Chapter 1

Discoveries and Essential Quantum Physics

In This Chapter

- Putting forth theories of quantization and discrete units
- Experimenting with waves acting as particles
- Experimenting with particles acting as waves
- Embracing uncertainty and probability

According to classical physics, particles are particles and waves are waves, and never the twain shall mix. That is, particles have an energy E and a momentum vector $\boldsymbol{p}$, and that's the end of it. And waves, such as light waves, have an amplitude A and a wave vector $\boldsymbol{k}$ (where the magnitude of $\boldsymbol{k} = \frac{2\pi}{\lambda}$, where λ is the wavelength) that points in the direction the wave is traveling. And that's the end of that, too, according to classical physics.

But the reality is different — particles turn out to exhibit wave-like properties, and waves exhibit particle-like properties as well. The idea that waves (like light) can act as particles (like electrons) and vice versa was the major revelation that ushered in quantum physics as such an important part of the world of physics. This chapter takes a look at the challenges facing classical physics around the turn of the 20th century — and how quantum physics gradually came to the rescue. Up to that point, the classical way of looking at physics was thought to explain just about everything. But as those pesky experimental physicists have a way of doing, they came up with a bunch of experiments that the theoretical physicists couldn't explain.

That made the theoretical physicists mad, and they got on the job. The problem here was the microscopic world — the world that's too tiny to see. On the larger scale, classical physics could still explain most of what was going on — but when it came to effects that depended on the micro-world, classical physics began to break down. Taking a look at how classical physics collapsed gives you an introduction to quantum physics that shows why people needed it.

Being Discrete: The Trouble with Black-Body Radiation

One of the major ideas of quantum physics is, well, *quantization* — measuring quantities in discrete, not continuous, units. The idea of quantized energies arose with one of the earliest challenges to classical physics: the problem of black-body radiation.

When you heat an object, it begins to glow. Even before the glow is visible, it's radiating in the infrared spectrum. The reason it glows is that as you heat it, the electrons on the surface of the material are agitated thermally, and electrons being accelerated and decelerated radiate light.

Physics in the late 19th and early 20th centuries was concerned with the spectrum of light being emitted by black bodies. A *black body* is a piece of material that radiates corresponding to its temperature — but it also absorbs and reflects light from its surroundings. To make matters easier, physics postulated a black body that reflected nothing and absorbed all the light falling on it (hence the term *black body,* because the object would appear perfectly black as it absorbed all light falling on it). When you heat a black body, it would radiate, emitting light.

Well, it was hard to come up with a physical black body — after all, what material absorbs light 100 percent and doesn't reflect anything? But the physicists were clever about this, and they came up with the hollow cavity you see in Figure 1-1, with a hole in it.

When you shine light on the hole, all that light would go inside, where it would be reflected again and again — until it got absorbed (a negligible amount of light would escape through the hole). And when you heated the hollow cavity, the hole would begin to glow. So there you have it — a pretty good approximation of a black body.

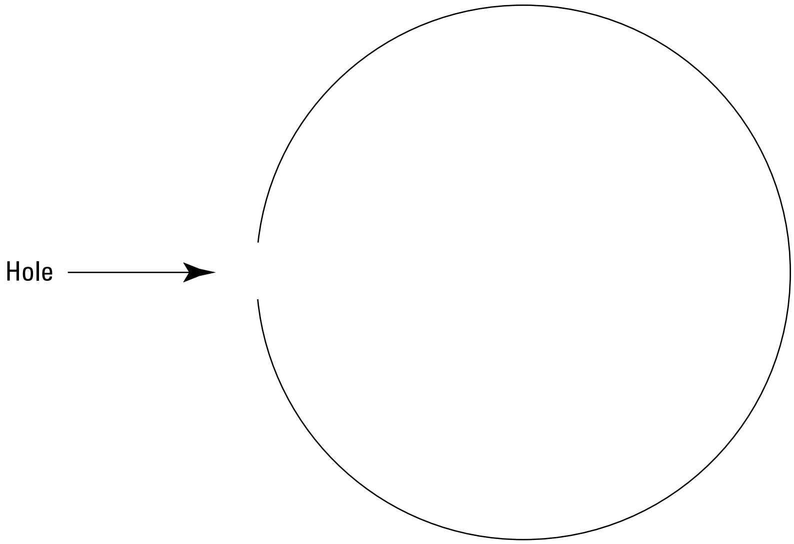

Figure 1-1: A black body.

