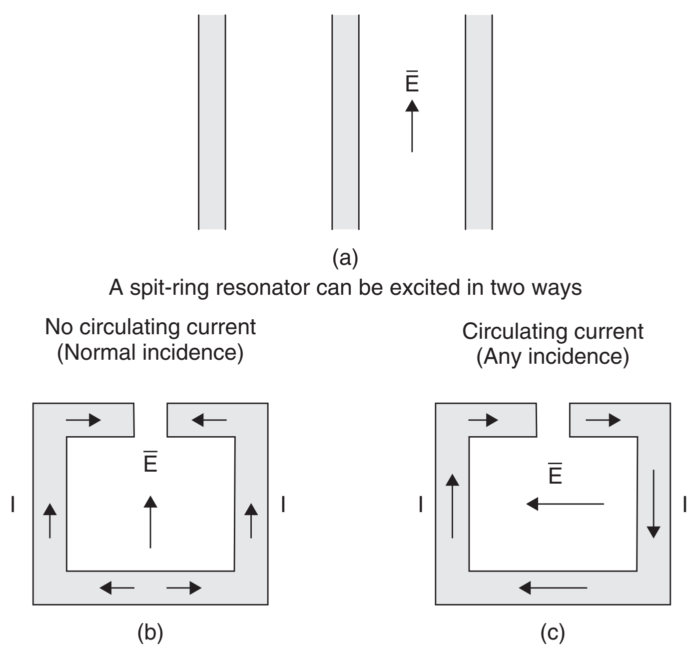


Figure 1.5 (a) Pendry suggested that a negative ε could be produced by an array of parallel wires; (b) and (c) similarly, a negative μ is expected from an array of loops with circulating currents.

can also be considered to be a negative capacitance, which again indicates the presence of a negative ε. Later, Pendry suggested that a negative μ could be obtained from a periodic structure of open split-ring resonators, as shown in Figure 1.5b and c. The idea here was that a circulating current was able to produce a negative μ [6–9]. However, we should note that the current induced is highly dependent on the orientation of the incident E-field. In the case shown in Figure 1.5b, the incident E-field is vertical, which for normal incidence will produce only push–push currents, as indicated in the figure, whereas for oblique incidence in the horizontal plane a weak circulating current will be present in addition to strong push–push currents. However, when the incident E-field is horizontal, as shown in Figure 1.5c, we will observe a circulating current for any angle of incidence unless $\bar{E}$ is perpendicular to the plane of the loop.

It was not long after Pendry's postulates that a group of physicists at the University of San Diego made a combination of flat wires and split-ring resonators, as shown in Figure 1.6 [10–13]. They then performed measurements on a wedge-shaped body as shown in Figure 1.7b. The idea was, as illustrated, that the refracted field would depend strongly on the sign of the refractive index, n_1. In fact, they measured the refracted

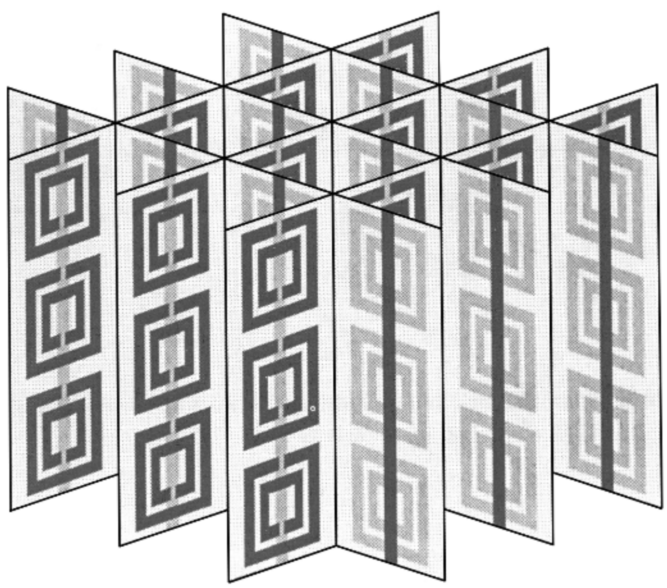


Figure 1.6 Original periodic structure used by the San Diego group to demonstrate the presence of negative refraction.

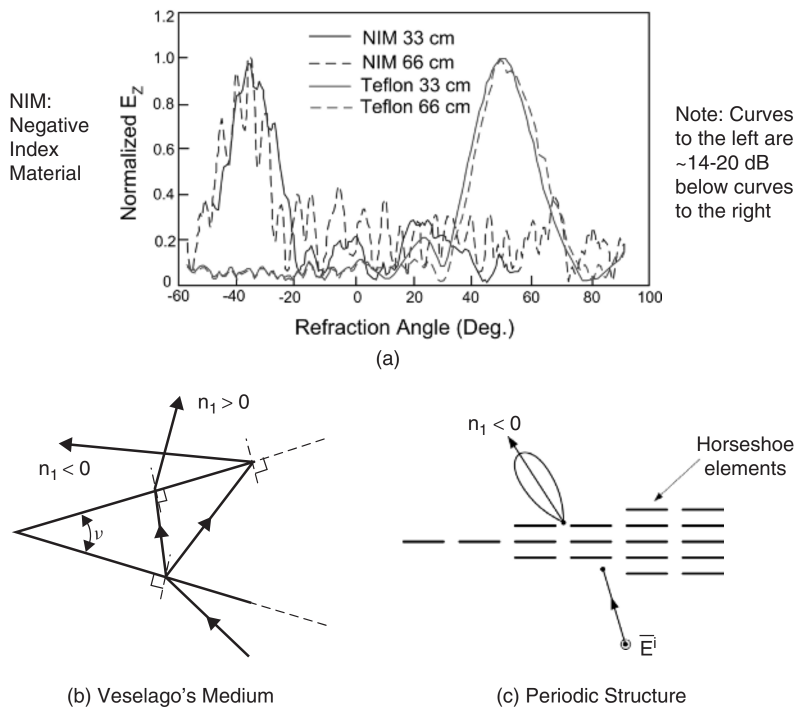


Figure 1.7 (a) Curves to the right represent the field refracted through a Teflon wedge as shown in part (b). Curves to the left are perceived as being the field refracted through a wedge made of wires and a split-ring resonator, as shown in part (c). Note that they are actually about 20 dB below (ca. 1% power) the curves to the right even if they are all shown normalized to 100%. (After ref. 14, with permission.)

field for a Teflon wedge ($\varepsilon \sim 2.1$) and obtained the refraction curve to the right in Figure 1.7a. They also measured the refracted field from a wedge-shaped assembly of wires and split-ring resonators, as shown in Figure 1.7c. Actually, the (measured) curves to the left in Figure 1.7a were not measured by the San Diego group but were obtained later by a group working at Boeing's "Phantom Works" [14]. They went to great lengths to obtain the exact refraction in both the far field [NIM (negative index material) 66 cm] and the near field (NIM 33 cm). Note how the sidelobes in the far-field pattern are almost gone for the near-field case, as is typically seen in antenna experiments. However, the most interesting feature is probably the fact that the refracted field for the synthesized material is about 14 to 20 dB or more below the refracted field for the Teflon case. Such a large loss cannot be attributed to either ohmic or dielectric loss for frequencies below 100 GHz. This fact and the presence of sidelobes in the far field suggested to this author that the refracted field for the synthesized material was actually not a refracted field but merely the radiation pattern for a surface wave that can exist only on a finite periodic structure. Such surface waves have not been demonstrated for split-ring resonators per se. However, they are well documented for simple straight wires (dipoles) [15–21]. In fact, a typical surface wave is shown in Figure 1.8b, where 50 dipoles, each of total length about 0.35λ, are exposed to an incident plane wave at 45°, as shown in the insert [20,21]. First, we note that the level of the surface-wave radiation is about the same as that of the blue refracted curve in Figure 1.8a (ca. 20 dB down). Next we note that the decay rate for the sidelobes is about the same for the two patterns. It should be emphasized further that the orientation of the E-field is as indicated in Figure 1.5b (see Figure 1.6). Thus, there were only very weak circulating currents such that μ would be weak according to Pendry. Nevertheless, negative refraction, although very weak ($<$14 to 20 dB below a Teflon wedge), was still claimed.

Finally, there are numerous papers in which negative refraction has been claimed for basically straight loaded or unloaded elements with no circulating currents, typical examples being shown in Figure 1.9 [22,23]. Note, in particular, Figure 1.9c, where the elements have been printed on each side of a thin substrate and the elements flipped to avoid any possible chiral or loop effect. All of these elements claim to have measured negative index of refraction, although with more than 20 dB loss.

The discussion above does not constitute a proof of whether we actually observe negative refraction or witness another phenomenon. We have suggested here that it is quite likely the radiation from a surface wave that typically exists over about 10% bandwidth. However, it could also simply be part of the sidelobes from the main beam of the field transmitted. Anyone with experience in measuring the fields scattered from a periodic structure will know how difficult such measurements are: in particular, if we are down 20 dB or more. In the next section we show that this phenomenon is almost certainly not due to refraction.

1.5 HOW DOES A PERIODIC STRUCTURE REFRACT?

1.5.1 Infinite Arrays

In this section some simple and well-known facts about periodic structures are pointed out. Unfortunately, they are too often overlooked, forgotten, or simply ignored! Consider an infinite × infinite array, as shown in

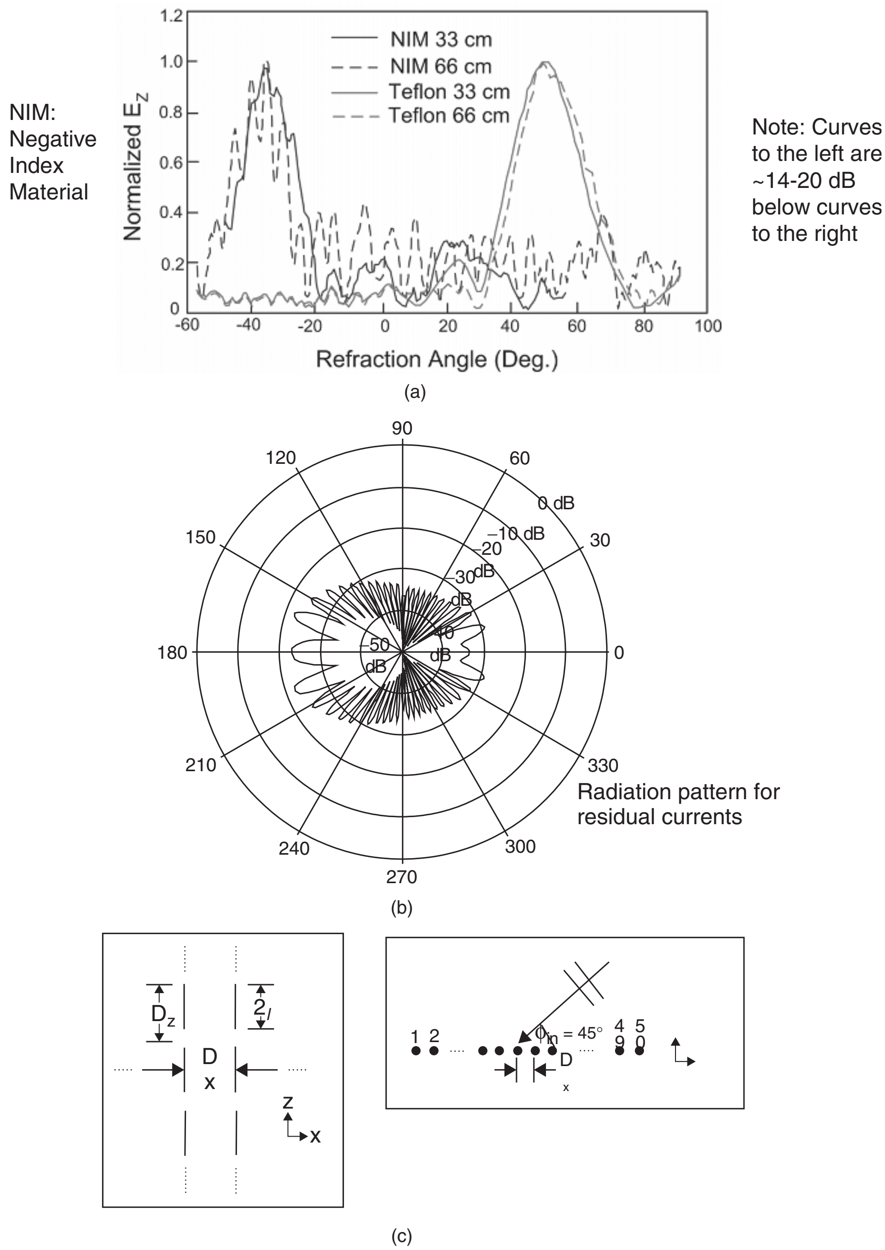


Figure 1.8 (a) Same curves as shown in Figure 1.7. However, note how the sidelobes of the curves to the left are similar to the sidelobes of surface-wave radiation shown in part (b). Further, the radiation intensity is about the same (ca. 20 dB below maximum).

