CHAPTER 1

Time Value of Money Toolbox

INTRODUCTION

One of the most important tools used in corporate finance is present value mathematics. These techniques are used to evaluate projects, make financial decisions, and evaluate investments. This chapter explains the time value of money, including present value (PV) and future value (FV), and how to adjust valuation formulas for various interest rate conventions. The chapter also presents several shortcuts to value a series of cash flows that fit a few standard patterns.

Readers should begin by developing an intuitive understanding of why it is necessary to incorporate interest rates into any analysis involving different periods of time. This understanding leads to a simple set of formulas expressing several time value relationships. After developing an intuitive understanding, readers will find it easy to incorporate interest rates by using the formulas for present value and/or future value in their analyses. Although this analysis shows up quite often, students will be relieved to find that its application is similar in most instances.

CASH FLOWS

Much of this text focuses on cash flows. Accountants realize the importance of cash; they devote an entire statement to the analysis of the sources and the uses of cash and cash balances. Accountants are interested in tracking cash flow in large measure because a company must have adequate cash to survive and prosper. Start-up companies may run out of cash before they have a chance to establish their businesses. Even established companies focus on both the profitability of the business and the flow of cash.

Corporate finance uses the same or similar measure of cash flow as accountants track in the statement of cash flows. However, this chapter and much of this book rely on cash flows for a completely different analysis and treat the cash flows from a project or even the cash flows of an entire corporation much like the cash flows of a bond. With a bond, investors transfer money today to borrowers, who in turn pay interest and eventually repay the loan. The size and timing of the cash payments and cash receipts determines the attractiveness of the bond investment. The techniques described herein will enable investors to evaluate the cash flows of any investment regardless of when the cash flows occur.

FUTURE VALUE

The future value of a cash flow is the value at some specified future time of a cash flow that occurs immediately. The concept of future value allows a company to decide whether cash flows that occur at two different times are equivalent. The way in which the two cash flows are equivalent is the subject of this chapter and will be explained subsequently.

Suppose that a company issues a bill that requires a customer to pay \$100 upon receipt. The customer asks for extra time to pay. The company can borrow at an 8 percent interest rate. The company tells the customer that it will accept \$102 instead in three months.

The company calculated the amount of cash it would accept that would be equivalent to getting \$100 immediately. If the delay in receiving payment causes the company to borrow \$100 for three months, the company must account for the interest on the loan. The formula for interest might look like Equation 1.1.

$$\text{Interest} = \$100 * 8\% * 3/12 = \$2 \tag{1.1}$$

This is a formula for simple interest. Simple interest applies the interest rate to a principal balance for a period of time. The formula begins with the principal balance multiplied by the annual interest rate of 8 percent or \$8. However, the rate applies only to three months or one-quarter of the year. Therefore, the interest for three months is \$2, and the amount of the delayed payment would have to be $\$100 + 2 = \$102$ to compensate the company for the delay in payment.

The immediate payment of \$100 in the preceding example is called the present value. The later payment is called a future value. As has been demonstrated, the two amounts are linked by the interest rate and the amount of time between the two payment dates.

In the preceding example, an 8 percent interest rate was used to determine an equivalent future payment from a present value. The method relied on a bank rate of interest. In fact, the company may still prefer the immediate payment of $100 to a deferred payment of $102. The deferred payment exposes the company to the risk of nonpayment for a longer period of time. The delay increases the amount the company must record as an account receivable in its financial statements and requires the company to include a liability on the balance sheet for the bank loan.

To address these concerns, the company may increase the interest rate used in determining the future value it will accept in lieu of the immediate payment of $100. Later, this text will explore factors that affect the interest rate or return that links present values to future values. This chapter, however, generally assumes that the company knows the required rate that incorporates these factors.

A more general formula for interest appears in Equation 1.2.

$$\text{Interest} = \text{Present Value} * \text{Rate} * \text{Time} \tag{1.2}$$

where Time is the interval in years between the time of the present value and the time of the future value and Rate is the annual interest rate.