You can see the spectrum of a black body (and attempts to model that spectrum) in Figure 1-2, for two different temperatures, T_1 and T_2. The problem was that nobody was able to come up with a theoretical explanation for the spectrum of light generated by the black body. Everything classical physics could come up with went wrong.

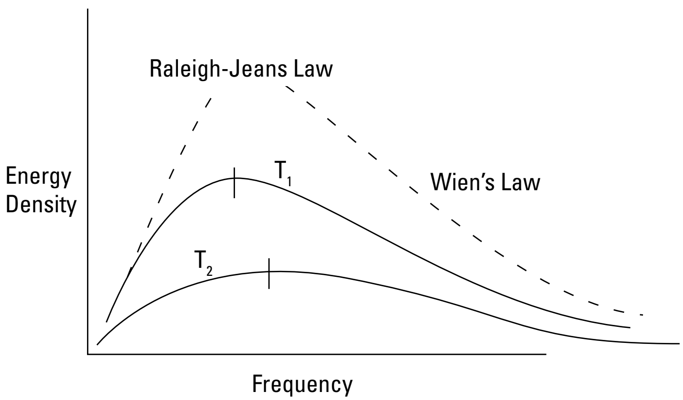

Figure 1-2: Black-body radiation spectrum.

First attempt: Wien's Formula

The first one to try to explain the spectrum of a black body was Willhelm Wien, in 1889. Using classical thermodynamics, he came up with this formula:

$$u(\upsilon, \mathrm{T}) = \mathrm{A}\upsilon^3 e^{-\beta\upsilon/\mathrm{T}}$$

where A and β are constants you determine from your physical setup, υ is the frequency of the light, and T is the temperature of the black body. (The spectrum is given by $u[\upsilon, \mathrm{T}]$, which is the energy density of the emitted light as a function of frequency and temperature.)

This equation, Wien's formula, worked fine for high frequencies, as you can see in Figure 1-2; however, it failed for low frequencies.

Second attempt: Raleigh-Jeans Law

Next up in the attempt to explain the black-body spectrum was the Raleigh-Jeans Law, introduced around 1900. This law predicted that the spectrum of a black body was

$$u(\upsilon, \mathrm{T}) = \frac{8\pi\upsilon^2}{c^3} k\mathrm{T}$$

where k is Boltmann's constant (approximately 1.3807×10^{-23} J·K^{-1}). However, the Raleigh-Jeans Law had the opposite problem of Wien's law: Although it worked well at low frequencies (see Figure 1-2), it didn't match the higher-frequency data at all — in fact, it diverged at higher frequencies. This was called the *ultraviolet catastrophe* because the best predictions available diverged at high frequencies (corresponding to ultraviolet light). It was time for quantum physics to take over.

An intuitive (quantum) leap: Max Planck's spectrum

The black-body problem was a tough one to solve, and with it came the first beginnings of quantum physics. Max Planck came up with a radical suggestion — what if the amount of energy that a light wave can exchange with matter wasn't continuous, as postulated by classical physics, but *discrete?* In other words, Planck postulated that the energy of the light emitted from the walls of the black-body cavity came only in integer multiples like this, where *h* is a universal constant:

$$E = nh\upsilon, \text{ where } n = 0, 1, 2, \ldots$$

With this theory, crazy as it sounded in the early 1900s, Planck converted the continuous integrals used by Raleigh-Jeans to discrete sums over an infinite number of terms. Making that simple change gave Planck the following equation for the spectrum of black-body radiation:

$$u(\upsilon, T) = \frac{8\pi\upsilon^2}{c^3} \frac{h\upsilon}{e^{h\upsilon/kT} - 1}$$

This equation got it right — it exactly describes the black-body spectrum, both at low and high (and medium, for that matter) frequencies.