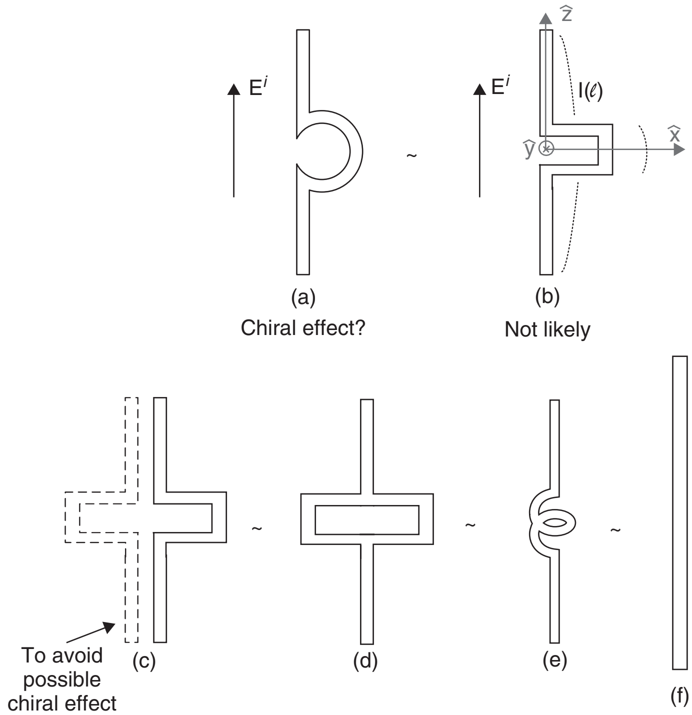


Figure 1.9 Some of many elements with and without circulating current where a negative index of refraction has been claimed, always 20 dB or more below reference.

Figure 1.10. It is being exposed to an incident plane wave with direction of propagation $\hat{s}$. For the sake of simplicity we will for the time being assume $\hat{s}$ to be contained in the yz-plane. For $\hat{s}$ pointing upward to the right as shown in the figure, it is clear by implication of Floquet's theorem that the voltages induced in row 1 will be delayed by $\beta D_z s_z$ compared to row 0. However, the fields re-radiated from row 1 will be ahead by the same amount, $\beta D_z s_z$, for waves propagating in the forward direction $\hat{s}$ as well as in the specular direction $\hat{s}_s = \hat{x}s_x - \hat{y}s_y + \hat{z}s_z$, as illustrated in Figure 1.10a and b, respectively. In other words, propagation in these two directions is always possible unless the element pattern has a null in any of these directions.

We now ask: Is it possible to reradiate a plane wave in an arbitrary direction $\hat{s}_a$? If so, the elements in row 1 will have a phase advance of $\beta D_z s_{az}$. Only if the sum of the delay and advance adds up to a multiple of 2π can a plane wave propagate in the direction $\hat{s}_a$. (Remember: Our array is infinite × infinite, not finite; see later.) Thus, the condition for

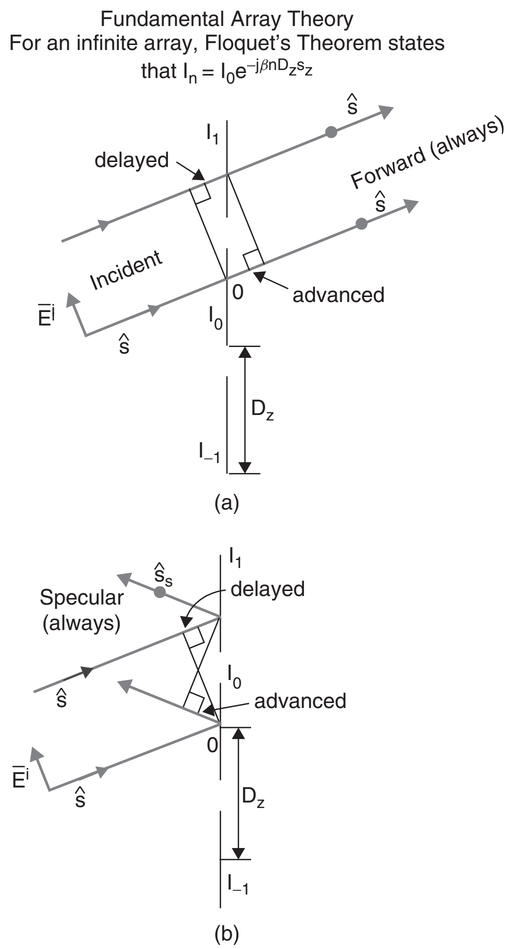


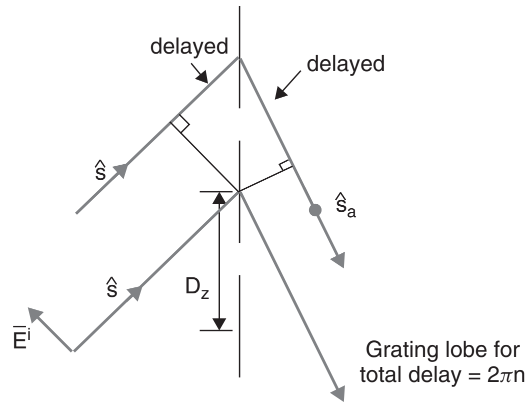
Figure 1.10 An incident plane wave with direction of propagation $\hat{s}$ will induce a voltage in element 1 that is delayed by $\beta D_z s_z$ compared to element 0. Conversely, the re-radiated field from element 1 will be advanced by $\beta D_z s_z$ compared to element 0 in the forward direction $\hat{s}$ (top) as well as in the specular direction $\hat{s}_s$ (bottom). Thus, propagation in the forward and specular directions is always possible. (Note the infinite arrays.)

reradiation in the arbitrary direction $\hat{s}_a$ is (see Figure 1.11)

$$\beta D_x(s_z - s_{az}) = 2\pi n_1 \qquad n_1 = 0, \pm 1, \pm 2, \ldots$$

or recalling that $\beta = 2\pi/\lambda$,

$$\frac{D_z}{\lambda} = \frac{n_1}{s_z - s_{az}} \qquad n_1 = 0, \pm 1, \pm 2, \ldots$$



Grating lobes possible only for $D_z > \lambda/2$.
Thus, NO negative refraction for "continuous"
medium (i.e., $D_z < 0.4\lambda$)

Note: Floquet's theorem is valid for ALL
element types (they merely change the
element pattern). This is at variance with the
theory for an artificial dielectric.

Figure 1.11 In contrast to the case in Figure 1.10, propagation in an arbitrary direction $\hat{s}_a$ is possible only if the total phase delay is $2\pi n$. These are simple grating lobe directions. (Note the infinite arrays.)

which shows that it can always be satisfied provided that we make D_z/λ sufficiently large. However, the smallest value of D_z/λ is obtained for $n_1 = +1$ and $s_z = 1$ (grazing incidence upward) and $s_{az} = -1$ (grazing re-radiation downward). In that case,

$$\frac{D_z}{\lambda} = \frac{1}{2}$$

In other words, for $D_z/\lambda < \frac{1}{2}$, re-radiation is possible *only* in the forward direction $\hat{s}$ and the specular direction s_s (infinite array only). For $D_z/\lambda > \frac{1}{2}$, propagation in other directions is possible (see Figure 1.11). In fact, these are simply the well-known grating lobe directions.

Note that the phase velocity along the z-direction is opposite for the incident and the lowest grating lobe direction. For that reason this grating lobe has sometimes mistakenly been denoted as a "backward"-traveling wave. These grating lobes are encountered in numerous microwave devices, such as the backward-traveling oscillator and the backward-traveling antenna, as well as in photonic bandgap materials. They have been known for a long time and are well understood. Again, we emphasize that these backward-traveling waves can exist only when the interelement spacings D_x and D_z exceed λ/2, and not for D_x, $D_z < \sim 0.4\lambda$.

The term *backward-traveling wave* was later suggested to mean a wave where the phase and group velocity were opposite each other [28–31]. It was thought that Veselago associated such waves with his findings for media with negative μ and ε when he concluded that the phase velocity and the Poynting vector were opposite each other. However, this writer is not aware that he ever used the term *backward-traveling wave*. Note: The grating lobes have identical phase and group velocity in a dispersionless medium. There is nothing "backward" about them.

We should note further that in the world of metamaterials, the interest seems to concentrate on two types of materials:

1. The interelement spacings D_x and D_z are somewhat smaller than λ/2, typically λ/4 or smaller. These cases are often denoted as "continuous," and the phase difference between adjacent elements is typically ignored (an approximation not allowed in the rigorous theory of periodic structures).
2. The interelement spacings are somewhat larger than λ/2, typically 0.7 to 1.5λ. These materials fall into a category usually called *photonic bandgaps* or *crystals*. They are often perceived as being able to propagate "backward"-traveling waves. Actually, these are nothing but grating lobes, as discussed above.

In other words, *the direction of refraction in air is determined solely by the interelement spacings D_x and D_z as well as the direction $\hat{s}$ of the incident field, never by the element type*.

These will determine where the structure resonates, the bandwidth, and to some extent, variation with angle of incidence as well as the amplitude in general of the scattered fields. Which leaves us with the following conclusion: The extensive discussion of whether surfaces with elements such as the split-ring resonator are blessed with negative refraction and others are not is somewhat misguided. In fact, a periodic structure (of infinite extent) of the continuous type D_x, $D_z < \lambda/2$ and no dielectric can only produce a refracted field with refraction at $n = +1$! You may forget entirely about negative refraction!

1.5.2 What About Finite Arrays?

The categorical denial above that any refraction other than the forward is specular if D_x, $D_z < \lambda/2$ is, rigorously speaking, true only for an infinite structure. In reality, all structures are, of course, finite. This fact will have certain consequences. Foremost, the signals in the forward and specular directions will occur in the form of main beams in each of these directions. They will be flanked by numerous sidelobes. Exact calculated examples of finite × infinite arrays is given in refs. 15–21.