The value of a cash payment that occurs immediately is the present value. The future value of this cash flow is the present value plus interest, as set forth in Equation 1.3.

$$\text{Future Value} = \text{Present Value} + \text{Interest} \tag{1.3}$$

Substitute the formula for interest in Equation 1.2 into the formula for future value in Equation 1.3 to produce Equation 1.4.

$$\text{Future Value} = \text{Present Value} + \text{Present Value} * \text{Rate} * \text{Time} \tag{1.4}$$

Finally, simplify Equation 1.4 by collecting terms. The result is Equation 1.5, which shows that the future value is related to the present value by a rate of interest that applies to the time from the present payments to the future payments.

$$\text{Future Value} = \text{Present Value}\,(1 + \text{Rate} * \text{Time}) \tag{1.5}$$

Compound Interest

The formula for future value in Equation 1.5 is correct for short intervals of time, but most investments pay interest every three months, every six months, or annually. When investments pay interest between the time of the present value and the time of the future value, the formula in Equation 1.5

is not correct. This section explains how these interim interest payments affect the calculation of the future value. First, it is necessary to explain the compounding process.

An old-fashioned passbook bank account illustrates the basic concepts of future value and compound interest. In the days before businesses had easy access to computers, banks used and reused a passbook as a simple ledger to account for customer deposits, withdrawals, and interest. Each time the customer deposited or withdrew funds, the new information was added to a running ledger. Modern monthly and quarterly statements work the same way, except that they include only a one-month or 3-month period of time. In contrast, the passbook included a running total of all deposits, withdrawals, and interest payments since the account was opened.

A customer could deposit an amount and see interest accumulate. Table 1.1 illustrates the process.

The investment of $1,000 on 3/14/20X1 grows to $1,040.28 by December 31, 20X1. In the absence of taxes, the cash amount on 3/14/20X1 is linked to the year-end balance of $1,040.28 by the amount of interest earned during the period.

The specific calculations in Table 1.1 require some explanation. In this example, interest is paid at the end of every calendar quarter. This example employs one commonly used method to calculate the number of days of interest—each month is assumed to have exactly 30 days and each year has 360 days. (See the appendix for a description of this method and other day-counting methods.) The first interest payment accumulates at 5 percent interest. If the rate applied for a full year (that is, the interest rate applied for a full year and was not compounded), the interest would be $50 ($1,000 times 5 percent). The interest for the 16-day period from March 14 to March 31 is a fraction of that annual amount equal to $50 * 16/360 or $2.22.

The period from March 31 to June 30 contains exactly 91 actual days but the counting convention used here assumes there are 30 days in each month or 90 in each quarter. The interest for this quarter is $1,002.22 * 5

TABLE 1.1 Passbook Investment at 5 Percent Quarterly Interest

Date	Deposit	Withdrawal	Days	Interest	Principal
03/14/20X1	1,000.00	—	—	—	1,000.00
03/31/20X1	—	—	16	2.22	1,002.22
06/30/20X1	—	—	90	12.53	1,014.75
09/30/20X1	—	—	90	12.68	1,027.43
12/31/20X1	—	—	90	12.84	1,040.28

percent ∗ 90/360 or $12.53. Notice that this old-fashioned way to count days calls for an interest payment of $12.53, whereas a more precise method would calculate $1,002.22 ∗ 91/365 or $12.49. The two values are generally fairly close, especially over a year or more. In some cases, the way interest is applied can have a big impact on the future value. It is important to understand the day-counting convention that is being used and to use the interest rate correctly.

Suppose a business orders some goods and could either pay the supplier $1,000 on March 14 or pay $1,040.28 at the end of the year when the goods will be delivered. Because the company can invest the $1,000 deposit, the customer could pay $1,000 now or invest the funds at 5 percent and pay $1,040.28 at year-end. Because the bank account provides exactly enough interest to pay the higher amount at the end of the year, the company does not prefer one alternative to the other. (Eventually, the comparison must include taxes and the risk of loss on the investment.)

Alternatively, the company could borrow $1,000 on March 14 at 5 percent and pay the lower invoice amount. The interest would accumulate to $1,040.28 by year-end. The company would need to pay $1,040.28 to repay the loan, which exactly equals the amount of the delayed payment.