This idea was quite new. What Planck was saying was that the energy of the radiating oscillators in the black body couldn't take on just any level of energy, as classical physics allows; it could take on only specific, *quantized* energies. In fact, Planck hypothesized that that was true for *any* oscillator — that its energy was an integral multiple of $h\upsilon$.

REMEMBER

And so Planck's equation came to be known as *Planck's quantization rule,* and *h* became *Planck's constant:* $h = 6.626 \times 10^{-34}$ Joule-seconds. Saying that the energy of all oscillators was quantized was the birth of quantum physics.

One has to wonder how Planck came up with his theory, because it's not an obvious hypothesis. Oscillators can oscillate only at discrete energies? Where did that come from? In any case, the revolution was on — and there was no stopping it.

The First Pieces: Seeing Light as Particles

Light as particles? Isn't light made up of waves? Light, it turns out, exhibits properties of both waves and particles. This section shows you some of the evidence.

Solving the photoelectric effect

The photoelectric effect was one of many experimental results that made up a crisis for classical physics around the turn of the 20th century. It was also one of Einstein's first successes, and it provides proof of the quantization of light. Here's what happened.

When you shine light onto metal, as Figure 1-3 shows, you get emitted electrons. The electrons absorb the light you shine, and if they get enough energy, they're able to break free of the metal's surface. According to classical physics, light is just a wave, and it can exchange any amount of energy with the metal. When you beam light on a piece of metal, the electrons in the metal should absorb the light and slowly get up enough energy to be emitted from the metal. The idea was that if you were to shine more light onto the metal, the electrons should be emitted with a higher kinetic energy. And very weak light shouldn't be able to emit electrons at all, except in a matter of hours.

But that's not what happened — electrons were emitted as soon as someone shone light on the metal. In fact, no matter how weak the intensity of the incident light (and researchers tried experiments with such weak light that it should have taken hours to get any electrons emitted), electrons *were* emitted. Immediately.

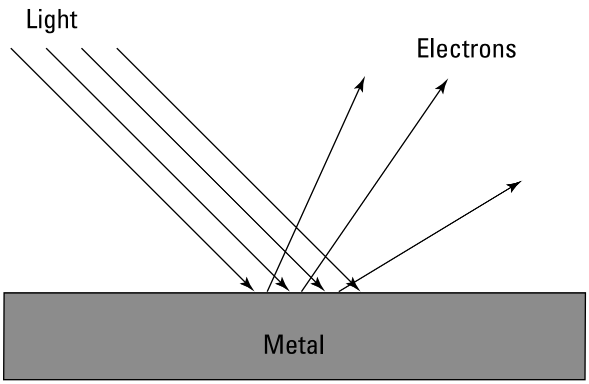

Figure 1-3: The photoelectric effect.

Experiments with the photoelectric effect showed that the kinetic energy, K, of the emitted electrons depended only on the frequency — not the intensity — of the incident light, as you can see in Figure 1-4.

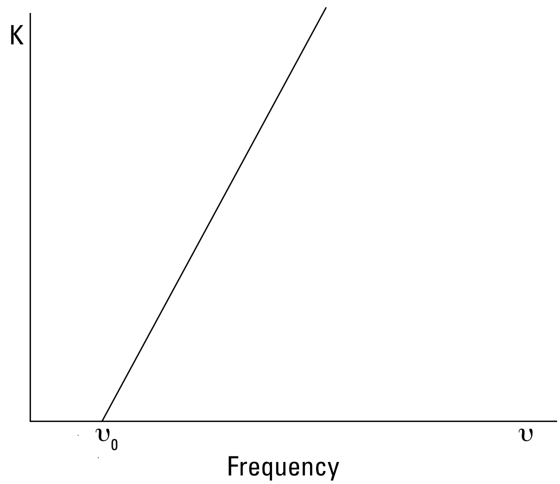

Figure 1-4: Kinetic energy of emitted electrons versus frequency of the incident light.

In Figure 1-4, υ_0 is called the *threshold frequency*, and if you shine light with a frequency below this threshold on the metal, no electrons are emitted. The emitted electrons come from the pool of free electrons in the metal (all metals have a pool of free electrons), and you need to supply these electrons with an energy equivalent to the metal's work function, W, to emit the electron from the metal's surface.