Further, a finite periodic structure is able to sustain certain types of surface waves not possible when the same structure is of infinite extent. Note: It is of utmost importance that the interelement spacing be less than $\lambda/2$ (i.e., the structure is of the "continuous" type). In that event we find that currents associated with the surface wave can be much stronger than currents associated with the mainbeams described above, typically over about 10% bandwidth and when the total element length is about 0.35λ for a simple dipole element [20; Figure 10 in Appendix A]. The surface wave current will, of course, re-radiate like any other element current. The good news is that the surface wave has a low radiation efficiency such that the reradiated field is typically about 14 to 20 dB or more below the amplitude of the mainbeam despite the higher current amplitudes. In addition to the pure surface waves, some currents will usually be associated with reflections from the edges. However, these radiations are usually small compared to those of pure surface waves (for details, see ref. 20).

The sum of the surface wave and the end currents are often referred to as *residual currents*. The re-radiation from these is shown by the radiation pattern in the middle of Figure 1.8b. Note that it has both the same level as the blue curves (about 20 dB below the Teflon wedge) and similar sidelobes: in short, a strong indication that we are seeing radiation from a surface wave and not a simple refraction. (In that case there would be *no* sidelobes!)

The discussion above emphasized the physical aspect of refraction. However, for those who prefer a more mathematical approach, in the next section we present the highlight of the plane-wave expansion [5]. This will demonstrate essentially two features:

1. The re-radiated field from a periodic structure is always right-handed, regardless of element shape or type.
2. The field both inside and outside a multilayered periodic medium is always right-handed.

1.6 ON THE FIELD SURROUNDING AN INFINITE PERIODIC STRUCTURE OF ARBITRARY WIRE ELEMENTS LOCATED IN ONE OR MORE ARRAYS

1.6.1 Single Array of Elements with One Segment

Consider a single planar array as shown in Figure 1.12. The elements are oriented along $\hat{p}^{1,1}$, where $\hat{p}^{1,1}$ is arbitrary except that it is contained in the plane of the array.* Further, we denote the infinitesimal element length by $dl^{1,1}$, the current by $\mathbf{I}^{1,1}$, and the reference point of the reference element by $\bar{R}^{1,1}$. This array, with interelement spacings D_x and D_z, is exposed to an incident plane wave with direction of propagation

$$\hat{s} = \hat{x}s_x + \hat{y}s_y + \hat{z}s_z \tag{1.2}$$

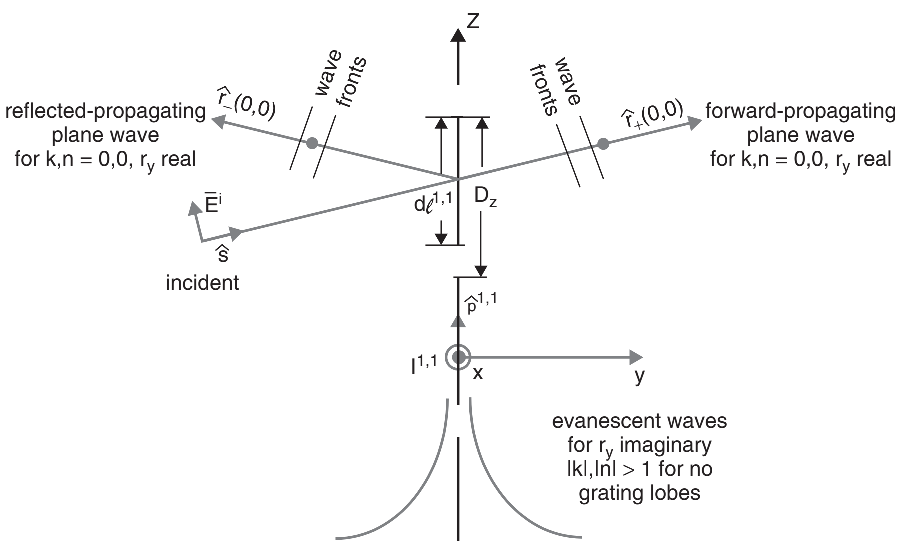


Figure 1.12 Plane wave with direction of propagation $\hat{s}$ incident upon an infinite array of single-segment elements with orientation $\hat{p}^{1,1}$, length $dl^{1,1}$, current $I^{1,1}$, and reference point $\bar{R}^{1,1}$. A plane wave will be scattered in the forward direction, $\hat{r}_+(0,0) = \hat{s}$, as well as the specular direction, $\hat{r}_-$ (0,0). Note: The total field in the forward direction is the sum of the incident and scattered fields. Further, there will be an infinite sum of evanescent (exponentially decreasing) waves. They make up the near field associated with the array.

*In the following, the first superscript refers to the array number, the second to the element section.

Denoting the element current in column q and row m by $I_{q,m}^{1,1}$, it follows from Floquet's theorem [5] that the element currents are given by

$$I_{q,m}^{1,1} = I_{0,0}^{1,1} e^{-j\beta q D_x s_x} e^{-j\beta m D_z s_z} \tag{1.3}$$

(i.e., they all have the same amplitude, and a phase which matches that of the incident plane wave with direction of propagation $\hat{s}$).

It has been shown rigorously that the electromagnetic fields from an infinite array are given by a spectrum $\hat{r}_\pm$ of inhomogeneous plane waves [5,24–27]:

$$d\bar{H}^{1,1} = I_{0,0}^{1,1} dl^{1,1} \frac{1}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{-j\beta(\bar{R}-\bar{R}^{1,1})\cdot\hat{\boldsymbol{r}}_\pm}}{r_y} [\hat{p}^{1,1} \times \hat{r}_\pm] \quad \text{for } y \gtrless 0 \tag{1.4}$$

$$d\bar{E}^{1,1} = I_{0,0}^{1,1} dl^{1,1} \frac{Z}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{-j\beta(\bar{R}-\bar{R}^{1,1})\cdot\hat{\boldsymbol{r}}_\pm}}{r_y} [\hat{p}^{1,1} \times \hat{r}_\pm] \times \hat{r}_\pm \quad \text{for } y \gtrless 0 \tag{1.5}$$

The spectrum $\hat{r}_\pm$ denotes the directions of the inhomogeneous plane waves emanating from the array. They are found to be [5]

$$\begin{aligned}\hat{r}_\pm &= \hat{x} r_x + \hat{y} r_y + \hat{z} r_z \\ &= \hat{x}\left(s_x + k\frac{\lambda}{D_x}\right) \pm \hat{y} r_y + \hat{z}\left(s_z + n\frac{\lambda}{D_z}\right) \qquad \text{for } y \gtrless 0\end{aligned} \tag{1.6}$$

where

$$r_y = \sqrt{1 - (s_x + k\frac{\lambda}{D_x})^2 - (s_z + n\frac{\lambda}{D_z})^2} \tag{1.7}$$

The fields expressed by equations (1.4) and (1.5) depend on r_y as follows: For the principal direction k, $n = 0{,}0$, we see from (1.7) that r_y is always real since $|s_x|,|s_z| \leq 1$ [see (1.2)]. This corresponds to a plane wave $\hat{r}_+(0, 0)$, transmitted in the forward direction $\hat{s}$ and another reflected in the specular direction $\hat{s}_s = \hat{r}_-(0, 0) = \hat{x}s_x - \hat{y}s_y + \hat{z}s_z$, as illustrated in Figure 1.10. For $|k|,|n| > 0,\ 0$, r_y may still be real provided that the interelement spacings D_x and D_z are large enough. These directions are termed *grating lobe directions*. They are discussed in Section 1.6.4 (see also Section 1.5).

However, for higher values of k and n, r_y will always be imaginary; that is, the exponent in the plane waves $e^{-j\beta(\bar{R}-\bar{R}^{1,1})\cdot\hat{\boldsymbol{r}}_\pm}$ will be real, depicting evanescent waves that go to zero as the point of observation $\bar{R}$ moves away from the array, as illustrated in Figure 1.12. (Formally, it is, of course, possible to choose the sign for r_y such that the evanescent field components would increase exponentially to infinity as we move away from the array. However, such a solution is obviously invalid since it violates fundamental physical laws.) The sum of these evanescent waves constitutes the near field surrounding the elements.

Note: Our array is located in an ordinary dispersionless media and not in Veselago's medium. Also, the field vectors $d\bar{E}^{1,1}$ and $d\bar{H}^{1,1}$are oriented along $[\hat{p}^{1,1} \times \hat{r}_\pm] \times \hat{r}_\pm$ and $[\hat{p}^{1,1} \times \hat{r}_\pm]$, respectively (i.e., $d\bar{E}^{1,1}$ and $d\bar{H}^{1,1}$ and the propagation $\hat{r}_\pm$ form a right-handed triplet). It is relatively simple to show that it holds as well when r_y becomes imaginary (i.e., for the evanescent waves).

Also, if the array is located in a dispersionless medium, Poynting's vector will coincide with the directions of propagation $\hat{r}_\pm$ as given by equations (1.6) and (1.7). Thus, the spectrum of plane waves radiated from this simple periodic structure will definitely be right-handed and never left-handed as is the case for "Veselago's medium."

1.6.2 Single Array of Elements with Two Segments

Next, we again consider a single array, but this time with elements made of two segments with arbitrary orientation $\hat{p}^{1,1}$ and $\hat{p}^{1,2}$, elements length $dl^{1,1}$ and $dl^{1,2}$, currents $I^{1,1}$ and $I^{1,2}$, and reference points $\bar{R}^{1,1}$ and $\bar{R}^{1,2}$, respectively. Obviously, the array with element orientation $\hat{p}^{1,2}$ has the same interelement spacings D_x and D_z as the first [i.e., the two arrays have the same spectrum $\hat{r}_\pm$; see equations (1.6) and (1.7)]. Thus, the fields from the array with orientation $\hat{p}^{1,2}$ are

$$d\bar{H}^{1,2} = I_{0,0}^{1,2} dl^{1,2} \frac{1}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{-j\beta(\bar{R}-\bar{R}^{1,2})\cdot\hat{\boldsymbol{r}}_\pm}}{r_y} (\hat{p}^{1,2} \times \hat{r}_\pm) \quad \text{for } y \gtrless 0 \tag{1.8}$$

$$d\bar{E}^{1,2} = I_{0,0}^{1,2} dl^{1,2} \frac{Z}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{-j\beta(\bar{R}-\bar{R}^{1,2})\cdot\hat{\boldsymbol{r}}_\pm}}{r_y} (\hat{p}^{1,2} \times \hat{r}_\pm) \times \hat{r}_\pm \quad \text{for } y \gtrless 0 \tag{1.9}$$

The total H-field from the combined array is obtained by addition of equations (1.4) and (1.8):

$$d\bar{H} = d\bar{H}^{1,1} + d\bar{H}^{1,2}$$
$$= \frac{1}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{-j\beta\bar{R}\cdot\hat{\boldsymbol{r}}_\pm}}{r_y} (\hat{p}^{1,1} dl^{1,1} I^{1,1} e^{j\beta\bar{R}^{1,1}\cdot\hat{\boldsymbol{r}}_\pm}$$
$$+ \hat{p}^{1,2} dl^{1,2} I^{1,2} e^{j\beta\bar{R}^{1,2}\cdot\hat{\boldsymbol{r}}_\pm}) \times \hat{r}_\pm \qquad \text{for } y \gtrless 0 \qquad (1.10)$$

Similarly, the total E-field from the combined array is obtained by addition of equations (1.5) and (1.9):

$$d\bar{E} = d\bar{E}^{1,1} + d\bar{E}^{1,2}$$
$$= \frac{Z}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{-j\beta\bar{R}\cdot\hat{\boldsymbol{r}}_\pm}}{r_y} [(\hat{p}^{1,1} dl^{1,1} I^{1,1} e^{j\beta\bar{R}^{1,1}\cdot\hat{\boldsymbol{r}}_\pm}$$
$$+ \hat{p}^{1,2} dl^{1,2} I^{1,2} e^{j\beta\bar{R}^{1,2}\cdot\hat{\boldsymbol{r}}_\pm}) \times \hat{r}_\pm] \times \hat{r}_\pm \qquad \text{for } y \gtrless 0 \quad (1.11)$$

Inspection of equations (1.10) and (1.11) shows readily that a single array with elements comprised of two segments will have a field where $d\bar{E}$, $d\bar{H}$, and $\hat{r}_\pm$ form a right-handed system.