A company cannot both borrow at 5 percent and invest at 5 percent as implied by the foregoing description. It is not necessary to be able to borrow and lend at the same rate. Rather, it is important to determine the relevant interest rate that can link the value on March 14 and the value on December 31. The calculation of future value using the appropriate interest rate provides a way to compare two different cash flows at different points in time.

The passbook investment detailed in Table 1.1 includes compound interest. In other words, the interest paid on March 31 also earns interest following that payment date. The value of the account rises on each payment date, and the base used to determine the interest payment is larger in later quarters. As a result, the interest payments are larger than if the bank had paid simple interest.

In contrast, if the bank paid simple interest on $1,000 using Equation 1.5, the customer would not benefit from earning interest on the quarterly interest payments. The interest rate would apply for 286/360 years or .794 years, taking the fraction of the year using the 30/360 day counting method shown in Table 1.1. The future value using Equation 1.5 would be

$$\text{Future Value} = \$1{,}000\,(1 + 5\% \ast .794) = \$1{,}039.72 \qquad (1.6)$$

Paying the customer quarterly (or even more frequent) interest has the effect of raising the return to the investor. It is important to follow the correct compounding assumption to determine the present or future value from a particular interest rate. This chapter will later address how to handle the difference in interest created by different compounding assumptions.

The formula for compound interest begins much like the formula for simple interest given in Equation 1.5. Suppose that the interest rate is 6 percent per year and an investment pays interest annually. The future value at the end of one year is shown in Equation 1.7.

$$FV_1 = PV * (1 + Rate * 1) = PV * (1 + Rate) \tag{1.7}$$

After one year, a passbook would contain PV * (1 + Rate). This amount from Equation 1.7 appears in Equation 1.8 in square brackets and is reinvested for another year.

$$\begin{aligned} FV_2 &= [PV * (1 + Rate)] * (1 + Rate) \\ FV_2 &= PV * (1 + Rate)^2 \end{aligned} \tag{1.8}$$

After two years, a passbook would contain $PV * (1 + Rate)^2$. The amount from Equation 1.8 appears in Equation 1.9 in square brackets and is reinvested for another year.

$$\begin{aligned} FV_3 &= \left[PV * (1 + Rate)^2\right] * (1 + Rate) \\ FV_3 &= PV * (1 + Rate)^3 \end{aligned} \tag{1.9}$$

Table 1.2 summarizes the growth in principal, now applying the specified rate of 6 percent and also showing the calculation of the interest generated in each period. In each case, the starting principal amount appears in the square brackets, and interest equals this updated principal amount times the annual interest rate.

The general formula for future value for an investment that pays annual interest appears in Equation 1.10.

$$FV_i = PV * (1 + Rate)^i \tag{1.10}$$

where i equals the number of years between the present and the future.

TABLE 1.2 Passbook Investment at 6 Percent Annual Interest

Date	Principal	Interest	Interest
3/15/20X1	1,000,000	60,000	[Deposit] * (Rate)
3/15/20X2	1,060,000	63,600	[Deposit(1 + Rate)] * (Rate)
3/15/20X3	1,123,600	67,416	[Deposit(1 + Rate)(1 + Rate)] * (Rate)
3/15/20X4	1,191,016	71,461	[Deposit(1 + Rate)(1 + Rate)(1 + Rate)] * (Rate)

TABLE 1.3 Passbook Investment at 6 Percent Annual Interest

Date	Principal	Interest	
3/15/20X1	1,000,000	60,000	
3/15/20X2	1,060,000	63,600	
3/15/20X3	1,123,600	67,416	
3/15/20X4	1,191,016	35,730	(prorated 50 percent of annual interest)
9/15/20X4	1,226,746		

Equation 1.10 makes it easy to calculate the future value for much longer periods without needing to calculate individual interest payments as was done in Table 1.2.

Some care should be exercised in using Equation 1.10 for fractional periods. For example, the future value after 3.5 years could be calculated successively, as in Table 1.3. In this case, the interest is prorated for half of a year.