The results were hard to explain classically, so enter Einstein. This was the beginning of his heyday, around 1905. Encouraged by Planck's success (see the preceding section), Einstein postulated that not only were oscillators quantized but so was light — into discrete units called *photons*. Light, he suggested, acted like particles as well as waves.

So in this scheme, when light hits a metal surface, photons hit the free electrons, and an electron completely absorbs each photon. When the energy, $h\upsilon$, of the photon is greater than the work function of the metal, the electron is emitted. That is,

$$h\upsilon = W + K$$

where W is the metal's work function and K is the kinetic energy of the emitted electron. Solving for K gives you the following:

$$K = h\upsilon - W$$

You can also write this in terms of the threshold frequency this way:

$$K = h(\upsilon - \upsilon_0)$$

So apparently, light isn't just a wave; you can also view it as a particle, the photon. In other words, light is quantized.

That was also quite an unexpected piece of work by Einstein, although it was based on the earlier work of Planck. Light *quantized*? Light coming in discrete energy packets? What next?

Scattering light off electrons: The Compton effect

To a world that still had trouble comprehending light as particles (see the preceding section), Arthur Compton supplied the final blow with the Compton effect. His experiment involved scattering photons off electrons, as Figure 1-5 shows.

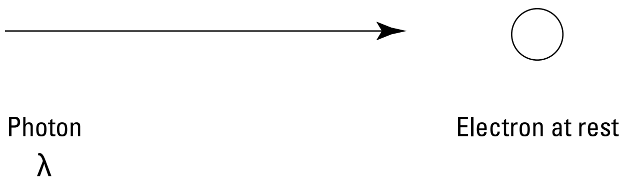

Figure 1-5: Light incident on an electron at rest.

Incident light comes in with a wavelength of λ and hits the electron at rest. After that happens, the light is scattered, as you see in Figure 1-6.

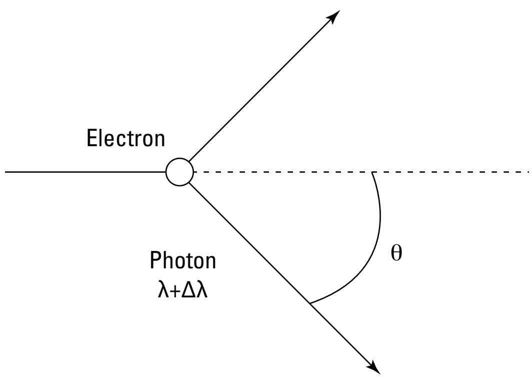

Figure 1-6: Photon scattering off an electron.

Classically, here's what should've happened: The electron should've absorbed the incident light, oscillated, and emitted it — with the same wavelength but with an intensity depending on the intensity of the incident light. But that's not what happened — in fact, the wavelength of the light is actually changed by $\Delta\lambda$, called the *wavelength shift.* The scattered light has a wavelength of $\lambda + \Delta\lambda$ — in other words, its wavelength has increased, which means the light has lost energy. And $\Delta\lambda$ depends on the scattering angle, θ, not on the intensity of the incident light.

Arthur Compton could explain the results of his experiment only by making the assumption that he was actually dealing with two particles — a photon and an electron. That is, he treated light as a discrete particle, not a wave. And he made the assumption that the photon and the electron collided elastically — that is, that both total energy and momentum were conserved.

Making the assumption that both the light and the electron were particles, Compton then derived this formula for the wavelength shift (it's an easy calculation if you assume that the light is represented by a photon with energy $E = h\upsilon$ and that its momentum is $p = E/c$):

$$\Delta\lambda = \frac{h}{m_e c}\left(1 - \cos\theta\right)$$

where h is Planck's constant, m_e is the mass of an electron, c is the speed of light, and θ is the scattering angle of the light.

You also see this equation in the equivalent form:

$$\Delta\lambda = 4\pi\lambda_c \sin^2\left(\theta/2\right)$$

where λ_c is the Compton wavelength of an electron, $\lambda_c = \hbar/m_e c$, where $\hbar = h/2\pi$. And experiment confirms this relation — both equations.