Note: There will, in general, be strong coupling between the two segmented arrays such that $I^{1,1}$ and $I^{1,2}$ may differ significantly from the single-segment cases. This coupling is incorporated in our theory and the PMM program* such that the array currents are always calculated correctly.

1.6.3 Single Array of Elements with an Arbitrary Number of Segments

Extension from two to an arbitrary number of element segments is done simply by induction. Again, we conclude that only right-handed waves will emanate from a single array, regardless of the shape of the elements.

*PMM stands for *periodic method of moments*. It is available from the U.S. Air Force. It was written by Lee Henderson as part of his dissertation at the Ohio State University. It is considered one of the fastest and most reliable programs available.

1.6.4 On Grating Lobes and Backward-Traveling Waves

When r_y is real, we experience propagating plane waves. We saw earlier that we always have two propagating waves for $k, n = 0{,}0$, corresponding to the forward and reflected waves shown in Figure 1.12 (these are also called the *principal waves*). However, as seen by inspection of equation (1.7), we may also obtain propagating waves for a limited number of values of k, n, depending on the interelement spacings D_x and D_z as well as s_x and s_z. The lowest-order grating lobe is obtained for either $s_z = 0$ with $k, n = -1, 0$ or $s_x = 0$ with $k, n = 0, -1$. The latter case is illustrated in Figure 1.13. Note that the component of the phase velocity along the z-direction is opposite for the incident and lowest grating lobe directions. For that reason this grating lobe has sometimes mistakenly been denoted as a *backward-traveling wave*. These grating lobes are encountered in numerous microwaves devices, such as the backward-traveling oscillator, the backward-traveling antenna, and photonic bandgap materials. They have long been known and are well understood. Again, we emphasize that these backward-traveling waves can exist only when the interelement spacings D_x and D_z exceed $\lambda/2$.

The term *backward-traveling wave* was later suggested to mean a wave where the phase and group velocities were opposite each other [28–31].

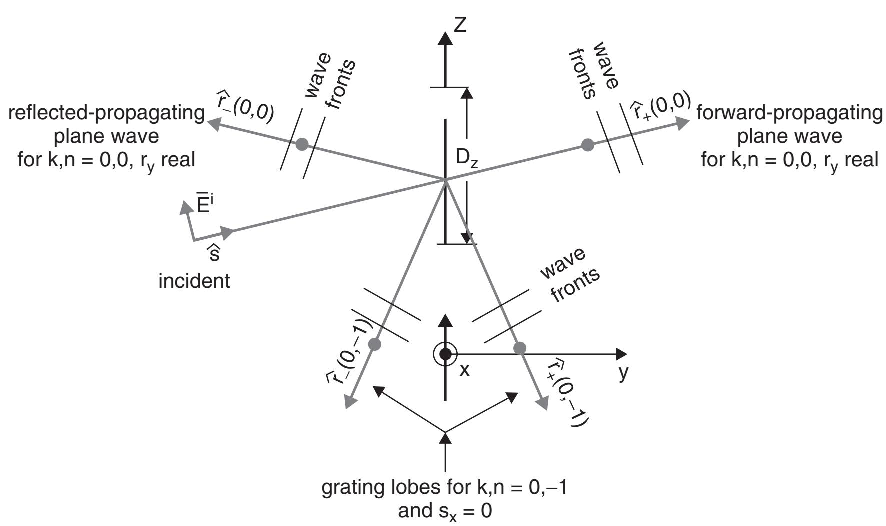


Figure 1.13 Plane wave incident upon an infinite array may also, in addition to the forward and specular reflected waves, produce plane waves in the grating lobe direction $\hat{r}_+(0, -1)$ and $\hat{r}_+(0, -1)$ if the interelement spacings $D_z > \lambda/2$ and $s_x = 0$. Note: All waves are right-handed.

It was thought that Veselago associated such waves with his findings for media with negative μ and ε when he concluded that the phase velocity and the Poynting vector were opposite each other. However, this writer is not aware that he ever used the term *backward-traveling wave*. It should be emphasized that Veselago and his followers, in general, consider media with interelement spacings of less than $\lambda/4$ (denoted *continuous*); that is, we are definitely not talking about grating lobes here. Furthermore, these newer backward-traveling waves can only exist in a highly dispersive medium. It is claimed that it is possible to construct these artificially by periodic loading of a transmission line [28–31] (see also Section 1.9). This writer is not aware that the equivalent was ever done in free space. At any rate, group velocity and phase velocity are the same for free space and a dispersionless medium. In other words, there is absolutely nothing "backward" about any of the plane waves emanating from a periodic structure in Figure 1.13 as long as it is placed in a medium without dispersion. The Poynting vector for all these plane waves points in the direction of propagation $\hat{r}_\pm$ regardless of the number of element segments or element shapes.

1.6.5 Two Arrays of Elements with an Arbitrary Number of Segments

So far we have considered only a single array with an arbitrary number of element segments. We found that the field emanating from such an infinite array consisted of a spectrum $\hat{r}_\pm$ of inhomogeneous plane waves, as given by equations (1.6) and (1.7):

1. A propagating wave in the forward and specular directions corresponding to k, $n = 0{,}0$ (also called the *principal directions*)
2. A finite number of grating lobes if the interelement spacings D_x and D_z are large enough, corresponding to a finite number of k, $n \neq 0{,}0$
3. An infinite number of evanescent waves that go to zero as we move away from the array

As shown earlier, all of these waves are right-handed. We now place another array a certain distance d_1 to the right of the first array, as illustrated in Figure 1.14. The interelement spacings D_x and D_z are the same as for array 1, but the number of element segments is arbitrary. Thus, the spectrum $\hat{r}_\pm$ is the same for the two arrays.

We now calculate the currents in all the element segments. Just as the coupling between the segments in one array can be significant, as noted above, it will also be significant between the segments in the two

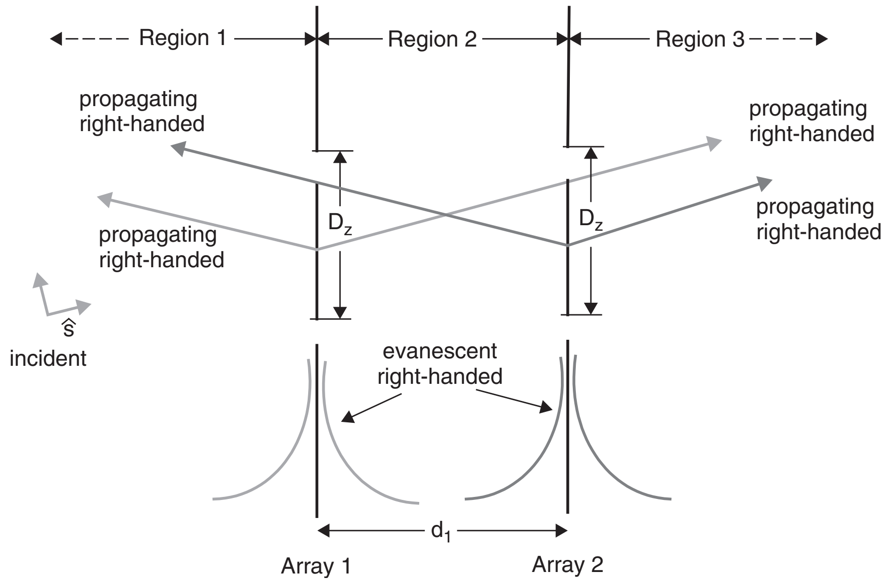


Figure 1.14 Plane wave with direction of propagation $\hat{s}$ incident upon two arrays with interelement spacings D_x and D_z. Each array emanates plane propagating waves in the forward as well as the specular directions; they are all right-handed and so are their sums, regardless of region. Further, there will be an infinite sum of evanescent (exponentially decreasing) waves that represent the near field associated with both arrays. Note: The arrays are located in a medium without dispersion.

arrays. We emphasize that this coupling is always taken rigorously into account both in the theory treated in refs. 5,24, and 25 and in the PMM program [26,27]. Once we find all the segment currents in both arrays in each other's presence, the determination of the fields emanating from each array is done precisely as was done for the single-array case treated earlier and as shown in Figure 1.14. We define three regions:

- Region 1 is the semi-infinite space to the left of array 1.
- Region 2 is the space between arrays 1 and 2.
- Region 3 is the semi-infinite space to the right of array 2.

In region 1 we observe left-going propagating waves radiating from the two arrays; similarly, we have right-going waves in region 3; and we have both left- and right-going waves in region 2, as shown. All of these waves are right-handed. The total field is obtained simply by superposition of the fields from the two arrays. There can be no doubt that in regions 1 and 3 the total field will be right-handed. Further, in region 2 we simply obtain a total field of two right-handed waves crossing each other. Neither one of these waves can ever turn into left-handed waves, since that would require the presence of Veselago's magic material, which everyone agrees does not exist in nature. Remember that our support medium is assumed to have no dispersion (i.e., linear).

1.6.6 Can Arrays of Wires Ever Change the Direction of the Incident Field?

Even for a multilayer, infinite array with identical interelement spacings of less than $\lambda/2$, the element currents in each array will always follow Floquet's theorem [see (1.3)]. As shown earlier, this can only lead to a plane-wave spectrum with directions $\hat{r}_{\pm}$ (i.e., never "bend" the incident field unless the waves are somehow slowed down). Molecular "dipoles" are a different matter (see also Sections 1.12.1 and 1.12.2 regarding artificial dielectrics.

1.7 ON INCREASING EVANESCENT WAVES: A FATAL MISCONCEPTION

The total evanescent field in Figure 1.14 is obtained by superposition of the evanescent waves from each array. However, these will, in general, not be in phase, and thus the total field cannot be obtained by simple addition of the magnitudes from the individual arrays. In fact, they could be out of phase and actually produce a null somewhere between the two arrays. Whatever the case, it is obvious from inspection of Figure 1.14 that the total field can increase only when the point of observation moves close to the elements, not all of a sudden because we are in a "Veselago medium." We are still in a medium without dispersion, and straightforward rules prevail. This writer is not aware of any demonstration of increasing evanescent waves except on capacitively loaded transmission-line models terminated in a resistive load [28,29].

It is, of course, quite possible to have a multiarray configuration as shown in Figure 1.15 or a transmission line where the last array has a much stronger current than that of the other arrays. (This situation could easily be obtained by loading the arrays in front of the last array either resistively or reactively.) Obviously, the total field will be dominated by the field from the last array, and this situation could be misinterpreted as an "evanescent" wave that "grows" as it moves through some "magic" material. Remember, you are in ordinary air between the elements where the classical laws of electromagnetics prevail.