Applying Equation 1.10 for the fractional period of 3.5 years yields approximately the same answer:

$$FV_{3.5} = PV * (1 + .06)^{3.5} = 1{,}226{,}226 \qquad (1.11)$$

More Frequent Compounding

Equations 1.7 to 1.10 develop a general formula for compounding an investment that pays interest annually. Most U.S. bonds, along with many other instruments, pay interest twice each year. Equation 1.12 shows the future value of a cash flow one year in the future for investments that pay interest twice each year:

$$\begin{aligned} \text{Future Value}_1 &= \text{Present Value} * \left(1 + \frac{\text{Rate}}{2}\right) * \left(1 + \frac{\text{Rate}}{2}\right) \\ &= \text{Present Value} * \left(1 + \frac{\text{Rate}}{2}\right)^2 \end{aligned} \qquad (1.12)$$

The term in the first set of parentheses adjusts the starting present value to the middle of the year using simple interest, much like the adjustment in Equation 1.5. Then the original amount plus the interest collected in six months is reinvested. The term in the second set of parentheses adjusts the starting present value plus interest from the first semiannual interest payment to the end of the year.

The future value one year in the future of a cash flow for investments that pay interest quarterly is presented in Equation 1.13.

$$\begin{aligned} FV_1 &= PV * \left(1+\frac{Rate}{4}\right) * \left(1+\frac{Rate}{4}\right) * \left(1+\frac{Rate}{4}\right) * \left(1+\frac{Rate}{4}\right) \\ &= PV * \left(1+\frac{Rate}{4}\right)^4 \end{aligned} \tag{1.13}$$

Like Equation 1.12 for semiannual interest, the year is divided into subperiods. At the end of each subperiod, the investment pays interest and the interest is available to be reinvested. Equation 1.14 presents a more general formula that can account for rates that compound at difference frequencies.

$$FV_{Years} = PV * \left(1+\frac{Rate}{Freq}\right)^{Years*Freq} \tag{1.14}$$

where Years represents the time between the present and the future value date and Freq refers to the number of compounding periods per year.

For example, an annual bond has a frequency or Freq of one. In this case, Equation 1.14 simplifies to Equation 1.10. A semiannual bond would use 2 for Freq, and there would be twice as many semiannual periods and years between the present and the future value date. Two other common frequencies are quarterly for many money market instruments and monthly for mortgage investments. Banks may pay interest with daily compounding, in which case Freq would be 365 or 366. Finally, the shortest possible compounding period could be a tiny fraction of a year. Compounding over infinitesimally small compounding periods, called continuous compounding, is discussed further along in the text.

THE IMPACT OF COMPOUNDING FREQUENCY ON FUTURE VALUE

Interest on interest increases the effective interest rate. In other words, compound interest raises the future value compared to simple interest. In general, the more frequently interest is compounded, the higher the effective interest rate and the higher the resulting future value.

Table 1.4 begins with a present value of $1,000. The interest rate is 6 percent and the future value 10 years later is determined for simple

TABLE 1.4 Compounding Frequency at 6 Percent Interest

Frequency	Description	Future Value
0	Simple	1,600.00
1	Annual	1,790.85
2	Semiannual	1,806.11
4	Quarterly	1,814.02
12	Monthly	1,819.40
365	Daily	1,822.03
∞	Continuous	1,822.12

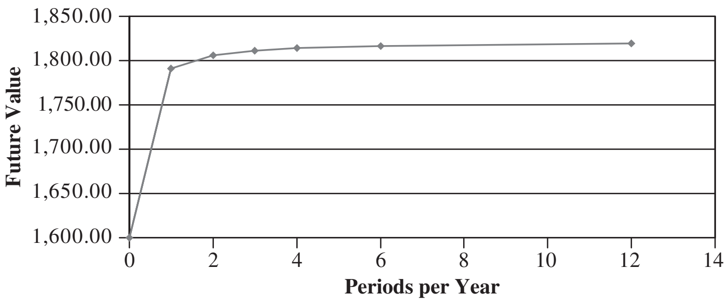

FIGURE 1.1 Future Value versus Compounding Frequency

interest (no compounding), and several commonly assumed compounding frequencies.