Note that to derive the wavelength shift, Compton had to make the assumption that here, light was acting as a particle, not as a wave. That is, the particle nature of light was the aspect of the light that was predominant.

Proof positron? Dirac and pair production

In 1928, the physicist Paul Dirac posited the existence of a positively charged anti-electron, the *positron*. He did this by taking the newly evolving field of quantum physics to new territory by combining relativity with quantum mechanics to create relativistic quantum mechanics — and that was the theory that predicted, through a plus/minus-sign interchange — the existence of the positron.

It was a bold prediction — an *anti-particle* of the electron? But just four years later, physicists actually saw the positron. Today's high-powered elementary particle physics has all kinds of synchrotrons and other particle accelerators to create all the elementary particles they need, but in the early 20th century, this wasn't always so.

In those days, physicists relied on cosmic rays — those particles and high-powered photons (called gamma rays) that strike the Earth from outer space — as their source of particles. They used *cloud-chambers*, which were filled with vapor from dry ice, to see the trails such particles left. They put their chambers into magnetic fields to be able to measure the momentum of the particles as they curved in those fields.

In 1932, a physicist noticed a surprising event. A pair of particles, oppositely charged (which could be determined from the way they curved in the magnetic field) appeared from apparently nowhere. No particle trail led to the origin of the two particles that appeared. That was *pair-production* — the conversion of a high-powered photon into an electron and positron, which can happen when the photon passes near a heavy atomic nucleus.

So experimentally, physicists had now seen a photon turning into a pair of particles. Wow. As if everyone needed more evidence of the particle nature of light. Later on, researchers also saw *pair annihilation:* the conversion of an electron and positron into pure light.

Pair production and annihilation turned out to be governed by Einstein's newly introduced theory of relativity — in particular, his most famous formula, $E = mc^2$, which gives the pure energy equivalent of mass. At this point, there was an abundance of evidence of the particle-like aspects of light.

A Dual Identity: Looking at Particles as Waves

In 1923, the physicist Louis de Broglie suggested that not only did waves exhibit particle-like aspects but the reverse was also true — all material particles should display wave-like properties.

How does this work? For a photon, momentum $p = h\upsilon/c = h/\lambda$, where υ is the photon's frequency and λ is its wavelength. And the wave vector, **k,** is equal to $\mathbf{k} = \mathbf{p}/\hbar$, where $\hbar = h/2\pi$. De Broglie said that the same relation should hold for all material particles. That is,

$$\lambda = \frac{h}{p}$$

$$k = \frac{p}{\hbar}$$

De Broglie presented these apparently surprising suggestions in his Ph.D. thesis. Researchers put these suggestions to the test by sending a beam through a dual-slit apparatus to see whether the electron beam would act like it was made up of particles or waves. In Figure 1-7, you can see the setup and the results.

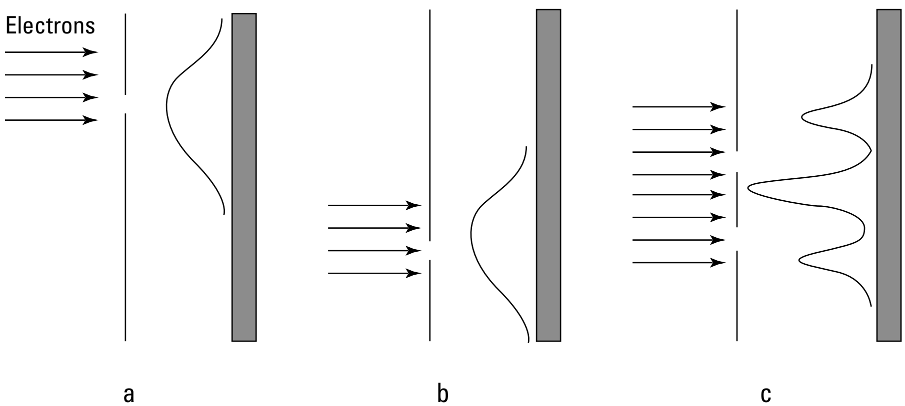

Figure 1-7: An electron beam going through two slits.