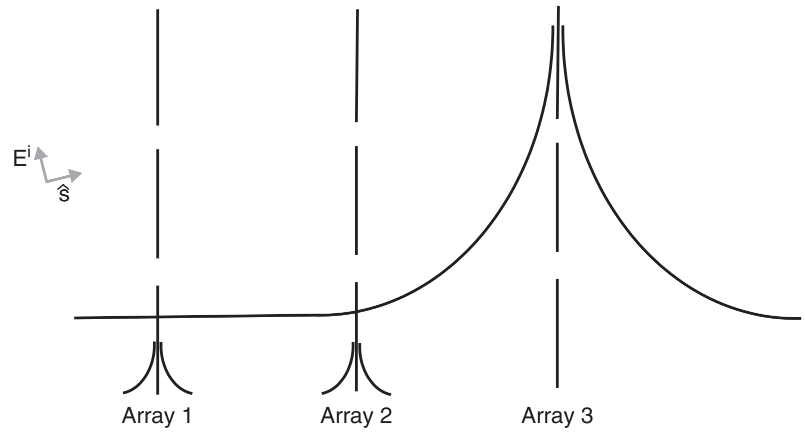


Figure 1.15 Several arrays where the one to the right is designed to have a much stronger element current than the others. This will produce a dominating evanescent field that often is misinterpreted as "proof" that the evanescent wave(s) can increase as you move from left to right.

1.8 PRELIMINARY CONCLUSION: SYNTHESIZING VESELAGO'S MEDIUM BY A PERIODIC STRUCTURE IS NOT FEASIBLE

In Section 1.2 we presented what is widely believed about Veselago's medium. We emphasize that the negative refraction was conceived by Veselago, but some of the other features were suggested by others. Although such materials have never been found in nature, it was suggested by Pendry that such materials could be synthesized by periodic structures with special elements. However, we found some troubling deviation between Veselago's theoretical material and what can de facto be obtained by using periodic structures. Regardless of element shape, the most prevalent factors were:

1. A negative index of refraction is observed between Veselago's medium and a medium with $\varepsilon,\ \mu > 0$. The phase match between the incident and refracted fields was explained by the concept of backward-traveling waves, as discussed in refs. 28–31. However, no trace of such waves was found in a lossless periodic structure, although they can exist on cables terminated in a proper load, as explained in Section 1.9. Experimental evidence of negative refracted fields in a finite periodic structure is plagued by persistent unexplained loss in excess of about −14 to 20 dB [10,14]. This writer has suggested that the field

observed is not a refracted field but radiation from a surface wave characteristic of finite periodic surfaces [20,21]. Further, we found no evidence that periodic structures with interelement spacings of less than $\lambda/2$ could change the direction of the incident field, as one would expect for an index of refraction $n \neq 1$ (however, see also Section 1.12.2).

2. It is widely believed that the input impedance of Veselago's medium mounted in front of a ground plane can rotate the "wrong" way (counterclockwise) in the Smith chart (see Figure 1.2 and refs. 2–4). We found absolutely no indication of such a phenomenon in lossless periodic structures suspended in a dispersionless medium (however, see also the discussion in Section 1.9).

3. Just as propagating waves in Veselago's medium can rotate the "wrong" way in a Smith chart, it is quite logical that evanescent waves might increase. In fact, it is generally believed that Veselago's material will support an evanescent wave that increases as you move away from the source (see, e.g., ref. 28, Fig. 3.27 and Sec. 3.7). We found that a periodic structure could only produce truly evanescent waves that would decrease as you move away from the individual arrays. Surely, a multiarray configuration could be designed such that a superficial look could give the impression that an evanescent wave increases as you go through the periodic structure (see Figure 1.15).

4. Veselago claims that a plane wave propagating through his material is left-handed; that is, $\bar{E}, \bar{H}$, and the direction of propagation (phase) form a left-handed triplet, while $\bar{E}, \bar{H}$, and Poynting's vector (energy direction) form a right-handed triplet as usual, regardless of the handedness of the medium. This implies that we will observe a time advance as we move away from the source (see Figure 1.2 as well as ref. 2). This concept is explained alternatively by backward-traveling waves [30,31]. (Note that very few of the classical textbooks treat this subject at all.)

However, we found from rigorous calculations that the field from an infinite periodic structure regardless of the element shape is always right-handed, both inside and outside the periodic structure. Further, there was never any trace of backward waves whatsoever. And as all experienced antenna engineers know, nothing ever moves backward in a Smith chart as long as our load impedance is purely imaginary (Foster's reactance theorem).

It should finally be emphasized that all impedance components in the discussion so far have been completely lossless, including the termination of the space behind the periodic structure. When resistive or dielectric loss is present, the situation changes radically, even if only the termination is lossy. Basically we will, in that case, move inside the rim of the Smith chart such that Foster's reactance theorem no longer holds. This case is discussed in the next section, where we illustrate a typical case in the form of a transmission line terminated in a complex load. This is a little easier than a periodic structure to comprehend, and it has already been discussed in several places [28,29]. Subsequent extension to periodic structures will be facilitated (see Section 1.9.2).

1.9 ON TRANSMISSION-LINE DISPERSION: BACKWARD-TRAVELING WAVES

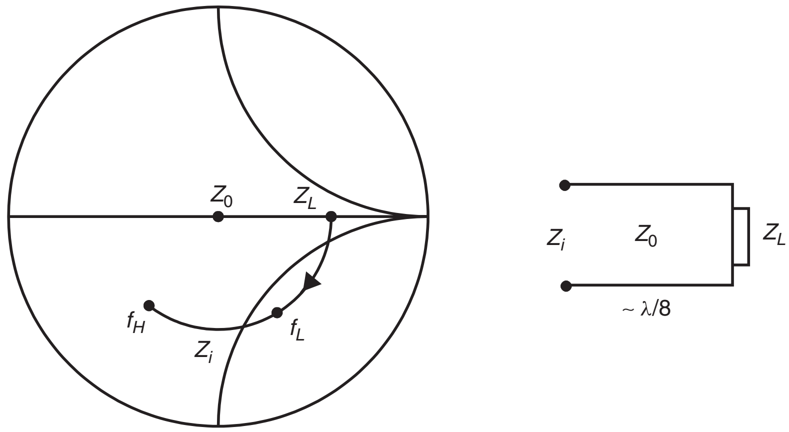
1.9.1 Transmission Lines

One of the most remarkable conclusions above was that the input impedance of a lossless transmission line terminated in a pure reactance is always located on the rim of the Smith chart and always runs clockwise with frequency (see Figure 1.2), never the other way around unless you *really* have a negative index of refraction. But what if the transmission line is terminated in a complex load rather than a pure reactance?

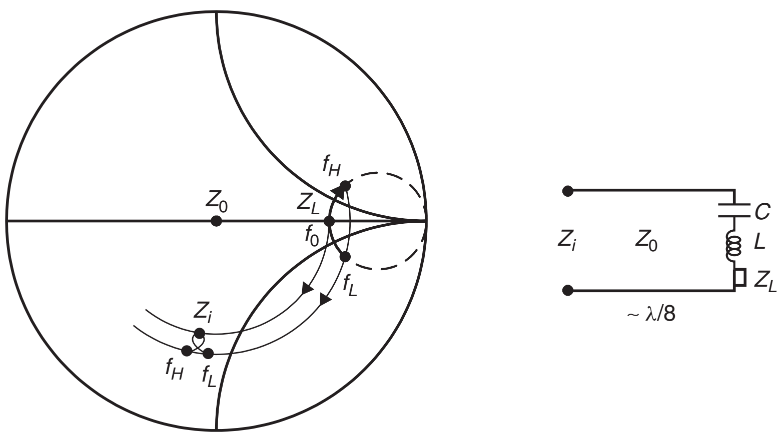
In fact, this problem has been investigated in numerous papers and at least four books [28–31]. The approach taken there is to start with the equivalent circuit for an ordinary transmission line (i.e., comprised of series inductors and parallel capacitors). The next step is to use duality to obtain an equivalent circuit with series capacitors and parallel inductors. By using simple first-order approximations, it is shown next that the dual circuit has a phase velocity equal to the negative of its group velocity. We shall not repeat the derivation here, since it suffers from several flaws, one being that the result is incorrect, and another that we end up with a dual circuit without a transmission line. This "essential" part could certainly be added later, but that approach leads to unnecessary complications and is still not satisfactory [28].

It is, in fact, usually much better to ask a direct question: What can be done to eliminate or at least reduce the dispersion of a transmission line? Actually, it has very little to do with duality. In fact, this problem is solved in the most direct way by use of the Smith chart, as illustrated by the following example.*

*The Smith chart is often frowned upon as being an approximate graphical approach. However, we should hasten to emphasize that the Smith chart represents a graphical illustration of an *exact* solution, not just some first-order approximate formulas. Most important, it depicts exactly what goes on in the complex plane and helps us in our thought process.



(a)



(b)

Figure 1.16 (a) Smith chart showing the input impedance Z as a function of frequency of a transmission line terminated in a load impedance Z_L (real). (b) The same as in part (a) above but with an LC series circuit to Z_L. It is seen to reduce the dispersion of Z_i; in fact, it does run backward over a limited frequency range. This does not violate Foster's reactance theorem because Z_L is lossy, so we are not on the rim of the Smith chart.

In the insert of Figure 1.16a we show a transmission line with characteristic impedance Z_0, length $\sim\lambda/8$, and terminated with a load impedance $Z_L \sim 2Z_0$. The input impedance Z_i as a function of frequency will typically look as shown in the Smith chart in the same figure, where the gap between the low frequency f_L and the high frequency f_H is an indication of the dispersion of the transmission line.

We ask a simple question: Is there any way in which this dispersion gap can be reduced or perhaps even reversed? As we shall see, there is indeed, but only if Z_L is located inside the Smith chart (i.e., has a resistive component), never when it is located on the rim of the Smith chart (i.e., is purely reactive). We illustrate this statement by the example shown in Figure 1.16b. As seen in the insert, we have added a series *LC* circuit to $Z_L \sim 2Z_0$ that resonates at the center frequency, $f_0 \sim \frac{1}{2}(f_L + f_H)$. In other words, the effective load impedance for the transmission line will be located on part of a circle going through Z_L and the infinity point, as shown (see Appendix B of ref. 32). Note that this impedance, when seen from the center of the Smith chart, will rotate counterclockwise (the "wrong" way) as the frequency increases from the low frequency f_L to the high frequency f_H. In other words, if we next add the clockwise rotation from the transmission line, we obtain an input impedance, Z_i, with a strongly reduced gap between f_L and f_H (i.e., we have reduced the dispersion for the transmission line and part of the curve is actually running "backward").

This approach can be extended and modified in many ways. If, for example, we extend the length of the cable from about λ/8 to about λ/4, as shown in Figure 1.17a, the new Z_i may have the high frequency f_H, somewhat ahead of the low frequency f_L (i.e., we see a moderate dispersion). However, if we note that the impedance of a parallel *LC* circuit is located on the rim of the Smith chart around the infinity point of the Smith chart, as shown, it is easy to see that adding this impedance in parallel with Z_i will result in a new Z_i where the dispersion even for this longer cable is strongly reduced, as shown in Figure 1.17b (see Appendix B in ref. 20).

We can extend this approach indefinitely, alternating between series and parallel *LC* circuits. It is easy to see that the waves on this composite cable can be considered as a combination of forward- and backward-traveling waves, where the first is always present and the relative strength of the second depends on the specific design. Although this is all well and good, it would be erroneous to think that we have produced a new exotic material. In fact, if we let the original load impedance, Z_L, go toward the rim of the Smith chart, we observe that only the forward-traveling wave will remain. Or put another way, if we cut a section out of our composite cable, it has no particular redeeming feature. We have simply demonstrated some old network tricks, well known for broadband matching technique [20,32].