The data is presented visually in Figure 1.1, which plots the future value for simple interest and various compounding frequencies.

The important point to remember is that it is necessary to know what compounding assumptions are associated with the rate of interest used in present value and future value calculations and to develop a value using the correct assumption.

EQUIVALENT INTEREST RATE

More frequent compounding has the same effect as using a higher interest rate. That is, for a given present value, the future value increases as the assumed interest rate rises. The future value also increases as the compounding frequency rises.

Not surprisingly, market participants adjust downward the stated rates of frequently compounded interest rates to compensate for the larger amount of interest on interest earned with frequent compounding. It is often necessary to determine how much to adjust a particular rate to produce the same results as a different interest rate with a different compounding frequency.

Suppose a 10 percent quarterly-compounded interest rate is used—that is, the interest rate for one year is 10 percent so that interest of 2.5 percent is paid quarterly. By applying Equation 1.14, it is possible to determine the annually compounded rate that is equivalent.

Applying Equation 1.14 to a $100 present value for one year, the future value would be $110.38 as shown in Equation 1.15.

$$FV_4 = 100 * \left(1 + \frac{10\%}{4}\right)^4 = 110.38 \qquad (1.15)$$

To determine the annual interest rate that produces the same result (i.e., produces in the same future value), set up Equation 1.14 again for an annually compounded rate.

$$FV_1 = 100 * \left(1 + \frac{Rate_1}{1}\right)^1 = 110.38 \qquad (1.16)$$

It is easy to see that the annually-compounded rate of 10.38 percent would produce the same future value as a 10 percent rate compounded quarterly. The rate calculated from Equation 1.16 is called the equivalent annual rate or the effective annual rate. (Note that the ‘‘annual percentage rate’’ that mortgage lenders are required to disclose to borrowers is not the same. That rate includes the impact of certain loan origination costs not included here.)

To calculate an equivalent semiannual rate, set up Equation 1.14 again for two compounding periods per year.

$$FV_2 = 100 * \left(1 + \frac{Rate_2}{2}\right)^2 = 110.38 \qquad (1.17)$$

This rate is a bit more challenging to calculate. By manipulating a few terms in Equation 1.17, it is possible to solve for the equivalent semiannual rate.

$$\left(1 + \frac{Rate_2}{2}\right)^2 = \frac{110.38}{100} \qquad (1.18)$$

Simplify the right side with division and take the square root of both sides as shown in Equation 1.19.

$$\left(1 + \frac{\text{Rate}_2}{2}\right) = \sqrt{1.1038} \tag{1.19}$$

Finally, rearrange with a little more algebra.

$$\text{Rate}_2 = 2\left(\sqrt{1.1038} - 1\right) = 10.125\% \tag{1.20}$$

The equations in this book evaluated with more decimal points of accuracy than appear on the page. Equation 1.20 equals 10.124 percent using the values printed due to rounding.

This method converts a rate with one compounding frequency to an equivalent rate with a different compounding frequency by determining the rate that produces the same future value as the future value associated with the known beginning rate.

CONTINUOUSLY COMPOUNDED INTEREST

Figure 1.1 makes clear that more frequent compounding results in a higher future value (or equivalently, a higher effective interest rate). Figure 1.1 also suggests that the impact of more frequent compounding has a larger impact going from one to two to four compounding periods than the impact of compounding more frequently than monthly. Table 1.4 demonstrates that, for a given annual rate, daily compounding produces a higher future value than monthly compounding, but the impact is smaller. Compounding more frequently than daily produces very little difference in the future value or the effective interest rate.

Although the impact is small, many people use interest rates that are continuously compounded. Continuously compounded rates assume that interest is paid and reinvested after each infinitesimally small step in time. These rates are used not to raise the effective interest rate but instead to simplify the math. For example, option pricing formulas that use continuously compounded rates may be simpler or easier to apply than rates with other compounding frequencies.