In Figure 1-7a, you can see a beam of electrons passing through a single slit and the resulting pattern on a screen. In Figure 1-7b, the electrons are passing through a second slit. Classically, you'd expect the intensities of Figure 1-7a and 1-7b simply to add when both slits are open:

$$I = I_1 + I_2$$

But that's not what happened. What actually appeared was an interference pattern when both slits were open (Figure 1-7c), not just a sum of the two slits' electron intensities.

The result was a validation of de Broglie's invention of matter waves. Experiment bore out the relation that $\lambda = h/p$, and de Broglie was a success.

The idea of matter waves is a big part of what's coming up in the rest of the book. In particular, the existence of matter waves says that you add the waves' amplitude, $\psi_1(r, t)$ and $\psi_2(r, t)$, not their intensities, to sum them:

$$\psi(r, t) = \psi_1(r, t) + \psi_2(r, t)$$

You square the amplitude to get the intensity, and the phase difference between $\psi_1(r, t)$ and $\psi_2(r, t)$ is what actually creates the interference pattern that's observed.

You Can't Know Everything (But You Can Figure the Odds)

So particles apparently exhibit wave-like properties, and waves exhibit particle-like properties. But if you have an electron, which is it — a wave or a particle? The truth is that physically, an electron is just an electron, and you can't actually say whether it's a wave or a particle. The act of *measurement* is what brings out the wave or particle properties. You see more about this idea throughout the book.

Quantum mechanics lives with an uncertain picture quite happily. That view offended many eminent physicists of the time — notably Albert Einstein, who said, famously, "God does not play dice." In this section, I discuss the idea of uncertainty and how quantum physicists work in probabilities instead.

The Heisenberg uncertainty principle

The fact that matter exhibits wave-like properties gives rise to more trouble — waves aren't localized in space. And knowing that inspired Werner Heisenberg, in 1927, to come up with his celebrated uncertainty principle.

You can completely describe objects in classical physics by their momentum and position, both of which you can measure exactly. In other words, classical physics is completely *deterministic*.

On the atomic level, however, quantum physics paints a different picture. Here, the *Heisenberg uncertainty principle* says that there's an inherent uncertainty in the relation between position and momentum. In the x direction, for example, that looks like this:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

where Δx is the measurement uncertainty in the particle's x position, δp_x is its measurement uncertainty in its momentum in the x direction and $\hbar = h/2\pi$.

That is to say, the more accurately you know the position of a particle, the less accurately you know the momentum, and vice versa. This relation holds for all three dimensions:

$$\Delta y \Delta p_y \geq \frac{\hbar}{2}$$

$$\Delta z \Delta p_z \geq \frac{\hbar}{2}$$

And the Heisenberg uncertainty principle is a direct consequence of the wave-like nature of matter, because you can't completely pin down a wave.

REMEMBER

Quantum physics, unlike classical physics, is completely undeterministic. You can never know the *precise* position and momentum of a particle at any one time. You can give only probabilities for these linked measurements.

Rolling the dice: Quantum physics and probability

In quantum physics, the state of a particle is described by a wave function, $\psi(r, t)$. The wave function describes the de Broglie wave of a particle, giving its amplitude as a function of position and time. (See the earlier section "A Dual Identity: Looking at Particles as Waves" for more on de Broglie.)

REMEMBER

Note that the wave function gives a particle's amplitude, not intensity; if you want to find the intensity of the wave function, you have to square it: $|\psi(r, t)|^2$. The *intensity* of a wave is what's equal to the probability that the particle will be at that position at that time.

That's how quantum physics converts issues of momentum and position into probabilities: by using a wave function, whose square tells you the *probability density* that a particle will occupy a particular position or have a particular momentum. In other words, $|\psi(r, t)|^2 d^3r$ is the probability that the particle will be found in the volume element d^3r, located at position r at time t.

Besides the position-space wave function $\psi(r, t)$, there's also a momentum-space version of the wave function: $\phi(p, t)$.

This book is largely a study of the wave function — the wave functions of free particles, the wave functions of particles trapped inside potentials, of identical particles hitting each other, of particles in harmonic oscillation, of light scattering from particles, and more. Using this kind of physics, you can predict the behavior of all kinds of physical systems.