It should finally be noted that the concept as presented here has some similarities with the circuit obtained by the duality concept: for example, the use of parallel capacitors. However, it fails to use the inductors, which give a greater variation with frequency. Also, it does not alternate between parallel and series *LC* circuits at every λ/4 separation. All in all, the duality approach is lacking compared to the circuit presented here. Actually, there is very little justification in using duality to deal with this problem.

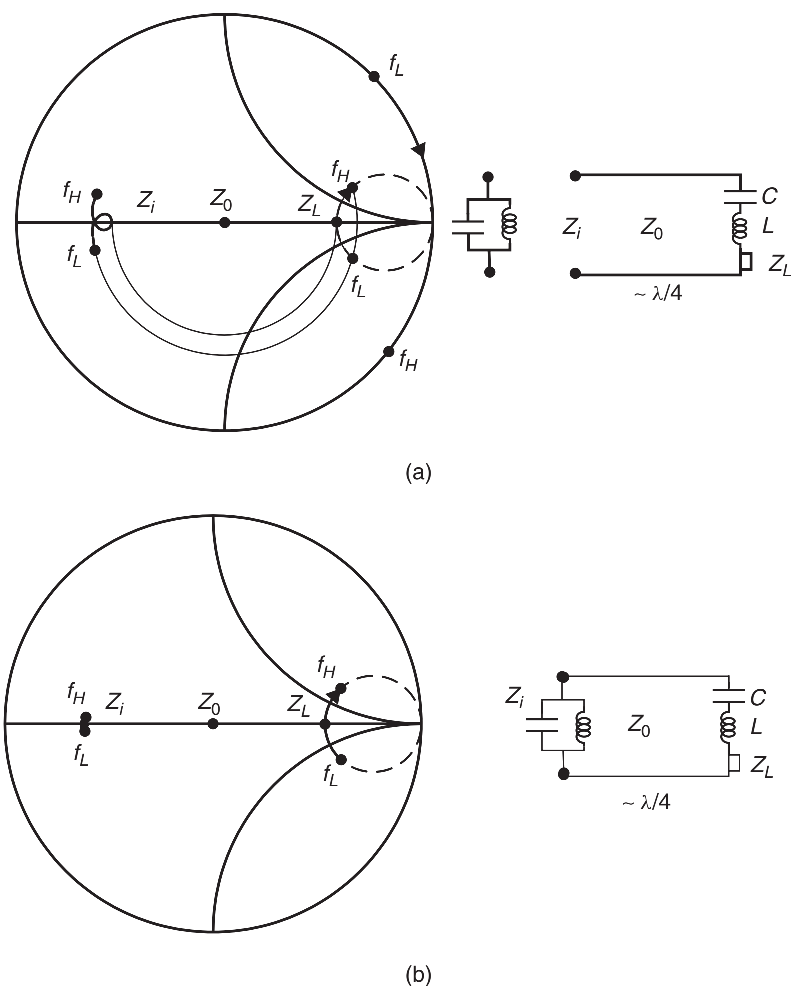


Figure 1.17 (a) The same as in Figure 1.16a but for a longer transmission line. Also indicated to the right in the Smith chart is the impedance of a parallel *LC* circuit. (b) When the parallel *LC* circuit is added to the left of the transmission line, we observe reduced dispersion of Z_i, even backward-traveling waves over a limited frequency range. Note: We have not "invented" a new "material," since it falls apart for Z_L reactive and other cases as well.

1.9.2 Periodic Structures

We investigated above the possibility of limiting or even reversing dispersion on a transmission line. This background will greatly facilitate our extension to periodic structures. An example is shown in Figure 1.18a, where we show a slotted frequency-selective surface (FSS) to the right and a dipole FSS to the left. This case differs from the transmission-line case in Figures 1.16 and 1.17 by the fact that the space to the right with intrinsic impedance Z_0 will put us right in the center of the Smith chart

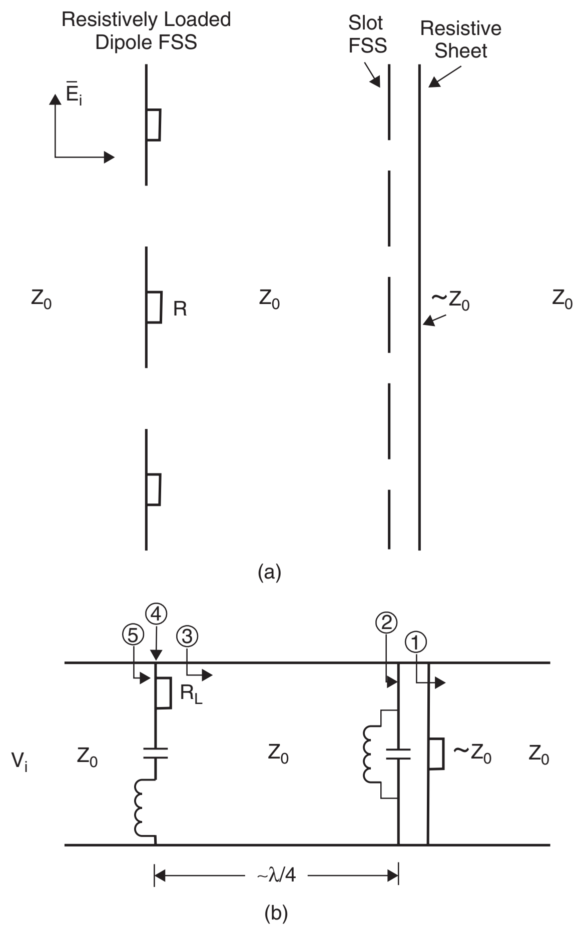
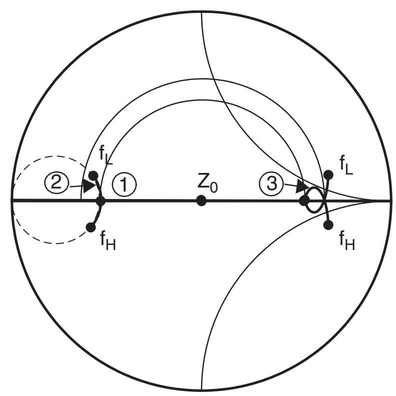


Figure 1.18 (a) Actual configuration of a combination of a slot FSS backed by a resistive sheet with a sheet resistance of about Z_0. To the left is a resistively loaded dipole FSS. The incident field is coming from the left as shown. (b) Equivalent circuit of the configuration shown in part (a). The circled numbers refer to the impedances looking to the right except that ④ refers simply to the loaded dipole FSS without space behind it.

and not at about $2Z_0$, as shown in Figure 1.16. The remedy for this dilemma is simply to add a resistance sheet of about Z_0 in parallel with the space impedance as shown in Figure 1.18a and also in the equivalent circuit in Figure 1.18b. This results in a total resistance ① equal to about $Z_0/2$. We now add the slotted FSS in parallel. Recalling that the equivalent circuit for a slotted FSS is a parallel LC circuit, we readily see that the total impedance ② looking to the right is merely located on a circle going through zero and about $Z_0/2$, as indicated in the Smith chart in Figure 1.19a. Note that when seen from the center of the Smith chart, this impedance curve runs the "wrong" way (counterclockwise). Thus, the impedance ③ obtained by clockwise rotation of ② has reduced dispersion.



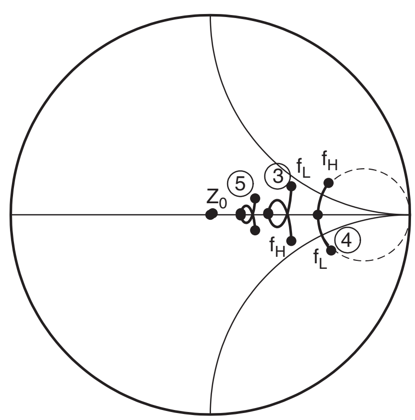


Figure 1.19 (a) The various impedances as denoted in Figure 1.16b shown in a Smith chart up to ③. (b) The remaining impedances from Figure 1.16b shown in a Smith chart.

Further reduction is obtained by adding a loaded dipole FSS. We recall that the equivalent circuit ④ for this configuration is a series *RLC* circuit, as shown in Figure 1.18b. As shown in the Smith chart in Figure 1.19b, this impedance is located on a circle going through R_L and the infinity point. Again we observe that the impedance ④ runs the "wrong" way (i.e., it will reduce the dispersion of impedance ③).

However, we see no particular reason at this point to continue this discussion concerning the possibility of creating new "materials" with a negative index of refraction. As mentioned earlier, we have really not created any new unique medium but merely applied a well-known approach from broadband matching techniques [20,32]. And it is far from lossless. Thus, you can forget about amplification of the evanescent waves.

1.10 REGARDING VESELAGO'S CONCLUSION: ARE THERE DEFICIENCIES?

1.10.1 Background

In 1968, Veselago asked a simple question: What would happen if both μ and ε for a material were negative [1]? He concluded that the index of refraction, n_1, between an ordinary medium with $\mu,\ \varepsilon, > 0$ and one with $\mu,\ \varepsilon < 0$ would be negative. Further, while Poynting's vector would propagate in the usual direction, the phase vector would point backward, later giving rise to the term *backward-traveling waves*. More extensions of Veselago's conclusions were added later by others. However, Veselago had conceived the most important aspect: a negative index of refraction. He stated quite correctly that no such material had ever been found or produced, and he very prudently added that there were, perhaps, very good reasons for the absence of such materials.

It was eventually suggested by Pendry almost 30 years later that materials with $\mu,\ \varepsilon < 0$ could be produced artificially by a periodic structure comprised of special elements [6–9]. We investigated that possibility earlier and concluded that *none* of the features characteristic of Veselago's medium could be produced by a periodic structure regardless of the type of element. Given that fact, it is natural to ask the simple question: Is Veselago's medium physically realizable?

1.10.2 Veselago's Argument for a Negative Index of Refraction

Veselago arrived at his conclusions by considering the boundary conditions between two media, 1 and 2, as shown in Figure 1.20. He first stated that the tangential components for the two media must be equal regardless of the sign of μ and ε in the two media; that is (using Veselago's notation),

$$E_{t1} = E_{t2} \qquad H_{t1} = H_{t2} \tag{1.12}$$

Further, the boundary conditions for the normal components states that

$$\varepsilon_1 E_{n1} = \varepsilon_2 E_{n2} \qquad \mu_1 H_{n1} = \mu_2 H_{n2} \tag{1.13}$$

Thus, we see clearly that if ε_1, μ_1 and ε_2, μ_2 have the same signs, the direction of propagation $\bar{k}_2$ in medium 2 will be as indicated in Figure 1.20 for $n_{12} > 0$. However, if ε_2, μ_2 has the sign opposite that of ε_1, μ_1, the normal components of $\bar{E}$ and $\bar{H}$ will be opposite each other according to (1.13), which means that the phase velocity k_{v2} in medium 2 will be as indicated in Figure 1.20: left-handed. However, we also note that

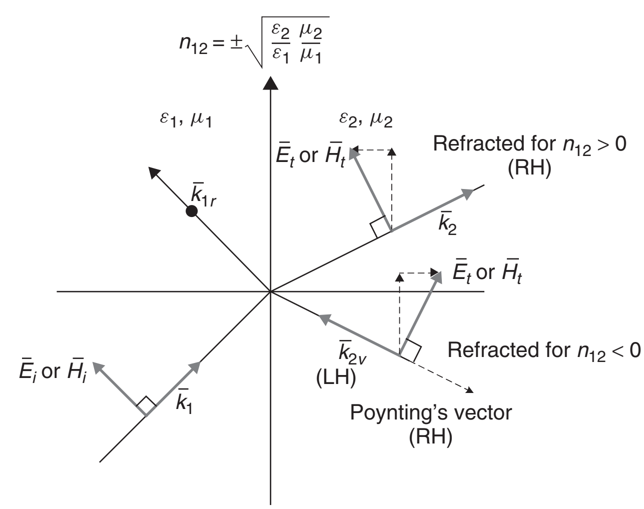


Figure 1.20 Veselago's proof that the index of refraction is negative for two media if ε_1, $\mu_2 > 0$ and ε_2, $\mu_2 < 0$. His argument is that the tangential field components must be the same regardless of handedness. However, the normal components change sign with $\varepsilon_1/\varepsilon_2$ and μ_1/μ_2 according to Maxwell's equations, as indicated in the figure. His proof is correct, but only mathematically, since it implies negative propagation constant β_2 and ultimately negative time (see the discussion related to Figure 1.21).