The future value of a cash flow using continuous compounding is shown in Equation 1.21.

$$\text{FV}_{\text{Continuous}} = \text{PV} * \text{e}^{\text{Rate}*\text{Time}} \tag{1.21}$$

TABLE 1.5 Future Value at 6 Percent Continuous Compounding

Time	Future Value	Excel Syntax
0	100.00	=100*exp(6%*0)
1	106.18	=100*exp(6%*1)
2	112.75	=100*exp(6%*2)
5	134.99	=100*exp(6%*5)
10	182.21	=100*exp(6%*10)

where Rate is a continuously compounded interest rate, Time again measures the difference in years between the future valuing date and the present, and e refers to the mathematical constant equal to 2.718.

Questions and answers that follow at the end of the chapter will use the continuously compounded rate and provide a more complete explanation of how to use continuous compounding. For a short explanation, refer to Table 1.5, which lists the future value of $100 for several years at 6 percent interest along with the syntax for instructing Excel to produce these values.

It is clear from the line in Table 1.5 showing the future value after one year that a 6 percent continuously compounded rate is equivalent to a 6.18 percent annually compounded rate.

PRESENT VALUE

So far, this chapter has introduced ways to find a value in the future that is worth the same as a cash payment that occurs immediately. Consider, now, the case of a single payment you must make one month (30 days) in the future. Assume the payment equals $1 million.* Suppose as well that you could make a somewhat smaller payment immediately instead of paying $1 million 30 days from now. The annual interest rate is 10 percent.

You may pay either $1 million in a month or $991,735.54 today. The interest in this case was calculated by multiplying a smaller amount (PV) times an interest rate of 10 percent. If the payment were deferred for an entire year, the interest would equal PV * 10 percent. In the preceding case, the annual interest rate was reduced to a daily rate by dividing by 360, the days per year in the 30/360 counting convention. Finally, the daily amount

*Later, it will be convenient to present the values in terms of a nominal $1.00 cash flow. The advantage of working with $1 in each case is that all results can then be simply multiplied by the relevant actual cash flows.

was converted to monthly by multiplying by 30. The calculation of interest is described in Equation 1.22.

$$\text{Interest} = \frac{\text{PV} * 10\% * 30}{360} \tag{1.22}$$

In the preceding example and in general, the 10 percent annual interest rate was reduced by the fraction of the year involved in the calculation. Equation 1.22 could be modified to reflect a different day counting convention. The appendix presents a brief description of day-counting alternatives.

The payment of PV plus the interest in 30 days must equal $1,000,000. Equation 1.23 shows how to calculate the immediate payment that is equivalent to $1,000,000 a month from now.

$$\begin{aligned} \$1{,}000{,}000 &= \text{PV} + \frac{\text{PV} * 10\% * 30}{360} \\ \$1{,}000{,}000 &= \text{PV} * \left(1 + \frac{10\% * 30}{360}\right) \\ \text{PV} &= \frac{\$1{,}000{,}000}{1 + \dfrac{10\% * 30}{360}} = \$991{,}735.54 \end{aligned} \tag{1.23}$$

If you have the money to pay the invoice now, you could pay the immediate amount of $991,735.54 or make the payment of $1,000,000 in 30 days. If you postpone payment and invest the money in a short-term investment (also at 10 percent), you will earn exactly enough to pay the later, higher amount.* If you really can earn exactly 10 percent, you should be completely indifferent as to whether you pay $991,735.54 now or $1,000,000 in a month from now.†

Alternatively, the company may prefer to borrow money now to make the immediate payment. If the cost of borrowing were 10 percent, you would owe $8,264.46 in interest ($991,735.54 * 10 percent * 30/360) if

*Again, taxes complicate the matter, of course. See Equations 1.28 and 1.29.

†Accountants would record the alternatives differently. They would recognize the explicit interest as interest revenue. They would not impute an interest expense for the deferred payment. Instead, they would likely record a higher cost of goods sold. Nevertheless, as long as the delayed payment amount is based on (exactly) the same interest rate as the earning rate, net income would be equal regardless of whether the company paid early or later.

you make an early payment while avoiding an equal charge on the invoice amount. The company must repay principal of $991,735.54 plus interest of $8,264.46 after 30 days, a total of $1 million. As long as the implicit interest rate on the invoice matches your borrowing rate, it doesn't matter whether you make an early payment or a payment in 30 days.