Poynting's vector (in cgs units) is given by

$$\bar{S} = \frac{c}{4\pi} \bar{E} \times \bar{H} \tag{1.14}$$

That is, $\bar{S}$ always forms a right-handed set with the vectors $\bar{E}$ and $\bar{H}$ and will therefore point in the direction opposite $\bar{k}_{2v}$ as also shown in Figure 1.20.

In other words, Veselago had shown that for two media, 1 and 2, where ε_1, μ_1 and ε_2, μ_2 have opposite signs, the index of refraction, n_{12}, would be negative. Furthermore, since the phase delay through a medium is given by $n_{12}\beta_0 d$, we observe immediately that for $n_{12} > 0$ we experience a phase delay and, similarly, a phase advance for $n_{12} < 0$, as illustrated in the Smith chart in Figure 1.2. Such conclusions should immediately raise questions about causality.* Indeed, some papers took issue with Veselago's conclusion, of which the most conspicuous was one by Valanju, Walser, and Valanju [33]. However, that merely led to an exchange of comments from Pendry and others, and eventually died out. In any case, Valanju et al. were never proven wrong. Meanwhile, the stream of papers concerning metamaterials continued unabated, and eventually at least four books on the same subject were published [2,28,29,46].

In this writer's opinion, Walser and associates were right and one may wonder why their paper did not have a greater impact. One reason probably is that it was a little intricate and not immediately understood. Thus, in the following we attempt a simpler explanation and show that Veselago's conclusions have physical deficiencies.

1.10.3 Veselago's Flat Lens: Is It Really Realistic?

The concept for Veselago's flat "lens" is by now well known, as shown in Figure 1.21. It consists of a flat slab where ε_2, μ_2 not only is negative but also $\varepsilon_2 = -\varepsilon_1$ and $\mu_2 = -\mu_1$ (i.e., $n_{12} = -1$) such that the refracted angle, according to Veselago, is always the negative of the angle of incidence. We show two rays emanating from the source point S located to the left. They cross inside the lens at a point denoted cross 1 and outside to the right at a point denoted cross 2. Such crossings are often thought to be focal points. However, more is required for such a classification. Foremost, we must require that all rays arrive with the same phase. Inspection of the two rays show clearly that ray SB is delayed in phase with respect to ray SA_2 by section A_1B. Further, section BA_3 is inside the metamaterials where the signal is advanced precisely by the same amount, according to Figure 1.2, such that the two rays will

*After all, how can a signal arrive before it starts?

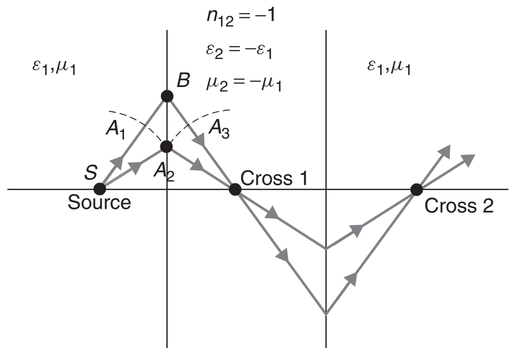


Figure 1.21 Veselago's flat lens with $\varepsilon_2 = -\varepsilon_1$ and $\mu_2 = -\mu_1$. The longest path ray will be delayed in phase corresponding to A_1B but be advanced in Veselago's medium corresponding to BA_3 (see also Figure 1.2). However, if the two rays are to arrive at the same time at cross 1, it must involve negative time in Veselago's medium. See also the discussion in the text in conjunction with equations (1.15) to (1.17) as well as Figures 6.1 and 6.2.

arrive at cross 1 in phase. However, we must also require the two rays to arrive at the crossing *at the same time*. Obviously, that would require the time delay A_1B to be canceled by a time advance BA_3 (i.e., negative time!). Although negative time does not "offend" mathematicians, it is definitely not an option open to physicists, particularly not to engineers.* So no wonder we have trouble synthesizing Veselago's medium!

1.11 CONCLUSIONS

When Veselago published his now famous paper in 1968 [1], he merely asked a simple question: What would happen if both μ and ε were negative? He came up with several interesting conclusions. The most important were:

1. The index of refraction between an ordinary medium and one with $\mu, \varepsilon < 0$ would be negative.
2. The field vectors $\bar{E}$ and $\bar{H}$ and the direction of phase propagation would form a left-handed triplet, whereas in an ordinary medium they are right-handed.

*Surely, it is possible for the two rays to arrive at different times and still be in phase, but only for a finite number of discrete frequencies. Thus, it fails for a general modulated signal. Similarly, a "static" case would consist of just one frequency. Since no modulation would be possible in this case, it would be of no practical interest (see also Section 6.5).

Apparently there was little interest in Veselago's work until Pendry in the mid-1990s suggested that material with negative ε and μ could be made artificially by use of periodic structures with special elements [6–9]. He and others subsequently came up with additional conclusions concerning materials with $\mu,\ \varepsilon < 0$. The most important were:

3. Evanescent waves would increase as they propagate through a medium with $\varepsilon,\ \mu < 0$, not decrease as they do in an ordinary medium.
4. The phase would advance in a medium with $\varepsilon,\ \mu < 0$ even if lossless [2–4], not be retarded as in an ordinary medium.

Conclusions 3 and 4 are identical from a mathematical point of view. (The exponent in the phase term goes from imaginary to positive real.) Strangely enough, some can accept one but not the other. (They are, of course, both wrong. The first leads to infinite energy at infinity; the other violates causality. See the comments below in conjunction with equations (1.15) to (1.17).] In this writer's opinion, the first of the conclusions above (i.e., negative index of refraction) has never been demonstrated satisfactorily, despite numerous claims in the literature. Most bothersome is the fact that the "negative refracted" power is always less than about 1 to 2% of the power transmitted through a low-loss dielectric reference material. This writer has suggested that the "refracted" field could simply be radiation from a surface wave characteristic for finite periodic structures with interelement spacings below $\lambda/2$ [20,21]. Or it could be a sidelobe from the mainbeam(s). But it certainly is *not* a refracted field! (See also Appendix D about lossy dielectric wedges.)

The second conclusion, that a material with $\mu,\ \varepsilon < 0$ must have $\bar{E}, \bar{H}$, and the propagation factor form a left-handed triplet, is probably the one that will be most difficult to synthesize by an infinite periodic structure. In fact, the field from such a structure was shown rigorously to always be right-handed, regardless of the element type. It would require rewriting Maxwell's equations to come up with a left-handed system!

Similarly, the field from an infinite periodic structure was shown always to consist of either propagating waves with phase retardation as you move away from the structure where they originate, or of evanescent waves that are attenuated as you move away. In other words, as claimed in conclusions 3 and 4, the fields simply could not be synthesized by an infinite periodic structure whether it consisted of a single array or of multiple arrays. Of all these conclusions, 3 and 4 are probably the ones that have been the most difficult for this author to accept. It appears that Pendry was quite comfortable with satisfying pure math and less concerned about physics. It simply makes no physical sense to have the amplitude of the electric field go to infinity as we go toward infinity. Nor can a signal arrive before we send it!

Similarly, having the propagation factor β go negative, as indicated in Figure 1.2, leads to fundamental physical problems. More specifically, let us consider a medium with propagation constant β and phase velocity v. Let us assume further that it will take t seconds to travel a distance of d meters. Then clearly we have

$$d = vt \qquad \text{meters} \tag{1.15}$$

We further have

$$v = \lambda f = \frac{\lambda 2\pi f}{2\pi} = \frac{\omega}{\beta} \tag{1.16}$$

Substituting equation (1.16) into (1.15) yields

$$d = \frac{\omega t}{\beta} \qquad \text{meters} \tag{1.17}$$

Inspection of (1.17) shows that if we assume that the distance d as well as the angular frequency ω are both positive (!), clearly β and t must have the same sign. In particular, if $\beta = n_1\beta_0 < 0$, as shown in Figure 1.2, then clearly time t must be negative as well. This observation supports our discussion in Section 1.10.3 about Veselago's flat lens. It also lends credence to the claim of Valanju et al. [33] that causality is violated for materials with μ, $\varepsilon < 0$ (see also Figures 6.1 and 6.2).

Certainly, Veselago was right when he stated in his original paper that material with μ, $\varepsilon < 0$ has never been found in nature. And he added (very prudently): "There are perhaps good reasons for this." He was, in this writer's opinion, also correct in his proof of negative index of refraction—however, only from a purely mathematical point of view. From a physical point of view, it was deficient because it leads to negative time. Walser et al. saw this very early, in 2002 [33].

This writer attended Engheta's oral presentation in Torino in 2001 [3]. He commented from the floor that he found the paper very interesting but that he did "not believe a word of it because it violated Foster's Reactance Theorem." It was followed by much discussion, but no agreement was reached.

We finally investigated the possibility of backward-traveling waves in transmission lines. These are deemed absolutely essential in obtaining the features characteristic of Veselago's medium. They have been investigated intensely by [28] and [29] using duality. We used a more direct approach here simply by applying a broadband matching technique. We found that it is indeed possible (and well known) to obtain an input impedance of a transmission line terminated resistively that makes a loop running the "wrong" way in the Smith chart, as seen from the center over a limited frequency band. This can be interpreted as a backward-traveling wave superimposed on a forward-traveling wave. But this is possible only if the transmission line is terminated in a resistive load in conjunction with a suitable reactance, never if the load impedance is purely imaginary. In other words, it is possible only when we are inside the Smith chart, where Foster's reactance theorem does not hold. We would therefore not characterize this as a special material (it "works" only when terminated with special loads) but, rather, as an application of the well-known broadband matching technique. And this solution is, of course, inherently lossy.

1.12 COMMON MISCONCEPTIONS

1.12.1 Artificial Dielectrics: Do They Really Refract?

Artificial dielectrics made of arrays of short conducting wires suspended either in free space or in a mother dielectric have been known for more than 50 years. W. E. Kock [34] is usually credited with being the originator of the fundamental idea: that an array of small metallic objects can delay a plane wave propagating through such a medium similar to what is observed in an ordinary dielectric medium compared to free space [34]. It is further believed, at least by some, that this delay can change the direction of propagation.

However, earlier in the chapter we stated categorically that a periodic structure of any conducting planar elements suspended in free space cannot change the direction of a plane wave incident upon such a structure. Obviously we owe the reader an explanation for this discrepancy. We are well aware that we disagree with the prevailing view regarding artificial dielectric.