By similar logic, your creditor is indifferent between receiving $991,735.54 immediately or $1 million in a month. If your creditor is a net borrower, it will incur additional borrowing costs of $8,264.46, because your delay in payment results in a larger loan balance for the creditor for a month (if the creditor borrows at 10 percent). Alternatively, if your early payment means your creditor has $991,735.54 more invested in interest-bearing investments, those investments would earn $8,264.46 over 30 days. Although the creditor receives $8,264.46 less from the customer, the creditor receives $8,264.46 additional interest income.

The interest calculated this way is not correct for longer time intervals. Contractual payments can extend over several years. For these uses, a compound interest method that calculates interest over years or even decades is required.

In all the preceding examples, both you and your company were able to either borrow or invest at the same rate. That is, of course, not realistic. In fact, trade credit is often a major motivation for sales. Large creditworthy manufacturers can pass some of their borrowing advantage on to customers as a sales inducement.

For the purposes of describing the time value of money, however, it is convenient to start with the simplifying assumption that everyone agrees on the interest rate. In the preceding examples, both you and your creditor are indifferent between an early payment and a larger payment 30 days later. If we calculate the future amount from an immediate cash flow, it is called a future value. The amount we could accept as immediate payment in lieu of a payment sometime in the future is called the present value. (See Figures 1.2 and 1.3.)

The present values of future cash flows are often called discounted cash flows because the present value is less than the actual cash flows occurring in the future. When an investment is valued from forecasted future cash flows and the future cash flows are all converted to present values, the analysis is called discounted cash flow analysis.

FORMULAS FOR PRESENT VALUE AND FUTURE VALUE

The general formula for the future value of an immediate cash flow appears in Equation 1.24.

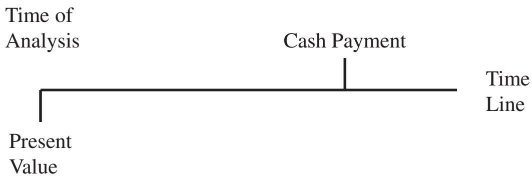

FIGURE 1.2 Present Value Time Line

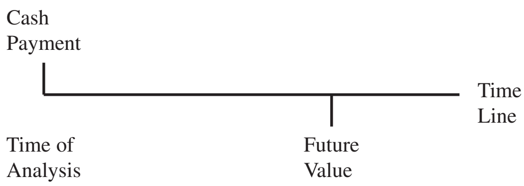

FIGURE 1.3 Future Value Time Line

$$\text{FV} = \text{Cash Flow}\,(1 + \text{Rate})^{\text{Time}} \quad \text{(Equation 1.10 restated slightly)} \qquad (1.24)$$

where Rate is the annual interest rate used and Time is the number of years until the future date. To repeat, this formula applies to interest rates that pay interest annually.

The general formula for the present value of a future payment appears in Equation 1.25.

$$\text{PV} = \frac{\text{Cash Flow}}{(1 + \text{Rate})^{\text{Time}}} \qquad (1.25)$$

where Rate is the annual interest rate used and Time is the number of years until the future date. Equation 1.25 is derived from Equation 1.24 by rearranging the times to adjust the value of a future cash flow back to the present instead of adjusting the value of an immediate cash flow to the future.

As with Equation 1.24, this formula applies to interest rates that pay interest annually. The formula can be altered in the same way that Equation 1.14 was modified to apply rates of different compounding periods by modifying the way the formula handles the rate and the number of compounding periods.

- **Example 1**: What is the present value of a payment of $40,000 made in 2.5 years if the annual interest rate is 8 percent?

$$PV = \frac{\$40{,}000}{(1+.08)^{2.5}} = 32{,}998.99 \tag{1.26}$$

Equation 1.26 contains the formula for the present value of Example 1. If 8 percent is the interest rate, the payment of \$40,000 in 2.5 years is worth only as much as an immediate payment of \$32,998.99.

- **Example 2:** What is the future value of a payment of \$25,000 made in four years if the annual interest rate is 6 percent?

$$FV = 25{,}000 * (1+.06)^4 = 31{,}561.92 \tag{1.27}$$

Equation 1.27 contains the formula for the future value of Example 2. Faced with a 4-year delay in receiving payment, you should insist on receiving \$31,561.92 instead of an immediate payment of \$25,000.

Taxes can easily be incorporated into the present value and future value models. Suppose that a corporation pays a marginal tax rate of 35 percent. This effectively lowers the corporation's borrowing cost by 35 percent, since an interest expense lowers taxable income and tax payments should go down by 35 percent of the interest expense. Similarly, the net return on an investment is 35 percent lower than the indicated interest rate.

To apply after-tax analysis to discounted cash flows, you could reduce the effective interest rate. For example, if interest rates are 10 percent and a corporation pays a 35 percent corporate income tax rate, the after-tax interest rate is 10 percent * (100 percent − 35 percent) = 6.5 percent.

The impact of corporate taxes effectively lowers the interest rate. The formulas for present value and future value can account for the impact of taxes on value. See Equations 1.28 and 1.29.

$$PV = \frac{1}{[1 + (1 - \text{Tax}) * \text{Rate}]^{\text{Time}}} \tag{1.28}$$

$$FV = [1 + (1 - \text{Tax}) * \text{Rate}]^{\text{Time}} \tag{1.29}$$

In the preceding formulas, a \$1 nominal cash flow was assumed in each case. The marginal corporate tax rate is included as "Tax." These formulations also assume that the cash flow is not a taxable cash flow or that the values from the formulas will be applied to cash flows already reduced by taxes.

The present value and future value models are tools that can be used to make certain financial decisions within a firm. One direct application is to compare alternative payment options as described previously. The company should select the alternative that maximizes shareholder value. The company should select the alternative that is cheaper when the appropriate discount rate is used.*

The present value and future value models can also assess simple investment alternatives. The formulas provide a basis for comparing a variety of simple investments (a single outflow at the beginning and a single, certain return later). These techniques are used frequently to evaluate a series of investments in order to ration scarce capital and invest in the most attractive alternatives.

CONCLUSION

The first step in reviewing cash flows is to make all individual cash flows equivalent. The largest impact is usually the adjustment for the time value of money. Present value and future value formulas provide a basis for comparing and combining the value of cash flows occurring at different times.

* Choosing the appropriate discount rate (also called the firm's cost of capital) is not a trivial matter. Many factors enter the calculation of the cost of capital, including the riskiness of the firm, the riskiness of a particular project, whether a new investment reduces or increases existing business risks, market risks, and more. Chapter 3 presents financial theories useful in determining the appropriate discount rate.

Questions

1.1. Your supplier asks you to pay your $300,000 invoice in 30 days. However, the supplier will allow you to pay $298,500 immediately. You can borrow at 5 percent (annual rate). Should you pay $298,500 immediately or $300,000 in 30 days? Ignore any impact of taxes.

1.2. Ignoring any impact of taxes, what borrowing rate would make you indifferent between paying the invoice in Question 1.1 immediately and paying $300,000 in a month?

1.3a. If you deposit money today into an account that pays 6.5 percent interest, how long will it take you to double your money if interest does not compound (simple annual interest)?

1.3b. If interest compounds annually?

1.3c. If interest compounds semiannually?

1.3d. If interest compounds quarterly?

1.3e. If interest compounds continuously?

1.4a. What is the daily compounded rate equivalent to a semiannual 6 percent rate?

1.4b. What is the monthly compounded rate equivalent to a 6 percent semiannual rate?

1.4c. What is the annually compounded rate equivalent to a semiannual 6 percent rate?

1.4d. What is the continuously compounded rate equivalent to a semiannual 6 percent rate?

1.5. Following a large decline in stock prices, David commented that he lost 100 percent of the value of his 401K; then he grinned and added, "continuously compounded." If David's 401K was worth $100,000 18 months ago, what is it worth today?