The concept for artificial dielectric is based on an equivalent transmission line loaded periodically with shunt impedances, Z_s, corresponding to each array as shown in Figure 1.22. It is further well known that for short wires ($2l < 0.3\lambda$) the equivalent shunt impedances Z_s are basically capacitive, resulting in a phase delay compared to that of free space, $\beta_0 d$ per array. This fact is usually taken into account by introducing the effective propagation constant β_{eff}, where in the present case, $\beta_{\text{eff}} > \beta_0$. The theory

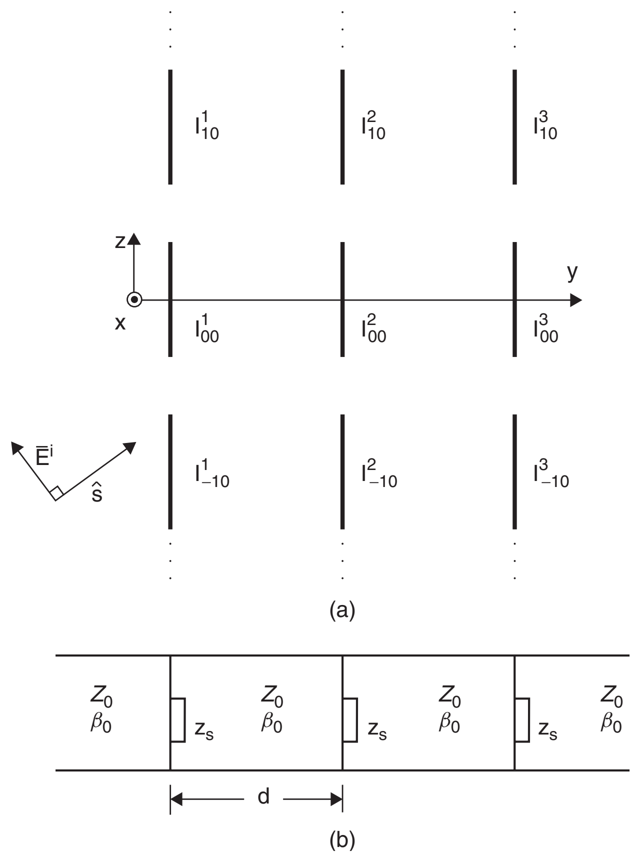


Figure 1.22 (a) Example of an artificial dielectric of small wires suspended in air; (b) equivalent circuit of the artificial dielectric shown in part (a).

for an artificial dielectric now states (defines) that the effective index of refraction is [35,36]

$$n_{\text{eff}} = \frac{\beta_{\text{eff}}}{\beta_0} \tag{1.18}$$

Certainly, had we considered a homogeneous dielectric material rather than an artificial dielectric of wires, the definition of the index of refraction as given by (1.18) would be correct. However, a more rigorous approach is needed when working with artificial dielectric or periodic structures.

First, we realize that when a plane wave with direction of propagation $\hat{s} = \hat{x}s_x + \hat{y}s_y + \hat{z}s_z$ is incident upon an infinite array in the x- and z-directions, the element currents in column k and row n of array 1 will, according to Floquet's theorem, be given as [5]

$$I_{kn} = I_{00}^{1} e^{-j\beta_0 k D_x s_x} e^{-j\beta_0 n D_z s_z} \tag{1.19}$$

Assuming that the interelement spacing D_x, $D_z < \lambda/2$, the element currents given by equation (1.19) will, according to fundamental array theory as shown in Section 1.5, produce propagating plane waves *only* in the forward direction $\hat{s}$ as well as the specular direction $\hat{s}_s = \hat{x}s_{sx} - \hat{y}s_{sy} + \hat{z}s_{sz}$. And the same statement holds for all the other arrays as well (see Chapters 4 and 8 of ref. 5 for details).

Certainly, we hasten to emphasize that the array currents I_{kn}^{a}, where $a = 1, 2, \ldots, N$, might indeed be delayed or advanced with respect to each other. However, that fact is by itself not capable of changing the *direction* of the radiation from the *individual* arrays. That depends only on the phase distribution across the individual arrays, and that is for an infinite array always given by (1.19) (i.e., Floquet's theorem) and thus will radiate only in the forward direction $\hat{s}$ as well as in the specular direction $\hat{s}_s$. However, see also Section 1.12.2.

If we fill the entire space between the arrays with a material with propagation constant β_1, there will be a change of propagation from $\hat{s}_0$ in air to $\hat{s}_1$ in the "mother" material. It is determined simply by matching phase velocities along the arrays and leads, as is well known from Snell's law. However, there is no additional change of direction due to the periodic structures (for details, see Chapters 4, 5, and 8 in ref. 5). More specifically: The arrays can only affect the propagation constant orthogonal to the arrays, not parallel to them. It appears that only a material with a propagation constant different from that of the incident space, β_0 in this case, can accomplish this.

A very important next step is to break the mother material up into arrays consisting of rectangular "flakes." Such an arrangement of elements that are not simply conducting but have permittivity and/or permeability open up new exciting possibilities, to be treated in a future paper by R. Walser et al.

1.12.2 Real Dielectrics: How Do They Refract?

Actually, what we said earlier about artificial dielectric is only approximately true if the number of parallel arrays is relatively small. In a real dielectric we work with periodic structures where the elements typically are molecular and where the number of arrays is very large indeed. This will result in essentially two things: (1) The delay caused by each array will eventually add up, resulting in a significant change of direction of propagation; and (2) the fields scattered from the individual arrays will add up to a wave propagating in the direction of the incident wave and, eventually, attain equal amplitude and be 180° out of phase. This statement is based on the *extinction theorem* presented by Ewald [45]. Thus, the field inside a real dielectric will consist only of a refracted wave propagating in a direction consistent with Snell's law.

There will, of course, be a somewhat similar effect in the artificial dielectric case. However, because the elements typically are larger, the number of arrays tend to be smaller, resulting in a weak effect. Just exactly what constitutes a "small" and a "large" number of arrays is an interesting problem to ponder. Probably the total field transmitted in the forward direction would have an amplitude following a spiral with decreasing radius as the number of arrays increases. It would require extensive computer runs with the PMM code, requiring help from students no longer available. Why? Because when you take on a student, your life expectancy should go beyond about five years, and I am past that limit!

Of course, in an artificial dielectric of small extent, the direction of propagation can change considerably when passing close to the individual element. However, the average direction is the same.

One thing is certain: Neither an artificial nor a real dielectric will produce negative refraction!

1.12.3 On the *E*- and *H*-Fields

It was originally suggested by Pendry that ε originates in parallel wires whereas μ is associated with split-ring resonators. It is often implied that the resulting *E*- and *H*-fields are independent of each other. This is fundamentally wrong. Only at dc can you control these two field vectors independently. At higher frequencies they become like the two sides of one piece of paper: You cannot have one side without the other. This is a simple consequence of Maxwell's equations.

More specifically, coupling between parallel wires and split-ring resonators is typically assumed to be about zero. Considering that the coupling is actually 100%, it is obvious that this will lead to both computational and conceptual mistakes. What actually takes place in a periodic structure of wires and split-ring resonators is discussed in Appendix A. We do not perform an actual calculation. Rather, to understand what really goes on, we explain the physics behind it, which is more important. Needless to say, we do not observe any negative μ or ε whatsoever.

1.12.4 On Concentric Split-Ring Resonators

The array of split-ring resonators is often made such that a smaller element is mounted concentrically inside a slightly larger one, whether circular or rectangular. The purpose of such an arrangement is, I am told, to obtain a broader bandwidth similar to staggered tuning. This expectation is based on the assumption that the coupling between the concentric elements is zero, or at least "not important." Nothing could be further from the truth. In fact, the coupling between concentric arrays is 100%. We do obtain two resonances, but instead of having a small valley between them, we find that they are separated by an infinite deep null (assuming that interelement spacing is small enough not to have grating lobes). Double tuning of arrays in general is discussed in detail in Chapter 9 of ref. 20.

1.12.5 What Would Veselago Have Asked if . . .

When Veselago asked his famous question in 1968 [1], he was obviously envisioning Maxwell's equations written in the usual form using μ and ε. However, as shown in Appendix A, it is also possible to write Maxwell's equations in a form that does not contain μ and ε, but instead, the propagation constant $\beta = \omega\sqrt{\mu\varepsilon}$ and the intrinsic impedance $Z = \sqrt{\mu/\varepsilon}$.* This makes quite a bit of sense since we typically measure β and Z from where we obtain μ and ε. Also, we are in general, from both a theoretical and a practical point of view, more interested in β and Z than in μ and ε (see Appendix A).

It is quite interesting to speculate what question Veselago would have asked had he used β and Z. Almost everyone agrees that a negative Z makes no sense unless we try to simulate a black hole in outer space. And a negative β could simply indicate a wave propagating in the negative direction but with a phase *delay* as we move away from the source. Although this case would be trivial, it would be quite a different story if a wave propagated with a phase advanced as we moved away from the source. I doubt that Veselago would have fallen into that trap. See also the discussion in connection with equations (1.15) to (1.17) as well as Section 1.5.

Well, Veselago did not use β and Z but μ and ε. And as we all know, his question started almost 30 years later, one of the most controversial subjects in our time. It has resulted in several books and literally thousands

*To the best of this writer's knowledge, this was first observed by W. Rotman in 1962 [37]. However, recently it was pointed out by the author's Swedish friend Per Erik Ljung that E. Hallén also considered this subject in his book *Elektricitetslära* (p. 109) as early as 1953.

of papers, but would we have been better off without these? At least I do not think we would be worse off. We have so far not seen any practical use (other than what we could design without any theoretical input from materials with a negative index of refraction).

It is often stated that we do not have the means to make such structures precise and lossless enough today but will perhaps in the future. I do not think so! Anything I have seen so far has been child's play compared to the highly sophisticated structures used in modern technology. No, the problem is pure and simple: The solution just does not exist! I hope this chapter has shed some light on this subject.

1.12.6 On "Magic" Structures

Every so often you see papers that claim a larger transmission through a periodic structure than expected. Typically, we are dealing here with simple structures such as circular holes (or squares, for that matter) in a thin perfectly conducting screen. The claims are based on the assumption that the transmission coefficient is given by the ratio between the sum of the physical area of the holes and the area of the entire screen: in other words, simple physical optics. Apparently, it is not always realized that periodic structures can exhibit resonances. A frequent explanation is based on the presence of a layer of "plasmons" adjacent to the screen. Although such a layer might be a reality at optical frequencies, we have never found it necessary to resort to such a mechanism at microwave frequencies.

True, a periodic structure of circular holes does possess a somewhat peculiar resonance. Actually, a *single* circular aperture does not resonate. What happens in an *array* is simply that just before onset of the lowest-order grating lobe, the lowest-order evanescent mode becomes extremely strong, which manifests itself in a lot of stored energy of such "polarity" that it makes the aperture holes resonate. This layer of stored energy is as close as we get to a "plasmon layer" at microwave frequencies.

Incidentally, any periodic structure with resonances governed primarily by the onset of grating lobes is usually undesirable because these vary so dramatically with frequency and angle of incidence. They were among the first type of periodic structures to be explored more than 40 years ago, and their bad features are well documented [38–40]. I was therefore surprised when I saw an article in *IEEE Transactions on Antennas and Propagation*, [41] in which researchers working in optics had written a paper about periodic structures with circular apertures in the hope that the FSS community would find this new "discovery" useful. For the record, we remind the reader that perfect transmission can be obtained for an array of slots of length about $\lambda/2$ and an arbitrary vanishing narrow slot width provided that the conductivity is 100% (i.e., there is virtually no physical area!) [42–44]—and a myriad of other element types (see Chapter 2 of ref. 5).

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